HW3 - Staff Planning

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Background

- Public transport company extending their network
- Estimated number of needed staff per time period
- 8 hour shifts
- Doubled salary from 22.00 to 06.00

➔ Minimize salary cost, while still satisfying staffing requirements

Time period	Staff
06–10	700
10–14	500
14–18	600
18–22	300
22–02	100
02–06	50

LP Problem

Let's assume that the base salary for a 8 hour shift is *2a*, then we can use the formulation:

$$
\begin{aligned}
\min \quad & 2 \cdot (x_{06-14} + x_{10-18} + x_{14-22}) + 3 \cdot (x_{02-10} + x_{18-02}) + 4 \cdot x_{22-06} \\
\text{s.t.} \quad & x_{06-14} + x_{02-10} \geq 700 \\
& x_{10-18} + x_{06-14} \geq 500 \\
& x_{14-22} + x_{10-18} \geq 600 \\
& x_{18-02} + x_{14-22} \geq 300 \\
& x_{22-06} + x_{18-02} \geq 100 \\
& x_{02-10} + x_{22-06} \geq 50
\end{aligned}
$$

- Between 6-10
- Between 10-14
- Between 14-18
- Between 18-22
- Between 22-02
- Between 02-06

From this it is easy to formulate the problem to a form:

$$
\begin{aligned}
\min \quad & f_{cost} = cx \\
\text{s.t.} \quad & Ax \leq b
\end{aligned}
$$

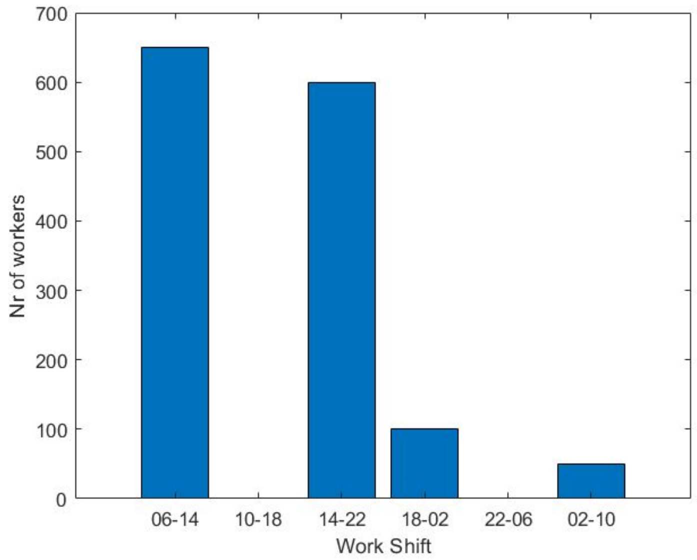
Approximation of the objective function

$$
\begin{array}{rl}
 & x_{06-14} + x_{02-10} \geq 700 \\
 & x_{10-18} + x_{06-14} \geq 500 \\
 & x_{14-22} + x_{10-18} \geq 600 \\
 & x_{18-02} + x_{14-22} \geq 300 \\
 & 2 \cdot (x_{22-06} + x_{18-02} \geq 100) \\
+ & 2 \cdot (x_{02-10} + x_{22-06} \geq 50) \\
\hline
 & 2 \cdot (x_{06-14} + x_{10-18} + x_{14-22}) + 3 \cdot (x_{02-10} + x_{18-02}) + 4 \cdot x_{22-06} \geq 2400
\end{array}
$$

Task 1-2

- Define problem in the above form
- Solve it with *linprog / intlinprog*
- *Using Simplex alg.*
- Solution: **[650, 0, 600, 100, 0, 50]**
- Cost: **2950*a**

- For task 2 we only need to change the b vector
- Solve it again with *linprog / intlinprog*
- *We get the same solution*




```
%% Task 1
options = optimset('Algorithm','dual-simplex','LargeScale','off');
% LP problem parameters
f = [2,2,2,3,4,3];
b = [-700;-500;-600;-300;-100;-50];
A = [-1, 0, 0, 0, 0, -1; ...
     -1,-1, 0, 0, 0, 0;...
     0, -1, -1, 0, 0,0;...
     0, 0, -1,-1, 0, 0;...
     0, 0, 0, -1,-1, 0;...
     0, 0, 0, 0, -1, -1];
lb = [0,0,0,0,0,0]; % lower bound

[x1,fval1] = linprog(f,A,b,[],[],lb,[],options)
```

```
%% Task 2
options = optimset('Algorithm','dual-simplex','LargeScale','off');
% LP problem parameters
f = [2,2,2,3,4,3];
b = [-700;-250;-600;-300;-100;-50];
A = [-1, 0, 0, 0, 0, -1; ...
     -1,-1, 0, 0, 0, 0;...
     0, -1, -1, 0, 0,0;...
     0, 0, -1,-1, 0, 0;...
     0, 0, 0, -1,-1, 0;...
     0, 0, 0, 0, -1, -1];
lb = [0,0,0,0,0,0]; % lower bound

[x1,fval1] = linprog(f,A,b,[],[],lb,[],options)
```

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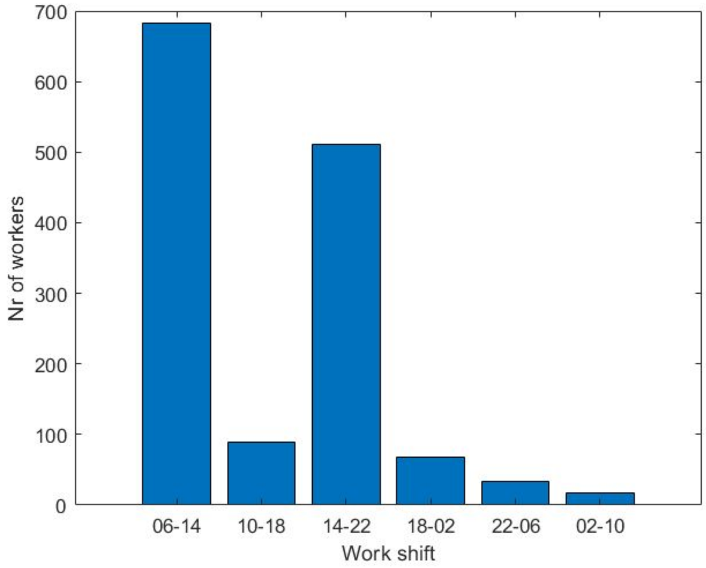
Task 3

- Now we need to change the solving method to the interior point method

```
options = optimset('Algorithm','interior-point','LargeScale','on');
```

- This method is gradient based so now we can expect that we may got a solution which is not an extreme points, but it can be in the middle of a side if *c* orthogonal to one of the sides of the polytope.
- Solution:

  [682.85, 88.76, 511.24, 67.15, 32.85, 17.15]
- Cost: **2950*a**



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