I) Cartesian Coordinate Systems

① 1D Mathematics

discrete mathematics — natural numbers, integers

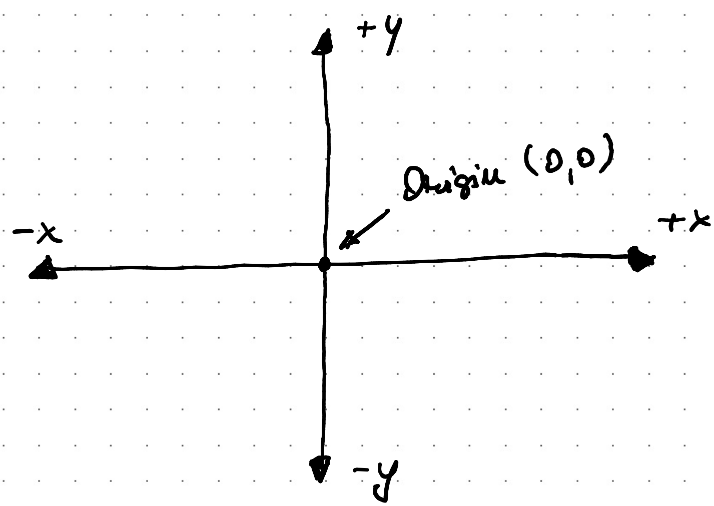
continuous mathematics — real numbers

* universe = discrete, finite

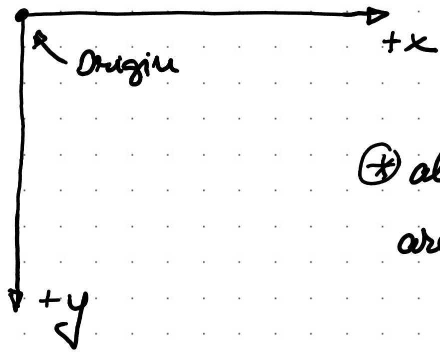
The First Law of Computer Graphics

If it looks right, it is right.

② 2D Cartesian Space

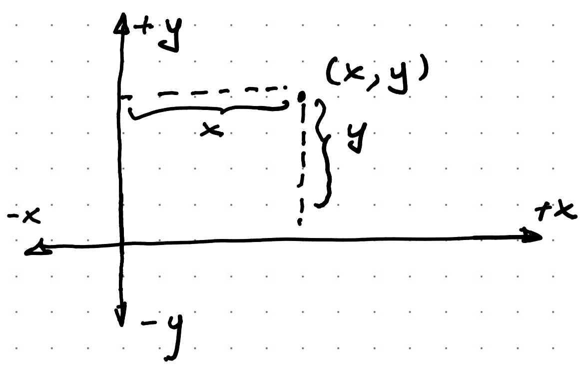


Screen Coordinate Space

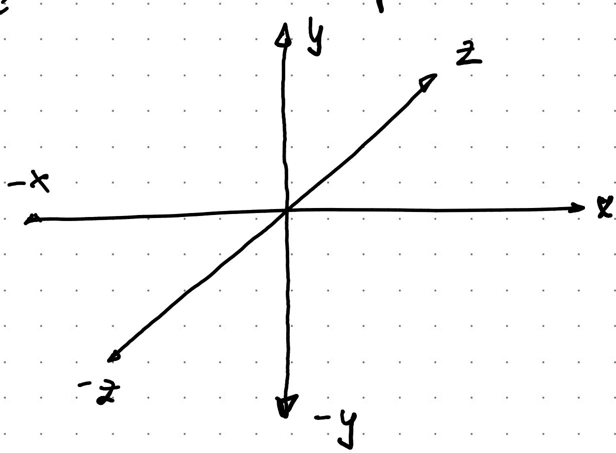


* all 2D coordinate systems are "equal"

! Cartesian Coordinates



(3) 3D Cartesian Space



(x, y, z)

- signed distance to yz, xz, xy

Left-handed versus Right-Handed Coordinate Spaces

(positive)

Rotation
- left handed — clockwise
- right handed — counter clockwise (positive)

(4) Odds and Ends

$$\sum_{i=1}^{6} a_i = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

$$\prod_{i=1}^{n} a_i = a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \cdot a_6 \ldots \cdot a_n$$

Interval notation

$[a, b] \Rightarrow a \leq x \leq b$

$(a, b) \Rightarrow a < x < b$

$[0, \infty)$

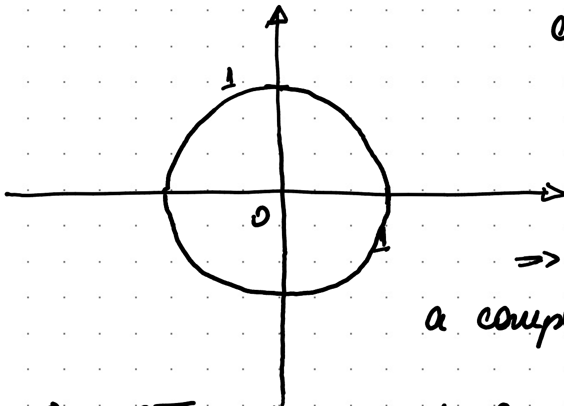
Angles, Degrees and Radians

° – degrees

$360° =$ complete revolution

rad – radians

Unit Circle



circumference $= 2\pi R$

$\boxed{R=1}$

$\Rightarrow 2\pi$ radians $=$
a complete revolution

$360^\circ = 2\pi$ rad $\Rightarrow 180^\circ = \pi$ rad

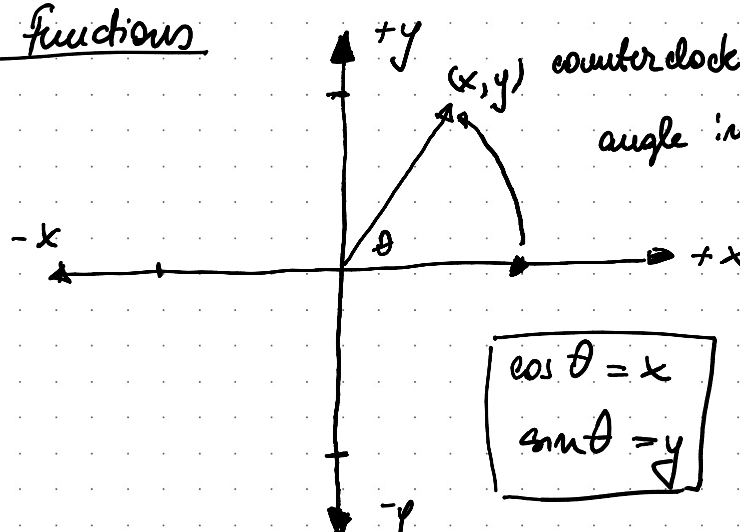
Conversions

from rad to degrees $\Rightarrow$ multiply by $180/\pi$
degrees to rad $\rightarrow$ $\pi/180$

1 rad $= (180/\pi)^\circ$

1 degree $= (\pi/180)$ rad

Trig functions



counterclockwise

angle in standard position

$\boxed{\begin{aligned}\cos\theta &= x\\ \sin\theta &= y\end{aligned}}$

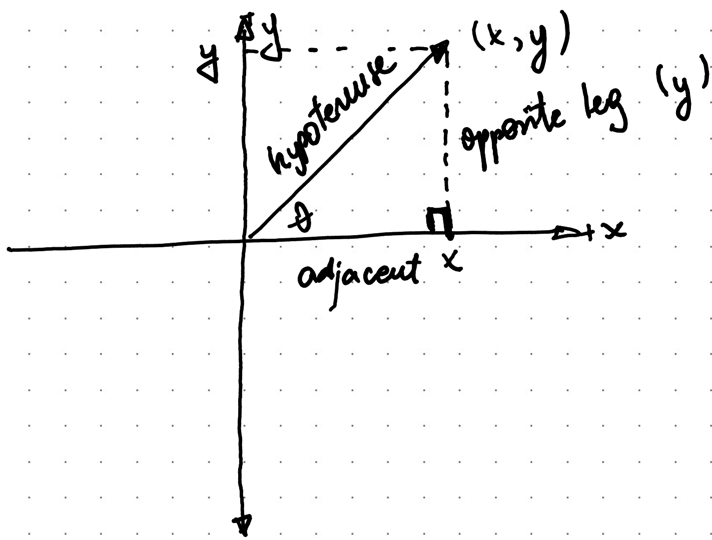
$R=1$
(unit circle)

$$\boxed{\sec\theta = \frac{1}{\cos\theta}}$$ secant

$$\boxed{\csc\theta = \frac{1}{\sin\theta}}$$ cosecant

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$ tangent

$$\boxed{\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}}$$ cotangent



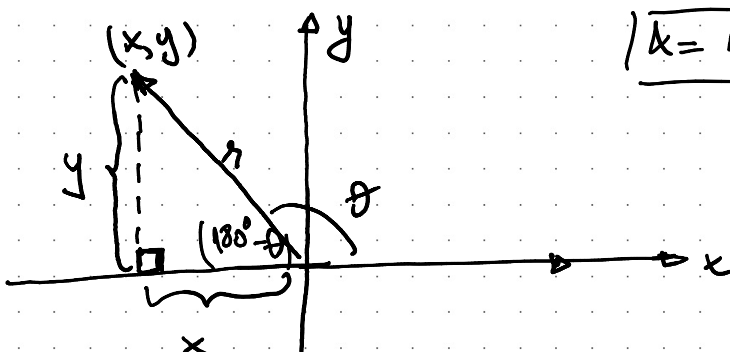
• the primary trig functions are defined by the following ratios:

$$\boxed{\cos\theta = \frac{adj}{hyp}, \quad \sin\theta = \frac{opp}{hyp}, \quad \tan\theta = \frac{opp}{adj}}$$

$$\boxed{\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{adj}{hyp}} = \frac{hyp}{adj}}$$

$$\boxed{\csc\theta = \frac{hyp}{opp}} \qquad \boxed{\cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp}}$$

* these properties do not apply when θ is obtuse (cannot form a right triangle with an obtuse interior angle)



$$\boxed{\alpha = 180° - \theta}$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

$$\tan\theta = \frac{y}{x}$$

5) Trig Identities

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

* pythagorean theorem

$$a^2 + b^2 = c^2$$

$$\boxed{\sin^2\theta + \cos^2\theta} = \frac{opp^2}{hyp^2} + \frac{adj^2}{hyp^2} = \frac{hyp^2}{hyp^2}$$

$= 1$

$$\boxed{1 + \tan^2\theta = \sec^2\theta} \quad \boxed{1 + \cot^2\theta = \csc^2\theta}$$

* trig function on the sum or difference of two angles

$$\boxed{\sin(a+b) = \sin a \cos b + \cos a \sin b}$$

$$\boxed{\sin(a-b) = \sin a \cdot \cos b - \cos a \cdot \sin b}$$

$$\boxed{\cos(a+b) = \cos a \cdot \cos b - \sin a \cdot \sin b}$$

$$\boxed{\cos(a-b) = \cos a \cdot \cos b + \sin a \cdot \sin b}$$

$$\boxed{\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}}$$

$$\boxed{\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}}$$

(*) obtuse angle

positive value

$$\boxed{\alpha = 180° - \theta} \Rightarrow \boxed{\cos\alpha = \frac{+x}{r}}$$

$$\cos(180° - \theta) = \cos(180°) \cdot \cos\theta + \sin 180° \cdot \sin\theta$$

$$= -1 \cdot \cos\theta + 0$$

$$\boxed{\cos\alpha = -\cos\theta}$$

$$\Rightarrow \cos\theta = \frac{-x}{r} \longrightarrow \text{coo}$$

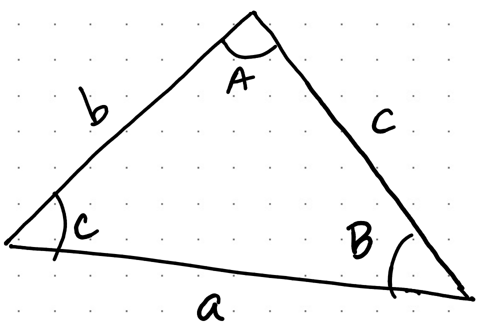
(*) if angles a and b are the same we have the following special cases

$$\boxed{\sin 2\theta = 2 \sin\theta \cos\theta}$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\boxed{\tan 2\theta = \frac{2 \tan\theta}{1 - \tan^2\theta}}$$

(*) law of sines and law of cosines



$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ law of sines

$$\boxed{\begin{aligned} a^2 &= b^2+c^2-2bc\cos A \\ b^2 &= a^2+c^2-2ac\cos B \\ c^2 &= a^2+b^2-2ab\cos C \end{aligned}}$$

law of cosines

Exercises

① $a=(-2.5,3)$ $\quad e=(0,0)$

$b=(1,2)$ $\quad f=(2,-0.5)$

$c=(2.5,2)$ $\quad g=(-0.5,-1.5)$

$d=(-1,1)$ $\quad h=(0,-2)$

$i=(-3,-2)$

② $a=(1,2,4)$

④ a) right handed

⑥ left-hand rule vs right hand rule

⑦ $\sum_{i=1}^{5} i = 1+2+3+4+5 = \frac{5\cdot 6}{2} = 15$

$\sum_{i=1}^{5} 2i = 2\cdot \sum_{i=1}^{5} = 30$

$$\prod_{i=1}^{5} 2\cdot i = 2\cdot 1\cdot 2\cdot 2\cdot 2\cdot 3\cdot 2\cdot 4\cdot 2\cdot 5$$

$$= 2^5\cdot 5! = 32\cdot 120 = 3840$$

$$\prod_{i=0}^{4} 7(i+1) = 7^5\cdot 5! = 2016840$$

$$\sum_{i=1}^{100} i = \frac{n(n+1)}{2} = \frac{100\cdot 101}{2} = 50\cdot 101$$

$$= 5050$$

(8) Convert from degrees to radians

$$2\pi = 360^\circ \Rightarrow 1^\circ = \frac{2\pi}{360} = \frac{\pi}{180}\ \text{rad}$$

(9) Convert from radians to degrees

$$2\pi\ \text{rad} = 360^\circ$$

$$\Rightarrow 1\ \text{rad} = \frac{360^\circ}{2\pi} = \left(\frac{180}{\pi}\right)^\circ$$

(11) a) $(\sin\alpha / \csc\alpha) + (\cos\alpha / \sec\alpha) = 1$

$$\csc\alpha = \frac{1}{\sin\alpha} \Rightarrow \frac{\sin\alpha}{\csc\alpha} = \frac{\sin x}{\frac{1}{\sin x}}$$

$$= \boxed{\sin^2\alpha}$$

$$\sec\alpha = \frac{1}{\cos\alpha} \Rightarrow \frac{\cos\alpha}{\sec\alpha} = \boxed{\cos^2\alpha}$$

$$\Rightarrow \boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

b) $(\sec^2\alpha - 1)/\sec^2\alpha = \sin^2\alpha$

$$\left(\frac{1}{\cos^2\alpha} - 1\right) / \frac{1}{\cos^2\alpha} = \sin^2\alpha$$

$$\left(\frac{1}{\cos^2\alpha} - 1\right) \cdot \cos^2\alpha = \sin^2\alpha$$

$$1 - \cos^2\alpha = \sin^2\alpha \Rightarrow \boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

c) $1 + \cot^2\alpha = \csc^2\alpha$

$$1 + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha} \quad / \cdot \sin^2\alpha$$

$$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$$

d) $\cos\alpha(\tan\alpha + \cot\alpha) = \csc\alpha$

$$\cos\alpha\left(\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}\right) = \frac{1}{\sin\alpha}$$

$$\sin\alpha + \frac{\cos^2\alpha}{\sin\alpha} = \frac{1}{\sin\alpha} \quad / \cdot \sin\alpha$$

$$\Rightarrow \boxed{\sin^2\alpha + \cos^2\alpha = 1}$$