Rating Stablecoins

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Abstract

We construct a rating method to assess the quality of stablecoins. Our evaluation measures for rating include price stability, liquidity, and volatility. In order to normalize values derived from the different measures to the same scale, we use the max normalization and provide its axiomatic characterization. Then we aggregate the normalized scores to a comprehensive rating value by the geometric mean, following the recent idea of welfare economics. Among the stablecoins we examined, we find that USDC is the best stablecoin and that the ranking of market cap is not equal to the ranking of rating. Our method is used in the project of Chainsight, which indexes various types of information about crypto assets.

Keywords: Stablecoin, Rating, Cryptocurrency, Time-series analysis, Max normalization

JEL Codes: G0, D7, D6.

1 Introduction

Since the advent of Bitcoin (Nakamoto, 2008), one of the main raison d'être of cryptocurrencies has been their independence from legal tender controlled by central banks. However, this need for independence does not imply complete separation. Rather, since both cryptocurrencies and legal tender are assets, it is preferable that they can transfer value to each other through exchange. However, cryptocurrencies are issued on blockchains, while legal tender is not, creating a gap between the two types of assets. Stablecoins bridge this gap by being cryptocurrencies issued on blockchains that represent the value of legal tender.

Stablecoins are issued on blockchains and have mechanisms to represent the value of another asset or a specific benchmark.[1] These mechanisms typically involve collateral, though some rely on certain stability algorithms.[2] Collateral can be broadly categorized into fiat currency, cryptocurrencies, or commodities like gold. Algorithm-based stablecoins, on the other hand, use algorithms to adjust supply in response to demand, ensuring their price remains pegged to a specific benchmark. They all serve as an alternative or a more

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[1]See, Mita, Ito, Ohsawa and Tanaka (2019) and Ante, Fiedler, Willruth, and Steinmetz (2023) for a survey of stablecoins.

[2]Jarno and Kołodziejczyk (2021) classify stablecoins into four categories: tokenized funds, off-chain collateralized stablecoins, on-chain collateralized stablecoins, and algorithmic stablecoins. However, since our study does not analyze differences among these categories, we simply distinguish stablecoins as either collateralized or algorithmic.

stable medium of exchange for purchasing unpegged cryptocurrencies, which are known for their exceptionally high volatility (Panagiotidis, Papapanagiotou, and Stengos 2022). Furthermore, stablecoins have recently gained attention as a faster and more cost-effective alternative to traditional banking systems for international money transfers.[3] Vidal-Tomás (2023) discusses the importance of stablecoins as payment infrastructure.[4] For a more detailed discussion on the roles and types of stablecoins, see Catalini, de Gortari, and Shah (2022).

Due to the favorable characteristics, hundreds of stablecoins have been launched across various blockchains.[5] However, it is not yet clear which stablecoins are superior in what respects. To the best of our knowledge, no rigorous analysis has been conducted on rating methodologies for comprehensively evaluating stablecoins based on various characteristics. Korobova and Fantazzini (2025) examine the assessment of stablecoin credit risks, while Jarno and Kołodziejczyk (2021) investigate volatility measures of stablecoins. Hoang and Baur (2024) analyze the returns, volatility, and trading volumes of stablecoins using high-frequency data. Although these studies do not provide a comprehensive evaluation, their motivation for quantitatively assessing stablecoins due to their growing importance aligns with ours.

One might think that the quality of a stablecoin can be assessed by its market cap. However, market cap does not reflect price-related characteristics of stablecoins, such as the degree of price deviation from the target or liquidity. Instead, market cap serves as an evaluation measure of the stablecoin's usability and accessibility, reflecting its convenience. To provide a comprehensive evaluation, we propose a rating method that incorporates various characteristics of stablecoins. Our evaluation measures are *the average deviation of prices, price volatility, persistence of price deviations, liquidity*, and *market cap.* Each of these five evaluation measures will be scored, and an overall rating will be calculated by aggregating the scores. For aggregation, we use the geometric mean, based on arguments in welfare economics.

A key step in scoring is the normalization of values derived from different evaluation measures. We use the max normalization and provide its axiomatic characterization to clarify its rationale (Theorem 1). We then compute the rating values of several stablecoins, including major and minor ones.

Our computational results indicate that USDT ranks highest in both mean and median during the first half of our data measurement period, but drops to second in the second half, whereas USDC rises from third to first. BUSD and DAI complete the top four, with TUSD consistently in fifth place. BUSD's performance was often better than USDC and DAI in the first half, and this fact aligns, to some extent, with Duan and Urquhart's (2023) observation that BUSD outperforms DAI in price stability. USDT, USDC, BUSD, and TUSD are backed by fiat reserves, whereas DAI is backed by crypto assets with a stabilization mechanism. The ratings of algorithmic stablecoins, FRAX, MIM, OUSD, and FEI, remain relatively

[3]For instance, Elon Musk's SpaceX utilizes stablecoins for this purpose. This was reported by Silicon Valley venture capitalist Chamath Palihapitiya in his podcast program "All-In". See, for example, https://www.bitrue.com/blog/spacex-uses-stablecoins-for-hedging.

[4]Apart from stablecoins, central bank digital currencies (CBDCs) represent a new class of currency as digital versions of legal tender. If issued on a blockchain, they could partially fulfill the role currently played by stablecoins. However, even if a major country were to introduce a CBDC, it is unlikely to be implemented on a blockchain, as central banks have no incentive to adopt decentralized management. Consequently, stablecoins and CBDCs are expected to coexist as key digital currencies in the future. See, Guseva, Gazi, and Eakeley (2024) for the possibility of such coexistence. For analyses of CBDCs, see Choi and Kim (2024) and Dionysopoulos, Marra, and Urquhart (2024).

[5]For example, see, https://coinmarketcap.com/view/stablecoin/

low. This observation aligns with the findings of Jarno and Kołodziejczyk (2021), who reported that algorithmic stablecoins exhibit higher volatility than other types, and Ma and Wu (2020), who found that USD-pegged stablecoins tend to perform well. We also observed that BUSD's performance has been deteriorating rapidly since 2024. This decline is likely due to Binance ceasing its support for BUSD. These observations highlight the importance of analyzing long-term data when evaluating stablecoins, as their performance can fluctuate over time. The relative standing of stablecoins is not always stable, as they fluctuate over time, experiencing both rises and declines.

The remainder of this paper proceeds as follows. Section 2 introduces evaluation measures. Section 3 defines how to derive easy-to-compare scores for the evaluation measures and how to aggregate the scores into a rating value. Section 4 computes scores and rating values for several stablecoins. Section 5 discusses ways of normalization and briefly explains Theorem 1, which provides an axiomatic foundation of the max normalization. The conclusion is presented in Section 6. A formal mathematical argument for Theorem 1 is offered in Appendix 1, and how to handle exceptional trouble cases is explained in Appendix 2.

2 Evaluation measures

A *list* is a non-empty set of stablecoins S. Given any stablecoin $s \in S$, a time-series price vector of s is

$$x^s \equiv (x_1^s, x_2^s, \ldots, x_T^s) \in \mathbb{R}_+^T.$$

The set $D \equiv \{1, 2, \ldots, T\}$ is called the *data collection period.* As usual, the mean of the components of x^s is defined by

$$\bar{x}^s \equiv \frac{\sum_{t=1}^T x_t^s}{T}.$$

In most cases, $\bar{x}^s$ is very close to 1 because of the nature of stablecoins, such as $\bar{x}^s = 0.99438$ or 1.00003.

As mentioned in Section 1, market cap is often used to evaluate stablecoins, but it does not necessarily indicate their quality or similarity to the associated legal tender. We therefore introduce the following evaluation measures: *average deviation* evaluates the price proximity of each stablecoin with its target price; *volatility* evaluates price stability over time; *persistence* evaluates the tendency to retain the deviation from one in each period after the deviation; *liquidity* measures how easily a stablecoin can be exchanged for other assets; *market cap* reflects its usability and overall convenience. The evaluation measures are formally defined as below:

Average deviation:

$$\texttt{avedev}(x^s) \equiv \frac{1}{T} \sum_{t=1}^{T} |x_t^s - 1|.$$

Usually $\texttt{avedev}(x^s)$ takes a small positive value by nature of stablecoins. For example, the values are

$$(0.00051, 0.00037, 0.00314, 0.00057, 0.00175),$$

each of which is the average deviation of USDT, USDC, FRAX, DAI, and TUSD, respectively, dated at October 31, 2024.[6] In Section 4, we explain details of data and

[6]The values are calculated using the last 30 observations. This applies to the rest of the calculations as well. However, the 30-day period is arbitrary, and the user can choose a different number of days depending on the evaluation horizons.

computation.

Volatility:

$$\texttt{volat}(x^s) \equiv \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t^s - \bar{x}^s)^2}.$$

Usually $\texttt{volat}(x^s)$ also takes a small positive value by nature of stablecoins.[7] For example, the values are

$$(0.00076, 0.00064, 0.00109, 0.00082, 0.00174)$$

each of which is the volatility of USDT, USDC, FRAX, DAI, and TUSD, respectively, dated at October 31, 2024.

Persistence: To measure the persistence of price deviation, we use the absolute value of the first-order autocorrelation of price deviation.

$$\texttt{persis}(x^s) \equiv \left| \frac{\sum_{t=2}^{T}(x_t^s - 1)(x_{t-1}^s - 1)}{\sum_{t=1}^{T}(x_t^s - 1)^2} \right|.$$

It always holds that $\texttt{persis}(x^s) \in [0, 1]$ by the definition of persistence. For example, the values are

$$(0.26375, 0.21109, 0.89567, 0.21217, 0.67237),$$

each of which is the persistence of USDT, USDC, FRAX, DAI, and TUSD, respectively, dated at October 31, 2024.

Market cap: The market cap of s at $t \in D$ is denoted by $y_t^s > 0$. Let $y \equiv (y_t^s)_{t\in D}$. *The market cap* of s at D is then defined by

$$\texttt{mcap}(y^s) \equiv \frac{\sum_{t=1}^{T} y_t^s}{T}.$$

The value $\texttt{mcap}(y^s)$ can be any non-negative integer. For example, the values are

$$(1183.7, 349.79, 6.4739, 52.848, 4.9503) \times 10^8$$

each of which is the market cap of USDT, USDC, FRAX, DAI, and TUSD, respectively, dated at October 31, 2024.

Liquidity: The transaction volume of s at $t \in D$ is denoted by $w_t^s \geq 0$, and liquidity of s at $t \in D$ is defined by $z_t^s \equiv w_t^s / y_t^s \geq 0$.[8] *Liquidity* of s at D is then defined by

$$\texttt{liq}(z^s) \equiv \frac{\sum_{t=1}^{T} z_t^s}{T}.$$

The value $\texttt{liq}(z^s)$ can be any non-negative value. For example, the values are

$$(0.38425, 0.18688, 0.01943, 0.02104, 0.0464),$$

[7] Esparcia, Escribano, and Jareño (2024) argue that stablecoins are the most affected tokens after the FTX collapse. Their discussion shows the importance of the instability of stablecoin prices as expressed by volatility. Grobys, Junttila, Kolari, and Sapkota (2021) empirically analyze volatility spillover of stablecoins.

[8] Bakhtiar, Luo, and Adelopo (2023) interpret the transaction volume as a measure of a network effect.

each of which is the liquidity of USDT, USDC, FRAX, DAI, and TUSD, respectively, dated at October 31, 2024.

3 Scoring and rating

We use the terms *scoring* and *rating* with a clear distinction. *Scoring* refers to the normalized numeration of five evaluation measures, while *rating* refers to the aggregation of five scores into a single, comprehensive evaluation value. This distinction is effective in avoiding confusion between the two types of calculations.

Step 1. Logarithmic transformation

As the five evaluation measures are about different properties of stablecoins, we must normalize the values of the evaluation measures so that they take scores in the unit interval $[0, 1]$. We first illustrate our method using average deviation as an example.

We transform

$$\texttt{avedev}(x^s),$$

which is usually a very small value that is close to zero, into the negative of its logarithm with base 10,

$$-\log_{10} \texttt{avedev}(x^s), \tag{1}$$

meaning the number of decimal places where deviation from 1 is shown. For example, if $\texttt{avedev}(x^s) = 0.001$, then $\log_{10} \texttt{avedev}(x^s) = -3$, so that $-\log_{10} \texttt{avedev}(x^s) = 3$.

Note that the smaller the average deviation, volatility, or persistence, the more desirable it is. On the other hand, the larger the market cap and the liquidity, the more desirable it is. All in all, we transform the value of each evaluation measure as follows:

$$\begin{aligned}
-\log_{10} \texttt{avedev}(x^s),\\
-\log_{10} \texttt{volat}(x^s),\\
1 - \texttt{persis}(x^s),\\
\log_{10} \texttt{mcap}(y^s),\\
\log_{10} \texttt{liq}(1 + 100z^s).
\end{aligned}$$

Four important remarks are in order:

- **Reason for using $\log_{10}$:** Network effects play an important role in the adoption of currencies, including stablecoins. Thus, the values of the evaluation measures can differ significantly by a factor of 10^2 or 10^3, not just by a factor of 2 or 3, across various stablecoins. Therefore, to compare the values, it is more appropriate to use $\log_{10}$ to clearly express the difference among the values of various stablecoins.
- **Reason for $1 - \texttt{persis}(x^s)$:** The transformation of persistence is exceptional in that we do not apply a logarithmic transformation. This is because the definition of persistence is based on the fraction of two numbers with similar numbers of digits, eliminating the need for a logarithmic transformation. Since the value $\texttt{persis}(x^s)$ is more desirable the closer it is to zero, we simply transformed it into $1 - \texttt{persis}(x^s)$ in the score design.

- **Reason for $\log_{10}$ liq$(1 + 100z^s)$:** The value of z^s ranges from a number very close to 0 to a large positive number greater than 1, depending on s. To ensure the score does not take both negative and positive sign, we add 1 before applying the logarithm. However, this addition could have a significant impact because z^s is mostly below 1. To mitigate this effect, we scale z^s by multiplying it by 100 to express in percentage, resulting in $100z^s$. This is why we apply the logarithmic transformation to $1 + 100z^s$ instead of directly to z^s.

- **Robustness against manipulative self-transactions:** Some issuer of a stablecoin s may engage in numerous self-transactions, causing w^s to seemingly increase and $z^s = w^s/y^s$ to also seemingly increase. However, because of the logarithmic transformation, increasing $\texttt{liq}(1 + 100z^s)$ through such manipulative self-transactions incurs significant transaction costs, particularly as z^s becomes larger.

- **How to deal with cases where the logarithmic transformation does not work well:** If $x_t^s = 1$ for all $t \in T$, then $\texttt{avedev}(x^s) = 0$, so that $\log_{10} \texttt{avedev}(x^s)$ is not well-defined. It is hard to imagine that this case actually happens unless the size of T is too small, but whenever it occurs, we simply add a small $\varepsilon > 0$ to $\texttt{avedev}(x^s)$, so that $\log_{10}(\texttt{avedev}(x^s) + \varepsilon)$ is well-defined. The same problem theoretically occurs with volatility and persistence, too. Appendix 2.1 discusses this issue and other theoretically possible but unrealistic issues related to the logarithmic transformation.

Step 2. Max normalization to define scores

Note that

$$\max_{u \in S} - \log_{10} \texttt{avedev}(x^u)$$

denotes the maximum value of (1) among the stablecoins in S. Therefore,

$$\text{score}(\texttt{avedev}(x^s)) \equiv \frac{-\log_{10} \texttt{avedev}(x^s)}{\max_{u \in S} - \log_{10} \texttt{avedev}(x^u)} \tag{2}$$

denotes the normalized value of (1), which is between 0 and 1.

We call this normalization method the *max normalization*. We offer its axiomatic foundation in Section 5.1 and Appendix 1 to explain the rationale of using it. Other types of normalization are discussed in Sections 5.2 and 5.3.

By the max normalization, we define other scores similarly

$$\text{score}(\texttt{volat}(x^s)) \equiv \frac{-\log_{10} \texttt{volat}(x^s)}{\max_{u \in S} - \log_{10} \texttt{volat}(x^u)}, \tag{3}$$

$$\text{score}(\texttt{persis}(x^s)) \equiv \frac{1 - \texttt{persis}(x^s)}{\max_{u \in S} 1 - \texttt{persis}(x^u)}, \tag{4}$$

$$\text{score}(\texttt{mcap}(y^s)) \equiv \frac{\log_{10} \texttt{mcap}(y^s)}{\max_{u \in S} \log_{10} \texttt{mcap}(y^u)}, \tag{5}$$

$$\text{score}(\texttt{liq}(z^s)) \equiv \frac{\log_{10} \texttt{liq}(1 + 100z^s)}{\max_{u \in S} \log_{10} \texttt{liq}(1 + 100z^u)}. \tag{6}$$

We do not need "$-$" to define the scores of (5) and (6) because larger values of $\texttt{mcap}(y^s)$ and $\texttt{liq}(z^s)$ are more desirable, unlike (2), (3) and (4).

Step 3. Aggregation to define rating

When calculating a single rating based on multiple evaluation measures, the geometric mean has a beneficial property: a low score on one evaluation measure cannot easily be compensated by a high score on another. The United Nations Development Program calculates the human development index by income, life expectancy, and education levels. Earlier, they used the arithmetic mean to calculate the index; however, in 2010 they switched to using the geometric mean.[9] They made this change because they considered the three evaluation measures to be essential for human life so that none of them can be easily substituted by another one. Similarly, we consider that our five evaluation measures are essential for evaluating stablecoins and define our rating function by the geometric mean as follows:

$$\begin{aligned}\mathrm{rating}(x^s, y^s, z^s) \equiv \mathrm{score}(\texttt{avedev}(x^s))^{\frac{1}{5}} \cdot \mathrm{score}(\texttt{volat}(x^s))^{\frac{1}{5}} \cdot \mathrm{score}(\texttt{persis}(x^s))^{\frac{1}{5}} \cdot \\ \mathrm{score}(\texttt{mcap}(y^s))^{\frac{1}{5}} \cdot \mathrm{score}(\texttt{liq}(z^s))^{\frac{1}{5}}, \end{aligned} \tag{7}$$

which is between 0 and 1. In this definition, we assign all criteria an equal weight of 1/5, reflecting the idea that the evaluation measures are equally important.[10] As an example, Table 1 shows rating values on October 31, 2024.

USDT	USDC	FRAX	DAI	TUSD	ALUSD	USDP
0.867	0.884	0.435	0.688	0.555	0.195	0.717

GUSD	BUSD	LUSD	MIM	SUSD	OUSD	FEI
0.664	0.228	0.524	0.495	0.676	0.448	0.319

Table 1: Rating values on October 31, 2024

4 Computation

4.1 List of stablecoins

To analyze long-term trends in scores and ratings, we selected stablecoins listed on CoinGecko between January 1, 2022, and October 31, 2024, that had few data gaps during this period.[11] Due to the relatively short history of most stablecoins, only 14 met these criteria: USDT, USDC, FRAX, DAI, TUSD, ALUSD, USDP, GUSD, BUSD, LUSD, MIM, SUSD, OUSD, and FEI. Thus our list of stablecoins is

$$\begin{aligned}S = \{&\text{USDT, USDC, FRAX, DAI, TUSD, ALUSD, USDP,} \\ &\text{GUSD, BUSD, LUSD, MIM, SUSD, OUSD, FEI}\}.\end{aligned}$$

The way to create S depends on the purpose of the analysis. Our purpose here is to clarify the nature of the rating method we have constructed through the derivation of long-

[9] See, Klugman, Rodríguez and Choi (2011) for a review and Herrero, Martínez, and Villar (2010) and Kawada, Nakamura, and Otani (2019) for properties of the human development index.

[10] Although we have never observed in reality, it is theoretically possible that $a = \texttt{mcap}(y^s)$ is negative, so that $a^{\frac{1}{5}}$ is not well-defined as a real-value. Then we need to let $a = 0$, so that $a^{\frac{1}{5}} = 0$. This issue is explained in Appendix 2.2.

[11] `https://www.coingecko.com/en/categories/usd-stablecoin`

term trends, and to observe the rise and fall of stablecoins used by people over a certain long period of time. An advantage of the max normalization is that, while not perfectly, it ensures the resulting scores remain largely independent of the choice of S, provided that S includes major stablecoins that attain the maximum scores.

4.2 Data

Take any $s \in S$. We obtained daily price data x^s, daily market cap y^s, and daily transaction volume w^s from CoinGecko[12] for the data measurement period spanning January 1, 2022, to October 31, 2024. The period consists of 1035 days.

We set $T = 30$, meaning that we treat a month as a single unit of time. Naturally, this choice is not absolute. Such settings can be determined based on the analyst's objectives. For example, if a quarter is considered a single unit, T could be set to 90. However, we analyzed both the $T = 30$ and $T = 90$ cases and found little difference in the results. Therefore, we focus on discussing the results for the $T = 30$ case.

For any sequence

$$x^s = (x^s_{\text{Jan 1 in 2022}}, x^s_{\text{Jan 31 in 2022}}, \ldots, x^s_{\text{Oct 31 in 2024}}) \in \mathbb{R}^{1035},$$

and any $t \in \{\text{Jan 30 in 2022, Jan 31 in 2022}, \ldots, \text{Oct 31 in 2024}\}$, we write

$$x^s(t) = (x^s_{t-29}, x^s_{t-28}, \ldots, x^s_t) \in \mathbb{R}^{30},$$

where "$t - 29$" denotes 29 days before from time t and January 1 in 2022 is 29 days before from January 30 in 2022. The average deviation of s on January 30 in 2022 is given by

$$\texttt{avedev}(x^s(\text{Jan 30 in 2022})) = \frac{\sum_{t=\text{Jan 1 in 2022}}^{\text{Jan 30 in 2022}} x^s_t}{30}.$$

Following (2), the score of

$$\texttt{avedev}(x^s(t)) \text{ for } t \in \{\text{Jan 30 in 2022, Jan 31 in 2022}, \ldots, \text{Oct 31 in 2024}\}$$

is

$$\texttt{score}(\texttt{avedev}(x^s(t))) = \frac{-\log_{10}(\texttt{avedev}(x^s(t)))}{\max_{u \in S} -\log_{10}(\texttt{avedev}(x^u(t)))}.$$

Then the time-series scores of the average deviation for s are given by

$$\{\texttt{score}(\texttt{avedev}(x^s(t)))\}_{t=\text{Jan 30 in 2022}}^{\text{Oct 31 in 2024}}.$$

The scores of the average deviation for s on any other day, volatility, persistence, market cap, and liquidity for s on any day are all defined by the same way.

4.3 Results

The scores of the five evaluation measures dated from January 30 in 2022 to October 31 in 2024, are shown in Figures 1–5. All the detailed scores can be found on the Chainsight website.[13] To enhance visual clarity, the figures display line graphs only for USDT, USDC, FRAX, DAI, and TUSD. Among the set of so-called stablecoins, USDT, USDC, and DAI

[12] `https://www.coingecko.com/`
[13] `https://chainsight.network/`

are three representative stablecoins that consistently hold the top three positions in market cap, while FRAX and TUSD are examples of second-tier stablecoins.

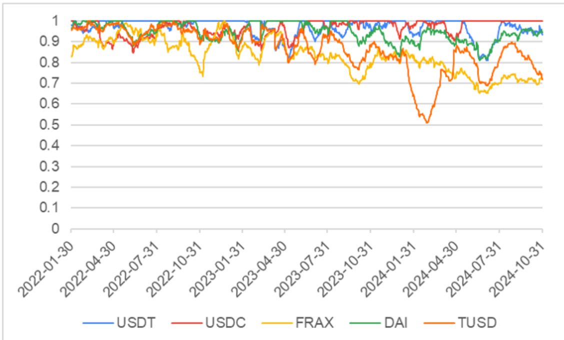

Figure 1: Scores of average deviation

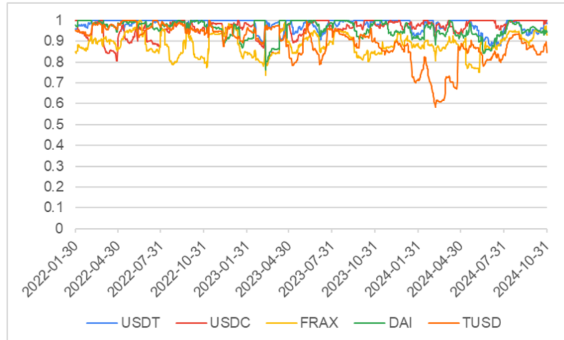

Figure 2: Scores of volatility

Persistence and liquidity scores fluctuate more than the other scores. This is because, by the nature of their definitions, theie values change greatly when the numerator and denominator change in opposite directions.

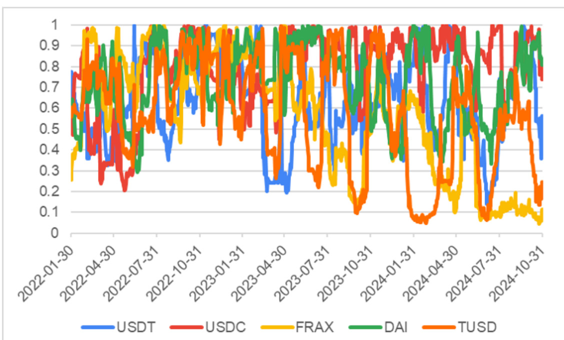

Figure 3: Scores of persistence

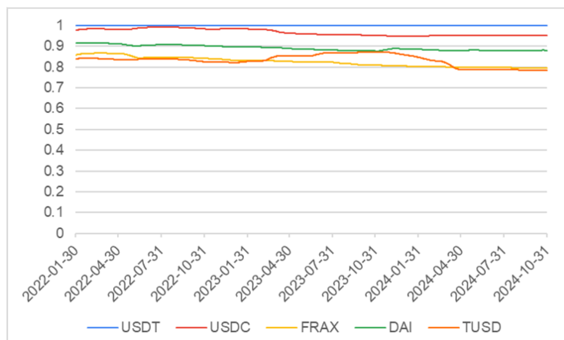

Figure 4: Scores of market cap

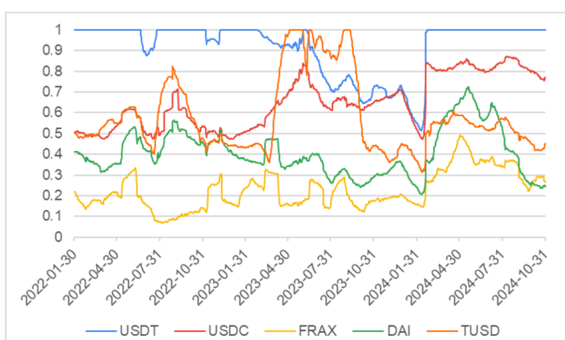

Figure 5: Scores of liquidity

The rating values and their rankings aggregated from the five scores are exhibited in Figures 6 and 7, while the mean and median of them are summarized in Table 2. The number of days each stablecoin ranked first is shown in Table 3. According to our observations and facts, we give some remarks. In the remarks, the "first half" refers to the initial 503 days out of the 1,006 days spanning from January 30, 2022, to October 31, 2024, while the "second half" refers to the following 503 days.

1. According to Table 2, USDT ranks first in both mean and median during the first half, whereas it ranks second in both mean and median during the second half. On the other hand, USDC ranks third in both mean and median during the first half, whereas it rises to first place in both mean and median during the second half. The remaining top four positions are occupied by BUSD and DAI, while TUSD consistently ranks fifth in both mean and median across both halves. We remark that USDT, USDC, BUSD, and TUSD are backed by fiat reserves, while DAI is backed by crypto assets with a price stabilization mechanism. The following point should be noted regarding BUSD.

2. BUSD is a stablecoin issued by Paxos and was supported by Binance. However, due to regulatory reasons, its usage has declined since Binance announced the suspension of BUSD support in September 2023. Figures 8 and 9 clearly illustrate that BUSD's rating has been rapidly declining since 2024. In fact, Table 3 shows that BUSD was ranked first on many days, but it has not held the top position since February 10, 2024.

3. According to Table 2, the ratings of algorithmic stablecoins, FRAX, MIM, OUSD, and FEI are relatively low, as none of their means or medians exceed 0.6, except for FRAX during the first half. This observation is consistent with the finding of Jarno and Kołodziejczyk (2021), who reported that algorithmic stablecoins exhibit higher volatility than other types.

4. Regarding Figures 6 and 7, the impact of the March 10, 2023, Silicon Valley Bank shock temporarily lowered USDC's rating, primarily due to increases in average deviation, volatility, and persistence. However, the swift announcement of deposit protection by the U.S. government restored confidence, enabling the rating to quickly return to its original level within a short period.

5. Although Figure 4 shows the market caps of USDT and USDC are almost always ranked first and second, respectively, during the data measurement period, Figures 6 and 7 indicate their ratings are not so. Market cap often garners the most attention, but our ratings, which consider various evaluation measures, suggest that market cap alone should not be the sole focus.

6. Over the 1,006-day period from January 30, 2022, to October 31, 2024, USDT had a higher rating than USDC on 554 days (55.1%). However, as Figure 10 shows, USDC has increasingly outperformed USDT in rating over time. This trend aligns with the fact that, as shown in Table 2, the USDT rating was higher than the USDC rating in both mean and median during the first half, whereas the situation reversed in the second half.

7. According to Table 3, out of the 1,006 days from January 30, 2022, to October 31, 2024, USDC ranked first for 418 days, BUSD for 301 days, and USDT for 270 days. However, among the 301 days of BUSD, only 10 days fall within 2024, due to Binance's suspension of support.

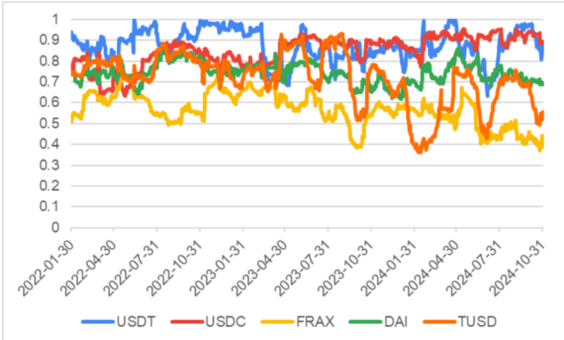

Figure 6: Rating

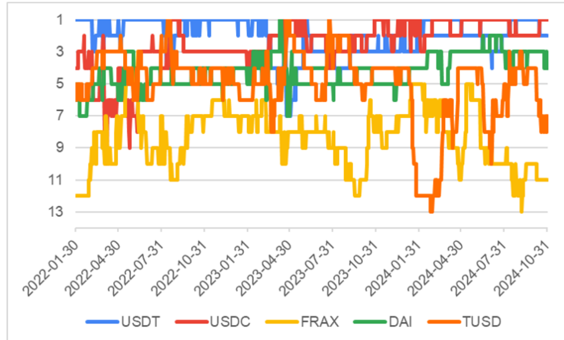

Figure 7: Ranking of rating

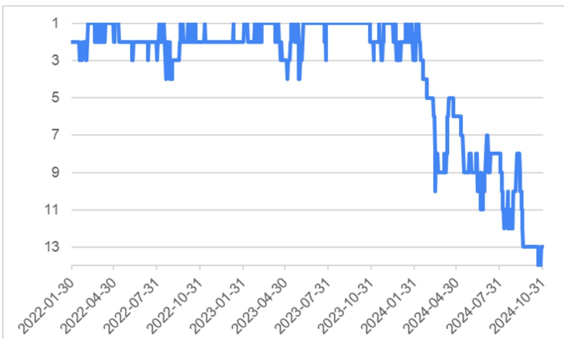

Figure 8: BUSD rating

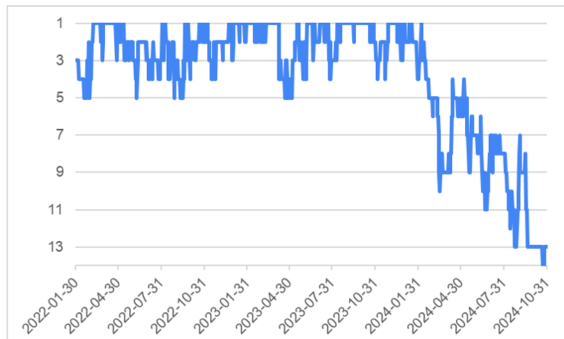

Figure 9: Ranking of BUSD rating

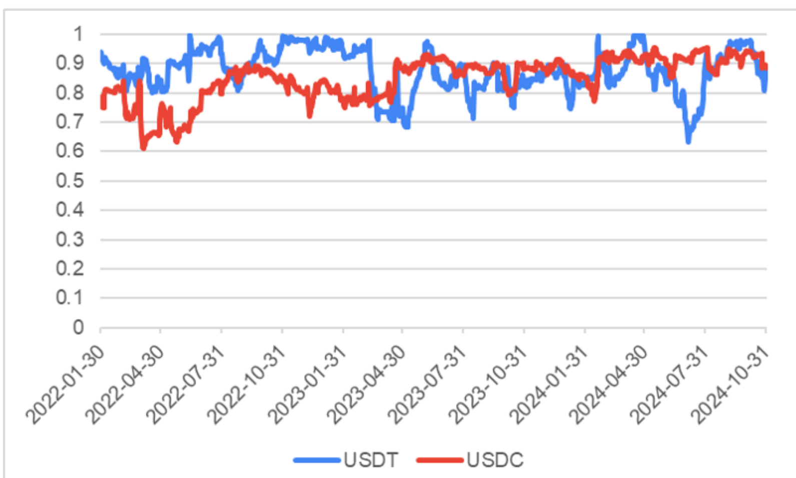

Figure 10: Over time, USDC has increasingly outperformed USDT in rating

Mean	USDT	USDC	FRAX	DAI	TUSD	ALUSD	USDP
First half	0.893	0.8	0.615	0.762	0.778	0.405	0.611
Second half	0.858	0.897	0.521	0.725	0.65	0.233	0.635
Total	0.875	0.849	0.568	0.743	0.714	0.319	0.623
Mean	**GUSD**	**BUSD**	**LUSD**	**MIM**	**SUSD**	**OUSD**	**FEI**
First half	0.537	0.873	0.416	0.578	0.637	0.512	0.535
Second half	0.536	0.691	0.481	0.451	0.566	0.495	0.319
Total	0.536	0.782	0.448	0.514	0.602	0.504	0.426
Median	**USDT**	**USDC**	**FRAX**	**DAI**	**TUSD**	**ALUSD**	**USDP**
First half	0.908	0.807	0.623	0.76	0.776	0.39	0.619
Second half	0.856	0.902	0.536	0.72	0.68	0.216	0.629
Total	0.875	0.87	0.572	0.743	0.73	0.282	0.625
Median	**GUSD**	**BUSD**	**LUSD**	**MIM**	**SUSD**	**OUSD**	**FEI**
First half	0.536	0.87	0.411	0.598	0.649	0.516	0.565
Second half	0.547	0.71	0.481	0.451	0.551	0.497	0.337
Total	0.542	0.857	0.454	0.504	0.628	0.505	0.37

Table 2: Mean and median of rating values dated from January 30, 2022 to October 31, 2024

	USDT	USDC	FRAX	DAI	TUSD	ALUSD	USDP
first half	308	45	0	4	11	0	0
second half	110	225	0	0	2	0	0
total	418	270	0	4	13	0	0
	GUSD	**BUSD**	**LUSD**	**MIM**	**SUSD**	**OUSD**	**FEI**
first half	0	135	0	0	0	0	0
second half	0	166	0	0	0	0	0
total	0	301	0	0	0	0	0

Table 3: The number of days when each stablecoin is ranked first among 1,006 days

5 Discussions on normalization

5.1 Rationale of the max normalization

The *max normalization* can be written as a function f that maps each non-negative vector $a = (a_1, \dots, a_k)$ to the normalized vector $f(a) = (f_1(a), \dots, f_k(a))$ such that

$$f_j(a) = \frac{a_j}{\max_{i \in \{1,\dots,k\}} a_i} \text{ for all } j \in S.$$

The max normalization satisfies several desirable properties as normalization functions, which offer the rationale of using it.

The first property is *ratio preservation*, stating that the ratio of differences of inputs is preserved under normalization: for each distinct a_i, a_j, a_k with $a_k \neq a_i, a_j$,

$$\frac{f_k(a) - f_i(a)}{a_k - a_i} = \frac{f_k(a) - f_j(a)}{a_k - a_j}.$$

The second property is *independence of null objects*, stating that adding an extremely low-quality stablecoin k whose unnormalized score a_k is zero to the list does not affect the normalized value of other stablecoins, and the normalized value of a_k is zero: for $a = (a_1, \dots, a_k, a_{k+1})$ with $a_{k+1} = 0$,

$$\begin{aligned} f_j(a) &= f_j(a_1, \dots, a_k) \text{ for all } j = 1, \dots, k, \\ f_{k+1}(a) &= 0. \end{aligned}$$

The third property is the *max-one property*, stating that the value of the stablecoin with the highest parameter is given one:

$$a_j = \max_{i \in \{1,\dots,k\}} a_i \Longrightarrow f_j(a) = 1.$$

The max normalization satisfies *ratio preservation*, *independence of null coins*, and the *max-one property*. In fact, we can establish a much stronger result that the max normalization is the *only* function satisfying the three properties. To prove this result, we need to define the properties more rigorously in a formal model. This will be done in the Appendix 1, but we state the result below for reference:

Theorem 1 (Informal statement). *The max normalization is the only function that satisfies ratio preservation, independence of null objects, and the max-one property.*

5.2 Alternative normalization: ratio to dynamic maximum

For each evaluation measure, the max normalization gives the value 1 to the stablecoin with the maximum value of the evaluation measure during the data collection period. Consequently, even when $f(x^s)$ reduces for all stablecoins through multiple data collection periods, the stablecoin with the maximum value at each data collection period is assigned the value 1. That is, our definition of score($\texttt{avedev}(x^s)$) does not capture such dynamic changes. Below we offer an alternative normalization that captures such changes.

Recall that the data collection period of time-series price data $x^s = (x^s_1, \dots, x^s_T)$ is defined as

$$D = \{1, \dots, T\} \text{ with } T \geq 1.$$

Let $D(1) \equiv D$. There can be "past" data collection periods

$$D(2), \ldots, D(K) \subset \{T-1, T-2, \ldots, 1, 0, -1, -2, -3, \ldots\}$$

and associated data

$$(x_t^s)_{t \in D(2)}, \ldots, (x_t^s)_{t \in D(K)}.$$

Then

$$\max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)}). \tag{8}$$

is the dynamically maximum value of $-\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)})$ among all data collection periods $D(1), \ldots, D(K)$ for s, and

$$\max_{s \in S} \max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)}).$$

is the maximum value of (8) among the stablecoins in S. Then,

$$\frac{-\log_{10} \texttt{avedev}(x^s)}{\max_{s \in S} \max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)})} \tag{9}$$

is the dynamically normalized score of x^s on the average deviation, which is between 0 and 1. The dynamic normalization of (9) is useful to capture time-series changes in values; however, we use the static normalization of (2) because we are still unclear about how values (9) and (2) can differ, and (2) is easier to use as there is no problem in choosing $D(2), ..., D(K)$. However, in the future with more data and experience, we may shift from static to dynamic normalization or something similar.

5.3 Alternative normalization: quantile

For any $s \in S$, the following set denotes the set of stablecoins whose average deviation is weakly greater than that of s:

$$\{u \in S : \texttt{avedev}(x^u) \geq \texttt{avedev}(x^s)\}.$$

Then

$$\frac{|\{u \in S : \texttt{avedev}(x^u) \geq \texttt{avedev}(x^s)\}|}{|S|} \tag{10}$$

denotes the quantile of s in the decreasing order. This value falls in the range $[0, 1]$, so quantile is also a method of normalization. However, there is a fundamental problem of appropriately selecting a list S. Perhaps we can easily agree that S should contain major stablecoins such as USDT or USDC, but we cannot do it for hundreds of minor stablecoins. Indeed, the boundary of stablecoins is not clear because there are multiple types of peg mechanisms and management organizations.[14] However, the value of (10) highly depends on the choice of S, and hence we do not use this quantile normalization.

[14] For example, whether TerraUSD (UST) on the Terra blockchain is a stablecoin has been debatable because of its exotic peg mechanism, until the collapse of LUNA in May 2022. See, Briola, Vidal-Tomás, Wang, and Aste (2023).

6 Conclusion

We have developed a method for rating stablecoins based on key evaluation measures, including average deviation, volatility, persistence, market capitalization, and liquidity. To ensure comparability, we transformed these values into standardized scores using logarithmic transformation and max normalization. Rating values were then computed by taking the geometric mean of these scores, following recent approaches in welfare economics. Additionally, we established an axiomatic characterization of max normalization to clarify the rationale for its use.

Our analysis identified USDC as the best-performing stablecoin among those examined. Notably, BUSD performed significantly better than many might have expected until Binance suspended its support. Our findings indicate that market capitalization does not necessarily align with rating, underscoring the importance of a more comprehensive evaluation beyond market size alone. To accurately assess the quality of a stablecoin, it is essential to focus on its fundamental evaluation measures rather than relying solely on market capitalization.

To the best of our knowledge, this study represents the first academically rigorous attempt to rate stablecoins. We hope that our work will contribute to further research aimed at assessing the quality of stablecoins and advancing the understanding of their stability and reliability.

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Appendix 1. Axiomatic characterization of the max normalization

Since the choice of the list S of stablecoins is controversial, we introduce a model with variable lists. The idea of our modelling comes from the theory of axiomatic resource allocation (e.g., Thomson 2003, 2019).

Let $\mathbb{N} = \{1, 2, \ldots\}$ be the set of infinitely many *objects*. A finite set $\hat{S} \subset \mathbb{N}$ is the set of *central objects*, which can be empty or non-empty. A *list* is a non-empty finite set S with $\hat{S} \subset S \subset \mathbb{N}$.

One can interpret that $\mathbb{N}$ is the set of various cryptocurrencies, $\hat{S}$ is the set of cryptocurrencies that everyone recognize as stablecoin such as USDC and USDT, and S is a set of cryptocurrencies that someone may recognize as stablecoin. A positive integer $M \geq 1$ indicates the minimum size of possible S, which can be chosen arbitrary.

Let $\mathscr{S}(M, \hat{S})$ be the set of lists whose size is at least M and contains $\hat{S}$; that is,

$$\mathscr{S}(M, \hat{S}) \equiv \{S \in 2^{\mathbb{N}} \setminus \{\emptyset\} : \hat{S} \subset S \text{ and } M \leq |S| < +\infty\}.$$

Our modelling with $\mathscr{S}(M, \hat{S})$ allows us to simultaneously work with any consideration on $\hat{S}$ and M.

Hereafter, for brevity of notation, we denote $\mathscr{S}(M, \hat{S})$ by $\mathscr{S}$ when there is no danger of confusion. For each $S \in \mathscr{S}$, let

$$D_S \equiv \mathbb{R}^S_+ \setminus \{\mathbf{0}\}$$

be the set of non-negative vectors $a = (a_i)_{i \in S} \in \mathbb{R}^S_+$ satisfying $a_j > 0$ for some $j \in S$, where the uninteresting vector of parameters $\mathbf{0} = (0, 0, \ldots, 0) \in \mathbb{R}^S_+$ is excluded. A *normalization function* on $\mathscr{S}$ is a function

$$f : \bigcup_{S \in \mathscr{S}} D_S \to \bigcup_{S \in \mathscr{S}} [0, 1]^S$$

such that for each $S \in \mathscr{S}$ and each vector of parameters $a \in D_S$, $f(a) = (f_i(a))_{i \in S} \in [0, 1]^S$. The *max normalization* on $\mathscr{S}$ is the normalization function f on $\mathscr{S}$ such that for each $S \in \mathscr{S}$ and each $a \in D_S$,

$$f_j(a) = \frac{a_j}{\max_{i \in S} a_i} \text{ for all } j \in S.$$

We define three axioms of normalization functions on $\mathscr{S}$ satisfied by the max normalization. The interpretation of the axioms has already been explained in Section 5.1.

Axiom 1 (Ratio preservation). *For each $S \in \mathscr{S}$, each $a \in D_S$, and each $i, j, k \in S$, whenever $a_k \neq a_i, a_j$,*

$$\frac{f_k(a) - f_j(a)}{a_k - a_j} = \frac{f_k(a) - f_i(a)}{a_k - a_i}.$$

Axiom 2 (Independence of null objects). *For each $S \in \mathscr{S}$, each $\ell \in \mathbb{N} \setminus S$, and each $a \in D_{S \cup \{\ell\}}$ with $a_\ell = 0$,*

$$\begin{aligned} &f_j(a) = f_j((a_i)_{i \in S}) \textit{ for all } j \in S, \\ &f_\ell(a) = 0. \end{aligned}$$

The idea of *independence of null objects* comes from "null claims consistency" in the literature of claims problem (see, Yeh 2008).

Axiom 3 (Max-one property). *For each $S \in \mathscr{S}$ and each $j \in S$, if $a_j = \max_{i\in S} a_i$, then $f_j(a) = 1$.*

We are now in a position to state and prove our axiomatization theorem.

Theorem 1 (Formal statement). *Let $\hat{S} \subset \mathbb{N}$ be any (possibly empty) set of central objects and let $M \geq 1$ be any positive integer. The max normalization is the only normalization function on $\mathscr{S}(M, \hat{S})$ that satisfies ratio preservation, independence of null objects, and the max-one property.*

Proof. Since it is trivial that the max normalization satisfies *ratio preservation*, *independence of null objects*, and *the max-one property*, we only prove the converse statement. Let f be any normalization function on $\mathscr{S}(M, \hat{S})$ satisfying the three axioms. We shall show that f is in fact the max normalization; that is, for each $S \in \mathscr{S}$ and each $a \in D_S$, $f_j(a) = \frac{a_j}{\max_{i\in S} a_i}$ for all $j \in S$.

Case 1. Any $\boldsymbol{S \in \mathscr{S}}$ and $\boldsymbol{a \in D_S}$ satisfying $\boldsymbol{\max_{i\in S} a_i = \min_{i\in S} a_i}$: In this case, by the *max-one property*,

$$f_j(a) = 1 = \frac{a_j}{a_j} = \frac{a_j}{\max_{i\in S} a_i} \text{ for all } j \in S,$$

as desired.

Case 2. Any $\boldsymbol{S \in \mathscr{S}}$ and $\boldsymbol{a \in D_S}$ satisfying $\boldsymbol{\max_{i\in S} a_i > \min_{i\in S} a_i}$:

Step 1. We shall show that there exists a real-value $g(S, a) \in \mathbb{R}$ such that

$$f_i(a) - g(S, a) \cdot a_i = f_j(a) - g(S, a) \cdot a_j \text{ for all } i, j \in S. \tag{11}$$

Let $k, \ell \in S$ be $a_k = \max_{i\in S} a_i$ and $a_\ell = \min_{i\in S} a_i$. Define

$$g(S, a) \equiv \frac{f_k(a) - f_\ell(a)}{a_k - a_\ell}. \tag{12}$$

We separately analyze several types of $j \in S$ below:

- For each $j \in S$ with $j \neq k$ and $a_j = a_k$, by the *max-one property*, $f_j(a) = 1 = f_k(a)$. Therefore,

$$f_j(a) - g(S, a) \cdot a_j = f_k(a) - g(S, a) \cdot a_k. \tag{13}$$

- For each $j \in S$ with $j \neq \ell$ and $a_j < a_k$, by *ratio preservation* and (12),

$$\frac{f_k(a) - f_j(a)}{a_k - a_j} = \frac{f_k(a) - f_\ell(a)}{a_k - a_\ell} = g(S, a),$$

so that

$$f_j(a) - g(S, a) \cdot a_j = f_k(a) - g(S, a) \cdot a_k. \tag{14}$$

- For ℓ, by (12),

$$f_\ell(a) - g(S, a) \cdot a_\ell = f_k(a) - g(S, a) \cdot a_k. \tag{15}$$

By (13), (14), and (15), we have (11).

Step 2: By Step 1, there exists a real-value $h(S,a) \in \mathbb{R}$ such that

$$h(S,a) = f_j(a) - g(S,a) \cdot a_j \text{ for all } j \in S. \tag{16}$$

Note that

$$f_j(a) = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S. \tag{17}$$

In the next step, we shall clarify the forms of g and h.

Step 3: Take any $\ell \in \mathbb{N} \setminus \{S\}$. Let $a_\ell \equiv 0$. Note $S \cup \{\ell\} \in \mathscr{S}$ and $(a, a_\ell) \in D_{S\cup\{\ell\}}$. Since $\max_{i\in S\cup\{\ell\}} a_i > \min_{i\in S\cup\{\ell\}} a_i$, by the same argument as Steps 1 and 2, there exist real-values $h(S \cup \{\ell\}, (a, a_\ell))$ and $g(S \cup \{\ell\}, a)$ such that

$$f_j(a, a_\ell) = h(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j + g(S \cup \{\ell\}, (a, a_\ell)) \text{ for all } j \in S \cup \{\ell\}.$$

By *independence of null objects*,

$$\begin{aligned} &f_j(a, a_\ell) = f_j(a) \text{ for all } j \in S, \\ &f_\ell(a, a_\ell) = 0, \end{aligned}$$

so that

$$\begin{gathered} g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j + h(S \cup \{\ell\}, (a, a_\ell)) = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S, \\ h(S \cup \{\ell\}, (a, a_\ell)) = 0, \end{gathered} \tag{18}$$

which imply

$$g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S. \tag{19}$$

Let $k \in S \cup \{\ell\}$ be such that $a_k = \max_{i\in S\cup\{\ell\}} a_i$. Since $a_\ell = 0$, we have $k \in S$, and so $a_k = \max_{i\in S} a_i$ as well.

By the *max-one property* and (18),

$$1 = f_k(a, a_\ell) = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_k + h(S \cup \{\ell\}, (a, a_\ell)) = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_k,$$

so that

$$g(S \cup \{\ell\}, (a, a_\ell)) = \frac{1}{a_k}. \tag{20}$$

Therefore by the fact $a_k = \max_{i\in S} a_i$, (20), and (19),

$$\frac{a_j}{\max_{i\in S} a_i} = \frac{a_j}{a_k} = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j = g(S,a) \cdot a_j + h(S,a) = f_j(a) \text{ for all } j \in S,$$

as desired. □

The axioms in Theorem 1 are tight; that is, if any one of them is dropped, then the axiomatic characterization is no longer valid.

- Dropping *ratio preservation*: Let f be the "quantile among positive values" such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$
\begin{aligned}
f_j(a) &= \frac{|\{i \in S : a_j \geq a_i > 0\}|}{|\{i \in S : a_i > 0\}|} \text{ if } a_j > 0, \\
f_j(a) &= 0 \text{ if } a_j = 0.
\end{aligned}
$$

  This f satisfies all the axioms except *ratio preservation*.

- Dropping *independence of null objects*: Let f be the "min-max normalization" such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$
f_j(a) = \frac{a_j - \min_{i \in S} a_i}{\max_{i \in S} a_i - \min_{i \in S} a_i}.
$$

  This f satisfies all the axioms except *independence of null objects*.

- Dropping *the max-one property*: Let f be a variant of the max normalization such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$
f_j(a) = q(\sum_{i \in S} a_i) \cdot \frac{a_j}{\max_{i \in S} a_i},
$$

  where $q : \mathbb{R}_{++} \to [0,1]$ is any function. Unless $q(\sum_{i \in S} a_i) = 1$ for all $\sum_{i \in S} a_i$, this f satisfies all the axioms except *the max-one property*. A simplest case is a constant function such as $q(\sum_{i \in S} a_i) = 0$ for all $\sum_{i \in S} a_i$.

  This example also suggests that *ratio preservation* and *independence of null objects* do not together imply any well-behaved form of f, since q is allowed to be very strange. For example, consider the following case:

$$
\begin{aligned}
q(\sum_{i \in S} a_i) &= 1 \text{ if } \sum_{i \in S} a_i \text{ is rational,} \\
&= 0 \text{ if } \sum_{i \in S} a_i \text{ is irrational.}
\end{aligned}
$$

  The normalization function f seems pathological in this case, but satisfies *ratio preservation* and *independence of null objects*. One may consider that *ratio preservation* is so strong that it implies certain linearity of f, but this f is even discontinuous everywhere and is not Lebesgue measurable on every D_S.

Finally, we remark that Theorem 1 provides a new axiomatization of a certain proportional rule in a new model. For studies on various proportional rules in other models, see, Ju, Miyagawa, and Sakai (2007).

Appendix 2 How to handle exceptional trouble cases

A2.1 Logarithmic transformation

Since our scoring method depends on the logarithmic transformation, it may happen that a value cannot be nicely transformed or be defined. Although such cases can rarely happen, we define how to deal with them.

- Whenever the average deviation, volatility, or persistence equals zero, their logarithmic transformation is undefined, posing a challenge in defining scores. To address this, consider the average deviation as an example: if $\texttt{avedev}(x^p) = 0$ for some $p \in S$, we transform x^s for each $s \in S$ using:

$$\log_{10}\left[\texttt{avedev}(x^s) + 0.1\right].$$

  This transformation allows us to proceed with calculating all scores as usual. The same approach is applied to volatility and persistence when necessary.

- Whenever $\texttt{mcap}(y^p) < 10$, its logarithmic transformation can be negative or undefined, which causes a trouble to define scores. Cases like this may only happen with very minor stablecoins for a short period of time, but here we define how to deal with it: if $\texttt{mcap}(y^s) < 10$ for some $s \in S$, then we simply let

$$\texttt{mcap}(y^s) = 10,$$

  so that $\log_{10}\texttt{mcap}(y^s) = 1$, and we calculate all scores as usual.

We do the same transformation to the number of active addresses and transaction volume.

A2.2 Negative scores

It is logically possible that $\texttt{avedev}(x^s) > 1$, although this may happen in unrealistic situations like $[x_t^s > 2 \text{ for all } t \in T]$. In this case,

$$\log_{10}\texttt{avedev}(x^s) > 0,$$

so that the score of average deviation becomes negative:

$$\texttt{score}(\texttt{avedev}(x^s)) = \frac{-\log_{10}\texttt{avedev}(x^s)}{\max_{u\in S} -\log_{10}\texttt{avedev}(x^u)} < 0.$$

This negative score itself is not problematic, but the derived property that

$$\texttt{score}(\texttt{avedev}(x^s))^{\frac{1}{5}}$$

is an imaginary number is problematic, since our rating function (7) is not well-defined as a real-valued function then. However, $\texttt{avedev}(x^s) > 1$ implies that the price of s is greater than 2 on mean over the data collection period, and this situation almost never happens with stablecoins. In the unlikely event that it happens, we will simply replace $\texttt{score}(\texttt{avedev}(x^s))$ with zero or a very small positive value, or we may simply remove s from the list S because such an excessively strange stablecoin is not worthy of rating. This negativity problem can theoretically occur for average deviation and volatility, but it is natural to consider that it never happens in reality because of the nature of stablecoins.