Priority Queues

- priority queue : remove the largest (or smallest) item

API   class MaxPQ <key extends Comparable>

  - void insert(key v)
  - key deleteMax()
  - boolean isEmpty()

- generalizes: stack, queue, randomized queues
- Challenge: find the largest M items in a stream of N items
  * not enough memory to store N items
  * use a min oriented priority queue

```
if (pq.size() > M)
        pq.delMin()
```

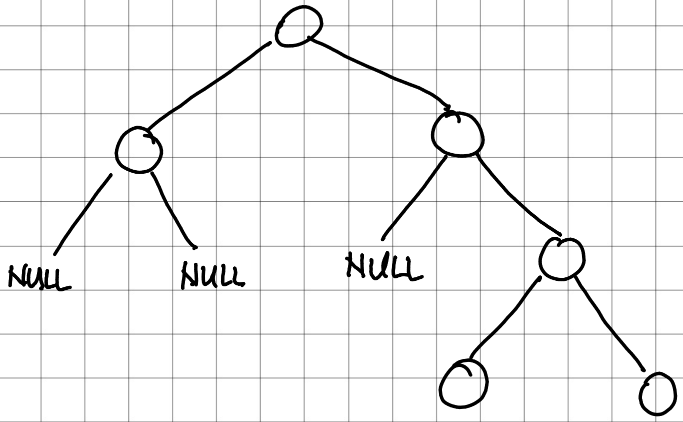
- can't sort because we can't store all N items
- elementary PQ   O(M*N) - too slow

- binary heap $O(N \log M)$ / space M

Binary Heap

* Complete Binary Tree

Binary Tree : empty or node with links to left and right binary trees



Complete tree : perfectly balanced, except for bottom level

Property : Height of a complete tree with N nodes is $\lfloor \lg N \rfloor$

* height only increases when N is a power of 2

Binary heap : array representation of a heap-ordered complete binary tree

Heap-ordered binary tree

- keys in nodes
- parent's key no smaller than children's key (>=)

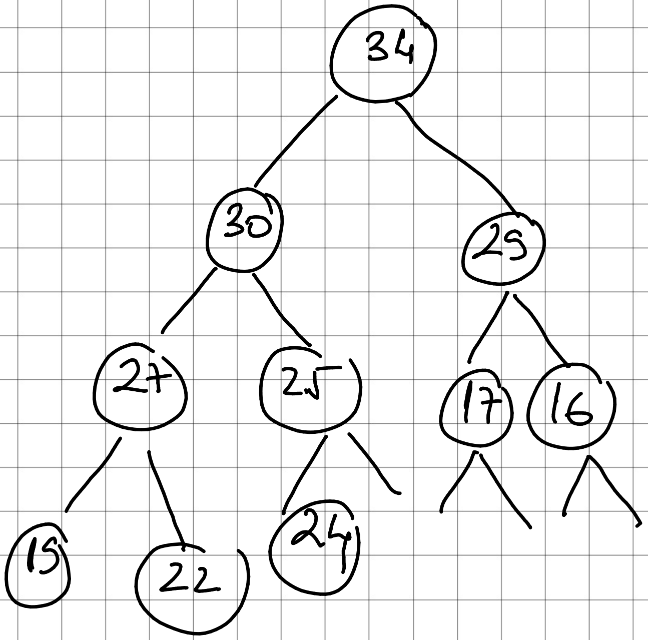
Array representation

- indices start at 1
- take nodes in level order
- no explicit links needed

Binary heap properties

- largest key is a[1] => root of the binary tree
- can use array indices to move through tree.
  - parent of node $\boxed{k}$ is at $\boxed{k/2}$ (int division)
  - children of node k are at $\boxed{2k}$, $\boxed{2k+1}$

34 30 29 27 25 17 16 15 22 24



Max oriented binary heap

Promotion in a heap

Scenario - child's key becomes larger than the parent's key

Fix : • exchange key in child with key in parent

◦ repeat until heap is ordered

Peter principle : node promoted to level of incompetence

- Insertion in a heap
  - add a new node at the end and swim it up
  - cost: at most $\boxed{1+\lg N}$ compares

Demotion in a heap

Scenario - parent's key becomes smaller than one (or both) of its children's

Fix - exchange key in parent with key in larger child
- repeat until order is restored

Delete the maximum in a heap

- exchange root node with node at the end, then sink the small node down
- at most $\boxed{2\lg N}$ compares

* Fibonacci max PQ => insert $O(1)$

del max $O(\lg N)$

max $O(1)$

- immutability of keys

- underflow and overflow
  - underflow: throw exception if deleting from empty PQ
  - overflow: add no-arg constructor and use resizing of array

* min oriented priority queue => replace less with greater