PhD Econometrics 1: Study Questions Week 2
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Question 1

(1.1) "Failure to reject H_0 means the null hypothesis is true", true or false? If true, explain why? If false, explain why.

(1.2) Is the statement, "A matrix is a projection matrix iff it is an idempotent matrix", true? If so, explain why? If not, explain when this can be true.

(1.3) "An idempotent matrix is always invertible", true or false?

(1.4) "A projection matrix is always invertible", true or false?

(1.5) In March 1994, Michigan voters approved a sales tax increase from 4% to 6%. In political advertisements, supporters of the measure referred to this as a two percentage point increase, or an increase of two cents on the dollar. Opponents of the tax increase called it a 50% increase in the sales tax rate. Explain which way of measuring the increase in the sales tax is more accurate.

Question 2 The model is:

$$y_i = \alpha + \gamma x_i + u_i \tag{1}$$

$$u_i \sim N(0, \sigma^2) \tag{2}$$

For $i = 1, \ldots, N$, then:

(2.1) How do you estimate the parameter of interest $\theta = \alpha\gamma$?

(2.2) How do you test significance of θ against zero?

Question 3 Given the regression model $y_i = x_i\beta + u_i$, with $x_i \in \mathbb{R}$ and that $\mathbb{E}(e_i|x_i) = 0$ and also $\sigma^2 = \mathbb{E}(e_i^2|x_i)$, then:

(3.1) Find $\mathbb{E}\left((\widehat{\beta} - \beta)^3|\boldsymbol{X}\right)$.

(3.2) Interpret the term in previous part.

Question 4 The model is:

$$y_i = \beta_1 + \beta_2 x_{1i} + \beta_3 x_{2i} + u_i \tag{3}$$

At each step, state any additional assumption you need to use:

(4.1) Derive the OLS estimators without using vectors/matrix notations.

(4.2) Show that OLS estimator is unbiased.

(4.3) Assume that,

$$\widehat{\beta} = \left(\sum_{i=1}^{n} \boldsymbol{x}_i\boldsymbol{x}_i' + \lambda \boldsymbol{I}_k\right)^{-1} \left(\sum_{i=1}^{n} \boldsymbol{x}_i y_i\right)$$

Find probability limit of $\widehat{\boldsymbol{\beta}}$ as $n \to \infty$.

(4.4) Comment on the previous part. In particular, can you think of a case where $\widehat{\beta}$ takes the form above, and what would be the main purpose of such regression?

Question 5 The model is:

$$\widetilde{y}_i = \alpha + \beta x_i + u_i \qquad (4)$$

assume all GM assumptions hold, and that $u_i \sim iid(0, \sigma_u^2)$ but we only observe $y_i = \widetilde{y} - v_i$ where $v_i \sim iid(0, \sigma_v^2)$ and that v_i and u_i are independent.

(5.1) Is OLS estimator of β consistent? What conditions are needed?

(5.2) Suppose $\mathbb{E}[u_i u_j] = 0$ but $\mathbb{E}u_i^2 = \sigma^2 z_i$ and derive the GLS estimator.

(5.3) Suppose the regression residual vector is $\boldsymbol{e} = \widehat{\boldsymbol{u}} - \widehat{\boldsymbol{v}}$ and that $\boldsymbol{M}_{\text{GLS}} = \boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{\Omega}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{\Omega}^{-1}$ then show that,

$$\text{var}(\boldsymbol{e}|\boldsymbol{X}) = \boldsymbol{M}_{\text{GLS}}(\sigma_u^2.diag(z_i) + \sigma_v^2 \boldsymbol{I})\boldsymbol{M}_{\text{GLS}}$$

Question 6 Consider the regression model:

$$y = X\beta_0 + u$$

where y is $T \times 1$, X is $T \times k$ and $rank(X) = k$, β_0 is the $k \times 1$ parameter vector, and $u \sim N(0_T, \sigma_0^2 I_T)$ where σ_0^2 is unknown but a positive constant.

(6.1) Using this result, propose a decision rule to test:

$$H_0 : R\beta_0 = r$$
$$H_A : R\beta_0 \neq r$$

where R and r are respectively a $q \times k$ matrix and a $q \times 1$ vector of constants. Define the test-statistic associated with this hypothesis testing in terms of R, r, etc. What would constitute a Type I error in this context and what is the probability of a Type I error associated with your decision rule?

(6.2) Define the p-value of the test in previous part.

Question 7 The model is:

$$y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + u_i$$

for $i = 1, \ldots, N$ and we wish to test the null hypothesis: $H_0 : \beta_1 = \beta_2 = 0$.

(7.1) What is the alternative hypothesis? Re-write the regression model, and the null hypothesis in terms of notations used in the lecture (R, r, etc.), indicating the size of each variable. Using the null hypothesis, what are the numerical values for elements in R, r, etc.

(7.2) What is the test statistic and its distribution when the variance of the error term is unknown?

(7.3) Represent elements[1] in $(X'X)^{-1} = \{c_{jk}\}$. What is $[R(X'X)^{-1}R']^{-1}$ in terms of c_{jk} elements?

(7.4) What is the test-statistic in terms of c_{jk} elements?

(7.5) Suppose the test conclusion is to reject the null, comment on this conclusion.

(7.6) Suppose the test conclusion is to fail-to-reject the null, comment on this conclusion.

[1] e.g. $\begin{pmatrix} c_{11} & c_{12} & \cdots \\ c_{21} & \cdots & \cdots \end{pmatrix}$ depending on the size of $(X'X)^{-1}$