El Farol Bar Problem

ENM140 Game Theory and Rationality
Project Report

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1 Introduction

The El Farol bar problem was introduced by Brian W. Arthur (1994) [1] as a simple model to investigate financial markets. It was inspired by the El Farol bar in Sante Fe, New Mexico which offered music on Thrusday nights.

The El Farol bar problem is stated as, N people must decide independently each week whether to go to the bar or not. The space in the bar is limited and the night is enjoyable if the bar is not overcrowded i.e, less than $R\%$ of the population N goes to the bar. Since there is no concrete way to know the number of agents going to the bar in advance; a agent deems it worthy of going to the bar if fewer than the $R\%$ of N agents show up in the bar or stays at home if the agents expects more than $R\%$ to show up.

The bar in the problem is a resource subject to congestion, therefore making the problem a version of problems in public economics, such as those in traffic congestion and congestion on the Internet. Since these congestion are experienced in modern economics a deeper understanding may help us develop methods for understanding how resources leading to congestion can be better managed in modern economics.

The El Farol bar Problem has been extended to the so called minority game where players must choose between two options and those on the minority side win. In this paper, the problem is modelled using an agent based simulation based on Arthur [1] and the following cases are observed:

- How symmetric and asymmetric Nash equilibria emerges in the finitely repeated game?
- How agents could benefit from playing a pure strategy when other agents play a mixed strategy?

2 Game Description

The El Farol bar problem is a simultaneous game. The capacity of the bar is represented by C.

- If less than C players choose to go the bar, then the players going to the bar receive a payoff of G, making it worthwhile for them going to the bar. The payoff received is strictly greater than the players staying at home.
- If C or greater players choose to go the bar, then the players going to the bar receive a payoff of B, making it not worthwhile for them going to the bar. The payoff received is strictly less than the payoff received had the players stayed at home.
- The players staying at home receive a payoff of S. The payoffs are strictly ordered as $G > S > B$. The payoff matrix is shown in Table 1

Table 1: El Farol Bar Payoff

		State of the Bar	
		Uncrowded	Crowded
Player *i*	Go to Bar	G	B
	Stay at home	S	S

3 Simulation Model

The game is modelled as an agent based simulation where each agent has decision making capability by looking at the game history.Explicitly, the agent can either choose to go to bar(action +1) or stay at home(action -1) with the help of "Agent Brain" during each game in this finitely repeated Bar problem setup. The payoff assigned to the agent at the end of every game is derived from the Table 1.

The strategy space for the game is predefined based on all possible outcomes and actions. The agents are allowed access all of the strategy space at any given time and also can rate each specific strategy after every game. In this game setup the number previous game outcomes(Memory size) an agent can look is fixed at the beginning because of the memory constraint. A clear representation of the how strategies can look like for a Memory size 2 can seen in the figure 1.

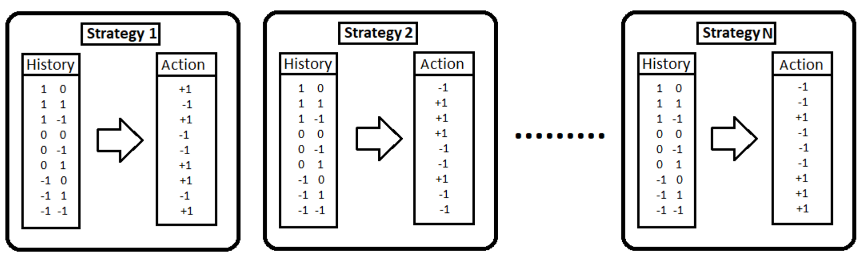

Figure 1: Memory size(m) = 2 and Number of strategies(N) = 2^{3^m}

Agent Brain:

The "Agent Brain" is divided into three sub-sections i.e. long term memory, short term memory, decision making unit and reward center [2].

- **Long term memory(LTM):** In the LTM all the strategies are stored where each strategy will have a score σ that starts at zero and will be updated depending on the outcome of each game when that certain strategy is used. These outcomes are either "win", "lose" or "neutral"

- **Short term memory(STM):** In the STM a list of the 'm' most recent outcomes are stored for each agent. Each agent will be given a list of the "m" most recent outcomes. An outcome could either be a "win" represented by a 1 in the list and happens when the agent goes to the bar and the bar is not crowded,"lose" that is represented by a -1 in the list and happens when the agent goes to the bar and it is crowded or "neutral" that is represented by a 0 in the list and happens when the agent stays home, the zero is not depending on if the bar is crowded or not.

- **Decision making unit(DMU):** In the DMU the agent uses the LTM and STM to make the decisions for the agents in each game. The first decision the agent needs to make is to choose a strategy to use. For this decision the agent looks at the point distribution in the LTM and chooses the strategy with the highest score and if there are more than one strategy with the highest score, it will pick one of the highest scoring strategies at random. The second decision it makes is the course of action, it uses the STM to look up the list of "m" previous outcomes and match them to the same history in the chosen strategy. The agent then selects the action linked to the history. Examples of histories and their corresponding actions can be observed in Figure **??**

- **Reward center:** In the Reward center the different strategies are updated depending on the outcomes of the game. A strategy will get one of three scores added to its current score. The score depends on the outcome of the game where the strategy get 1 for "win", 0 for "neutral" and -1 for "lose". Each agent has its own payoff that starts at zero and is updated after each game. After each game a score is added to the agents payoff depending on the outcome of the game which is either 1 for a "win", 0 for a "neutral" and -1 for a "lose".

4 Results

The agent-based model was used to simulate 100 agents deciding whether going to the bar will be rewarding. Going to the bar is rewarding if not more than 60 people go there. The agents' strategies converge towards a mixed strategy when all the agents play mixed strategy based on their memory. The mixed strategy converges towards similar point even during different runs, pointing to a unique mixed strategy equilibrium (as seen in Figure 2 3). The payoff of all the agents also seem to converge when they play the game for large number of times (≈ 10 million games).

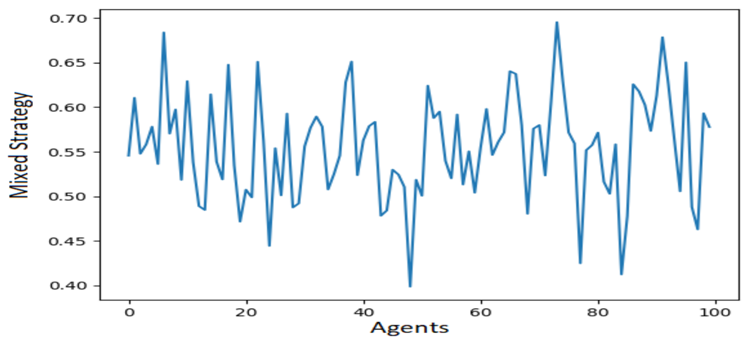

Figure 2: After 10k games

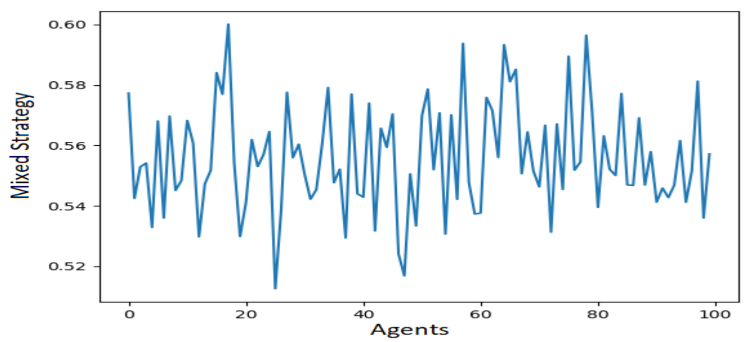

Figure 3: After 100k games

Figure 4: Mixed strategy of agents converge as number of games played increases

5 Discussion

Since there is no interaction between agents and hard to predict the attendance of a large population, a large combination of strategies are possible. According to Whitehead[3], this game has three types of Nash Equilibrium,

- **1. Pure Strategy Nash Equilibria:** The number of pure strategy Nash equilibria is the combination of agents playing pure strategy of either always going to the bar or staying at home. Whitehead showed that the number of such pure strategies is finite.
- **2. Symmetric Mixed Strategy Nash Equilibrium:** According to Whitehead, there is only 1 unique mixed strategy Nash equilibrium when each agent play mixed strategy. Our simulation also confirms that if the

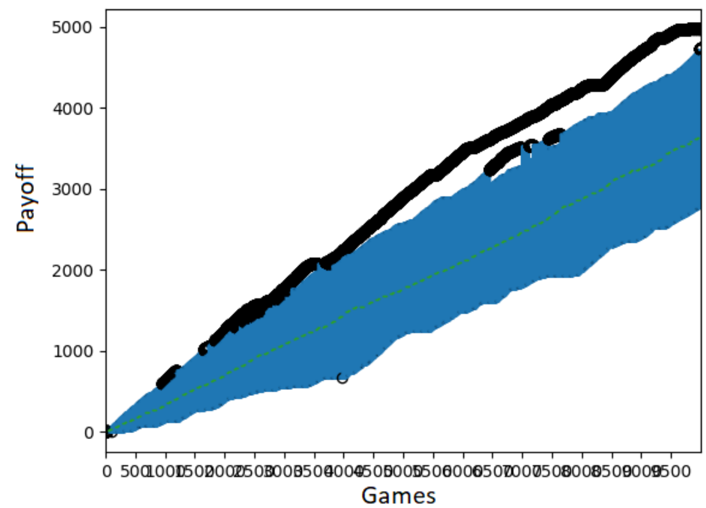

Figure 5: Box plot of payoff distribution over 10k games

game is played large number of times, mixed strategy of all agents converge to single mixed strategy (Figure 2 3).

- **3. Asymmetric Mixed Strategy Nash Equilibria:** Asymmetric mixed strategy Nash Equilibria happen when some agents play pure strategy and others play mixed strategy. Whitehead proved such number of Nash equilibria are countable. We plan to investigate this scenario.

We also prepared model to see: when all agents are playing mixed strategy and approaching Nash Equilibrium, does it pay well to play a pure strategy of going to the bar. The results of the same are not ready yet.

6 Possible extensions

- The present model and the classic El Farol bar game does not allow interaction/communication between agents. It will be interesting to observe results when agents communicate with each other.

- A more complicated form will be with agents who can broadcast their opinion or cheat other agents during communication. From game theory point of view, it will be worthwhile to observe when and how cheating is most effective.

References

[1] W. Brian Arthur. Inductive reasoning and bounded rationality. *The American Economic Review*, 84(2):406–411, 1994.

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[3] Duncan Whitehead. The El Farol Bar Problem Revisited: Reinforcement Learning in a Potential Game. (186), September 2008.