

GAME THEORY MODELS OF PEACE AND WAR

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Game theory's relevance to peace and war was controversial from the start. John McDonald (1949) praised the new discipline as "more avant-garde than Sartre, more subtle than a Jesuit," and quoted one Pentagon analyst, "We hope it will work, just as we hoped in 1942 that the atomic bomb would work." P.M.S. Blackett, who had guided British operational research during the war, was a sceptic: "I think the influence of game theory has been almost wholly detrimental." If it had practical relevance, he claimed, investors and card players would have snapped it up, but since they have not, "it is clearly useless for the much more complicated problems of war" (1961).

The debate has continued up to the present day, but it has been conducted mostly in the abstract, with critics trying to prove a priori that game theory is inapplicable. The present chapter surveys the work actually done on peace and war and uses it to rebut some common beliefs. It focuses on the subjects studied and the techniques of application rather than the mathematics. The international relations (IR) section aims to be comprehensive, and while this means including some obsolete or less-than-high-quality papers, it gives the reader an unfiltered account of what has been done. The military section is not comprehensive but includes the main subjects modelled so far, and notes the interaction of game models with new military strategy and technology.

1. International relations game theory

"Philosophical" discussions of game theory in IR are by Deutsch (1954, 1968), Waltz (1959), Quandt (1961), R. Snyder (1961), Boll (1966), Shubik (1968, 1991), Robinson (1970), Forward (1971), Rosenau (1971), Junne (1972), George and Smoke (1974), Plon (1976), Martin (1978), Wagner (1983), Maoz (1985), Snidal (1985a), Hardin (1986b), Larson (1987), Jervis (1988), O'Neill (1989b), Rapoport (1989), Hurwitz (1989), Gates and Humes (1990), Hollis and Smith (1990), Ludwig (1990), Bennett (1991), Abbey (1991), Riker (1992), Wagner (1992), Nicholson (1992), Morrow (1992b), and Leilzel (1993).

1.1. Game analyses of specific international situations

Compared to games in IR theory, studies of specific conflicts tend to use uninnovative mathematics, but they can be interesting when the authors are writing about their own countries and concerns. The goal is not prediction, of course, since the outcome is already recorded, but a game organizes the account of each nation's decisions and shows the essence of the conflict. The favorite subjects have been the 1962 Cuban Missile crisis, the United States/Soviet arms competition and the Middle East wars.

Economics game models involving goods or money allow assumptions about continuity, risk aversion, or conservation of the total after a division. Without the mathematical handle of an extensive commodity, IR theorists have often used 2×2 matrices and looked at only the ordinal properties of the payoffs to avoid having to justify specific values. Rapoport and Guyer's taxonomy (1966) of 78 ordinal games without tied payoffs has been used repeatedly. The most prominent study is by Snyder and Diesing (1977) who set up 2×2 matrices for sixteen historical situations. Other examples are Maoz (1990) on Hitler's expansion, and on Israel/Syria interactions, Measor (1983), Boulding (1962), Nicholson (1970), and Brown (1986) on the arms race, Gigandet (1984) on German/Soviet relations between the World Wars, Dekmajian and Doran (1980) on the Middle East conflict, Ravid (1981) on the decision to mobilize in the 1973 War, Forward (1971) and Weres (1982) on the Cuban missile crisis, Levy (1985) on the war in Namibia, Satawedin (1984) on the Thai-American relations in the early 1960s, Schellenberg (1990) on the Berlin and Formosa Straits Crises of 1958, and Kirby (1988) and Call (1989) on antimissile systems. Scholars from South Korea and West Germany used matrix games to discuss reunifying their countries [Pak (1977), Gold (1973), Krelle 1972 see also Cook (1984)]. Hsieh (1990b) modelled the U.S. Iraq confrontation over Kuwait.

To find cardinal payoffs, some researchers have interviewed the real players and scaled their responses. Delver (1986, 1987, 1989) contacted senior Dutch naval officers and applied Dutch developments in bargaining theory. His interest was the effect of NATO maritime strategy on superpower relations. Lumsden (1973) interviewed Greek students about the Cyprus crisis, but was unable to talk with their Turkish counterparts. Plous (1985, 1987, 1988, 1993) sent questionnaires to all members of the US Senate, the Israeli Knesset, the Australian Parliament and the Politburo, asking them to estimate the benefits if each superpower reduced its nuclear arsenal. The Politburo was unresponsive in those pre-glasnost days, but thirty-two senators sent back data that would make good examples for introductory classes.

Two-by-two matrices often miss the conflict's essence. In the arms reduction questionnaire, for example, it is implausible that one side would be acting with no information about what the other did. Snyder and Diesing portray their situations as matrices, but their text often interprets the moves as sequential. Many applications should have considered games in extensive form or with incomplete information. One extensive form example on the Falklands/Malvinas War is by Sexton and Young (1985), who add an interesting analysis of the consequences of misperception. Young (1983) argues that the technique is no more complex but more adequate than 2×2 matrices. Johr (1971), a Swiss political scientist, uses the extensive form to discuss his country's strivings for neutrality in the face of Nazi Germany. With perfect information game trees, Joxe (1963) describes the Cuban Missile Crisis, O'Neill (1990c) examines Eisenhower's 1955 offer to Chiang Kai-shek to blockade China's coast, Hsieh (1990a) discusses Taiwan's contest with

China for diplomatic recognition from the rest of the world, and Bau (1991) models the changing relations of the two nations over forty years.

Game trees with perfect information are simpler models than they appear. Players are called on to make only a few of the possible payoff comparisons, so the modeller has to commit to few assumptions about utilities. For example, a two-person binary tree where each player makes two moves, has sixteen endpoints. A modeller ranking all outcomes for each player's preference would face $16!16!$ possibilities, equivalent in information to $\log_2(16!16!) = 89$ binary decisions on which of two outcomes is preferred. In fact by working backwards through the tree, only $8 + 4 + 2 + 1 = 15$ two-way decisions are enough to fix the solution. This is near the $\log_2(4!4!) = 9.2$ bits needed to fill in a 2×2 ordinal matrix, which has only half as many moves. Trees can be solved using very sparse information about the payoffs.

Another example of moving beyond 2×2 matrices is Zagare's $3 \times 3 \times 3$ game on the Vietnam war negotiations (1977). Incomplete information analyses include Guth (1986) on negotiations over an intermediate missile ban in Europe, Mishal et al. (1990) on the PLO/Israel conflict, and Niou (1992) on Taiwan signalling its resolve to the People's Republic. Wagner on the Cuban missile crisis (1989a) gives a mathematically significant model that also takes the facts seriously. Hashim (1984), at the University of Cairo, uses mostly three-person games to treat the Middle East conflict.

Responding to the limits of 2×2 games and Nash equilibria, some writers have devised their own solution theories, prominent ones being Howard's *metagames* (1971), *conflict analysis* [(Fraser and Hipel (1984), Fraser et al. (1987)], which are related to *hypergames* [Bennett and Dando (1982), and Brams and Hessel's *theory of moves* (1984) with its non-myopic equilibria. [For summaries and comparisons, see Hovi (1987), Powell (1988), Nicholson (1989), Zagare (1986) and Brams (1994).] Conflict analysis and hypergames were applied to the Suez crisis [Wright et al. (1980), Shupe et al. (1980)], the fall of France [Bennett and Dando (1970, 1977)], the 1979 Zimbabwe conflict [Kuhn et al. (1983)], the Cuban Missile Crisis [Fraser and Hipel (1982)], the 1973 Middle East War [Said and Hartley (1982)], the Falklands/Malvinas conflict [Hipel et al. (1988)], arms control verification [Cheon and Fraser (1987)], the nuclear arms race [Bennett and Dando (1982)], the Armenian/Azerbaijan conflict [Fraser et al. (1990)] and post-Cold War nuclear stability [Moore (1991)]. Some metagame analyses are by Richelson (1979) on nuclear strategy, Alexander (1976) on Northern Ireland, Benjamin (1981) on Angola, the Kurdish uprising and various other conflicts, Benjamin (1982) and Radford (1988) on the Iranian hostage crisis, and Howard (1968, 1972) on the Vietnam war, the testban negotiations and the Arab/Israeli conflict [see also Schelling (1968)]. With the theory of moves Leng (1988) studied crisis bargaining; Mor (1993), deception in the 1967 Middle East War; Brams studied the Cuban missile crisis (1985, 1994), the bombing of North Vietnam and the Middle East conflict (1994); Brams et al. (1994), international trade negotiations; Brams and

Mattli (1993) the Iranian Hostage Crisis; and Zagare the Middle East conflicts of 1967 and 1973 (1981, 1983), the Geneva conference of 1954 (1979), and the nuclear arms race (1987a). Brams (1976) compared standard and metagame analyses of the Cuban missile crisis, and Gekker (1989a) looked at the 1948 Berlin crisis through sequential equilibria and the theory of moves.

Advocates of these alternatives object to the mainstream theory for the wrong reasons, in my view, since the difficulties are treatable within it. Hypergame analyses involve one side misunderstanding the other's goals, as do incomplete information games. Metagame theory prescribes general policies of play, as do repeated games. Non-myopic equilibria deal with players who can switch back and forth in the cells of a matrix, like Danskin's matrix-differential games [Washburn (1977)] or some parts of repeated game theory. The theory of moves translates a 2×2 matrix into a particular extensive-form game and makes substantial assumptions about who can move at each point. These need to be justified within a given application but often they are not.

Advocates of these methods can point out that they are simple enough for decision makers to comprehend, unlike repeated game theory, so they can recommend moves in real decisions. To this end, Howard as well as Fraser and Hipel developed large interactive software programs, and government agencies have shown interest in such programs.

Only one other group has focused on methodology for converting the judgements of IR experts into games. The *Schwaghof Papers* [Selten et al. (1977)] summarize a 1976 Bonn conference where political analysts and game theorists met to model Middle East issues. Their method of *scenario bundles* estimated governments' goals, fears, strengths and plausible coalitions, and generated games of perfect information. Their first scenario in 1976 had Iraq tempted to invade Kuwait but deterred by the prospect of an Iran/Saudi/U.S. coalition and a Soviet withdrawal of support. In 1990 Iraq really invaded, and the common wisdom was that Saddam went ahead *in spite of* the Soviet Union's retirement as his patron. The model suggests the opposite possibility, that Soviet absence may have given Iraq less to lose. The little-known report is available from the first author at the University of Bonn.

1.2. The debate on realism and international cooperation

Turning to IR theory, a prevalent framework is "realism", sometimes refined as "neorealism" or "structural realism" [Keohane (1986a)]. It takes the main actors to be states, which pursue their interests within an anarchic world. To avoid the continuing danger of subjugation, they seek power, which in a competitive system is defined *relative to* other states and so can never be achieved mutually and finally. Significant cooperation is very difficult. The basic explanation of international events lies in the structure of this system, the resources held by its members and their alliances. Alternatives to realist theory allow more hope for cooperation,

either through international institutions and interdependence (traditional idealism), a dominant power who keeps order (hegemonic stability theory), the common interests of subregions (regional integration theory), international institutions and standards of behavior (regime theory), or transnational class interests (Marxist structuralism). The more basic critique of postmodernism or reflectivism emphasizes that the actors' identities and interests are social creations.

Participants in the debate have increasingly used game models to state their arguments. Prisoner's Dilemma (PD) games have formalized a central issue between realists and regime theorists: How can cooperation develop in an anarchic world? Axelrod (1984) discussed cooperation in repeated PD with probabilistic termination, comparing it with war and competition between species. His ideas were connected to political realism by Stein (1982a, 1990), Keohane (1985, 1986b), Lipson (1984) and Grieco (1988, 1990). Informal work on game models of cooperation was done by Axelrod and Keohane (1985), Brams (1985), Jervis (1988), Larson (1987), Nicholson (1981), Snidal (1985a,b), Cudd (1990), Iida (1993b) and the authors in Oye's volume (1986). Taylor's work (1976, 1987), although not addressed to IR, has been influential. Snidal (1985b, 1991a) uses *n*-person and non-PD games to analyze international cooperation, McGinnis (1986) considers issue linkage, Sinn and Martinez Oliva (1988) policy coordination, Stein (1993) international dependence, and Pahre (1994) and Martin (1992b), the choice between bilateral and multilateral arrangements, all central ideas of regime theory.

Axelrod's "theory of cooperation" has triggered much interaction between mathematical and non-mathematical researchers, and produced papers of different degrees of formalization. It has influenced many non-formal policy writings on superpower dealings, e.g., Goldstein and Freeman (1990), Evangelista (1990), Weber (1991) and Bunn (1992). The simplified model was accessible and appeared just when the related political theory questions were prominent. Axelrod incorrectly believed that tit-for-tat was in effect a perfect equilibrium and he overlooked actual equilibria, but these errors were fortunate in a way since technicalities would have reduced his readership. An analysis of repeated PD with significant discounting turns out to be quite complicated [Stahl (1991)].

The study of cooperation connects with international institutions. One neglected area is game-theoretical analyses of voting to propose decision rules for international organizations, a rare example being Weber and Wiesmeth's paper (1994). Snidal and Kydd (1994) give a summary of applications of game theory to regime theory.

Relevant to the realist debate on cooperation are game models in international political economy, in the sense of international trade. Some studies interweave historical detail, conventional theory and simple matrix games. Conybeare's work (1984, 1987) separates trade wars into different 2×2 games with careful empirical comparison of the models. Josko de Gueron (1991) investigates Venezuela's foreign debt negotiations. Alt et al. (1988), Gowa (1986, 1989), Snidal (1985c), Krasner (1991) and Kroll (1992), Gowa and Mansfield (1993) analyze hegemony and regimes. In Iida's model (1991) the United States drastically cuts its contribution to the

economic order to signal that it will no longer act as the hegemon. Wagner (1988a) uses Nash bargaining theory to analyze how economic interdependence can be converted to political influence, and Kivikari and Nurmi (1986) and Baldwin and Clarke (1987) model trade negotiations. A frequent theme has been the Organization of Petroleum Exporting Countries [Bird and Sampson (1976), Brown et al. (1976), Bird (1977), Sampson (1982)], often analyzed as a coalitional game.

A central realist concept is power. Selten and Mayberry (1968) postulate two countries who build weapons at linear costs, and share a prize according to the fraction that each holds of the total military force. The prize might involve political compliance from the adversary or third countries. Their model's form was identical to Tullock on rent-sharing twelve years later (1980). Harsanyi (1962) uses bargaining theory to define the notion of power, and compares the result with Robert Dahl's famous treatment. Brito and Intriligator (1977) have countries invest in weapons to increase their share of an international pie, as determined by a version of Nash bargaining theory, but their arsenals also increase the risk of war. Shapley and Aumann (1977) revise standard bargaining theory so that threats involve a non-refundable cost, e.g., the money spent on arms, and Don (1986) applies their model. Brito and Intriligator (1985) have countries that commit themselves to a retaliatory strategy to be implemented in the second period in order to induce a redistribution of goods in the first. In Garfinkel's model (1991) each side divides its resources between domestic consumption versus arms. She postulates that the resource quantities vary randomly and relates the state of the two economies to their armsbuilding. Matsubara (1989) gives a theoretical discussion of international power. Zellner (1962), Satawedin (1984) and Gillespie and Zinnes (1977) discuss large countries' attempts to gain influence in the Third World, the latter authors applying differential games. Mares (1988) discusses how a large power can force its will on a middle one, examining the United States versus Argentina in the Second World War and later Mexico. Gates (1989) looks at American attempts to prevent Pakistan from acquiring nuclear weapons, and Snidal (1990a) discusses how economic investment in the People's Republic of China by smaller but richer Taiwan, may aid the latter's security. Bueno de Mesquita (1990) and Bueno de Mesquita and Lalman (1990; 1992) examine how wars change hegemony in the international system, exemplified by the Seven Weeks War of 1866. Wagner (1989b) argues from bargaining theory that economic sanctions are ineffective at compelling a state to change its policy, and Tsebelis (1990) is similarly negative. Martin (1992a) uses 2×2 games to define varieties of international cooperation on sanctions. A subsequent model (1993) investigates influencing another state to join in sanctions against a third. Her approach represents a recent positive trend of combining game models with empirical statistics or case studies [e.g., James and Harvey (1989, 1992), Bueno de Mesquita and Lalman (1992), Gates et al. (1992), Gowa and Mansfield (1993), Fearon (1994)].

Realist explanations of international events take states as having goals, but is this assumption justified? Kenneth Arrow devised his social choice theorem after Olaf Helmer, a colleague at the RAND Corporation, asked him whether strategic

analysis was coherent in treating countries as single actors [Feiwei (1987) p. 193]. Thirty years later, Achen (1988) gave conditions on the decision-making process of a state's competing domestic groups such that the state would possess a utility function. Seo and Sakawa (1990) model domestic forces in a novel way, by assigning fuzzy preference orderings to states. Downs and Rocke (1991) discuss the agency problem of voters who want to prevent their leaders from using foreign interventions as a reelection tactic. McGinnis and Williams (1991, 1993) view correlated equilibria as a technique to link domestic politics and foreign policy. Whereas Achen's approach treated the entities within a country as unstructured, McGinnis and Williams suggest extending his idea by finding assumptions that yield a national utility function *and* reflect the structure of a government's decision-making institutions. Other game analyses involving domestic and international levels are by Bendor and Hammond (1992), Mayer (1992) and Lohmann (1993).

The IR mainstream tends to perceive game theory as innately realist-oriented in that it casts governments as the players and their choices as "rational". Rosenau (1971), for one, writes, "The external behavior called for in game-theoretical models, for example, presumes rational decision-makers who are impervious to the need to placate their domestic opponents or, indeed, to any influences other than the strategic requisites of responding to adversaries abroad". This is a misunderstanding. Many current game models fit this pattern, but this reflects realism's prevalence, not the theory's requirement, and other studies have used players within or across states. Intriligator and Brito (1984) suggest a coalitional game with six players: the governments, electorates and military-industrial complexes of the United States and the Soviet Union. Although the two military-industrial complexes cannot talk directly, they coordinate their activities toward their goals. In 1968, Eisner looked at Vietnam War policy, the players being North Vietnam, President Johnson, and the American public. Iida (1993a,c) and Mo (1990) formalize Putnam's notion of two-level games (1988), with negotiations conducted at the international level that must be ratified domestically. Bueno de Mesquita and Lalman (1990a), Ahn (1991) and Ahn and Bueno de Mesquita (1992) investigate the influence of domestic politics on crisis bargaining. Morrow (1991a) has the US administration seeking a militarily favorable arms agreement with the Soviet Union, but hoping to please the US electorate, which cares only about the economy and whether there is an agreement. Sandretto's players (1976) include multinational corporations, and he extends the ideas of Lenin and Rosa Luxemburg on how transnational control of the means of production influences world politics. These examples show that nothing precludes domestic or transnational organizations as players.

There is an irony in another misunderstanding, that game theory requires confidence in everyone's "rationality": the first significant IR game model, Schelling on pre-emptive instability (1958b, 1960), had each side worrying that the other would attack irrationally. The confusion derives from the ambiguous word "rationality", which perhaps could be dropped from our discourse, if it means that players' goals are always soberly considered and based on long-term self-

centered “national interests”; game models do not assume that. O’Neill’s NATO competitors in nuclear escalation (1990a) act from transitory emotions of anger and fear. It is not game theory that determines who are the players and what are their goals; that is up to the modeller. The essence of game theory is that each player considers the other’s view of the situation in its own choice of strategy.

A cluster of papers debating a realist issue comprises Powell (1999a,b), Snidal (1990b, 1991a,c, 1993), Morrow (1990b), Niou and Ordeshook (1992), Grieco (1993) and Busch and Reinhardt (1993). They discuss whether pursuit of resources and power is absolute or relative, how immediate goals derive from long-term ones, and whether they preclude cooperation. Relevant also to realist theory, Niou and Ordeshook have developed a set of models of alliance behavior, regimes, the balance of power and national goals (1990b, 1990c, 1991, and others treated below in the alliance section).

Although reflectivist writers are usually hostile to game theory and regard it as the mathematical apology for realism, in fact the two approaches agree on the importance of socially-defined acts and subjective knowledge. Kratochwil (1989) uses matrix games in a complex account of the development of norms. My work on international symbols (O’Neill, 1994b) tries to define symbolic events in game terms and analyze symbolic arms contests, symbolic tension-provoking crisis events and symbolic leadership. Other models of honor, insults and challenges (O’Neill, 1991d, 1994b) emphasize the power of language acts to change players’ goals.

1.3. *International negotiations*

The first applications to arms bargaining were mathematically auspicious. Herbert Scoville, Director of the U.S. Arms Control and Disarmament Agency, funded many prominent theorists to investigate the topic, and their papers pioneered the study of repeated games with incomplete information [Aumann et al. (1966, 1967, 1968)]. Saaty (1968) summarizes one of their models. Their work was too technical for political science, and the approach has been taken up only recently when Downs and Rocke (1987, 1990) studied arms control negotiations using repeated Prisoner’s Dilemma and simulation, in an excellent interweaving of the factual and theoretical.

A fine conceptual use of sample 2×2 games for arms bargaining is Schelling’s 1976 paper. When he revised it for the widely-read periodical *Daedalus* (1975), the editor insisted that he omit the matrices. The article has strongly influenced US thinking on arms control. No doubt policymakers found it easier to read in its bowdlerized form, but they missed the path that led Schelling to his ideas.

Guth and van Damme’s interesting “Gorbi-games” (1991) analyze when arms control proposals show honorable intentions. Picard (1979), Makins (1985) and Bunn and Payne (1988) study strategic arms negotiations as elementary games, and Jonsson (1976) looks at the nuclear test ban. Hopmann (1978) uses Harsanyi’s

model of power to discuss conventional arms talks. Guth (1985, 1991) treats an issue that dogged NATO, that negotiating with a rival requires negotiating with one's allies. Brams (1990a) applies a model of sophisticated voting.

No studies have yet compared game models of bargaining with real international negotiations, probably because the earlier mathematical bargaining theory focused on axioms for the outcomes and not dynamics. Several applications to IR have suggested clever mechanisms to reach an accord. Kettelle (1985) proposes that each side feed its best alternative to an agreement into a computer that simply announces whether any compromise is possible. A "go ahead" might prompt more effort to agree. Unaware of the theoretical literature, he did not ask whether each side would tell the computer the truth. Green and Laffont (1994) outline an interesting procedure in which the parties have only one agreement available, to sign or not sign. Each is unsure whether the agreement will help or hurt it, and has only partial knowledge about its own and the other's gains. In a series of simultaneous moves, each says "yes" or "no" until both accept together or time runs out. At the equilibrium each can infer some of the other's knowledge from the other's delays in accepting.

Other procedures to promote agreement include the use of coalitional Shapley values for the practice of European arms negotiators of meeting in caucuses before the plenary [O'Neill (1994a)]. Frey (1974) discusses an "insurance system" to convert PD traps of arms-building into mechanisms that prompt agreements to reduce arms. Vickrey (1978) applies the Clarke tax to induce governments to state their true interests, and Isard's (1965) technique uses metagames. Brams (1990b) puts his negotiating mechanisms in an international context. Singer (1963) and Salter (1986) expand an application originally proposed by Kuhn to make divide-and-choose symmetrical: each side separates its arms into n piles, and each chooses one of the other's piles to be dismantled. The goal is that neither can claim that it has had to disarm more, but what assumptions about how weapons holdings interact to produce security would guarantee a complaint-proof process?

1.4. Models of armsbuilding

When does armsbuilding dissuade the other from arming and when does it provoke? Jervis' answer (1978) involved the "security dilemma", in which each side tries to increase its own security, and thereby lowers the other's. If technology could provide a purely defensive weapon that could not be used for attack, the dilemma would disappear, but most weapons can be used for protection or aggression. Jervis modelled the security dilemma by the Stag Hunt game shown below, where the moves might be passing up some new military system or building it. Governments do end up at the poor equilibrium, each fearing the other will choose the second strategy. (Payoffs are ordinal with "4" the most preferred and the two equilibria are underlined.)

	Refrain	Build
Refrain	<u>4,4</u>	1,3
Build	3,1	<u>2,2</u>

Jervis' paper drew attention to the Stag Hunt, sometimes called Assurance or Reciprocity, which had been overshadowed by PD and Chicken. Sharp (1986), Snyder (1984) and Nicholson (1989) also discuss the Stag Hunt vis-a-vis the security dilemma. It is as old as PD: back in 1951 Raiffa had presented it to experimental subjects and discussed it in never-published parts of his Ph.D. thesis (1952). Sen (1967, 1973, 1974) had applied it in welfare economics, and Schelling (1960) combined it with incomplete information to model preemptive attacks, but otherwise it was ignored. Political scientists like Jervis saw its importance while game theorists were dismissing it as trivial on the grounds that one equilibrium is ideal for both players. Some recent theoretical papers have looked at it following Aumann's (1990) suggestion that the upper left equilibrium may not be self-enforcing [O'Neill (1991c)].

Avenhaus et al. (1991) let each government build offence- or defence-oriented weapons which the other will observe to judge aggressive intentions. My model [O'Neill (1993b)] relates the degree of offensive advantage and the tendency to military secrecy, another theme of the security dilemma literature. Kydd (1994) presents an interesting model where a player's probability distribution over the other's payoff for war is the former's private information.

Lewis Frye Richardson's differential equations for an arms race have fascinated so many scholars, it is not surprising that they have been incorporated into differential games [Deyer and Sen (1984), Gillespie and Zinnes (1977), Simaan and Cruz (1973, 1975a,b, 1977), van der Ploeg and de Zeeuw (1989, 1990), Gillespie et al. (1975, 1977), see also Case (1979), Moriarty (1984), Saaty (1968) and Amit and Halperin (1987)]. Melese and Michel (1989) analyze the closed-loop equilibrium of a differential game of arms racing, and show that reasonable assumptions about a player's payoff stream as a function of its own rate of buildup, the other's holdings, and a time discount factor, can lead to the traditional solutions of Richardson's equations.

A substantial literature addresses repeated PD models of arms races [Brams (1985), Downs et al. (1985), Lichbach (1988, 1989, 1990), Majeski (1984, 1986a,b), Schlosser and Wendroff (1988), McGinnis (1986, 1990), Stubbs (1988)]. "When do arms races lead to war?" is a fundamental question in the non-modelling literature, but it is hard to find a game model that relates mutual armsbuilding and the decision to attack, an exception being Powell (1993). Engelmann (1991) discusses how a country might change its preferences continuously from a PD game to achieve a cooperative outcome. Sanjian (1991) uses fuzzy games for the superpower competition through arms transfers to allies.

Another issue is whether to keep one's arsenal a secret [Brams (1985)]. Should you keep the adversary uncertain, or will your silence be seen as a token of weakness? In Nalebuff's model (1986), McGuire's (1966) and Hutchinson's (1992) secrecy is harmful, and in Sobel's (1988), extended by O'Neill (1993b), keeping secrets would help both, but is not an equilibrium. At the sensible equilibrium the two sides reveal their holdings, even though they would be better off by keeping them secret.

Sobel assumes two aggressive governments, each possessing a military strength known to itself and verifiably revealable to the other. Each decides whether to reveal, and then chooses either to attack or to adopt a defensive posture. If there is a war, whoever has the larger force wins, and losing is less harmful if the loser has a defensive posture. The payoffs to X decrease in this order: (1) X and Y attack, X wins; (2) X attacks and Y defends, X wins; (3) both defend, so there is no war; (4) X defends and Y attacks, X loses; and (5) X attacks and Y attacks, X loses. The payoffs to Y are analogous. A strategy for each must involve a cutoff such that a government will reveal its strength if and only if it is greater than the cutoff. However an equilibrium pair with a non-zero cutoff cannot exist, because a government just slightly below it will not want its strength estimated as that of the average government below. It will reveal its strength, and thus the set of keeping secrecy will peel away from the top end, like the market for lemons.

Some multistage games on the choice of guns or butter are by Brito and Interiligator (1985), Powell (1993), and John et al. (1993). A body of work not surveyed in detail here looks at strategies of weapons procurement [e.g., Laffont and Tirole (1993), Rogerson (1990)]. Also, I will only mention samples from the many experiments or simulations on game matrices representing the arms race. Alker and Hurwitz paper (n.d.) discusses PD experiments; Homer-Dixon and Oliveau's After-MAD game [Dewdney (1987)] is primarily educational; Alker et al. (1980) analyze a computer simulation on international cooperation based on repeated PD; experiments like those of Pilisuk (1984), Pilisuk and Rapoport (1963) and Plous (1987) interpret game moves as arming and disarming, escalating or deescalating; and Lindsfold's experiment (1978) studies GRIT, the tit-for-tat tension-reducing procedure.

1.5. Deterrence and signalling resolve

The puzzle of deterrence has been how to make threats of retaliation credible. When they involve nuclear weapons, they are usually not worth carrying out. One example was NATO's traditional threat to use tactical nuclear arms in Europe if it was losing a conventional war. Far from saving Europe, this would have devastated it beyond any historical precedent. The difficulty is this: to trust deterrence is to believe (1) that the adversary will not act suicidally, and (2) that the adversary

thinks that you might react suicidally. Game models did not cause this tension, they just made it more obvious.

One response is Chain Store-type models of “reputation”, where the other holds some small credence that threatener would actually prefer to retaliate, and the threatener is careful not to extinguish this possibility as the game is repeated [Wilson (1989)]. However, the assumptions of repeated plays and evolving reputations do not fit nuclear crises. Issues change from one to the next, government leaders come and go, and in the case of the ultimate nuclear threat, the game will not be repeated. Security scholars have taken other approaches to the credibility problem, following especially Schelling (1960, 1966a, 1967). He gave two alternatives to brandishing threats of total destruction. The first is to perform a series of small escalations that gradually increase the pain suffered by both antagonists. The second method involves setting up a mechanism that repeatedly generates a possibility of total disaster, allowing the two sides to compete in risk-taking until one gives in, like two people roped together moving down a slope leading to a precipice. A threat to commit murder-and-suicide might be ignored, but one to incur a chance of it might work. These schemes assume too much information and control to really be implemented [O'Neill (1992)], but they became the paradigms in nuclear deterrence theory. For United States nuclear strategy they generated attempts to set up “limited nuclear options”, and deploy weapons that fill in “gaps in the escalation ladder”.

Powell (1987a,b, 1989a,b,c, 1990, 1994) makes Schelling's ideas mathematically rigorous with sequential games of incomplete information. A government's resolve can be high or low, and if it stays in the conflict, the other will raise its estimate of this resolve. The mainstream literature uses term “resolve” vaguely but Powell's models justify its definition as a ratio of certain utility increments, the ratio involved in determining who will prevail. He discusses how perceptions of resolve, not just actual resolve, influences the outcomes how the information held by the parties affects the likelihood of inadvertent war, and how a change in the size of the escalatory steps alters the likelihood of war. Nalebuff's related model (1986) has a continuum of escalatory moves.

The common concept of *showing* resolve is full of puzzles: Clearly the weaker you are the more important it is to show resolve, so why are aggressive actions not interpreted as signs of weakness [Jervis (1988)]? In many cases, weak and strong alike would make a display of resolve, perhaps because the action costs little, and because no action would be taken surely to mean weakness. But if the weak always act, what rational basis is there for inferring weakness from inaction? Inferences from moves no one would make are the domain of equilibrium refinement theory, and Nalebuff (1991) applies it in a simple model of promoting reputation. A situation presents you with the choice of Acting or Not Acting, and acting confers on you a benefit of x , known to all, and a cost of c , known only to you. Letting $c = c_A$ or c_N be everyone else's estimate of your cost following their

observation of you choosing Act or Not Act, you will receive a "reputation for toughness" payoff of $a(1 - c)$, where $a > 0$. Thus, Act yields you $x - c + a(1 - c_A)$ and Not Act, $a(1 - c_N)$.

Nalebuff's structure is not strictly a game since one side simply observes, but it shows how different equilibrium theories restrict others' belief about your cost. Assuming that $x = \frac{1}{4}$, $a = \frac{1}{2}$, and that the audience holds a uniform distribution for c before they observe your move, one sequential equilibrium calls for Acting if $c < \frac{1}{2}$, for Not Acting otherwise, so those who can act cheaply will do so. In another, you should choose Act no matter what your type, since it is assumed that others are disposed to believe someone choosing Not Act has the maximum cost of 1. A further sequential equilibrium calls for Not Acting for all types, since it is assumed that if you were to Act, others would believe that your cost is 1, that your eagerness to prove yourself betrays weakness. The requirement of rationalizability rejects the equilibrium Not Act for any cost. The most that Act could benefit you through reputation is $\frac{1}{2}$, so if others see you Act, they should decide that your cost must be no higher than $\frac{1}{2}$, certainly not 1. A cost of $\frac{1}{2}$ is the same cost as they would estimate for you if you chose Not Act, so even when others' beliefs are worst for you, Acting does not change your reputation, so Act only if it is worth it disregarding reputation, i.e., if $c < \frac{1}{4}$. Nalebuff eliminates other equilibria using universally divine equilibria and perfect sequential equilibria. His approach adds to our understanding of both refinement theories and deterrence. Looking at a specific application, some government's attempt to signal, we can ask whether the refinement is reasonable to admit or reject some beliefs, and we are led to seek out historical evidence on how the target of the signal actually received it. The political science literature has mostly neglected the latter question, and the few who have examined it, e.g., Thies (1980) on U.S. signalling during the Vietnam war and George and Smoke (1974) on Chinese intervention in Korea, have not supported the elaborate schemes of military buildups and violence that governments have used as signals.

Referring to the 1990 war of insults between Bush and Saddam Hussein, I discuss displays of resolve, including a volatile "war of face", where the sides use amounts of their prestige as "bids" for a prize [O'Neill (1994b)]. The game is like a war of attrition with incomplete information about the values of the prize, except that only the loser must pay. Fearon (1992b,c) develops a model where the loser alone must pay, but has the outside option of going to war. Fearon (1990) and Brito and Intriligator (1987) present costly signalling models, and Banks (1990) gives a general theorem on when greater resolve leads to success in a crisis. O'Neill (1991d) suggests a mechanism behind large powers' preoccupation with commitment over issues of no innate importance, based on an analysis of the chivalric custom of challenges to honor in the Middle Ages.

Kilgour and Zagare (1991a,b, 1993) set up an incomplete information game to compare the conventional literature's concept of credibility with game theory's, and Kilgour (1991) uses it to theorize about whether democratic states are more

warprone. Wagner (1988b, 1992) and Zagare (1990a) use sequential equilibria to answer those political scientists who have criticized deterrence theory from psychological and historical perspectives, an issue also addressed by Mor (1993). Bueno de Mesquita and Lalman (1990a,b) discuss how each side's incomplete information about the other's intentions can lead to a war desired by neither, and argue that the confirmation of deterrence theory may be in part an artifact. Quester (1982) uses matrices to categorize ways that wars can start, using historical examples. Stein (1980, 1982b) looks at various incidents from the viewpoint of conditional rewards and punishments, and at types of misperception. Wilson (1986) gives a repeated game model of attacks and counterattacks in which a sequential equilibrium directs each side to maintain a positive probability of striking at each trial. Zagare (1985, 1987b) axiomatizes the structure of mutual deterrence situations to derive that it is a PD game, but Brams and Kilgour (1988b) critique his assumptions and end up at Chicken.

To analyze partial credibility of a deterrent threat, O'Neill (1990a) posits a threatener whose probability of retaliation increases with the damage done by the adversary and decreases with the cost to the threatener of retaliating. The model assesses one justification put forward for "coupling", NATO's conscious attempt to ensure that a conventional war in Europe would trigger a global nuclear war. Wagner (1982) and Allan (1980, 1983) think of deterrence as negotiation over the boundaries of power, over what rivals may do without facing retaliation. Examining multi-sided threats, Scarf (1967) assumes that each government allocates some weapons specifically against each of the others, and that each configuration of all parties' allocations yields a vector of ordinal utilities. Adapting his n -person solution theory (1971), he states assumptions guaranteeing a core allocation of military threats. His theory is developed by Kalman (1971) and modified to a sequential game by Selten and Tietz (1972).

In O'Neill (1989a, 1994b) I extend Kull's "perception theory" explanation (1988) of why the superpowers built far more nuclear weapons than needed for deterrence and chose weapons designed for unrealistic counterforce wars. Each side knows that warfighting ability and superiority in numbers are irrelevant, and knows that the adversary knows this, but they are unsure that the adversary knows that they know it. Using Aumann's representation of limited common knowledge (1988), the model exhibits belief structures that induce governments to build militarily useless armaments.

Balas (1978) calculates the appropriate size of the U.S. Strategic Petroleum Reserve, proposed to reduce vulnerability to oil embargoes. Compared to a non-game model based on an exogenous likelihood of an embargo, his approach calls for smaller reserves, because it takes account that reserves tend to deter an embargo. Several authors look at deterrence and negotiation with "terrorists" [e.g., Sandler and Lapan (1988)].

Intriligator (1967) and Brito (1972) model a missile war as a differential game, with bounds on the rates of fire and the attrition of missiles and economic targets

governed by differential equations. Intriligator's approach has been widely cited. It was extended by Chaatopadhyay (1969), and applied in many subsequent articles to derive a lower limit on armaments needed to guarantee deterrence.

Two French scholars, Bourqui (1984) and Rudnianski (1985, 1986), considered their country's worry, how a small nuclear power can deter a much larger one. Threats and deterrence in elementary games are treated also by Dacey (1987), Maoz and Fiesenthal (1987), Moulin (1981) Nalebuff (1988), Reisinger (1989), Schelling (1966b), Snyder (1971), Stefanek (1985), Nicholson (1988), Bueno de Mesquita and Lalman (1988, 1989), Fogarty (1990), Gates and Ostrom (1991), Langlois (1991a,c) Rudnianski (1991) and Cederman (1991). Some matrix games portray the ethical dilemmas of nuclear weapons [Hardin (1983, 1986a), Gauthier (1984), Woodward (1989)]. More general discussions of games and deterrence are by Nicholson (1990), Kugler and Zagare (1987, 1990), P. Bracken (1984), P. Bracken et al., (1984), and Gekker (1989b), and O'Neill (1989b) surveys the goals of game models of deterrence.

McGinnis (1992) critiques signalling applied to deterrence, and O'Neill (1992) suggests that game models are biased towards armsbuilding in that they usually start after the crisis is in progress, and also ignore the promise half of deterrence: if you don't attack me, I won't attack you.

1.6. *The myth that game theory shaped nuclear deterrence strategy*

One myth about game models and deterrence is worth refuting in detail. It is that in the late 1940s and 1950s thinking on nuclear strategy was molded by game theory. By the end of the Cold War this claim was so widely believed that no evidence was needed to support it, and it is often used to dismiss the game theory approach as tried and rejected. Nye and Lynn-Jones (1988) write that as the field of strategic studies was developing, "Deterrence theory and game theory provided a powerful unifying framework for those central issues, but often at a cost of losing sight of the political and historical context" (p.6). The current view, they say, holds that "the abstract formulations of deterrence theory – often derived from game theory – disregard political realities" (p.11). Evangelista (1988, p.270) writes, "the early study of the postwar Soviet–American arms race was dominated by game theory and strategic rational-actor approaches". Horowitz (1970, p.270) states that game theory became the "operational codebook" of the Kennedy Administration. According to Steve Weber (1990), its mathematical elegance made it "almost irresistible" to those trying to untangle nuclear strategy, and it promoted a basic delusion of American policy, that the United States must match Soviet weapons type-for-type. In his highly-praised history of nuclear strategy, Freedman (1983, p.181) states, "for a time, until the mid-1960s, the employment of matrices was the *sine qua non* of a serious strategist".

In fact, with a couple of exceptions, substantial game modelling of international strategy started only in the later 1960s, after the tenets of nuclear strategy had

already developed. Nye and Lynn-Jones give no supporting references, and when Freedman asserts that any serious strategist had used matrix games, can we believe that non-users Brodie, Kahn, Kaufmann, Kissinger or Wohlstetter were less than serious? Hedley Bull, who had associated with the American nuclear strategists, wrote (1967, p.601), “the great majority of civilian strategists have made no use at all of game theory and indeed would be at a loss to give any account of it”. Licklider’s survey of 177 non-governmental nuclear strategists found only two mathematicians (1971). Donald Brennan (1965), a prominent strategist at the Hudson Institute, estimated that his center’s reports, classified and unclassified, might fill twenty feet of shelf space, but “the number of individual pages on which there is any discussion of concepts from game theory could be counted on one hand; there has been no use at all made of game theory in a formal or quantitative sense... gaming is much used by nuclear strategists, game theory is not”.

Aiming to show game theory’s influence, Freedman presents the 2×2 Chicken matrix as one used by strategists of the 1950s and early 1960s. Chicken did not appear until the mid-1960s, and it is ironical that its sources were not nuclear planners, but two noted peace activists, Bertrand Russell and Anatol Rapoport. In 1957 Russell used the teenage dare as an analogy for the arms race, and in 1965 Rapoport defined “Chicken” as the now-familiar 2×2 matrix, [O’Neill (1991c)]. Russell’s highway metaphor was prominent in the early discussions of strategists Schelling, Kahn and others, but the matrix game had not yet been defined.

When one tracks down cited examples of game theory’s influence, most fall into one or another type of non-application. Some employ wargaming or systems analysis, rather than game theory. Some deal with military tactics, like submarine search or fire control, rather than strategy and foreign policy. Some use parlor games like monopoly, chess or poker as metaphors for international affairs, but give no formal analysis. Some only explain the principles of mathematical games and hope that these might lead to a model [e.g., Morton Kaplan, (1957)]. Others construct matrices, but analyze them using only decision theory. In particular, Ellsberg (1961) and Snyder (1961), writing on deterrence, and Thomas Schelling (1958a, 1960) on threat credibility, calculated no game-theoretical equilibria. A decision-maker in their models maximized an expectation against *postulated* probabilities of the adversary’s moves. It was just to avoid assuming such probability values that minimax strategies and equilibria were developed. The adversary’s move probabilities are not assumed but *deduced* from the situation. George and Smoke’s influential book explains Snyder’s 1961 deterrence model in a “modified version” (1974, p.67). Referring back to the original, one finds that the modification was to drop Snyder’s postulated probabilities of a Soviet attack, thereby turning his decision model into a game. Their treatment gives the impression that IR game theory began earlier than it really did.

Fred Kaplan’s lengthy attempt (1983) to link nuclear strategy to game theory focuses on the RAND Corporation. He describes the enthusiasm of mathematician

John Williams to assemble a team there, but evidently those hopes never materialized; RAND's applications addressed mainly military tactics. The only arguable exceptions that have appeared were Schelling's preemptive instability model (1958a) written during a sabbatical year at RAND, and the borderline game model of Goldhamer, Marshall and Leites' secret report on deterrence (1959), which derived no optimal strategies. In Digby's insider history of strategic thought at RAND (1988, 1990), game theory is barely mentioned [see also Leonard (1991)].

A milder claim was that game theory set an attitude; strategists approached their problem with game theory principles in mind, by considering the adversary's perspective. As an example of game reasoning, Kaplan recounts Albert Wohlstetter's early 1950s argument that American bomber bases near the Soviet Union were vulnerable to surprise attack. This is no more than thinking from the enemy's viewpoint, which must be as old as warfare.

As a criterion for what counts as a "game model", one should expect at least a precise statement of assumptions and a derivation of the parties' best strategies, but there are almost none of these in the early nuclear strategy literature. The only such models published were Morton Kaplan's on deterrence (1958) and Schelling's use of 2×2 matrices to explicate trust and promises, and study first-strike instability, in *The Strategy of Conflict*. Schelling's book as a whole has been enormously influential, but its formal models had small effect and were left untouched by other authors for many years [e.g., Nicholson, 1972]. Schelling interspersed the models with shrewd observations about tacit bargaining and coordination in crises, but he did not derive the latter from any formal theory. His informal ideas were beyond extant mathematics and in fact the reverse happened: his ideas spurred some later mathematical developments [Crawford (1990)].

The claimed link between game theory and the bomb began in the late 1940s with suggestions that the new logic of strategy would reveal how to exploit the new weapon. It was bolstered by von Neumann's visible activity in defense matters, as an advisor to the US government on nuclear weapons and ICBMs [Heims (1980)], and later Chairman of the Atomic Energy Commission. The Hollywood movie *War Games* depicted a mathematical genius whose supercomputer controls the U.S. missile force and solves nuclear war as a poker game. Likewise Poundstone's *Prisoner's Dilemma* (1992) intersperses chapters on von Neumann's life, nuclear strategy and game theory basics. A reader might conclude that von Neumann connected the two activities, but nowhere does the author document von Neumann ever applying games to the Cold War. On the contrary, according to his colleague on the Air Force Scientific Advisory Board, Herbert York (1987, p.89), "although I frequently discussed strategic issues with him, I never once heard him use any of the jargon that game theorists and operations analysts are wont to use – 'zero-sum game', 'prisoner's dilemma' and the like – nor did he ever seem to use such notions in his own thinking about strategy... he always employed the same vocabulary and concepts that were common in the quality public press at the

time". Morgenstern's writings on military matters included parlor game analogies and mathematical-sounding terms (e.g., 1959), but no models.

During the 1960s the debate on the validity of nuclear strategy reinforced the myth. The critics attacked game theory and nuclear strategy together, as acontextual, ahistorical, technocratic, positivistic and counter to common sense. Supporters saw game theory as right for the atomic era, when wars must be planned by deduction rather than based on historical cases. The antis opposed a method that apparently prescribes behavior in complex situations with no role for any experiences, intuitions or goals that cannot be formalized. They saw its mathematical character as a warning that the same technocrats that gave the world the nuclear arms race, now wanted control over decisions on war. The unfortunate name "game theory" suggested that international politics was a frivolous pastime played for the sake of winning, and the idea of a "solution" seemed arrogant. [Examples include Maccoby (1961), Aron (1962), and Green (1966) as attackers; Wohlstetter (1964) and Kaplan (1973) as defenders; and Schelling (1960) and Rapoport (1964, 1965, 1966) as reformers.]

The debate proceeded along these lines, but neither group pinpointed just how game theory had influenced nuclear thinking. The best critiques of strategic thought [Rapoport (1964), Green, 1966] did not assert that game theory was widely used to formulate it. Rapoport's thesis was more subtle, that knowledge of formal conflict theory's existence creates an expectation that nuclear dilemmas can be solved by a priori reasoning. Many other writers, however, asserted a link without specifying just how game theory was involved [e.g., Zuckermann (1961), Horowitz (1962, 1963a,b, 1970), Heims (1980), Morris (1988)].

The notion that game theory has already been tried probably dampens current interest and provides an excuse to deemphasize it in graduate programs. Some appropriate mathematical techniques, like incomplete information, equilibrium selection in extensive games and limitations on common knowledge, were invented fairly recently, and have seen application only in the last few years.

1.7. First-strike stability and the outbreak of war

In his model of preemptive instability (1960), Schelling published the first non-trivial game application to international strategy, a full-fledged game of incomplete information. Starting with a Stag Hunt game, he assumed that each side assigns some probability that the other might attack irrationally, and traced the payoff-dominating equilibrium as this fear grows. Nicholson (1970) extended his model.

Game theory has contributed by sharpening existing informal concepts, sometimes suggesting a numerical measure. Examples are Harsanyi's definition of power (1962), Axelrod's formula for conflict of interest (1969), O'Neill's measure of advantage in a brinkmanship crisis (1991c), Powell's formula for resolve (1987a,b, 1990), and various measures for the values for the worths of weapons, discussed

later. O'Neill offers a formula for degree of first-strike instability (1987), the temptation for each government to attack due to fear that the other is about to, fuelled by a military advantage from attacking first. The decision is cast as a Stag Hunt, and the measure is applied to the debate on space-based missile defences. Scheffran (1989) uses this formula as part of a computer simulation of arms-building dynamics, and Fichtner (1986a) critiques it. Bracken (1989b) proposes an alternative measure. Interest in missile defences in space has generated about three dozen quantitative papers asking whether these systems increase the temptation to strike [Oelrich and Bracken (1988)], and more recent work has analyzed the stability of arms reduction proposals [Kent and Thaler (1989)]. In many cases their authors were grappling with a basically game-theoretical problem, but were not ready to use the right technique, and built inconsistencies into their models [O'Neill (1987)].

Brown (1971) and Callaham (1982) offer early uses of game theory to analyze instability. In Nalebuff's model of first-strike instability (1989), government A holds a conditional probability function stating a likelihood that B will strike expressed as a function of B's assessed likelihood that A will strike. Some exogenous crisis event impels these beliefs up to an equilibrium. Powell (1989a) gives assumptions on a game of escalation will complete information such that there will be no temptation to strike, and Bracken and Shubik (1993a) extend his analysis. Franck's (1983) work is related. Wagner (1991) gives a model intended to rationalize US counterforce policy.

Responding to the depolarization of the world, Bracken and Shubik (1991, 1993b) and Best and Bracken (1993a,b) look at the war consequences of all possible coalitions of the nuclear powers in regard to preemptive stability, the later study including many classes of weapons, using very large and fast computers to solve game trees with tens of millions of nodes. They sometimes reproduce a result of the theory of truels, three-person duels, that the weakest party may hold the best position. A different approach is Avenhaus et al. (1993).

Greenberg (1982) represents the decision of how to launch a surprise attack, and O'Neill (1989c) models the defender's decision when to mobilize in the face of uncertain warning, using the stochastic processes methods on quickest detection. The goal is to produce a measure of the danger of a war through a false alarm, expressed as a function of the probability parameters of the warning systems.

1.8. *Escalation*

Two views of escalation regard it as either a tool that each side manipulates, or a self-propelling force that takes control of the players. Models with complete information and foresight tend to the former picture, but those that place limits

on the governments' knowledge and judgement usually predict that they will be swept up into the spiral of moves and responses. An example of the former is the dollar auction. This simple bidding game resembles international conflict in that both the winner and the loser must forfeit their bids. O'Neill (1985, 1986, 1990b), Leininger (1989), Ponsard (1990) and Demange (1990) discuss the solution, in which one bids a certain amount determined by the sums of money in the pockets of each bidder, and the other resigns. The rule is unintuitive and counter to the experimental finding that people do bid up and up, but it shows one role of game theory, to establish what constitutes rational behavior within a set of assumptions, in order to identify a deviation to be explained.

Langlois (1988, 1989) develops a repeated continuous game model postulating limited foresight in escalation: each side acts not move by move, but chooses a reaction function, a rule for how strongly it will respond to the other side's aggressiveness. He derives conditions for an equilibrium pair of functions, investigates when a crisis would escalate or calm down, and argues for the repeated game approach in general. Langlois (1991b) discusses optimal threats during escalation. O'Neill (1988) portrays limited foresight using the artificial intelligence theory of game-playing heuristics, to show that under some conditions players fare worse when they consider more possible moves or look further down the tree. Zagare (1990b, 1992) gives some complete information games of escalation.

In a series of models, most collected in their book, Brams and Kilgour (1985, 1986b, 1987a,c,d, 1988a,b, 1994) ask how threats of retaliation can support a cooperative outcome. They investigate multistage games involving plays of continuous versions of a Chicken or Prisoner's Dilemma matrix, where players adopt degrees of cooperation, followed by possible retaliation. In an empirical test, James and Harvey (1989) compare the Brams and Kilgour model with several dozen incidents between the superpowers, and Gates et al. (1992) compare an extended deterrence model with military expenditure data. Uscher (1987) compares repeated PD and Chicken models of escalation. Powell (1987a,b; 1990) analyzes crisis bargaining in the context of his escalatory model, and Leng (1988) uses the theory of moves to discuss the learning of bargaining stances from the experience of past crises. Further studies on escalation are by Brown, et al. (1973) and Ackoff et al. (1968). Hewitt et al. (1991) generate predictions from a simple extensive game model, and test them on 826 cases over the last seventy years.

Morrow (1989b) considers bargaining and escalating as complementary moves during a crisis, and has each side learn about the other's capacity for war through its delay of an agreement or its willingness to fight a small conflict. In a follow-on paper (1992a), he asks whether each state can learn enough about the adversary to avoid the next crisis. A block to resolving a crisis is that any side initiating a compromise may be seen as weak. Morrow (1989a) looks for information conditions that lessen this problem, which is the diplomatic analogue of a military first-strike advantage.

1.9. *Alliances*

Game models of alliances have focused on four issues: who joins in coalitions, which alliance structures promote peace, who pays for the "public good" of defense and how much do they buy, and where do allies set the tradeoff between giving up their freedom of action and acquiring support from others.

Arguing informally from coalition-form game theory, William Riker (1962) suggested that a smallest winning coalition will be the one that forms. He discussed the implications for national and international affairs. In the theory of national politics his idea has been very influential, but has been followed up only sparingly in the international context [Siverson and McCarty (1978), Diskin and Mishal (1984)].

How will nations respond to an increase in one party's power? Will they "bandwagon," i.e., join the powerful country to be on the winning side, or will they "balance," form a countervailing alliance against the threat? Much of U.S. foreign policy has followed the notion that small countries will bandwagon. Arosalo (1971), Chatterjee (1975), Young (1976), Don (1986), Guner (1990, 1991), Maoz (1989a,b), Nicholson (1989), Luterbacher (1983), Hovi (1984), Yu (1984), Dudley and Robert (1992), Niou and Ordeshook (1986, 1987, 1989a,b) and Niou et al. (1989), Lieshout (1992), Linster (1993) and Smith (1993) have addressed this and related questions using mostly coalitional games, the last-mentioned authors stating a solution theory developed from the nucleolus and bargaining set. Their book operationalizes and tests the theory using large European power relations from 1871 to 1914. The recent formal literature on coalition formation exemplified in Roth's volume (1988) has not been applied to military alliances, nor to "peaceful alliances," like the signators of the treaty banning certain nuclear tests and the treaty against proliferation of nuclear weapons. Gardner and Guth's paper (1991) is the closest work.

Wagner (1987) addresses the traditional question of the optimal number of players to produce balance-of-power stability. Selten (1992) invented a board game that reproduces balance-of-power phenomena and is simple enough to be analyzed mathematically. Zinnes et al. (1978b) give a differential game model of n -nation stability. Kupchan (1988) represents relations within an alliance by several 2×2 matrix games, while Sharp (1986) and Maoz (1988) discuss the dilemma between gaining support through an alliance versus getting entangled in a partner's wars. O'Neill (1990a) uses games of costly signalling to clarify the idea of "reassurance" in alliances. Allies invest in militarily pointless arms to show each other that they are motivated to lend support in a crisis. Brams and Mor (1993) consider the victor in a war who may restore the standing of the loser in hopes of a future alliance.

Weber and Wiesmeth (1991a,b) apply the notion of egalitarian equivalent allocations, and Sandler and Murdoch (1990) set a Lindahl allocations model and a non-cooperative game model against actual contributions to NATO. Guth (1991) applies his resistance avoidance equilibrium refinement. Bruce (1990) gives the

most fully game theoretical model of burden sharing: unlike most economic models he includes the adversary's reactions as well as the allies'. Morrow (1990a, 1994) and Sorokin (1994) deal with the decision to give up one's autonomy in an alliance in return for a less than fully credible promise of help.

1.10. Arms control verification

Game-theoretical studies of verification divide into two groups. The first and earlier involves decisions about allocating inspection resources or a quota of inspections limited by treaty. The second ask whether to cheat and whether to accuse in the face of ambiguous evidence.

The first group, on how to allocate inspections, began in the early 1960s. In the nuclear test ban talks the American side claimed seven yearly on-site inspections were necessary, but the Soviets would not go above three, citing the danger of espionage. The negotiations stalled and a complete ban has still not been achieved. Maximizing the deterrent value of each inspection should please a government worried about either compliance or espionage, and this goal motivated the work of Davis (1963), Kuhn (1963), Dresher (1962) and Maschler (1966, 1967). Their inspector confronts a series of suspicious events, and each time decides whether to expend a visit from a quota, knowing that there will be fewer left for the next period. Rapoport (1966) gives a simple example. The problem is somewhat like a search game extended over time rather than space, and Kilgour (1993) in fact has the players select places, rather than times to violate. Extensions involve non-zero-sum payoffs [Avenhaus and von Stengel (1991)] or several illegal acts [von Stengel (1991)]. Maschler's papers showed the advantage to an inspector who can publicly precommit to a strategy, a theme developed by Avenhaus and Okada (1992), Avenhaus et al. (1991) and Avenhaus and von Stengel (1993). Moglewer (1973), Hopfinger (1975), Brams, et al. (1988) and Kilgour (1990) extend quota verification theory, and Filar (1983, 1984) and Filar and Schultz (1983) treat an on-site inspector who has different travel costs between different sites, these being known to invader who plans accordingly. Some recent work has examined the monitoring of materials in nuclear energy plants, in support of the Non-Proliferation Treaty [Avenhaus (1977, 1986), Avenhaus and Canty (1987), Avenhaus et al. (1987), Bierlein (1983), Zamir (1987), Avenhaus and Zamir (1988), Canty and Avenhaus (1991, Canty et al. (1991), Petschke (1992)].

A challenging puzzle from the first group has a violator choose a time to cheat on the $[0,1]$ interval, simultaneously with an inspector who allocates n inspection times on the interval. The inspector gets a free inspection at $t = 1$. The payoff to the violator is the time from the violation to the next inspection, while the inspector receives the negative of this. Diamond (1982) derives the mixed strategy solution, not at all trivial, where the draw of a single random variable fixes all n inspection times [see also von Stengel (1991)].

Almost all studies in the first group are zero-sum, like Diamond's game. At first it seems odd that a government would be happy to find that the other side has violated, but the model's viewpoint is not the inspecting government's but that of its verification organization. The latter's duty is to behave *as if* the game were zero-sum, even to assume that the other side will violate, as Diamond's game and others do. Verification allocations are to peace what military operations are to war, and are more oppositional than the grand strategy of the state.

The second group of verification studies takes the governmental point of view, with non-zero-sum payoffs and a decision of whether, rather than how, to violate and whether to accuse. Maschler's early paper (1963) is an example. Evidence is sometimes represented as a single indicator that suggests innocence or guilt [Frick (1976), Avenhaus and Frick (1983), Fichtner (1986b), Dacey (1979), Wittman (1989), Weissenberger (1991)], dichotomously or on a continuum of evidential strength [O'Neill (1991b)]. Brams, Kilgour and Davis have developed several models [Brams (1985), Brams and Davis (1987), Brams and Kilgour (1986a, 1987a), Kilgour and Brams (1992)], as has Chun (1989), Bellany (1982) and Avenhaus (1993). Weissenberger's model (1990) jointly determines the optimal treaty provisions and the verification system. O'Neill (1991b) proves the odd result that with more thorough verification, the inspector may be less certain about the other's guilt after observing guilty evidence. With better verification technology, the inspector's prior, before the evidence was that the other would not dare to violate, and this aspect can outweigh the greater strength of evidence of guilt from the verification scheme.

Most research has concentrated on these two areas, but many more verification issues are amenable to game theory. Future arms treaties will more often be multilateral and it would be impractical for each government to send inspectors to monitor every treaty partner, but relying on third parties introduces principal-agent issues. A second example involves the claim that as stockpiles are greatly reduced, verification must be more and more thorough since a small evasion will give a bigger advantage. How would a game model develop this idea? Another example arose from past treaties: Should you make an accusation that requires proof when that proof compromises intelligence sources needed to spot a future violation? A final topic involves a Soviet concern that was recurrent since the testban talks: how to balance between effective verification and military secrecy.

2. Military game theory

Progress in military game theory has gone mostly unknown beyond that community. While some studies are available in *Operations Research and Naval Research Logistics Quarterly*, many have been issued as reports from government laboratories, strategic studies institutes or private consulting firms. Much of the American work is not publicized or is stamped secret, sometimes placed in classified journals like *The Proceedings of the Military Operations Research Society Symposium*

or *The Journal of Defense Research*. Secrecy obstructs the circulation of new ideas so even game theorists with a security clearance can be unaware of their colleagues' work.

Most of the research below is from the United States. I do not know how much has gone on elsewhere, hidden by classification. Some writers claimed that the Soviet Union was advanced in military game applications, but there seems to be no direct evidence about this either way. Some Soviet texts included military game theory [Ashkenazi (1961), Suzdal (1976), Dubin and Suzdal (1981)], but their treatment was in no way deeper or more practical than Western works (Rehm, n.d.). This review will concentrate on applications in the United States, where access to information is the freest.

The social value of military game theory has been controversial. To use mathematics to make killing more efficient strikes one as a travesty, and it also seems pointless, since each side's gains can usually be imitated by the adversary, with both ending up less secure. On the other hand, a logical analysis might counter governments' natural instincts to build every possible weapon in great numbers, since, for all anyone can say, it might confer some benefit some day. Analysis could prompt officials to think clearly about their goals and evaluate the means more objectively. One policy document that used game theory, Kent's Damage Limitation Study, had this effect, and helped to slow the arms race.

General discussions with military examples are given by Dresher (1961b, 1968), Shubik (1983, 1987), Thomas (1966), Finn and Kent (1985) and O'Neill (1993c). The NATO symposium proceedings edited by Mensch (1966) give a good sample of problems. Leonard (1992) recounts the history of military interest in game theory, and Mirowski (1992) argues, unconvincingly in my view, that military concerns have shaped game theory as a whole.

2.1. Analyses of specific military situations

In 1917 Thomas Edison came remarkably near to devising the first game model of a real military situation. His game was well-defined and analytically solvable, but practically minded as he was and lacking a solution theory, he had people gain experience playing it and he examined their strategies [Tidman (1984)]. The problem was how to get transport ships past German U-boats into British ports. He divided a chart of the harbor waters into squares, forty miles on a side, to approximate the area over which a submarine could spot the smoke rising from a ship's funnel. One player placed pegs in the squares corresponding to a path of entry, and the other player on a separate board placed pegs representing waiting submarines, defining a game of ambush [Ruckle (1983)]. He concluded that the last leg of the Atlantic crossing was not as dangerous as had been thought.

The next world war prompted a full solution theory of a game of anti-submarine patrols, possibly the first full game study on record of a real problem beyond

board games and gambling. Philip Morse, who had overseen the wartime Anti-submarine Warfare Operations Research Group, posited a sub running a corridor. To recharge its batteries it must travel on the surface for some total time during its passage [Morse and Kimball (1946)]. The corridor widens and narrows, making anti-submarine detection less and more effective. Mixed strategies were calculated for how to surface and how to concentrate the search. The measure-theoretic foundations were complex and it took Blackwell (1954) to add the details. Allentuck (1968) believes that anti-submarine game theory was decisive in winning the Battle of the North Atlantic, but his evidence is indirect, and the theory most likely never guided a real decision [Morse (1977), Tidman (1984), Waddington (1973)].

Perhaps the most influential analysis of a specific situation was Goldhammer, Marshall and Leites' secret report on nuclear counterforce policy [1959; see also O'Neill (1989b)]. They polled fellow RAND strategists to get their estimates of Soviet utilities for various nuclear war outcomes, and described a tree of several stages. However they kept their tree unsolvable by omitting many payoffs and the probabilities of chance moves, leaving out in particular the utility for an American surrender. A more recent extensive form model on overall strategic policy is by Phillips and Hayes (1978).

Successfully capturing simplicity and reality is Hone's matrix analysis (1984) of the naval arms race during the years between the world wars. Each side could build ships emphasizing armor, firepower, speed or range, each quality being differentially effective against the others, as in Scissors, Paper and Stone. He applied Schelling's technique (1964) of categorizing payoffs as simply high, medium or low. [(A similar method had been devised independently for a secret 1954 British Army report on the optimum gun/armor balance in a tank, Shephard (1966)]. Schellenberg (1990) terms these *primitive* games and lists the 881 2×2 types.

One "historical" episode was analyzed as a game but has now been exposed as fabricated to preserve a military secret. The 1944 defeat of the German Seventh Army near Avranches had been attributed to the astute foresight of the American commander, but documents released later showed that the Allies had secretly intercepted Hitler's message to his general [Ravid (1990)]. The analyses of Haywood (1950, 1954), Brams (1975) and Schellenberg (1990) would be tidy applications if the event had ever happened.

Game theorists sometimes apologize for recommending mixed strategies, but Beresford and Preston (1955) cite how real combatants used them. In daily skirmishes between Malaysian insurgents and British overland convoys, the insurgents could either storm the convoy or snipe at it, and the British could adopt different defensive formations. Each day the British officer in charge hid a piece of grass in one hand or the other, and a comrade chose a hand to determine the move. It is not clear whether they generated their procedure themselves or owed it to von Neumann and Morgenstern.

2.2. Early theoretical emphases

In the late 1940s at the RAND Corporation, Lloyd Shapley, Richard Bellman, David Blackwell and others worked on the theory of *duels*. Two sides approach each other with bullets of specified accuracies. Each must decide when to fire: shooting early gives you a lower chance of hitting but waiting too long risks that your opponent shoots first and eliminates you. In other versions, the bullets are silent (the opponent does not know when one has been fired), or noisy (the opponent hears your shots, and will know when your ammunition is gone), or there is continuous fire, or several weapons on each side, or a time lag between firing and hitting. The pressing problem then was defence against atomic bombers: When should an interceptor open fire on a bomber, and vice versa [Thomas (1966)]? The destructiveness of atomic weapons justified zero-sum, non-repeated-game assumptions: the payoff was the probability of destroying the bomber, with no regard for the survival of the fighter. Dresher (1961b) gives the basic ideas and Kimmeldorf (1983) reviews recent duelling theory. A related application was the optimal detonation height of attacking warheads facing optimally detonating anti-missile interceptors [Kitchen (1962)].

Also close to duels were *games of missile launch timing*, investigated at RAND in the late 1950s. The first intercontinental missiles were raised out of protective underground silos or horizontal “coffins,” prepared and launched. During the preparation they would be more vulnerable to attack. Several papers [Dresher (1957, 1961a, pp. 114–115), Johnson (1959), Dresher and Johnson (1961), Thomas (1966) and Brown’s camouflaged piece (1957)] considered a missile base that chooses a series of firing times, versus an attacker who chooses the times for its warheads to arrive at the base. A warhead destroys a missile with a given probability if it is still in its silo, and with higher likelihood if it is outside in preparation for firing.

By 1962 the Strategic Air Command had begun deploying missiles fired directly out of silos, so the launch timing question lost some relevance. Now SAC began worrying about “pin-down”. Soviet submarine missiles lacked the accuracy to destroy silos but they could explode their warheads above the North American continent in the flyout paths of US missiles. This would create an “X-ray fence” damaging the electronics of missiles flying through it, so US weapons would be stuck in their silos until Soviet ICBMs arrived. Keeping up a complete blockade would exceed Soviet resources, so the sub missiles would have to be used prudently. How should SAC choose an optimal fire-out strategy against an optimal Soviet fire-in strategy? For many years the game’s solution guided actual plans for launching rates [Zimmerman (1988), p. 38].

Games of aiming and evasion [Isaacs (1954)] are still relevant today, and still largely unsolved. In the simplest version a bomber attacks a battleship that moves back and forth on a one-dimensional ocean. It turns and accelerates to its maximum velocity instantly. The plane drops a bomb in a chosen direction; the bomb reaches

the surface after a time lag, destroying anything within a certain distance. The ship knows when the bomb is released but not where it is heading. Finding the approximate mixed strategy solutions is not too difficult: if the radius of destruction is small compared to the interval the ship can reach, the aim point and ship's position should be chosen from a uniform distribution. However a slight change makes the problem more interesting: suppose the ship does not know when the bomb is dropped. It must then make its position continuously unpredictable to the bomber, who is trying to guess where it will be a time lag from now [Washburn (1966, 1971)]. A discrete version where the ship moves on a uni-dimensional lattice yields some results when the duration of the bomb's fall equals one, two or three ship movements [Lee and Lee (1990, and their citations)]. The modern story would involve an intercontinental missile attacking a missile-carrying train or a submarine located by underwater sensors.

Several papers addressed the strategy of naval *mine-laying and detecting*. In Scheu's model (1968), the mine-layer leaves magnetic influence mines or time-actuated mines to destroy the searcher, and the searcher may postpone the search and the advance of ships until some of the mines have exploded. Another post-war focus involved *ordnance selection*, such as the armament of a fighter plane that trades off between cannons and machine guns. Fain and Phillips' working paper (1964) gives an example on the optimal mix of planes on an aircraft carrier.

Reconnaissance problems received much attention [Belzer (1949), Blackwell (1949), Bohnenblust et al. (1949), Danskin (1962a), Drescher (1957), Shapley (1949), Sherman (1949)] in regard to the value of acquiring information for a later attack, and the value of blocking the adversary's reconnaissance attempts. Tompkins' *Hotspot game* [Isbell and Marlow (1956)] is an elegant abstract problem suggested by military conflict, involving allocation of resources over time. Each side sends some integral number of resources to a "hotspot" where one or the other loses a unit with a probability determined by the two force sizes. When a unit disappears, either side sends more until one player's resources are exhausted.

A class of conflicts suggested by warfare is *Colonel Blotto*, where players simultaneously divide their resources among several small engagements. Borel stated a simple version in 1921, not long before he became France's Minister of the Navy. Each player divides one unit among three positions and whoever has assigned the greater amount to two out of the three positions, wins. Although the game is easy to state, some mixed strategy solutions found by Gross and Wagner (1950) have as their supports intricate two-dimensional Cantor sets. Soviet researchers contributed to the theory [Zauberman (1975)], which is reviewed by Washburn (1978).

Military applications of Blotto games turned out to be rarer than expected. There seem to be few real contexts where unforeseen numbers of forces from both sides show up at several locations. Two applications are Shubik and Weber (1979, 1982) on the defence of a network, in which they introduce payoffs that are not additive in the engagements, and Grotte and Brooks' (1983) measure of aircraft

carrier presence. An application of Blotto-like games involves missile attack and defence, treated next.

2.3. *Missile attack and defence*

Missile defence theory has had an important influence on arms policy. Some work had been done at the RAND Corporation [Belzer et al. (1949), Dresher and Gross (1950), Gross (1950a,b)] but the most significant model has been Prim–Read theory, named for its two originators. In a simple version, an attacker sends missile warheads to destroy a group of fixed targets, and a defender tries to protect them using interceptors, which are themselves missiles. At the first stage the defender divides its interceptors among sites, and plans how many to send up from each targeted site against each incoming warhead. The attacker, knowing the allocation and the firing plans, divides its warheads among the sites, and launches the missiles in each group sequentially at their target. If an attacking warhead gets through, it destroys the site with certainty, but each interceptor succeeds only with a probability. There is no reallocation: if the attacker has destroyed a site, further weapons directed there cannot be sent elsewhere, nor can any interceptors be shifted. The payoff is the sum of the values of the target destroyed.

The recommended rule for allocating the defence interceptors, the Prim–Read deployment, typically does not divide the defences in proportion to the values of the targets, but induces the attacker to send warheads in roughly those proportions. A remarkable result is that defensive deployments exist that are best independent of the number of attacking weapons, so the defender need not know the opposing force size. The basics were laid out by Read (1957, 1961), and Karr (1981) derives some uniqueness and optimality results. Examples of modern applications are by Holmes (1982) and Bracken and Brooks (1983).

Prim–Read theory was used in a report that may have helped dampen the arms competition. In 1964, US Air Force General Glenn Kent completed his secret “Damage-Limitation Study” on how to make nuclear war less destructive. One proposal was to build a vast anti-missile system. With Prim–Read theory, Kent’s group derived a supportable estimate of a defence’s effectiveness, and the verdict was negative: an antimissile system would be expensive to build, and cheap to thwart by an adversary’s increasing the attacking force. Kent’s study bolstered the case for a ban on large missile defence systems [Kaplan (1983)], and in 1972 the two powers signed the Anti-Ballistic Missile Treaty. It continues in force today, probably the greatest success of nuclear arms control, a move to an equilibrium Pareto-superior to a more intense race in defensive and offensive weapons.

Few policy decisions are on record as influenced by game theory, but we have to separate innate reasons from institutional ones. Perhaps the opportunity was there to apply the theory, but there was no “inside client”, no right person in place

to carry it through. Kent had a reputation for innovativeness, had associated with Schelling at Harvard and had written on the mathematics of conflict (1963). When he gained a position of influence, he used the theory effectively.

Prim-Read theory in Kent's study assumes that the attacker knows the defender's firing schedule. This is questionable, but the opposite premise would require a mixed strategy solution. Mixed strategies are more complicated, and were alien to the operations researchers working on these problems, who were trained on optimization and programming. McGarvey (1987), however, investigated the case. Another branch of missile defence theory alters another part of Prim-Read, by assuming that the attacker is unaware of the defender's allocation. The attitude has been that Prim-Read theory treats attacks against population centers and the alternative branch deals with attacks on missile sites. Strauch's models (1965, 1967) also assumed perfectly functioning interceptors, and thus were formally Blotto games. Other versions [Matheson (1966, 1967)] had imperfect interception, so each side had a strategic decision beyond the Blotto allocation, of how many interceptors to send up against each attacking warhead, not knowing how many more would be descending later on the particular target. Unlike the Prim-Read case, a deployment that is ideal against an all-out attack may be poor against a smaller one, but Bracken et al. (1987) found "robust" defence allocations that for any attack level stay within a certain percentage of the optimum. Burr et al. (1985) find solutions without the approximation of continuously divisible missiles. Matlin (1970) reviewed missile defence models, about a dozen of which involved games, and Eckler and Burr (1972) wrote a thorough report on mathematical research up to that time. Many of the studies should have treated their situations as games, but used one-person optimization, and so were forced to assume inflexible or irrational behavior by the other player.

Ronald Reagan's vision of anti-missile satellites shielding an entire country bypassed the mathematically fertile study of allocation among defended sites. New game models appeared more slowly than one would expect from the bountiful funding available, but recent examples are by O'Meara and Soland (1988), as well as several authors they reference, on the coordination problem of guarding a large area. Another theme is the timing of the launch of heat decoys against a system that attacks missiles just as they rise from their silos.

2.4. Tactical air war models

Strategy for tactical air operations has preoccupied many military operations researchers and for good reason. Each side would be continually dividing its fighter-bombers among the tasks of attacking the adversary's ground forces, intercepting its aircraft and bombing its airfields. The two sides' decisions would be interdependent, of course, since one's best choice today depends on what it expects the other to do in the future. Through the Cold War there was concern

that the aircraft allocation rule would play a large role in a conventional war in Europe, but dynamic analyses of the “conventional balance” sidestepped it. Some large computer models stressed air activity, some the ground war, often as a function of the sponsoring military service, but few addressed their interaction. An exception was the program TAC CONTENDER, forerunner of the current RAND program TAC SAGE [Hillstead (1986)], which was thought to find an optimal strategy, until Falk (1973) produced a counterexample.

Satisfactory tactical air models might bolster each side’s feelings of security and prompt it to reduce its forces, but so far the problem has been solved only in simplified versions. Research began quite early [Giamboni et al (1951), and Berkovitz and Drescher later succeeded in solving a complicated sequential game of perfect information [(1959, 1960), Drescher (1961b)]. The dilemma is to make a problem realistic and computationally manageable, and there have been three approaches: Lagrange multipliers as in TAC CONTENDER, which provide only a sufficient condition for optimality; grid methods, applied by Dantzig [Control Analysis Corporation (1972)], with the same shortcoming; and solutions of trees of matrix games [Bracken et al. (1975), Schwartz (1979)], which are sure to be optimal but allow models of fewer stages.

2.5. *Lanchester models with fire control*

Lanchester’s widely-known equations mean to describe the rates at which two armies in combat destroy each other. They assume a homogeneous force on each side. Extending them to a mixed force, like infantry and artillery, introduces the question of how to allocate each component against each of the adversary’s components. Almost always the approach has been to divide the forces in some arbitrary proportion that stays fixed throughout the battle, but a clever opponent might beat this by concentrating forces first on one target and later on the other. Accordingly, one stream of work treats the allocation-of-fire decision as a differential game constrained by Lanchester’s equations. In Weiss’s fine initiating study (1957, 1959), each side could direct its artillery against the other’s artillery or ground forces, but the ground forces attacked only each other. Whereas air war applications are usually discrete, with planes sent out daily, fire control games tend to be continuous. Taylor (1974) made Weiss’s results more rigorous, and Kawara (1973), followed up by Taylor (1977, 1978) and Taylor and Brown (1978), adds an interesting variant in which the attacker’s ground forces shoot while they move towards the defender’s line, but with “area fire”, not precisely aimed because they are moving. The artillery bombardment must cease when the two sides meet, so the each side tries for the highest ratio of its forces to the other’s by that time. Moglewter and Payne (1970) treat fire control and resupply jointly [see also Sternberg (1971) and Isaacs’ *Game of Attack and Attrition* (1965)]. Taylor (1983, Ch.8) gives an excellent summary of the field.

2.6. Search and ambush

In a *search game*, a Searcher tries to find a Hider within some time limit or in the shortest time. Sometime the Hider is moving, like a submarine, sometimes fixed, like a mine. As described earlier, the research began in World War II but has found few applicable results, perhaps because of the difficulty of incorporating two-or three-dimensional space. The bulk of applied search theory today is only "one-sided", postulating a non-intelligent evader located in different places with exogenous probabilities, but several dozen papers solving abstract games are listed by Dobbie (1968) and the Chudnovskys (1988). Gal's book (1980) finds some strategies that approach minimax as the search area becomes large with respect to the range of detection. Lalley and Robbins (1987) give examples of the peculiar hide-and-seek behavior that can arise as a minimax solution. One variant requires the evader to call at a port within a time interval [Washburn (1971)], and Dobbie (1968) suggested a game where both sides are searchers, a ship trying to rendezvous with its convoy. A problem apparently not yet investigated as a game allows the hider to become the seeker, such as a submarine versus a destroyer.

Related to search games are continuous *games of ambush*, which Ruckle investigates in his book (1983) and a series of articles in *Operations Research*. Usually the Ambusher does not move, so the game is easier to solve than a search. One or both players typically choose some geometrical shape. A simple example has the Evader travelling from one side of the unit square to the other, while the Ambusher selects a set of prescribed area in the square, and receives as payoff the length of the path that lies in the area. A less elegant but more practical example [Randall (1966)] involves anti-submarine barriers. Danskin on convoy routing (1962b) is also relevant.

Simple to state but difficult to solve is the *cookie-cutter game*. The target chooses a position within a circle of unit radius while the attacker chooses the center of a circle with given radius $r < 1$, and wins if that circle contains the target. The case of $r = 0.5$ is elegantly solvable [Gale and Glassey (1975)], but for r less than approximately 0.4757 little is known [Danskin (1990)].

2.7. Pursuit games

Pursuit problems [Isaacs (1951, 1965), Hayek (1980)], studied in the 1950s to improve defence against atomic bombers, sparked the whole area of differential games. Pursuit game theory has grown steadily and a recent bibliography [Rodin (1987)] lists hundreds of articles. Typically the Pursuer tries to come within capture distance of the Evader, but in some games the angle of motion is important, as in the case of maneuvering fighters. Solutions of practical pursuit problems are hard to obtain and hard to implement, since human behavior cannot be programmed second-by-second. Aerial dogfights involving complete turns are too complicated, and add

the wrinkle that the Evader is trying to switch roles with the Pursuer. However, strategies optimal for sections of an engagement have been calculated, with the goal of abstracting general rules [Ho (1970)]. A more feasible application is the control of maneuvering nuclear warheads and interceptor missiles. The short engagement times and the interceptors' high accelerations mean less maneuvering, so their movements can be represented by simpler kinematic laws. Even though they can be analysed mathematically, the weapons are not in wide use; as of recent years the only maneuvering warheads facing antimissile interceptors were British missiles aimed at Moscow. Discussions of differential games in military operations research are given by Isaacs (1975), Ho and Olsder (1983) and Schirlitzki (1976).

2.8. *Measures of effectiveness of weapons*

An issue of practical interest is how to assign numbers to represent the military worths of weapons. The goal is to generate indices of overall strength, in order to assess the "military balance", to investigate doctrine that directs battle decisions of what should be "traded" for what, to score war games, or to set fair arms control agreements. Most non-game-theory procedures work from the bottom up, estimating the qualities of individual weapons based on their design features, then adding to evaluate the whole arsenal, but clearly additivity is a questionable assumption here since some weapons complement each other and others are substitutes. A game-theoretical approach might start with the benefits of having a certain arsenal and infer back to the worths of the components. Pugh and Mayberry (1973)[see also Pugh (1973) and Assistant Chief of Staff, USAF Studies and Analysis (1973)] treat the question of the proper objective function of war, following Nash's general bargaining model, to argue that each side will try for the most favorable negotiated settlement by conducting war as strictly competitive. Anderson (1976) critiques their approach, and discusses games where the payoffs are the ratios of military strengths. O'Neill (1991a) applies the Shapley value, regarding the weapons as the players in a coalitional game. The characteristic function displays non-monotonicity whenever the enemy's armaments join the coalition. Robinson's important papers (1993) link the traditional eigenvalue approach in which values are defined recursively as functions of the values of the adversary's weapons destroyed, with the values calculated from payoffs of a game. Both methods face the problem of non-uniqueness. These models do not lead to a measure for specific decisions, but do clarify the informal debate.

2.9. *Command, control and communication*

Some writers have analyzed the contest of a jammer versus a transmitter/receiver team. Fain's early report (1961) on the tradeoff between offensive forces and

jamming units was belittled by the other conference participants, but more studies followed. Some discuss a jammer who chooses a signal with a limit on its bandwidth and power [Weiss and Schwartz (1985), Stark (1982), Helin (1983), Basar (1983), Basar and Wu (1985), Bansal and Basar (1989)]. Others posit a network where the transmitter can route the communication a certain way, and the jammer attacks certain nodes or links [Polydoros and Cheng (1987)]. Other game analyses have looked at the authentication of messages [Simmons (1981), Brickell (1984), and the identification of an aircraft as friend or foe [Bellman et al. (1949)]. These papers may be the tip of a larger classified literature.

McGuire (1958) assumed unreliable communication links connecting ICBMs and discussed the tradeoff between retaliating in error, e.g., in response to an accidental breakdown of a link, versus not retaliating after a real attack. For a given network, the attacker plans what to target and the defender plans what instructions for retaliation to issue to base commanders who find themselves incommunicado. Independently, Weiss (1983) considered missiles that retaliate against target of differing values, where each launch control officer may retarget based on partial knowledge of what other missiles have been destroyed. The retaliator issues plans for the missile commander, contingent on the latter's knowledge of what others remain, and the stiker decides what to attack. In Weiss' view, implementing the theory would bolster a government's deterrent threat without adding weapons that it could use in a first strike; in other words, it would ease the security dilemma. Notwithstanding, the Strategic Air Command and its successor STRATCOM did not transfer this kind of control down to the level of a missile base commander.

2.10. *Sequential models of a nuclear war*

"Nuclear exchange" models' goal has been to suggest the consequences of changing employment plans, adding new weapons or signing an arms agreement. Several models involve the attacker dividing its weapons between the adversary's population and retaliating missiles. The sides maximize some objective functions of the damage to each after a fixed number of attacks. Usually the solution method is a max-min technique, but one exception is Dalkey's study (1965). Early papers used Lagrange techniques on two-stage models and so guaranteed only local optimality, but Bracken et al. (1977) found a method for the global optimum in a class of reasonable problems. Grotte's paper (1982) gives an example with many references. A model by Bracken (1989a) shows the philosophy behind this work, trying to add as much detail as possible. It has twelve stages, where each side allocates missiles among the roles of attacking population targets, attacking military targets and saving as reserve forces, and also decides how much of its missile defence system to hold back for use against future strikes. This discrete game of perfect information has 20 000 000 possible paths, and takes several minutes on a Cray computer.

The Arsenal Exchange Model is the program most used by US government agencies to investigate nuclear war strategies, and it is not game-theoretic. An attempt was made to include optimal choices by both sides, but the task turned out to be too complex, and it is still the user who chooses levels of attack and priorities of targets, whereupon a linear programming routine assigns weapons to targets.

2.11. *Broad military doctrine*

An army ceases to fight before its physical endurance is gone; its psychological breakpoint involves partly the mutual trust and expectations of the soldiers. No one seems to have applied the Stag Hunt, although Molander (1988) used the Prisoner's Dilemma to analyze the cohesion of society as a whole in wartime.

A unique study is by Niou and Ordeshook (1990a) on the military thinking of Sun Tzu. Many of his teachings, set down over two thousand years ago, can be reproduced formally, although he sometimes cut off the logic of think and doublethink at an arbitrary level; for example, he neglected the consequences if your opponent knows your opportunities for deception.

3. **Comparisons and concluding remarks**

Military game theory is structured more like a field of mathematics, each writer's theorems building on those of others. IR game models have tended to take their problems from the mainstream literature rather than each other, and so have touched on an immense number of questions, as this survey shows. Only recently have identifiable research streams of game models formed, such as PD models of cooperation, signalling analyses of deterrence in a crisis, and models of relative versus absolute gains as national interests.

The early theory of military tactics emphasized simple games with continuous strategy spaces, as appear in Karlin's (1959) and Drescher's (1961d) books. Now the growing practice is to approximate very detailed problems as finite and solve them by computer, but the old theorems were elegant and helped show a situation's strategic structure more than a giant linear program could. IR models, in contrast, are becoming more mathematically sophisticated. Most authors began as social scientists and learned the mathematics later on their own, so many of the earlier pieces surveyed here are at the level of "proto-game theory" [O'Neill (1989b)], using the concepts but no formal derivations, often setting up the game but not solving it. Now techniques like signalling models are percolating through from economics and the papers state their assumptions precisely and derive theorems.

While IR models have tried to clarify theoretical debates, military models have been used as sources of practical advice. The view has been that the models can never include enough factors to be precisely true, but they give general principles

of good strategy [Dresher (1966)], resonant with the idea of military doctrine. The target-defence principle of Prim–Read theory, for example, says never assign defences that induce the adversary to assault a defended target while sending nothing against an undefended one. It is related to the “no soft-spot principle” in general studies of defence allocation. Tactical air war models continually suggest that the superior force should split its attack, while the inferior should concentrate on a place randomly chosen.

Like these examples of military principles, game models have impressed some important truths on the IR discipline: international struggles are not zero-sum, but justify cooperative acts; good intentions are not enough for cooperation, which can be undermined by the structure of the situation; building more weapons may increase one's danger of being attacked.

The reference section shows about twenty-five publications with models from the 1960s, sixty from the 1970s and over two hundred from the 1980s. The nuclear strategy debate of the 1950s and 1960s made the theory controversial and more prominent. Despite the accusation that game theory was a tool of the Cold War, many in the peace research community became users. After a lull in the early 1970s, the activity of Brams and his colleagues helped keep games visible to IR theorists. In the 1980s worries over Reagan's buildup and hawkish statements triggered a boom in security studies in general, and more interest in game models. The staying influence of Schelling's 1960 book, still required reading in most graduate programmes, has given game theory a presumption of importance, and Axelrod's work has promoted access to it by the non-mathematical majority. Articles are appearing more often in the main political science journals.

With the end of the Cold War, attention is moving to subjects like international political economy, alliances and verification, and new models are appearing at a steady or increasing rate. However the attitude of many political scientists is still hostile. The need for mathematical knowledge blocks progress: tens of thousands of undergraduates hear about the Prisoner's Dilemma each year, but those graduate students who want serious applications usually find no mentor in their department. Many IR scholars misunderstand game theory profoundly, rejecting it as the ultimate rational actor theory, repeatedly explaining its “innate limitations”, or, like Blackett quoted above, castigating it because it cannot make precise numerical predictions, a test no other theoretical approach can pass.

How should we judge Blackett's point that if game theory were practical, practical people would be using it? He claims that if it does not apply to a simple context like gambling, it must be irrelevant to complicated international issues. In fact game theory clarifies international problems exactly because they are more complicated. Unlike card games, the rules of interaction are uncertain, the aims of the actors are debatable and even basic terms of discourse are obscure. What does it mean to “show resolve”? What constitutes “escalation”? What assumptions imply that cooperation will emerge from international anarchy? The contribution of game models is to sort out concepts and figure out what the game might be.

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