Macro-Finance: Class 5

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Today's Outline

- Global interest rates model
- Asset pricing re-cap
- Problems for next week

You are encouraged to try the problems at the end beforehand for the next session. Let me know if you have any queries:

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A two-country model of equilibrium interest rates

Suppose now that there are two countries (domestic and foreign) and in each country there is a representative agent that lives for two periods and has preferences given by

$$U(c_1^j, c_2^j) \quad = \quad \ln c_1^j + \beta \ln c_2^j \tag{1}$$

where $j = d$ for the domestic country and $j = f$ for the foreign country Assume also that:

- the domestic representative agent receives as income y_1^f units of the consumption good in period 1, and zero in period 2
- the foreign representative agent receives zero income in period 1 and y_2^f units of the consumption good in period 2;
- agents can borrow or lend at the world gross interest rate $1 + r$.

1. Find the optimal consumption for each of the two agents as a function of r and β and their own incomes.
2. Find the level of utility of each agent as a function of r and β and their own incomes. How does the utility of the two agents change as r changes? Why?
3. Assume that these are the only two countries in the world and solve for the equilibrium interest rate as a function of β and the endowments. (Hint: you should impose the market clearing condition that total consumption in each period is equal to the total endowment in that period). How does r change a y_2^f changes? Why?

Intertemporal Budget

Domestic (d)

$$c_1^d + \underbrace{s^d}_{\text{investment}} = \underbrace{y_1^d}_{\text{income today}} \tag{2}$$

$$c_2^d = \underbrace{(1+r)s^d}_{\text{investment plus interest}} \tag{3}$$

$$\underbrace{c_1^d + \frac{c_2^d}{1+r}}_{\text{lifetime consumption present value}} = \underbrace{y_1^d}_{\text{income present value}} \tag{4}$$

Foreign (f)

$$c_1^f + \underbrace{s^f}_{\text{credit or borrowing}} = 0 \tag{5}$$

$$c_2^f = y_2^f + (1+r)s^f \tag{6}$$

$$\underbrace{c_1^f + \frac{c_2^f}{1+r}}_{\text{lifetime consumption present value}} = \underbrace{\frac{y_2^f}{1+r}}_{\text{income present value}} \tag{7}$$

Combine (2) and (3) to write (4). Combine (5) and (6) to write (7).

Application

Use the intertemporal budgeting to discuss what happens in the following scenario:

Financial Crisis

In an unexpected condition, e.g. financial crisis

1. a negative shock leads to many bankruptcies in the business sector
2. an increasing number of non-performing loans occur
3. the banking system is unable to honour depositors in full
4. the Federal Deposit Insurance Corporation (FDIC) intervenes to repay depositors

Let's start from financial sector before focusing on government and assume that a government bond B_t is risk-free. At date-1:

Assets	**Liabilities**
Bond (B_1)	Debt/Deposit (Q_1)
Loan (L_1)	Equity (E_1)
Investments	Resources

Typically we observe that $i^B < i^L$ but bonds serve an important role as a safe investment. At date-2:

Assets	**Liabilities**
Bond $B_2 = (1 + i_2^B)B_1$	Debt/Deposit $D_2 = (1 + i_2^q)Q_1$
Loan $L_2 = (1 + i_2^L)L_1$	Equity (E_2)
Investments	Resources

Assets		Liabilities	
Bond	(B_2)	Deposit	(Q_2)
Loans	(L_2)		
Non-performing Loans		Equity	(E_2)
		Resources	

Assets		Liabilities	
Bond	(B_2)	Deposit	(Q_2)
Loans	(L_2)		
Non-performing Loans		Equity	(E_2)
		Resources	

When non-performing investment increases (beyond deposits), there are two effects arising:

- Inability to repay deposits in full has contagious effects e.g. monetary policy interventions
- Bankruptcy cost worsens the situation

Optimal Consumption

Domestic

$$\begin{aligned} \max \quad & \ln c_1^d + \beta \ln c_2^d \\ s.t. \quad & c_1^d + \frac{c_2^d}{1+r} \leq y_1^d \end{aligned}$$

derivative w.r.t. c_1^d and c_2^d:

$$\frac{1}{c_1^{d*}} = \beta(1+r)\frac{1}{c_2^{d*}}$$

Foreign

$$\begin{aligned} \max \quad & \ln c_1^f + \beta \ln c_2^f \\ s.t. \quad & c_1^f + \frac{c_2^f}{1+r} \leq \frac{y_2^f}{1+r} \end{aligned}$$

derivative w.r.t. c_1^f and c_2^f:

$$\frac{1}{c_1^{*f}} = \beta(1+r)\frac{1}{c_2^{f*}}$$

Optimal Consumption

The domestic (foreign) agent is a net lender (borrower), as r increases her utility increases (decreases) due to a positive (negative) income effect.

$$\begin{aligned} c_1^{d*} &= \frac{1}{1+\beta} y_1^d; \qquad c_1^{f*} = \frac{1}{1+\beta} \frac{y_2^f}{1+r} \\ c_2^{d*} &= \frac{1}{1+r} \frac{\beta}{1+\beta} y_1^d; \quad c_2^{f*} = \frac{\beta}{1+\beta} y_2^f \end{aligned}$$

In equilibrium markets clear, total consumption in each period should be equal to total output in that period:

$$c_1^d + c_1^f = y_1^d; \; c_2^d + c_2^f = y_2^d;$$

Using any of these equilibrium conditions (by Walras' law, if $n-1$ markets are in equilibrium, the n^{th} market will be in equilibrium too)

the optimal solutions for consumption we get an endogenous interest rate:

$$1 + r = \frac{y_2^f}{y_1^d \beta}.$$

Implications:

- as y_2^f increases, r increases
- this is due to the fact that when y_2^f increases the foreign agents is overall richer and would like to increase consumption in both periods of her life
- But to increase consumption in the first period she needs to borrow more, and this increase in demand increases the equilibrium price of borrowing r.
- *The endogenous interest rate depends also on the preferences (in this case through β). As β decreases, both agents would like to increase consumption in the first period relative to the second period (since the contribution of second period consumption to their utility is reduced). As a consequence, the supply of credit is reduced and the borrowing demand is increase, and for the credit market to clear the interest rate has to increase.*

Discounting

For an equity (dividend D), bond (coupon), real estate (rent), etc:

(I) Deterministic (constant rate)

$$\underbrace{P_t}_{\text{Present Value}} = \sum_{i=1}^{\infty} \frac{\overbrace{CF_{t+i}}^{\text{Dividend}}}{\underbrace{(1+r)^i}_{\text{Discount Rate}}}$$

1. business sector generates cashflow: higher CF_{t+i}'s imply higher prices
2. higher discount rate (low DF '$\frac{1}{1+r}$'), lowers prices

(II) Deterministic (time-varying rates)

$$P_t = \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \frac{1}{(1+r_{t+i})^i} \right] D_{t+i} \overset{\text{e.g.}}{=} \underbrace{\frac{1}{1+r_{t+1}}}_{\text{discounting once}} D_{t+1} + \underbrace{\frac{1}{1+r_{t+1}} \frac{1}{1+r_{t+2}}}_{\text{discounting twice}} D_{t+2}$$

e.g. if only tomorrow and the day after tomorrow pay dividend.

Discounting (Stochastic)

(III) Stochastic (time-varying but exogenous) — Expectations are (also) drivers of prices today

$$P_t = \mathbb{E}_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \frac{1}{(1+r_{t+i})^i} \right] D_{t+i} \right\}$$

(IV) Stochastic (time-varying, endogenous) — Incorporate investors-borrowers interaction whose joint behaviour prices the assets

$$P_t = \mathbb{E}_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \underbrace{\beta^i \frac{u'(C_{t+1+i})}{u'(C_{t+i})}}_{\text{SDF } (m)} \right] D_{t+i} \right\}$$

1. Investors (e.g. households) own capital funds:
   interested in earning return on investment to be able to consume
2. Borrowers run enterprises:
   attract capital to finance operations

Stochastic Discount Factor

Investors-borrowers agree on r_{t+1}:

- e.g. promised return on a bond or expected return on equity contracts
- more investment opportunities increase r_{t+1} and more saving lowers r_{t+1}
- next year we can be either in a recession or an expansion (uncertain)

Bad economy

- incomes are low (low C_{t+1})
  e.g. more corporate budgeting and bankruptcies (low D_{t+1}), etc.
- investors are concerned about finances
- any asset that pays \$1 is very valuable
- DF is high (implies $u'(C_{t+1})$ is high) or r_{t+1} is low
  e.g. bond's seniority over equity in bankruptcy proceedings

Good economy

- investors are less concerned and income is *relatively* less valued
- DF is low (low $u'(C_{t+1})$) or r_{t+1} is high

Stochastic Discount Factor

Difficult to deliver in bad economy

- high m (bad condition), coupled with with high payoff
- asset price is high

$$P_t = \underbrace{\mathbb{E}_t[m_{t+1}]}_{\text{discounting}} \times \underbrace{\mathbb{E}_t[P_{t+1} + D_{t+1}]}_{\text{expected payoff tomorrow}} + \underbrace{\text{cov}[m_{t+1}, P_{t+1} + D_{t+1}]}_{\text{risk adjustment}}$$

Elevated likelihood of a recession

- drives up savings today
- lowers discount rates
- business sector, on average, is less able to generate dividends (profits are low)

Risk-free Rate

An asset that pays \$1 regardless of the state of the world:

$$P_t = \mathbb{E}_t[m_{t+1}]\mathbb{E}_t[P_{t+1} = \$1]$$

P_t is the price of a risk-free asset today:

- $1 + r_f = \$1/P_t$
- high m, low r_f

$$1 + r_f = \frac{1}{\mathbb{E}_t[m_{t,t+1}]}$$

Pensions

Why pension funds are struggling to deliver?

- Contracted to deliver high r
- mid-way deviation from expectation

Next Weeks

- Next week: discuss dividend-yields, feedback on problem set 1 and R
- in two weeks: debt sustainability, and exchange rates
- in three weeks: feedback on textual analysis
- last week: revision

Debt Sustainability

What follows is very typical of what happens when a country turns to the IMF for help with its sovereign debt and therefore applies to any country facing these problems.

Before the Cyprus bail-in in March 2013, there was a substantial discussion about government debt sustainability ahead of the agreement with the Troika. Cyprus was excluded from borrowing in international capital markets since May 2011 when long term government interest rates went above 7%. At the time, the total deficit (as % of GDP) was around 6%, the primary deficit was around 3%, inflation was 2% and real GDP growth was around -1%. Public debt was approaching 88%.

1. Using these numbers, compute the projected next year end debt as a percent of GDP and decompose it to the three components: the real interest rate part, the part coming from growth and the primary deficit part. Why do you think Cyprus stopped borrowing internationally when interest rates moved above 7%?
2. When a country cannot borrow internationally but still needs to borrow to finance its operating needs, it usually enters into an agreement with the IMF. The IMF puts certain conditions on the country so that the debt can be repaid. This is known as IMF conditionality. The country gets the benefit of a lower interest rate loan from the IMF (relative to the interest rate capital markets would offer). The country pays the cost of some restructuring and austerity to be able to repay the loan. Assume in the Cyprus case the total deficit is reduced to 3% and the primary deficit to zero percent. In exchange the IMF/Troika gives a loan at 2.5%. Without changing any other numbers, re-compute the year end debt as a percent of GDP. Roll this forward one year to forecast next year's year end debt. What do you learn?

Exchange Rates and Monetary Policy

I — Read the following article on the FT and answer the following questions:
`Will the Riksbank do whatever it takes?`
(by Martin Sandbu on January 6, 2016)

1. What is causing the upward pressure on the Swedish krona?
2. Why is the upward pressure on the krona affecting the Riksbank's inflation targeting goal?
3. Why might ECB's QE be affecting the euro?
4. How was Switzerland defending the currency peg between the franc and the euro?
5. What are the costs of defending a peg which led Switzerland to abandon the explicit currency peg of the Swiss franc to the euro?

II — Read the following article on the FT and comment why is China trying to maintain currency stability? Do you think China will succeed?
`Global assets shaken by China market turmoil`
(by Dan McCrum and Gabriel Wildau, January 8, 2016)

N.B. articles can be accessed by logging on to the FT via college account (a copy is available on the hub).