Trajectory Prediction Based on Improved Sliding Window Polynomial Fitting Prediction Method

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***Abstract*—Aiming at the problem of high precision trajectory prediction in air traffic control automation system, a new method based on improved sliding window polynomial fitting prediction is proposed. This method improves the traditional sliding window polynomial fitting method and selects an appropriate historical data group for every predicted value, which provides every prediction with a suitable polynomial fitting equation and make the most use of the current value, and its prediction precision is higher than the traditional sliding window polynomial fitting prediction method. The simulation results show that the improved sliding window polynomial fitting prediction is better than traditional sliding window polynomial fitting prediction method and it can predict the civil aviation flight trajectory effectively.**

Keywords- trajectory prediction; sliding window polynomial fitting; improvement

I. INTRODUCTION

The accurate and efficient aircraft trajectory prediction is an important part of air traffic control automation system in the field of air traffic management, and it is critical to improve the air traffic management efficiency and safety. Especially, the accurate trajectory forecast technology plays a key role in aircraft flight conflict detection and resolution process.

In recent years, the researches and applications of the trajectory prediction problem are mainly divided into two categories, one is to establish the aircraft dynamics and kinematics model, using simulation of the performance parameters of various aircraft and meteorological information for the implementation of aircraft flight trajectory [1,2], some scholars using ADS-B to obtain the status and intent information of the cooperative aircraft to predict the trajectory [3,4], generally speaking, it is difficult to collect complete information of the aircraft and the flight environment, especially the intent information of non-cooperative aircraft; the other is the data mining method such as neural network and grey theory for trajectory prediction [5-7], the data mining method is simple and easy to use, but its accuracy is not high, so this method does not have wide applicability.

In this paper, an improved sliding window polynomial fitting method is proposed based on the idea of real time modeling for the flight path prediction of non-cooperative aircraft. The basic idea of the real time modeling and predicting is that the future value is only decided by the current value and the N historical values before, and it has no relation with the further past values. When the improved sliding window polynomial fitting prediction method predicts the multiple consecutive future value simultaneously, an appropriate historical data group will be selected in the sliding window aiming at each predicted value, to obtain polynomial linear minimum variance estimation model to predict the target value. The simulation results show that the improved sliding window polynomial fitting prediction method is more accurate than the traditional sliding window polynomial fitting method in the prediction of flight trajectory.

II. SLIDING WINDOW POLYNOMIAL FITTING PREDICTION METHOD

A. Least Squares Fitting Principle

Considering the measurement of the error $r_i = f(t_i) - X_i$ wholly from approximate function $f(t_i)$, $t_i = i \bullet T$ and given data points $\{X_i \mid i = 0,1,\cdots,n\}$, there are three common methods: 1) the maximum absolute of the error $r_i = f(t_i) - X_i$, $\max_{0 \le i \le n} |r_i|$, which means the ∞ −norm of error vector $r = [r_0, r_1, \cdots, r_n]^{\mathrm{T}}$; 2) the sum of the absolute error $\sum_{i=0}^{n} r_i^2$, the 1−norm of the error vector; 3)the error quadratic sum $\sum_{i=0}^{n} r_i^2$, and its arithmetic square root is 2−norm of error vector. The first two methods are simple and natural, but is not fit for differential operation, the last method is equivalent to considering the 2-normof error vector. So it is often used

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the error square $\sum_{i=0}^{n} r_i^2$ to compute the overall value of the error $r_i\left(i=0,1,\cdots,n\right)$.

The specific method of data fitting is: seeking the $f\left(t_i\right)\in\Phi$ and obtaining minimum of the sum of the square of the error $r_i = f\left(t_i\right)-X_i\left(i=0,1,\cdots,n\right)$ for the given data $\left\{X_i \mid i=0,1,\cdots,n\right\}$ and the set Φ , that is:

$$I=\sum_{i=0}^{n}\left[f\left(t_i\right)-X_i\right]^2=\sum_{i=0}^{n}\left[\sum_{j=0}^{m}a_j t_i^j-X_i\right]^2=\min \quad (1)$$

The geometric sense of formula(1) is to seek the curve $f\left(t\right)$ that can get the least square of the distance between the curve and the fixed point $\left\{X_i \mid i=0,1,\cdots,n\right\}$. The function $f\left(t\right)$ is called the fitting function or the least square solution, and the method of solving the fitting function $f\left(t\right)$ is called the least square method of curve fitting.

B. Sliding Window PolynomialPprediction Theory

Assuming a given data points $\left\{X_i \mid i=0,1,\cdots,n\right\}$, Φ is the function class consist of the polynomial that the degree is not more than n , the $f\left(t_i\right)=\sum_{j=0}^{m}a_j t_i^j,\quad \left(j=0,1,\cdots,m,\quad m\le n\right)$ is calculated out to solve following formula:

$$I=\sum_{i=0}^{n}\left[f\left(t_i\right)-X_i\right]^2=\sum_{i=0}^{n}\left[\sum_{j=0}^{m}a_j t_i^j-X_i\right]^2=\min \quad (2)$$

The $f\left(t_i\right)$ in the formula(2) is called least square fitting polynomial, which is the problem solving the extreme value of $I=I\left(a_0,a_1,\cdots,a_m\right)$. Considering the necessary condition for the extremum of a multivariate function, we can get:

$$A=\left(P^{\mathrm{T}}\bullet P\right)^{-1}\bullet P^{\mathrm{T}}\bullet X \quad (3)$$

In the formula (3):

$$A=\begin{bmatrix}\hat{a}_0\\ \hat{a}_1\\ \vdots\\ \hat{a}_{m-1}\\ \hat{a}_m\end{bmatrix},\ p=\begin{bmatrix}1 & t_1 & \cdots & t_1^{m-1} & t_1^m\\ 1 & t_2 & \cdots & t_2^{m-1} & t_2^m\\ \vdots & \vdots & & \vdots & \vdots\\ 1 & t_{n-1} & \cdots & t_{n-1}^{m-1} & t_{n-1}^m\\ 1 & t_n & \cdots & t_n^{m-1} & t_n^m\end{bmatrix},\ X=\begin{bmatrix}X_1\\ X_2\\ \vdots\\ X_{n-1}\\ X_n\end{bmatrix}$$

.

$\left(P^{\mathrm{T}}\bullet P\right)$ in formula (3) proves to be a symmetric positive definite matrix easily, so the solution is unique. The $a_j\left(j=0,1,\cdots,m\right)$ in formula (3) can be solved, and we can get the polynomial:

$$f\left(t\right)=\sum_{j=0}^{m}a_j t^j \quad (4)$$

We can get the below formula by formula(4) to predict the future target value,

$$f\left(t_n+dt\right)=\sum_{j=0}^{m}a_j\left(t_n+dt\right)^j \quad (5)$$

And sliding window method is used to update the latest n data constantly in the prediction process, to realize real-time forecast of the target value in the next moment.

III. THE IMPROVEMENT OF SLIDING WINDOW POLYNOMIAL FITTING PREDICTION METHOD

The advantage of the traditional sliding window polynomial fitting prediction method is that it can make the most use of the current value by the sliding window, to provide the prediction of every time point with an adaptable polynomial fitting equation[8, 9], and it is further strengthened through the improvement of the traditional sliding window polynomial fitting prediction method.

When the improved sliding window polynomial fitting prediction method predicts the multiple consecutive future value simultaneously, an appropriate historical data group will be selected aiming at every predicted value, the specific method is to regard the distance between the future value and the current value as sampling interval, and acquired the historical data group, including the current value, by sampling the historical data set according to the sliding window width. If the current value is set as X_k , the sliding window width is n , the number of multiple consecutive future value that is predicted simultaneously is q , and the sampling interval is q and the historical data group is $\left\{X_{k-q(n-1)},X_{k-q(n-2)},\cdots,X_{k-q},X_k\right\}$ aiming at the q future prediction .

The improved sliding window polynomial fitting prediction method selects an appropriate historical data group for every predictive value, the polynomial fitting equations that obtained by the historical data group will be more targeted, every prediction will take the full advantage of the current value, theoretically speaking, the trajectory prediction accuracy of improved sliding window polynomial fitting prediction method will be more higher than the traditional sliding window polynomial fitting prediction method.

IV. SIMULATION CALCULATION AND ANALYSIS

A. Data Sources

Simulation data is derived from the climb segment and some route flight segment trajectory information of the CHH7711 flight in June 11th, 2014, and their position in the rectangular coordinate system is shown in the following figure:

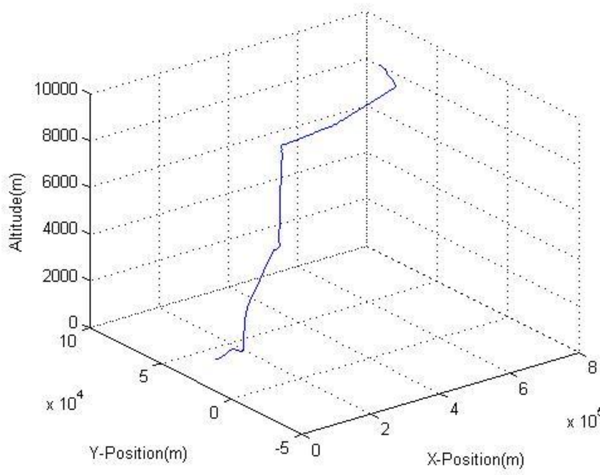


Figure 1. Reflections on the trajectory prediction absolute error of improved sliding window polynomial fitting prediction method.

B. Parameter Setting

1) Determination of sliding window width

Because the prediction bases on the finite history data, we must make a choice of the time window width. If the time window width is large, and the historical data is too much, the computational complexity of the algorithm will be enhanced and the prediction speed will be slow, and if the target trajectory changes rapidly, the fitting effect between the actual trajectory and time polynomial function will be bad, and the prediction deviation will increase. If the time window width is too small, which means the amount of data is too small to calculate out a polynomial function that describes the target's motion characteristics perfectly and the prediction effect is not ideal, we must select the appropriate data length. Generally speaking, we select the data length that make the forecast error be minimum according to the actual measurement data. Set the historical data number $n = 3$, the prediction number $q = 3$.

2) Determination of the number of degree

According to the principle of least squares fitting polynomial, firstly, the degree of polynomials in the prediction model must be determined. Determination of the number of degree should base on the analyses of the actual measurement data and the sliding window width, and the moving target can not be uniform due to various limitations. Set the polynomial degree $m = 2$.

C. Error Index

Mean absolute error

$$mar = \frac{1}{N}\sum_{t}\sqrt{(x_q(t) - x_r(t))^2 + (y_q(t) - y_r(t))^2 + (z_q(t) - z_r(t))^2} \quad (6)$$

Maximum absolute error

$$mmar = max\sqrt{(x_q(\mathrm{t}) - x_r(\mathrm{t}))^2 + (y_q(\mathrm{t}) - y_r(\mathrm{t}))^2 + (\mathrm{z}_q(\mathrm{t}) - z_r(\mathrm{t}))^2} \quad (7)$$

Mean relate error

$$mrr = \frac{1}{N}\sum_{t}\sqrt{\left(\frac{x_q(t) - x_r(t)}{x_r(t)}\right)^2 + \left(\frac{y_q(t) - y_r(t)}{y_r(t)}\right)^2 + \left(\frac{z_q(t) - z_r(t)}{z_r(t)}\right)^2} \quad (8)$$

In the formula: $x_q(t)$ $y_q(t)$ $z_q(t)$ are prediction value of the $(k+q)$ moment after the current time on the xyz axis in the rectangular coordinate system. $x_r(t)$ $y_r(t)$ $z_r(t)$ are true trajectory of the $(k+q)$ moment after the current time on the xyz axis in the rectangular coordinate system.

D. Results Analysis

The simulation is in the MATLAB2014a, according to the parameter settings before, trajectory prediction absolute error of flight CHH7711 based on improved sliding window fitting method and the traditional sliding window fitting prediction method are shown as below:

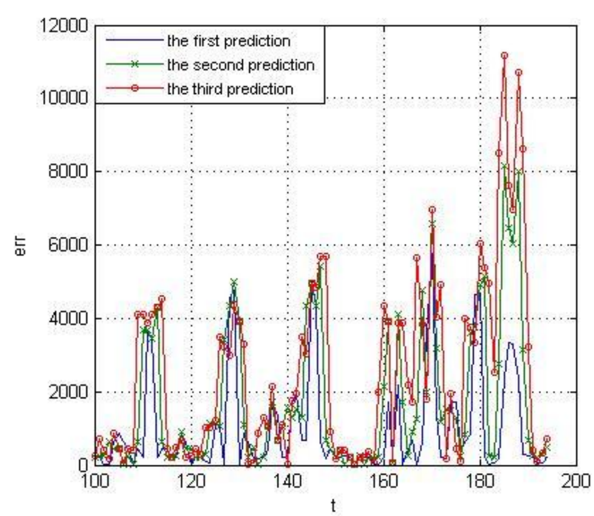
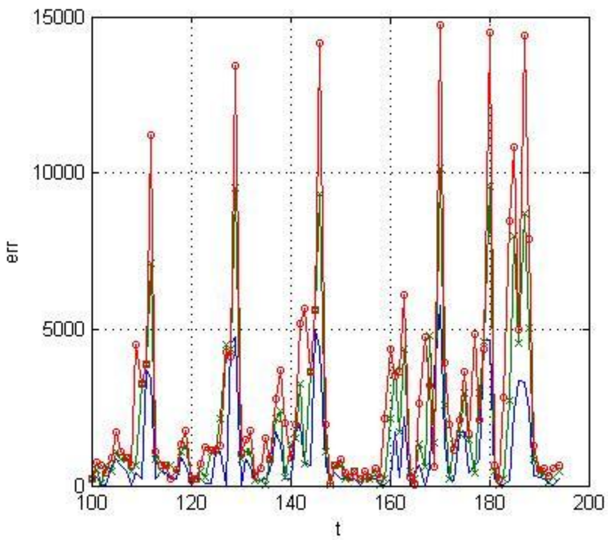


Figure 2. Reflections on the improved prediction absolute error of traditional sliding window polynomial fitting prediction method.



Note: the meaning of the all the lines in above figures is same.

Figure 3. Reflections on the trajectory prediction absolute error of traditional sliding window polynomial fitting prediction method.

According to the error index before, the calculation results are shown in the following table:

TABLE I. ERROR INDEX VALUES OF THE TWO METHOD PREDICTIONS

	The improved method			The traditional method		
	q=1	q=2	q=3	q=1	q=2	q=3
MAR	991.2	1768.2	2507.2	991.2	1888.3	2776.9
MMAR	5772.5	8142.4	11155	5772.5	10152	14737
MRR	0.0025	0.0049	0.0073	0.0025	0.0050	0.0076

In the absolute error value curve of the above figures, the waves reflect the flight maneuver point, the highest peak corresponds to the $mmar$ value in the table, the different of three prediction errors obtained from the traditional polynomial fitting prediction at every peak position is larger than the proved polynomial fitting prediction.

In the table, the index values mar $mmar$ mrr are less than the traditional method. It is clear that the proved polynomial fitting prediction method is more excellent than the traditional polynomial fitting prediction method.

It is applicable to the civil aviation aircraft flying on the route from the mean of the error due to the state update time t=17s, and the any time of preceding all the prediction and evaluation program is less than 2 second in the simulation, it meets the requirement of the real time online calculation, but the flight flying maneuver trajectory prediction needs to be further studied.

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