A Diffusion Model for Computer Animation of Diffuse Ink Painting

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Abstract

The multidimensional diffusion model for computer animation of diffuse ink painting opens up a new dimension in painting. In diffuse painting final image is a result of ink diffusion in absorbent paper. A straightforward diffusion model however is unable to provide very specific features of real diffuse painting. In particular, it can not explain the appearance of certain singularities in intensity of color in the image which are very important features of diffuse ink painting. In our previous work [1], a model based on physical analysis of paper structure was proposed. Although this model provided an adequate simulation of many diffuse ink painting properties, it was still insufficient to explain the singularities of intensity distribution precisely. Now we solve this problem. A multidimensional diffusion model which we propose proves to provide exactly the same intensity distribution as in real images. The method was applied to animate ink diffusion 'Nijimi' of traditional Japanese ink painting 'Sumie'.

Keywords: Multidimensional diffusion processes, Nijimi, Colloidal liquid, Computer animation.

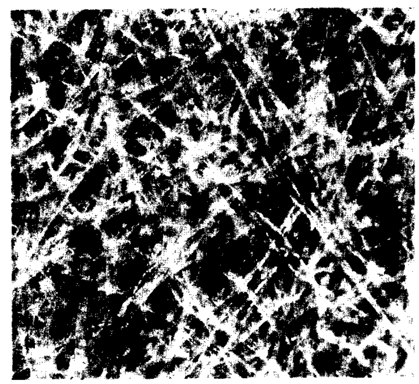

Figure 1: Paper fibers structure. Picture was taken through a microscope with the magnification of 500

1 Introduction

Diffuse ink painting (in Japanese - 'Sumie') is a kind of ink painting with very specific features. It requires a special paper with high absorbency and a special ink. The ink used in 'Sumie' is a colloidal liquid which consists from water and solid particles of carbon distributed in it. Glue is also added

Diffuse ink painting phenomenon is a new topic in computer graphics. Although some previous papers discussed the modeling of painting strokes ([4, 3, 6]) and liquid flow ([2, 5]), the diffuse ink effect was not analyzed.

When a drop of diffuse ink falls on the surface of highly absorbent paper, it begins to spread throughout the paper. As a result of this process, the final image appears to be sufficiently bigger then the initial zone to which the ink was directly applied. See Fig. 2.

The remarkable feature of diffuse ink image is a kind of black border which appears along the edge of the *initial zone* (i.e. zone where ink was directly applied to paper). One can see that the intensity of

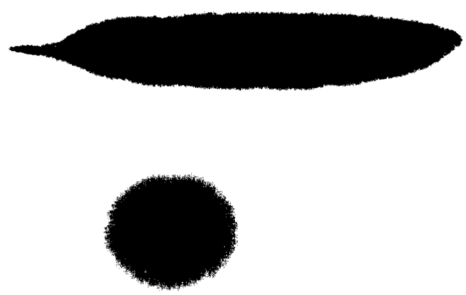

Figure 2: Examples of diffuse ink painting

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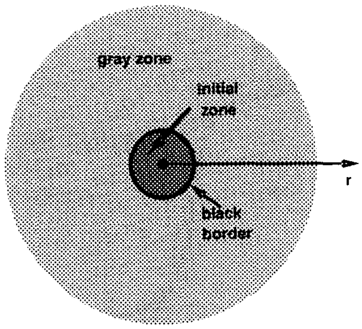

Figure 3: Distribution of color intensity in a stain which was made on the paper by diffuse ink. Three zones of different intensity appear in the image: 1. initial zone where ink was directly applied to paper; 2. black border - a dark line along the border of the initial zone; 3. gray zone - the area where solid particles of the ink collect as a result of diffusion.

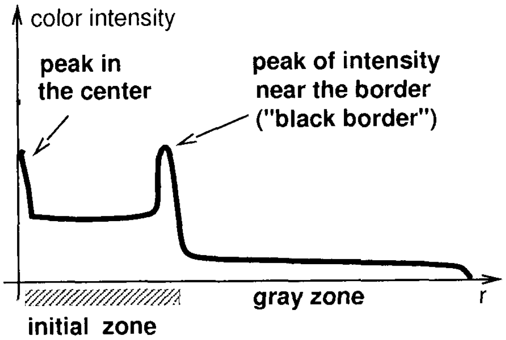

Figure 4: Typical diagram of surface density of carbon particles within a certain point of the image for real 'Sumie' painting. The horizontal axis corresponds to the distance r of a certain point from the center of the image, and vertical axis represents the value of gray color intensity within that point.

color along the border of the initial zone is higher then inside the zone.

Outside the initial zone there finally appears a sufficiently large *gray zone* with not very high, but more or less homogeneous intensity of color. The border of this zone appears to be rather irregular, 'feathery'. This gray zone is one where ink is not directly applied to paper. Carbon particles collect there as a result of diffusion.

We will call such distribution of color intensity in the image as *initial zone - black border - gray zone distribution.*

The typical intensity diagram of diffuse ink image (i.e. the diagram of surface density of carbon particles within a certain point of the image) is shown in Fig. 4. This figure correspond to the case when ink was initially applied in a central symmetric area on the paper (i.e initial zone is a disk). Note, that in this case, there appears a peak in intensity, right in the center.

This article is concerned with a problem of numerical simulation and computer animation of the phenomena of diffuse ink painting. In order to solve this problem it seems necessary to suggest an adequate phenomenological model which can explain the above mentioned initial zone - black border - gray zone intensity distribution in the image area (see Fig. 4).

2 A phenomenological model for initial zone - black border - gray zone distribution of intensity in diffuse ink painting image

One can try to explain the phenomenon of initial zone - black border - gray zone distribution by means of a filtering process inside the paper. Indeed, imagine that largest particles of carbon can not diffuse in the paper because of their size. In this case they will each always remain at their initial location, while particles of little size diffuse freely. This might result in some specific distribution of color intensity of the image. Such an approach was realized in [1].

But experiments show in reality that the distribution of carbon particle size in 'Sumie' ink is rather narrow. It means that almost all carbon particles are approximately of the same size. In this situation filter effect can not be used to explain the diffuse ink painting phenomena. If all particles are approximately of the same size, then they all can diffuse or not diffuse simultaneously. In such a case the result of filtering is trivial. It simply allows or disallows the diffusion - depending on the typical size of carbon particle and on the paper structure.

In the present paper we suggest a multidimensional diffusion model which proves to explain the initial zone - black border - gray zone distribution phenomenon of diffuse ink painting.

First of all we stress that the simple model of diffusion of a liquid (liquid ink) in a paper is not adequate in the case of 'Sumie'. It is clear that the phenomenon of initial zone - black border - gray zone distribution could not be obtained under the assumption that ink (colloidal liquid) diffuses in the paper 'as a whole', i.e. remaining a stable substance during this diffusion process. If this assumption is true, then the final density of color in the image would, in any case, decrease monotony in the vicinity of the border of initial zone. So, the effect of the *black border* which one can see in 'Sumie' painting would be impossible.

It is necessary to construct a more complicated diffusion model of diffuse ink painting. This model must take into account that different components of the diffuse ink (i.e. water and carbon particles) behave in a different way during the diffusion process. It means that we must consider a multidimensional state space

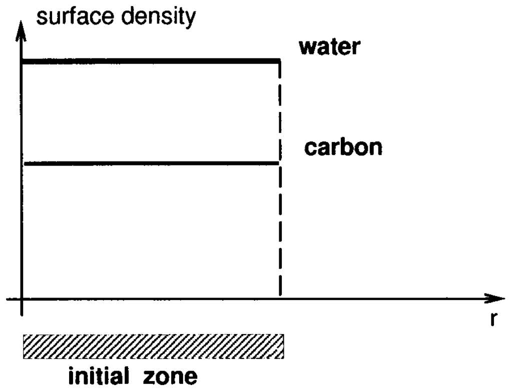

Figure 5: Initial distribution of water and carbon in a round stain made by a drop of diffuse ink on the surface of the paper.

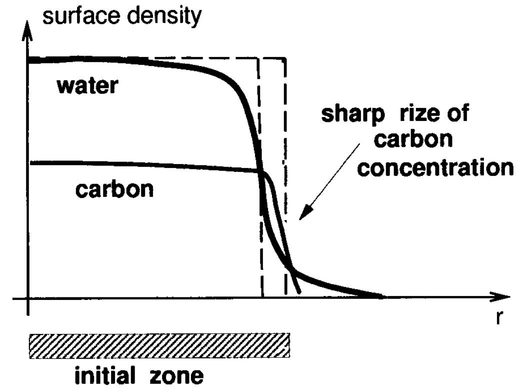

Figure 6: Distribution of water and carbon in the stain made by diffuse ink on the surface of the paper after the diffusion process starts.

for the ink density at every point of the image. Each dimension in this space corresponds to the surface density of a component of the ink at that point.

Indeed, the nature of diffusion of water and of carbon particles is quite different. Water spreads in the paper mainly due to microscopic capillary effects. The summary macroscopic effect can be satisfactory described in terms of diffusion. The diffusion coefficient in this case is determined mainly by the structure of the paper and so is approximately constant. And it might be different in different directions - depending on the manufacturing method of the paper.

As to carbon particles, they are much bigger than molecules of water. Their motion is not based on capillary effect. Motion of carbon particles in water is a well-known Brownian motion. It's nature is due to microscopic collisions between molecules of water and carbon particles. The faster such collisions occur and the higher the average speed of water molecules is - the bigger diffusion coefficient for Brownian motion of carbon particles will be. Thus, the diffusion coefficient of the motion of carbon particles depends on the temperature and on the local concentration (in the zone where this concentration is low) of these particles in water. Diffusion of carbon particles does not depend directly on paper structure.

Let us assume that the temperature is constant. Then the diffusion coefficient for carbon particles depends only on a concentration of carbon particles in water. The higher this concentration is - the slower diffusion might be. But the concentration of carbon in water changes with time in each point of the image. Thus, the diffusion coefficient for the motion of carbon particles can strongly depend on time and on the point of the image.

Consider an initial distribution of water and carbon particles density as shown in Fig. 5 .

Due to spread of water outside of initial zone, the density of carbon in water will decrease inside this zone. The sharpest decrease of this density will occur near the border of initial zone because gradient of density function is maximal there. Consequently, it seems that distribution of water density would soon become like the distribution shown in Fig. 6 .

As to carbon particles, they can diffuse only in water (not in the paper itself). As mentioned above, the diffusion coefficient for the motion of carbon particles can depend on a concentration of these particles in water. Diffusion slows down when concentration is too high.

As mentioned above, density of water along the border of initial zone begins to fall very rapidly with time. Consequently, the concentration of carbon particles in water (i.e. ratio carbon to water) rises sharply along the boundary of initial zone. Thus, along this boundary a *'stop-diffusion'* phenomena can take place. It means that in the points with high concentration, diffusion of carbon particles will slow down. It results in the appearance of a *barrier* for carbon particles near the border of initial zone. Many carbon particles which diffuse to the boundary of initial zone from inner parts of this zone, begin to slow down there and can not leave the vicinity of the border. After water dries up they remain near the boundary. That is how above mentioned *black boundary* effect may occur.

Some carbon particles which are able to overcome this barrier, appear outside the initial zone. Those particles immediately fall into a zone where there is much more water than carbon. Although there might be little water outside the initial zone, the decrease in the number of carbon particles can be even greater than decrease of water density. In this case, concentration of carbon particles in water appears to be sufficiently low outside of initial zone which results in a diffusion with maximal rate for carbon outside the initial zone. Carbon particles there diffuse freely in water and draw a *gray zone* around initial zone. In this gray zone, intensity of gray color will be approximately constant. Due to the effects described above, the gray zone will be separated from initial zone by a dark line - *black boundary.* Therefore the density distribution function will look as shown on Fig. 4 .

Thus, the initial zone - black boundary - gray zone distribution phenomenon, which could be observed in read diffuse images, gets it's explanation in terms of two-component diffusion process.

3 Mathematical formalisation

Let us choose some Cartesian coordinate system (x, y) on the surface of the paper. Time variable will be denoted by t. Consider a touch of brush with diffuse ink to the surface of the paper at the moment $t = 0$. Assume that at the first moment $t = 0$, densities of water and carbon particles are constants C_1 and C_2 respectively, in the initial zone D_0 (i.e. in the area where brush touched the paper).

Let $g(x, y, t)$ denote the surface density of water in the point (x, y) on the paper at the moment t, $(t >= 0)$. Similarly, let us denote by $f(x, y, t)$ the surface density of carbon particles on the paper.

At the first moment of time, the distributions of water and carbon in the paper are as follows

$$g(x,y,0) = C_1, \quad f(x,y,0) = C_2 \qquad (x,y) \in D_0;$$

$$f(x,y,0) = 0, \quad f(x,y,0) = 0 \qquad (x,y) \notin D_0.$$

Now we write a system of differential equations which determines the change of density of water and carbon particles on the paper surface with time. We take in account that this process of density change is determined by diffusion and drying.

$$\frac{\partial g}{\partial t} = \nabla(a^2 \nabla g) - \frac{1}{\tau} g + p\frac{\partial g}{\partial x} + q\frac{\partial g}{\partial y}, \qquad (1)$$

$$\frac{\partial f}{\partial t} = \nabla(Z(\frac{g}{f})\nabla f). \qquad (2)$$

Boundary conditions are zero as point (x, y) tends to infinity. Practically we assume that a sheet of paper is large enough so that water will dry up before reaching the edge of the paper.

In above equations coefficient $\frac{1}{\tau}$ characterize the speed of drying process for water. Coefficients p and q determine a flow of water on the surface of paper which might occur due to special paper structure (or due to gravitational effects if the paper is placed vertically). In present work we do not take into account such effects and assume that $p, q = 0$. Parameter a determines diffusion of water on the surface of paper. Here we assume that it is constant. But it might be not so. If the paper structure is not homogeneous (for example, if the paper was manufactured in such a way that there exist some typical directions of paper fibers, which are different in different parts of paper) then coefficient a would depend on coordinates (x, y).

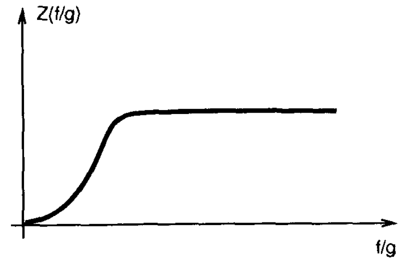

Figure 7: Behavior of a function $Z(\frac{g}{f})$ which results in appearance of initial zone - black boundary - gray zone distribution phenomenon in diffuse ink painting simulation.

The most important point in the model is a choice of function $Z(\frac{g}{f})$ which determine how diffusion rate of carbon particles depend on the concentration of carbon in water. Our calculations show that properties of intensity distribution in the final image can strongly depend on the choice of function Z.

We performed a number of numerical experiments in order to determine the appropriate shape of function Z. It turns out that the initial zone - black boundary - gray zone distribution phenomenon occurs only in the case when function Z is concave in the interval of it's rise (i.e. in the vicinity of zero, because for sufficiently large values of argument, function Z should be constant: rate of diffusion for the motion of carbon particles will not change when water is added provided that there was already enough water).

4 Results

In order to obtain computer animation of evolution of diffuse ink image on the paper, we solve the system of partial differential equations (1),(2) numerically on a grid. A sequence of steps of numerical solution gives us values of carbon density in each pixel of the image for for corresponding sequence of time moments. Then specially developed animation software was used to display the results of animation on the screen.

Fig. 8 and Fig. 9 show the result of calculations in the case of round stain which was made on the paper by a drop of diffuse ink. This case is most difficult for numerical simulation because real images in this case show the sharpest singularities in the intensity distribution.

Fig. 8 shows an initial ink image on a paper (top) and the final step of animation of the ink diffusion process (bottom). In this figure one can observe all features of the initial zone - black boundary - gray zone distribution phenomenon in the final image (bottom).

Several steps of animation are shown in Fig. 9 .

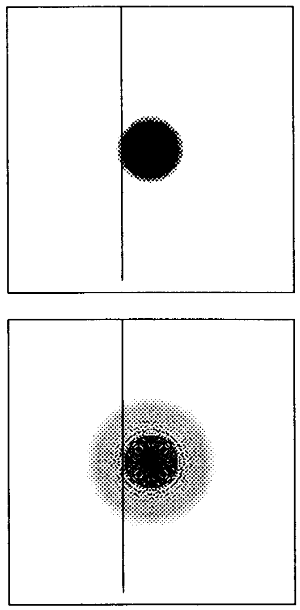

Figure 8: Initial (top) and final (bottom) images of round stain, produced on the paper by a drop of diffuse ink. Vertical line shows where the edge of the image was located at the beginning. Computer animation.

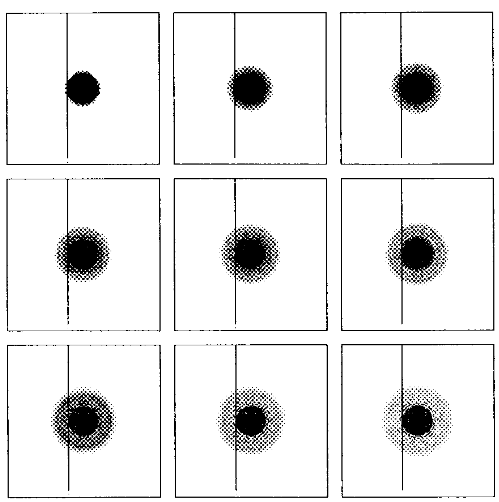

Figure 9: Several steps of computer animation of the evolution of the round stain made on the paper by diffuse ink. Vertical line shows where the edge of the stain was located at the beginning.

5 Summary and Conclusions

A new approach to computer animation of complicated phenomenon of diffuse ink painting was proposed. According to this approach, diffusion of diffuse ink (colloidal liquid) on the surface of the paper should be considered as two separate diffusion processes. In other words, diffusion of water in the paper and diffusion of the motion of solid particles in water should be treated as different (but interlinked) processes.

Such an approach leads to mathematical model with multidimensional diffusion. This model appears to be able to simulate precisely very specific features of intensity distribution in diffuse ink painting images.

One of the important advantages of new model is that it can produce images with singularities in color intensity - just as it is in real diffuse ink painting images. The proposed mathematical model is able to provide exactly the same color intensity distribution as in real images made by diffuse ink.

The method was applied to animate ink diffusion 'Nijimi' of traditional Japanese ink painting 'Sumie'. The results of computer animation appear to be very close to real 'Sumie' painting.

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