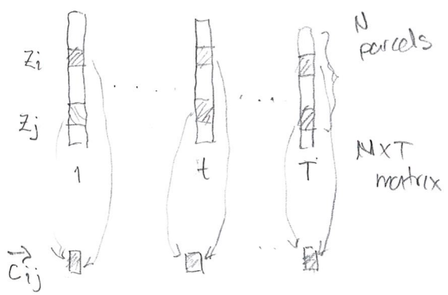


$$\mathbb{Z} = [z_{ij}]_{i \in \{1,N\},\ j \in \{1,T\}}$$ (time.)

NULL:

Columns of $\mathbb{Z} \sim N(0, \mathbb{R})$ ↑ NxN matrix.

time readings

$$\vec{C_{ij}} = (z_i(1)\cdot z_j(1),\ z_i(2)\cdot z_j(2),\ \ldots,\ z_i(T)\cdot z_j(T))$$

$C_{ij}(t) = z_i(t)\cdot z_j(t)$ (t^{th} element of $\vec{C_{ij}}$)

N^2 elements

$$\vec{C_{11}} = (z_1(1)\cdot z_1(1),\ z_1(2)\cdot z_1(2),\ \ldots,\ z_1(T)\cdot z_1(T))$$
$$\vec{C_{12}} = (z_1(1)\cdot z_2(1),\ z_1(2)\cdot z_2(2),\ \ldots,\ z_1(T)\cdot z_2(T))$$
$$\vdots$$
$$\vec{C_{1N}} = (z_1(1)\cdot z_N(1),\ z_1(2)\cdot z_N(2),\ \ldots,\ z_1(T)\cdot z_N(T))$$
$$\vec{C_{21}} = (z_2(1)\cdot z_1(1),\ z_2(2)\cdot z_1(2),\ \ldots,\ z_2(T)\cdot z_1(T))$$
$$\vdots$$
$$\vec{C_{2N}} = (z_2(1)\cdot z_N(1),\ z_2(2)\cdot z_N(2),\ \ldots,\ z_2(T)\cdot z_N(T))$$
$$\vdots$$
$$\vec{C_{N1}} = (z_N(1)\cdot z_1(1),\ z_N(2)\cdot z_1(2),\ \ldots,\ z_N(T)\cdot z_1(T))$$
$$\vdots$$
$$\vec{C_{NN}} = (z_N(1)\cdot z_N(1),\ z_N(2)\cdot z_N(2),\ \ldots,\ z_N(T)\cdot z_N(T))$$

eFC.

EDGE TIME SERIES ??

$$\begin{bmatrix} z_1(t)\cdot z_1(t) & z_1(t)\cdot z_2(t) & \cdots & z_1(t)\, z_N(t) \\ z_2(t)\cdot z_1(t) & z_2(t)\cdot z_2(t) & \cdots & z_2(t)\cdot z_N(t) \\ \vdots & & & \\ z_N(t)\cdot z_1(t) & z_N(t)\cdot z_2(t) & \cdots & z_N(t)\cdot z_N(t) \end{bmatrix}$$

NxN symmetric. only $\binom{N}{2} = \frac{N\cdot(N-1)}{2}$ are different.

$$C_{ij}(t) = z_i(t)\cdot z_j(t)$$

$$eFC_{jk,lm} = \sum_t C_{jk}(t)\, C_{lm}(t) \Big/ (\quad)$$

$$= \sum_{t=1}^{T} z_j(t)\cdot z_k(t)\cdot z_l(t)\cdot z_m(t) \Big/ (\quad)$$

$$r_{ij} = \frac{1}{T-1} \sum_{t=1}^{T} z_i(t)\cdot z_j(t)$$

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1N} \\ r_{21} & r_{22} & & r_{2N} \\ & \vdots & & \\ r_{N1} & r_{N2} & \cdots & r_{NN} \end{bmatrix}$$

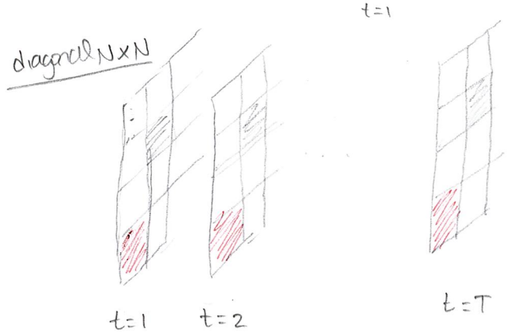
$$M_1 = \begin{bmatrix} z_1^1 \cdot z_1^1 & z_1^1 \cdot z_2^1 & \cdots & z_1^1 \cdot z_N^1 \\ z_2^1 \cdot z_1^1 & z_2^1 \cdot z_2^1 & \cdots & z_2^1 z_N^1 \\ \vdots & & & \\ z_N^1 \cdot z_1^1 & z_N^1 \cdot z_2^1 & \cdots & z_N^1 \cdot z_N^1 \end{bmatrix} \quad N \times N \text{ symmetric} \quad \binom{N}{2} \times \binom{N}{2}$$

$$\cdots \quad M_\tau = \begin{bmatrix} z_1^\tau \cdot z_1^\tau & z_1^\tau \cdot z_2^\tau & \cdots & z_1^\tau \cdot z_N^\tau \\ z_2^\tau \cdot z_1^\tau & z_2^\tau \cdot z_2^\tau & \cdots & z_2^\tau \cdot z_N^\tau \\ \vdots & & & \\ z_N^\tau \cdot z_1^\tau & z_N^\tau \cdot z_2^\tau & \cdots & z_N^\tau \cdot z_N^\tau \end{bmatrix} \quad \cdots \quad M_T = \begin{bmatrix} z_1^T \cdot z_1^T & z_1^T \cdot z_2^T & \cdots & z_1^T \cdot z_N^T \\ z_2^T \cdot z_1^T & z_2^T \cdot z_2^T & \cdots & z_2^T \cdot z_N^T \\ \vdots & & & \\ z_N^T \cdot z_1^T & z_N^T \cdot z_2^T & \cdots & z_N^T \cdot z_N^T \end{bmatrix}$$

time z_{percel}

$$\sum_{t=1}^{T} z_1^t \cdot z_1^t \qquad \sum_{t=1}^{T} z_1^t \cdot z_2^t$$

diagonal $N \times N$



$$z_j^1 \cdot z_k^1 \cdot z_l^1 \cdot z_m^1 + z_j^2 \cdot z_k^2 \cdot z_l^2 \cdot z_m^2 + \ldots + z_j^T \cdot z_k^T \cdot z_l^T \cdot z_m^T$$

$$\sum_{t=1}^{T} z_1^t \cdot z_1^t \cdot z_1^t z_1^t \qquad \sum_{t=1}^{T} z_1^t z_1^t z_1^t z_2^t \qquad \cdots \qquad \sum_{t=1}^{T} z_1^t \cdot z_1^t \cdot z_1^t z_N^t \cdot$$

$$\binom{N}{2} \times \binom{N}{2}$$

N terms | N-1 terms | 2 terms | 1 term

$$\begin{bmatrix}
\sum_{z=1}^{T} z_1^z z_1^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_1^z z_1^z z_1^z z_N^z & \sum_{z=1}^{T} z_1^z z_1^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_1^z z_1^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_1^z z_1^z \cdot z_{N-1}^z z_{N-1}^z, \sum_{z=1}^{T} z_1^z z_1^z \cdot z_{N-1}^z z_N^z & \sum_{z=1}^{T} z_1^z z_N^z z_N^z z_N^z \\
\sum_{z=1}^{T} z_1^z z_2^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_1^z z_2^z z_1^z z_N^z & \sum_{z=1}^{T} z_1^z z_2^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_1^z z_2^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_1^z z_2^z \cdot z_{N-1}^z z_{N-1}^z, \sum_{z=1}^{T} z_1^z z_2^z \cdot z_{N-1}^z z_N^z & \sum_{z=1}^{T} z_1^z z_2^z z_N^z z_N^z \\
\vdots & \vdots & & \vdots & \vdots \\
\sum_{z=1}^{T} z_1^z z_N^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_1^z z_N^z z_1^z z_N^z & \sum_{z=1}^{T} z_1^z z_N^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_1^z z_N^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_1^z z_N^z \cdot z_{N-1}^z z_{N-1}^z, \sum_{z=1}^{T} z_1^z z_N^z \cdot z_{N-1}^z z_N^z & \sum_{z=1}^{T} z_1^z z_N^z z_N^z z_N^z \\
\sum_{z=1}^{T} z_2^z z_1^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_2^z z_1^z z_1^z z_N^z & \sum_{z=1}^{T} z_2^z z_2^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_2^z z_2^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_2^z z_2^z \cdot z_{N-1}^z z_{N-1}^z, \sum_{z=1}^{T} z_2^z z_2^z \cdot z_{N-1}^z z_N^z & \sum_{z=1}^{T} z_2^z z_N^z z_N^z z_N^z \\
\vdots & \vdots & & \vdots & \vdots \\
\sum_{z=1}^{T} z_2^z z_N^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_2^z z_N^z z_1^z z_N^z & \sum_{z=1}^{T} z_2^z z_N^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_2^z z_N^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_2^z z_N^z \cdot z_{N-1}^z z_{N-1}^z, \sum_{z=1}^{T} z_2^z z_N^z \cdot z_{N-1}^z z_N^z & \sum_{z=1}^{T} z_2^z z_N^z z_N^z z_N^z \\
\vdots & \vdots & & \vdots & \vdots \\
\sum_{z=1}^{T} z_{N-1}^z z_{N-1}^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_{N-1}^z z_{N-1}^z z_1^z z_N^z & \sum_{z=1}^{T} z_{N-1}^z z_{N-1}^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_{N-1}^z z_{N-1}^z z_2^z z_N^z & \cdots & \sum_{z=1}^{N} z_{N-1}^z z_{N-1}^z \cdot z_{N-1}^z z_N^z, \sum_{z=1}^{T} z_{N-1}^z z_{N-1}^z \cdot z_N^z z_N^z & \sum_{z=1}^{T} z_{N-1}^z z_N^z z_N^z z_N^z \\
\sum_{z=1}^{T} z_{N-1}^z z_N^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_{N-1}^z z_N^z z_1^z z_N^z & \sum_{z=1}^{T} z_{N-1}^z z_N^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_{N-1}^z z_N^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_{N-1}^z z_N^z z_{N-1}^z z_{N-1}^z; \sum_{z=1}^{T} z_{N-1}^z z_N^z \cdot z_{N-1}^z z_N^z, \sum_{z=1}^{T} z_{N-1}^z z_N^z \cdot z_N^z z_N^z & \\
\sum_{z=1}^{T} z_N^z z_N^z z_1^z z_1^z \cdots \sum_{z=1}^{T} z_N^z z_N^z z_1^z z_N^z & \sum_{z=1}^{T} z_N^z z_N^z z_2^z z_2^z \cdots \sum_{z=1}^{T} z_N^z z_N^z z_2^z z_N^z & \cdots & \sum_{z=1}^{T} z_N^z z_N^z z_{N-1}^z z_{N-1}^z; \sum_{z=1}^{T} z_N^z z_N^z \cdot z_{N-1}^z z_N^z, \sum_{z=1}^{T} z_N^z z_N^z \cdot z_N^z z_N^z &
\end{bmatrix}$$

N rows.

N-1 rows.

2 rows

1 row

Total rows

$N+(N-1)+\cdots+1$

$= \binom{N}{2}$