PhD Econometrics 1: Study Questions Class 3
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Question 1 Suppose the outcome variable y_i is linearly modelled in the following way:

$$y_i = \sum_{j}^{2} X_{ji}\beta_j + u_i \tag{1}$$

for $i = 1, \ldots, N$ and stochastic regressors X_{ji}'s are scalars such that $u_i|X_{j,i} \sim \mathcal{N}(o, \sigma^2)$. Suppose ϵ_i is the regression residuals from equation (1):

(1.1) Find the least squares estimator for β_2 and show that it is unbiased.

(1.2) Show that $\text{var}(\widehat{\beta}_2|\boldsymbol{X}_j) = \text{TSS}_2^{-1}\sigma^2/(1-R_1^2)$ where $\text{TSS}_2 = \sum_i (X_{2i} - \overline{X}_2)^2$ and R_2^2 is obtained from partial regression of X_{2i} on X_{1i}.

(1.3) Show the implications of mutually orthogonal regressors condition on efficiency of least squares estimator for β_2 when $\boldsymbol{X}_1$ is unobservable and excluded from equation (1).

(1.4) Under what condition the variance of regression in equation (1) (strictly) decreases relative to the case without X_{2i}?

Question 2 Consider the model $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{u}$ satisfying all GM assumptions including full rank-k regressors $\boldsymbol{X}$, except the exogeneity $\mathbb{E}[x_i|u_i] \neq 0$ but an instrument $\boldsymbol{Z}$ with rank $l > k$ exists that satisfies both exclusion $\mathbb{E}[z_i|u_i] = 0$ and relevance conditions $\text{cov}(z_i, x_i) \neq 0$. Derive the asymptotic distribution of IV estimator for β using z_i as an instrument.

Question 3: Consider the system of two simultaneous equations,

$$y_{i1} = y_{i2}\alpha_1 + x_i\beta_1 + u_{i1} \tag{2}$$

$$y_{i2} = y_{i1}\alpha_1 + w_i\beta_1 + u_{i2} \tag{3}$$

where y_{i1} and y_{i2} are endogenous variables, and x_i and w_i are two exogenous regressors. There are four scalar structural parameters α_1, α_2, β_1 and β_2. The reduced form equations are,

$$y_{i1} = x_i\pi_{11} + w_i\pi_{21} + \epsilon_{i1} \tag{4}$$

$$y_{i2} = x_i\pi_{12} + w_i\pi_{22} + \epsilon_{i2} \tag{5}$$

where π_{11}, π_{12}, π_{21} and π_{22} are the reduced form parameters.

(3.1) Assume that $\alpha_1\alpha_2 \neq 1$. Find expressions for the reduced form parameters in terms of the structural parameters.

(3.2) Assume that $\pi_{11} \neq 0$ and $\pi_{22} \neq 0$. Show that all structural parameters are identified. Find expressions for the structural parameters in terms of the reduced form parameters.

(3.3) Consider estimation of α_1 and β_1 by applying 2SLS to the first structural equation,using w_i as an instrument for y_{i2}. Why is the condition $\pi_{22} \neq 0$ important for this 2SLS estimation? Can the parameters α_1 and β_1 be consistently estimated when π_{22}?