The Peace War Game

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1 Introduction

This report presents the results from the project work of group 10, in the course Game Theory and Rationality at Chalmers University of Technology. The general topic of this report is conflicts between countries, a topic of general concern in these days. Game theory has been used as a general framework for analysis of questions related to conflicts between countries since at least the 1940's when von Neumann analysed issues related to nuclear war. Since then, game theory has been used inter alia for identification of suitable military strategies [3, 6] as well as for analysis of whether war should be waged or not [2, 6]. The latter type of analyses take into account that both war and trade can be used as means to obtain goods, and are called Peace war (PW) games. PW games vary with respect to complexity and type of analysis and can be analysed both in lab experiments as well as with mathematical models. Commonly, PW games are analysed as Prisoner's Dilemmas (PD). Many different aspects of peace and war have been analysed already (for an overview see O'Neill (1994)) but one aspect that seems missing is the uncertainty of outcomes if waging war. Experiences from wars waged in Vietnam (1962-1972), Afghanistan (1979-1989 & 2001-present), and Iraq (2003-2011) show that the outcome when waging war is uncertain, event though one country is military superior. It is therefore relevant to study the impact of uncertainty in outcomes in a PW game.

In addition to earlier studies of the PW game this report introduce uncertainty of outcomes if waging war. What makes this addition interesting is that it adds the possibility to analyse whether uncertainty of outcomes from war will give different results than in standard PD-games, i.e. if peace can be an equilibrium solution. Do the changes that are implemented in the game affect the analysis and if so, then which strategy establishes peace? Is it possible to find a stable strategy that leads to peace? The question asked in this project is: Can Peace be a stable strategy in a stochastic Peace War game?

Due to time constraints the study is limited to two agents with two simultaneous actions in a two stage repeated PD game. The utility from an action is not only dependent on the action of the other player as in standard PD games, but also on stochasticity. The stochasticity represents the chance of winning a war. Given the stochasticity of the utility, the game is also characterised by imperfect information. The game is analysed in numerical form, analytical form, and in an evolutionary simulation using replicator dynamics. This type of game, which includes aspects of repetition as well as stochasticity, is not directly represented in the course literature on game theory. We therefore had to adapt existing theories on repeated and stochastic games to our study.

2 Theory

2.1 Repeated Games

A repeated game is a game which is played multiple times by the same players, the game which is being repeated is referred to as a *stage game*. A finitely repeated game is a game which is played a finite amount of times, where each player does not know what the other player is doing in current stage, but afterwards they do. The strategy of a player can therefore depend on the history of previous plays. The payoff function in a finitely repeated game is usually accumulated. [4]

In a finitely repeated game if can be argued that the dominant strategy in the last stage can be propagated backwards to earlier stages by backwards induction. This is common practise to find Subgame Perfect Equilibrium (SPE) as well as Nash Equilibrium in finitely repeated games. [4]

2.2 Stochastic Games

A stochastic game is a collection of normal-form games played for more than one stage and the outcome at a given iteration depends on the history of the actions the players had chose to play [4]. Neyman and Sorin talked about Stochastic games as a multi-stage game played in discrete time, the stage game played depends upon a parameter called state. The value of the state evolves as a function of its current value and the actions of the players [5].

Neymann and Sorin state that a full set of pure and mixed strategies in these games are rather cumbersome, since they take much information into account [5]. Theorem 6.36 in [4] state that "Every two-player, general-sum, average reward, irreducible stochastic game has a *Nash equilibrium*.".

2.3 Replicator Dynamics

A common approach to analyse games and the behaviour of different strategies over time is to use an evolutionary approach, which is commonly implemented using *Replicator Dynamics*. In general, this method evaluate different individuals performance over time, by playing the game to be analysed against all other players multiple times, where each round is referred to as a generation. An individual in this context refers to an agent with a strategy profile, without memory between generations, as well as the agents abundance of a fictional population size. The abundance reflect how well a certain individual is performing compared to the rest of the population, and affect the individuals possibility to replicate. The abundance is initiated equally between all individuals, then updated after each generation relative to each individual's fitness in comparison to the mean fitness of the entire population throughout that generation. [1, 7]

When the abundance for a certain individual is lower than a threshold, that individual is considered extinct. The replicator dynamics is usually simulated until there is only one individual left alive, or until the abundance individuals reach a steady state. [1, 7]

The general form of the replicator dynamics is given by

$$\dot{x}_i = x_i\Big[f_i(x) - \phi(x)\Big], \tag{1}$$

where $x(i)$ is the abundance of type i in the population $x = [x_1, x_2, ..., x_n]$, f_i is the fitness if type i and ϕ is the average population fitness, given by

$$\phi(x) = \sum_{j=1}^{n} x_j f_j(x). \tag{2}$$

Since the elements in the abundance vector x should sum to a constant in each timestep, the equation is defined on the n-dimensional simplex. This equation can be simplified using matrix notation for easier analysis according to

$$\dot{x}_i = x_i\Big((Ax)_i - x^T Ax\Big), \tag{3}$$

where the payoff matrix A hold precalculated fitness information for the population. The expected payoff can be written as $(Ax)_i$ and the average population fitness as $x^T Ax$. If the abundance of an individual is lower than a threshold, it is commonly considered extinct and its abundance is set to zero, resulting that the individual will not be able to replicate to the next generation. [1, 7]

3 Method

3.1 The Game

As presented earlier, the game explored in this project is a two-player, simultaneous two-action, two-stage PD game with a stochastic utility function. Given the stochasticity, the game is an imperfect information game and suitable actions are identified based on the expected utility (EU) of the action.

The players (P1 & P2) can choose between two actions in each stage game: either to go to War or stay in Peace. If War is chosen, there is a probability (p) of winning the war. The probability is in our analysis directly proportional to relative wealth, as shown in Equation 4,

$$p\text{-win}(P1) = W_{P1}/(W_{P1} + W_{P2}) \tag{4}$$

where W_x is the wealth for player x and p-win is the probability that player wins.

The potential utility for each stage game is seen in Figure 1 and the expected utility for each action if $p = 50\%$ is seen in Figure 2. Expected utility for each action a is for P1 calculated as

$$EU_a = U_{win} * p_a + U_{loose} * (1 - p_a), \tag{5}$$

where U_{win} is the utility if the player wins in an action profile involving war and U_{loose} is the utility if player loose in an action profile involving war.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">P2</td></tr>
<tr><td>War</td><td>Peace</td></tr>
<tr><td rowspan="4">P1</td><td rowspan="2">War</td><td>8 , -6</td><td>12 , 0</td></tr>
<tr><td>-6 , 8</td><td>-5 , 0</td></tr>
<tr><td rowspan="2">Peace</td><td>0 , -5</td><td rowspan="2">2 , 2</td></tr>
<tr><td>0 , 12</td></tr>
</table>

Figure 1: Potential utility, utility if Player 1 wins War (by any player) is given in grey cells.

		P2	
		War	Peace
P1	War	1 , 1	4 , 0
	Peace	0 , 4	2 , 2

Figure 2: Expected utility if both players have the same wealth (p = 50%), rounded values.

As can be seen in Figure 2, the expected utility is identical to a standard prisoner's dilemma ($EU_{w,p} > EU_{p,p} > EU_{w,w} > EU_{p,w}$).

If war is chosen by either player in stage game 1, the probability of winning war in round 2 is updated following the impact on wealth from the outcome of stage game 1. If both players choose to remain in peace the potential utility has probability 1.

In our game, the initial wealth for each player is set to 15. Correspondingly, the wealth after stage game 1 can for each player vary between 8 and 27, dependent on chosen action by both players and the outcome of chance. The wealth after stage 1 is used to calculate the probability of winning war in stage 2. Using equation 1 and values from Figure 1 together with an initial wealth of 15 for each player we can see that the highest possible probability of winning war in stage 2 is 72% and the lowest possible is 28% (if both play War in stage 1). If both players play Peace in stage 1, p for stage game 2 = 50%.

At the end of both stage games (one round) the player with the most wealth wins the game. Maximum wealth (play War against Peace two times and win both times) is 39, and minimum wealth (play War against War two times and lose both times) is 3. Figure 3 shows all 49 potential outcomes of one round in extended form.

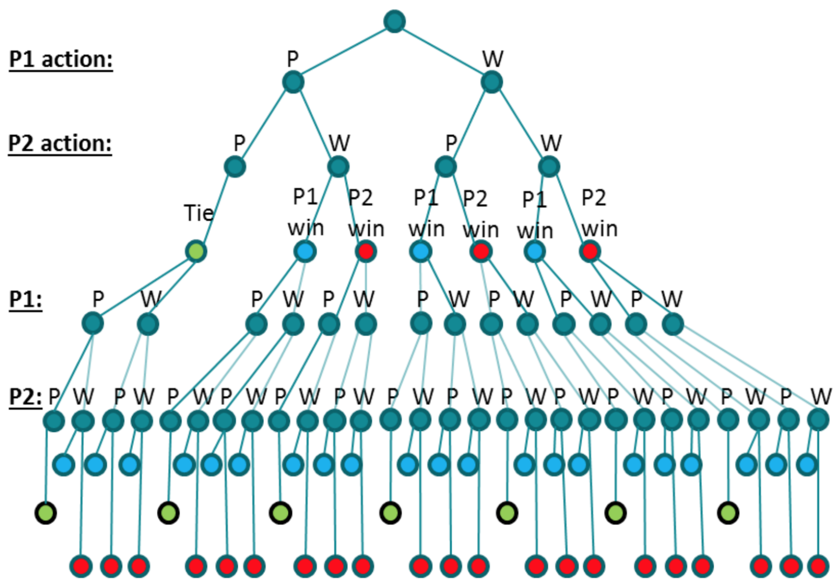

Figure 3: Extended form of the 49 outcomes of the 2-player, 2-action, 2-stage, stochastic Prisoners' Dilemma used for analysing the PW game. Green nodes indicates a tie (peace & peace), blue nodes that P1 wins a war (regardless of who played war), red nodes that P2 wins.

In this report we analyse the game with different approaches. In the first approach we analyse the game based on the numerical estimates of utility, in the second approach we identify the analytical solution of the game, and in the third we analyse the game based on a evolutionary simulation using replicator dynamics. The analytical solution focus on identifying potential Subgame perfect equilibria, Nash equilibria and Pareto optimal strategies. The goal of the evolutionary simulation is to find a strategy that will win the most games and survive for the longest period of time. All approaches used the same setup of the PD, utility Matrix, and probability function as described above.

The game described above is a simplification of other Peace war games. The most important necessary simplifications of our approach are: the description of the economy including only wealth and no production factors; the inclusion of only two countries; finitely repeated in only two stages. To have a simple description of the economy simplifies the possible decisions but also limits the rationales for staying in peace. The inclusion of only two countries makes it impossible to analyse the potential peace-inducing mechanisms of alliances and the previously observed occurrence of trust as a strategy to remain in peace [2].

3.2 The Analytical Solutions

In Game theory Nash equilibrium and Pareto optimality are properties of a strategy profile. In this report however, the Nash equilibria are calculated through the identification of dominating actions based on expected utility, assuming that dominating actions are analogous to dominating strategies in [4] Definition 3.3.1. If both players have strictly dominating actions, the Nash equilibrium is considered strong. Correspondingly, weakly dominating actions will give mixed Nash.

A dominating action is identified through the Boolean operation AND, as in the example for Player 1 playing War below:

Action War by player 1 is dominating if

$$\Big[EU_{p1}(\text{War},\text{War}) > EU_{p1}(\text{Peace},\text{War})\Big] \; AND \; \Big[EU_{p1}(\text{War},\text{Peace}) > EU_{p1}(\text{Peace},\text{Peace})\Big]. \tag{6}$$

Having identified dominating actions for both players, the Nash equilibrium is identified as equal to the dominating action profile for both players. Nash equilibrium is identified for each stage game as well as for each round (both stage games). Corresponding calculations were done for weakly dominating actions if applicable.

Similarly to how the Nash Equilibrium is identified above, pareto optimality is identified for each stage game. To find the pareto optimality, the strategy profiles are identified and if there does not exist another strategy profile that Pareto dominates the other strategy profile then that profile is pareto optimal. This is a method from [4] and definitions 2.1.1 and 2.1.2 in this book are used to identify the Pareto optimality in the game. [4]

Sub-game perfect equilibrium / Sequential equilibrium

According to [4] Subgame-perfect equilibrium is *definite* "The subgame-perfect equilibria (SPE) of a game G are all strategy profiles s such that for any subgame G of G, the restriction of s to G is Nash equilibrium of G" [4]. This means that if a player follows in both stage games the pre-described Nash equilibrium they have a Subgame perfect equilibrium. To have a subgame perfect Nash equilibrium of the whole game then you must find a solution that no player wants to deviate from his action.

This Peace War game has a high complexity level that gives a good reason to have different approaches to analyse the game. The second one will be explained here. In this part a two round game has been put into a Normal form with every possible utility of the game.

3.3 Simulation

Game Representation

The game is evaluated by comparing the respective strategies for the two players. The strategies of War and Peace are represented as 0 and 1 respectively. If both players have strategy 0, the corporation payoff will be returned to both players deterministically. But if at least one player have strategy 1, the outcome will be determined probabilistically. The payoff will be returned together with a state variable depending on which terminal state the first round ended. There are seven terminal state in the first stage, two states for each of the probabilistic strategy pairs, and one state for the deterministic state. The initial state, the state where the player makes its first choice of strategy is labeled state 0, the terminal states are labeled from 1 to 7, shown in Figure 4, resulting in 8 state variables.

Both players have equal probability of winning in the first stage, and the winning and losing payoffs according to Figure 1 will be returned. The utility of each player will be stored and used in the second stage game, where the probability of winning a probabilistic game is relative to the respective utility, according to Equation 4. The utilities of stage two will be added to the utility from stage one for each player, thus creating an accumulated two stage utility, which will be the outcome of the two-stage round.

Strategies

As mentioned in previously, each strategy is represented by 0 for War and 1 for Peace. There is one choice in the first round of the game, but the choice for the second round depend on the outcome in the first round.

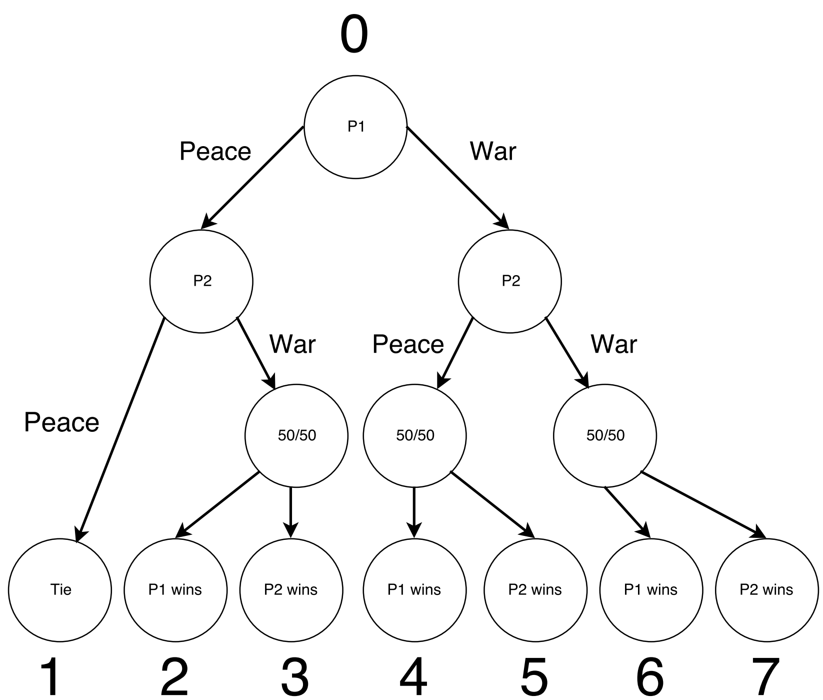

Figure 4: The first stage in the peace war game marking each of the eight states.

There exist seven terminal states in round one, hence seven different strategy possibilities for the second round. One strategy profile therefore consist of eight choices - one choice for round one, and seven for round two, one for each terminal state. The strategy profiles are thereby chosen to be represented by an eight-bit binary array. The possible strategy profiles include all combination of strategies in the eight-bit array, even those containing abundant information, e.g. if peace is chosen by one player in the first round, state four to seven will never be reached. This is for simplicity and completeness reasons. The amount of possible strategies are therefore $2^8 = 256$.

Replicator Dynamics

This project evaluates the strategies in the peace game by replicator dynamics, introduced in Section 2.3. The matrix notation seen in Equation 3 was used for easier and faster analysis, giving the abundance on the following form after finished simulation

$$x = \begin{bmatrix} x_1(1) & x_2(1) & \dots & x_m(1) \\ x_1(2) & x_2(2) & \dots & x_m(2) \\ \dots & \dots & \dots & \dots \\ x_1(n) & x_2(n) & \dots & x_m(n) \end{bmatrix}, \tag{7}$$

with n individuals simulated for m timesteps.

The score matrix A was pre-calculated to speed up simulation time by taking the average score of 500 games played pairwise between all individuals in the population. Both players had an initial utility of 15, which was subtracted from the final scores to only display the relative average utility. The fictional population size was chosen to $N = 1000$, with the amount of individuals being $n = 2^8 = 256$. When calculating the change in abundance, a scaling factor α was used to affect the convergence rate during the evolution, initially set to $\alpha = 0.2$, since it with gave an appropriate convergence after 5000-10000 generations. When having calculated the change in abundance in each timestep, abundance vector was normalised to the chosen population size N to maintain fixed population size, which the update could not guarantee alone. When normalised each individual was evaluated by the threshold $1/N$, individuals with an abundance lower than the threshold was considered extinct and their abundance set to zero. The abundance vector was then re-normalised to account for the potential changes caused by extinct individuals.

4 Results

In this section we first present the single round analysis of the Peace war game, followed by a presentation of the evolutionary simulation. The single round game results are presented in numerical form.

4.1 Numerical results

The results are first presented for stage game 1, followed by stage game 2 and the normal form representation of the entire round. Figure 3 presents how to read the result tables.

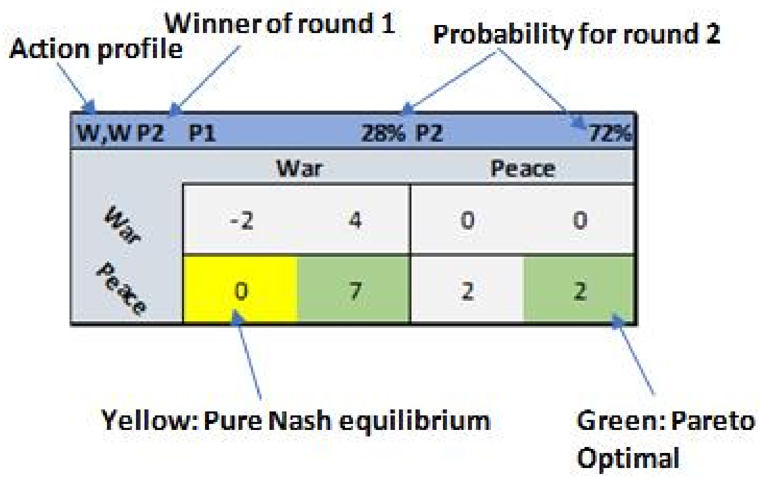

Figure 5: The expected utility matrix for one round, with the action profile and winner from stage game 1 (W,W P2) and probability of winning stage game 2 (28% and 72%) presented on the top row. Yellow cells present the Strict Nash equilibrium solution and green cells present the pareto optimal solution.

The numerical results of the two stage game are as follows:

Stage Game 1

In the expected payoff matrix for stage game 1 there is one pure Nash equilibrium (War, War). This is a pure Nash equilibrium because no player wants to deviate from this strategy. In this payoff matrix there are 3 pareto optimal actions because all the action profiles except the Nash are pareto optimal as seen in Figure 6.

Stage Game 2

Given the stochasticity of the game, there are 7 different states that are potential starting points for Stage game 2, see Figure 4. We present the outcomes in the following stage game 1 order: (W,W), (W,P) & (P,W), (P,P). The results are presented from the perspective of player 1.

W,W

If both players played War in stage game 1, (Peace, War) and (War, Peace) gives expected payoff matrices that are mirror images to each other as seen in Figure 7. The winner of the war in stage game 1 will

Starting Matrix - Expected payoff				
	War		Peace	
War	1	1	4	0
Peace	0	4	2	2

Figure 6: Expected utility for stage game 1.

have War as a dominating action in stage game 2, while the loser will have Peace as a dominating action. Correspondingly, the Nash Equilibrium for Stage game 2 is (War,Peace) and (Peace, War).

W,W P1	P1	72%	P2	28%
	War		Peace	
War	4	-2	7	0
Peace	0	0	2	2

W,W P2	P1	28%	P2	72%
	War		Peace	
War	-2	4	0	0
Peace	0	7	2	2

Figure 7: Expected utility for stage game 2 if both players played War in Stage game 2

P,W & W,P

If Player 1 plays Peace and Player 2 plays War and Player 1 wins Stage game 1, The Nash equilibrium solution is for Player 1 to play War, and for Player 2 to play Peace. This is because Player 1 has a dominant strategy of War so this player will never mix strategies in a solution. Pareto optimal actions are (Peace, Peace) and (Peace, War). The mirror image solution is found if the roles are reversed as seen in Figure 8.

The mirror image solutions are played out for the action profiles (P,W) with P2 as a winner and (W,P) with P1 as a winner, although with slightly different payoff.

As can be seen, if in stage game 1 either of the players play War while the other plays Peace, the Stage game 2 expected utility matrix is no longer a prisoners dilemma.

P,P

The last payoff matrix is a result of both players choosing Peace in the first round and get the action profile of (Peace, Peace). This profile gives both players a guarantee payoff of 2 which means that they have equal wealth in round 2 and the expected payoff matrix is the same one has in stage game 1 with the same Nash equilibrium and pareto optimal as described in Table 6. In this scenario, the Nash equilibrium is (War, War), and (Peace, War), (War, Peace) and (Peace, Peace) are Pareto optimal.

Subgame perfect equilibrium / Sequential equilibrium

In this game the Subgame perfect Nash equilibrium for the whole game is. ((War, War), (Peace, War) and (War, War), (War, Peace)). This outcome gives the Subgame Nash equilibrium and is the Subgame-perfect equilibrium for the whole game.

P,W P1	P1	60%	P2	40%
	War		Peace	
War	2	0	5	0
Peace	0	2	2	2

W,P P2	P1	40%	P2	60%
	War		Peace	
War	0	2	2	0
Peace	0	5	2	2

Figure 8: Expected utility in Stage game 2 if in stage game 1 (Peace,War) player 1 Wins & Expected utility in stage game 2 if in stage game 1 (War,Peace) player 2 wins.

P,P	P1	50%	P2	50%
	War		Peace	
War	1	1	4	0
Peace	0	4	2	2

Figure 9: Expected utility in stage game 2 if in stage game 1 both played peace.

Player 1/Player 2		Round 1		W		W		P		P	
Round 1		Round 2		W		P		W		P	
Winner R:	Action	Winner R:	Action	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2
Player 1	War	Player 1	War	2	2	5.75	-0.25	4.5	-0.25	7	0
Player 1	War	Player 2	War								
Player 2	War	Player 1	War								
Player 2	War	Player 2	War								
Player 1	War	Player 1	Peace	1	4.5	3	3	3.5	3.5	5.5	2
Player 1	War	Player 2	Peace								
Player 2	War	Player 1	Peace								
Player 2	War	Player 2	Peace								
Player 1	Peace	Player 1	War	1	5.75	2.5	3.5	3	3	5.5	2
Player 1	Peace	Player 2	War								
Player 2	Peace	Player 1	War								
Player 2	Peace	Player 2	War								
Player 1	Peace	Player 1	Peace	0	7	2	5.5	2	5	4	4
Player 1	Peace	Player 2	Peace								
Player 2	Peace	Player 1	Peace								
Player 2	Peace	Player 2	Peace								

Figure 10: Summary of the Nash equilibrium and Pareto optimal solutions for the round.

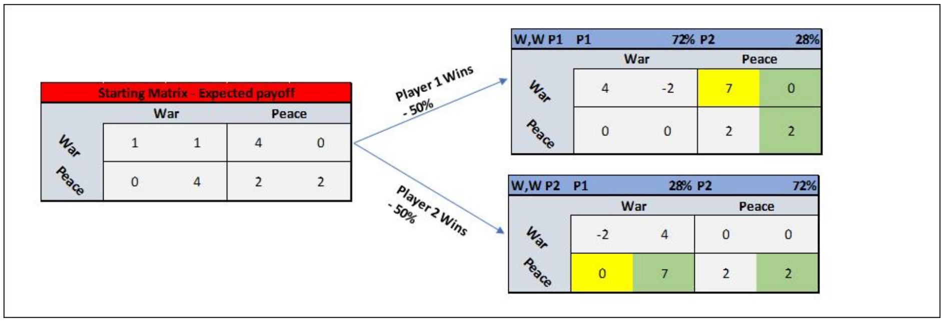

Figure 11: Subgame -perfect Nash equilibrium for the expected value method.

Stochastic normal form

In this part, the theoretical analysis for the entire game is presented in Normal form shown in the Figure 12.

Player 1/Player 2		Round 1		W		W		P		P	
Round 1		Round 2		W		P		W		P	
Winner R:	Action	Winner R:	Action	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2
Player 1	War	Player 1	War	2	2	5.75	-0.25	4.5	-0.25	7	0
Player 1	War	Player 2	War								
Player 2	War	Player 1	War								
Player 2	War	Player 2	War								
Player 1	War	Player 1	Peace	1	4.5	3	3	3.5	3.5	5.5	2
Player 1	War	Player 2	Peace								
Player 2	War	Player 1	Peace								
Player 2	War	Player 2	Peace								
Player 1	Peace	Player 1	War	1	5.75	2.5	3.5	3	3	5.5	2
Player 1	Peace	Player 2	War								
Player 2	Peace	Player 1	War								
Player 2	Peace	Player 2	War								
Player 1	Peace	Player 1	Peace	0	7	2	5.5	2	5	4	4
Player 1	Peace	Player 2	Peace								
Player 2	Peace	Player 1	Peace								
Player 2	Peace	Player 2	Peace								

Figure 12: Normal form representation of both stage games total expected utility.

As is shown in Figure 12, the Actions that both players play War in both rounds strictly dominates all other actions. (War, War)(War, War) is thereby a sub-perfect equilibrium. Based on that result then the subgame-perfect equilibrium is a strict Nash equilibrium and a Evolutionarily Stable Strategy (ESS).

Overall, the results show that the player that wins the War in the first round should play War in the second round and the other player best response is to play Peace. However, the results also show that the stage game 2 is no longer a Prisoners' dilemma, although the Expected Utility of the stage game 1 and the entire round is.

4.1.1 Simulation

The game was simulated using the game and strategy representations introduced in Section 3.3, with payoff values as given by the payoff matrix in Figure 1. The payoff matrix A was precalculated as described in Section 3.3 and can be seen in Figure 13. Replicator dynamics was implemented as described in Section 3.3 and the simulation was run for 20000 generations, reaching a steady state after about 10000 generations, as seen in Figure 14. The remaining individuals had strategy profiles and resulting abundance as shown in Table 1.

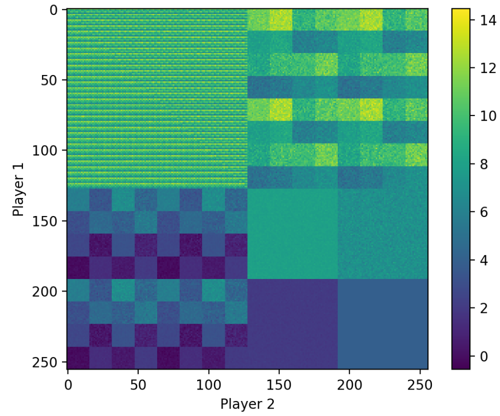

Figure 13: The payoff matrix A as used in Equation 3, shown using imshow where the colour at each position represent the average payoff over 500 games for Player 1 when playing against Player 2. The player index is the decimal representation of the binary strategy profiles.

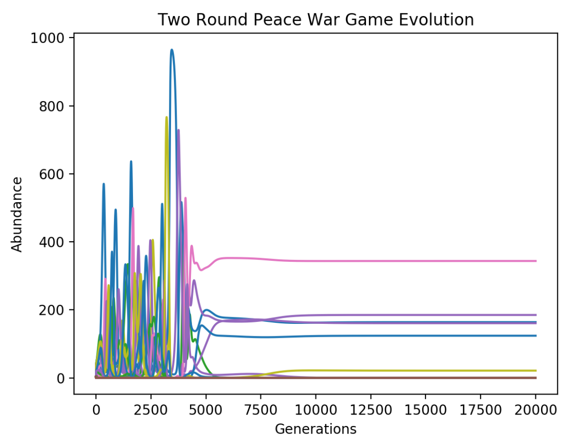

Figure 14: The abundance for each individual playing the two stage peace war game simulated using replicator dynamics, simulated for 20000 generations. The abundance of all players with abundance larger than zero is presented in Table 1.

Table 1: The abundance for all individuals with an abundance greater than zero after 20000 generations simulated.

Strategy	**Abundance**
00010100	164
00011000	185
00101100	160
00111000	343
00111100	124
01000100	21

5 Discussion

5.1 Regarding the results from the analysis of a one round game

In this two-stage, Peace War game there are multiple equilibria that can be identified. In the first stage game the Nash equilibrium is for both players to choose War.

As in every game, there are multiple strategies for each player to choose. The interesting aspect of this specific game is the introduction of stochasticity. A strategy where Player 1 choose "War" and Player 2 choose "Peace" in both rounds, and the peacemaker loses both the first and the second round, provides the chance to Player 1 to get a wealth of 39, while Player 2 stays at 15. In another scenario where both players choose "War" in both stages and one of them wins both stages then the one who lost gets the lowest payoff of the game, leading to a wealth of 3 points. Thus, for the one who loses it is preferable to choose "Peace" as in this case his payoff is higher.

In order for both players to get equal points and as much as possible the required strategy is for stage 1, one player chooses "War" and the other player chooses "Peace" and in round 2 both players change their actions, so the war maker stays in peace and the peacemaker choose to go to war. In this specific scenario and in case in both rounds the one who chooses "War" is the winner of the round then both players end up with a payoff of 21 points, which is higher than the expected points if both players play Peace both rounds (19 points). However, the expected utility of this fortunate outcome of a risky strategy is 18 & 19 points respectively.

All the above mentioned strategies include the element of stochasticity as the outcome depends on the probability for each player to win the round. If both players want to avoid the uncertainty, then they both have to choose to stay in peace and they will both end up with 19 points each. However for the players to choose Peace, trust that the other player will do the same is required, otherwise the peacemaker gets lower payoff. Trust is a strategy not possible to analyse in our project as our game consists of only two stage games.

When looking at the results one can see clearly that the optimal strategy is best characterised as a behavioural strategy, since the utility maximizing choice in stage 2 does not depend on one's own actions in stage 1, but on chance. The key criteria for selecting strategy is to identify whether stage 1 resulted in a win or a loss. To adapt a behavioural strategy gives higher expected utility than to play the pure war strategy. It is however important to remember that if one player plays War and the other player plays Peace, the expected utility of stage game 2 no longer represents a Prisoner's dilemma.In relation to the question asked in this report, the one round analysis of Nash equilibria and sub-game equilibria in our stochastic PW game shows that even though peace can not be considered a stable strategy, it will be a superior strategy in stage 2 for at least one of the players if war was played by any player in stage 1.

5.2 Simulation

The simulation results showed six surviving strategies after 20000 generations, reaching steady state after approximately 10000 generations. If analysing the surviving strategies, it can be seen that all strategies start by defecting in the first round, seen by 0 in place 0 in the strategy profile. This means that state 1-3 will never be reached, no matter what the other player is choosing, rendering those positions in the strategy profile uninteresting since they do not affect the score of the strategy profile. Looking at position 4-8 in the profiles, we can see that all the profiles chose defect in state 7 and 8, and a mix of defect and cooperate in states 4 and 5. So if both players defect in the first stage, the best response is to also defect in the second stage.

The two profiles with highest score share that both play Peace in state 4 and War in state 5. So if player 1 chose War, and player 2 chose Peace, if player 1 wins he will choose peace in stage two, otherwise war.

The results acquired in the simulations use threshold $1/N$ and scaling factor $\alpha = 0.2$, other parameters during simulation result in different results. Whilst different α will furthermost affect the rate of convergence, other threshold will affect the outcome of the surviving strategies. Multiple different values for α and the threshold were tested after the initial results were acquired. The results of those simulations always showed defect in the first stage, as well as in state 7 and 8, then mixed values for the other states.

A final comment is that the results from the analysis of the one-round game are contradicting our results from the simulation. In the one-round analysis it is found that the loser will play Peace in stage 2, while in the simulation some of the surviving strategies are strategies in which the winner of stage 1 will play Peace in stage 2. These are interesting results that requires further analysis not possible to include in this report.

6 Conclusion

Our one round analysis show that the introduction of stochasticity can induce a sub-game perfect equilibrium and a Nash equilibrium that includes one of the players playing Peace in stage 2. Chance is going to determine which of the players will choose to stay in peace, instead of their choices in stage 1.

The simulation showed that the ESS always play War in the first stage, as well in the second stage if the opponent also played War in the first stage, remaining states in the strategy profiles were mixed. Two players playing optimal strategies would therefore always result in a constant war situation. In cases where an optimal player plays against a non-optimal player that cooperate in the first round, it is possible to reach a state where both cooperate in the second round.

Our general conclusion from the project is that Peace will not be a stable strategy only through the introduction of stochasticity of War outcomes. Introducing stochasticity does however induce motivations to play Peace as sub-game solutions.

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