A highly accurate iterative PIV technique using a gradient method

2000 Meas. Sci. Technol. 11 1666

(http://iopscience.iop.org/0957-0233/11/12/303)

Meas. Sci. Technol. **11** (2000) 1666–1673. Printed in the UK PII: S0957-0233(00)15511-0

A highly accurate iterative PIV technique using a gradient method

Yasuhiko Sugii†, Shigeru Nishio‡, Taketoshi Okuno§ and Koji Okamoto†

† Nuclear Engineering Research Laboratory, University of Tokyo, 2-22 Shirane, Shirakata, Tokai-mura, Naka-gun, Ibaraki, 319-1106, Japan
‡ Kobe University of Mercantile Marine, 5-1-1, Fukae-minami, Higashinada, Kobe, 658-0022, Japan
§ Department of Marine System Engineering, Osaka Prefecture University, 1-1, Gakuen-cho, Sakai, Osaka, 599-8531, Japan

Received 17 July 2000, in final form and accepted for publication 25 September 2000

Abstract. An iterative PIV technique in which the combination of the iterative cross-correlation technique and three-point Gaussian peak fitting for a sub-pixel analysis has been used can improve the spatial resolution and accuracy of measurement. It is reported that the root-mean-square (RMS) error of the technique is of the order of only 0.04 pixels. However, a large interrogation window, typically 32 × 32 pixels or larger, should be taken, resulting in a low resolution, in order to achieve the high sub-pixel accuracy. The high accuracy is not compatible with high spatial resolution. In this paper, a new high-resolution PIV technique based on a gradient method is proposed. Initially, the pixel unit displacement is detected by the iterative method. Then, the sub-pixel displacement is evaluated by the use of the gradient method instead of the three-point Gaussian peak fitting technique. The error of the proposed technique is analytically assessed by Monte Carlo simulations. The RMS error is of the order of 0.01 pixels even with a small interrogation window, for instance 13 × 13 pixels or less. Thus, the method can achieve high sub-pixel accuracy and high spatial resolution compatibility.

Keywords: gradient method, sub-pixel analysis, high-resolution technique, Monte Carlo simulations

1. Introduction

Particle image velocimetry (PIV) is a quantitative method for measuring velocity fields instantaneously in experimental fluid mechanics (Raffel *et al* 1998). One of the popular methods is the cross-correlation method, which is well known to be a robust method, which evaluates a cross-correlation coefficient using Fourier transformation in a small region of images (Keane and Adrian 1992). In this method, the highest peak on the correlation plane identifies the local volume averaged velocity. The dynamic range of the velocity is limited by the size of its interrogation window because a displacement larger than the window size cannot be obtained by cross-correlation based on fast Fourier transformation (FFT). Therefore the technique commonly requires a rather large interrogation window, typically 32 × 32 pixels or larger. Since the displacement vectors can be obtained only in pixel units, several sub-pixel analysis techniques have been proposed. In these techniques, the cross-correlation functions are fitted to some known functions, such as parabolic ones, the Gaussian function and so on, to obtain the sub-pixel peak.

Several methods for improving the spatial resolution and accuracy have recently been proposed. One of the methods is the so-called iterative correlation method. The technique uses the translation of the interrogation windows with an offset derived from an iterative procedure (Westerweel *et al* 1997, Scarano and Riethmuller 1999). Also, super-resolution PIV has been proposed. This involves correlating a window of standard size and splitting it into smaller sub-windows before re-correlating from the initial correlation for a predictor of the subsequent correlation (Hart 1998). In most of these methods, the accuracy of particle image displacement by a pixel unit is improved, although a sub-pixel displacement is also calculated by the use of the Gaussian fitting.

The error of a PIV algorithm is usually assessed by means of Monte Carlo simulations (Raffel *et al* 1998), in which particle images with known displacements are generated synthetically. The precision and reliability of measurement are evaluated by comparing the results with the given displacements to analyse the synthetic images. Keane and Adrian (1990) have examined the effects on the conventional correlation method of experimental conditions such as the relative in-plane displacement, the velocity gradient, the particle image density and so on. Willert and Gharib (1991) have also reported that three-point Gaussian fitting with optimum experimental parameters produces measurements with an uncertainty level of about 0.1 pixels.

0957-0233/00/121666+08$30.00

Lourenco and Krothapalli (1995) have studied the effects of various correlation peak-finding schemes. Westerweel (1999) developed a theoretical analysis of the correlation method using a discrete window offset with sub-pixel analysis by Gaussian fitting. However, the minimum root-mean-square (RMS) error of the techniques is of the order of only 0.04 pixels. Furthermore, the Gaussian fitting requires a very high particle density so that a rather large interrogation window has to be taken. Therefore, the spatial resolution must be low in order to achieve a high accuracy.

On the other hand, in the gradient method that is occasionally called the spatio-temporal derivative method, the governing equation is derived using a Lagrangean derivative, which means that it is a technique with high spatial resolution. It has been developed to evaluate an optical flow in computer vision and measure the visual motion (Horn and Schunck 1981, Hildreth 1984, Ando 1986). It has also been applied to flow field measurements (Okuno and Nakaoka 1991) and wall shear stress measurements (Okuno 1995), with the advantage of being able to measure very small displacements. The method is applicable to various visualized images for dye, oil, particles and so on. However, it cannot measure large displacements because of its small dynamic range.

In this research, a new high-resolution PIV technique is proposed in order to improve the dynamic range, spatial resolution and accuracy of measurement. A pixel unit displacement is obtained using the iterative cross-correlation method while a sub-pixel displacement is calculated by the use of the spatio-temporal derivative method. By combining these two methods, highly accurate measurements have been achieved, with inaccuracies of as little as 0.01 pixels. The technique can be performed with a low particle density to obtain high-resolution measurement of a distribution.

2. PIV techniques

2.1. The iterative cross-correlation method

The cross-correlation coefficient $R(\xi, \eta)$ for the relationship between the two consecutive images $f(x, y, t)$ and $f(x, y, t + \Delta t)$ is given by

$$R(\xi, \eta) = \frac{1}{A} \sum_{A} f(x, y, t) f(x + \xi, y + \eta, t + \Delta t) \qquad (1)$$

where $f(x, y, t)$ denotes the luminance function, (x, y) is the position in the image plane, (ξ, η) is the position in the correlation plane and A is the interrogation window.

The highest value in the correlation plane identifies the average displacement in the interrogation window. However, an erroneous vector frequently occurs due to mismatching, when a small interrogation window is taken. Thus, the use of a larger window, typically 32×32 pixels or more, causes the velocity distribution to become one of low spatial resolution. In order to obtain data with much higher resolution, the iterative cross-correlation method has been developed (Raffel *et al* 1998). Initially, the cross-correlation method is applied with a large interrogation window to estimate the displacement correctly. Then, the cross-correlation method is applied again for a smaller window using the estimated displacement. Since the displacement vector is roughly estimated using a large window, the vector of the smaller window can be estimated with high accuracy. By repeating the above process with a decreasing window size, the high-resolution velocity can finally be calculated. Furthermore, the erroneous vector can be reduced significantly by the iteration process.

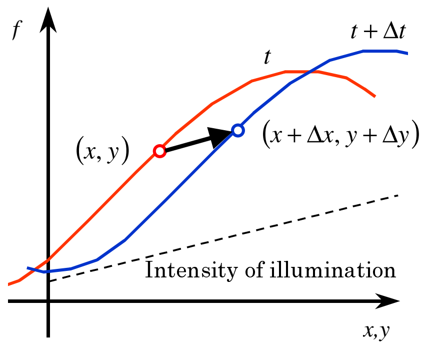


Figure 1. The principle of the spatio-temporal derivative method.

Since the image $f(x, y, t)$ consists of the discrete function in pixel units, the displacement vector (ξ, η) is discrete. Thus, the location of the peak on the correlation plane is also in pixel units. Several methods for improving the dynamic range have been developed, such as sub-pixel analysis. One of the approaches is peak fitting, in which the correlation peak is fitted to some function with a sub-pixel unit. A fitting technique using the three adjoining values, such as parabolic peak fit, the Gaussian peak fit and so on, is widespread. Three-point Gaussian peak fitting is widely used for sub-pixel analysis, in which a sub-pixel displacement ε is estimated from

$$\varepsilon = \frac{1}{2} \frac{\ln R(\xi - 1) - \ln R(\xi + 1)}{\ln R(\xi - 1) + \ln R(\xi + 1) - 2 \ln R(\xi)} \qquad (2)$$

where ξ denotes the position of the peak in the one-dimensional case.

Keane and Adrian (1992) have reported the effects on the RMS error of parameters such as the relative in-plane displacement, the velocity gradient, the particle image density and so on. The uncertainty level is about 0.1 pixel units under the optimum conditions. The theoretical investigation revealed that the lowest RMS error is about 0.04 pixels (Westerweel *et al* 1997).

2.2. The spatio-temporal derivative method (the gradient method)

Figure 1 illustrates the image intensity in one dimension. Assuming that the visual motion is exactly the same as the fluid motion and that a particle moves from (x, y) at t to $(x + \Delta x, y + \Delta y)$ at $t + \Delta t$, a difference in luminance between these two points is expressed as follows:

$$f(x + \Delta x, y + \Delta y, t + \Delta t) - f(x, y, t) = h(x, y, t, \Delta x, \Delta y, \Delta t) \qquad (3)$$

where h is the variation in background intensity.

Furthermore, assuming that the visualized image pattern varies smoothly like stationary motion or selectively homogenous motion and that the luminance function $f(x, y, t)$ is expressed as a Taylor series in time and space, it can be expressed in terms of a Lagrangean derivative,

$$f(x+\Delta x, y+\Delta y, t+\Delta t) \\ = f(x, y, t) + \Delta x f_x + \Delta y f_y + \Delta t f_t + O(\Delta^2) \quad (4)$$

where

$$f_x = \frac{\partial f}{\partial x} \qquad f_y = \frac{\partial f}{\partial y} \qquad f_t = \frac{\partial f}{\partial t}.$$

Equation (3) is rewritten as

$$f_t + u f_x + v f_y = e \quad (5)$$

where

$$\lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = u \qquad \lim_{\Delta t \to 0}\left(\frac{\Delta y}{\Delta t}\right) = v$$

and e is the backgrounds, i.e. the error. Using the first-order approximation, equation (5) is the governing equation for the spatio-temporal derivative method, the so-called optical flow constraint equation for the computer visual motion. There remains the finite value e in equation (5) as a function of time and space, when the intensity of the background illumination is not constant in the measurement area as shown in figure 1. Usually it can be assumed to converge to zero. Since equation (5) contains two unknowns (u and v), it is not possible to solve the equation without any other constraints.

Assuming that the right-hand term in equation (5) is white noise, the least squares method can be applied for the image in an interrogation window A. If the velocities (u and v) are assumed to be constant in the window, the norm of the left-hand term,

$$\sum_A (f_t + u f_y + v f_y)^2$$

is minimized. Then, the velocity components (u and v) can be obtained by solving the following equation:

$$\begin{pmatrix} \sum_A f_x^2 & \sum_A f_x f_y \\ \sum_A f_x f_y & \sum_A f_y^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = -\begin{pmatrix} \sum_A f_x f_t \\ \sum_A f_y f_t \end{pmatrix} \quad (6)$$

where A is the interrogation window. The spatial derivative terms f_x and f_y are obtained by using the forward difference approximation with a third-order accuracy.

3. The new high-resolution PIV technique

As was pointed out in the previous section, the advantages of the iterative correlation method can be summarized as follows:

(i) a large displacement can be measured using a discrete window offset; and
(ii) the magnitude of the erroneous vector can be reduced by using the iterative cross-correlation technique.

These advantages have led to the iterative cross-correlation technique becoming a *de facto* standard. However, the well-known Gaussian fitting technique limits the dynamic range in the sub-pixel analysis as follows:

(i) the sub-pixel accuracy is only to within 0.04 or more pixel units even if optimal parameters are used (Westerweel *et al* 1997);
(ii) a large interrogation window, typically 32 × 32 pixels or more, and a high particle image density are required; and
(iii) under certain conditions, a systematic error appears in the fractional displacement, which biases the measurement towards integer values, which is called the peak-locking effect.

Therefore, the current PIV evaluation was limited to about 0.1 pixels, while the spatio-temporal derivative method has the following advantages:

(i) very small displacements, as small as 0.01 pixels, can be measured with high accuracy; and
(ii) the technique can provide high accuracy even if a particle image density is very small, such as only one or two particles within the window.

However, the spatio-temporal derivative method also has the following disadvantages, which result in the technique being of limited use for specific targets:

(i) the error ratio increases rapidly with increasing displacement; and
(ii) the errors are generated from the temporal variation of the particle luminance by the gradient in background illumination, out-of-plane motions and so on.

In this study, a new high-resolution technique has been proposed in order to take full advantage of both methods. For the pixel unit detection, the iterative cross-correlation method has been applied. By decreasing the interrogation window and the search region by iterating the process, a local displacement (ξ, η) can be evaluated in pixel units. The process is repeated iteratively until the displacement between two images converges to ±1 pixel. Then, the spatio-temporal derivative method is applied for the sub-pixel displacement instead of the Gaussian fitting. Since the pixel unit displacement is already known from the iterative cross-correlation method, the spatio-temporal derivative method is applied to the shifted first image $f(x, y, t)$ and the second image $f(x+\xi, y+\eta, t+\Delta t)$. Using the displacement (ξ, η) in pixel units obtained by the iterative method, the temporal derivative term f_t and spatial derivative terms f_x and f_y in equation (6) are obtained by using the following equations:

$$f_t = f(x+\xi, y+\eta, t+\Delta t) - f(x, y, t) \quad (7)$$

$$f_x = \frac{1}{2}\left(\frac{\partial f(x, y, t)}{\partial x} + \frac{\partial f(x+\xi, y+\eta, t+\Delta t)}{\partial x}\right) \quad (8)$$

$$f_y = \frac{1}{2}\left(\frac{\partial f(x, y, t)}{\partial y} + \frac{\partial f(x+\xi, y+\eta, t+\Delta t)}{\partial y}\right). \quad (9)$$

The spatial derivative terms are also obtained by using the forward difference approximation with a third-order accuracy.

Because the sub-pixel displacement $(\delta x, \delta y)$ is less than 0.5 pixel, the spatio-temporal derivative method can detect the sub-pixel displacement very accurately. In addition, the interrogation window for sub-pixel analysis can be relatively small, typically 13 × 13 pixels or less, which indicates that the high resolution for iterative correlation remains. Therefore, the present method can produce the velocity distribution with a high accuracy and a wide dynamic range.

4. Numerical evaluation for sub-pixel accuracy

The uncertainty and systematic errors of measurement are assessed using a Monte Carlo simulation numerically (Raffel *et al* 1998). The particle image is generated with known parameters such as the particle displacement, diameter, size of interrogation window, out-of-plane velocity, displacement gradient and so on. The individual particle is described with a Gaussian intensity profile at a randomly distributed location in space. The image is of size 256 × 256 pixels with an 8-bit (256) intensity level for a single pixel. Since the sub-pixel analysis is focused in this study, the pixel unit detection is assumed to be correctly evaluated by the iterative cross-correlation. Then, a sub-pixel displacement from 0.0 to 0.5 pixels in steps of 0.01 pixel is examined with the assumption. The present method has been compared with three-point Gaussian peak fitting, which has been used widely by many researchers. The Gaussian fitting has been examined regarding the accuracy and optimum parameters (Westerweel *et al* 1997, Raffel *et al* 1998). Therefore, we have compared the present technique with the Gaussian fitting as the representative technique of current PIV. In order to take various sizes of interrogation window, the coefficients are obtained by the direct cross-correlation method instead of the FFT based method.

4.1. Optimization of the particle image diameter

Figures 2(*a*) and (*b*) show the RMS errors of the present method and three-point Gaussian peak fitting as functions of the particle diameter. The error on the vertical axis is defined as the average value of the deviation from the given displacements for samples. The image parameters are the number density of particles C and the source density N_s. The number density of particles C and the source density N_s are defined (Adrian 1984) as

$$C = \frac{N}{V}\Delta z_0 \tag{10}$$

$$N_s = C\frac{\pi d_i^2}{4} \tag{11}$$

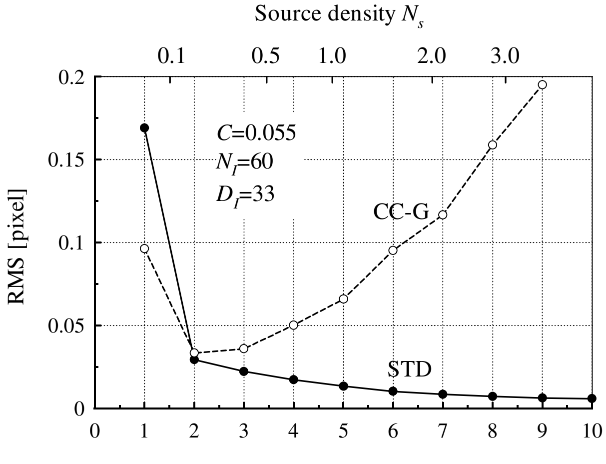
where N is the number of particles, V is the volume, Δz_0 is the thickness of the sheet of light and d_i is the average particle diameter defined by the e^{-2} intensity value of the Gaussian bell. All length scales are defined in pixel units. The number density of particles C denotes the number of particles in one pixel. The source density N_s means the ratio of the particle image area to the whole image area.

The image density N_I within the interrogation window is also defined (Adrian 1984) as

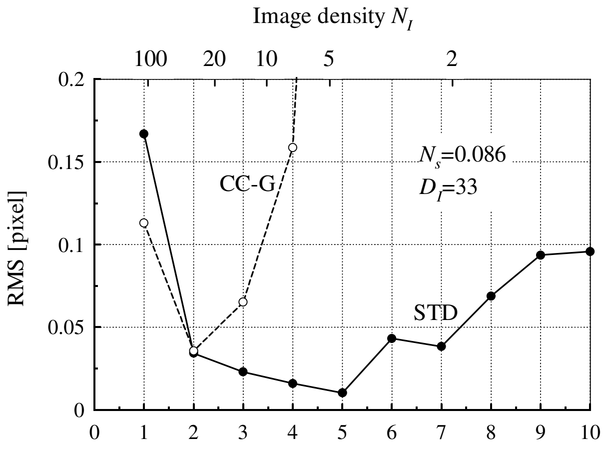
$$N_I = CD_I^2 \tag{12}$$

where D_I is the size of the interrogation window. The image density N_I is the number of particles within the interrogation window.

In figure 2(*a*) the number density of particles per unit area C and the source density N_I are fixed. The source density is also fixed in figure 2(*b*). The results of the present method and the Gaussian fitting are shown as full and broken lines respectively in figure 2. The interrogation window of both methods is taken as 33 × 33 pixels with constant $C = 0.055$, i.e. $N_I = 60$ in figure 2(*a*). The errors of the present method are acceptably small except for particle diameters d_i below 2.0 pixels. Because the method is based on image gradients, larger particles give spatial derivative terms with higher accuracy. The gradient of small particle diameter contains a larger error because the spatial derivative terms lead to large discrete errors. In contrast, the error of the Gaussian fitting drops off with increasing diameter until it reaches a minimum error at a diameter d_i about slightly more than 2.0 pixels and increases again for larger diameter.



(*a*)



(*b*)

Figure 2. The RMS uncertainty error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) with respect to varying particle image diameter: (*a*) the number density of particles for $C = 0.055$ (number of particles per unit area) and (*b*) the source density $N_s = 0.086$.

In figure 2(*b*) the interrogation window is also taken as 33 × 33 pixels for both methods with constant $N_s = 0.086$. The present method has also a high accuracy between 2 and 7 pixels, which corresponds to $N_I = 2.5$–30. The error of the Gaussian fitting also has a minimum at a diameter d_i of slightly more than 2.0 pixels, which corresponds to

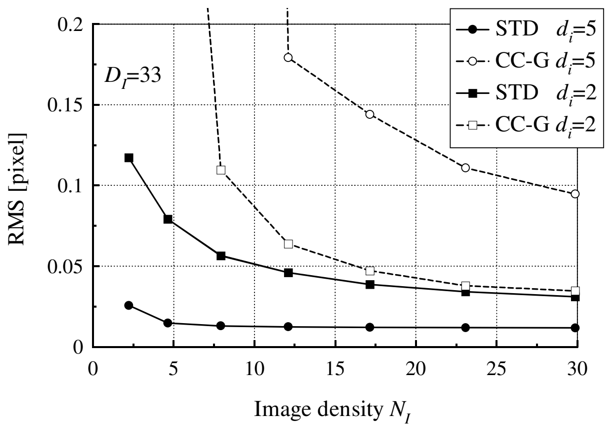


Figure 3. The RMS uncertainty error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) as a function of the image density per interrogation window, N_I (the simulation parameters are particle image diameters $d_i = 2.0$ and 5.0 pixels).

$N_I = 30$. The Gaussian fitting is more sensitive to the particle diameter.

The results show that the error of the present method is less than 0.01 pixels whereas that for the Gaussian fitting is of the order of 0.04 pixels. Furthermore, the optimum particle image diameter for the Gaussian fitting is slightly more than 2.0 pixels, which corresponds to values found in previous work (Raffel *et al* 1998), whereas that for the present method is about 5.0 pixels. Therefore, the results prove the robustness of the present method for measuring the particle diameter. In the following sections, the image density N_I, the average particle diameter d_i and the size of the interrogation window D_I are taken as the parameters.

4.2. The effect of the particle density

The Gaussian fitting provides a high performance for a high particle image density. Figure 3 shows the simulation results for the uncertainty of measurement as a function of the image density per interrogation window N_I and the number of particles within the interrogation window. The particle diameters d_i are 2.0 and 5.0 pixels without out-of-plane motion. The RMS error of the Gaussian fitting is too large to display in figure 3 for fewer than ten particles for both of those particle diameters. The error decreases with increasing density. The technique requires more than 30 particles to achieve an error of 0.1 pixel with $d_i = 5.0$. Thus, even if the optimal diameter $d_i = 2.0$ pixels is used, ten particles are required. In contrast, the error of the present method is nearly constant (about 0.01 pixel) with $d_i = 5.0$ pixels. The error is also small, even with a very small density such as two or three particles. The result indicates that the present method requires only a few particles within the interrogation window so that a smaller interrogation window can be taken. The size of the interrogation window generally determines the spatial resolution. Therefore, the spatial resolution of the vector obtained in this method can be much higher with more accurate sub-pixel analysis.

4.3. The accuracy of sub-pixel displacement

In the previous section, the errors for certain sub-pixel displacements from 0.0 to 0.5 pixels with a step of 0.01 pixel are averaged to give the RMS error. The sub-pixel error is examined in detail in this section. Figure 4 shows the uncertainty of measurement as a function of the sub-pixel displacement. The particle image diameter d_i is fixed to be 5.0 pixels and the number density of particles C varies to take the values 0.016, 0.027 and 0.049. As shown in figure 3, it is found that the present method requires a relatively smaller window D_I, whereas the Gaussian fitting needs a relatively larger window. In this comparison D_I, is set to be 33 and 13 for the Gaussian fitting and the present method respectively. The image density N_I is 2.8, 4.6 and 8.3 for the present method and 17.9, 29.9 and 53.8 for the Gaussian fitting. The errors increase with decreasing density C and are nearly constant (about 0.07–0.1 pixel) for all displacements. The smallest error is about 0.07 pixels with a highest density of $N_I = 53.8$. Furthermore, a certain value remains even at the displacement $\delta = 0$ because the coefficients have been obtained by the direct cross-correlation method instead of the FFT based method with the larger diameter. The dotted line in figure 4 shows the results of the Gaussian fitting with the optimal diameter $d_i = 2.0$ and $C = 0.027$ and $N_I = 29.9$. The minimum error of the Gaussian fitting is 0.02 pixels at the displacement $\delta = 0$ and increases with increasing displacement, which corresponds to the results of previous work (Raffel *et al* 1998). On the other hand, the error of the present method is smaller than that of the Gaussian fitting with both diameters at all displacements in spite of the smaller particle density. Furthermore, the present method has almost the same error at all densities C without biased error, which also shows that it is insensitive to the particle density.

4.4. The effect of displacement gradients

A displacement gradient across the window is likely to cause biased errors because any particle within the first interrogation window will not be found within the second window. Considering the domain distortion induced by two-dimensional shearing flow, the rate of the normalized vorticity $\omega = (\partial u/\partial y)\Delta t$ is adopted to quantify the extent of the deformation ($\partial v/\partial x = 0$). For instance, $\omega = 0.1$ means that neighbouring pixels are shifted by 0.1 pixel with respect to the others due to the shear flow. Figure 5 shows the uncertainty of measurement as a function of the displacement gradient. The error of the Gaussian fitting increases sharply with the gradient strength, particularly above a displacement gradient of about $\omega = 0.1$, because the correlation method using a linear approximation of the domain transform gives significantly larger uncertainties because of the gradient effect. Those of the present method increase linearly with increasing displacement gradient ω. When the acceptable error is about 0.1 pixel, the displacement gradient ω should be less than 0.02 for the Gaussian fitting and 0.05 for the present method.

4.5. The effect of out-of-plane motion

Most PIV techniques are inherently two-dimensional measurement techniques using a single camera and the sheet

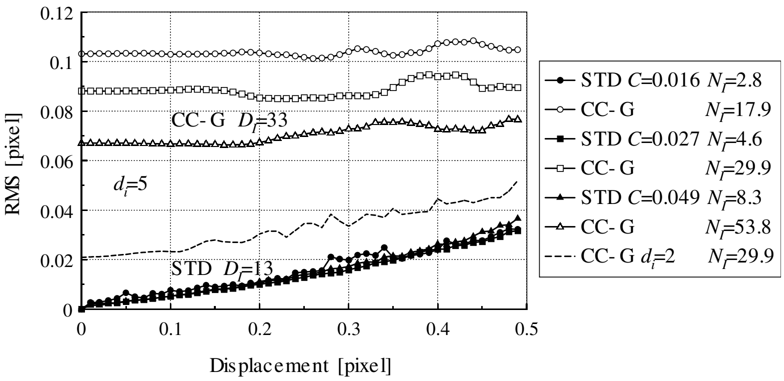


Figure 4. The RMS error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) as a function of the sub-pixel displacement (the simulation parameter is the particle image diameter $d_i = 5.0$ pixels).

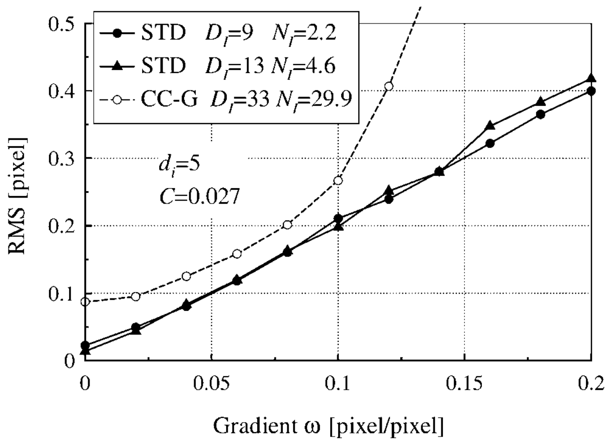


Figure 5. The RMS error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) as a function of the velocity gradient (the simulation parameters are the particle image diameter $d_i = 5.0$ pixels and the number density of particles $C = 0.027$).

of laser light. However, the flow is often fully three-dimensional, involving turbulence, wing-tip vortices, the flow around some structures and so on. Thus, it is important to consider the robustness of the present method with respect to out-of-plane motions. Figure 6 shows the results for the uncertainty of measurement as a function of the out-of-plane displacement $W = (w/\Delta z_0)\Delta t$ normalized by the sheet thickness Δz_0. The luminance of the laser light sheet is assumed to be a Gaussian function. Out-of-plane displacement $W = 0.0$ means that all particles in the first image remain in the second image with the same particle luminance whereas $W = 1.0$ means that no original particle remains in the second image. The error of the Gaussian fitting increases with the out-of-plane velocity, which should be less than 0.125 to achieve an error within 0.1 pixel. On the other hand, that of the present method, which increases with the out-of-plane velocity for every window size, D_I is sensitive to the size of the interrogation window. Since the luminance of the particle is assumed to be invariant, as shown by equation (6), the temporal differential, f_t, is spoiled by the variation of the particle luminance caused by the out-of-plane motion, resulting in larger errors. With increasing N_I, these effects decrease, making the error lower. When the acceptable error is about 0.05 pixel, the out-of-plane displacement W should be less than 0.03 and 0.15 for $N_I = 5$ and 30, respectively.

4.6. The effect of background noise

In the previous sections, the synthetic images are clean; however, any actual PIV image includes noise. Thus, the effect of noise is examined in this section. Figure 7 shows the results for the uncertainty of measurement as a function of the background noise. Normally distributed noise at a specified fraction of the dynamic range, [0:255] pixel intensity range, is linearly added to each pixel. The particle image diameter d_i is set to be 2.0 and 5.0 pixels and the number density of particles C is fixed to 0.027. The error of the Gaussian fitting increases linearly with increasing noise, similarly to the case of out-of-plane motion. While the error of the present method is smaller than that of the Gaussian fitting with $d_i = 5.0$, the error increases sharply with above 5% noise. The results show that the present method is as sensitive to noise as it is to out-of-plane motion. However, if parameters such as the particle image diameter and size of interrogation window are chosen appropriately, the technique can provide a high performance.

4.7. Discussion

A new sub-pixel analysis using the gradient method with the iterative cross-correlation technique has been compared with the conventional Gaussian fitting in this study. Assuming that the iterative cross-correlation method guarantees the accuracy of the displacement with the pixel unit, the sub-pixel analysis has been investigated. The simulation results show that the RMS error of the present method is of the order of 0.01 pixel whereas that of the Gaussian fitting is of 0.04 pixels or more. In addition, the optimum particle image diameter for the present method is 2–7 pixels. While the Gaussian fitting requires a high particle density, the present method can provide a high accuracy even with only two or three particles within the interrogation window.

For a very small particle diameter such as less than 2.0 pixels, the present method gives a large error because

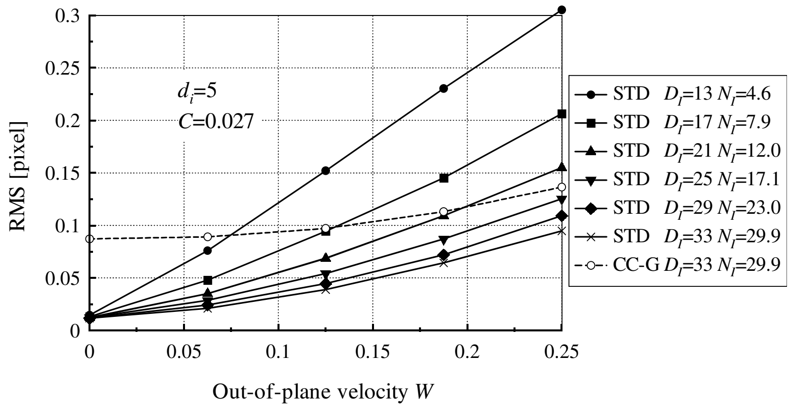


Figure 6. The RMS error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) as a function of the out-of-plane velocity W (the simulation parameters are the particle image diameter $d_i = 5.0$ pixels and the number density of particles $C = 0.027$).

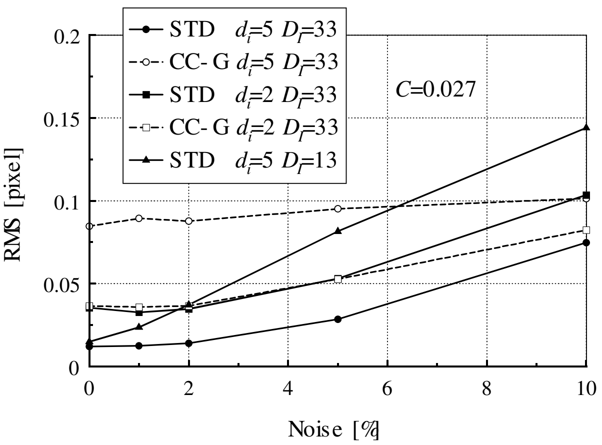


Figure 7. The RMS error of measurement for the spatio-temporal derivative method (STD) and three-point Gaussian fitting based on cross-correlation (CC-G) as a function of noise (the simulation parameters are the particle image diameters $d_i = 2.0$ and 5.0 pixels and the number density of particles $C = 0.027$).

errors in the spatial derivative terms f_x and f_y in equation (6) increase. It also has a disadvantage when the temporal differential, f_t in equation (6), is spoiled by variation of the particle luminance, which is caused by out-of-plane motion or the background noise, which results in a larger error. Because a larger interrogation window should be used for these cases, it is apparent that the present method is superior to the Gaussian fitting for sub-pixel analysis.

The high ability of the present method is also explained by considering the information of a particle image. The sub-pixel analysis in the present method directly uses the variation of luminance in every pixel whereas some luminance information is lost due to the integration for calculating the correlation in the Gaussian fitting. Therefore, the present method can obtain a high accuracy even in a small interrogation window

5. Conclusion

A new high-resolution PIV technique has been proposed in order to take advantage of the iterative cross-correlation method and the spatio-temporal derivative method. Initially, the pixel unit displacement is obtained by the iterative cross-correlation method. Then, the spatio-temporal derivative method determines the sub-pixel displacement by the use of a window offset. By combining these two methods, highly accurate and wide-dynamic-range measurements have been completed.

The error of the proposed technique is analytically evaluated as a function of the sub-pixel displacement, the particle image diameter, the out-of-plane displacement and the displacement gradient using Monte Carlo simulations. The results show that the RMS error of the present method is of the order of 0.01 pixel even with a very low particle number density, in which the size of the interrogation window could be reduced to 13 × 13 pixels or less. Thus, the present method is superior to the combination of the iterative cross-correlation method and the Gaussian fitting. Because the method can achieve a high sub-pixel accuracy and a high spatial resolution compatibility, it can offer possibilities for overcoming the limitations of PIV techniques.

References

Adrian R J 1984 Scattering particle characteristics and their effect on pulsed laser measurements of fluid flow: speckle velocimetry vs particle image velocimetry *Appl. Opt.* **23** 1690–1

Ando S 1986 A vector field measurement system based on spatio-temporal image derivative *Trans. Soc. Instrum. Control Engineers* **22** 1330–6

Hart D P 2000 Super-resolution PIV by recursive local-correlation *J. Visualization* **3** (2) 187–94

Hildreth E C 1984 Computation underlying the measurement of visual motion *Artificial Intelligence* **23** 309–54

Horn B K P and Schunck B G 1981 Determining optical flow *Artificial Intelligence* **17** 185–203

Keane R D and Adrian R J 1990 Optimization of particle image velocimeters. Part 1: double pulsed system *Meas. Sci. Technol.* **1** 1202–15

——1992 Theory of cross-correlation analysis of PIV images *Appl. Sci. Res.* **49** (3) 191–215

Lourenco L and Krothapalli A 1995 On the accuracy of velocity and vorticity measurements with PIV *Exp. Fluids* **13** 421–8

Okuno T 1995 Image measurement by means of spatio-temporal derivative method *Proc. Int. Workshop on PIV '95, Fukui* pp 167–73

Okuno T and Nakaoka J 1991 Velocity field measurement by spatio-temporal derivative method *J. Kansai Soc. Naval Architects* **215** 69–74

Raffel M, Willert C E and Kompenhans J 1998 *Particle Image Velocimetry* (Berlin: Springer)

Scarano F and Riethmuller M L 1999 Interactive multigrid approach in PIV image processing with discrete window offset *Exp. Fluids* **26** 513–23

Westerweel J M 1999 Theoretical analysis of the measurement precision and reliability in PIV *Proc. 3rd Int. Workshop on PIV '99* pp 9–14

Westerweel J, Dabiri D and Gharib M 1997 The effect of a discrete window offset on the accuracy of cross-correlation analysis of digital PIV recordings *Exp. Fluids* **23** 20–8

Willert C E and Gharib M 1991 Digital particle velocimetry *Exp. Fluids* **10** 181–93