Taylor Approximation: Let $f(z)$ be any real-valued continuous (nonlinear) function then we can approximate $f(z)$ around a *(i) given point* z_0, up to a certain *(ii) order* n if the function is n-times differentiable:

$$f(z) \approx \underbrace{\underbrace{f(z_0) + f'(z_0)(z - z_0)}_{\text{1st order } (n=1)} + \frac{1}{2}f''(z_0)(z - z_0)^2}_{\text{2nd order } (n=2)} + \dots \tag{1}$$

where $f'(.)$ and $f''(.)$, the first and second order derivatives of $f(.)$, respectively, exist. The first order provides a linear approximations and the second order a quadratic approximation of $f(z)$, and so on. The approximation expresses the original nonlinear function in terms of base polynomials and the approximation error decreases as further higher order base polynomials are considered.

Example (Log): Let $g(z) = \log(1 + z)$ then $g'(z) = \frac{1}{z}$ and $g''(z) = -\frac{1}{z^2}$ then approximating g around point $z_0 = 0$ up to the first order yields[1],

$$\begin{aligned} g(1+z) &\approx g(1+z_0) + g'(1+z_0)(z - z_0) \\ &\approx \log(1+0) + \frac{1}{1+0}(z-0) \\ &\approx z \end{aligned}$$

this implies that in a small neighbourhood of 1.00 then $log(1 + z) \approx z$ if $z \approx 0$, for instance when $z = 0.01$ then:

$$\log\left(1 + \frac{1}{00}\right) \approx \frac{1}{100}$$

Example (Exponential): Consider the following[2],

$$\log\left(\frac{\widetilde{y}}{y}\right) = \beta_1$$

exponentiating both sides yields $\widetilde{y}/y = \exp(\beta_1)$ which can be re-written as $(\widetilde{y} - y)/y = \exp(\beta_1) - 1$, then let $h(\beta_1) = \exp(\beta_1)$ and approximating around point $\beta_1^{(0)} = 0$ up to the first order by applying (1) and knowing that $h(\beta_1) = h'(\beta_1) = h''(\beta_1)$ yields:

$$\begin{aligned} \exp(\beta_1) &\approx \exp\left(\beta_1^{(0)}\right) + \exp\left(\beta_1^{(0)}\right)\left(\beta_1 - \beta_1^{(0)}\right) \\ &\approx 1 + 1 \times (\beta_1 - 0) \\ &\approx 1 + \beta_1 \end{aligned}$$

then the expression $(\widetilde{y} - y)/y = \exp(\beta_1) - 1$ can be substituted for in the following way:

$$\begin{aligned} (\widetilde{y} - y)/y &= \exp(\beta_1) - 1 \\ &\approx (1 + \beta_1) - 1 \\ &\approx \beta_1 \end{aligned}$$

[1] Lecture 1 OLSCEF page 68
[2] Lecture 2 OLSCEF page 69