PhD Econometrics 1: Study Questions Class 5
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Question 1 Suppose that the dependent variable y_t is a proportion, so that $0 < y_t < 1$, $t = 1, \ldots, T$. An appropriate model for such a dependent variable is:

$$\log\left(\frac{y_t}{1-y_t}\right) = \boldsymbol{X}_t\boldsymbol{\beta} + u_t \tag{1}$$

where $\boldsymbol{X}_t$ and $\boldsymbol{\beta}$ are k-dimensional vectors of exogenous variables and regression parameters, respectively. Write down the loglikelihood function for this model under the assumption $u_t \sim i.i.d.\mathcal{N}(0, \sigma^2)$.

Question 2 The model is:

$$y_i = \beta_1 + \beta_2 x_i + \beta_3 z_i + u_i \tag{2}$$

Assume that the population disturbance term is independently and identically distributed with Normal density, and we wish to examine the null hypothesis $H_0 : \beta_3 + 1 = 0$. Define the likelihood ratio (LR), Wald (W) and Largrange multiplier (LM) test-statistics, discuss their asymptotic and finite sample relationships and examine the null hypothesis using each testing methodology.

Question 3 Consider the linear regression model:

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{X}_2\boldsymbol{\beta}_2 + \boldsymbol{u} \tag{3}$$

$$\boldsymbol{u} \sim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I}) \tag{4}$$

(3.1) Derive the Wald Statistic for the hypothesis that $\boldsymbol{\beta}_2 = \boldsymbol{0}$.

Question 4 Suppose a random sample of N values drawn from a uniform distribution is represented by $\{x_i\}_{i=1}^N$ such that

$$f(x) = \frac{1}{\theta_2 - \theta_1} \quad \theta_1 \leq x \leq \theta_2 \tag{5}$$

Derive the maximum likelihood estimator for θ_1 and θ_2.