```
KerrGeoMinoFrequencies[(1 | 1.), p_, e_, x_] :=
 Module[{M = 1, a = 1, En, L, Q, r1, r2, r3, r4, ϵ0, zm,
   a2zp, ϵ0zp, zmOverZp, kr, kθ, ϒr, ϒθ, rp, rm, hr, hM, ϒϕ, Γ},

{En, L, Q} = Values[KerrGeoConstantsOfMotion[a, p, e, x]];

{r1, r2, r3, r4} = KerrGeoRadialRoots[a, p, e, x, En, Q];
ϵ0 = a^2 (1 - En^2) / L^2;
zm = 1 - x^2;
a2zp = (L^2 + a^2 (-1 + En^2) (-1 + zm)) / ( (-1 + En^2) (-1 + zm));

ϵ0zp = -((L^2 + a^2 (-1 + En^2) (-1 + zm)) / (L^2 (-1 + zm)));

(*zmOverZp=
    If[a==0,0,zm/((L^2+a^2 (-1+En^2) (-1+zm))/(a^2 (-1+En^2) (-1+zm)))];*)
zmOverZp = zm / ((L^2 + a^2 (-1 + En^2) (-1 + zm)) / (a^2 (-1 + En^2) (-1 + zm)));


kr = Sqrt[(r1 - r2) / (r1 - r3) (r3 - r4) / (r2 - r4)]; (*Eq.(13)*)
kθ = Sqrt[zmOverZp]; (*Eq.(13)*)
ϒr = (Pi Sqrt[(1 - En^2) (r1 - r3) (r2 - r4)]) / (2 EllipticK[kr^2]);
  (*Eq.(15)*)
ϒθ = (Pi L Sqrt[ϵ0zp]) / (2 EllipticK[kθ^2]); (*Eq.(15)*)

hM = ((r1 - r2) (r3 - M)) / ((r1 - r3) (r2 - M));

hr = (r1 - r2) / (r1 - r3);

(*ϒϕ and Γ from Appendix B for a=M case*)

ϒϕ = (2 ϒθ) / (π Sqrt[ϵ0zp]) EllipticPi[zm, kθ^2] +
    (2 a ϒr) / (π Sqrt[(1 - En^2) (r1 - r3) (r2 - r4)])
     ((2 M En) / (r3 - M) (EllipticK[kr^2] - (r2 - r3) / (r2 - M) EllipticPi[hM, kr^2]) +
       (2 M^2 En - a L) / (2 (r3 - M)^2) ((2 - ((r1 - r3) (r2 - r3)) / ((r1 - M) (r2 - M)))
            EllipticK[kr^2] + ((r1 - r3) (r2 - r4) (r3 - M)) / ((r1 - M) (r2 - M) (r4 - M))
            EllipticE[kr^2] + (r2 - r3) / (r2 - M) ((r1 - r3) / (r1 - M) +
              (r2 - r3) / (r2 - M) + (r4 - r3) / (r4 - M) - 4) EllipticPi[hM, kr^2]));

Γ = 4 M^2 En + (2 a^2 En a2zp ϒθ) / (π L Sqrt[ϵ0zp]) (EllipticK[kθ^2] - EllipticE[kθ^2]) +
    (2 ϒr) / (π Sqrt[(1 - En^2) (r1 - r3) (r2 - r4)])
     (En / 2 ((r3 (r1 + r2 + r3) - r1 r2) EllipticK[kr^2] + (r2 - r3) (r1 + r2 + r3 + r4)
            EllipticPi[hr, kr^2] + (r1 - r3) (r2 - r4) EllipticE[kr^2]) +
       2 M En (r3 EllipticK[kr^2] + (r2 - r3) EllipticPi[hr, kr^2]) +
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        (2 M (4 M^2 En - a L)) / (r3 - M)
         (EllipticK[kr^2] - (r2 - r3) / (r2 - M) EllipticPi[hM, kr^2]) +
        (M^2 (2 M^2 En - a L )) / (r3 - M)^2 ((2 - ((r1 - r3) (r2 - r3)) / ((r1 - M) (r2 - M)))
            EllipticK[kr^2] + ((r1 - r3) (r2 - r4) (r3 - M)) / ((r1 - M) (r2 - M) (r4 - M))
            EllipticE[kr^2] + (r2 - r3) / (r2 - M) ((r1 - r3) / (r1 - M) +
              (r2 - r3) / (r2 - M) + (r4 - r3) / (r4 - M) - 4) EllipticPi[hM, kr^2]));

<| "\!\(\*SubscriptBox[\(ϒ\), \(r\)]\)" -> ϒr,
 "\!\(\*SubscriptBox[\(ϒ\), \(θ\)]\)" -> Abs[ϒθ],
 "\!\(\*SubscriptBox[\(ϒ\), \(ϕ\)]\)" -> ϒϕ,
 "Γ" -> Γ |>
]
```