

例 6.4

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$$\text{依 } E(X_i) = \mu \quad V(X_i) = \sigma^2 = E(X_i^2) - \mu^2$$

$$\text{則 } E(\bar{X}) = \mu \quad V(\bar{X}) = \frac{\sigma^2}{n} = E(\bar{X}^2) - \mu^2$$

$$E(\hat{\theta}_1) = E\left[\frac{\sum_{i=1}^n (X_i - \bar{X})}{n}\right] = \frac{1}{n} \left(\sum_{i=1}^n X_i^2 - n\bar{X}^2 \right)$$

$$= \frac{1}{n} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \sigma^2 + \mu^2 - \frac{\sigma^2}{n} - \mu^2$$

$$= \sigma^2 - \frac{\sigma^2}{n}$$

$$= \frac{n-1}{n} \sigma^2 \Rightarrow E(\hat{\theta}_1) \text{ 未滿足 } \sigma^2 \text{ 之 unbiased 估計量} \Rightarrow \text{biased 估計量}$$

$$E(\hat{\theta}_2) = E\left[\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}\right] = \frac{1}{n-1} E\left[\sum_{i=1}^n (X_i - \bar{X})^2\right]$$

$$= \frac{1}{n-1} (n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2)$$

$$= \left(\frac{n\sigma^2}{n-1} + \frac{n\mu^2}{n-1} - \frac{\sigma^2}{n-1} - \frac{n\mu^2}{n-1} \right)$$

$$= \frac{n\sigma^2 - \sigma^2}{n-1}$$

$$= \sigma^2 \Rightarrow E(\hat{\theta}_2) \text{ 滿足 } \sigma^2 \text{ 之 unbiased 估計量} \Rightarrow \text{unbiased 估計量}$$