Learning Dynamics / Computational Game Theory Assignment 1

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Exercise 1: The Hawk-Dove Game

The Hawk-Dove game has the following payoff matrix:

Row Player \ Column Player	Hawk	Dove
Hawk	$\frac{V-D}{2}$, $\frac{V-D}{2}$	V, 0
Dove	0, V	$\frac{V}{2} - T$, $\frac{V}{2} - T$

Judging by the values in the payoff matrix, four configurations will be analysed to get the best response for each of them. The parameters' D, T and V order will be interchanged, to get the different configurations, assuming these parameters are bigger than 0.

Case 1: $V > D$ and $-T < \frac{V}{2}$

Best response matrix:

Row Player \ Column Player	Hawk	Dove
Hawk	$\boxed{\frac{V-D}{2}}$, $\boxed{\frac{V-D}{2}}$	$\boxed{V}$, 0
Dove	0, $\boxed{V}$	$\frac{V}{2} - T$, $\frac{V}{2} - T$

In this case, the **Hawk strategy, strictly dominates the Dove Strategy,** because any player would get a better payoff using the Hawk strategy. In this symmetric game, **{Hawk, Hawk} is a Nash Equilibrium.** This game resembles the Prisoner Game.

Case 2: $V > D$ and $-T > \frac{V}{2}$

Best response matrix:

Row Player \ Column Player	Hawk	Dove
Hawk	$\boxed{\frac{V-D}{2}}$, $\boxed{\frac{V-D}{2}}$	V, 0
Dove	0, V	$\boxed{\frac{V}{2} - T}$, $\boxed{\frac{V}{2} - T}$

In this case, there are **no dominant strategies**, however the players' best option is to play the same strategy at the same time. There are two **Nash Equilibria : {Hawk, Hawk} and {Dove, Dove}**. This game resembles the Stag-Hunt Game.

Case 3: $V < D$ and $-T < \frac{V}{2}$

Best response matrix:

		Column Player	
		Hawk	Dove
Row Player	Hawk	$\frac{V-D}{2}$, $\frac{V-D}{2}$	V, 0
	Dove	0, V	$\frac{V}{2} - T$, $\frac{V}{2} - T$

The Dove strategy strictly dominates the Hawk strategy, therefore **in this case it's more beneficial to display rather than to escalate** (ex 1.2). The only **Nash Equilibrium of this configuration is**, therefore **{Dove, Dove}**. This game, again resembles the Prisoner Game.

Case 4: $V < D$ and $-T > \frac{V}{2}$

Best response matrix:

		Column Player	
		Hawk	Dove
Row Player	Hawk	$\frac{V-D}{2}$, $\frac{V-D}{2}$	V, 0
	Dove	0, V	$\frac{V}{2} - T$, $\frac{V}{2} - T$

In this case, the pure strategy is to play the opposite strategy of the opponent. Therefore **two pure Nash Equilibria** exist in this game: **{Hawk, Dove} and {Dove, Hawk}**. This game resebmles the Chicken Game.

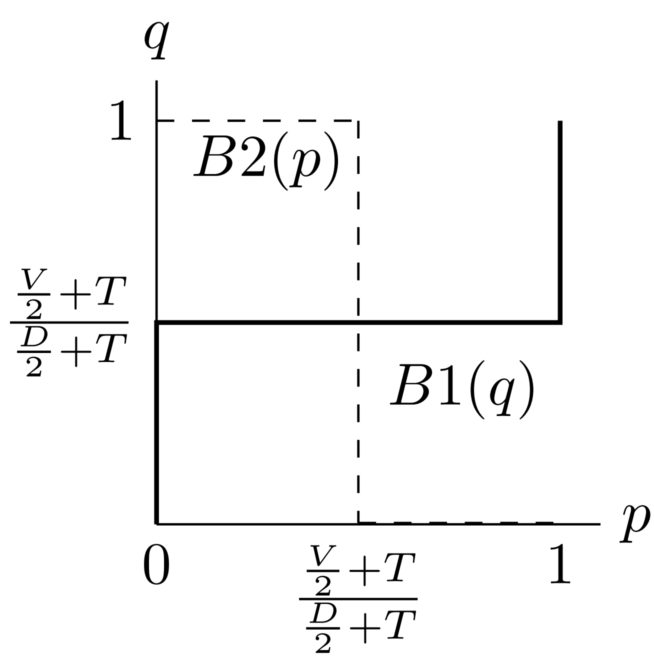
Mixed Strategies

As the game is symmetrical let's calculate the mixed strategy expected payoff. Let p be the probability for the first player to play Hawk. This probability is found after equalising the second player's utilities which are the following:

$$\begin{cases} U_{Hawk} = p * \frac{V-D}{2} + (1-p) * V \\ U_{Dove} = (1-p) * (\frac{V}{2} - T) \end{cases}$$

$$=> p = \frac{\frac{V}{2} + T}{\frac{D}{2} + T}$$

Therefore, in this particular symmetrical game we can state that displaying (Dove strategy) becomes more beneficial than escalating when the fitness cost of injury (D) is bigger than the fitness value of winning the resources (V) which in turn is bigger than twice the fitness cost of wasting time (-T), as it can be seen in the plot below:



Validating the results using NashPy

The results obtained after manipulating the sliders were as predicted in the analysis. This proves that the assessment was correct. Furthermore, the Nash equilibria found with NashPy for V = 2, D = 3 and T = 1 are the following: Player 1 pure Hawk - player 2 pure Dove (and vice-versa) and both players playing Hawk 80% of the time and Dove 20% of the time. This holds true when replacing the variables in our equation, it becomes:

$$p = \frac{\frac{V}{2} + T}{\frac{D}{2} + T} = \frac{\frac{2}{2} + 1}{\frac{3}{2} + 1} = 0.8$$

Exercise 2: Which Social Dilemma?

First of all, the best response matrices for each game are as follows:

Prisoner game

	C	D
C	2 2	5 0
D	0 5	1 1

Stag-Hunt game

	C	D
C	5 5	2 0
D	2 1	1 1

Stag-Hunt game

	C	D
C	2, 2	1, 5
D	5, 1	0, 0

We can compile a payoff table based on player 1's (row player) beliefs by comparing their strategy against the column player's three possible strategies. Since the row player does not know in which game they are, player 1's payoffs are as follows:

	CCC	CCD	CDC	CDD	DDD	DDC	DCD	DCC
C	3	2 + 2/3	1 + 1/3	1	1/3	2/3	2	2 + 1/3
D	4	2 + 1/3	3 + 2/3	2	2/3	2 + 1/3	1	2 + 2/3

For each best response in the previous table, in order to find the Nash Equilibria, we need to find which of these plays will contain the other player's (column player) best response for each game as well. We can see that if the row player plays **D** and the column player plays **(DDC)**, we have best responses for both players. This is the case as well when player 1 plays **C** and the other plays **(DCD)**.
Therefore, the two Nash Equilibria found in this game are: **{D, (DDC)}** and **{C, (DCD)}**

Exercise 3:
Evolutionary Dynamics in the Hawk-Dove game

3.1 Infinite Populations

The plot of the gradient selection for infinite populations is the plot for:
$G(x_i) = \dot{x}_i = x_i[(Ax)_i - x^T Ax]$, where $\dot{x}_i = x_i[f(x_i) - \sum_{i=1}^{n} x_i f(x_i)]$.
We can see there are **three saddle points**, only one of which is stable at x = 0.8, and it's **representing the mixed strategy Nash Equilibria**. The other two saddle points are unstable and they represent the **pure strategies Dove and Hawk for 0.0 and 1.0 respectively**.
This **behaviour is the same with** the one we could see in **exercise 1**, with the same mixed strategy Nash equilibrium - playing Hawk 80% of the time. So it can be safely stated that there are **no changes to be expected whether the population is finite or infinite**.

3.2 Finite Populations

For finite populations, dynamics are affected as follows:

Changing population size Z

When population reaches a very high number, stochastic effects due to mutation and genetic drift become very small, and all the weight of the stationary distribution tends to be put on the singular point 0.8. Therefore, the distribution becomes strongly peaked at this point, recovering the results found from the previous points, players always preferring to play Hawk. As population decreases to very small numbers, the genetic drift increases and therefore the distribution will lose the aforementioned peak, but still the Hawk strategy will prevail.

Changing the intensity of selection β

As the intensity of the selection approaches 0, the strategies will have an equal chance of being chosen, and as it approaches 1, players will always choose Hawk. This happens because when the intensity of selection is 0, there is no correlation between the selection process and the strategy chosen.

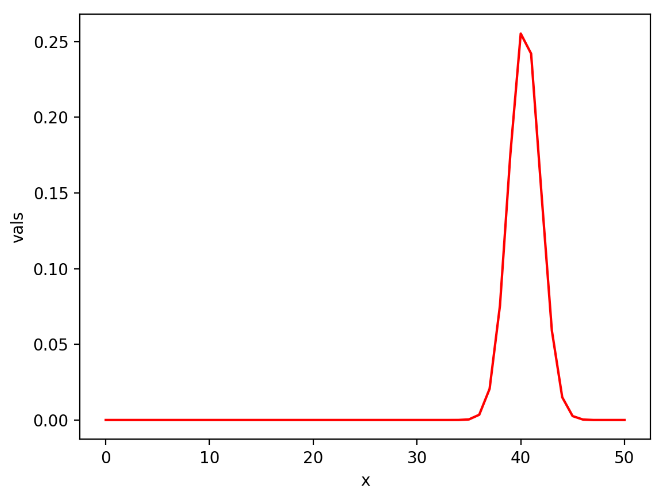
Changing the probability of mutation μ

As the probability of mutation reaches very small values, the stationary distribution resembles the previous cases, peaking at 0.8. However, when the mutation probability is high, the peak will approach 0.5. This is because as more and more players mutate, the less and less the other factors are taken into account, this is resembling randomness.

3.3 Moran Process with Pair-wise Comparison

The Moran process has been implemented using python3. The moran_step function was implemented as presented in class, with added parameter μ (mutation chance). The Monte Carlo simulation was ran 10 times with a transitory period of 1000 and a number of generations of 100000. The following plot was outputtted:

Figure 1: The x axis represents the amount of population that played the strategy in the vals axis. Strategies are as follows: 0 for Hawk and 1 for Dove.



Conventions used

Best response for column player: Best response

Best response for row player: Best response