PhD Econometrics 1: Study Questions Class 4
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Question 1 Consider the model $y_i = \alpha + \exp(x_i\beta) + u_i$. Derive the NLS estimators for α and β.

Question 2 Let $\boldsymbol{g}_i(\boldsymbol{\theta}_0)$ denote the score function and $\boldsymbol{H}(\boldsymbol{\theta}_0)$ denote the hessian matrix. Show that $\mathbb{E}[\boldsymbol{g}_i(\boldsymbol{\theta}_0)] = 0$ and that $\text{var}(\boldsymbol{g}_i(\boldsymbol{\theta}_0)) = -\mathbb{E}[\boldsymbol{H}_i(\boldsymbol{\theta}_0)]$.

Question 3 Consider the probability density function, $f(x;\theta) = \lambda e^{-\lambda x}$. Find the MLE of λ and its variance (assuming that the sample is i.i.d.).

Question 4 Consider a simple linear regression model with non-stochastic regressors and $i = 1, \ldots, n$:

$$y_i = \alpha + \beta x_i + u_i \tag{1}$$

$$u_i \sim i.i.d\, \mathcal{N}(0, \sigma^2) \tag{2}$$

(4.1) Define the ML estimator for α and β.

(4.2) Clearly stating any assumption you need, derive the ML estimators for α and β.

(4.3) Is this estimator BLUE? Derive the asymptotic distribution of the vector $\widehat{\boldsymbol{\theta}} = (\widehat{\alpha}\ \widehat{\beta})'$ providing an expression for its asymptotic covariance matrix.