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Lab — Iterative Methods

Iterative solution of the soap film problem

Introduction

In many cases it is better to solve sparse linear systems using iterative methods. Typically, an iterative method uses less memory than LU factorization, and it is often faster.

To solve the linear system $Ax = b$ with an iterative method, you begin by “guessing” a solution, $x^{(0)}$. You then use some algorithm to iteratively modify and hopefully improve the solution. The solution after iteration k is denoted by $x^{(k)}$.

There are many iterative methods to choose from. They differ in how they compute $x^{(k+1)}$, given $x^{(k)}$. In this part of the lab assignment, you will compare three iterative methods: Jacobi’s method, Gauss-Seidel’s method, and the Conjugate Gradient method. Each method will be applied to the problem of computing the soap film surface (which, as you may have noted is modeled by the Laplace equation).

Experiment

For these experiments, we will use the Interactive Educational Modules web site, by Michael Heath et al. Go to
`http://heath.cs.illinois.edu/iem/index.html`.
Choose
`Partial Differential Equations` → `Laplace Equation`.
For boundary conditions, use Set #4. Select 16 for the number of points in each space direction.

You should first solve the problem with the Jacobi method, then with Gauss-Seidel, and finally with the Conjugate Gradient method. For each method you use, press `iterate` to carry out an iteration. You will then see how the solution is updated. You will also see the residual obtained for that solution. Make one iteration at a time, and continue until the residual reaches 0.05. Make a note about the number of iterations that were required for each method.

Fallback

If the interactive eductional modules are not working on your computer, you may instead run the experiment above in Matlab. Download the matlab file `laplace_equation.m`. Run `laplace_equation(N,method)` where `N` is the number of interior grid points in each dimension and `method` is one of `'jacobi'`,`'gauss-seidel'`,`'conjugate-gradient'`.