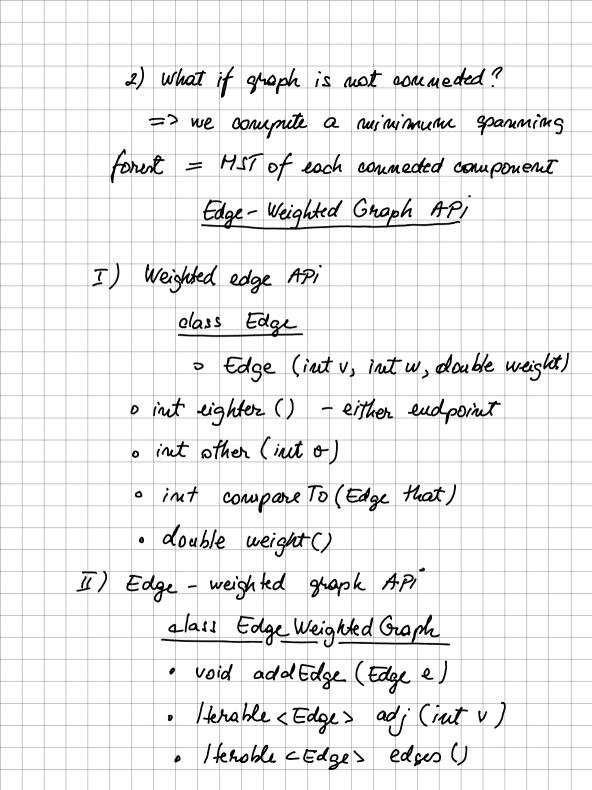
Minimum Spanning Tree Undirected Graph G with positive edge weight (connected) definition [ A sparming tree of G is a Subgraph T that is both a tree (connected and acyclic) and spanning (includes all vertices) Goal = find a min weight spanning thee Brute force: try all spanning trees? Greedy Algorithm Simplifying Assumptions: - edge weights are distinct - graph is connected Consequence: MST exists and it is unique Cut Proporty A (cut) in a graph is a partition of its

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o int V() · ent E() Conventions = allow self-loops and parallel edges III) Minimum Spanning Tree API class MST · MST (Edge Weighted Graph 6) Iterable < Edge > edges () o double weight () Kruskol's Algorithme a consider edges in ascending order of · add next edge to tree T when doing so would create a cycle Proof: Kruskal's algorithm is a special case of the greedy MST algorithm

o Suppose kruskal's algorithm wors the edge e (v-w) black · act = set of vertices connected to v in O no crossing edge is block o no orossing edge has lower weight Challenge · Would adding edge V -> w to tree T deste a cycle? If not, add it! 1) . hum D75 from v and check if w is reachable - O(V) 2) · use mion-find ] - to see it wand ware commected -> O (log V) \* maintain a set of each connected comp. I if I and w over in the same set =>

adding (v-w) would create a cycle - otherwise add (o-w) to T and morge the sets containing vand w | Kruskal | - > O (E log E) Prum's Algorithme · start with vertex 0 and greedily grow · add to T the ruin weight edge with exactly oue endpoint in T • repeat until (V-1) edges Proof: Prince's algorithme is a special case of the geedy 457 algorithm => Suppose edge e = min weight edge commeding a vertex on the tree to a vertex not on the tree.

o cut = set of vertices connected on thee o no drossing edge is black o no crossing edge has lower weight => use a priority guere! Losy inuplementation -> find the min weight edge with exactly one endpoint in T => Maintain a PQ of edges with (at look) oue endpoint in T · key = edge , priority = weight of edge o deletetin to detornine the next edge e = (v-w) to add to T · disrigard edge that has both endpoints vaud w in the T

o is on the tree

wherevise

w not on the tree - add to PQ any edge incident to w

(assuming the other endpoint not in thee) - add w to T - Running Time O (ElogE) Eager Implementation - maintain a PQ of vertices connected by an edge to T => priority of vortex v = weight of the shortest edge connecting v to T o delete The min vortex or and add its associated edge (e = 0-0 w) to T o update PQ by couridoring all edges e = v -x incident to v - ignore if x is abready in T - add x to PQ if not already ou it - decrease priority of x if v-x, becomes shortest edge connecting & to T

Indexed Priority Queue - associate are index between I and N-1 for each key in the priority guene - dient con insert and delek the minimum - dieut con change the key by specifying the index hidex Min PQ · void insert (init i, key key) · wid decreosekey (nut i, key key) · booleau contains (int k) = int del Min () · boolean is Empty () · sint size () - start with the sauce code for Min PQ] - maintain parallel arrays keys [],

pail and apti so that: - beys[i] = priority of i - pa [i] = index of the key in heap pesition i - gp[i] - heap position of the key with · use swinn (gp[k]) inuplement deorese key MST Context is there a linear time MST algorithme? - Eudideau MST - Clustoring - fingle - link dustering algorithme