PhD Econometrics 1: Study Questions Week 1

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Question 1 Consider the regression model ($\boldsymbol{y}$ and $\boldsymbol{u}$ each is $N \times 1$, $\boldsymbol{X}$ is $N \times k$ and $\boldsymbol{\beta}$ is $k \times 1$):

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}$$

and that we additionally wish to examine $\boldsymbol{R}\boldsymbol{\beta} = \boldsymbol{r}$ where $\boldsymbol{R}$ is $q \times k$ and $\boldsymbol{r}$ is $q \times 1$. Let RSS_U and RSS_R denote the unrestricted and restricted sum of squared residuals, respectively.

(1.1) Write a formal expression for the null and alternative hypotheses.

(1.2) Write the problem in terms of a constrained problem (Lagrange problem).

(1.3) Derive the first order conditions and solve.

(1.4) What the value of Lagrange multiplier? Interpret the Lagrange multiplier. What is the sign?

(1.5) What are the equations for RSS_U and RSS_R?

(1.6) Derive an expression in terms of regression residuals for, $RSS_R - RSS_U$.

(1.7) Interpret the term $RSS_R - RSS_U$. What is the sign and why?

Question 2 You have a random sample from the model:

$$\begin{aligned} y_i &= x_i\beta_1 + x_i^2\beta_2 + u_i \\ \mathbb{E}[u_i|x_i] &= 0 \end{aligned}$$

where y_i is wages (dollars per hour) and x_i is age. Describe how you would test the hypothesis that the expected wage for a 40-year-old worker is \$20 an hour. Formulate the hypothesis in terms of null and alternative hypotheses.

Question 3 Categorical variables $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$, and $\boldsymbol{1}_n$ each is a vector of size $n \times 1$, and that $\boldsymbol{d}_2 = \boldsymbol{1}_n - \boldsymbol{d}_1$ with $n = n_1 + n_2$ (n_1: number of men and n_2: number of women) such that:

$$d_{1,i} = \begin{cases} 1 & \text{if man} \\ 0 & \text{if woman} \end{cases}$$

suppose:

$$\boldsymbol{y} = \boldsymbol{d}_1\widehat{\gamma}_1 + \boldsymbol{d}_2\widehat{\gamma}_2 + \widehat{\boldsymbol{u}}$$

(2.1) Show that $(\widehat{\gamma}_1,\ \widehat{\gamma}_2)' = (\overline{y}_1,\ \overline{y}_2)'$.

(2.2) Compare $\widehat{\gamma}_1$ and $\widetilde{\gamma}_1$ from two OLS regressions:

$$\widehat{\boldsymbol{y}} = \boldsymbol{d}_1\widehat{\gamma}_1 + \boldsymbol{d}_2\widehat{\gamma}_2 \tag{1}$$

$$\widehat{\boldsymbol{y}} = \boldsymbol{d}_1\widetilde{\gamma}_1 \tag{2}$$

Question 4 You estimate a least-squares regression:

$$y_i = \boldsymbol{x}_{1i}'\overline{\boldsymbol{\beta}}_1 + \tilde{u}_i \tag{3}$$

and then regress the residuals on another set of regressors:

$$\tilde{u}_i = \boldsymbol{x}_{2i}'\overline{\boldsymbol{\beta}}_2 + \tilde{e}_i \tag{4}$$

Does this second regression give you the same estimated coefficients as from estimation of a least-squares regression on both set of regressors?

$$y_i = \boldsymbol{x}_{1i}'\widehat{\boldsymbol{\beta}}_1 + \boldsymbol{x}_{2i}'\widehat{\boldsymbol{\beta}}_2 + \widehat{e}_i \tag{5}$$

In other words, is it true that $\overline{\boldsymbol{\beta}}_2 = \widehat{\boldsymbol{\beta}}_2$? Explain your reasoning.