Optimal Financial Regulation

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Abstract

I show that when the banking sector's assets comprise large excess reserves and loans subject to non-diversifiable risk, a tighter risk-weighted capital regulation provides social value only when interest-on-excess-reserves (IOER) is positive. In general equilibrium, very low or possibly negative real IOER is associated with an increasingly flatter response by deposit rate that leads to, first, faster fall in bank's interest incomes than its interest expenses, and second lower bank valuation due to higher default risk. Bank's substitution of further lending with reserves to maintain value strains credit flow and loosens RW-capital regulation. I show that an optimal joint financial regulation trades off social benefits of lower costs associated with under-capitalized bank failure against social benefits of credit expansion to real sector.

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1 Introduction

Over the past decade to present day, oversized excess reserves of banking sector comprised over one-third of the total assets of major central banks in charge of 40% of world economy[2]. Interest-on-excess-reserves (IOER) is one of the levers used by policymakers to regulate reserves. The transmission mechanism from IOER to capital requirement regulation is an important consideration with welfare implications because conflicting impacts of the two policies may lead to over-regulation of the banking sector and disruptions in credit flow to the real sector. Alternatively, two policies may lead to under-regulation and re-expose the banking sector to heightened default risk and possibly bank failure with socially undesirable outcomes.

This paper provides a general equilibrium framework to understand the interconnection between risk-weighted capital regulation[3] and IOER. First, risk-weighted capital requirement regulation offers welfare gains by lowering likelihood of bank failure and its associated distortions that are ultimately borne by society. Tighter capital constraint requires the bank to raise capital from equity market but simultaneously lowers bank demand for debt financing leading to lower deposit rate. Lower interest cost is associated with lower bank default risk which enables the bank to extend lending until on the marginal increase in bank net worth from lending equates marginal decrease in its net worth due added risk to the asset side.

Nonetheless, welfare gains provided by optimal capital requirement that is decided in isolation of IOER lead to welfare cost when lending is negatively affected. In equilibrium, lower IOER is followed by an almost proportional lower deposit rate when IOER is positive and away from zero lower bound. The transmission mechanism from IOER to deposit rate leads to lower default risk within the banking sector because banks invest only a fraction of deposits in reserves. Any proportional decrease in IOER and deposit rates, leads to a faster reduction in interest expenses on deposit than in interest incomes from reserves as long as extensive margin in deposits exceeds the extensive margin in reserves. However, equilibrium deposit rate remains strictly positive when deposit investors' valuation arises endogenously because investors always require a compensation for time preference to forgo consumption. Subsequently, any further reduction in IOER around zero lower bound, or possibly below zero, is followed by a weak response by the deposit rate limiting the reduction in interest expenses whereas interest incomes fall more rapidly leading to heightened default risk. Because investor price this risk, bank substitutes lending with further reserves until on the margin, reduced net worth, due to lower earnings from assets, equates increased net worth due to lower default risk.

I show that a joint regulation including tighter capital regulation and lower IOER provides

[2]In September 2019, depository institutions in the United States hold more than \$1.35T of funds in excess reserves that accounts for more than 40% of total balance sheet size of the Federal Reserves. The ECB holds over €1.9T in excess reserves forming a similar share relative to consolidated balance sheet of the Eurosystem

[3]Bank capital requirement regulation has formed an integral component of global financial regulatory architecture. Regulators' wider economic outlook is conveyed to banking system through partnerships with banks to ensure their capital structure meet certain standards. The primary source of regulatory guidance for such regulation has been the voluntary set of standards suggested by the Basel Committee on Banking Supervision.

social value when IOER remains above zero bound. The join policy addresses distortion associated costly bank failure and expands credit flow to the real economy. On the contrary, when IOER is below zero, the policymaker is able to provide social benefits by loosening the capital requirement regulation. This non-monotonic relationship provides support for an integration between two policymakers in charge of IOER and capital regulation[4]. Each lever is able to address one distortion to provide welfare gains whereas a joint policy that considers the interconnections between both levers is able to provide further benefits. Particularly, an optimal IOER policy addresses overreliance on idle excess reserves while capital regulation addresses inefficiencies of costly bank failure.

Interest expenses, or alternatively interest incomes[5], associated with oversized excess reserves are an integral consideration for the policymaker. A narrower spread between lending rate and IOER is an incentive for the banks to invest further funds in reserves. However, large quantities of interest payments are ultimately financed from taxation which strain government funds that are intended to serve multiple purposes. Specifically, in this paper, deposit insurance is a tax-financed service that provides guarantee for deposits held at the banks by deposit investors when banks default. This service offers social benefits by preventing self-confirming runs on bank deposits due to lack of confidence, even if not originally justified by fundamentals, in bank ability to meet its debt liabilities in full. I show that as IOER increases, first, the policymaker increases taxation in order to finance interest expenses which leads to lower size of financial sector and lower real economic activity. Second, credit flow by the banking sector to the real economy is further decreases because on the margin, risky lending becomes less attractive relative to reserves.

When IOER is below zero, reserves provide interest incomes for the policymaker leading to lower taxation because part of funds intended for deposit insurance are financed from paying negative interests. This mechanism increases the size of financial sector but leads to lower output because credit flow to the real sector is substituted with further reserves investment. In particular, the banking sector's overall response to increase share of total assets invested in reserves is driven by the trade-off between lower net worth valuation—due to heightened default risk as a result of widened net interest expenses—against gains in net worth valuation due to lower default risk associated with lowered lending.

This result relies on the assumption that banks are unable to hold cash and therefore find it optimal to store large quantities of their funds in reserves even if the interest rate on this investment falls below zero. Irving Fisher argued that when a commodity can be stored costlessly

[4]In the U.S, The Federal Open Market Committee (FOMC) is in charge of monetary policy that includes setting IOER, whereas capital regulation is implemented by Financial Supervision Committee, in the United Kingdom, interest rate policy is decide by the Bank of England while bank regulation is implemented by the Financial Services Authority (FSA).

[5]During 2018:Q3-2019:Q3, excess reserves balances of depository institutions in the U.S. received nearly \$2.43B in net interest incomes given an average IOER of 1.85% which is equivalent to approximately 10% to total excess reserves balance in 2008:Q3. A central bank's interest earnings ordinarily are transferred as tax revenues to the Treasury, by the Federal Reserves or other major central banks, whereas interest expenses on reserves need to be financed from the Treasury.

over time, then the lower bound in terms of units of that commodity will always remain positive or at least zero[6]. However, generalization of this result to this context is less straightforward because storage of large quantities of funds is costly even for a banking sector. The model in this paper does not explicitly introduce storage cost for funds but relies on the following empirical observations that excess reserves balance of banking sectors across 25% world economy remained positive when central banks lowered IOER to negative territory.

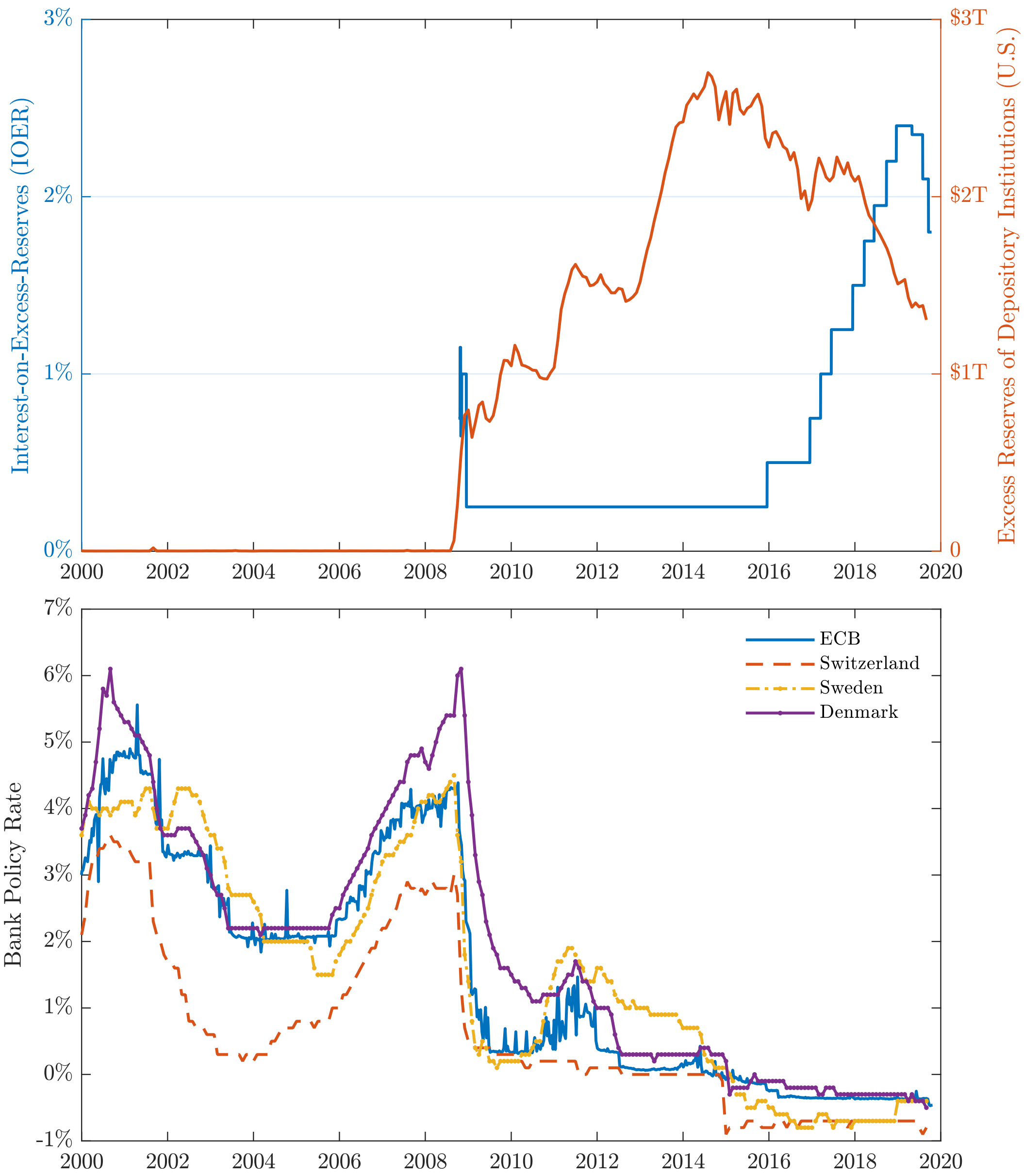


Figure 1: The top figure illustrates excess reserves balances of the depository institutions in the U.S. on the left axis, and interest-on-excess-reserves paid by the Federal Reserves on the right axis. The figure on the bottom shows the base rate paid on excess reserves by the European, Switzerland, Sweden and Denmark central banks.

In this paper, household's valuation of deposit and equity arises endogenously. The equilibrium price of equity is determined by households' preference for earning from bank dividend against

[6]The Theory of Interest, Iving Fisher, pp52.

its default risk. As bank extends lending, on the one hand its share price is bid up due to higher embedded cashflow but on the other hand, increased exposure to aggregate uncertainty lowers its expected share prices through default risk. When the bank is highly leveraged, each additional unit of equity provide sizable contribution to its net worth because default risk is relatively a more important driver of its share price. As bank's capital structure comprises further equity relative to total assets, marginal contribution of equity to lower default risk diminishes and equity's higher cost relative to debt become a more important consideration for its net worth.

The general equilibrium framework in this paper shows that equity premium compensation to risk-averse investor falls as equity-to-asset ratio in bank capital structure increases. When the bank raises capital through equity market, first, its share price falls due to higher demand for capital because the equity investor requires compensation to forgo consumption. However, fall in share price is less steep because a risk-averse equity investor prices lower riskiness of their investment.

This paper is organized to provide a brief overview on existing and ongoing studies that examine interconnections between capital regulation and IOER in Section (2). I develop a dynamic general equilibrium model in Sections (3)-(6) to study the implications of financial regulation on welfare, real economy and fragility of banking sector. Section (7) provides a numerical solution and Section (8) discusses welfare and asset pricing implications. Section (9) concludes.

2 Background

Financial regulation provides social value by addressing distortions that intermediaries fail to internalize. Diamond & Dybvig (1983) show that deposit insurance provides social value by preventing self-confirming runs on bank debt, especially when panic-based runs are not originally justified by fundamentals. However, deposit insurance increases bank's willingness to over-rely on debt financing because heightened default risk as a result of under-capitalization is no longer captured by the cost of debt.

pass-through Bank capital requirement regulation has formed an integral component of global financial regulatory architecture. Regulators' wider economic outlook is conveyed to banking system through partnerships with banks to ensure their capital structure meet certain standards[7] in relation to risk profile of their assets. A bank with higher exposure to riskier borrowers is required to hold more capital with the intention of increasing bank's ability to meet its debt liabilities should the borrowers become unable to repay their liabilities to the bank[8]. Anat Admati and Martin Hellwig have written comprehensive studies to recommend capital requirement policies

[7] The primary source of regulatory guidance for such regulation has been the voluntary set of standards suggested by the Basel Committee on Banking Supervision

[8] These standards have drawbacks, for example, their heavy reliance on privately provided credit ratings leads to inaccuracies and creates distortions. Credit ratings are less accurate than credit spreads and the standards neither distinguish among issues within a particular rating category nor among issues with different spreads.

that set forth stricter risk-weighted capital structures to increase bank's stake in risk taking to decrease the bank failures that are socially undesirable[9].

Bank failures have important implications for welfare consideration by the financial regulation because bankruptcies in the banking system is associated with realized losses that are estimated to be about 30% of ex-post total assets. These losses include expenses that arise only when a bankruptcy is triggered which involves lengthy legal processes, costly liquidation and sale of assets, lost charter value, past realized losses[10]. James (1991) estimates that a bankruptcy process is associated with 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, Andrade & Kaplan (1998) and Korteweg (2010) show that bankruptcy cost can vary between 10% to 23% of total assets within non-financial firms and between 15% to 30% of total assets for financial firms. Almeida & Philippon (2007); Acharya et al. (2007) and Glover (2016) provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs can account for more than 30 cents on the dollar.

Financial regulation internalizes bankruptcy costs that is otherwise ignored by individual banks and sets a minimum (risk-weighted) capital requirement policy to lower the possibility of bank failure and by this means its associated deadweight loss. Academic and professional literature has studied the impact of capital requirement on banks within macroeconomic settings (Gertler et al., 2012; Bianchi et al. (2016), Christiano and Ikeda, 2014; Landvoigt & Begenau (2016); Chari and Kehoe, 2015) to show how the financial accelerator effect slows down when bank's capital is subject to less fluctuations due to the introduction of capital requirements. The key mechanism that motivates setting capital requirement works through the bank decision to that fails to internalize the effect their own borrowing on financial stability and its feedback effect.

The first contribution of this paper is to extend the finding of existing literature with a general equilibrium approach in which banking system is exposed to uninsurable uncertainty through loans to borrowers. The introduction of aggregate uncertainty is a key ingredient as it creates a close resemblance to an economy that faces potential loss of productivity and financial crises due to inability of borrower to raise further funding at sector level to meet debt contracts. General equilibrium framework has significant implications to incorporate interrelated feedback between lenders whose decision to provide financing to the banking system is dependent on profitability of equity investment and dividend payouts under defaults and solvencies, and borrowers whose valuation of future cashflow incorporates their shareholder's preferences.

Another consideration that factors into regulator's decision to set capital requirement is take into account lender's ability and willingness to participate in equity investment of banking system. Setting stricter capital requirement implies that households, as ultimate provider of financing, need to take a smaller position in risk-free investments such as deposits, and larger position in risky

[9]Such policies are yet to be adopted by the regulators into the financial system. Kern Alexander (2015) addresses international efforts to regulate bank capital requirements and leverage which are negated by factors such as asymmetric lobbying against stricter rules.

[10]Losses on assets that occur prior to the bank's failure but are not reported on the bank's balance sheet at the time of the failure.

investment which eventually forms bank's capital. When households are reluctant to participate in stock market or purchase risky equities, increasing capital requirements amounts to additional resistance by banking sector because marginal price of capital has to increase significantly to convince holders of risk-free assets to rebalance their portfolios which leads to falling risk-free rate (deposit rate) and widening equity premium.

The last ingredient that regulator takes into account when deciding on capital regulation is the efficiency of financial market that intermediates funds from investors to equity borrowers. Although deposit investment is costless in most economies[11], equity investment requires services from financial intermediaries such as investment banks and brokers. These costs include underwriting fees, broker's bid-ask spreads, etc. that are charged to lenders or borrowers throughout intermediation process which lower the ultimate equity investment's return to lenders or dampens raised capital that reaches borrowers. In this context, regulator considers such costs through intermediation process as a secondary deadweight loss that is socially undesirable and intends to lower when a decision on capital regulation is evaluated. Specifically, when financial markets are perfectly efficient and intermediations fees are zero, regulator is only concerned with recommending sufficiently high level of capital that eliminates bank failures. This sets an upper bound on capital regulation, however, as intermediation fees increases, regulator considers this deadweight loss against costly bankruptcies and recommends a capital regulation policy that balances welfare gains of higher bank capital associated with less frequent failures versus gains associated with lower funds channelled through costly intermediation.

Bank asset holding include cash or its equivalents, reserves, Treasuries and other risk-free investments and loans to borrower. Stricker risk-weighted capital regulation requires more equity per unit of risky investment limits the share of risky asset holding and increases the share of assets in risk-free investments. In this context, reserves deposit facility that is available to the banking system serves as a risk-free investment that increases when bank's capital regulation amounts to lower loans. Since 2008, excess reserves[12] held by the banking system with the central banks dramatically increased in the U.S. banking system from \$45 billions in September 2008 to nearly \$1 Trillion by January 2009. Keister & McAndrews (2009) and Ennis (2018) show that part of such changes in holding reserves is explained by the implications of heightened uncertainty and low productivity that lead the banking system to seek out a safe investment to avoid bankruptcies that rose during the 2007-2008 financial crisis. Unlike required reserves that are mandatory deposits, excess reserves are voluntary deposits that receive interest-on-reserves (IOR) paid by central bank to the banking system which can be positive, zero or negative. As IOR is decreased, excess reserves become a less attractive investment which are substituted for by loans to business sector. However, this portfolio rebalancing due to IOR is interrelated with risk-weighted capital

[11] Fees, minimum deposit limits, and many transaction costs are in place but in general deposit investment is a more accessible financial investment relative to equity purchases that incurs intermediation fees

[12] Excess reserves are fund that are deposited to central banks in additional to required reserves, that are mandated to be held as a proportional of total assets for legal requirements. While required reserves increases grew modestly over the past decade, excess reserves have grown with an unprecedented rate.

requirement across bank's balance sheet which leads to a tighter capital requirement.

The dependencies between IOR and risk-weighted capital requirement has important implication that calls for a joint response by the monetary authority in charge of IOR and financial regulator in charge of capital requirement. From welfare perspective, capital requirement is a policy tool that is able to counter with the deficiency caused by bankruptcy cost and intermediation fees that the banking sector fails to internalize. Without further deficiencies, any IOR that results in changes in capital requirement is irrelevant to welfare. However, a joint policy tool that include IOR needs to consider that although reserves provide financial stability, they are an unproductive investment and interest payments on reserves has to be funded from taxes. A financial regulator that is in charge of IOR and capital regulation and offer deposit insurance raised taxes or ex-ante insurance premium to be able to pay any interest expense on reserve and compensate deposit holder when the banks fails. The choice of taxation is an important factor that determines whether deposit insurance can provide full compensation in any default state.in particular, when taxes are equivalent to outstanding deposits less the reserves then depositors are guaranteed to receive their funds even if the bank fails due to an adverse shock to its borrowers who become unable to repay their liabilities to the bank. As taxes fall short of the amount deposits less reserves, the deposit insurance is able to offer only partial insurance to depositors in real terms.

This deficiency arises due to the choice of taxation in relation to the capital structure and asset allocation of banking system. A joint financial regulatory policy that compensates reserves by IOR has to take into account that interest expenses are a further force that lower taxation which limit the ability of deposit insurance to compensate deposit holders. Consequently, the interaction between IOR and other policy tools is associated with a welfare implication which has to be considered when deciding an optimal IOR that has interactions with capital requirement regulation. The social value of bank equity and social value of reserves are two consideration to policy makers that counter bankruptcy cost and deposit insurance's ability and have to decided jointly.

dynamics

3 The Model

Assume time is discrete, with dates $t = 0, 1, 2, \ldots$. The economy consists of three sectors including a representative household, a representative bank (commercial bank or intermediary) and a financial regulator. First, the household is an infinitely-lived dynasty who lives off financial wealth. At each date-t, the household chooses optimal consumption-saving and portfolio allocation to two investment opportunities, deposits and equity. The deposit is a risk-free investment compensated at gross interest rate $R_{D,t+1}$ by the banking sector and benefits from deposit insurance guarantee.

The equity is a risky investment that is subject to stochastic return $\mathbb{E}_t[R_{E,t+1}] > R_{D,t+1}$ and is protected by limited liability such that in a default state, investor is only responsible up to its original investments. Section (x) discusses the households' preferences in more details.

Second, a representative banking sector is in charge of intermediating funds from the households to borrowers by accepting deposits and issuing equity to raise capital. The bank invests its financings in two purposes: issue a commercial loan portfolio that earns stochastic return $R_{L,t+1}$ per each unit of investment, or invest in reserves held at the central bank to earn risk-free interest-on-reserves $R_{X,t+1}$. At the end of each period, Bank's liabilities consist of deposits plus interest which must be honored for the bank to remain bank solvent in which case earnings from loans and reserves are transferred to deposit holders, and then equity investors. The bank, however, is able to declare bankruptcy when it is unable to meet its liabilities in which case deposit holders are compensated partially by the bank and equity value is zero. Section (x) discusses the banking sector's valuation problem and defaults in details.

Third, a financial regulator provides the following three services: offers deposit insurance, sets the minimum risk-weighted capital requirement, and accepts reserve deposits from the banking sector. Deposit insurance is a guarantee that compensates depositors in full in default states. The minimum risk-weighted capital requirement considers a welfare maximizing objective that internalizes costly bankruptcy that both the households and banking sectors ignore. Lastly, accepting deposits from the banking system is a form of reserves deposit facility. Section (X) discusses the regulators problem in details.

The methodology section is organized with the following set-up. Subsection (X1) presents the households and the banking problems without regulator's minimum capital requirement intervention and discusses general equilibrium implications, followed by the optimal interest-on-reserves policy at the end of the subsection. Second, the regulator's problem to set the minimum capital requirement is presented together with bank's problem subject to the regulatory constraint, followed by general equilibrium implications, given an exogenous interest-on-reserves rate in subsection (X2). Lastly, subsection (X3) presents a general equilibrium model with both the optimal interest-on-reserves and minimum capital requirement policies and discusses the interactions. Deposit insurance service is provided by the regulator across subsections (X1)-(X3).

3.1 Preferences

An infinitely-lived representative household finances consumption from financial wealth and maximizes her preferences described by recursive utility proposed by Epstein & Zin (1991) and Weil

(1990):

$$\{C_t^*, D_{t+1}^*, E_{t+1}^*\}_{t=0}^{\infty} \in \underset{\{D_{t+1},E_{t+1}\}}{\arg\max}\ \mathbb{E}_0\left[U(C_t, \mathbb{E}_t U_{t+1})\right] \tag{3.1}$$

$$U(C_t, \mathbb{E}_t U_{t+1}) = \left\{(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{3.2}$$

where at each date-t, the investor decides on optimal consumption and portfolio allocation subject to the intertemporal budget constraint described below, receives utility from real consumption C_t, $\beta \in (0,1)$ is the subjective discount factor, γ is the coefficient of relative risk aversion, and ψ is the elasticity of intertemporal substitution (EIS). The investor's attitude towards static risk is separated from intertemporal substitution of consumption and the investor is assumed to have preferences for early resolution of uncertainty such that $\gamma > \frac{1}{\psi}$ throughout the model. The conditional expectation operator $\mathbb{E}_t[.]$ evaluates investor's probabilistic assessment of different outcomes one-step ahead.

The investment environment includes risk-free deposit investment D_{t+1} that is chosen at date-t backed by deposit insurance receiving gross deposit interest R_{t+1}^D, and risky bank equity E_{t+1} with stochastic gross return protected by limited liability,

$$R_{E,t+1}^{+} = \max\left\{\frac{P_{E,t+1} + \Pi_{E,t+1}}{P_{E,t}}, 0\right\} \tag{3.3}$$

where $P_{E,t}$ and $\Pi_{E,t}$ are the price of equity and dividend, respectively. Equity investment is assumed to be subject to a linear underwriting cost $\kappa \in (0,1)$. The intertemporal budget constraint is,

$$P_{C,t}C_t + \underbrace{D_{t+1} + E_{t+1}}_{\text{Saving}} = \underbrace{(1-\tau_{t+1})}_{\text{Premium}}\Big(\underbrace{\overline{R_{D,t}D_t}}_{\text{Deposit Insured}} + \underbrace{R_{E,t}^{+}(1-\kappa)E_t}_{\text{Limited Liability}}\Big) + \underbrace{Tr_{t+1}}_{\text{Transfer}} \tag{3.4}$$

where τ_{t+1} is a fraction of household income that is taxed and $Tr_t \geq 0$ is a transfer that the household receives from the regulator described in section (x). The right-hand-side of equation (3.4) describes investor's wealth W_t which evolves at rate $R_{W,t+1}$ between two consecutive dates t and $t+1$ according to:

$$R_{W,t+1} = (1-\tau_{t+1})(1-\theta_{t+1})R_{D,t+1} + (1-\tau_{t+1})\theta_{t+1}(1-\kappa)R_{E,t+1} + \frac{Tr_{t+1}}{W_t} \tag{3.5}$$

where θ_{t+1} is the portfolio weight on risky asset. The household's value function is,

$$V_t = \left\{(1-\beta)\left(\frac{C_t}{W_t}\right)^{1-\frac{1}{\psi}} + \beta\left(1-\frac{C_t}{W_t}\right)^{1-\frac{1}{\psi}}\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}R_{W,t+1}^{1-\gamma}\right]\right)^{1-\frac{1}{\psi}}\right\}^{\frac{1}{1-\psi}} \tag{3.6}$$

3.2 Bank Problem

The representative risk-neutral bank serves a central role in the economy. First, the bank finances its operations by accepting deposits $\mathcal{D}_{t+1}$ and issuing equity to form its capital $\mathcal{E}_{t+1}$. Second, the bank acts as a conduit to intermediate funds from the households to borrowers[13].

The investment environment that the bank faces includes loans as a form of risky investment and risk-free reserves deposited at a deposit facility offered by the regulator. Therefore, at each date-t the bank decides how to finance its operations by choosing an optimal capital structure and a portfolio allocation to maximize the present value of the following cashflow described by:

$$\underbrace{\mathcal{E}_{t+1}R_{E,t+1}}_{\text{Shareholder Value}} = \max\left\{\underbrace{\underbrace{R_{X,t+1}X_{t+1}}_{\text{Reserve Income}} + \underbrace{R_{L,t+1}L_{t+1}}_{\text{Loan Portfolio}}}_{\text{Total Reveune}} - \underbrace{R_{D,t+1}\mathcal{D}_{t+1}}_{\text{Total Cost}}, 0\right\}$$

where X_{t+1} and L_{t+1} are investments made by the bank in the reserves deposit facility and loans granted, each receiving gross interest-on-reserves and stochastic loan rate, respectively. The right-hand-side of equation (3.7) is the total shareholder value that bank is able to generate after paying out its deposits and its interest. The bank balance sheet at each date consists of debt $\mathcal{D}_{t+1}$, capital $\mathcal{E}_{t+1}$, reserves X_{t+1} and loans L_{t+1}:

$$L_{t+1} + X_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{3.7}$$

Let η_{t+1} and ω_{t+1} denote equity-to-assets and loan-to-assets ratios derived from banks balance

Assets		Liabilities	
Reserves $(1-\omega_{t+1})$	X_{t+1}	Deposits $(1-\eta_{t+1})$	D_{t+1}
Loans (ω_{t+1})	L_{t+1}	Shareholder Value (η_{t+1})	E_{t+1}
Balance Sheet Size	A_{t+1}		

Table 1: The table describes bank's balance sheet with deposits and equity forming the liabilities side and reserves and loans forming the assets side.

sheet at each period, respectively[14], such that $(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1]$. The risk-neutral bank maximizes economic profit according to,

$$\max_{\eta_{t+1}, A_{t+1}, \omega_{t+1}} \int_{\Delta_h} M_{t,t+1} div_{t+1} dF(z) \tag{3.8}$$

[13]The intermediation process is a key service that reduces potential costs which lenders and borrowers would have faced throughout a dis-intermediated economy. A banking sector offers an important welfare improving implication by minimizing searching and monitoring costs across sectors. Although such associated cost are only assumed implicitly in this study, the welfare improving implications form the basis for the presence of a banking sector.

[14]As section (x) shows, in equilibrium the optimal capital structure decision η^*_{t+1} and portfolio allocation by the bank ω^*_{t+1} take interior solutions. This implies that in equilibrium bank's capital structure includes both debt and capital and its portfolio allocation includes both reserves and loans.

Subject to,

$$X_{t+1} + L_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{3.9}$$

$$\eta_{t+1} \geq \overline{\eta}_{t+1} \tag{3.10}$$

$$(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1] \tag{3.11}$$

where A_{t+1} is the total balance sheet size and $M_{t,t+1}$ is the stochastic discount factor[15] of the households who own bank's equity that is taken as given from bank's perspective when making decision on its capital structure. The bank discounts expected economic profit at date-$t+1$ with respect to probability space $(\Omega, \mathscr{F}, F)$ to choose decisions given the price of equity and deposit interest.

Equation (3.9) is bank's balance sheet constraint where X_{t+1}, L_{t+1}, $\mathcal{D}_{t+1}$ and $\mathcal{E}_{t+1}$ are reserves, loans, deposits and equity components of the balance sheet, respectively. The bank chooses total balance sheet size, and the following two fractions, equity-to-asset and loan-to-assets ratios, over the solvency region. Equation (3.10) is the minimum capital requirement constraint that stipulates for any balance sheet size, the bank must raise a certain fraction $\overline{\eta}_{t+1}$ of its total liabilities through equity funding.

Defaults — The bank is only concerned with the solvency region defined by Δ_h. The solvency region is determined by the end-of-period loan rate that breaks even between revenues and outstanding liabilities formulated according to the following ex-post condition,

$$\underbrace{R_{p,t+1}A_{t+1}}_{\text{Total Revenues plus Interest Income/Expense}} = \underbrace{R_{D,t+1}\mathcal{D}_{t+1}}_{\text{Total Liabilities plus interest payment}} \tag{3.12}$$

where $R_{p,t+1} = (1-\omega_{t+1})R_{X,t+1} + \omega_{t+1}R_{L,t+1}$ denotes the gross return on bank portfolio. Given bank's decisions η_{t+1}, A_{t+1} and ω_{t+1}, equation (3.12) pins down a unique gross loan rate $R_{b,t+1}$ in the state space that makes the bank break-even or just able to pay off its debt-holders. Specifically, at loan rate $R_{b,t+1}$, the bank is collecting only a fraction of its outstanding loans which together with reserves enables the bank to remain just solvent with the value of its shareholders equal to zero:

$$R_{E,t+1} = \frac{R_{p,t+1}A_{t+1} - R_{D,t+1}\mathcal{D}_{t+1}}{\mathcal{E}_{t+1}} = 0 \tag{3.13}$$

Assuming a strictly positive beginning-of-period equity value $\mathcal{E}_{t+1} > 0$, then condition (3.12) implies that because $A_{t+1} > \mathcal{D}_{t+1}$ then $R_{p,t+1} < R_{D,t+1}$. The threshold loan rate is given by:

$$R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1}) = \max\left\{\frac{1-\eta_{t+1}}{\omega_{t+1}}R_{D,t+1} - \frac{1-\omega_{t+1}}{\omega_{t+1}}R_{X,t+1}, 0\right\} \tag{3.14}$$

[15] Details in section (x)

Henceforth the shorthand break-even loan rate $R_{b,t+1}$ specifies the default and solvency regions $\Delta_f := [0, R_{b,t+1})$ and $\Delta_s := [R_{b,t+1}, \infty)$, respectively over the possible loan outcome in the state space. The default threshold is known at date-t when the bank decides on its funding and asset allocation decisions. Specifically, higher equity-to-asset ratio (ceteris paribus) enables the bank to withstand a greater adverse shock e.g. higher non-performing loans, and remain solvent, thus $R_{b,t+1}$ is weakly decreasing in η_{t+1}. In an extreme case, when the bank is over-capitalized such that it is able to cover its exposure to risky loans with capital alone ($\omega_{t+1} < \eta_{t+1}$), then $R_{b,t+1}$ is equal to zero and constant in η_{t+1}. Conversely, higher loan-to-asset ratio (ceteris paribus) worsens bank's ability to withstand adverse outcomes and therefore $R_{b,t+1}$ is weakly increasing in ω_{t+1}. Similarly, in an extreme case when the bank is over-capitalized then $R_{b,t+1}$ is equal to zero for any $\omega_{t+1} < \eta_{t+1}$. The break-even loan rate $R_{b,t+1}$ is increasing in deposit rate because higher deposit rate increases interest payments to bank's debt holders which increases the likelihood of ending up in a default outcome. Conversely, $R_{b,t+1}$ is decreasing in interest-on-reserves because higher interest-on-reserves contributes as an interest income to bank and extends its ability to meet its liabilities. Interestingly, $R_{b,t+1}$ is independent of bank's balance sheet size A_{t+1} because of implicit constant return to scale (CRS) assumption on loan section[16]. Intuitively, this implies that the bank may choose any balance sheet size but the key driver of its default depends on η_{t+1}, ω_{t+1}, $R_{X,t+1}$ and $R_{D,t+1}$ only, because the compositions inside the balance sheet determines ability to withstand adverse outcomes for any arbitrary balance sheet size.

The bank faces bankruptcy when its end-of-period revenues $R_{p,t+1}A_{t+1}$ is strictly less than its outstanding liabilities $R_{D,t+1}\mathcal{D}_{t+1}$. The probability of default depends on the properties of aggregate shock to bank's borrowers who repay their own liabilities to the bank:

$$\mathbb{P}\left(\text{Default}_{t+1}\right) \;=\; 1 - \mathbb{P}\left(R_{p,t+1}A_{t+1} \geq R_{D,t+1}\mathcal{D}_{t+1}\right) \tag{3.15}$$

In a default state, realized loan rate is strictly less than the threshold[17] $R_{b,t+1}$ and subsequently the bank is forced into bankruptcy and its proceeds are distributed to the debt holders on pro rata basis[18]. Limited liability condition prevents equity investors to internalize losses beyond their initial equity investments which indicates that in any default state, the bank is subsequently unable to fully compensate its debtors and the risk is partially passable to deposit accounts. This introduces the possibility of Diamond-Dybvig financial panic where depositors may start

[16]When, however, the loan section exhibits a decreasing return to scale framework such that larger scale is associated with lower effective return per unit, the balance sheet size matters to $R_{b,t+1}$ precisely because larger A_{t+1} implies lower return on loan section which accordingly limits bank's ability to meet its liabilities for a given adverse outcome (I have solved for this case as well but now in the process of including it for the subsequent draft).

[17]For instance, a default threshold $R_{b,t+1} = 0.75$ indicates that the (minimum) net loan rate that a bank can withstand to remain solvent is $R_{b,t+1} - 1 = -25\%$. When the bank realizes an ex-post loan rate $R_{L,t+1} < R_{b,t+1}$ then its total assets value falls below total liabilities and has to declare bankruptcy.

[18]The bankruptcy definition is stipulated by the debt contract between the bank and its debt-holders. Particularly when the bank denies repaying debt-holders in full, it can be forced into bankruptcy. The term liquidation within this context refers to enforcing debt contract to seize bank's (end-of-period) assets in the event of bankruptcy. It is worth mentioning that in the discrete-time model presented in this section, after bankruptcy a new bank is set up and continues its service by raising debt and capital.

to withdraw their funds in anticipation of a potential default. Deposit insurance offered by the regulator rules out this specific financial panic by promising depositors a guarantee on their risk-free investments.

The bank solves the problem in (3.8) by choosing first, total balance sheet size (A_{t+1}) and funding composition η_{t+1} given the price of equity and deposit rate[19]. The solution to the bank problem on the funding side thus are two demand functions or 'twin demands' for capital that are jointly determined by price of equity, deposit rate, and also any asset allocation choice ω_{t+1} from the assets side of the bank balance sheet. The bank trades with the households to pin down equilibrium capital structure and their prices, given any ω_{t+1}. Third the bank considers interest-on-reserves and loan rate to pin down its portfolio allocation which overall solve the bank problem.

Bank Borrowers and Production — For tractability, I assume bank grants loans to borrowers who have no alternative access to financing and engage in production activities in a non-financial sector. This assumption maintains bank's central role to act as an intermediary between households and ultimate borrowers, however, indicates that the non-financial sector is all-externally financed. First, the non-financial sector engages in a static production process which requires financing at the beginning of each period and pays off a stochastic outcome at the end of the period. Second, production process in the non-financial sector is subject to aggregate uncertainty which is non-diversifiable at sector level therefore bank as the lender is unable to diversify its commercial loan portfolio's risk across the non-financial sector. The underlying loan contract between the bank and its borrowers stipulates that a loan is considered non-performing when the borrower fails to repay the original borrowed amount plus interest that is decided between two counter-parties ex-ante. In any default state, the bank is allowed to seize borrower's total assets which together with the non-diversifiable risk profile implies the bank's loan section is non-performing in that default state.

In this context, because the non-financial sector in unable to raise financing directly from the households, it is also unable to redistribute dividends (if any) to households in a solvency state, and the banks also receives any dividend from the non-financial sector which effectively implies the bank serves as the owner of the non-financial sector. The outcome of the non-financial sector is the real economic output that is consumed in the goods market by the households.

Technology — The bank faces a log-normally distributed shock per unit of investment in loan section with the following Cobb-Douglas production technology that is subject to an exogenous aggregate shock z_{t+1},

$$h(L_{t+1}, z_{t+1}) \;=\; z_{t+1} L_{t+1}^{\alpha} \tag{3.16}$$

[19]Subject to boundary constraints to ensure interior or corner solution.

where $\log z_t = \mu_z + \sigma_z \epsilon_{t+1}$, $\epsilon_t \sim \mathcal{N}(0,1)$ and $\alpha \in (0,1]$.

3.3 Financial Regulator

The banking sector described in the previous section is only concerned with the solvency region. However, bank's capital structure includes funding that is raise through debt contracts which allow debt holders to force the bank into bankruptcy due to inability to honor debt contracts in full. James (1991) estimates that a bankruptcy process is associated with 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, Andrade & Kaplan (1998) and Korteweg (2010) show that bankruptcy cost can vary between 10% to 23% of total assets within non-financial firms and between 15% to 30% of total assets for financial firms. Almeida and Philippon (2007); Acharya et al. (2007) and Glover (2012) provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs can account for more than 30 cents on the dollar.

In this context, bankruptcy cost is characterized by a proportional fraction $1-\chi \in (0,1)$ of banking sector's total assets when a default occurs. Therefore, the financial regulator who is concerned with both solvency and default outcomes, considers such costs and sets a minimum (risk-weighted) capital requirement to maximize the following social welfare function.

$$\max_{Q_{X,t},\overline{\eta}_{t+1}} \mathbb{E}_0 \left[U(C_t, \mathbb{E}_t U_{t+1}) \right] \tag{3.17}$$

subject to,

$$P_{C,t}C_t + D_{t+1} + E_{t+1} = (1-\tau_{t+1}) \left(\frac{1-\overline{\eta}_{t+1}}{Q_{D,t}} + \overline{\eta}_{t+1}(1-\kappa)R_{E,t} \right) + Tr_{t+1} \tag{3.18}$$

and $\overline{\eta}_{t+1} \in [0,1]$ where the transfer function is

$$\frac{\mathrm{Tr}_{t+1}}{W_t} = \begin{cases} \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} & \text{if } z_{b,t+1} \leq z_{t+1} \text{ (non-default)} \\ \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} - \Lambda_{t+1} & \text{if } z_{s,t+1} \leq z_{t+1} < z_{b,t+1} \text{ (default)} \\ 0 & \text{if } z_{t+1} < z_{s,t+1} \text{ (inadequate deposit insurance)} \end{cases}$$

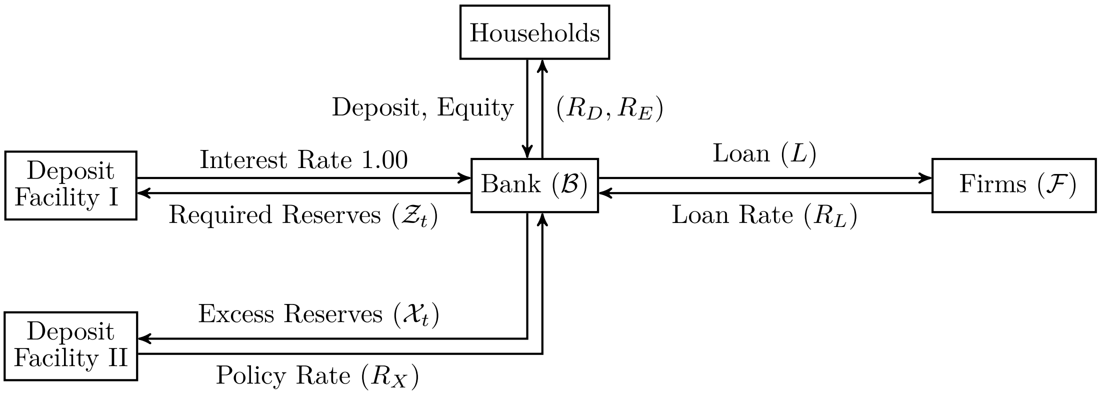
the term Λ_{t+1} shows uncovered share of debt contracts (uncompensated deposits in relation to the whole deposits plus promised interests) from the banking sector,

$$\Lambda_{t+1} = (1-\tau_{t+1}).\left(\frac{1-\eta_{t+1}}{Q_{D,t}} - \chi.R_{p,t+1}A_{t+1} \right)$$

where $r_{X,t+1} \equiv R_{X,t+1} - 1 = 1/Q_{X,t} - 1$ and that $r_{X,t+1} \lesseqgtr 0$ is the net interest-on-reserves offered on reserves deposit facility offered by the regulator to the banking sector, and $\overline{\eta}_{t+1}$ is the minimum (risk-weighted) capital requirement set on the banking sector.

First, social welfare function in (3.17) is identical to utility function of the households which regulator maximizes considering regulatory tools available in this context. Equation (3.18) characterizes regulators resource constraint that internalizes transfers to households.

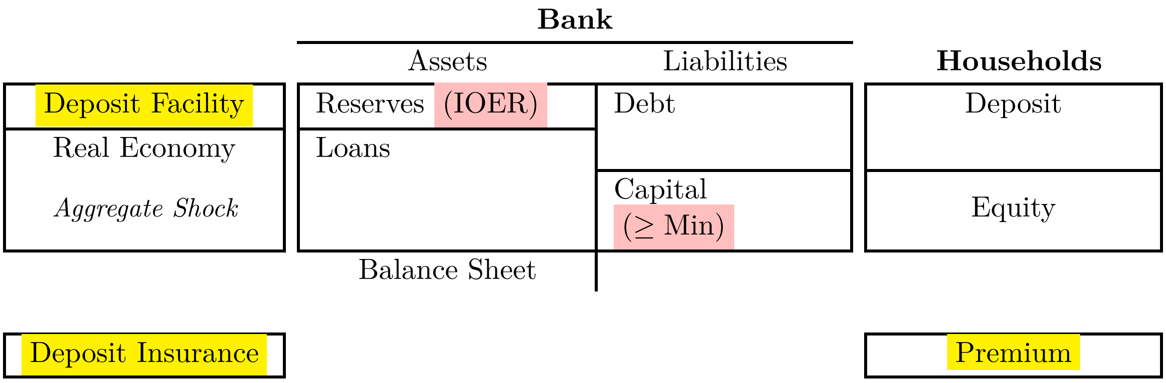
Second, the regulator raises funds through a proportional taxation[20] τ_{t+1} from the households. These funds are available to the regulator to offer deposit insurance[21] in a default state and to cover interest expenses on reserves when interest-on-reserves are positive. The transfer function has no interaction with interest-on-reserves when interest-on-reserves is zero. When interest-on-reserves is negative, then reserves deposit facility provide an interest income to the regulator since the proportion of reserves $1 - \omega_{t+1}$, scaled by after tax resources $1 - \tau_{t+1}$ earns interest income when $r_{X,t+1} < 0$. Third, the regulator considers three possible outcome interval when



considering the transfer. The non-default region is characterized by the the aggregate shock outcome $z_{b,t+1} \leq z_{t+1}$ specifying that the banking sector remains solvent. The default region is characterized by $z_{s,t+1} \leq z_{t+1} < z_{b,t+1}$ specifying that due to realizing an large adverse shock, the banking sector's total assets falls below its debt liabilities. In this case, the bank defaults and its post bankruptcy process is described by $\chi.R_{p,t+1}A_{t+1}$ and the regulator compensates depositors out of its available resources which implies that although deposits are risk-free, households receive a smaller transfer. From a welfare perspective, the regulator considers fraction $(1-\chi).R_{p,t+1}A_{t+1}$ as a deadweight loss that is socially undesirable to the economy. Fourth, the choice of taxation is taken as given and the solution section considers the following two possible cases. When taxation is sufficiently large enough to provide full insurance on deposits. This case requires taxes to be equal to deposits (plus promised interest) less the reserves (plus interests) such that any resulting uncovered deposits within the banking sector can be covered by the taxes and reserves, for example, in an extreme case when the entire loan section of the banking sector is eliminated due to an adverse large shock. However, when taxation is insufficient to cover deposits in real terms, the regulator can offer only partial insurance on deposits.

[20]The notion of taxation in this context is to simplify the analysis. The regulator uses these funds for the purpose of deposits insurance, which indicates that taxes serves as a ex-ante premium, and also to pay interest payments.

[21]Deposit insurance fee can be charged directly from the banking sector, however, in this context with a representative households sector and banking sector, applying the charges directly to household offers tractability.



4 Laissez-faire Intermediation

4.1 Supply of Financing

The household maximizes expected utility of future consumption stream subject to the intertemporal budget constraint[22]. At each date-t, the household chooses optimal consumption and portfolio choice. The first order condition with respect to consumption yields the Euler equation,

$$1 = \mathbb{E}_t \left[M_{t,t+1} R_{W,t+1} \right] \tag{4.1}$$

where

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{\gamma-1}{1-\rho}} \frac{V_{t+1}}{[\mathbb{E} V_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \tag{4.2}$$

where $M_{t,t+1}$ is household's stochastic discount factor. Return on household's wealth includes both the equity and deposit returns in the solvency state and deposit rate only in the default state.

Giovannini and Weil (1989) and Campbell and Viceira (1999) show that the consumption-saving policy function is constant over time when the stochastic process governing equity return is i.i.d. therefore the following conjecture,

$$C_t = (1 - \varphi) R_{W,t} W_t \tag{4.3}$$

solves the intertemporal problem as a special case with i.i.d. uncertainty[23] where φ is the marginal propensity to save (MPS). Solving for the value of MPS gives the following saving to wealth ratio in logarithmic units:

$$\log(\mathrm{MPS}_{t+1}) = \psi \log(\beta) + \frac{1 - \psi^{-1}}{\psi^{-1}} \left[\mathbb{E}_t r_{W,t+1}(\theta_{t+1}) + \frac{1}{2}(1-\gamma)\sigma_{rW}^2 \right] \tag{4.4}$$

shows that the log investment-to-wealth ratio is positively related to investor's subjective discount

[22] No-Ponzi condition is assumed implicitly to be satisfied.

[23] Within the context of the model the uncertainty is assumed to be i.i.d. throughout the discussion. Section (x) provides further details of stochastic process that governs equity return.

factor or impatience parameter β, such that higher patience implies higher saving if $\psi < 1$ and $\gamma > 1$.

The first order condition with respect to portfolio choice θ_{t+1} together with (4.4) determines household's optimal investments in deposits and equity given the prices of equity and deposits:

$$\begin{aligned} D_{t+1}(Q_{D,t}, P_{E,t}) &= \left[\text{MPS}_{t+1}(\theta^*_{t+1}) \times S^*_t\right] \times (1 - \theta^*_{t+1}) \\ E_{t+1}(Q_{D,t}, P_{E,t}) &= \left[\text{MPS}_{t+1}(\theta^*_{t+1}) \times S^*_t\right] \times \theta^*_{t+1} \end{aligned}$$

where household's allocation to risky asset is given by:

$$\theta^*_{t+1} = \underbrace{\frac{1}{2\gamma} + \frac{\mathbb{E}_t \log R_{E,t+1} - \log R_{D,t+1}}{\gamma\sigma_E^2}}_{\text{Merton's myopic demand}} + \underbrace{\frac{1}{\gamma\sigma_E^2} \log \Phi(R_{E,t+1} > 0)}_{\text{default disincentive}}$$

the first two terms on the right-hand-side are (rational) myopic allocations to risky asset derived by Merton (1979) and later in discrete-time by Campbell & Viceira (1999). Essentially, household's investment in risky asset does not depend on elasticity of intertemporal substitution when the shock to economy follows an i.i.d. process. The term Φ is the probability of solvency of the underlying risky asset that the household holds which appears in logarithmic units and there for is a negative factor to lower household's investment when defaults are possible. However, as the likelihood of solvency increases, the demand for the risky asset increases.

4.2 Demands for Financing

The risk-neutral expected present value problem[24] in (3.8) indicates that bank's funding and asset allocation decisions affect both the (discounted) dividend and the solvency region over which the bank is able to collect the dividend. Approximating the problem in (3.8) to separate the probability channel from the (discounted) dividend channel gives[25]:

$$\max_{\theta_{t+1}, A_{t+1}, \omega_{t+1}} \underbrace{\Phi\left[\lambda\Big(R_{b,t+1}\Big)\right]}_{\text{Probability Channel}} \times \underbrace{\mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right]}_{\text{Discounted Dividend Channel}} \tag{4.5}$$

where the first term quantifies the explicit probability of solvency and the second term quantifies the discounted dividend[26]. The logarithmic quantile[27]

$$\lambda(R_{b,t+1}) = \frac{\mu_z + \sigma_z^2 - \log(R_{b,t+1})}{\sigma_z} \tag{4.6}$$

[24] Subject to balance sheet constraint (3.9) and boundary constraints (3.11).

[25] Proof to be included — approximation with 10^{-4} accuracy.

[26] Doing so implies that (first) dividend is treated as a random variable without max[.] operator and can be both positive or negative, (second) $\mathbb{E}_t[.]$ is over both solvency and default regions. As an illustration consider that the kink on $\max[div, 0]$ function over the state space is the exact break-even loan rate

[27] Because the loan is log-normally distributed, the problem can be written in terms of a standard Normal cumulative distribution function with standardized logarithmic quantiles.

henceforth λ_{t+1}, is associated with log-normally distributed loan rate threshold $R_{b,t+1}$.

First, because $R_{b,t+1}$ is weakly decreasing in η_{t+1} (ceteris paribus), then $\Phi(\lambda_{t+1})$ is weakly increasing in η_{t+1} indicating that a higher equity-to-assets ratio increases the probability of solvency. This is because a higher equity-to-assets ratio lowers break-even threshold $R_{b,t+1}$ which corresponds to a lower standardized quantile $\lambda(.)$. Note that both functions $\Phi(.)$ and $\lambda(.)$ are strictly monotonic in their arguments. Second, because $R_{b,t+1}$ is weakly increasing in ω_{t+1} (ceteris paribus), then $\Phi(\lambda_{t+1})$ is weakly decreasing in ω_{t+1} indicating higher loan-to-assets ratio lowers the probability of solvency.

The second term in (4.2) can be written as,

$$M_{t,t+1}div_{t+1} = M_{t,t+1}\Bigg[\underbrace{\frac{1-\omega_{t+1}}{Q_{X,t}}A_{t+1}}_{\text{Reserves plus IOR}} + \underbrace{\omega_{t+1}z_{t+1}A_{t+1}}_{\text{Loan plus interest}} - \underbrace{\frac{1-\eta_{t+1}}{Q_{D,t}}A_{t+1}}_{\text{Deposit Financing}} - \underbrace{\frac{\eta_{t+1}}{P_{E,t}}A_{t+1}}_{\text{Equity Investment}}\Bigg] \tag{4.7}$$

where $Q_{D,t} = 1/R_{D,t+1}$ and $Q_{X,t} = 1/R_{X,t+1}$ are the prices of deposits and reserves, respectively. The constant return to scale technology implies that A_{t+1} does not affect the probability channel and the risk-neural property implies that balance sheet size is linear in dividend channel.

First-Order-Condition (Balance Sheet Size) The first order condition of bank problem with respect to A_{t+1} is given by,

$$0 = \left[\frac{d}{dA_{t+1}}\Phi(\lambda_{t+1})\right].\,\mathbb{E}_t\Big[M_{t,t+1}div_{t+1}\Big] + \Phi(\lambda_{t+1})\,.\frac{d}{dA_{t+1}}\mathbb{E}_t\Big[M_{t,t+1}div_{t+1}\Big] \tag{4.8}$$

decomposition in (4.2) results in the product rule in the first order condition above that tracks in impact of balance sheet size on marginal changes in present value of dividend, keeping probability of solvency constant, and marginal changes in probability of solvency while keeping the dividend channel constant. Re-arranging (4.16) gives:

$$\frac{d}{dA_{t+1}}\log\Phi(\lambda_{t+1}) = \frac{d}{dA_{t+1}}\log\mathbb{E}_t\Big[M_{t,t+1}div_{t+1}\Big] \tag{4.9}$$

an optimal balance sheet size decision $A^*(\eta_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t})$ by bank that solves problem (4.2) trades off percentage change in probability of solvency[28] $\%\Delta_A\Phi(.)$, against percentage change in expected present value of dividend $\%\Delta_A\mathbb{E}_t[M_{t,t+1}div_{t+1}]$.

First, the probability channel always motivates the bank to choose a smaller balance sheet size due to decreasing return to scale feature of the loan section. This is indicated by the sign of the term $\%\Delta_A\Phi(.)$ that is always negative for any balance sheet size. As the bank increases its balance sheet size, the marginal loan rate falls which reduces its ability to meet deposit expenses. Further, $\%\Delta_A\Phi(.)$ is increasing in price of deposit and price of equity because higher funding prices lower cost of financing, for example, when the bank is able to raise debt through deposits

[28] A positive (negative) but constant $\%\Delta_A\Phi(.)$ indicates that probability of solvency increases (decreases) at a fixed rate, and when $\%\Delta_A\Phi(.)$ is zero then probability of solvency remains fixed.

at a lower deposit rate then it faces a higher $\%\Delta_A \Phi(.)$ which indicates that the balance sheet size can increase on the margin. Similarly, when the degree of decreasing return to scale (α) falls, the probability channel become a stronger motivation to decrease balance sheet size because a lower α reduces marginal loan rate. As a special case when $\alpha = 1$ the probability channel become irrelevant to bank's decision making because the choice of balance sheet is independent of marginal return from loan section. In this special case the first order condition with respect to size only interacts with the dividend channel and the probability of solvency remains constant for any choice of size. Intuitively, this case indicates that the solvency is only driven by the composition of components inside the balance sheet and not the size itself and therefore any size is therefore optimal. More formally, the expectation operator[29] on the right hand size of equation (5.1) does not depend on endogenous variables and that the bank optimal decisions takes $M_{t,t+1}$ as given then,

$$0 \quad = \quad \Phi\Big[\lambda\Big(R_{b,t+1}\Big)\Big]\mathbb{E}_t\Big[M_{t,t+1}\frac{d}{dA_{t+1}}div_{t+1}\Big] \tag{4.10}$$

Since probability of solvency is always strictly positive because for any equity-to-assets and loan-to-asset ratios the bank can always remain solvent for an arbitrarily large loan rate outcome, then:

$$0 \quad = \quad \mathbb{E}_t\Big[M_{t,t+1}\frac{d}{dA_{t+1}}div_{t+1}\Big] \tag{4.11}$$

which results in the following first order condition that indicates, on the margin, the expected present value of cost of financing should be equal to the expected present value of one unit of investment return from bank's portfolio,

$$\mathbb{E}_t\left[M_{t+1}\left(\frac{1-\omega_{t+1}}{Q_{X,t}} + \omega_{t+1}z_{t+1}\right)\right] \quad = \quad \mathbb{E}_t\left[M_{t+1}\left(\frac{1-\eta_{t+1}}{Q_{D,t}} + \frac{\eta_{t+1}}{P_{E,t}}\right)\right] \tag{4.12}$$

the balance sheet size is always at its optimum when PV of financing cost equal PV of portfolio return. When, however, the PV of financing cost is greater than that of the portfolio return, the bank chooses balance sheet size equal to zero and when the PV of financing cost is lower than that of the portfolio return the bank choose a size that grows without bounds. Equilibrium mechanism, however, specifies that equation (4.12) must hold with equality which then establishes a condition between the prices of deposits, equity and reserves (and moments of loan).

In a more general case when $\alpha \in (0, 1)$, right-hand-side of equation (5.1) summarizes the effect of dividend channel with the term $\%\Delta_A \mathbb{E}_t[M_{t,t+1}div_{t+1}]$. Specifically, this term is monogenically decreasing in size because as the balance sheet grows (absent probability channel) lower marginal rate from loan section reduces the expected value of profit in resent value terms. The term $\%\Delta_A \mathbb{E}_t[M_{t,t+1}div_{t+1}]$ is very large when size is small and begins to fall as the size increases. When the marginal loan rate, together with bank's income from reserves become equal to cost of

[29] Integral boundaries are the support for random variable over $[0, \infty)$.

financing then $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is zero which corresponds to the maximum present value of bank profit. Any further increase in the size beyond this limit amounts to a negative expected profit.

Further, the bank faces lower cost of financing when price of deposit and equity increase which accordingly enable the bank to increase the balance sheet size that is associated with a lower marginal loan rate. In a special case, when $\alpha = 1$ the dividend become linear in size which implies that the bank faces an indeterminate choice with respect to size. In this case, the expected return on bank portfolio must be equal to expect cost of financing, otherwise the optimal size increases without bound when investing in portfolio is always marginally more profitable than marginal cost of financing, or the size is zero when expected portfolio return is lower than cost of financing.

The solution to first-order-condition (5.1) is a unique choice of balance sheet that equates percentage change in probability of solvency and percentage change in expected dividend channel. When $\%\Delta_A \Phi(.) < \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ bank is able to increase the size to obtain more profit at the expense of lowering the probability of solvency. When $\%\Delta_A \Phi(.) > \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ then the balance sheet must shrink such that the solvency increases at the expense of lower dividend. Since $\%\Delta_A \Phi(.)$ is always negative and $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is monotonically decreasing in size, the optimal balance sheet size in a general case when $\alpha \in (0, 1)$ is always smaller than the case when $\alpha = 1$.

Before discussing the optimal capital structure choice it is worth examining the relationship between optimal size and any funding composition on the liabilities side. Higher choice of equity-to-asset ration η_{t+1} increases bank's ability to withstand more adverse shock outcomes thus $\%\Delta_A \Phi(.)$ is increasing in η_{t+1} which indicates that the bank can increase its balance sheet size when its equity-to-assets ratio increases (ceteris paribus).

First-Order-Condition (Capital Structure) The first order condition with respect to capital structure is given by,

$$0 = \left[\frac{d}{d\eta_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{d}{d\eta_{t+1}} \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{4.13}$$

using the decomposition in (4.2), the expression above is re-arranged as the following,

$$\frac{d}{d\eta_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{d}{d\eta_{t+1}} \log \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{4.14}$$

where similar to the previous part left-hand-side summarizes percentage change in probability channel due to changes in equity-to-asset ratio $\%\Delta_\eta \Phi(.)$. As the bank increases η_{t+1} probability of solvency increases because higher equity-to-asset ratio increases bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of the term $\%\Delta_\eta \Phi(.)$ that is always positive for any choice of η_{t+1}. Further, increasing equity-to-asset ratio monotonically

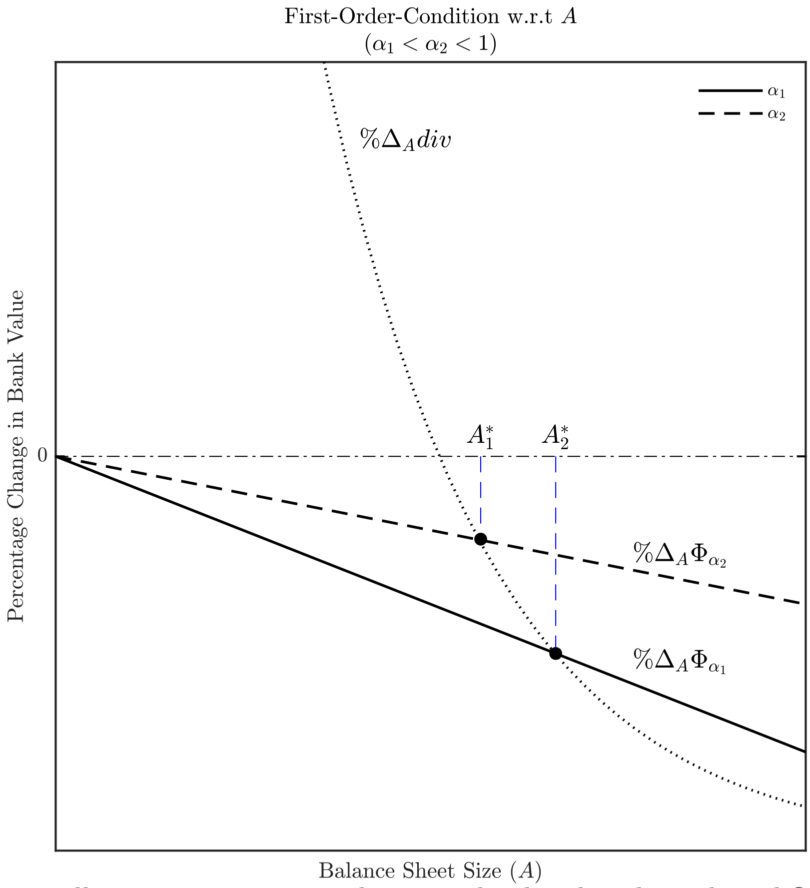


Figure 2: This figure illustrates percentage change in bank value through cashflow and solvency components when balance sheet size changes. The dotted lines shows that as balance sheet size grows, bank value increases at a decreasing rate when $\%\Delta_A div > 0$. When $\%\Delta_A div = 0$ increasing balance sheet size amount to no changes in cashflow channel. The solid line shows the solvency effect of increasing bank balance sheet size on its value

improves the chance of solvency however, when the bank is overcapitalised[30] the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by the slope of $\%\Delta_\eta\Phi(.)$ which is decreasing in η_{t+1}, specifically, when η_{t+1} is very small, the percentage change in probability of solvency is large because each additional unit of equity can considerably lower defaults. As η_{t+1} increases, $\%\Delta_\eta\Phi(.)$ decreases upto the point at which $\%\Delta_\eta\Phi(.)$ become very close to zero showing that the probability of solvency is reaching one[31] . Increasing equity-to-asset ratio beyond this limit has to impact on the solvency channel and as a result $\%\Delta_\eta\Phi(.)$ is weakly decreasing in η_{t+1}. Furthermore, $\%\Delta_\eta\Phi(.)$ is highly dependant on the price of deposits as the end-of-period interest expenses is an important determinant whether the bank remains solvent. Thus $\%\Delta_\eta\Phi(.)$ is decreasing in the price of deposit because the bank is able to withstand

[30]Over-capitalization is a relative term with respect to loan-to-asset ratio discussed in the next subsection. However, it is innocuous in to assume a bank is overcapitalised when its equity-to-asset and loan-to-asset ratios are close to each other which reflect a bank that hold enough equity to withstand a very large adverse shock to loans and remain solvent.

[31]In this case, the break-even threshold $R_{b,t+1}$ is equal to zero indicating that there is no possible loan outcome that set bank's total assets below its total liabilities. Note that when $R_{b,t+1} = 0$ (net ex-post loan rate is -100% i.e. all of bank loan section disappears in the extreme case) then $\lim \lambda(R_{b,t+1}) = \infty$ and the associated CDF is equal to one in the limit.

relatively more adverse shock when deposit interest expenses fall. Interestingly, the term $\%\Delta_\eta \Phi(.)$ is independent of the price of equity because defaults is only driven by debt contracts.

The right-hand-side of equation (4.14) summarizes the effect of capital structure choice on expected present value of bank profit. In particular, $\%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is negative and monotonically decreasing[32] in η_{t+1} when $P_{E,t} < Q_{D,t}$ as the bank considers equity more expensive relative to deposits due to its riskiness.

When $\%\Delta_\eta \Phi(.) < \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is a driver to increase η_{t+1} which results in lower expect present value of dividend but increases the probability of solvency. When $\%\Delta_\eta \Phi(.) = \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of equity to solvency against expected dividend.

The marginal contribution of equity to expected economic profit through probability channel is a factor that bids up equity price against deposits price from bank's perspective. As the equity become mores scarce, the bank is willing to accept lower price today as equity's marginal probability contribution is very high.

Solving equation (4.14) for η^*_{t+1} gives,

$$\eta^*_{t+1} \quad = \quad \eta^*(A_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t}) \tag{4.15}$$

which is bank's optimal capital structure for any $Q_{D,t}$ and $P_{E,t}$. Particularly, η^*_{t+1} specifies that when the price of equity at date-t increases (ceteris paribus), bank increases its demand for equity financing because it is able to raise more funding per share. Conversely, when the price of deposit increases (ceteris paribus) the bank lowers η^*_{t+1} as equity becomes relatively more expensive than deposit financing and bank shifts its liabilities towards more debt[33]. Bank's funding decision is fully characterized by equations (5.1) and (4.14) which are solved for in equilibrium for deposit and equity prices in the following subsection.

First-Order-Condition (Asset Allocation) The first order condition with respect to capital structure is given by,

$$0 \quad = \quad \left[\frac{d}{d\omega_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \ \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{d}{d\omega_{t+1}} \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{4.16}$$

using the decomposition in (4.2), the expression above is re-arranged as the following,

$$\frac{d}{d\omega_{t+1}} \log \Phi\left(\lambda_{t+1}\right) \quad = \quad \frac{d}{d\omega_{t+1}} \log \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{4.17}$$

[32]The assumption $P_{E,t} < Q_{D,t}$ relies on equilibrium outcome discussed in the subsequent sections but since suppliers of funding are risk-averse, then it is reasonable to restrict the discussion to cases in which price of equity is always below the price of deposits.

[33]In an extreme case when the deposits (equity) price is very high, the bank finds optimal to raise more debt (equity) even outside $\eta^*_{t+1} \in [0, 1]$ interval. These cases are discussed later and eliminated as the funding composition can include zero equity at the very least.

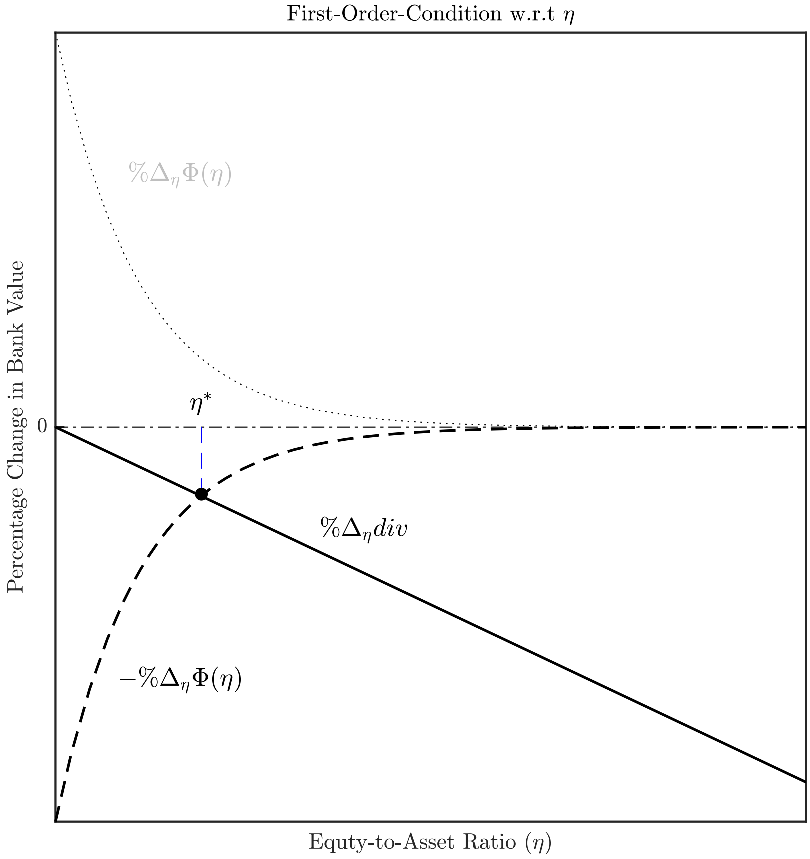


Figure 3: The figure illustrates percentage change in bank's value through cashflow and probability components. The solid line illustrates that bank's value falls when bank raises further financing through equity when equity is more costly than debt financing. The dashed line illustrates (times a negative to depicts first-order-condition) that bank's value increases through higher likelihood of solvency but at a decreasing rate because each additional unit of equity provide lower marginal contribution to solvency likelihood.

the left-hand-side summarizes percentage change in probability channel due to changes in loan-to-asset ratio $\%\Delta_\omega \Phi(.)$. As the bank increases $\omega t + 1$ probability of solvency decreases because higher loan-to-asset ratio increases exposure to shock outcomes and lowers bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of term $\%\Delta_\omega \Phi(.)$ that is always negative for any choice of $\omega t + 1$. Further, increasing loan-to-asset ratio monotonically worsen chance of solvency however, when the bank is overcapitalised the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by $\%\Delta_\omega \Phi(.) = 0$ when $\omega_{t+1} \leq \eta_{t+1}$. As ω_{t+1} increases, $\%\Delta_\omega \Phi(.)$ increases monotonically reflecting growing chance of default due further exposure to aggregate shock.

The right-hand-side of equation (4.17) summarizes the effect of asset allocation on expected present value of bank profit. When $\%\Delta_\omega \Phi(.) < \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank increases $\omega t + 1$ which results in lower expect present value of dividend at the expense of increasing the probability of default. When $\%\Delta_\omega \Phi(.) = \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of asset allocation (to loan) to solvency against expected dividend.

Solving equation (4.17) for $\omega t + 1^*$ gives,

$$\omega^*_{t+1} \quad = \quad \omega^*(A_{t+1}, \eta_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t}) \tag{4.18}$$

which is bank's optimal asset allocation choice for any $Q_{D,t}$ and $P_{E,t}$.

4.3 Laissez-faire Equilibrium

Fixed-point I (cross-agent dependencies in optimal schedules)

Special case when $\alpha = 0.99$ Market clearing conditions on the deposits and equity markets establish the following equilibrium conditions:

$$\underbrace{D_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household deposits)}} \quad = \quad \underbrace{\mathcal{D}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for Capital (bank debt)}} \tag{4.19}$$

$$\underbrace{E_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household equity)}} \quad = \quad \underbrace{\mathcal{E}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for equity (bank capital)}} \tag{4.20}$$

First, equation (4.12) from bank's first order condition with respect to balance sheet size determines a relationship between the price of deposits in terms of price of equity (and other variables that are determined later, e.g. allocation to loans, etc.). This condition clears the deposits market for a specific deposit prices, give any price of equity, $Q_{D,t}(P_{E,t})$. Second, the resulting market clearing deposit price $Q_{D,t}(P_{E,t})$ is solved for jointly with the equity market clearing condition for a specific price of equity, given other variables that are determined outside the funding markets.

5 Intermediation with Capital Regulation

The financial regulator maximizes social welfare function in (3.17) with respect to minimum capital requirement choice set on liabilities of the banking sector. This regulatory policy takes bank's asset allocation decision as given to find the optimal capital requirement conditional on ω_{t+1}, or henceforth the optimal risk-weighted capital requirement $\overline{\eta}^*_{t+1}(\omega_{t+1})$.

First-Order-Condition (RW-Capital Requirement) Marginal changes in $\overline{\eta}_{t+1}(\omega_{t+1})$ give the following FOC over the default and solvency regions, respectively:

$$0 \quad = \quad \int_0^{z_{b,t+1}} M_{t,t+1}\left[\frac{d\mathrm{Tr}_{t+1}}{d\overline{\eta}_{t+1}} - \frac{1}{Q_{D,t}}\right] dF + \int_{z_{b,t+1}}^{\infty} M_{t,t+1}\left[(1-\kappa)R_{E,t+1} - \frac{1}{Q_{D,t}}\right] dF + \mathscr{U}$$

where the first term shows the present value of marginal changes in $\overline{\eta}_{t+1}(\omega_{t+1})$ over the default region where the realization of shock is low $z_{t+1} < z_{b,t+1}$ such that banking sector's total assets fall below its debt liabilities. The regulator considers that equity income to households is zero and deposits plus interest is the only financial income households earn.

Further, over the default region, regulator evaluates changes in transfer function because bankruptcy requires the deposit insurance service to compensate depositors for any uncovered fraction of their deposit investments which is funded from regulator's resources. Once the bank defaults, its ex-post total assets falls further below its total liabilities due to bankruptcy cost that incurs in any default state which increases the amount that deposit insurance needs to pay to depositor to guarantee their investment in full.

The second term on the right-hand-side of equation (5.1) shows the present value of marginal changes due $\overline{\eta}_{t+1}(\omega_{t+1})$ over the solvency region where the households are able to receive financial income from equity and deposit investments. The transfer function remains unchanged over solvency because the bank honors its debt contracts and deposit insurance need not to intervene. The last term in equation (5.1) summarizes the direct welfare effect associated with bankruptcy cost exactly at the default threshold by comparing just-solvency against just-defaults outcome. Re-arranging equation (5.1) using the decomposition lemma discussed above gives[34],

$$0 \;=\; \underbrace{(1-\overline{\eta}_{t+1}+\kappa\overline{\eta}_{t+1})}_{\text{after-purchase investment}} \int_0^\infty M_{t,t+1}\left\{\chi.(1-\Phi(\lambda))+\Phi(\lambda)\right\}R_{p,t+1}dF$$

where the first term is less than one when purchasing bank equity incurs fee $1-\kappa$ leading to lower savings. When equity purchasing is costless $\kappa = 1$ and the first term has no interaction with $\overline{\eta}_{t+1}$. The second term on the right-hand-side shows the marginal effect of capital regulation on probability of solvency through $\Phi(\lambda)$ which increases as $\overline{\eta}_{t+1}$ increases. Let $\varpi(\chi) = \chi.(1-\Phi(\lambda))+\Phi(\lambda)$ denote the probability effect where χ shows the ex-post liquidation proceeds ($1-\chi$ is the proportional bankruptcy cost) that occurs over the default region. When $\chi = 1$ then $\varpi(\chi) = 1$ showing that probability factor $\varpi(\chi)$ is irrelevant to regulator's decision because there is no deadweight loss associated with defaults therefore the likelihood of default region $1-\Phi(\lambda)$ is immaterial to welfare. As χ decreases (proportional bankruptcy cost increases) $\varpi(\chi)$ becomes smaller showing the welfare loss in regulator's value function due to deadweight loss through probability channel. When $\chi \in (0,1)$ the regulator always is concerned with costly bankruptcies because increasing $\overline{\eta}_{t+1}$ amounts to increasing the probability of solvency that lowers its associated distortions. As $\overline{\eta}_{t+1}$ monotonically (weakly) increases $\varpi(\chi)$, the regulator recommends higher $\overline{\eta}_{t+1}$, and in an extreme case when equity purchase is costless ($\kappa = 1$), optimal capital requirement[35] is 100%. The optimal capital RW-capital requirement trades-off social costs of equity purchase fee against social benefits of less bank failure and its associated bankruptcy cost and is given by,

$$\overline{\eta}^*_{t+1} \;=\; 1+\frac{1-\omega_{t+1}}{Q_{X,t}}+\varphi_0(\mu_L,\sigma).B.\left(\varphi_1(\mu_L,\sigma)-\log\left(\frac{1-\kappa}{1-\chi}.B\right)\right)$$

[34] See Appendix

[35] When $\overline{\eta}_{t+1} > \omega_{t+1}$ the bank is always solvent as it owns more equity than its loans therefore increasing the capital requirement beyond this limit leads to no further welfare gains as any optimal capital requirement is associate with $\Phi(\lambda) = 1$ and $\overline{\eta}^*_{t+1}$ has multiple solutions.

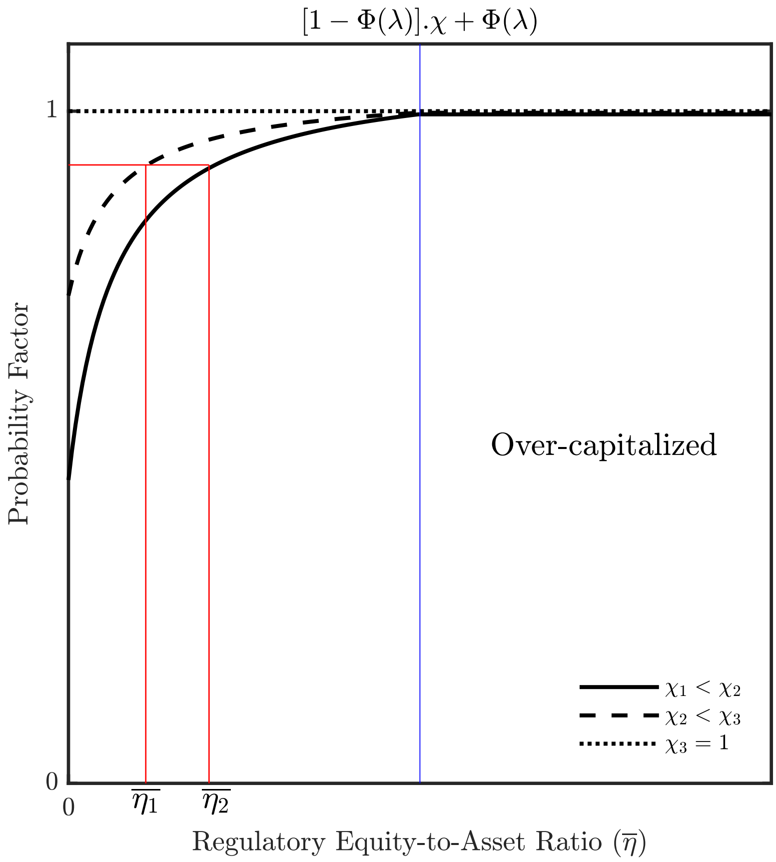


Figure 4: The probability factor $\varpi(\chi) = \chi.(1-\Phi(\lambda)) + \Phi(\lambda)$ increases when bank's capital structure includes more equity (ceteris paribus). As bankruptcy become more costly (lower $\chi 1$) then $\varpi(\chi)$ increases more sharply when capital structure includes more equity. When bankruptcy is costless ($\chi = 1$) then changes in capital structure leads to no welfare gain through probability factor $\varpi(\chi)$.

where $B = \frac{Q_{D,t}\omega_{t+1}^{\alpha}}{A^{1-\alpha}}$, $\varphi_0(\mu_L, \sigma) < 0$ and $\varphi_1(\mu_L, \sigma) > \frac{1-\kappa}{1-\chi} > 0$. The solution specifies that when equity purchase fee increases (lower κ) then $\overline{\eta}^*_{t+1}$ decreases leading to lower deadweight loss. When bankruptcy cost increases (lower χ) then $\overline{\eta}^*_{t+1}$ increases to lower the probability of default where distortion lowers the welfare. When the price of deposits $Q_{D,t}$ increase $\overline{\eta}^*_{t+1}$ decreases because the bank needs to pay lower interest payments to depositors. When the balance sheet size increases the optimal capital requirement increases because of the decreasing return to scale on bank's loan section. When the price of reserves increase, the capital requirement increases because the bank is able to earn lower interest income from its reserve investment. This effect becomes smaller as loan-to-asset ration increases which lowers bank's exposure to interest income from or expenses due to reserves.

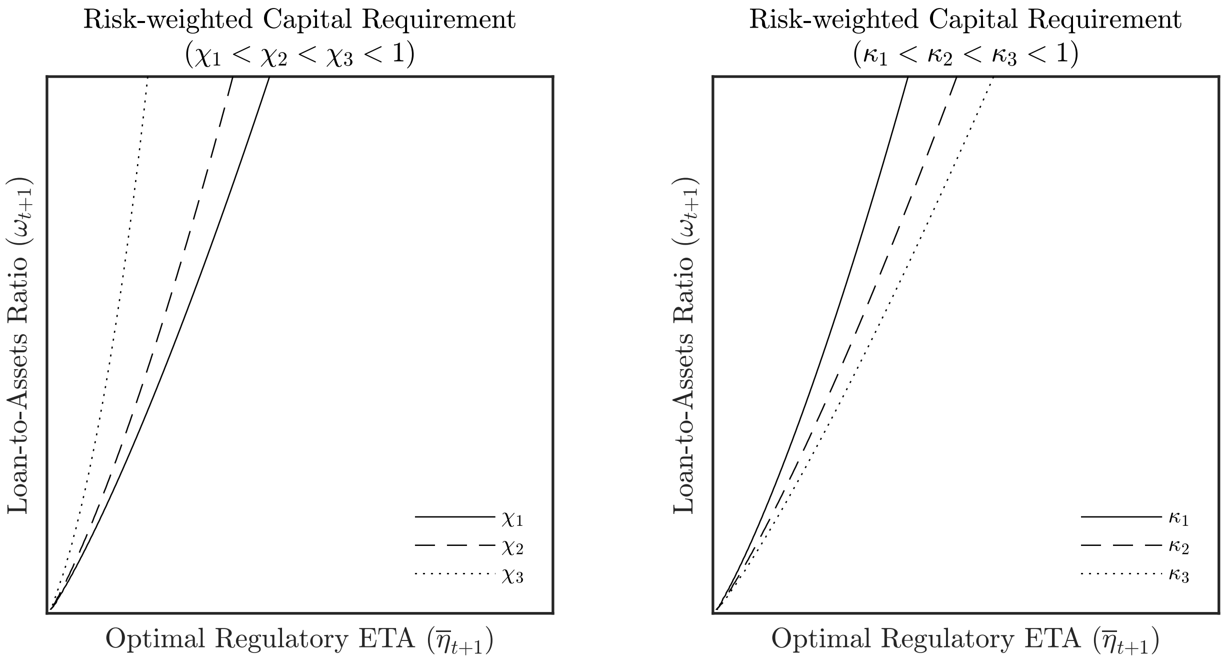
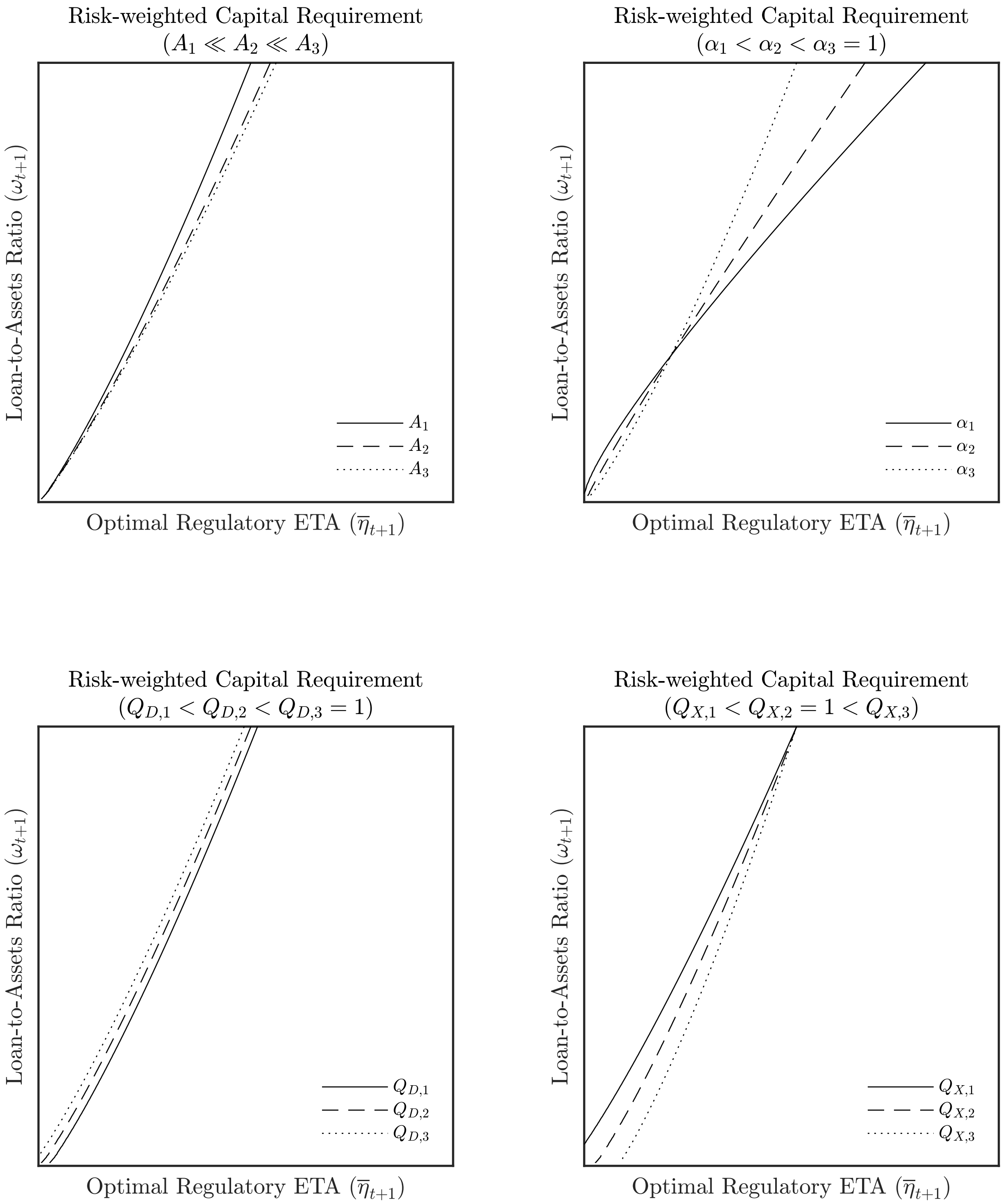


Figure 5: The slope of curves are RW-capital requirement in the space $(\overline{\eta}_{t+1}, \omega_{t+1})$. The graph illustrates changes in RW-capital requirement when bankruptcy cost parameter value changes (left) and when equity purchase fee changes (right).

Figure (5) illustrates changes in optimal capital requirement for given loan allocation by bank for three different bankruptcy cost parameter values $(\chi_1 < \chi_2 < \chi_3 < 1)$ and equity purchase parameter values $(\kappa_1 < \kappa_2 < \kappa_3 < 1)$. In particular, the slope of curves describe the ratio $\overline{\eta}_{t+1}/\omega_{t+1}$ which is the RW-capital requirement. As bankruptcy cost decreases, slope of curves in the left diagram become steeper which show lower lower equity requirement per unit of loan because the regulator is less concerned with costly defaults. As the fee associated with equity purchases decreases, the slopes of curves in the right diagram become flatter showing higher equity requirement per unit of loan because the regulator is less concerned with deadweight loss during equity purchases.

The price of equity is irrelevant to the optimal capital requirement because regulator's consideration is focused on distortions related to defaults that are determined by debtholder's contracts and not shareholders. The regulator is concerned with the welfare of the economy that includes both the debtholders and shareholders, however, the welfare improvement is achieved by reducing distortions so that households obtain higher consumption due to minimal deadweight losses.



5.1 Demands for Financing under Capital Regulation

First-Order-Condition (Balance Sheet Size under Capital Regulation) Given the capital requirement, choosing the balance sheet size also determines the capital structure on funding side. Substituting $\overline{\eta}^*_{t+1}$ into the dividend function and probability of solvency gives the following first order condition:

$$\frac{d}{dA_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{d}{dA_{t+1}} \log \mathbb{E}_t \left[M_{t,t+1} div_{t+1} \right] \tag{5.1}$$

the first order condition shows the trade offs between marginal gain in dividend against lowering probability of solvency due to lower marginal return from the loan section that is subject to decreasing return to scale. This first order condition is the same as the case without capital regulation, however, the decision $\overline{\eta}^*_{t+1}$ is predetermined.

5.2 Equilibrium with Capital Regulation

Capital structure of the bank complies with RW-capital requirement for any balance sheet size. First, in order for the deposits and equity market to clear, the bank raises funds by choosing its balance sheet size considering the capital regulation $A^*_{t+1}(\overline{\eta}_{t+1})$ through total savings by the households S^*_{t+1}:

$$A^*_{t+1}(\overline{\eta}_{t+1}) = S^*_{t+1} \tag{5.2}$$

Second, in equilibrium, the portfolio choice of the households including deposits and equity must be equal to the capital structure of the bank that is predicated by RW-capital requirement,

$$\overline{\eta}_{t+1} = \theta^*_{t+1} \tag{5.3}$$

As a special case with logarithmic utility, the first market clearing condition simplifies to:

$$A^*_{t+1}(\overline{\eta}_{t+1}) = \underbrace{(1-\tau_{t+1}).(1-\theta^*_{t+1}+\kappa.\theta^*_{t+1}).(1-\beta).W_t}_{\text{supply of funds}} \tag{5.4}$$

where the first term on the right-hand-side shows the effect of equity purchase fee on lowering the total supply fund to the economy. When $\kappa = 1$ then equity purchase is costless and the supply of funds is fixed. The term $1 - \tau_{t+1}$ show household's disposable income after paying proportional taxation to the regulator.

6 Optimal Financial Regulation

$$0 = \frac{\partial}{\partial r_{X,t+1}} \left\{ \int_0^{z_{b,t+1}} \underbrace{M_{t,t+1}\Big[r_{X,t+1}(1-\omega_{t+1}) - \Lambda_{t+1}\Big]}_{\substack{\text{discounted transfer value} \\ \text{in default}}} dF + \int_{z_{b,t+1}}^{\infty} \underbrace{M_{t,t+1}\Big[r_{X,t+1}(1-\omega_{t+1})\Big]}_{\substack{\text{discounted transfer value} \\ \text{in Solvency}}} dF \right\} \tag{6.1}$$

where the first term on the right-hand-side shows the (marginal) social value of a unit of transfer given changes in IOER in default. More precisely, the regulator considers the following trade off: in any default state, higher IOER increases ex-post liquidation proceeds within the banking sector which implies a lower deadweight loss in that state. However, IOER is financed from taxation indicating that transfer to households falls as IOER rate paid on excess reserves increases. The regulator considers the aforementioned opposable effects determine trade-offs over the default region. In solvency, the regulator is only concerned with lowering interest expenses associated with excess reserves because the banking sector is able to meet its deposit liabilities and losses are equal to zero. Re-writing the condition given in (6.1) using lemma (x) yields the following result:

$$0 = \frac{\partial}{\partial r_{X,t+1}} \left\{ \mathbb{E}_t M_{t,t+1} \left[r_{X,t+1}(1-\omega_{t+1}) - \underbrace{(1-\Phi(z_{b,t+1}))}_{\substack{\text{endogenous} \\ \text{default probability}}} \Lambda_{t+1} \right] \right\} \tag{6.2}$$

where an explicit default probability term $1 - \Phi(z_{b,t+1})$ show the impact of IOER decision on likelihood of default[36]. Particularly, this term captures a non-symmetric role of loss term Λ_{t+1} which arises only in default. As the regulator lowers IOER, defaults become more likely because equilibrium deposit rate decreases in IOER but at a diminishing rate leading to a growing net interest expenses in the banking sector.

6.1 Equilibrium Analysis

The optimal risk-weighted capital requirement determines the share of equity in banks capital structure which together with household's portfolio decision on risky asset pin down the following first equilibrium condition:

$$\overline{\eta}_{t+1} = \theta^*_{t+1} \tag{6.3}$$

Household's decision on aggregate saving together with bank's optimal decision on total balance sheet size, given the prie of equity and deposit rate, determine the second equilibrium condition

[36] Appendix D.

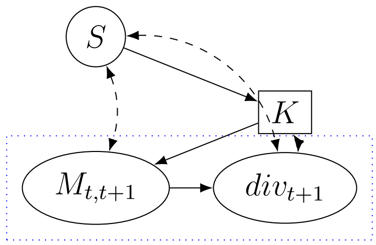
given by:

$$A^*_{t+1}(\overline{\eta}_{t+1}) \quad = \quad S^*_{t+1} \tag{6.4}$$

In equilibrium, regulator's decision on optimal IOER considers the following channels: first, an optimal IOER trades off welfare gains of lower deadweight losses associated with costly bank failure, because in any default state, bank's higher revenues due to IOER increases ex-post liquidation proceeds. Second, regulators decision on optimal IOER equates marginal benefits of lower costly defaults against marginal benefits of higher transfers to households. Thirds, IOER interacts with banks asset allocation decision through its impact on solvency channel that is priced in bank's net worth. Specifically, an optimal IOER maximizes welfare gains associated with credit flow to the real economy against marginal cost of heightened default risk due to extended lending. Decision to lower IOER when the rate is above zero bound is negatively related to optimal capital regulation because lower net interest incomes increases bank's ability to meet debt liabilities and therefore increases probabilistic cost of default. Regulator's decision on optimal IOER incentivises banking sector to extend lending with an expansionary impact while tighter capital requirement provides welfare gains by reducing (expected) bank failure cost.

7 Numerical Exercise

This section fixed point theorem



Panel A: Structural Parameters					
Description	Par.	Value	Description	Par.	Value
Household discount factor	β	0.99	Aggregate shock mean	μ	0.10
Household risk-aversion	γ	1	Aggregate shock std. err.	σ	0.15
Household EIS	ψ	1	Firm capital share	α	0.95

Table 2: Calibration Parameterization

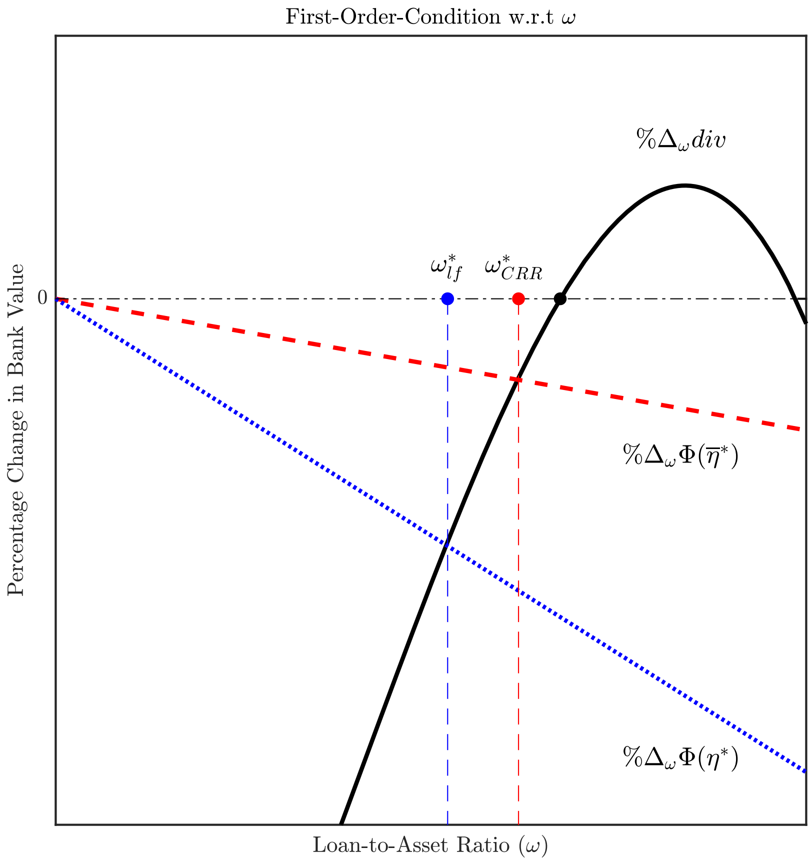


Figure 6: The figure illustrates bank's ability to extend loans when capital regulation requires the bank to hold additional equity per unit of loan. The solid line shows percentage change in bank's value function given a unit change in allocation to loan, the dotted line shows percentage change in bank's solvency when its portfolio holding of loan increases, and dashed line shows percentage change in solvency when bank complies with capital regulation.

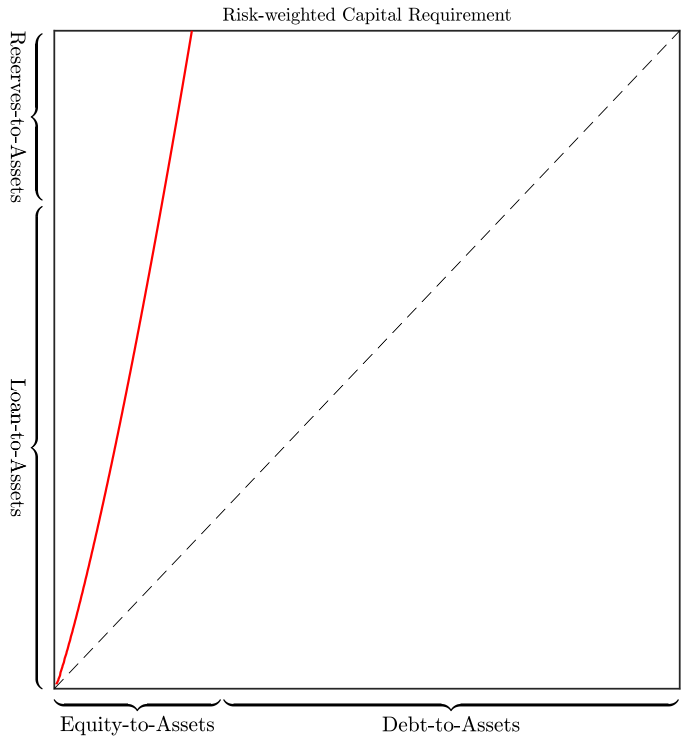


Figure 7: The figure illustrate equilibrium deposit rate in response to given interest-on-reserves rate. Variations within higher interest-on-reserves rate is associated with changes in equilibrium deposit rate in the same direction and comparable magnitude, however, as interest-on-reserves fall, equilibrium deposit rate becomes less responsive to changes in interest-on-reserves and remains strictly positive.

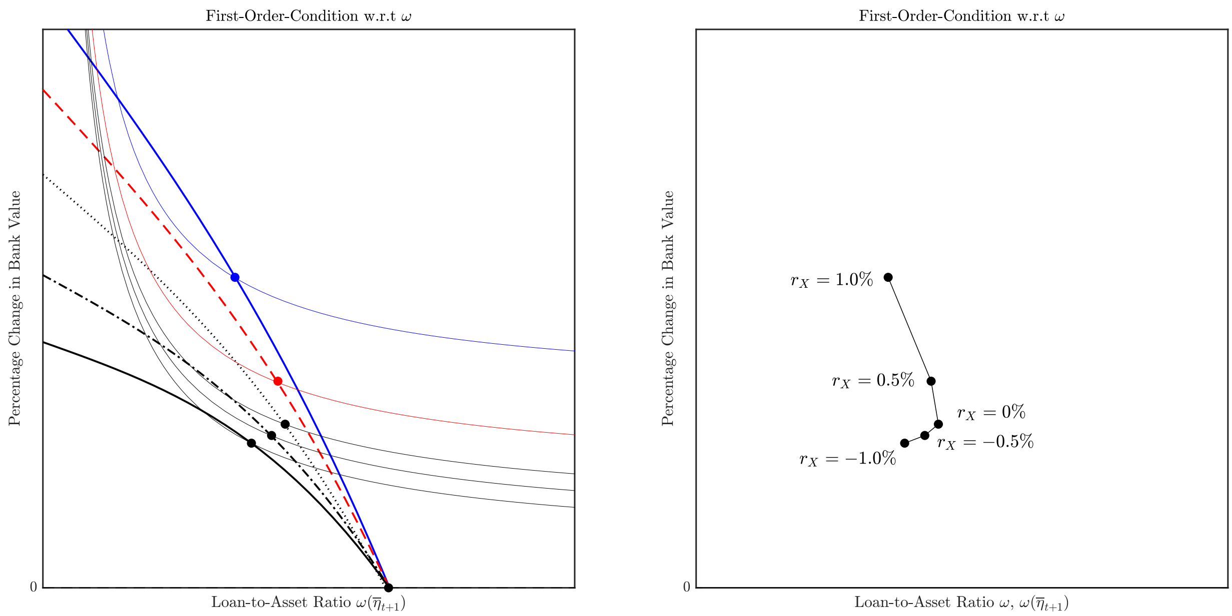


Figure 8

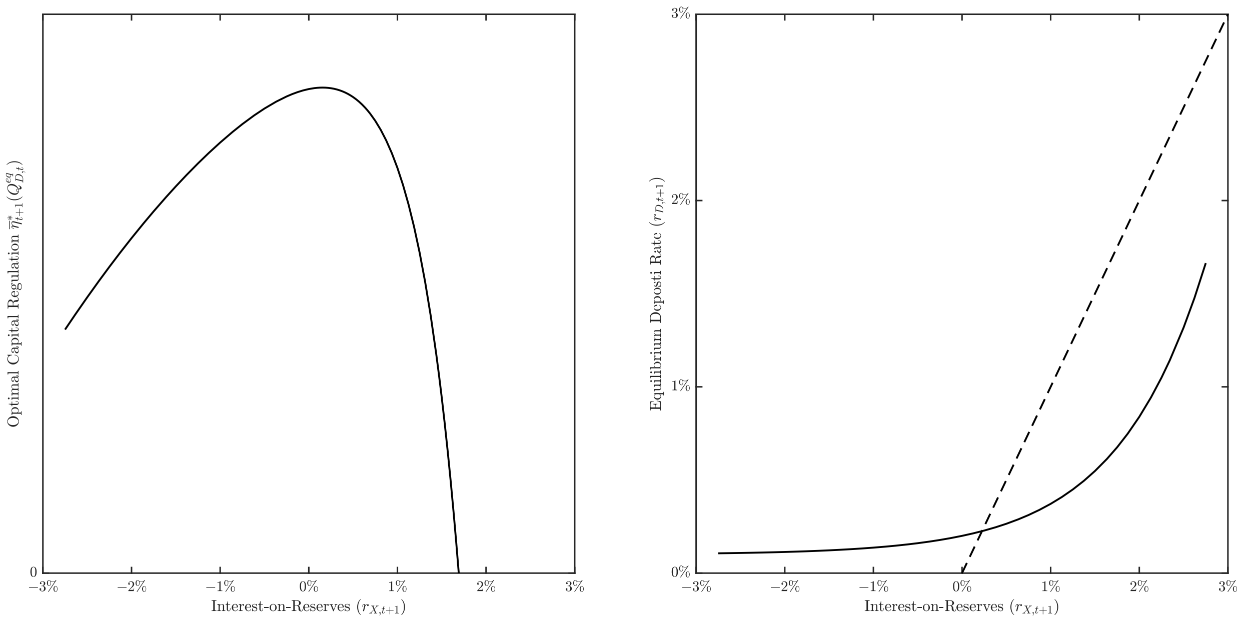


Figure 9: The figure illustrates that for a given interest-on-reserves rate, optimal capital regulation falls when bank interest expenses fall faster than the reduction in interest incomes from reserves. Conversely, the relationship between optimal capital regulation and interest-on-reserves reverses when bank's default risk increases as a results of loss of interest income from reserves and nonresponsive changes in deposit rate. The figure illustrates bank's ability to extend loans when capital regulation requires the bank to hold additional equity per unit of loan. The solid line shows percentage change in bank's value function given a unit change in allocation to loan, the dotted line shows percentage change in bank's solvency when its portfolio holding of loan increases, and dashed line shows percentage change in solvency when bank complies with capital regulation.

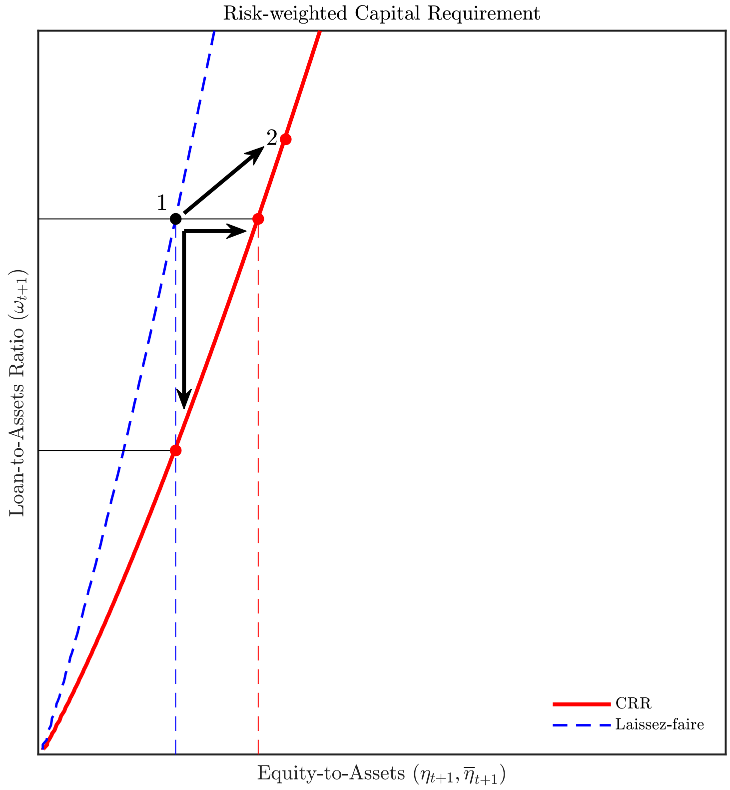


Figure 10: The figure illustrates bank's portfolio rebalancing when RW-capital regulation requires the bank to hold higher equity per loans. The solid dashed line described bank's laissez-faire loan-to-equity schedule and the solid line describes regulated loan-to-assets schedule which is always toward the outer right side of unregulated schedule.

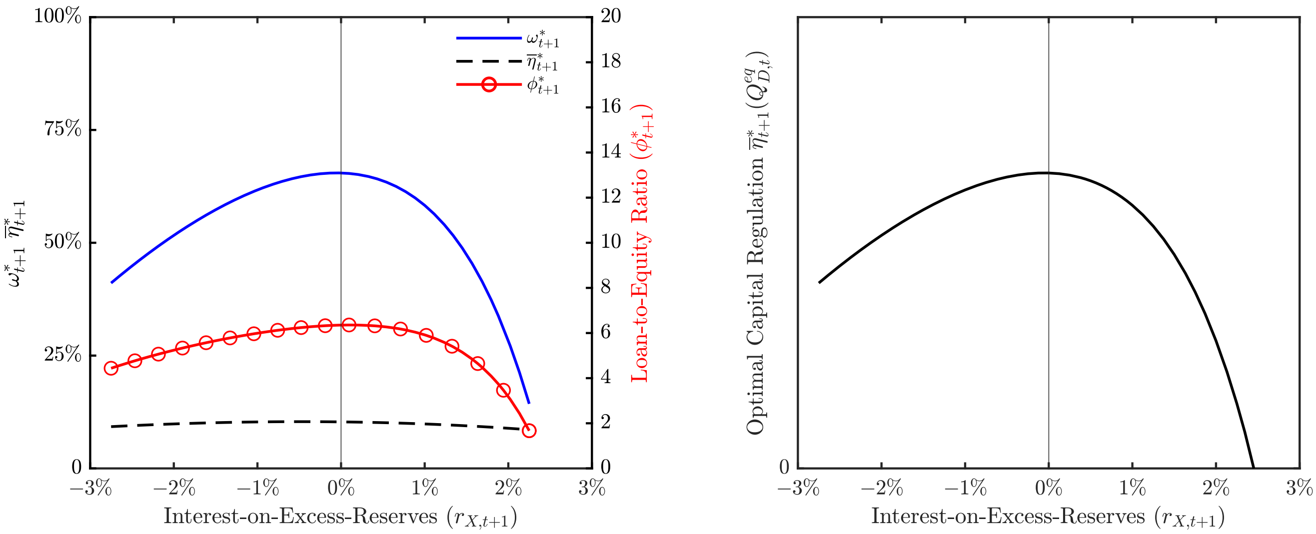


Figure 11: The figure illustrates bank's portfolio rebalancing when RW-capital regulation requires the bank to hold higher equity per loans. The solid dashed line described bank's laissez-faire loan-to-equity schedule and the solid line describes regulated loan-to-assets schedule which is always toward the outer right side of unregulated schedule.

8 Discussion

The 2007-2008 financial crisis and its aftermath prompted policymakers to re-evaluate regulatory instruments that were intended to address banking system's negative externalities to society. The model in this paper considered a financial regulatory policy together with a monetary policy tool that are available to policymaker to address distortions in banking system generated by costly bankruptcies and overreliance on interest-bearing reserves as safe assets that strain credit flow to real economy.

Section (4) shows that when interest-on-reserves is taken as given, capital regulation is able to lower the likelihood of bank failure by requiring the bank to maintain a higher equity per loan ratio. From bank's perspective this implies that in order to comply with the regulation, further capital per unit of loan must be raised through equity market which is more expensive, in terms of price per each unit of fund, relative to debt. The general equilibrium implication of section (4) indicates that as the bank seeks to raise financing from equity market, the price of equity falls which further increases the cost of meeting the capital regulation. The model in section (4) sheds light on three channels that entail important implications when considering costly equity financing.

First, raising funds through equity market comes at a more expensive price at purchase which ultimately narrows cashflow generated by the difference between bank's revenues less its costs, but on the margin, each additional unit of equity also increases likelihood of bank solvency which is priced by bank investors. The decomposition in section (4) shows bank's expected profit is determined by the product of cashflow component and solvency component indicating that although bank's equity return falls due to higher cost of funding, the fall is partly offset by marginal contribution of each additional unit of equity to solvency and therefore bank value. Results in solution methodology show that the contribution of solvency component to increase expected profit is substantial when equity is scarce and fades as bank's equity-to-assets increases because it becomes less likely for the bank to declare bankruptcy due to delinquencies among borrowers. Although the bank is risk-neutral, it is still concerned about pecuniary implications of holding equity for (expected) profitability and therefore never chooses to finance all of its funds from debt. This is because marginal contribution of equity to expected profit is higher than that of the cashflow when bank's capital structure includes limited equity. In laissez-faire equilibrium, marginal contribution of equity to solvency and cashflow are equal, but in equilibrium with capital regulation, bank equity has lower marginal contribution to solvency channel than it has to cashflow channel which indicates capital regulation is always binding.

The third channel furthers this equilibrium analysis and shows that higher bank capital leads to lower riskiness of bank equity and lowers the risk compensation that bank has to pay to raise funds from risk-averse households. When capital regulation is levied, the bank considers that its market share price is bound to fall because of increased demand for capital but as each equity unit is added to its capital structure, lower risk compensation bids up the share price which dampens the increasing cost of capital as further equity is raised to comply with capital regulation[37].

Household's are unable to internalize higher non-financial income through the transfers that they receive from the regulator when defaults are less likely. However, household's financial income comprises the present value of deposit income in default and the present value of excess return in solvency which increases when solvency becomes more likely. This effect lowers the stochastic discount factor which has two effects. First, this implies that the deposit rate increases because the household marginal utility of consumption becomes flatter with added income and requires higher risk-free[38] compensation to invest in the deposits, and second, higher stochastic discount factor is associated with lower bank valuation which subsequently lower's households demand for bank equity. As a result, the equity premium narrows under equilibrium with capital regulation. However, this effect is dampened because on the margin, the bank holds more debt which bids down its share price due to higher required compensation for additional default risk.

When the regulator exogenously lowers interest-on-reserves, the equilibrium deposit rate falls. First, lower interest-on-reserves implies that, because the spread between the expected loan and reserves widens, the bank substitutes reserves with loans on its asset side. This re-allocation must be accompanied by higher equity on the liabilities side to satisfy regulator's risk-weighted capital requirement which is ensued by a lower equity price, and accordingly, a lower deposit rate because the bank demand for debt financing is reduced. This transmission mechanism across bank assets-liabilities implies that exogenous changes in interest-on-reserves moves the equilibrium deposit rate in the same direction, however, as falling interest-on-reserves nears zero, or possibly below zero, the equilibrium deposit rate become less responsive. This is because households are endogenously forming their valuations about investments and as long as they require a minimal compensation for time preference, they always require a strictly positive deposit rate and subsequently, falling interest-on-reserves is associated with an increasingly flatter response by equilibrium deposit rate particularly when interest-on-reserves is very low or negative.

When considering extensive margins, bank's funding from deposits is always larger than bank's investment in reserves. As equilibrium deposit rate falls, bank's interest expenses on deposits fall faster that reduced interest incomes from reserves due to higher relative extensive margins in deposits than reserves. This mechanism indicates that bank's default risk falls thereby, first extending its ability to meet debt liabilities at the end of the period and, second, the bank is able to increase lending to its borrowers until the marginal gain from loan revenues become equal to

[37] Transitioning from laissez-faire to equilibrium with capital regulation.

[38] Although section (4) takes the choice of taxation as given, the level of taxation is still an important decision for the equilibrium asset prices. First, it is important for the regulator to raise an adequate level of taxation to be able to provide guarantees on deposits so that deposit insurance eliminates possibility of bank runs. Any value of taxation higher than the difference between outstanding loans plus interest less the reserves plus interest is irrelevant to bank runs specifically because deposits are always guaranteed in real terms. However, as taxation falls below this certain limit, there exists some states of the world in which extremely adverse negative shock to bank borrowers can bankrupt the bank such that the deposit insurance fund becomes unable to cover the depositors in full. This study does not examine the welfare implication of taxation and assumes that deposit insurance is provide in real terms by taxing the economy in anticipation of worst-case scenario shock outcome.

increased default risk due to increased loans.

However, flattening response of deposit rate to falling interest-on-reserves narrows the difference between interest expenses and interest incomes that allows the bank to extend its lending. When interest-on-reserves is close to zero, falling deposit rate offers limited reduction in bank interest expenses which together with sharper drop in interest income from reserves, amounts to a net decrease in interest incomes that leads to higher bank default risk. The bank optimally reacts to added default risk by lowering its lending which then lowers the real output. The underlying hump-shaped relationship between interest-on-reserves specifies that RW-capital regulation needs to tighten as interest-on-reserves falls from a positive level to close to zero and the needs to loosen if interest-on-reserves falls further to zero or below zero. Optimal capital regulation in response to any interest-on-reserves value considers welfare benefits of higher equity per loan, relative to laissez-faire allocation.

Section (6) shows capital regulation addresses distortions associated with costly bankruptcy at the expense of strains on credit flow to the real sector when interest-on-reserves is very low. The regulator considers the non-monotonic interaction between two policies to choose an optimal interest-on-reserves rate that provides social value by expanding credit while capital regulation is at its optimum. First, high interest-on-reserves, given an optimal capital regulation, is associated with high remuneration of reserves that has to be paid from regulator's resources to the banking sector. Regulator's resources are financed from taxation of the economy to cover interest expenses but also are intended to compensate depositors in any default state to as a part of government guarantee provided by the deposit insurance service. The equilibrium analysis in this section shows that overreliance on excess reserves together with high interest burdens the regulators resources and therefore taxes must increase to maintain guarantees in real term, otherwise, depositors' loss of confidence in given guarantees, even if not originally justified by fundamentals, will tend to be self-confirming. Lastly, an optimal interest-on-reserves policy considers credit flow to output sector against added default risk due to exposure of the banking sector to extended lending when interest-on-reserves is above zero. Conversely, very low or negative interest-on-reserves trades off social costs of lower lending against lower default risk within the banking sector.

Paradox of Risk-aversion

9 Conclusion

Since September 2008 to present day, oversized excess reserves consistently comprised nearly half of the total assets of central banks in charge of 40% of world economy and policymakers used IOER as a lever (inter alia) to address banks' overreliance on excess reserves. The transmission mechanism between IOER policy rate and capital requirement regulation is an important consideration with welfare implications because conflicting policies may effectively lead to under-regulation of banking sector and therefore re-exposure to default risk, or over-regulation that disrupts credit flow to the real economy.

First, this paper provides a foundation to understand this interaction and show that policymaker's decision to lower IOER provides social benefits only when this policy rate is above zero. In general equilibrium, falling IOER is followed by an almost proportional fall in the equilibrium deposit rate when IOER is above zero but as this rate becomes very low or possibly negative, equilibrium deposit rate remains positive and nonresponsive to further changes in IOER. Because the banking sector has only a fraction of deposits invested in reserves, a proportional decrease in equilibrium deposit rate in response to falling IOER leads to a faster drop in interest expenses on deposits than loss of interest incomes from reserves. The banking sector extends lending to the real economy as a result of lower default risk when IOER falls and subsequently the optimal capital regulation tightens to adjust for the added risk to banks' assets.

However, when IOER becomes very low, or possibly negative, equilibrium deposit rate exhibits an increasingly flatter response to further changes in IOER because deposit investors require a marginally positive compensation for time preference to forego consumption. When equilibrium deposit rate is increasingly nonresponsive to any further reduction in IOER, loss of interest incomes from reserves exceeds lowered interest expenses on deposits. Bank's optimally responds to increased default risk due to higher net interest expenses by lowering lending in order to maintain its shareholder value and subsequently optimal capital regulation loosen. The analysis in Section (4) shows that lower IOER dissuades the banking sector from overrelying on idle excess reserves and stimulate lending only when lower rates lead to lower default risk, otherwise lowering IOER generates counterproductive results by worsening this overreliance problem.

Second, the analysis in Section (6) shows that for any given IOER rate, optimal capital regulation constantly addresses distortions associated with costly bank failure by requiring the banking sector to hold higher equity per unit of loan. Particularly, as IOER falls within positive territory, optimal capital regulation responds negatively, and positively when IOER becomes very low or below zero. The social value provided by the capital regulation, however, is able to address one distortion at a time at the expense of disruptions of credit flow to the real sector. An optimal IOER policy, when considered in conjunction with the optimal capital regulation, is able to provide further social value by maximizing gains from boosting the real economy while addressing costly bank failure distortions.

An optimal joint financial regulation that considers this non-monotonic relationship between two levers provides support for an integration between the monetary authority in charge of reserves management and financial regulatory body in charge of capital regulation. The analysis in Section (6) also sheds light on the interconnectedness of IOER to government guarantees that protect deposits held at the banking sector. The results show that a positive IOER when combined with oversized excess reserves leads to large interest expenses and strains government resources that are intended to compensate depositors in any default state, whereas low or below zero IOER can relax government funds, raised from the households or the banking sector, and provide social benefits by increasing the size of the financial sector and ultimately credit flow to the real economy.

Finally, this paper shows a motivation for the monetary and financial regulatory policymakers to act jointly to provide further welfare gains to the society. Nonetheless, future work on joint financial regulation of banking system confronted with aggregate uncertainty needs to consider the welfare implications of deposit insurance funding regimes, an aspect that remains an open question in this paper. Under-funded deposit insurance system provides partial insurance to depositors leading to higher equilibrium cost of debt for the banking system because rational investors price potential bank defaults as well as sovereign defaults. Although sovereign default is unlikely to occur when government guarantees are met in nominal terms, nominal implications become an important consideration that extend the scope of this study to models beyond real economy. Alternatively, a government guarantee is met by borrowing against future which raises fiscal implications, or by borrowing from foreign with potentially a downward pressure on exchange rates and international finance considerations.

10 Appendix

A. Cashflow and Solvency Channels

Let $g(k)$ denote expectations function over a subset of x support:

$$g(k) = \int_k^\infty x f(x) dx \tag{10.1}$$

where $f(x)$ is the lognormal distribution, therefore:

$$g(k) = \int_k^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) dx \tag{10.2}$$

Using a change of variables $y = \frac{\ln x - \mu}{\sigma}$, $dx = \sigma \exp(\sigma y + \mu) dy$ gives:

$$\int_{y=(\ln k-\mu)/\sigma}^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{2}y^2)\sigma \exp(\sigma y + \mu) dy \tag{10.3}$$

Completing the square

$$\begin{aligned}
&= \int \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}y^2 + \sigma y + \mu) dy & (10.4)\\
&= \int \frac{1}{\sqrt{2\pi}} \exp[-\frac{1}{2}(y-\sigma)^2 + (\mu + \frac{1}{2}\sigma^2)] dy & (10.5)\\
&= = \exp(\mu + \frac{\sigma^2}{2}) \frac{1}{\sqrt{2\pi}} \int_{y=(\ln K-\mu)/\sigma}^\infty \exp(-\frac{1}{2}(y-\sigma)^2) dy & (10.6)
\end{aligned}$$

Apply the change of variable $v = y - \sigma$ and $dy = dv$ to re-write the original expectation as:

$$\begin{aligned}
g(k) &= \exp(\mu + \frac{\sigma^2}{2}) \frac{1}{\sqrt{2\pi}} \int_{v=(\ln k-\mu)/\sigma-\sigma}^\infty \exp(-\frac{1}{2}v^2) dv & (10.7)\\
&= \exp\left(\mu + \frac{\sigma^2}{2}\right) \left[1 - \Phi(\frac{\ln k - \mu - \sigma^2}{\sigma})\right] & (10.8)\\
&= \exp\left(\mu + \frac{\sigma^2}{2}\right) \Phi\left(\frac{-\ln k + \mu + \sigma^2}{\sigma}\right) & (10.9)
\end{aligned}$$

where $1 - \Phi(x) = \Phi(-x)$.

B. Stochastic Discount Factor and the Equity Premium

The stochastic discount factor is (when $\psi = 1$ and i.i.d. return):

$$\begin{aligned}
M_{t,t+1} &= \frac{R_{h,t+1}^{-\gamma}}{\mathbb{E}_t[R_{h,t+1}^{1-\gamma}]} & (10.10)\\
\log M_{t,t+1} &= -\gamma \log R_{h,t+1} - \log \mathbb{E}_t[R_{h,t+1}^{1-\gamma}] & (10.11)\\
&= -\gamma \log R_{h,t+1} - \mathbb{E}_t \log R_{h,t+1}^{1-\gamma} - \frac{1}{2}\mathbb{V}_t \log R_{h,t+1}^{1-\gamma} & (10.12)\\
&= -\gamma \log R_{h,t+1} - (1-\gamma)\mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} & (10.13)
\end{aligned}$$

the (logarithmic) moments are:

$$\mathbb{E}_t \log M_{t,t+1} = -\mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} \tag{10.14}$$

$$\mathbb{V}_t \log M_{t,t+1} = \gamma^2 \mathbb{V}_t \log R_{h,t+1} \tag{10.15}$$

using the Euler equation with respect to deposit investment, $1 = R_{D,t+1}\mathbb{E}_t M_{t,t+1}$, the deposit rate (household's risk-free rate) is:

$$\begin{aligned}
0 &= \log R_{D,t+1} + \log \mathbb{E}_t M_{t,t+1} && (10.16)\\
&= \log R_{D,t+1} + \mathbb{E}_t \log M_{t,t+1} + \frac{1}{2}\mathbb{V}_t \log M_{t,t+1} && (10.17)\\
&= \log R_{D,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} + \frac{\gamma^2}{2}\mathbb{V}_t \log R_{h,t+1} && (10.18)\\
&= r_{D,t+1} - \mathbb{E}_t r_{h,t+1} + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (10.19)\\
&= r_{D,t+1} - (1-\pi_{t+1})r_{D,t+1} - \pi_{t+1}\mathbb{E}_t r_{E,t+1} + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (10.20)\\
&= -\pi_{t+1}(\mathbb{E}_t r_{E,t+1} - r_{D,t+1}) + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (10.21)
\end{aligned}$$

The equity premium is:

$$\mathbb{E}_t r_{E,t+1} - r_{D,t+1} = \left(\gamma - \frac{1}{2}\right)\mathbb{V}_t r_{E,t+1} \tag{10.22}$$

C. Present Value of Equity Return

Monotonicity of the following present value problem yields ($\mathbb{E}_t M_{t,t+1} R_{E,t+1} = \int_\Delta M_{t,t+1} R_{E,t+1} dF$):

$$\arg\max\ \mathbb{E}_t M_{t,t+1} R_{E,t+1} = \arg\max\ \log \mathbb{E}_t M_{t,t+1} R_{E,t+1} \tag{10.23}$$

then,

$$\log \mathbb{E}_t M_{t,t+1} R_{E,t+1} = \mathbb{E}_t \log(M_{t,t+1} R_{E,t+1}) + \frac{1}{2}\mathbb{V}_t \log(M_{t,t+1} R_{E,t+1}) \tag{10.24}$$

the first term on the RHS of equation (10.24) is:

$$\begin{aligned}
\mathbb{E}_t \log(M_{t,t+1} R_{E,t+1}) &= \mathbb{E}_t \log R_{E,t+1} + \mathbb{E}_t \log M_{t,t+1}\\
&= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} && (10.25)
\end{aligned}$$

the second term (without 1/2) on the RHS of equation (10.24) is:

$$\begin{aligned}
\mathbb{V}_t \log(M_{t,t+1} R_{E,t+1}) &= \mathbb{V}_t \log R_{E,t+1} + \mathbb{V}_t \log M_{t,t+1} + 2\mathrm{Cov}_t(\log M_{t,t+1}, \log R_{E,t+1})\\
&= \mathbb{V}_t \log R_{E,t+1} + \gamma^2 \mathbb{V}_t \log R_{h,t+1} + 2\mathrm{Cov}_t(\log M_{t,t+1}, \log R_{E,t+1})\\
&= (1-\gamma\pi_{t+1})^2 \mathbb{V}_t \log R_{E,t+1} && (10.26)
\end{aligned}$$

Re-writing equation (10.24) using (10.25) and (10.26):

$$\begin{aligned}
&= \log \mathbb{E}_t M_{t,t+1} R_{E,t+1} \\
&= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2} \mathbb{V}_t \log R_{h,t+1} + \frac{(1-\gamma\pi_{t+1})^2}{2} \mathbb{V}_t \log R_{E,t+1} \\
&= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{1}{2}\left[(1-\gamma)^2 \pi_{t+1}^2 - (1-\gamma\pi_{t+1})^2\right] \mathbb{V}_t \log R_{E,t+1} \\
&= (1-\pi_{t+1})(\mathbb{E}_t r_{E,t+1} - r_{D,t+1}) - \frac{1}{2}(1-\pi_{t+1})\left[(2\gamma-1)\pi_{t+1} - 1\right] \mathbb{V}_t \log R_{E,t+1} \qquad (10.27)
\end{aligned}$$

D. Deposit Insurance Premium

First-Order Condition (5) w.r.t. to deposit insurance taxations:

$$0 = \mathbb{E}_t\left\{M_{t,t+1}\left(\underbrace{-\left[(1-\theta_{t+1})R_{D,t+1} + \theta_{t+1}R_{E,t+1}\right]}_{\text{loss of credit to output market}} \underbrace{+\frac{d\mathrm{Tr}_{t+1}}{d\tau_{t+1}}}_{\text{gain from lower bankruptcy cost}}\right)\right\}$$

$$\begin{aligned}
0 = & \int_0^\delta \left\{M_{t,t+1}\left(-\left[(1-\theta_{t+1})R_{D,t+1}\right] + \frac{d\mathrm{Tr}_{t+1}}{d\tau_{t+1}}\right)\right\} dF + \\
& \int_\delta^\infty \left\{M_{t,t+1}\left(-\left[(1-\theta_{t+1})R_{D,t+1} + \theta_{t+1}R_{E,t+1}\right] + 1\right)\right\} dF
\end{aligned}$$

E. Optimal Capital Regulation

The social welfare function is evaluated by household's utility function given regulator's resources,

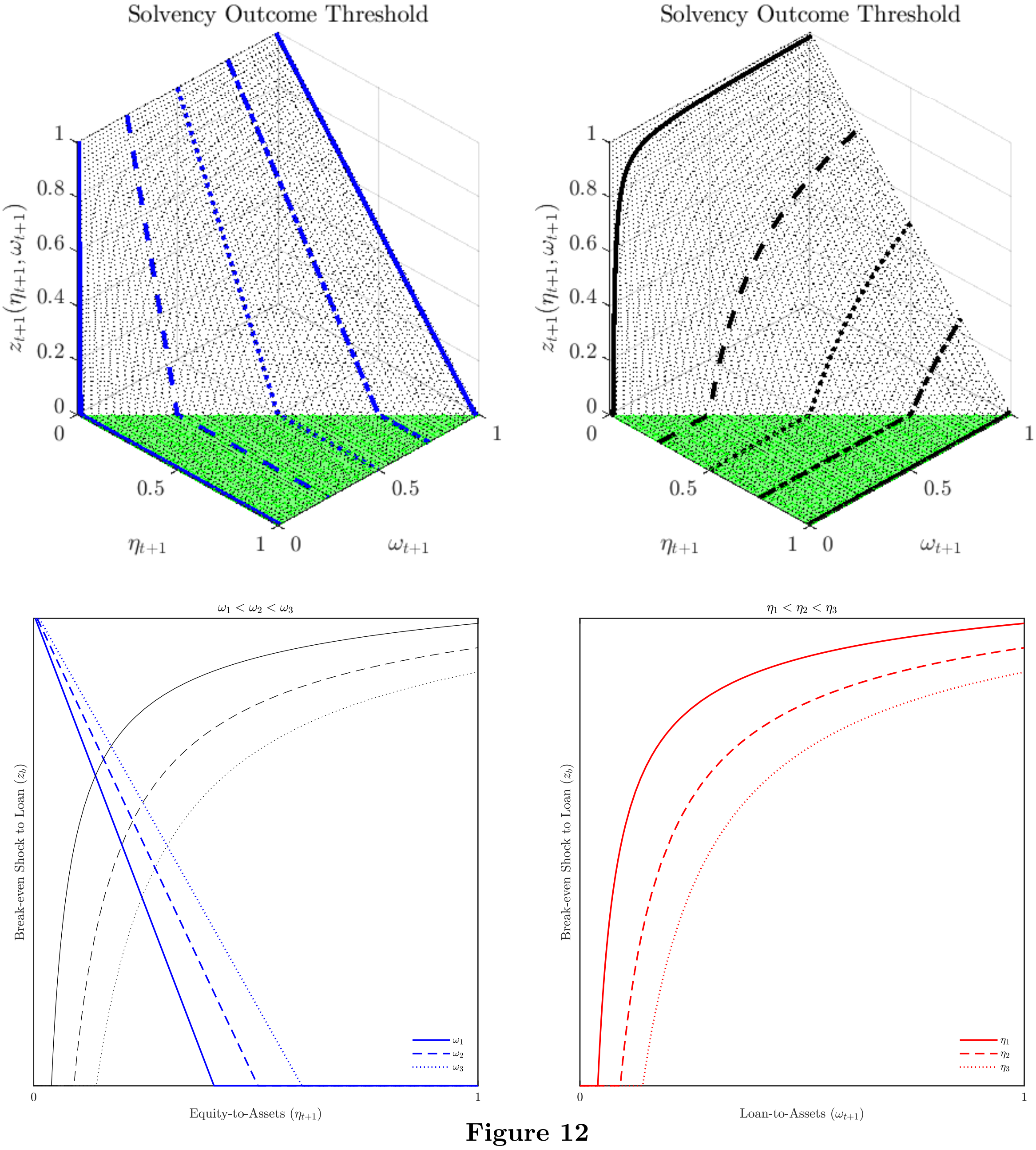
$$g = \begin{cases} (1-\tau).\left[\frac{1-\overline{\eta}}{Q_{D,t}} + (1-\kappa).\overline{\eta}.R_{E,t+1}\right] + \left[\tau - (1-\tau).(1-\omega_{t+1}).r_x\right] & \text{in solvency} \\ (1-\tau).\frac{1-\overline{\eta}}{Q_{D,t}} + \left[\tau - (1-\tau).\left[\left(\frac{1-\overline{\eta}}{Q_{D,t}} - \chi.R_{p,t+1}\right) - (1-\omega_{t+1}).r_x\right]\right] & \text{in default} \end{cases}$$

The first derivative of w.r.t. capital regulation choice $\overline{\eta}_{t+1}$ is

$$0 = \frac{\chi + \Phi(\lambda_{t+1}).(1-\chi)}{1-\chi} - \frac{1-\overline{\eta}_{t+1}(1-\kappa)}{1-\kappa}\left(-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}\right)\frac{\partial z_{b,t+1}}{\partial\overline{\eta}_{t+1}}$$

Approximating the term $-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}$ with the following exponential affine function, $e^{a_0+a_1 z_b}$ where a_0 and a_1 are functions of μ and σ.

F. Solvency Condition



Solvency Outcome Threshold
Solvency Outcome Threshold
$z_{t+1}(\eta_{t+1};\omega_{t+1})$
η_{t+1}
ω_{t+1}
$\omega_1 < \omega_2 < \omega_3$
$\eta_1 < \eta_2 < \eta_3$
Break-even Shock to Loan (z_b)
Equity-to-Assets (η_{t+1})
Loan-to-Assets (ω_{t+1})
ω_1
ω_2
ω_3
η_1
η_2
η_3

Figure 12

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