Brewer's Problem

Linear Programming = problem solving model for optimal allocation of scarce resources among a number of competing activities that encompasses: shortest paths, max flow, MST ...

Example

- small brewery produces ale and beer
- production limited by scarce resources:
  - corn
  - barley malt
  - hops
- recipes for ale and beer require different proportions of resources
- different profitability for ale and beer

Linear programming formulation

- let A be the number of barrels of ale

• let B be the number of barrels of beer

	ALE		BEER			
maximize	$13A$	$+$	$23B$			profits
subject to the constraints	$5A$	$+$	$15B$	$\leq$	480	corn
	$4A$	$+$	$4B$	$\leq$	160	hops
	$35A$	$+$	$20B$	$\leq$	1190	malt

$$A, B \geq 0$$

* inequalities define halfplanes => feasible region is a convex polygon

=> objective function is also a line

=> where the objective function line (or its parallel) intersects the convex polygon => max profit

=> optimal solution occurs at an extreme point

- extreme point = intersection of 2 constraints in 2D

<u>Standard form</u>

c is a vector

maximize → $C^T x$ $(c_1x_1 + c_2x_2 + .. c_nx_n)$

subject to the constraints ⇒ $Ax = b$

$x \geq 0$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & & & & a_{2n} \\ a_{31} & & & & a_{3n} \\ \vdots & & & & \cdots \\ a_{m1} & & & & a_{mn} \end{bmatrix}$$

$m \times n$

- Maximize = objective function - Z
- add slack variable to convert each inequality to an equality
- a set is convex if for any two points in the set (a and b) so is $\boxed{\frac{1}{2}(a+b)}$

- extreme point = point in a set that can't be written as $1/2(a+b)$ where a and b are two distinct points in the set
- greedy property = extreme point optimal iff no better adjacent extreme point

Simplex Alghorithm

Generic Algo

- start at some extreme point
- pivot from one extreme point to an adjacent one
- repeat until optimal

Basis : subset of m of the n variables

BFS = Basic Feasible Solution

- set $(n-m)$ nonbasic variables to 0

→ solve for remaining m variables

→ solve m equations in m unknowns

- if unique & feasible => BFS
- BFS <=> extreme point

! Look at Simplex Algorithm and linear programming

Simplex Implementations

- encode standard form LP in a single Java 2D array

Linear Programming Reductions

- reduction to standard form