Stochastic Peace war game

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Peace War game - About the game

Two agents

Two actions (Peace and War)

The outcome is probabilistically dependant

The probabilities in the second round are influenced by the decisions of the players in the first round

Peace and War Game- Payoff matrix

In the matrix are presented the various outcomes of our game

In the upper line, Player 1 wins

		Player 2			
		War		Peace	
Player 1	War	8	-6	12	0
		-6	8	-5	0
	Peace	0	-5	2	2
		0	12		

Peace and War Game- Expected payoff Matrix

This matrix presents the expected payoff with 0.5 probability for each player to win.

		Player 2			
		War		Peace	
Player 1	War	1	1	4	0
	Peace	0	4	2	2

Peace and War Game- Stochastic payoff matrix

Expected payoff - Stochastic game

Pure Nash Equilibrium
Mixed Nash equilibrium
Pareto Optimal

(Player 1, Player 2)

Starting Matrix - Expected payoff

	War		Peace	
War	1	1	4	0
Peace	0	4	2	2

100% (Peace,Peace)

P,P P1 50% P2 50%

	War		Peace	
War	1	1	4	0
Peace	0	4	2	2

(Peace,War) 50% 50%

P,W P2 P1 36% P2 64%

	War		Peace	
War	-1	3	1	0
Peace	0	6	2	2

P,W P1 P1 60% P2 40%

	War		Peace	
War	2	0	5	0
Peace	0	2	2	2

(War,War) 50% 50%

W,W P1 P1 72% P2 28%

	War		Peace	
War	4	-2	7	0
Peace	0	0	2	2

W,W P2 P1 28% P2 72%

	War		Peace	
War	-2	4	0	0
Peace	0	7	2	2

(War,Peace) 50% 50%

W,P P1 P1 64% P2 36%

	War		Peace	
War	3	-1	6	0
Peace	0	1	2	2

W,P P2 P1 40% P2 60%

	War		Peace	
War	0	2	2	0
Peace	0	5	2	2

Peace and War Game- Subgame-perfect equilibrium

For the whole game

Starting Matrix - Expected payoff				
	War		Peace	
War	1	1	4	0
Peace	0	4	2	2

Player 1 Wins - 50%

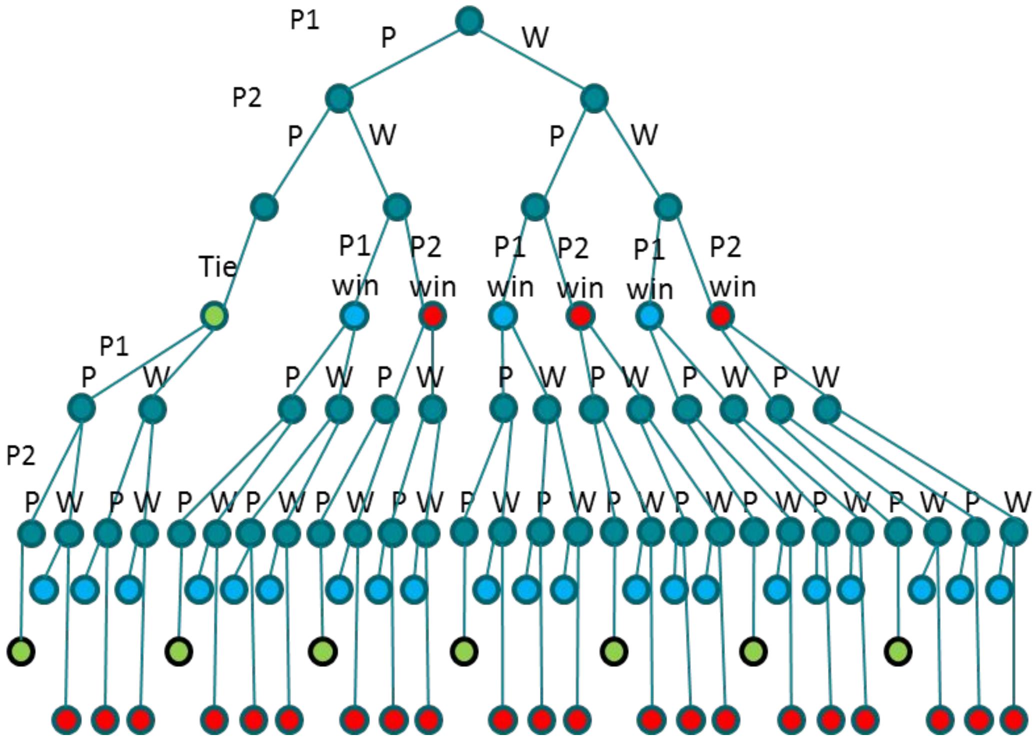
W,W P1	P1	72%	P2	28%
	War		Peace	
War	4	-2	7	0
Peace	0	0	2	2

Player 2 Wins - 50%

W,W P2	P1	28%	P2	72%
	War		Peace	
War	-2	4	0	0
Peace	0	7	2	2

Simulation

EXTENDED FORM



STRATEGIES

[0,0,0,0,0,0,0,0]

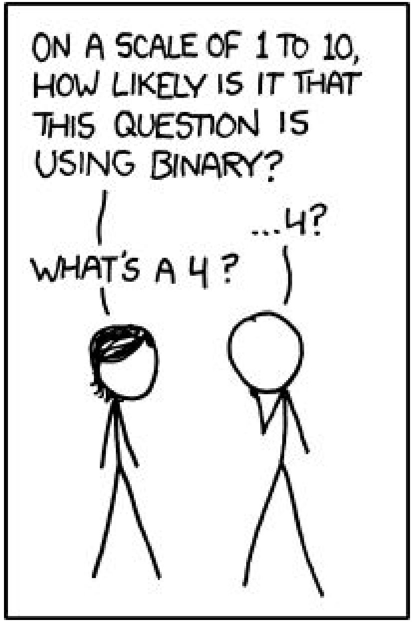
[0,0,0,0,0,0,0,1]

[0,0,0,0,0,0,1,0]

...

[1,1,1,1,1,1,1,1]

Total: $2^8 = 256$



METHOD

Play the game!! (stochastic peace and war)

Players: n = 256

Everyone play against everyone!

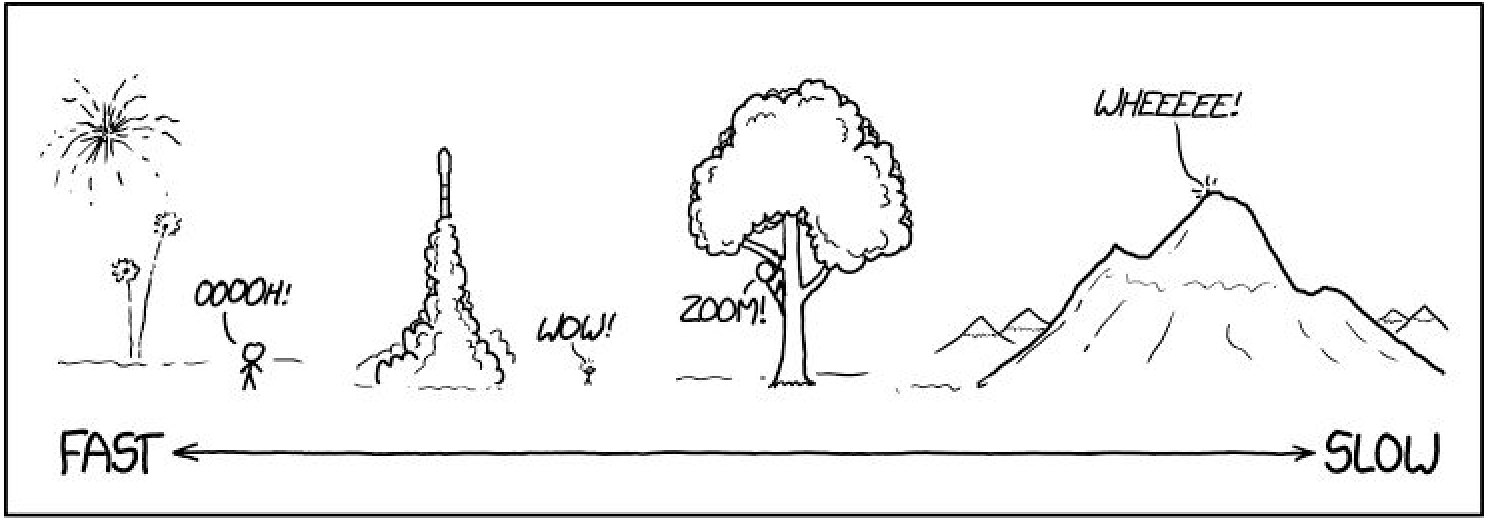
Games: $\sum_{i=0}^{n-1}(n-i) = \frac{n(n+1)}{2} = 32896$

Many games… How to analyse over time?

Evolutionary stable strategy?

Naïve approach? Nope, use replicator dynamics

Replicator dynamics



MOST OF MY INTERESTS FALL UNDER "THINGS RISING UP FROM THE GROUND, HANGING IN THE AIR, AND THEN DRIFTING AWAY ON THE BREEZE," JUST ON VERY DIFFERENT TIMESCALES.

Replicator dynamics

$$\text{Abundance: } x = \begin{bmatrix} x_1(1) & x_2(1) & \dots & x_m(1) \\ x_1(2) & x_2(2) & \dots & x_m(2) \\ \dots & \dots & \dots & \dots \\ x_1(n) & x_2(n) & \dots & x_m(n) \end{bmatrix}$$

$\alpha \approx 0.2$

$$x_t^*(i) = x_{t-1}(i)\Big[1 + \alpha(s_{t-1}(i) - \bar{s}_{t-1})\Big]$$

Fitness

Average population fitness: $\bar{s}_{t-1} = \sum_i s_{t-1}(i) x_{t-1}(i)$

Replicator dynamics

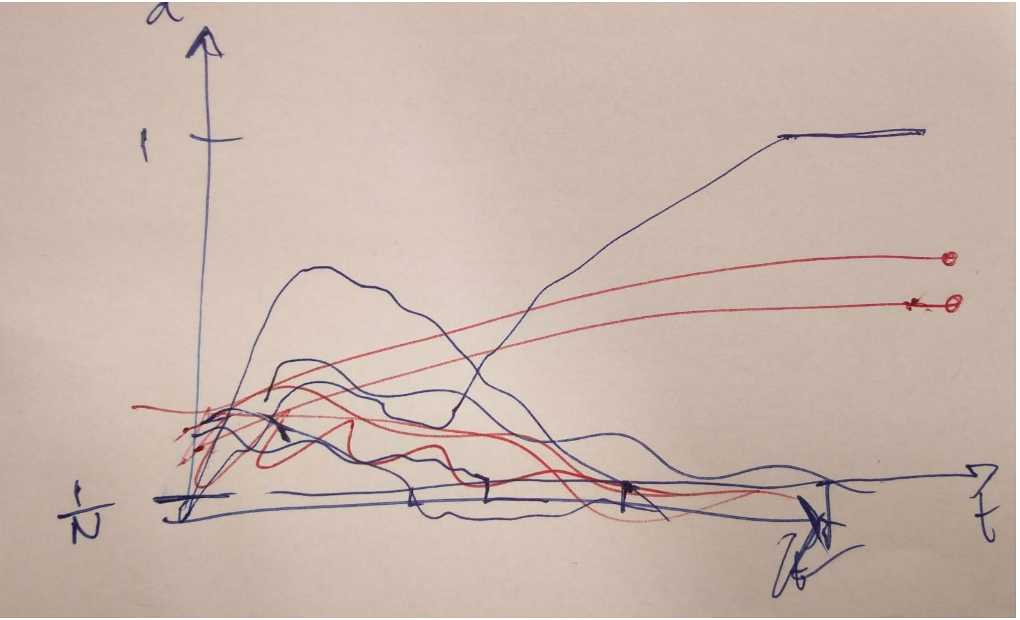
$$x_t^*(i) = x_{t-1}(i)\Big[1 + \alpha(s_{t-1}(i) - \bar{s}_{t-1})\Big]$$

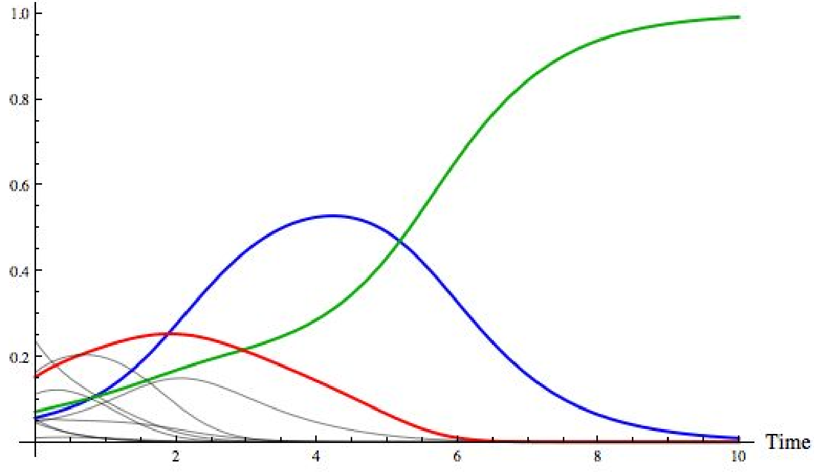
$$x_t^{**}(i) = \frac{x_t^*(i)}{\sum_i x_t^*(i)}$$

$x_t(i) =$ all individuals with $x_t^{**}(i) > \dfrac{1}{N}$, normalised to new population size

$N =$ artificial population size

Expected results of simulation





Peace and War Game

Questions ?