Neuroinformatics Journal Club Dynamic Brain Networks (HMM) Electrophysiology

Horacio Gómez-Acevedo Department of Biomedical Informatics University of Arkansas for Medical Sciences

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Today's paper

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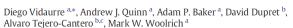
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Spectrally resolved fast transient brain states in electrophysiological data



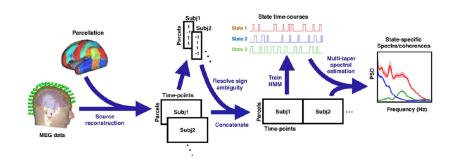


b MRC Brain Network Dynamics Unit, Department of Pharmacology, University of Oxford, UK



Computational Neuroscience, Department Biologie II Ludwig Maximilian, University Munich, Germany

Workflow



Modeling ingredients

The paper follows some of the same themes as the previous one

- Hidden Markov Models
- Autoregressive models

To make things "interesting" it adds some more topics

- Bayesian methods
- Some Fourier Analysis

Basic Setup

We consider K hidden states, and T time points.

Let \mathbf{y}_t^N be the multichannel source signal.

The construction of the multivariate autoregressive (MAR) model is different than before.

$$\mathbf{y}_t'|x_t = k \sim \mathcal{N}\left(\sum_{l \in A} \mathbf{y}_{t-1}' \mathbf{W}_l^{(k)}, \Sigma^{(k)}\right)$$
 (1)

Don't panic (just yet)

- $\mathbf{y}'_t|x_t=k$ This part refers to as the emission at time t when you are in the hidden state k. It is the transpose of the vector of N entries.
- ▶ The $\mathcal N$ refers to the multivariate normal distribution. In the 1-dimensional case $\mathcal N(\mu,\sigma)$ the μ is transformed into a vector, and the σ by a matrix (variance-covariance).

Basic Setup (cont)

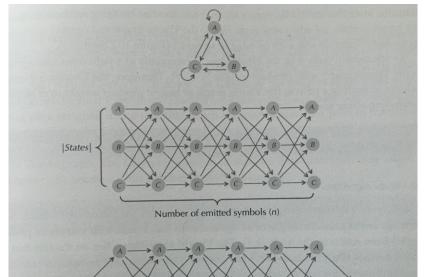
- $\mathbf{y}_{t-1}'\mathbf{W}_{l}^{(k)}$ refers to as the MAR component. The matrix $\mathbf{W}_{l}^{(k)}$ is of type $N \times N$ for the kth (hidden) state for the lag l. We add all the individual contributions of each lag step.
- $\triangleright \Sigma^{(k)}$ is the noise covariance matrix.

The Markovian property

$$P(x_t = k_1 | x_{t-1} = k_2) = \Theta_{k_1 k_2}, \quad P(x_t = k) = \eta_k$$
 (2)



Viterbi's representation HMMs



MAR model

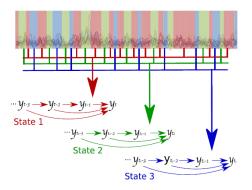


Fig. 2. Graphical representation of the HMM–MAR. The time series (background) is partitioned into three states denoted by the blue, red and green slabs. Each state is characterised by a different set of dynamics, determined by the linear historical interactions between data points y, (small arrows).

MAR model (cont)

The paper key component for the MAR section is the ${\cal A}$ set. It contains the number of previous steps to be considered

$$A = \{P_0 + 1, P_0 + Q, P_0 + \left\lfloor \frac{1}{2} Q^2 \right\rfloor, \dots, P\}$$
 (3)

It calls it exponential due to the Taylor representation of the exponential function

$$\exp(X) = 1 + X + \frac{1}{2!}X^2 + \frac{1}{3!}X^3 + \cdots$$



Oh Bayes where art thou?

The precision matrix is modeled with a Wishart distribution

$$\Omega^{(k)} \sim \mathcal{W}(t_0, \mathbf{B}_0) \tag{4}$$

An example: Sample covariance for idd multivariate normal. You can think about it as a multivariate version of the Gamma distribution.

Priors.

We expect each state to be characterized by a certain set of connections and certain frequency profile... $\sigma_{ij}^{(k)}$ weight the presence of a specific connection between nodes (i,j) when in state k. The precision $\alpha_j^{(k)}$ adaptively weight to the presence of interaction at a certain lag I for all nodes when in state k.



Bayes cont

$$W_{l_{ij}}^{(k)} \sim \mathcal{N}(0, \sigma_{ij}^{(k)} \alpha_j^{(k)}) \tag{5}$$

where

$$\sigma_{ij}^{(k)} \sim \Gamma(\phi_0, d_0) \tag{6}$$

$$\alpha_j^{(k)} \sim \Gamma(\xi_0, d_0) \tag{7}$$

Bayes cont

$$\Theta_k \sim Dir(\nu_0) \quad \eta \sim Dir(\psi_0)$$
 (8)

The Dirichlet distribution is the conjugate prior for the parameters of the normal multinomial distribution.

