Global Tracking Controllers for Flexible-joint Manipulators: a Comparative Study

Hossam Arafat - 1803850

Abdallah Kobrosly - 1806548

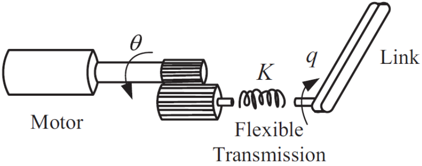
Omar Salem - 1797978

Sapienza Università di Roma

Underactuated Robotics

Elastic Joint System Model

- Explicit modelling of stiffness creates underactuated system
- Simplified model proposed by Spong (1989) [6]
- Assumes that the angular part of the motor's kinetic energy is due only to its own rotation
- No Inertial or Centrifugal coupling; only the joint's elasticity



Dynamics

$$D(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) = K(q_2 - q_1) \tag{1}$$

$$J\ddot{q}_2 + K(q_2 - q_1) = u$$

- where $q_1 \in R^n$ link angles $q_2 \in R^n$ motor angles $g(q_1)$ gravity term $D(q_1)$ $n \times n$ link inertia matrix $C(q_1,\dot{q}_1)\dot{q}_1$ Coriolis & centrifugal forces
- Dynamical model in compact form,

$$\bar{D}\ddot{q} + \bar{C}\dot{q} + \bar{g} + \bar{K}q = Mu$$

$$q^T = \begin{bmatrix} q_1^T & q_2^T \end{bmatrix} \bar{D} = \begin{bmatrix} D & 0 \\ 0 & J \end{bmatrix} \bar{C} = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \bar{g} = \begin{bmatrix} g \\ 0 \end{bmatrix} \bar{K} = \begin{bmatrix} K & \underbrace{-K} \\ \underbrace{-K} & K \end{bmatrix} M = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \tag{2}$$

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Decoupling based controller

Theoretical Background

- A simple input transformation reconstructs system in to two cascaded subsystems
- The name is due to the resulting closed loop equation is also cascaded
- The cascaded subsystems are proven stable, when the system orbits are proven bounded as seen in lemma 1 introduced by[3]
- The reduced dynamic model is partially decoupled except in the elasticity term
- It is considered the easiest controller

Decoupling based controller

Implementation

- Use trangularization property to obtain two GAS subsystems:
  - The motor (unforced) drives the link by q_2
  - The link (forced)
- Main idea: Find a function that will drive $q_1 \to q_{1d}$ when $q_2 \equiv q_{2d}$
- Several choices possible for q_{2d}
- We chose the q_{2d} proposed by author in [4]; it will unify the implementation for all controllers and give us a common framework:

$$q_{2d} = K^{-1} u_r + q_1 \tag{3}$$

- with the error signals as

$$s_1 = \dot{q}_1 + \Lambda_1 \widetilde{q}_1 \qquad , \Lambda_1 > 0 \qquad\qquad \widetilde{q}_1 = q_1 - q_{1d}$$

Decoupling based controller

Implementation

- The control law designed for the rigid part of the body by [4]

$$u_r = D(q_1)\ddot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1) - B_1 s_1 \tag{4}$$

- Inverse dynamics also possible for the definition of u_R instead of [4]
- The velocity reference signal

$$\dot{q}_{1r} = \dot{q}_{1d} - \Lambda_1 q_1 \tag{5}$$

- Apply $\widetilde{q}_2 = q_2 - q_{2d}$ as an input perturbation to the model results in the closed loop dynamic model

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\widetilde{q}_2 \tag{6}$$

Decoupling based controller

Implementation

- The perturbed closed loop system was proved GAS by [5], where setting the perturbation in the motor as $\tilde{q}_2 \equiv 0$ results in the rigid Lyapunov

$$V_R = \frac{1}{2} s_1^T D s_1 + \widetilde{q_1^T} \Lambda b_1 \tilde{q}_1 \tag{7}$$

- is shown to be a strict Lyapunov function
- The control law decouples the motor dynamic and makes system GAS:

$$u = K(q_2 - q_1) - K_1 \tilde{q}_2 - K_2 \dot{\tilde{q}}_2 + J \ddot{q}_{2d} \tag{8}$$

- This control will yield a decoupled linear error equation

$$J \ddot{\tilde{q}}_2 + K_2 \dot{\tilde{q}}_2 + K_1 \tilde{q}_2 = 0 \tag{9}$$

Decoupling based controller

Implementation

- The proof that the full decoupled system is GAS comes from the Lyapunov equation

$$V_{DB} = V_R + \frac{1}{2} z^T P z$$

  Where P is a positive definite matrix, and z = $\begin{pmatrix} \tilde{q}_2^{\mathrm{T}} & \dot{\tilde{q}}_2^{\mathrm{T}} \end{pmatrix}$
- where the V_{DB} was proven to be a strict Lyapunov as z is bounded, and decays to zero, proving system is GAS.

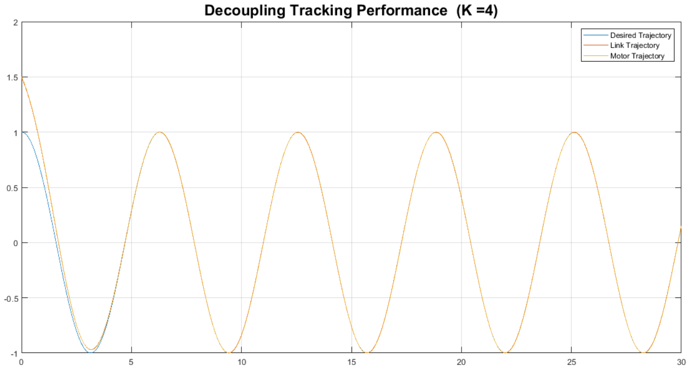
Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Decoupling - Tracking Performance

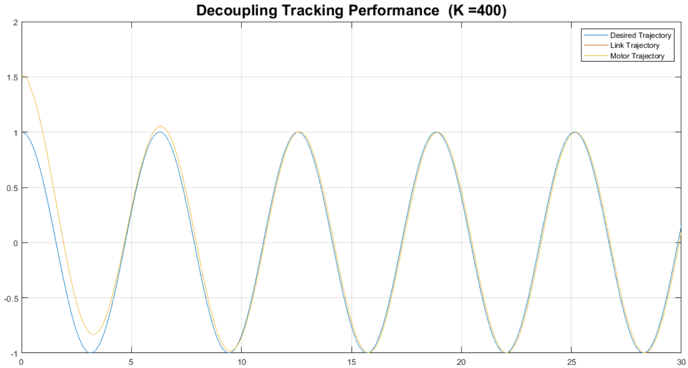
K=4



Settling time = $4.1223sec$

Decoupling - Tracking Performance

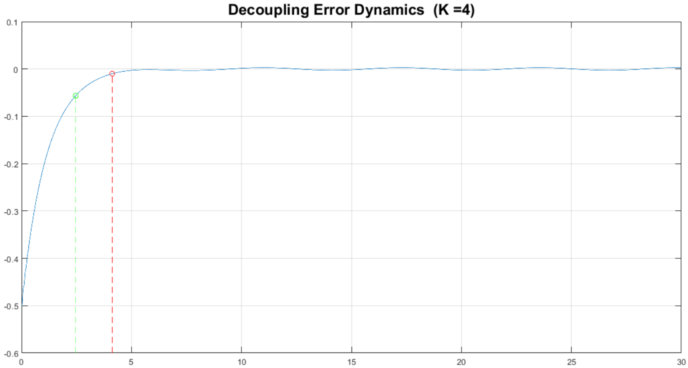
K=400



Settling time = $4.1223sec$

Decoupling - Error Dynamics

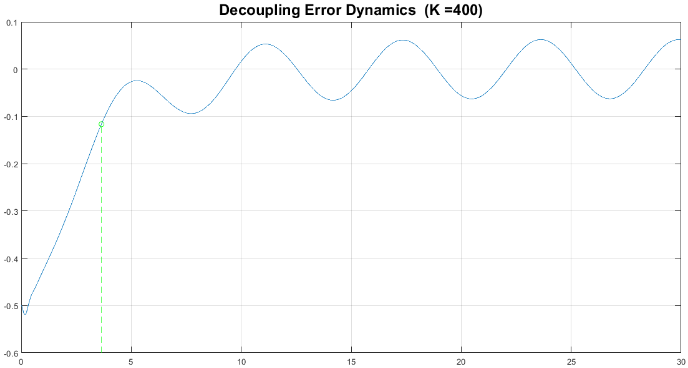
K=4



Rise time = 2.4657*sec*

Decoupling - Error Dynamics

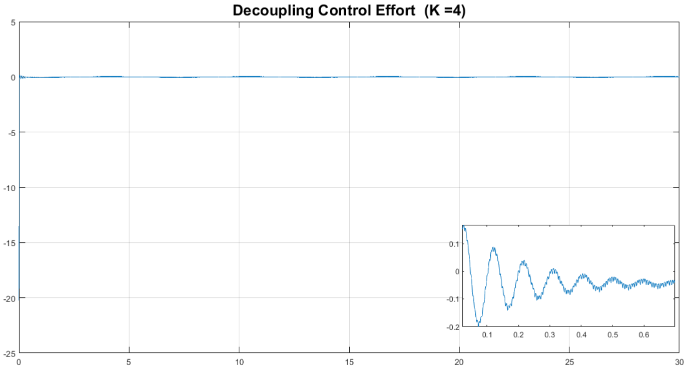
K=400



Rise time = $2.4657 sec$

Decoupling - Control Effort

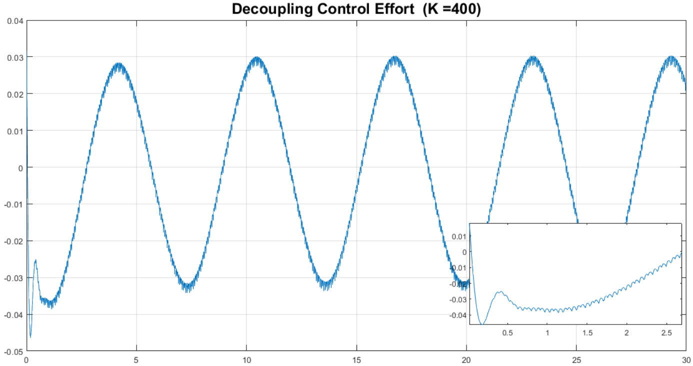
K=4



Gains: kp = 18.0, kd= 20.0

Decoupling - Error Dynamics

K=400



Gains: kp = 0.80, kd= 0.20

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Backstepping based controller

Theoretical Background

- The most systematic procedure presented by the authors
- Based on Integrator Backstepping

1. Select imaginary input
2. Choose a suitable Lyapunov candidate
3. Prove strict Lyapunov(system is GAS)
4. Integrate input
5. Repeat until real input appears

Backstepping based controller

Implementaion

- Applying the following feedback law

$$u = J\nu - K(q_1 - q_2) \tag{10}$$

- reduces robot's dynamics to the cascaded form:

$$\begin{aligned} D\ddot{q}_1 + C\dot{q}_1 + g + Kq_1 &= Kq_2 \\ \ddot{q}_2 &= \nu \end{aligned} \tag{11}$$

- Systematic application of Integrator Backstepping can begin

Backstepping based controller

Step 1

- Assume q_2 is the first virtual input of the first equation in (11)
- by means of feedback choose $q_2 = q_{2d} = K^{-1}u_r + q_1$
- Closed-loop equation becomes

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\widetilde{q}_2$$

- We know it's GAS when $\widetilde{q}_2 = 0$ with Lyaponuv V_R (refer to (6) & (7))
- Since q_2 is not the real input, we have an error $\widetilde{q}_2 = -q_{2d} + q_2$
- We add an integrator before the input $\widetilde{q}_2$ and proceed to step 2

Backstepping based controller

Step 2.1

- Assume $\dot{q}_2$ is now the second virtual input
- Augment the Lyaponuv function candidate from the first step , by adding a term in the first virtual input resulting in

$$V_2 = V_R + \frac{1}{2}\widetilde{q_2}^T\widetilde{q_2} \tag{12}$$

- Its derivative is given by

$$\dot{V}_2 = -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Lambda_1^{\mathrm{T}} B_1 \Lambda_1 \tilde{q}_1 + \underbrace{s_1^{\mathrm{T}} K \tilde{q}_2 + \tilde{q}_2^{\mathrm{T}} \left(\dot{q}_2 - \dot{q}_{2\mathrm{d}}\right)}_{not\ -ve\ definite} \tag{13}$$

- Choose $\dot{q}_2$ in such a way to cancel cross terms, and creating a quadratic term in $\tilde{q}_2$

$$\dot{q}_2 = -Ks_1 - \tilde{q}_2 + \dot{q}_{2d}$$

Backstepping based controller

Step 2.2

- This cancels undesired terms & adds quadratic term in $\tilde{q}_2$ to ensure GAS

$$\dot{V}_2 = ... + \cancel{s_1^{\mathrm{T}} K \tilde{q}_2} - \cancel{\tilde{q}_2^{\mathrm{T}} K s_1} - \underbrace{\tilde{q}_2^{\mathrm{T}} \tilde{q}} + \cancel{\dot{q}_{2d}} - \cancel{\dot{q}_{2\mathrm{d}}}$$

- Since $\dot{q}_2$ is not the real input, we have an error $e_2 = \dot{q}_2 - e_{2d}$

$$e_{2d} = -Ks_1 - \tilde{q}_2 + \dot{q}_{2d} \tag{14}$$

- We add an integrator before the 2^{nd} virtual input ($\dot{q}_2$) and proceed to step 3, where the overall error equations

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\tilde{q}_2 \tag{15}$$

$$\dot{\tilde{q}}_2 = e_2 - Ks_1 - \tilde{q}_2 \qquad \dot{e}_2 = -\dot{e}_{2d} + \ddot{q} = -\dot{e}_{2d} + \nu$$

Backstepping based controller

Step 3

- The real input ν appears in equation (15) !
- Augment the Lyaponuv function candidate from the second step , by adding a term in the second virtual input resulting in

$$V_3 = V_2 + \frac{1}{2}|e_2|^2 \tag{16}$$

- Its derivative is given by

$$\dot{V}_3 = -\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Lambda_1^{\mathrm{T}} B_1 \Lambda_1 \tilde{q}_1 - |\tilde{q}_2|^2 - |\tilde{e}_2|^2 \tag{17}$$

- Set $\nu = -e_2 + \dot{e}_{2d} - \widetilde{q}_2$
- The proposed backstepping controller is

$$u = K(q_2 - q_1) + J[\ddot{q_{2d}} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - K(\dot{s} + s)] \tag{18}$$

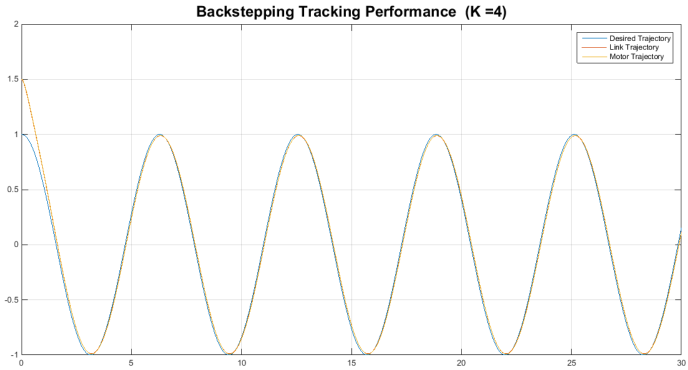
Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Backstepping - Tracking Performance

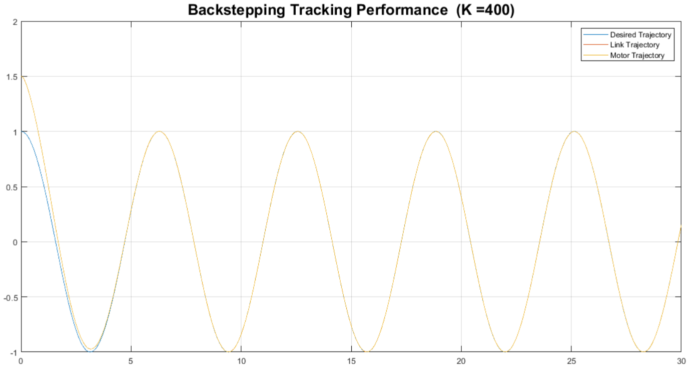
K=4



Settling time = $3.2776 sec$

Backstepping - Tracking Performance

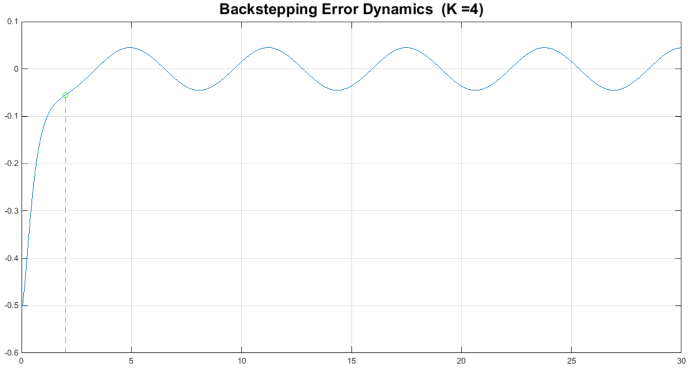
K=400



Settling time = $4.1035 sec$

Backstepping - Error Dynamics

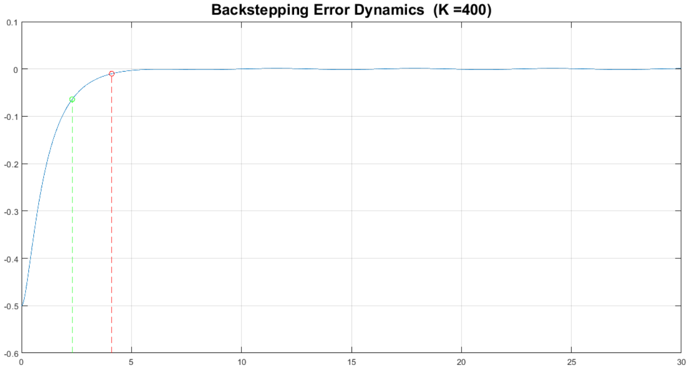
K=4



Rise time = 2.9997*sec*

Backstepping - Error Dynamics

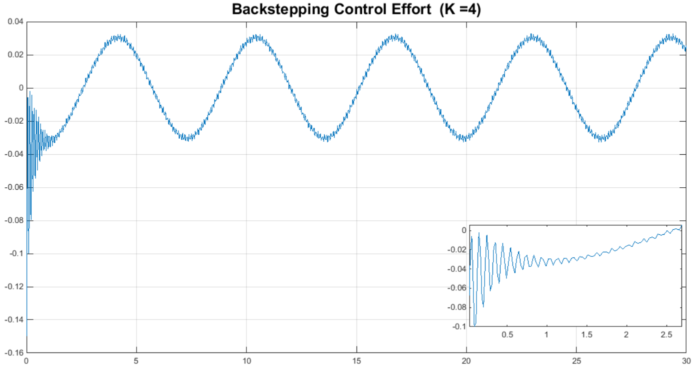
K=400



Rise time = 2.3017*sec*

Backstepping - Control Effort

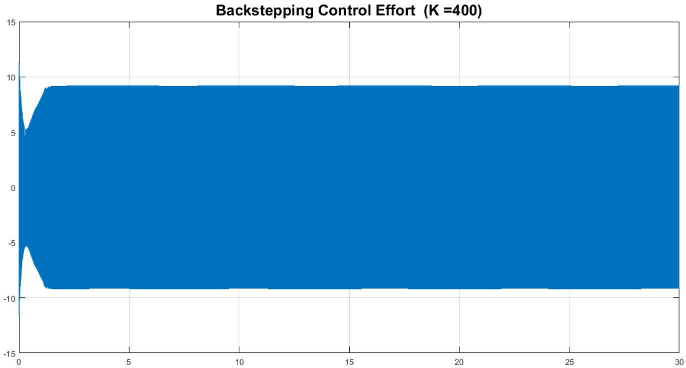
K=4



Gains: kp = 0.80, kd= 0.08

Backstepping - Control Effort

K=400



Gains: kp = 10.0, kd= 2.00

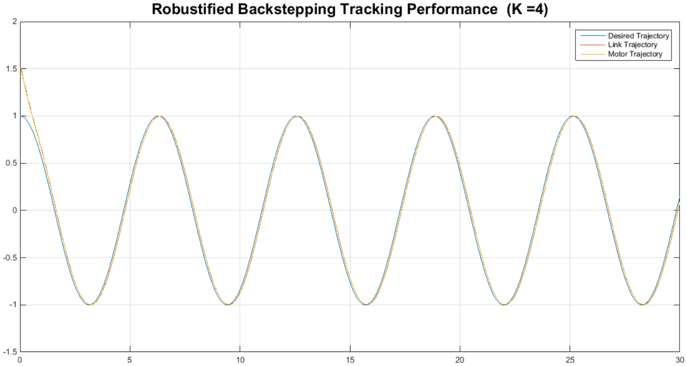
Robustification

- Introduced due to the required high gain for backstepping controller
- In step 2.2 use $V_2 = V_R + \frac{1}{2}\widetilde{q_2}^T K \widetilde{q_2}$ instead of $V_2 = V_R + \frac{1}{2}\widetilde{q_2}^T \widetilde{q_2}$
- Continue the remaining steps of the procedure as is
- As a result, The stiffness K is removed from the final term $(\dot{s} + s)$ of the backstepping method
- Resulting controller

$$u = K(q_2 - q_1) + J[\ddot{q_{2d}} - 2\dot{\tilde{q}}_2 - 2\tilde{q}_2 - (\dot{s} + s)] \qquad (19)$$

Rob. Backstepping - Tracking Performance

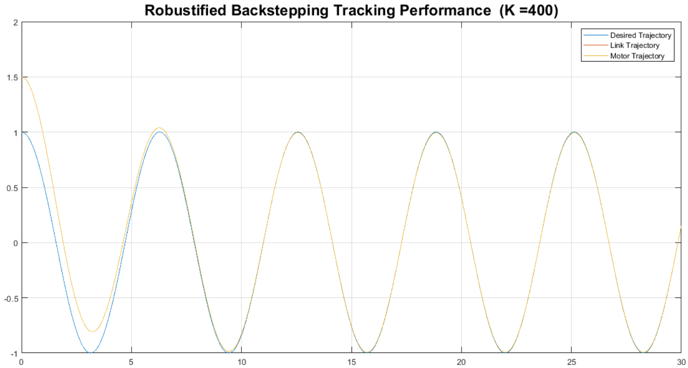
K=4



Settling time = NA

Rob. Backstepping - Tracking Performance

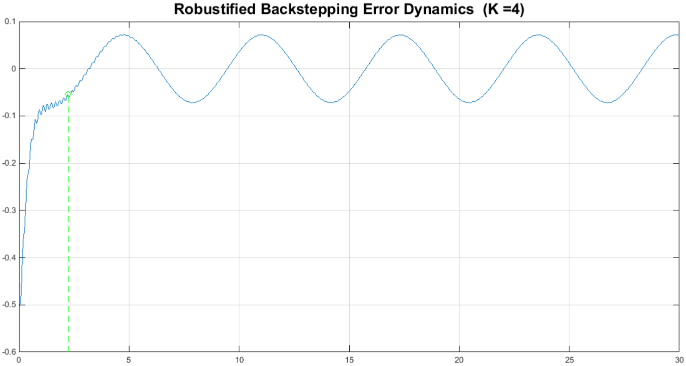
K=400



Settling time = 10.2635

Rob. Backstepping - Error Dynamics

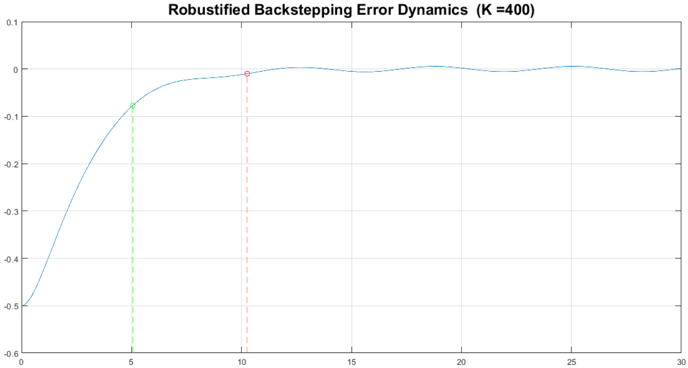
K=4



Rise time = 3.3290*sec*

Rob. Backstepping - Error Dynamics

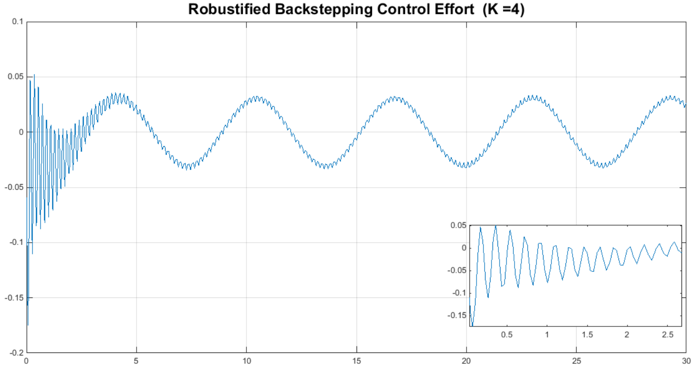
K=400



Rise time = 5.0564sec

Rob. Backstepping - Control Effort

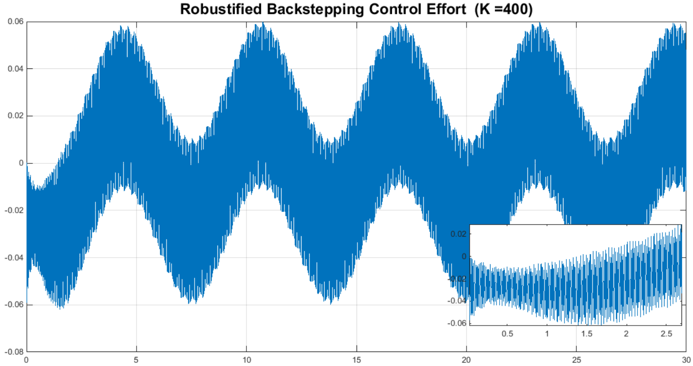
K=4



Gains: kp = 43.0, kd= 49.00

Rob. Backstepping - Control Effort

K=400



Gains: kp = 10.0, kd= 11.00

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Passivity based controller

Theoretical Background

- Passivity-based control is based on concept of energy
- Passivity intuitively means that something does not produce internal energy
- Describes the energy flow in the system
- The signal q_{2d} here has a different purpose: it ensures system's energy shaping, where the system total energy matches the desired energy, however in other controllers it represents desired input to link dynamical equation
- Elastic Joint model; we have to add elastic potential energy to our total energy function, and damping term to apply strict passivity

Passivity based controller

Implementation

- We start by choosing the desired energy function as
$H_d = \frac{1}{2}s^T\bar{D}s + \frac{1}{2}[\int_0^t s^T(\tau)d\tau]\bar{K}[\int_0^t s(\tau)d\tau]$
$$\dot{H}_d = \nu^T s + s^T\bar{B}s$$
recalling the compact dynamic system in terms of error signals after adding damping term we find

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\int_0^t s(\tau)d\tau = \psi \qquad (20)$$

Where $\bar{B}$ is the damping coefficient, and the ψ represents the desired error system perturbation term and is equal to

Passivity based controller

Implementation

$$\psi = \bar{u} - (\bar{D}\ddot{q}_r + \bar{C}\dot{q}_r + \bar{K}q_r + \bar{g}) + \bar{B}s - \dot{K}\tilde{q}(0) \tag{21}$$

with $q_r = q_d - \bar{\Lambda}[\int_0^t \bar{q}(\tau)d\tau]$, where the energy function derivation results in

$$\dot{H}_d = -s^T\bar{B}s + s^T\psi$$

which follows the passivity property.At this point, q_{2d} and u are used to match the desired total energy of the system ensuring perturbation is zero s.t $\psi \equiv 0$.

Passivity based controller

Implementation

- The proposed q_{2d} and controller u are as follows

$$\begin{aligned} q_{2d} = p\,(pI + \Lambda_2)^{-1} \Big\{ & K^{-1} u_R + q_{1d} \\ & + K\,[\tilde{q}_1(0) - \tilde{q}_2(0)] - \int_0^t (\Lambda_1 \tilde{q}_1 - \Lambda_2 q_2)\, d\tau \Big\} \end{aligned} \tag{22}$$

where p = $\frac{d}{dt}$

$$u = -B_2 s_2 + J\ddot{q}_{2r} - K(q_{1r} - q_{2r}) \tag{23}$$

leading to closed loop equation 20, with perturbation ψ identically equal zero

Passivity based controller

Drawbacks

- Firstly the passivity based control is based on lemma 3 [2], which proof convergence of error, which is a weaker property than asymptotic stability.
- The controller has a complex dynamic part which will remove the ability to compare between schemes.
- The controller consider the initial conditions in the control law.
- Due to these drawbacks we choose a different passivity controller, by changing the desired total energy function and we call it the Modified(simplified) passivity controller.

Modified Passivity based controller

Implementation

- The problems faced in the passivity controller are due to the choice of an integral term in the desired energy function.
- The new desired total energy will be

$$H_d = \frac{1}{2} s^T \bar{D} s + \frac{1}{2} \tilde{q}^T \bar{K} \tilde{q}$$

- From this we find the new perturbed error dynamics as

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\tilde{q} = \psi$$

Modified Passivity based controller

Implementation

- And the perturbation term becomes

$$\psi = \bar{u} - (\bar{D}\ddot{q}_r + \bar{C}\dot{q}_r + \bar{K}q_r + \bar{g}) + \bar{B}s \tag{24}$$

- where to set the perturbation term to zero, we use the following q_{2d} and the control law

$$u = -K_2 s_2 + J(\ddot{q}_{2d} - \Lambda_2 \dot{\tilde{q}}_2) - K(q_{1d} - q_{2d})$$
$$q_{2d} = q_{1d} + K^{-1} u_R$$

- Then Lyapunov candidate function:

$$V_{PB} = \frac{1}{2} s^T \bar{D} s + \tilde{q}_1 \Lambda_1^T k_1 \tilde{q}_1 + \tilde{q}_2^T \Lambda_2^T K_2 \tilde{q}_2 + \frac{1}{2} \tilde{q}^T \bar{K} \tilde{q}$$

- It can be shown that $\dot{V}_{PB} \leq -\alpha V_{PB}$ for some $\alpha > 0$. Hence we conclude GAS of the equilibrium.

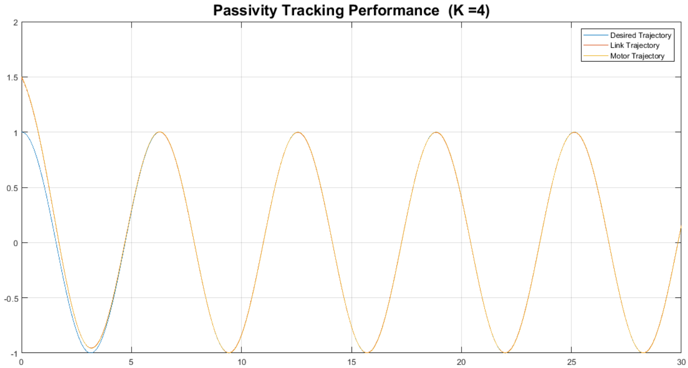
Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison

Passivity - Tracking Performance

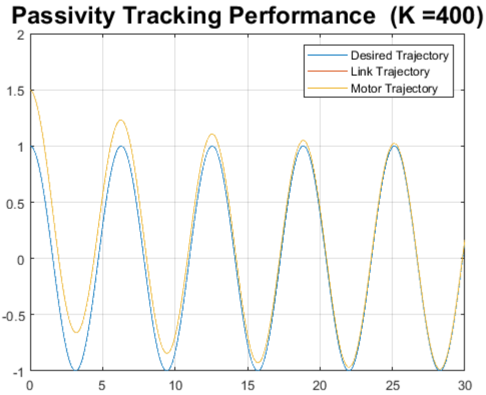
K = 4



Settling time = $4.7711 sec$

Passivity - Tracking Performance

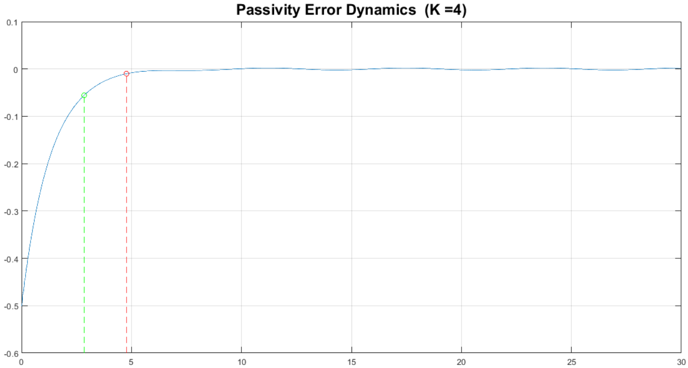
K = 400



Settling time = 25.2321*sec*

Passivity - Error Dynamics

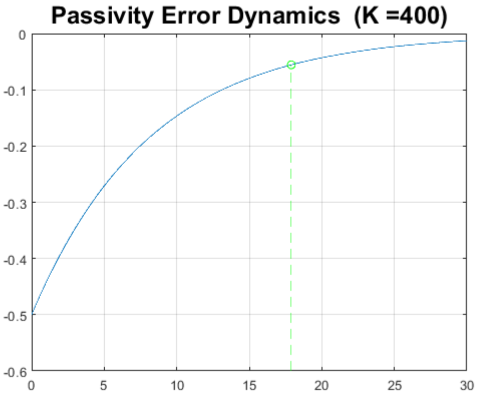
K = 4



Rise time = 2.8472*sec*

Passivity - Error Dynamics

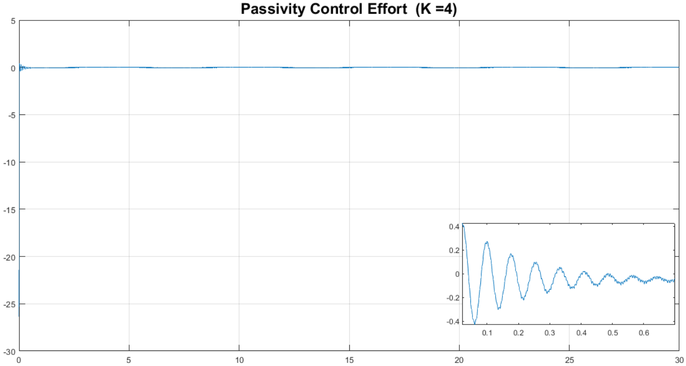
K = 400



Rise time = 17.9032*sec*

Passivity - Control Effort

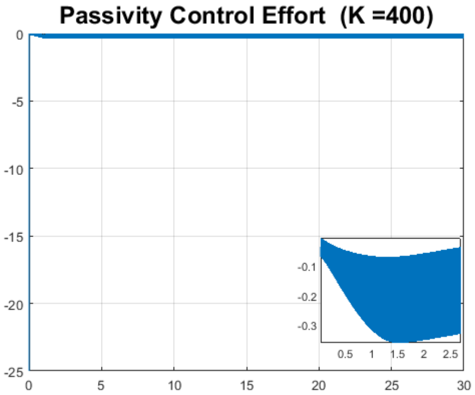
K = 4



Gains: kp = 19.0, kd= 20.0

Passivity - Control Effort

K = 400



Gains: kp = 290, kd= 400

Simulation Parameters

- We used simulation parameters provided by İsmail H. Akyuz *et. al* [1]

Parameter	Value
Mass	$0.03Kg$
Distance to Center of Mass	$0.06m$
Motor Inertia	$0.0075Kgm^2$
Link Inertia	$0.004\ Kgm^2$
Stiffness	$4N/m$
Gravity Acceleration	$-9.81m/s^2$

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison
   - Control Effort
   - Speed of Convergence
   - Possibility of Adaptive implementation

Comparison

K = 4

Controllers	Performance	Gains
Decoupling	Rise = 2.4657 Settling = 4.1223	$kp = 18.0$ $kd = 20.0$
Back-stepping	Rise = 1.9879 Settling = 3.50	$kp = 65.0$ $kd = 12.0$
Robustified Back-stepping	Rise = 3.3290 Settling = 3.20	$kp = 65.0$ $kd = 12.0$
Modified Passivity	Rise = 2.8472 Settling = 4.7711	$kp = 19.0$ $kd = 20.0$

Comparison

K = 400

Controllers	Performance	Gains
Decoupling	Rise = 2.4657 Settling = 4.1223	$kp = 0.80$ $kd = 0.20$
Back-stepping	Rise = 2.3017 Settling = 4.1035	$kp = 10.0$ $kd = 2.00$
Robustified Back-stepping	Rise = 5.0564 Settling = 10.2635	$kp = 10.0$ $kd = 11.0$
Modified Passivity	Rise = 17.9032 Settling = 25.0	$kp = 290$ $kd = 400$

Control Techniques

- Decoupling based
- Backstepping based
- Passivity based

1. Theoretical Background
2. Implementation
3. Simulation Results
4. Comparison
   - Control Effort
   - Speed of Convergence
   - Possibility of Adaptive implementation

Comparison

Adaptive extension - Assuming K is known

- *Decoupling based*

- Making the input ”u” LP would make it difficult to guarantee convergence. Authors do not ”confirm or deny” possibility if adaptive extension

- *Backstepping based*

- Authors provide clear steps regarding the adaptive extension of the controller, but it is beyond the scope of this study

- *Passivity (or Energy Shaping) based*

- Problem will be the choice of q_{2d}. Reader is referred to the authors' 1992 paper on Adaptive version of Energy Shaping techniques

Comparison

Summary and Remarks

- *For low joint stiffness*

- Performance of all controllers is very satisfactory and very similar
- Results inline with the paper

Comparison

Summary and Remarks

- *For increasing values of joint stiffness*

- Backstepping controller becomes a high gain controller because of the term $K(\dot{s} + s)$
- Robustified Backstepping controller designed to solve this issue; better tuning of gains will clearly show this
- As $K \to \infty$, the passivity based controller converges to the "FeedForward + PD Controller" $u = J\ddot{q}_{2r} - B_2 s_2$ introduced by [4]
- Decoupling & Backstepping feed directly into the loop $\ddot{q}_1$ while passivity uses $\ddot{q}_1$ and better noise sensitivity can be expected as a result

References

[1] Ismail Akyuz, Ersin Yolacan, H.M. Ertunc, and Z. Bingul.
Pid and state feedback control of a single-link flexible joint robot manipulator.
pages 409 – 414, 05 2011.

[2] R. Lozano B. Brogliato, R. Ortega.
Global tracking controllers for flexible-joint manipulators: a comparative study.
31(7):941–956, 1995.

[3] Peter Seibert and Rodolfo Suarez.
Global stabilization of nonlinear cascade systems.
Systems & Control Letters, 14(4):347–352, 1990.

[4] J-JE Slotine and Li Weiping.
Adaptive manipulator control: A case study.
IEEE transactions on automatic control, 33(11):995–1003, 1988.

[5] Mark W Spong, Romeo Ortega, and Rafael Kelly.
Comments on ”adaptive manipulator control: a case study”.
IEEE Transactions on Automatic Control, 35(6):761–762, 1990.

[6] Mark W. Spong and M. Vidyasagar.
Robot dynamics and control.
Wiley, 1989.