Rating Stablecoins: An Approach by Chainsight*

Shumpei Koike†, Tatsuyoshi Okimoto‡, and Toyotaka Sakai§

May 2, 2024

Abstract

We construct a rating methodology to assess the quality of stablecoins. The rating criteria are based on proximity to legal tender, including price stability, liquidity, and network effects. In order to convert values derived from different rating criteria to the same scale, we use the max normalization and give its axiomatic characterization to clarify the rationale of using it. Based on the idea of welfare economics, we define a rating function that gives each stablecoin a rating value from 0 to 1. Rating values are computed for several stablecoins.

Keywords: Stablecoin, Rating, Cryptocurrency, Time-series analysis, Welfare economics

1 Introduction

Since the advent of Bitcoin (Nakamoto 2009), one of main raison d'être of cryptocurrencies has been independence of any legal tender controlled by the central bank. However, independence does not imply separation. The value of cryptocurrencies and legal tender can be converted to each other through exchange. A special characteristic of cryptocurrencies is that they can only be transferred on the blockchain that issues them. Stablecoins overcome this problem.

Any stablecoin is a cryptocurrency issued on a blockchain and pegged with some legal tender. For example, USDC is a cryptocurrency issued on various blockchains and is pegged with the United States Dollar (USD). In this way, USDC on a blockchain smoothly links the value system of USD with the value system on the blockchain. Because of such importance, hundreds of stablecoins have been issued on a variety of blockchains.[1] However, it is not yet clear which stablecoins are superior in what respects. In fact, as far as we know, there does not seem to be rigorous discussion about measures to evaluate stablecoins.

One might think that the issuance volume and market capitalization are the most important for stablecoins. However, these values do not indicate how accurately any stablecoin reflects the nature of legal tender. In fact, neither of them represents the core properties of stablecoins, such as the degree of price deviation from legal tender, the speed with which the deviation shrinks, liquidity, or network effects on a particular blockchain. Given this situation, we shall establish a method to assess qualities of stablecoins based on the core

*This white paper is published by Chainsight, which indexes on-chain data and makes it widely available. `https://chainsight.network/`

†CTO, Chainsight. Correspondence: `hawk@chainsight.network`

‡Professor of Economics, Keio University and Economist, Chainsight

§Professor of Economics, Keio University and Economist, Chainsight

[1] For example, see, `https://coinmarketcap.com/view/stablecoin/`

properties. Specifically, we introduce several measures for assessment and a rating function that aggregates the scores for the measures. The rating function is constructed based on recent arguments of welfare economics. A key step in constructing the rating function is the normalization of values of different measures. We use the max normalization and provide its axiomatic characterization that clarifies the rational of using it (Theorem 1). We then compute the rating values of several stablecoins, including major and minor ones, by using the rating function. A quantitative comparison is essential for users to make informed choices and for service providers to engage in healthy competition. Our method contributes to facilitate this comparison.

Section 2 introduces evaluation measures. Section 3 defines how to derive easy-to-compare scores for the criteria and how to aggregate the scores into a rating value. Section 4 provides remarks on our scoring and rating as well as alternative methods of normalization. Section 5 computes scores and rating values for several stablecoins. Conclusions are provided in Section 6. A formal mathematical argument for Theorem 1 is summarized in the Appendix.

2 Evaluation measures

A *list* is a non-empty set of stablecoins S. Given any stablecoin $s \in S$, a time-series price vector of s is

$$x^s \equiv (x_1^s, x_2^s, \ldots, x_T^s) \in \mathbb{R}_+^T.$$

The set $D \equiv \{1, 2, \ldots, T\}$ is called the *data collection period.* As usual, the mean of the components of x^s is defined by

$$\bar{x^s} \equiv \frac{\sum_{t=1}^T x_t^s}{T}.$$

In many cases, $\bar{x^s}$ tends to 1 because of the nature of stablecoins, such as $\bar{x^s} = 0.9992$ or 0.998.

As we noted in Section 1, although issuance volume and market capitalization are often used to evaluate stablecoins, they do not necessarily represent how well a given stablecoin mimics legal tender. We therefore introduce the following evaluation measures: *average deviation* evaluates the price proximity of each stablecoin with its associated legal tender; *variance* evaluates price stability over time; *auto-correlation* evaluates how the rate at which prices return to 1 after deviating it; *DEX liquidity* evaluates an exchange liquidity in DEX; *Number of active addresses* and *transaction volume* represent the strength of network effects, which is a key factor of user convenience.[2] The evaluation measures are formally defined as below:

Average deviation:

$$\texttt{avedev}(x^s) \equiv \frac{1}{T} \sum_{t=1}^{T} |x_t^s - 1|.$$

Variance:

$$\texttt{var}(x^s) \equiv \frac{1}{T} \sum_{t=1}^{T} |x_t^s - \bar{x^s}|^2.$$

[2]Bakhtiar, Luo, and Adelopo (2023) interpret *the number of active addresses* and *transaction value* as measures of the strength of the network effect.

Auto-correlation:

$$\texttt{autcor}(x^s) \equiv \left| \frac{\sum_{t=2}^{T}(x_t^s - 1)(x_{t-1}^s - 1)}{\sum_{t=1}^{T}(x_t^s - 1)^2} \right|.$$

DEX liquidity: To measure the liquidity of a stablecoin s on a blockchain, we use the amount of s paired with the native currency of the blockchain in the representative DEX pool for swapping.[3] We define *DEX liquidity* of s at $t \in D$ by this amount of s at t, which is denoted by $y_t^s \geq 0$. Similarly, *DEX liquidity* of s at D is given by

$$\texttt{dexliq}(y^s) \equiv \frac{\sum_{t=1}^{T} y_t^s}{T}.$$

Number of active addresses The number of addresses that have transacted more than a certain amount of s at $t \in D$ is denoted by $z_t^s \geq 0$. Let $z \equiv (z_t^s)_{t \in D}$. *The number of active addresses* of s at D is then defined by

$$\texttt{address}(z^s) \equiv \frac{\sum_{t=1}^{T} z_t^s}{T}.$$

Transaction volume The transaction volume of s at $t \in D$ is denoted by $w_t^s \geq 0$. Let $w \equiv (w_t^s)_{t \in D}$. *The transaction volume* of s at D is then defined by

$$\texttt{txvol}(w^s) \equiv \frac{\sum_{t=1}^{T} w_t^s}{T}.$$

3 Scoring and rating

3.1 Logarithmic transformation to compare values

As the six evaluation measures are about different properties of stablecoins, we must normalize the values of the criteria so that they take scores in the unit interval $[0, 1]$. We first explain our method for average deviations.

We transform

$$\texttt{avedev}(x^s)$$

into the negative of its logarithm with base 10,

$$-\log_{10} \texttt{avedev}(x^s), \tag{1}$$

meaning the number of decimal places where deviation from 1 is shown. For example, if $\texttt{avedev}(x^s) = 0.001$, then $\log_{10} \texttt{avedev}(x^s) = -3$, so that $-\log_{10} \texttt{avedev}(x^s) = 3$. Note that the smaller the average deviation, the more desirable it is. Therefore, we consider $-\log_{10} \texttt{avedev}(x^s)$ instead of $\log_{10} \texttt{stdev}(x^s)$ itself. Strictly speaking, (1) is not well-defined for $\texttt{avedev}(x^s) = 0$, but such situations can hardly happen. Section 4.1 discusses how to deal with such situations, and interested readers are referred there.

Two important remarks are in order:

- **Reason of using $\log_{10}$:** Network effects play an important role in the adoption of currencies, including stablecoins. Thus, the values of the evaluation measures can

[3] For example, ETH is the native currency of the Ethereum blockchain, and Uniswap is the representative DEX.

differ significantly by a factor of 10^2 or 10^3, not just by a factor of 2 or 3, across various stablecoins. Therefore, to compare the values, it is more appropriate to use $\log_{10}$ to clearly express the difference among the values of various stablecoins.

- **Robustness against strategic manipulation:** Some issuers of a stablecoin may attempt to make false transactions to increase scores. For example, they may do huge transactions within their own address to increase $\texttt{txvol}(w^s)$. However, because of the logarithmic transformation of $\texttt{txvol}(w^s)$, increasing $\log_{10}\texttt{txvol}(w^s)$ through such strategic manipulation incurs significant transaction costs. The marginal cost also increases drastically as $\log_{10}\texttt{txvol}(w^s)$ increases.

3.2 Max normalization to define scores

Note that

$$\max_{u\in S} -\log_{10}\texttt{avedev}(x^u) \tag{2}$$

denotes the maximum value of (1) among the stablecoins in S. Therefore,

$$\text{score}(\texttt{avedev}(x^s)) \equiv \frac{-\log_{10}\texttt{avedev}(x^s)}{\max_{u\in S} -\log_{10}\texttt{avedev}(x^u)} \tag{3}$$

denotes the normalized value of (1), which is between 0 and 1. We call this normalization method the *max normalization*. We explain its rationale in Section 3.3 and the Appendix. Other types of normalization are discussed in Sections 4.3 and 4.4.

By the same logic as for the average deviation, we define

$$\text{score}(\texttt{var}(x^s)) \equiv \frac{-\log_{10}\texttt{var}(x^s)}{\max_{u\in S} -\log_{10}\texttt{var}(x^u)}, \tag{4}$$

$$\text{score}(\texttt{autcor}(x^s)) \equiv \frac{-\log_{10}\texttt{autcor}(x^s)}{\max_{u\in S} -\log_{10}\texttt{autcor}(x^u)}, \tag{5}$$

$$\text{score}(\texttt{dexliq}(y^s)) \equiv \frac{\log_{10}\texttt{dexliq}(y^s)}{\max_{u\in S}\log_{10}\texttt{dexliq}(y^u)}, \tag{6}$$

$$\text{score}(\texttt{address}(z^s)) \equiv \frac{\log_{10}\texttt{address}(z^s)}{\max_{u\in S}\log_{10}\texttt{address}(z^u)}, \tag{7}$$

$$\text{score}(\texttt{txvol}(w^s)) \equiv \frac{\log_{10}\texttt{txvol}(w^s)}{\max_{u\in S}\log_{10}\texttt{txvol}(w^u)}. \tag{8}$$

We do not need "−" to define the scores of (6), (7) and (8) because larger values of $\texttt{dexliq}(y^s)$, $\texttt{address}(z^s)$ and $\texttt{txvol}(w^s)$ are more desirable, unlike (3), (4) and (5).

3.3 Rationale of max normalization

The *max normalization* can be written as a function f that maps each non-negative vector $a = (a_1, \ldots, a_k)$ to the normalized vector $f(a) = (f_1(a), \ldots, f_k(a))$ such that

$$f_j(a) = \frac{a_j}{\max_{i\in\{1,\ldots,k\}} a_i} \text{ for all } j \in S.$$

The max normalization satisfies several desirable properties as normalization functions, which offer the rationale of using it.

The first property is *ratio preservation*, stating that the ratio of differences of inputs is preserved under normalization: For each distinct a_i, a_j, a_k with $a_i \le a_j < a_k$,

$$\frac{f_k(a) - f_j(a)}{a_k - a_j} = \frac{f_k(a) - f_i(a)}{a_k - a_i}.$$

The second property is *independence of null objects*, stating that adding an extremely low-quality stablecoin whose parameter is zero to the list does not affect the values of other stablecoins, and the value of the extremely low-quality stablecoin is chosen to be zero: For $a = (a_1, \dots, a_k, a_{k+1})$ with $a_{k+1} = 0$,

$$\begin{aligned} f_j(a) &= f_j(a_1, \dots, a_k) \text{ for all } j = 1, \dots, k, \\ f_{k+1}(a) &= 0. \end{aligned}$$

The third property is the *max-one property*, stating that the value of the stablecoin with the highest parameter is given one:

$$a_j = \max_{i \in \{1,\dots,k\}} a_i \Longrightarrow f_j(a) = 1.$$

The max normalization satisfies *ratio preservation*, *independence of null coins*, and the *max-one property.* In fact, we can establish a much stronger result that the max normalization is the only function satisfying the three properties. To prove this result, we need to define the properties more rigorously in a formal model. This will be done in the Appendix, but we state the result below for reference:

Theorem 1 (Informal statement). *The max normalization is the only function that satisfies ratio preservation, independence of null objects, and the max-one property.*

3.4 Aggregation to define rating

When calculating a single rating based on multiple criteria, the geometric mean has a beneficial property: a low score on one criterion cannot easily be compensated by a high score on another. The United Nations Development Program calculates the so-called human development index by income, life expectancy, and education level. Earlier, they used the arithmetic mean to calculate the index; however, in 2010 they switched to using the geometric mean instead.[4] They made this change because they considered all three criteria to be essential for human life so that none of them can be substituted by another one. Similarly, we consider that our six measures are essential for evaluating stablecoins and define our rating function by the geometric mean as follows:

$$\begin{aligned} \mathrm{rating}(x^s, y^s, z^s, w^s) \equiv{}& \mathrm{score}(\texttt{avedev}(x^s))^{\frac{1}{6}} \cdot \mathrm{score}(\texttt{var}(x^s))^{\frac{1}{6}} \cdot \mathrm{score}(\texttt{autcor}(x^s))^{\frac{1}{6}} \cdot \\ & \mathrm{score}(\texttt{dexliq}(y^s))^{\frac{1}{6}} \cdot \mathrm{score}(\texttt{address}(z^s))^{\frac{1}{6}} \cdot \mathrm{score}(\texttt{txvol}(w^s))^{\frac{1}{6}}, \end{aligned} \tag{9}$$

which is between 0 and 1.[5]

[4] See, Klugman, Rodríguez and Choi (2011) for a review and Herrero, Martínez, and Villar (2010) and Kawada, Nakamura, and Otani (2019) for properties of the human development index.

[5] In this definition, we assign all criteria an equal weight of $\frac{1}{6}$, reflecting the idea that the evaluation measures are equally important.

4 Remarks on scoring and rating

4.1 Logarithmic transformation of zero

One caveat to our transformation

$$\log_{10} a$$

is that $\log_{10} 0$ cannot be defined, but average deviation, variance, and auto-correlation can be zero. In particular, auto-correlation might be relatively prone to zero due to the nature of its definition. Nonetheless, whenever the data collection period is sufficiently long, these value are hardly zero. Overall, we need not take this issue too seriously, but we explain how to deal with such problems.

We consider auto-correlation as an example. If $\texttt{autcor}(x^p) = 0$ for some $p \in S$, then for each $s \in S$, we transform x^s by

$$\log_{10} [\texttt{autcor}(x^s) + 0.1]$$

instead of $\log_{10} \texttt{autcor}(x^s)$, so that

$$\log_{10} [\texttt{autcor}(x^p) + 0.1] = -1.$$

Then we let

$$\begin{aligned} \text{score}(\texttt{autcor}(x^s) + 0.1) &\equiv \frac{-\log_{10} [\texttt{autcor}(x^s) + 0.1]}{\max_{u \in S} - \log_{10} [\texttt{autcor}(x^u) + 0.1]} \\ &= \frac{-\log_{10} [\texttt{autcor}(x^s) + 0.1]}{-\log_{10} [\texttt{autcor}(x^p) + 0.1]} \\ &= -\log_{10} [\texttt{autcor}(x^s) + 0.1] . \end{aligned}$$

4.2 Negative scores

It is logically possible that $\texttt{avedev}(x^s) > 1$. In this case,

$$\log_{10} \texttt{avedev}(x^s) > 0,$$

so that the score of average deviation is negative:

$$\texttt{score}(\texttt{avedev}(x^s)) = \frac{-\log_{10} \texttt{avedev}(x^s)}{\max_{u \in S} - \log_{10} \texttt{avedev}(x^u)} < 0.$$

This negative score itself is not problematic, but the derived property that

$$\texttt{score}(\texttt{avedev}(x^s))^{\frac{1}{6}}$$

is an imaginary number is problematic, since our rating function (9) is not well-defined as a real-valued function then. However, $\texttt{avedev}(x^s) > 1$ implies that the price of s is greater than 2 on average over the data collection period, and this situation almost never happens with stablecoins. In the unlikely event that it happens, we will simply replace $\texttt{score}(\texttt{avedev}(x^s))$ with zero or a very small positive value, or we may simply remove s from the list S because such an excessively strange stablecoin is not worthy of rating. This negative score problem can logically occur only for average deviation and variance. This is because

$$\texttt{autcor}(x^s) \in [0, 1]$$

by its definition, and because the scores of $\texttt{dexliq}(y^s)$, $\texttt{address}(z^s)$, and $\texttt{txvol}(w^s)$ are defined not by using "$-$".

4.3 Alternative normalization: ratio to dynamic maximum

For each evaluation measure, the max normalization gives the value 1 to the stablecoin with the maximum value of the evaluation measure during the data collection period. Consequently, even when $f(x^s)$ reduces for all stablecoins through multiple data collection periods, the stablecoin with the maximum value at each data collection period is assigned the value 1. That is, our definition of score($\texttt{stdev}(x^s)$) does not capture such dynamic changes. Below we offer an alternative normalization that captures such changes.

Recall that the data collection period of time-series price data $x^s = (x_1^s, \ldots, x_T^s)$ is defined as

$$D = \{1, \ldots, T\} \text{ with } T \geq 1.$$

Let $D(1) \equiv D$. There can be "past" data collection periods

$$D(2), \ldots, D(K) \subset \{0, -1, -2, -3, \ldots\}$$

and associated data

$$(x_t^s)_{t \in D(2)}, \ldots, (x_t^s)_{t \in D(K)}.$$

Then

$$\max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)}). \tag{10}$$

is the dynamically maximum value of $-\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)})$ among all data collection periods $D(1), \ldots, D(K)$ for s, and

$$\max_{s \in S} \max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)}).$$

is the maximum value of (10) among the stablecoins in S. Then,

$$\frac{-\log_{10} \texttt{avedev}(x^s)}{\max_{s \in S} \max_{k \in \{1,\ldots,K\}} -\log_{10} \texttt{avedev}((x_t^s)_{t \in D(k)})} \tag{11}$$

is the dynamically normalized score of x^s on the standard deviation, which is between 0 and 1. The dynamic normalization of (11) is useful to capture time-series changes in values; however, we use the static normalization of (3) because we are still unclear about how values (11) and (3) can differ, and (3) is easier to use as there is no problem in choosing $D(2), ..., D(K)$. However, in the future with more data and experience, we may shift from static to dynamic normalization or something similar.

4.4 Alternative normalization: quantile

For any $s \in S$, the following set denotes the set of stablecoins whose average deviation is weakly greater than that of s:

$$\{u \in S : \texttt{avedev}(x^u) \geq \texttt{avedev}(x^s)\}.$$

Then

$$\frac{|\{u \in S : \texttt{avedev}(x^u) \geq \texttt{avedev}(x^s)\}|}{|S|} \tag{12}$$

denotes the quantile of s in the decreasing order. This value falls in the range $[0, 1]$, so quantile is also a method of normalization. However, there is a fundamental problem of appropriately selecting a list S. Perhaps we can easily agree that S should contain major stablecoins such as USDC or USDT, but we cannot do it for hundreds of minor stablecoins. Indeed, the boundary of stablecoins is not unclear because there are multiple types of peg mechanisms.[6] However, the value of (12) highly depends on S, and hence we do not use this quantile normalization.

5 Computation

As an example, we compute the scores and ratings of several stablecoins to better understand our scoring and rating. However, our data is for a short period of time, and we note that this computation is for reference only. We soon plan to analyze long-term time-series data for a larger number of stablecoins; however, it is outside of the scope of this study.

In this study, we have selected USDT, USDC, DAI, FDUSD, USDe, LUSD, crvUSD, FEI, and GHO on the Ethereum network, all of which are pegged to USD. Thus the list of stablecoins is

$$S = \{\text{USDT, USDC, DAI, FDUSD, USDe, crvUSD, LUSD, FEI, GHO}\}.$$

USDT, USDC, DAI, FDUSD, USDe are the top five stablecoins based on market capitalization.[7] crvUSD and LUSD are chosen as examples of "middle class" stablecoins, whose market capitalization consistently ranks in the top 20. FEI and GHO have even smaller market capitalization but are in the top 100.

Data are obtained as follows:

- We obtained daily price data from CoinGecko between March 28–April 25, 2024[8] and calculated the average deviation, variance, and auto-correlation and their scores. For example,

$$\begin{aligned}
x^{\text{DAI}} &= (0.997491, 0.997741, \ldots, 0.999625),\\
\texttt{avedev}(x^{\text{DAI}}) &= 0.02143693,\\
-\log_{10}(\texttt{avedev}(x^{\text{DAI}})) &= 1.66883739,\\
\text{score}(\texttt{avedev}(x^{\text{DAI}})) &= 0.8071409734.
\end{aligned}$$

[6] For example, whether TerraUSD (UST) on the Terra blockchain is a stablecoin has been debatable because of its exotic peg mechanism, until the collapse of LUNA in May 2022.

[7] `https://coinmarketcap.com/view/stablecoin/`

[8] `https://www.coingecko.com/en/coins/usdc/historical_data`

- When a user swaps an amount of ETH to a stablecoin s at Uniswap, the Uniswap system selects a "best pool" to minimize the swap's price impact. Our dexliq(x^s) denotes the amount of s in the pool, which is crucial for realizing the price impact. Therefore, we choose this amount as a measure of liquidity. The daily data of the amount are obtained from on-chain data of Uniswap from April 9–17, 2024. For example,

$$\begin{aligned} y^{\text{DAI}} &= (5558588.81, 6043658.45, \dots, 7111106.29), \\ \texttt{dexliq}(y^{\text{DAI}}) &= 396107801.48, \\ \log_{10}(\texttt{dexliq}(y^{\text{DAI}})) &= 6.597813396, \\ \text{score}(\texttt{dexliq}(y^{\text{DAI}})) &= 0.8294125817. \end{aligned}$$

- An active address s is defined as an address with a daily transaction volume of more than 100 units of s. Transaction volume is defined as the daily volume of all transactions of s. These data were collected on-chain from April 9–14, 2024. For example,

$$\begin{aligned} z^{\text{DAI}} &= (1615, 2625, \dots, 2399), \\ \texttt{address}(z^{\text{DAI}}) &= 2004, \\ -\log_{10}(\texttt{dexliq}(z^{\text{DAI}})) &= 3.301789, \\ \text{score}(\texttt{dexliq}(z^{\text{DAI}})) &= 0.691840. \end{aligned}$$

- The transaction volume is the daily volume of all transactions. These data were collected on-chain from April 9–14, 2024. For example,

$$\begin{aligned} w^{\text{DAI}} &= (11869832525, 8445518282, \dots, 19339245814), \\ \texttt{address}(w^{\text{DAI}}) &= 10308168285, \\ \log_{10}(\texttt{dexliq}(w^{\text{DAI}})) &= 10.013181, \\ \text{score}(\texttt{dexliq}(w^{\text{DAI}})) &= 0.995929. \end{aligned}$$

Calculation results are summarized in the next table.

	Average deviation				Variance			
	Original value	-log_10	Score	Ranking	Original value	-log_10	Score	Ranking
USDC	0.01792148	1.74662606	0.8447638268	2	0.00000088	6.05502165	0.9363001763	2
USDT	0.00855872	2.06759097	1	1	0.00000034	6.46696626	1	1
DAI	0.02143693	1.66883739	0.8071409734	4	0.00000098	6.00879492	0.9291520445	3
FDUSD	0.06941672	1.15853589	0.5603312749	5	0.00001011	4.99515384	0.7724106855	7
USDe	0.02074541	1.68307790	0.8140284618	3	0.00000110	5.95969935	0.9215602977	4
LUSD	0.08557848	1.06763542	0.5163668406	6	0.00000488	5.31121425	0.8212837422	5
crvUSD	0.10920097	0.96177352	0.4651662418	7	0.00001075	4.96873289	0.7683251601	8
GHO	0.15774552	0.80204297	0.3879118185	8	0.00000564	5.24900737	0.811664567	6
FEI	0.57370090	0.24131447	0.1167128677	9	0.00055947	3.25222181	0.5028975998	9

	Auto-correlation				DEX liquidity			
	Original value	-log_10	Score	Ranking	Original value	log_10	Score	Ranking
USDC	0.110919108222	0.954993630763	0.7980630371	2	75,383,053.28	7.877273724	1	1
USDT	0.121126750036	0.916759935221	0.7661121442	3	5,733,059.40	6.758386441	0.857960086	3
DAI	0.355143587601	0.449596022452	0.3757155604	6	3,961,078.01	6.597813396	0.8375757435	4
FDUSD	0.231232454989	0.635951209899	0.5314476847	5	350,716.99	5.544956807	0.7039182593	6
USDe	0.160327819783	0.794991113142	0.6643531452	4	8,341,119.31	6.921224333	0.878631945	2
LUSD	0.841656200976	0.074865272526	0.06256293743	8	447,070.41	5.650375924	0.7173009498	5
crvUSD	0.814868271955	0.088912591734	0.0743019123	7	24,341.60	4.386349041	0.5568359301	7
GHO	0.934372797871	0.029479813539	0.02463550412	9	3,189.56	3.503730204	0.4447896984	9
FEI	0.063585875301	1.196639345988	1	1	4,388.04	3.642270972	0.4623770989	8

Figure 1: Scores of average deviation, variance, auto-correlation, and DEX liquidity

	Number of active addresses				Transaction volume			
	Original value	log_10	Score	Ranking	Original value	log_10	Score	Ranking
USDC	24,695	4.392606	0.920404	1	11,326,884,961	10.054110	1.000000	1
USDT	59,221	4.772477	1.000000	3	8,863,141,591	9.947588	0.989405	3
DAI	2,004	3.301789	0.691840	2	10,308,168,285	10.013181	0.995929	2
FDUSD	39	1.591065	0.333383	4	690,015,168	8.838859	0.879129	4
USDe	2,661	3.425018	0.717660	5	677,532,587	8.830930	0.878340	5
LUSD	96	1.983777	0.415670	8	10,160,864	7.006931	0.696922	8
crvUSD	239	2.378701	0.498421	6	136,159,402	8.134048	0.809027	6
GHO	98	1.992701	0.417540	7	13,352,800	7.125572	0.708722	7
FEI	6	0.790050	0.165543	9	115,236	5.061587	0.503435	9

	Rating	Ranking
USDC	**0.9134701042**	**2**
USDT	**0.9307983231**	**1**
DAI	**0.7387977574**	**4**
FDUSD	**0.6016982282**	**5**
USDe	**0.8069101607**	**3**
LUSD	**0.4203075573**	**7**
crvUSD	**0.42583359**	**6**
GHO	**0.317321881**	**9**
FEI	**0.3623053788**	**8**

Figure 2: Scores of the number of active addresses and transaction volume, and rating values

6 Conclusion

We have developed a method for rating stablecoins based on their key evaluation measures such as average deviation, variance, auto-correlation, DEX liquidity, number of active addresses, and transaction volume. We have transformed those values into an easy-to-compare score between 0 and 1 using the logarithmic transformation and the max normalization. We then computed rating values by calculating the geometric mean of the scores, so that a stablecoin with balanced scores has a higher rating than a stablecoin with unbalanced scores. We focused on balancing scores because all of these measures are essential to mimic legal tender. We also established an axiomatic characterization of the max normalization to clarify the rational of using it. As far as we know, this study is the first academically rigorous attempt to rate stablecoins. We hope this study will facilitate further research to assess the quality of stablecoins.

References

Bakhtiar, T., Luo, X., and Adelopo, I. (2023) “Network Effects and Store-of-Value Features in the Cryptocurrency Market,” *Technology in Society*, Vol. 74.

Grobys, K., Junttila, J., Kolari, J. W., and Sapkota, N. (2021) “On the Stability of Stablecoins,” *Journal of Empirical Finance*, Vol. 64, pp. 207–223.

Herrero, C., Martínez, R., and Villar, A. (2010) “Multidimensional Social Evaluation: An Application to the Measurement of Human Development,” *Review of Income and Wealth*, Vol. 56, pp. 483—97.

Kawada, Y., Nakamura, Y., and Otani, S. (2019) "An Axiomatic Foundation of the Multiplicative Human Development Index," *The Review of Income and Wealth*, Vol. 65-4, pp. 771–784.

Klugman, J., Rodríguez, F. and Choi, H.-J. (2011) "The HDI 2010: New Controversies, Old Critiques," *The Journal of Economic Inequality*, Vol. 9, pp. 249–288.

Nakamoto, S. (2008) "Bitcoin: A Peer-to-Peer Electronic Cash System," https://assets.pubpub.org/d8wct41f/31611263538139.pdf

Thomson, W. (2003) "Axiomatic and Game-Theoretic Analysis of Bankruptcy and Taxation Problems: a Survey," *Mathematical Social Sciences*, Vol. 45-3, pp. 249–297.

Thomson, W. (2019) *How to Divide When There Isn't Enough*, Cambridge University Press.

Yeh, C.-H. (2008) "Secured Lower Bound, Composition Up, and Minimal Rights First for Bankruptcy Problems," *Journal of Mathematical Economics*, Vol. 44, pp. 925—932.

Appendix: Axiomatic characterization of the max normalization

Since the choice of the list S of stablecoins is controversial, we introduce a model with variable lists. The idea of our modelling comes from the theory of axiomatic resource allocation (e.g., Thomson 2003, 2019).

Let $\mathbb{N} = \{1, 2, \ldots\}$ be the set of infinitely many *objects*. A finite set $\hat{S} \subset \mathbb{N}$ is the set of *central objects*, which can be empty or non-empty. A *list* is a non-empty finite set S with $\hat{S} \subset S \subset \mathbb{N}$.

One can interpret that $\mathbb{N}$ is the set of various cryptocurrencies, $\hat{S}$ is the set of cryptocurrencies that everyone recognize as stablecoin such as USDC and USDT, and S is a set of cryptocurrencies that someone may recognize as stablecoin. A positive integer $M \geq 1$ indicates the minimum size of possible S, which can be chosen arbitrary.

Let $\mathscr{S}(M, \hat{S})$ be the set of lists whose size is at least M and contains $\hat{S}$; that is,

$$\mathscr{S}(M, \hat{S}) \equiv \{S \in 2^{\mathbb{N}} \setminus \{\emptyset\} : \hat{S} \subset S \text{ and } M \leq |S| < +\infty\}.$$

Our modelling with $\mathscr{S}(M, \hat{S})$ allows us to simultaneously work with any consideration on $\hat{S}$ and M.

Hereafter, for brevity of notation, we denote $\mathscr{S}(M, \hat{S})$ by $\mathscr{S}$ when there is no danger of confusion. For each $S \in \mathscr{S}$, let

$$D_S \equiv \mathbb{R}^S_+ \setminus \{\mathbf{0}\}$$

be the set of non-negative vectors $a = (a_i)_{i \in S} \in \mathbb{R}^S_+$ satisfying $a_j > 0$ for some $j \in S$, where the uninteresting vector of parameters $\mathbf{0} = (0, 0, \ldots, 0) \in \mathbb{R}^S_+$ is excluded. A *normalization function* on $\mathscr{S}$ is a function

$$f : \bigcup_{S \in \mathscr{S}} D_S \to \bigcup_{S \in \mathscr{S}} [0,1]^S$$

such that for each $S \in \mathscr{S}$ and each vector of parameters $a \in D_S$, $f(a) = (f_i(a))_{i \in S} \in [0,1]^S$. The *max normalization* on $\mathscr{S}$ is the normalization function f on $\mathscr{S}$ such that for each $S \in \mathscr{S}$ and each $a \in D_S$,

$$f_j(a) = \frac{a_j}{\max_{i \in S} a_i} \text{ for all } j \in S.$$

We define three axioms of normalization functions on $\mathscr{S}$ satisfied by the max normalization. The interpretation of the axioms has already been explained in Section 3.3.

Axiom 1 (Ratio preservation). *For each $S \in \mathscr{S}$, each $a \in D_S$, and each $i, j, k \in S$, whenever $0 \leq a_i \leq a_j < a_k$,*

$$\frac{f_k(a) - f_j(a)}{a_k - a_j} = \frac{f_k(a) - f_i(a)}{a_k - a_i}.$$

Axiom 2 (Independence of Null Objects). *For each $S \in \mathscr{S}$, each $\ell \in \mathbb{N} \setminus S$, and each $a \in D_{S \cup \{\ell\}}$ with $a_\ell = 0$,*

$$f_j(a) = f_j((a_i)_{i \in S}) \text{ for all } j \in S,$$
$$f_\ell(a) = 0.$$

The idea of *independence of null objects* comes from "null claims consistency" in the literature of claims problem (see, Yeh 2008).

Axiom 3 (Max-one property). *For each $S \in \mathscr{S}$ and each $j \in S$, if $a_j = \max_{i \in S} a_i$, then $f_j(a) = 1$.*

We are now in a position to state and prove our axiomatization theorem.

Theorem 1 (Formal statement). *Let $\hat{S} \subset \mathbb{N}$ be any (possibly empty) set of central objects and let $M \geq 1$ be any positive integer. The max normalization is the only normalization function on $\mathscr{S}(M, \hat{S})$ that satisfies ratio preservation, independence of null objects, and the max-one property.*

Proof. Since it is trivial that the max normalization satisfies *ratio preservation*, *independence of null objects*, and *the max-one property*, we only prove the converse statement. Let f be any normalization function on $\mathscr{S}(M, \hat{S})$ satisfying the three axioms. We shall show that f is in fact the max normalization; that is, for each $S \in \mathscr{S}$ and each $a \in D_S$, $f_j(a) = \frac{a_j}{\max_{i \in S} a_i}$ for all $j \in S$.

Case 1. Any $S \in \mathscr{S}$ and $a \in D_S$ satisfying $\max_{i \in S} a_i = \min_{i \in S} a_i$: In this case, by the *max-one property*,

$$f_j(a) = 1 = \frac{a_j}{a_j} = \frac{a_j}{\max_{i \in S} a_i} \text{ for all } j \in S,$$

as desired.

Case 2. Any $S \in \mathscr{S}$ and $a \in D_S$ satisfying $\max_{i \in S} a_i > \min_{i \in S} a_i$:

Step 1. We shall show that there exists a real-value $g(S, a) \in \mathbb{R}$ such that

$$f_i(a) - g(S, a) \cdot a_i = f_j(a) - g(S, a) \cdot a_j \text{ for all } i, j \in S. \tag{13}$$

Let $k, \ell \in S$ be $a_k = \max_{i \in S} a_i$ and $a_\ell = \min_{i \in S} a_i$. Define

$$g(S, a) \equiv \frac{f_k(a) - f_\ell(a)}{a_k - a_\ell}. \tag{14}$$

We separately analyze several types of $j \in S$ below:

- For each $j \in S$ with $j \neq k$ and $a_j = a_k$, by the *max-one property*, $f_j(a) = 1 = f_k(a)$. Therefore,

$$f_j(a) - g(S, a) \cdot a_j = f_k(a) - g(S, a) \cdot a_k. \tag{15}$$

- For each $j \in S$ with $j \neq \ell$ and $a_j < a_k$, by *ratio preservation* and (14),

$$\frac{f_k(a) - f_j(a)}{a_k - a_j} = \frac{f_k(a) - f_\ell(a)}{a_k - a_\ell} = g(S, a),$$

  so that

$$f_j(a) - g(S, a) \cdot a_j = f_k(a) - g(S, a) \cdot a_k. \tag{16}$$

- For ℓ, by (14),

$$f_\ell(a) - g(S, a) \cdot a_\ell = f_k(a) - g(S, a) \cdot a_k. \tag{17}$$

By (15), (16), and (17), we have (13).

Step 2: By Step 1, there exists a real-value $h(S,a) \in \mathbb{R}$ such that

$$h(S,a) = f_j(a) - g(S,a) \cdot a_j \text{ for all } j \in S. \tag{18}$$

Note that

$$f_j(a) = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S. \tag{19}$$

In the next step, we shall clarify the forms of g and h.

Step 3: Take any $\ell \in \mathbb{N} \setminus \{S\}$. Let $a_\ell \equiv 0$. Note $S \cup \{\ell\} \in \mathscr{S}$ and $(a, a_\ell) \in D_{S\cup\{\ell\}}$. Since $\max_{i\in S\cup\{\ell\}} a_i > \min_{i\in S\cup\{\ell\}} a_i$, by the same argument as Steps 1 and 2, there exist real-values $h(S \cup \{\ell\}, (a, a_\ell))$ and $g(S \cup \{\ell\}, a)$ such that

$$f_j(a, a_\ell) = h(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j + g(S \cup \{\ell\}, (a, a_\ell)) \text{ for all } j \in S \cup \{\ell\}.$$

By *independence of null objects*,

$$\begin{aligned} f_j(a, a_\ell) &= f_j(a) \text{ for all } j \in S, \\ f_\ell(a, a_\ell) &= 0, \end{aligned}$$

so that

$$\begin{gathered} g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j + h(S \cup \{\ell\}, (a, a_\ell)) = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S, \\ h(S \cup \{\ell\}, (a, a_\ell)) = 0, \end{gathered} \tag{20}$$

which imply

$$g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j = g(S,a) \cdot a_j + h(S,a) \text{ for all } j \in S. \tag{21}$$

Let $k \in S \cup \{\ell\}$ be such that $a_k = \max_{i\in S\cup\{\ell\}} a_i$. Since $a_\ell = 0$, we have $k \in S$, and so $a_k = \max_{i\in S} a_i$ as well.

By the *max-one property* and (20),

$$1 = f_k(a, a_\ell) = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_k + h(S \cup \{\ell\}, (a, a_\ell)) = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_k,$$

so that

$$g(S \cup \{\ell\}, (a, a_\ell)) = \frac{1}{a_k}. \tag{22}$$

Therefore by the fact $a_k = \max_{i\in S} a_i$, (22), and (21),

$$\frac{a_j}{\max_{i\in S} a_i} = \frac{a_j}{a_k} = g(S \cup \{\ell\}, (a, a_\ell)) \cdot a_j = g(S,a) \cdot a_j + h(S,a) = f_j(a) \text{ for all } j \in S,$$

as desired. □

The axioms in Theorem 1 are tight; that is, if any one of them is dropped, then the axiomatic characterization is no longer valid.

- Dropping *ratio preservation*: Let f be the "quantile among positive values" such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$f_j(a) = \frac{|\{i \in S : a_j \geq a_i > 0\}|}{|\{i \in S : a_i > 0\}|} \text{ if } a_j > 0,$$
$$f_j(a) = 0 \text{ if } a_j = 0.$$

  This f satisfies all the axioms except *ratio preservation*.

- Dropping *independence of null objects*: Let f be the "min-max normalization" such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$f_j(a) = \frac{a_j - \min_{i \in S} a_i}{\max_{i \in S} a_i - \min_{i \in S} a_i}.$$

  This f satisfies all the axioms except *independence of null objects*.

- Dropping *the max-one property*: Let f be a variant of the max normalization such that for each $S \in \mathscr{S}$, $a \in D_S$, and $j \in S$,

$$f_j(a) = q(\sum_{i \in S} a_i) \cdot \frac{a_j}{\max_{i \in S} a_i},$$

  where $q : \mathbb{R}_{++} \to [0, 1]$ is any function. Unless $q(\sum_{i \in S} a_i) = 1$ for all $\sum_{i \in S} a_i$, this f satisfies all the axioms except *the max-one property*. A simplest case is a constant function such as $q(\sum_{i \in S} a_i) = 0$ for all $\sum_{i \in S} a_i$.

  This example also suggests that *ratio preservation* and *independence of null objects* do not together imply any well-behaved form of f, since q is allowed to be very strange. For example, consider the following case:

$$\begin{aligned} q(\sum_{i \in S} a_i) &= 1 \text{ if } \sum_{i \in S} a_i \text{ is rational,} \\ &= 0 \text{ if } \sum_{i \in S} a_i \text{ is irrational.} \end{aligned}$$

  The normalization function f seems pathological in this case, but satisfies *ratio preservation* and *independence of null objects*. One may consider that *ratio preservation* is so strong that it implies certain linearity of f, but this f is even discontinuous everywhere and is not Lebesgue measurable on every D_S.