Uppsala University, 2022
1TD184 Optimisation

Assignment 1 – basic option
October 19, 2022

Assignment 1: Ice on Mars

Background

It has been more than 50 years since the first spacecraft Mariner 4 flew by the planet Mars on July 1965. In 2015, NASA confirmed that there is liquid water flows on Mars and the evidences of ice were found during the last ten years.

Mars is currently a cold, hyper-arid global desert planet where water on the surface is mainly stored in the extensive polar caps, and very small amounts in the atmosphere. In 2008, the high-resolution images of the Martian surface showed the evidence of ice in low latitude area between 30 to 50 degrees [1]. After that, more than 1300 glacier-like forms are identified in the mid-latitude area [2]. The study by the Shallow Radar sounding experiment on Mars Reconnaissance Orbiter proved that the deposits are mainly consist of water ice [3]. However, the water ice is not stable on the low latitude area and can sublimate to vapor at a very low temperature due to the low atmospheric pressure on Mars. Therefore, the existing glaciers are now protected from sublimating by debris covers.

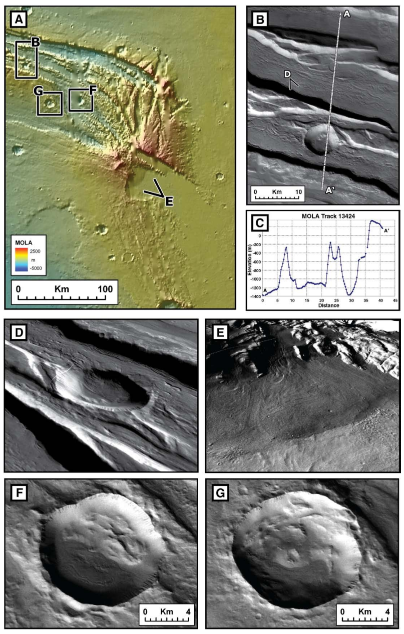

Figure 1: Some glaciers in Acheron Fossae region centered at 37.67° north latitude and 135.87° west longitude on Mars [4].

Ice Sheet Modeling

The ice sheets and glaciers are considered as incompressible Newtonian fluid which is governed by the Full Stokes equations [5]. However, there are several numerical difficulties in solving the Full Stokes equations and a commonly used approach is the Shallow Ice Approximation (SIA). The assumption in SIA is that the ice sheet is very thin in vertical direction comparing to the horizontal directions such that a few terms in Full Stokes equations are neglected. In two dimensional, the longitudinal section along the flow line of the ice sheet is considered, as shown in Figure 2.

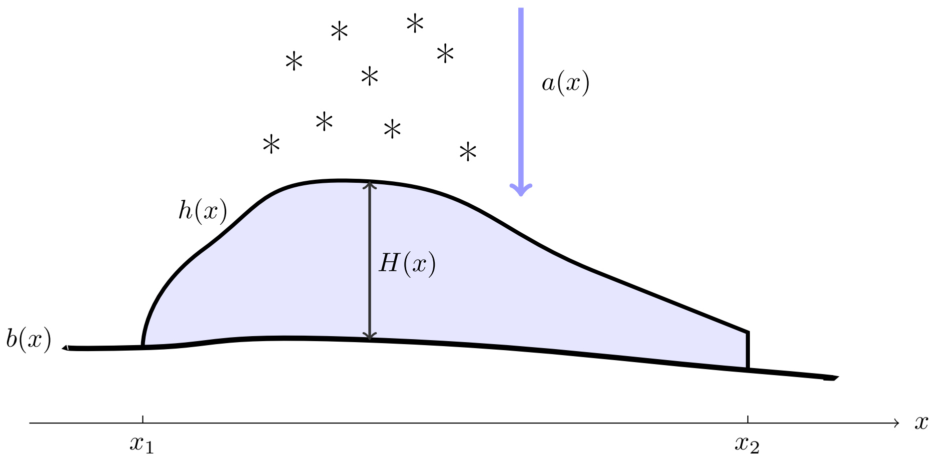

Figure 2: A two dimensional ice sheet.

A nonlinear relationship can be established between the thickness $H(x)$ and the mass balance function on the surface $a(x)$ that

$$a(x) = -\frac{2A}{n+2}(\rho g)^n H(x)^{n+2} \left|\frac{\mathrm{d}h}{\mathrm{d}x}\right|^{n-1} \frac{\mathrm{d}h}{\mathrm{d}x}, \tag{1}$$

where the ice sheet covers the area from $x = x_1$ to $x = x_2$, ρ is the density of ice and g is the gravitational acceleration. A is called the rate factor which relates to the temperature, the value (in $\mathrm{s}^{-1}\mathrm{Pa}^{-3}$) is usually between 10^{-27} to 10^{-23} which corresponds to $-50°$C to $0°$C on Earth. n is the flow law exponent which indicates the stiffness of the ice, commonly in the range $[1, 4]$.

The slope of the ice surface $\frac{\partial h}{\partial x}$ is known from the satellite measurement. Therefore, given all the physical parameters and the mass balance function, the thickness of the ice sheet can be computed by (1). On the other hand, if the ice thickness is known, you can estimate the unknown physical parameters, e.g. the rate factor A or the flow law exponent n. The rate factor A is commonly estimated according to the temperature of the glacier. However, the flow law exponent n is difficult to know since it requires filed work or experiments in lab.

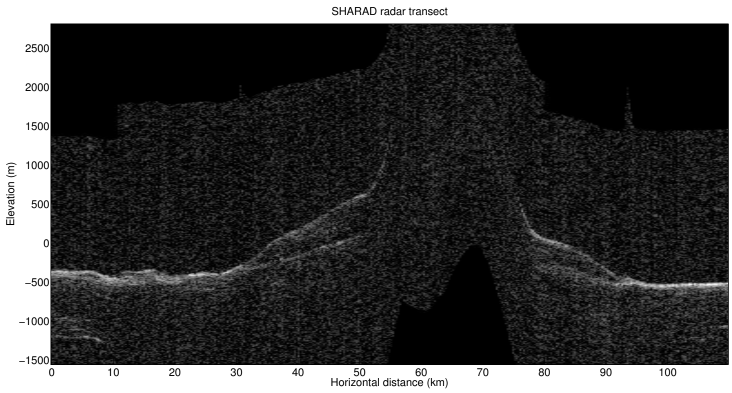

Figure 3: The Shallow Radar data of a glacier on Mars. The white curves are the interface between two different materials [1].

Shallow Radar Data

The original Shallow Radar data is shown in figure 3. **Your task** is to use the processed data in `Sharad.mat` to estimate the flow law exponent of ice on Mars. In the data file, we only consider the left part of the glacier in figure 3. The vector `x` contains the x-coordinates, vector `a` is the mass balance function in (1), vector `dhdx` contains the surface slope of the glacier and vector `H_obs` is the thickness observed by the Shallow Radar. The scalar `rho` is the density of the ice (kg/m^3) and `g` is the gravitational acceleration on Mars (m/s^2).

Task 1

Formulate the objective function to minimize the 2-norm of the errors between the model solution and the observation that

$$f = \|H(x) - H_{obs}(x)\|_2^2.$$

Use the nonlinear unconstrained optimization function `fminunc` in Matlab to find n which minimize the objective function f. Use the rate factor $A = 10^{-25}$ in this task which corresponds to the surface temperature at about $-20°$C.

Task 2

Use the value of n you find in Task 1, try to find A to minimize f. Did you get a different A value than 10^{-25}?

Task 3

Change the initial guess in Task 1 and 2, redo the tasks. Do you get different optimal solutions?

Task 4

It is also possible to find A and n with the given data at the same time. Try to modify your code to find the optimal values for A and n. Take $A = 10^{-25}$ and $n = 1.1$ as an initial guess.

Task 5

Redo Task 4 with some other initial guess. What can you say about your result in Task 4? Which result from all these tasks is the optimal solution?

Task 6

Is it possible to do task 1 and task 2 iteratively to find the optimal solution for both A and n? To understand this, you may need to plot the value of the objective function against A and n in a certain range in 3D plot.

Task 7

This type of problem is also called curve fitting which can be formulated as a nonlinear least square(NSL) problem. Write down the problem in the NSL form and solve Task 1 to 4 with the `lsqnonlin` in Matlab and compare your results with those solved by `fminunc`.

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References

[1] John W Holt, Ali Safaeinili, Jeffrey J Plaut, James W Head, Roger J Phillips, Roberto Seu, Scott D Kempf, Prateek Choudhary, Duncan A Young, Nathaniel E Putzig, et al. Radar sounding evidence for buried glaciers in the southern mid-latitudes of mars. *Science*, 322(5905):1235–1238, 2008.

[2] NB Karlsson, LS Schmidt, and CS Hvidberg. Volume of martian midlatitude glaciers from radar observations and ice flow modeling. *Geophysical Research Letters*, 42(8):2627–2633, 2015.

[3] Jeffrey J Plaut, Ali Safaeinili, John W Holt, Roger J Phillips, James W Head, Roberto Seu, Nathaniel E Putzig, and Alessandro Frigeri. Radar evidence for ice in lobate debris aprons in the mid-northern latitudes of mars. *Geophysical research letters*, 36(2), 2009.

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[5] Ralf Greve. *Dynamics of Ice Sheets and Glaciers*. 2009.