

Finite difference method (FDM)

Is a numerical method for solving differential equations by setting the derivatives with finite differences.

$$y'_i \approx \frac{y_{(i+1)} - y_{(i-1)}}{(2h)}$$

$$y''_i \approx \frac{y_{(i+1)} - 2y_i + y_{(i-1)}}{(h^2)}$$

$$h = \frac{b-a}{N}$$

N is the number of points between a and b

Part 1

In the code, FDM will be used to solve the differential equation and to obtain the temperature at the outer walls of the cylinder. The program will come until the difference between the temperature becomes less than the margin, which is assumed to be 0.1.

Answer should be:

u (outer wall) = 200 grader

Part 2

The outside wall temperature is known and K required is calculated. Using the FDM and matrix elements from Part 1 at r = 2, the matrix can be solved.

$$k = \frac{2r(n)}{h^2} + \frac{1}{1 + \frac{2r(n)}{h}} * (u(n-1) - u(n)) \frac{(1)}{u(n) - 20}$$

Answer should be:

K=3.3338 at n=25

Part 3

Same as Part 1 except that the temperature of the hot liquid is 460 degrees.

Answer should be:

K (460) = 3.429 vid n=25

K (450) = 3.3338 vid n =25

Inhomogeneous metal

In this case, the new vector can be cooled as follows:

Assume that:

$$D(r) = ar^3 + br^2 + cr + d$$

From the boundary conditions

$$D'(1)=0 \rightarrow 3a + 2b + c = 0$$

$$D'(2)=0 \rightarrow 12a + 4b + c = 0$$

$$D(2)=2D(1) \rightarrow 2a + 2b + 2c + 2d = 8a + 4b + 2c + d$$

Which gives

$$b = -\frac{9}{2}a, c = 6a, d = -3a$$

$$1 + \frac{D'(r)}{D(r)} = 1 + \frac{3r^2 - 9r + 6}{r^3 - \frac{9}{2}r^2 + 6r - 3}$$

Part 4

Part 1 and consideration for inhomogeneous metal.

Answer should be:

$$u(\text{outer wall}) = 174 \text{ grader}$$

Part 5

Part 2 and consideration for inhomogeneous metal.

Answer should be:

$$K = 3.3338$$