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Under-actuated Robotics

Global Tracking Controllers for Flexible-joint Manipulators: A Comparative Study

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Abstract

In this paper we implement three different tracking controller design techniques demonstrated in [2] as applied to the problem of trajectory tracking on a robot with a single flexible joint: decoupling-based, passivity-based and backstepping-based. They are then compared using the following performance indicators: speed of convergence, control effort, and if the controllers can be implemented in an adaptive framework. MATLAB Simulations are carried out and their results presented to clearly demonstrate the similarities and/or difference between the controllers.

Contents

1 Introduction

The problem of trajectory tracking on non-linear systems has often been addressed by applying feedback linearization techniques. However, when it comes to robots with flexible joints, the elasticity between the motor and the link must be explicitly modeled. This means that the number of degrees of freedom becomes larger than the number of available control inputs and feedback linearziation is no longer applicable. Ortega et. al [2] introduced new control techniques to address the problem of global tracking of the under-actuated robots with elastic joints.
More specifically, the problem addressed in this paper will be finding the internally stable control laws (global tracking controllers) allowing us to solve the global tracking problem for the described model mentioned in equation 1.

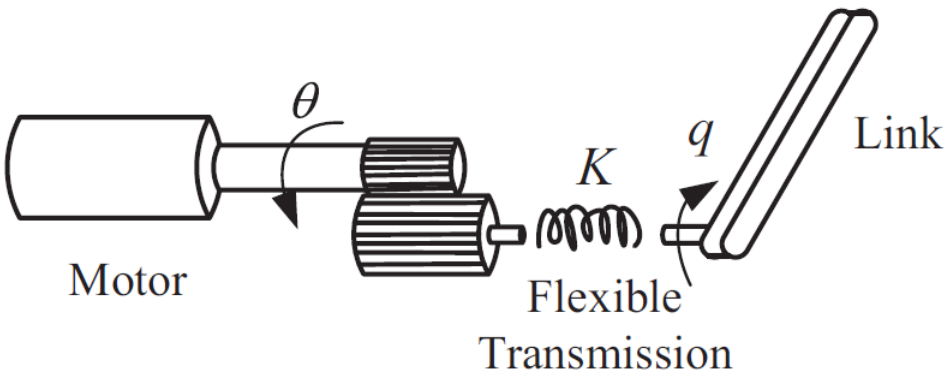


Figure 1: Flexible joint

1.1 System Model

The underactuated n-D.o.F robot manipulator dynamical model is divided in to two equations which were presented by [9], the link dynamics, and the motor dynamics respectively:

$$D(q_1)\ddot{q_1} + C(q_1, \dot{q_1})\dot{q_1} + g(q_1) = K(q_2 - q_1) \tag{1}$$

$$J\ddot{q_2} + K(q_2 - q_1) = u$$

where $q_1 \in R^n$ representing the link angles, and $q_2 \in R^n$ representing the motor angles, $D(q_1)$ is the link inertia matrix with dimensions $n \times n$, $C(q_1, \dot{q_1})\dot{q_1}$ is the Coriolis and centrifugal forces, $g(q_1)$ gravity term, and the $K(q_2 - q_1)$ is the joint's elasticity term, with K a diagonal and positive definite matrix.
This is a simplified model that assumes that the angular part of the motor's kinetic energy is due only to its own rotation. This means that there is **no** Inertial or Centrifugal coupling; only the joint's elasticity couples the motor and link dynamics.

1.2 Theoretical Background

In this paper, we will compare three different global tracking controllers which were proposed by [2], on the globally feedback linearizable flexible joint model in equation 1. These globally stable controllers will be derived using the classical technique which requires the acceleration and jerk measurements. It was proposed to acquire them from the model without differentiation, however, this method is not flexible, where the system will be highly sensitive to non-linearities cancellation and parameters uncertainty. The parameters adaptation technique which will be introduced later by author will solve this problem, proving global convergence. The system model having a diagonal inertia matrix gives the property of transforming the system into two cascaded subsystems with a suitable input transformation (which is not unique), where there was two input transformations introduced by the author, the first is $u = Kq_1 + \nu$, and the second is u $= J\nu + k(q_2 - q_1)$ which can be used in controllers to creating two cascaded subsystems.

Decoupling-based controller

The decoupling based controller name comes from the fact that the closed loop system that appears when using this controller is also cascaded, where its stability is confirmed by the lemma1 in [6], which states the following:

lemma 1: If the systems $\dot{x}$ = F(x) and $\dot{y}$ = G(0, y) are globally asymptotically stable (GAS), and if every orbit of the cascaded system is bounded, then overall system is GAS, where the overall system is presented as $\dot{x}$ = F(x) and $\dot{y}$ = G(x, y)

Backstepping-based controller

The backstepping-based techniques also use the cascade decomposition property of the model along with the integrator augmentation stabilization of Kokotovic [3] which addresses the problem of stabilizing a nonlinear system in cascade with an integrator chain and states the following

lemma 2: If a system $\dot{x}$ = F(x) is smoothly stabilizable then the system

$$\dot{x} = f(x, \xi) \quad , \quad \dot{\xi}_1 = \xi_2, ..., \dot{\xi}_k = \nu$$

obtained by cascading the original system with a chain of integrators is smoothly stablizable as well.
Simply put, the main idea of the backstepping technique is to take a smooth feedback signal, $\xi_{1d}(x)$, such that $\dot{x} = f(x, \xi_{1d}(x))$ is GAS with a known Lyapunov function and design an error equation comprising of the GAS system and a perturbation term as follows: $\xi_1(x) - \xi_{1d}(x)$. Finally, an integrator is added to its input.

Passivity-based controller

The third and final approach we shall consider in this paper is passivity-based control techniques originally proposed by [5], where we will be able to control and stabilize under-actuated Euler-Lagrange systems. Passivity control techniques are exhaustively used to solve the rigid body regulation problem by adding a damping term and taking into account the shape of the system's total potential energy. It was proved that a classical control law such as static state feedback is obtained from choosing a different desired potential energy (closed-loop). The passivity controller is proved to be exponentially stable where it was evolved to work on system robustness where we do not need to cancel non-linearities.
For this section, the robot's dynamical equations will be presented in compact form as

$$\bar{D}\ddot{q} + \bar{C}\dot{q} + \bar{g} + \bar{K}q = Mu \tag{2}$$

Where the matrices are as follows

$$q^T = \begin{bmatrix} q_1^T & q_2^T \end{bmatrix} \bar{D} = \begin{bmatrix} D & 0 \\ 0 & J \end{bmatrix} \bar{C} = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \bar{g} = \begin{bmatrix} g \\ 0 \end{bmatrix} \bar{K} = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} M = \begin{bmatrix} 0 \\ I_n \end{bmatrix}$$

lemma 3: Equation 2 defines a passive operator $\Sigma : \mathcal{L}_{2e}^n \to \mathcal{L}_{2e}^n : u \mapsto \dot{q}_2$. That is, there exists $\beta \in \mathbb{R}$ such that $\langle u|\dot{q}_2\rangle \geq \beta$, for all $u \in \mathcal{L}_{2e}^n$. Furthermore, for all $q_1, \dot{q}_1 \in \mathcal{L}_{2e}^n$, the following system

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\int_0^t s(\tau)d\tau = \psi \tag{3}$$

with $\bar{B} = \bar{B} > 0$, defines an output strictly passive operator $\Sigma_d : \mathcal{L}_{2e}^{2n} \to \mathcal{L}_{2e}^{2n} : \psi \mapsto s$, i.e. there exists $\beta \in \mathbb{R}$ and $\alpha > 0$ such that $\langle \psi|s\rangle \geq \alpha\|s\|_2^2 + \beta$ for all $\nu \in \mathcal{L}_{2e}^{2n}$. Consequently, if $\psi \equiv 0$, we have $s \in \mathcal{L}_{2e}^{2n}$.
Recalling the results of [7], the energy shaping property of the passivity controller will facilitate solving the global tracking problem where the rigid part of the desired system's energy is $\frac{1}{2}s_1 D s_1^T$, and the additional damping term is $B_1 s_1$ with $B_1 > 0$ to ensure passivity as explained before.
Towards this end, we introduce the following error signals:

$$s_1 = \dot{q}_1 + \Lambda_1\widetilde{q}_1 \quad , \Lambda_1 > 0 \qquad\qquad \widetilde{q}_1 = q_1 - q_{1d} \tag{4}$$

This will allow the controller to coincide the system's total energy with the desired one.
It is important to note that the analysis of Lemma 3 only guarantees the global **convergence** of the error signals; a weaker property than asymptotic stability. Furthermore, to more clearly provide a comparison between the previously introduced control techniques and this one, the controller ought to be simplified. Consequently, we introduce the modified passivity-based controller in the following section.

Modified Passivity-based controller

By simply updating our choice of the system's desired potential energy to

$$H_d = \frac{1}{2}s^T \bar{D}s + \frac{1}{2}\widetilde{q}\bar{K}\widetilde{q} \tag{5}$$

for which we have the perturbed desired error dynamics

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\tilde{q} = \psi \tag{6}$$

with perturbation term

$$\psi = \bar{u} - (\bar{D}\ddot{q}_r + \bar{C}\dot{q}_r + \bar{K}q_d + \bar{g}) + \bar{B}s \tag{7}$$

which eliminates the integral terms in the total energy function, where a problem will be faced due to the inclusion of the initial conditions.And to render the perturbation term ψ to zero using a similar framework of other controllers.

2 Control Techniques

2.1 Decoupling based

The decoupling based controller acts where it takes a cascaded system and produce a cascaded output too, where the idea is that we have two systems, the motor(un-forced subsystem) which is driven by q_2, and the link (forced), where applying $q_2 \equiv q_{2d}$ will lead to $q_1 \to q_{ld}$. For this to be implementable, the link dynamics must be GAS when q_2 goes to q_{2d}. The proposed parameters adaptation technique by author in [2] generalize the form of acquiring our controller which will help in implementing adaptive cases, which is presented as follows.
The desired motor position

$$q_{2d} = K^{-1}u_r + q_1 \tag{8}$$

The control law designed for the rigid part of the body by [7]

$$u_r = D(q_1)\ddot{q}_{1r} + C(q_1, \dot{q}_1)\dot{q}_{1r} + g(q_1) - B_1 s_1 \tag{9}$$

The velocity reference signal

$$\dot{q}_{1r} = \dot{q}_{1d} - \Lambda_1 q_1 \tag{10}$$

Where if we applied the above equations on our model 1, with an input of perturbation term in the motor angles such that $\widetilde{q_2} = q_2 - q_{2d}$ will lead to a system in the form of

$$D\dot{s}_1 + Cs_1 + B_1 s_1 = K\widetilde{q_2} \tag{11}$$

Where author in [8] proved that it is globally stable at $\widetilde{q_2} \equiv 0$, where the function

$$V_R = \frac{1}{2}s_1^T D s_1 + \widetilde{q_1^T}\Lambda b_1 \widetilde{q_1}$$

Is proved to be a strict Lyaponov function by [8]
The proposed decoupling controller acting on model and rendering it to GAS is as follows

$$u = K(q_2 - q_1) - K_1\widetilde{q_2} - K_2\dot{\widetilde{q_2}} + J\ddot{q}_{2d} \tag{12}$$

Resulting in a closed loop function as

$$J\ddot{\widetilde{q_2}} + K_2\dot{\widetilde{q_2}} + K_1\widetilde{q_2} = 0$$

showing that the system will be GAS, using equation 4, and controller 12.

2.2 Backstepping based

The application of the backstepping-based scheme is a very systematic method that starts by applying Lemma 2, namely, expressing the system as a cascade connection of integrators and the link dynamics. This can be done using the following feedback law

$$u = J\nu - K(q_1 - q_2)$$

As a result, the robot's dynamics are reduced to the following cascaded form

$$\begin{aligned} D\ddot{q}_1 + C\dot{q}_1 + g + Kq_1 &= Kq_2 \\ \ddot{q}_2 &= \nu \end{aligned} \tag{13}$$

At this point we start implementing our backstepping controller, where it is divided to several steps, where we change the control command by integration and is described as follows

- The q_2 in the first equation 13, is considered as our first virtual controller, where when considering that $\widetilde{q_2} \equiv 0$ such that $q_2 = q_{2d}$,where q_{2d} is the same as the q_{2d} is calculated in 8,where will have the same Lyapunov candidate function as equation V_R, which was proved GAS . However, in reality $q_2 \neq q_{2d}$, which implies that $\widetilde{q_2} \neq 0$, which defines the input in 11, then we add an integrator to the input signal producing

  $$\dot{\widetilde{q}} = -\dot{q_{2d}} + \dot{q_2}$$

- Now we assume our new input to be $\dot{q_2}$. At this point, multiple Lyapunov candidate functions were discussed in [2], we select the one recommended by the author

  $$V_2 = V_R + \frac{1}{2}\widetilde{q_2}^T\widetilde{q_2}$$

  where in it's differentiation appears an undesired cross-term

  $$\begin{aligned} \dot{V}_2 = &-\dot{\tilde{q}}_1^{\mathrm{T}} B_1 \dot{\tilde{q}}_1 - \tilde{q}_1^{\mathrm{T}} \Lambda_1^{\mathrm{T}} B_1 \Lambda_1 \tilde{q}_1 \\ &+ s_1^{\mathrm{T}} K \tilde{q}_2 + \tilde{q}_2^{\mathrm{T}} \left(\dot{q}_2 - \dot{q}_{2\mathrm{d}}\right) \end{aligned} \tag{14}$$

  at this point we choose our input controller $\dot{q_2} = -Ks_1 - \widetilde{q_2} - \dot{q}_{2d}$ which will automatically cancel the undesired terms and add a quadratic term to the equation ensuring system's GAS. As previously stated, due to the assumption of the input we look at the error equation where the $e_2 = \dot{q_2} - e_{2d}$ where we now consider e_{2d} as our second virtual input and will be considered as

  $$e_{2d} = -Ks_1 - \widetilde{q_2} + \dot{q}_{2d}$$

  At this point, we add an integrator obtaining the overall error equation as

  $$D\dot{s_1} + Cs_1 + B_1 s_1 = K\widetilde{q_2} \tag{15}$$

  $$\dot{\widetilde{q_2}} = e_2 - Ks_1 - \widetilde{q_2}$$

  $$\dot{e_2} = -\dot{e}_{2d} + \nu$$

- The originally proposed control input of the cascaded model ν appeared in the error equation, we consider a new Lyapunov candidate function where $V_{BS} = V_2 + \frac{1}{2}|e_2|^2$ setting our input $\nu = -e_2 + \dot{e}_{2d} - \widetilde{q_2}$, where the Lyapunov function proves GAS. The author proposed controller , which if used with equations 8,9 will solve the global trajectory problem, where the Backstepping controller is as follows

$$u = J[\ddot{q}_{2d} - 2\widetilde{q_2} - 2\dot{\widetilde{q}}_2 - K(\dot{s}_1 - s_1)] + K(q_2 - q_1) \tag{16}$$

$$q_{2d} = K^{-1}(u_r + q_1) \tag{17}$$

2.2.1 Robustification

At this point, author at [2] introduced a more robust technique of backstepping, where the Lyapunov candidate will change such that,

$$V_2 = V_R + \frac{1}{2}\widetilde{q_2^T} K \widetilde{q}_2$$

Which will result in the GAS controller

$$u = J[\ddot{q}_{2d} - 2\widetilde{q}_2 - 2\dot{\widetilde{q}}_2 - (\dot{s}_1 - s_1)] + K(q_2 - q_1)$$

2.3 Passivity based

The passivity controller is different than the previous controllers in term of goal, where here the signal q_{2d} ensures that the system total energy matches the chosen desired energy, however in the other controllers, q_{2d} represents the input for ensuring the link tracks the desired trajectory. At this point the author [2] introduced an energy function with the inclusion of the elastic potential energy and it is introduced as

$$H_d = \frac{1}{2}s^T \bar{D} s + \frac{1}{2}[\int_0^t s^T(\tau)d\tau]\bar{K}[\int_0^t s(\tau)d\tau]$$

with it's differentiation with respect to time is

$$\dot{H}_d = \nu^T s + s^T \bar{B} s$$

Which proves strict passivity, where the differentiation show the supply rate plus the dissipation rate term, where it proves that no internal energy was created. The closed loop error function responding to this total energy is already mentioned as equation 3, where the $\bar{B}$ is the damping term and ψ is the perturbed input which is formalized as

$$\psi = \bar{u} - (\bar{D}\ddot{q}_r + \bar{C}\dot{q}_r + \bar{K}q_r + \bar{g}) + \bar{B}s - \dot{K}\tilde{q}(0) \tag{18}$$

Where our goal is to render this perturbation term to be identically zero i.e. $\psi \equiv 0$. The proposed controller and q_{2d} by [2] are as follows

$$\begin{aligned} q_{2d} = p\,(pI + \Lambda_2)^{-1}\{&K^{-1}u_R + q_{1d} \\ &+K\left[\tilde{q}_1(0) - \tilde{q}_2(0)\right] - \textstyle\int_0^t (\Lambda_1\tilde{q}_1 - \Lambda_2 q_2)\, d\tau\Big\} \end{aligned} \tag{19}$$

where p = $\frac{d}{dt}$

$$u = -B_2 s_2 + J\ddot{q}_{2r} - K(q_{1r} - q_{2r}) \tag{20}$$

leading to closed loop equation 3, with perturbation ψ identically equal zero. However the introduced passivity controller is complex, due to the appearance of extra states, the lyapunov analysis of the system will be almost impossible, also the dependence on the initial conditions which appears to be constrained, and finally for comparison reasons between mentioned controllers,we would like to create a common framework to compare them fairly. At this point a simplified model of the passivity controller was established and was named **Modified passivity**

2.4 Modified Passivity based

The modified passivity-based control was introduced to overcome the drawbacks that were faced while working with the passivity controller. As the former is a simplified version of the latter, is easier to implement, and is not expected to provide significantly better results than its more complex counterpart. At this point, the author introduced a nee total energy function, eliminating the integral terms(that caused the appearance of the initial conditions), and replacing them by $\tilde{q}$ as follows

$$H_d = \frac{1}{2}s^T \bar{D} s + \frac{1}{2}\tilde{q}^T \bar{K}\tilde{q}$$

Where the corresponding perturbed closed loop model will be

$$\bar{D}\dot{s} + (\bar{C} + \bar{B})s + \bar{K}\tilde{q} = \psi$$

with perturbation term will change as follows

$$\psi = \bar{u} - (\bar{D}\ddot{q}_r + \bar{C}\dot{q}_r + \bar{K}q_r + \bar{g}) + \bar{B}s \tag{21}$$

As a result, we proceed by rendering the new perturbation term to zero by the following control law:

$$u = -K_2 s_2 + J(\ddot{q}_{2d} - \Lambda_2 \dot{\tilde{q}}_2) - K(q_{1d} - q_{2d}) \tag{22}$$

$$q_{2d} = q_{1d} + K^{-1}u_R \tag{23}$$

Which lead the system to global convergence as the passivity controller Furthermore, we can also prove Lyapunov stability with the Lyapunov function candidate:

$$V_{PB} = \frac{1}{2}s^T\bar{D}s + \tilde{q}_1\Lambda_1^T k_1 \tilde{q}_1 + \tilde{q}_2^T\Lambda_2^T K_2\tilde{q}_2 + \frac{1}{2}\tilde{q}^T\bar{K}\tilde{q} \tag{24}$$

for which it can be shown that $\dot{V}_{PB} \leq -\alpha V_{PB}$ for some $\alpha > 0$. Hence we conclude GAS of the equilibrium.

3 Simulation Results

A total of four controllers were implemented in SimuLink and MATLAB:

1. Decoupling-based Controller
2. Backstepping-based Controller
3. Robustified Backstepping-based Controller
4. Modified Passivity-based Controller

Parameters for the robot were obtained from İsmail H. Akyuz *et. al* [1] and are shown in the table below.

Parameter	Value
Mass	$0.03Kg$
Distance to Center of Mass	$0.06m$
Motor Inertia	$0.0075Kgm^2$
Link Inertia	$0.004\ Kgm^2$
Stiffness	$4N/m$
Gravity Acceleration	$-9.81m/s^2$

Table 1: The chosen values for the robot parameters

Furthermore, the values for the controller gains were chosen as follows:

Controllers	Gains
Decoupling	$k_p = 18.0$ $k_d = 20.0$
Backstepping	$k_p = 0.80$ $k_d = 0.08$
Robust Backstepping	$k_p = 43.0$ $k_d = 49.0$
Modified Passivity	$k_p = 19.0$ $k_d = 20.0$

Table 2: The chosen values for the Gains

The controllers were made to follow a sinusoidal trajectory with an initial error of $0.5rads$; the robot and link are made to start at $1.5rads$ while the sinusoidal trajectory is a cosine wave that begins at exactly $1rad$.

3.1 Decoupling based

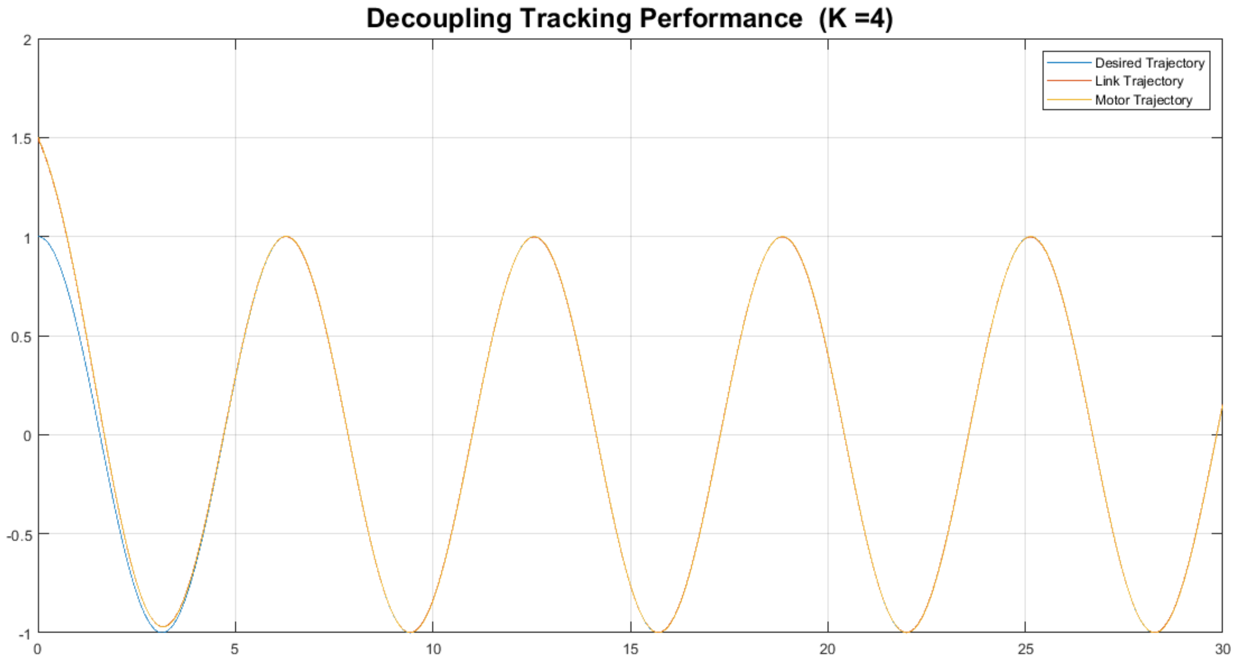


Figure 2: The decoupling-based controller demonstrates an excellent tracking performance with a steady-state error of 4×10^{-3} rads.

3.2 Backstepping based

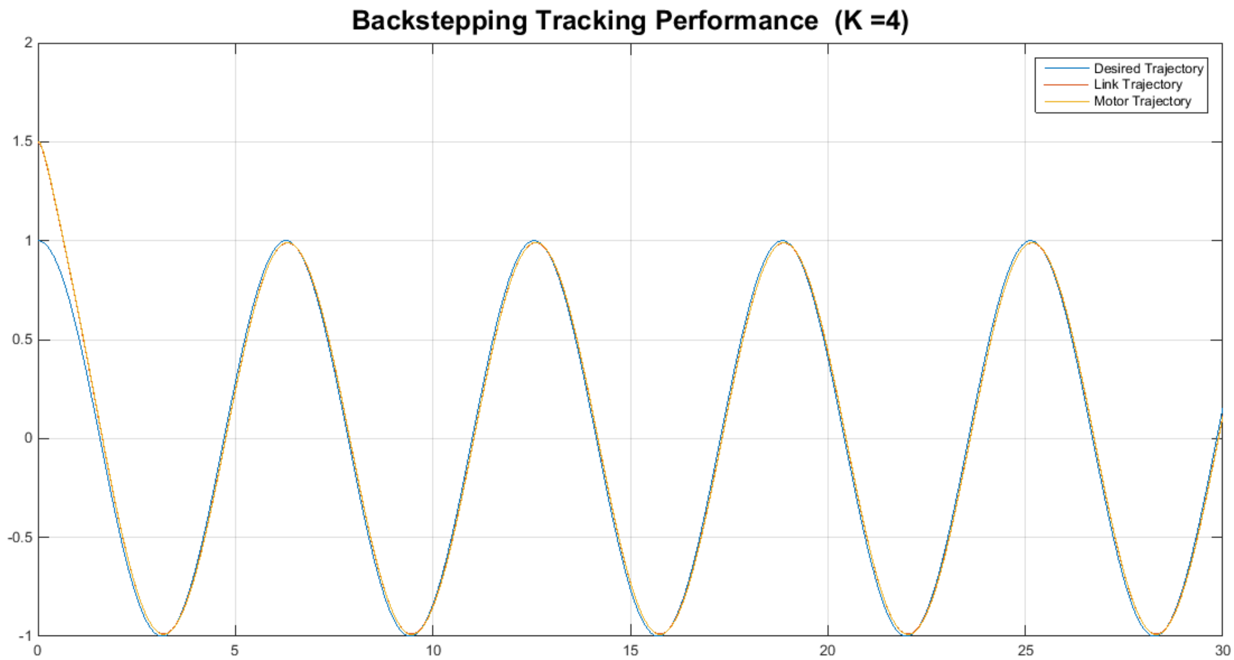


Figure 3: The backstepping-based controller quickly recovers the initial errors and is able to follow the trajectory quite while, only sustaining a steady state error of around $\pm 0.06 rads$

3.3 Robustified Backstepping based

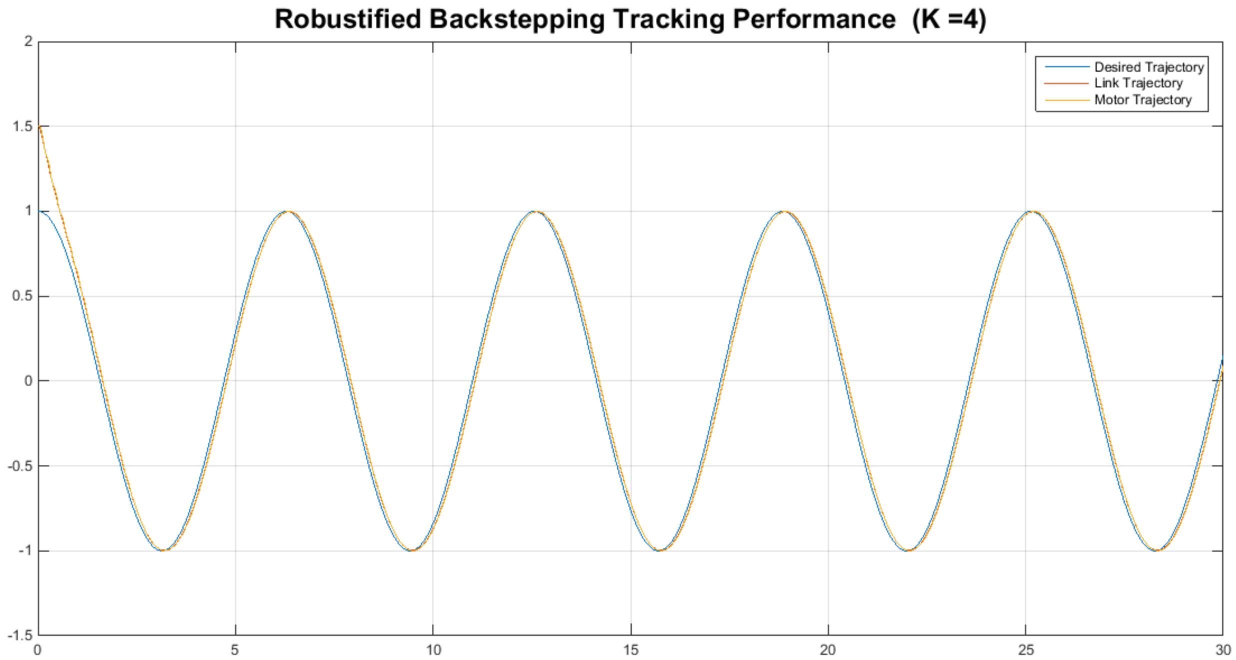


Figure 4: The Robustified backstepping technique is quite similar to it's non-robust counterpart

3.4 Passivity based

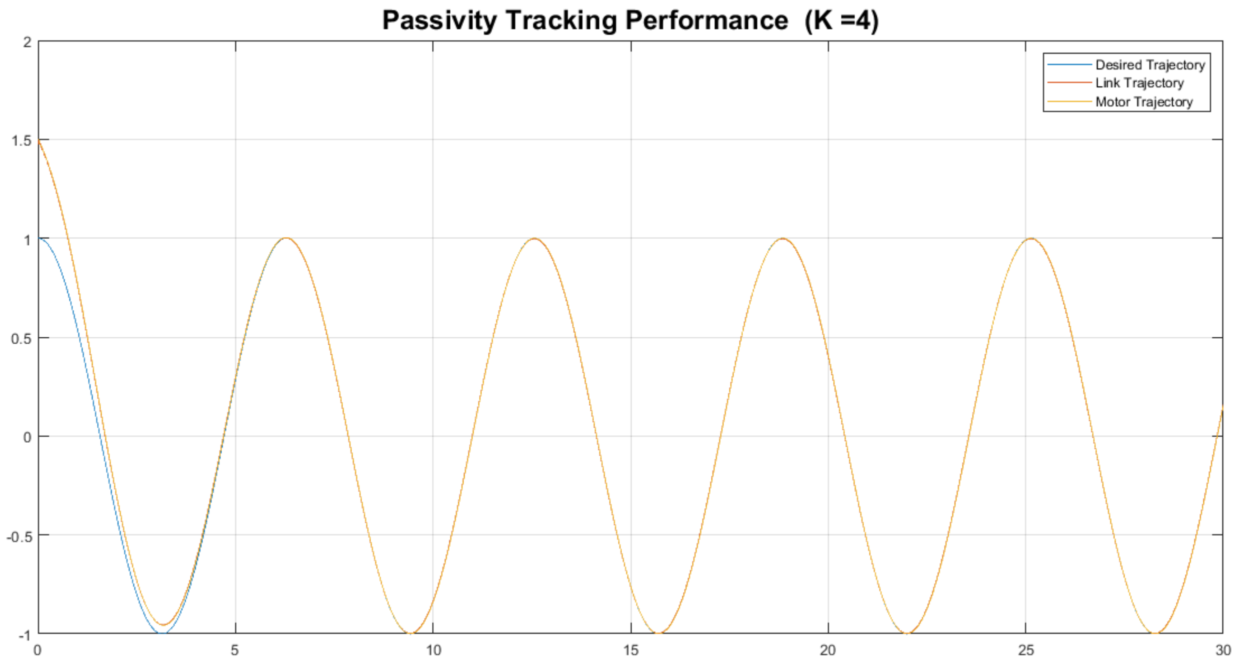
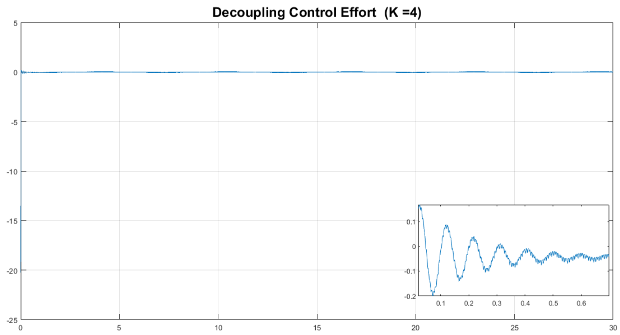


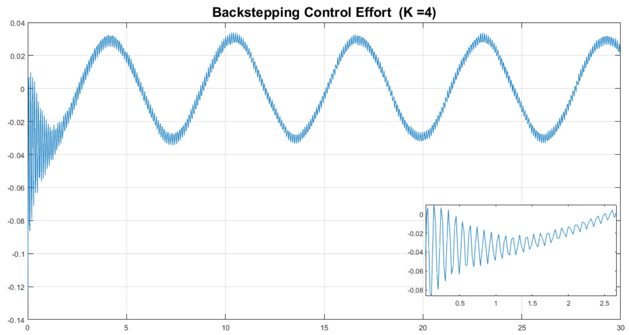
Figure 5: The Passivity-based controller shows a very good trajectory tacking of with a steady-state error of 6×10^{-3} rads.

4 Comparison

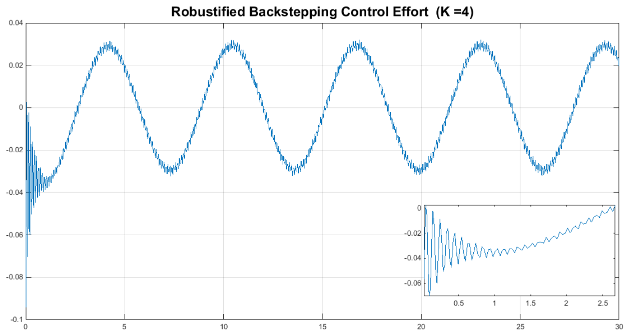
4.1 Control effort



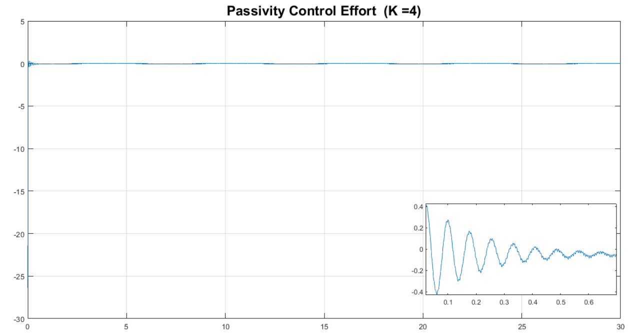
(a) Decoupling controller control effort



(b) Backstepping controller control effort

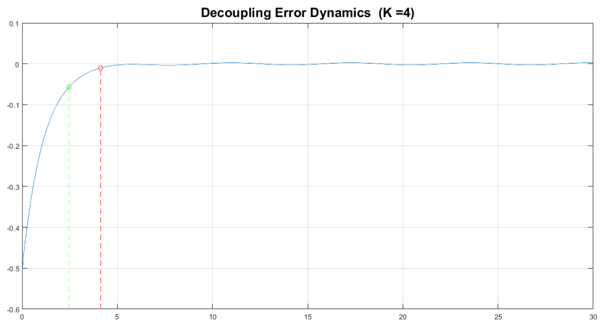


(c) Robustified Backstepping controller control effort

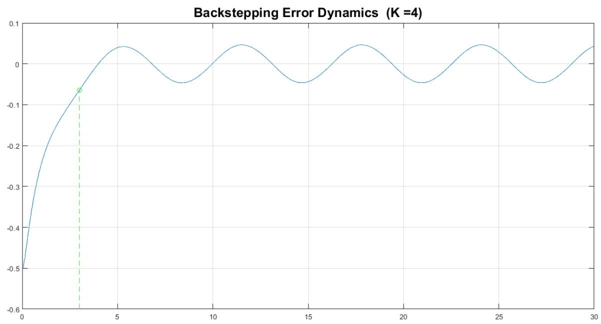


(d) Passivity controller control effort

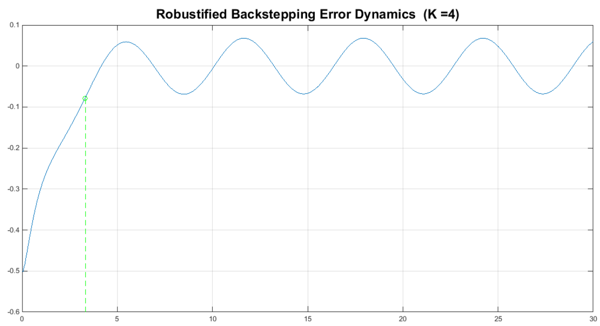
4.2 Speed of Convergence



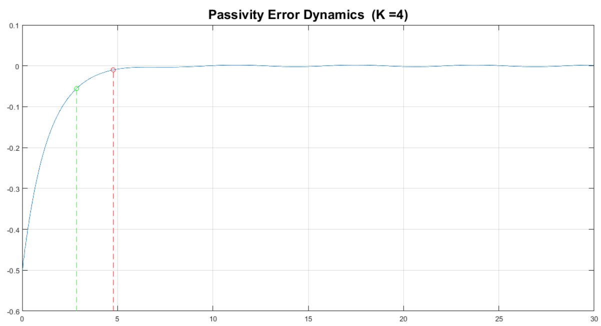
(a) Decoupling controller convergence



(b) Backstepping controller convergence



(c) Robustified Backstepping controller convergence



(d) Passivity controller convergence

4.3 Adaptive Implementation

4.3.1 Decoupling based

The author in [2] didn't confirm if the scheme can be extended to the adaptive case due to the fact that to perform adaptive implementation, it is needed to change the whole scheme that was proposed, which will deflect from the main point of the paper, however it's not stated that it is easily applicable.

4.3.2 Backstepping based

The adaptive implementation was established by a series of complex yet straightforward equations where the system at the end was proven to be adaptive as it was shown that the input ν was linearly parameterized, and the error signals between real and virtual inputs where presented to be always available online, due to that they are used in the system's update law.

4.3.3 Passivity based

The adaptive implementation of the passivity based control was established by author [4], where the choice of the energy function was changed, however it is a bit complex and needs further work.

5 Conclusion

In this project, we have studied 3 different controllers: decoupling-based, backstepping-based, and passivity based, implemented on a robot with a single flexible joint. We started by discussing the general dynamic model of a robot and then discussed the theoretical background as well as the practical implementation of each of the controllers. The performance of the controllers was compared through simulations that were carried out on MATLAB.

The modified Passivity-based as well as the Decoupling based controllers take somewhat longer to converge, but do so with the minimum error. On the other hand, the backstepping-based controller and its robustified version converge much faster but sustain a steady state error of $\pm$ 2 degrees.

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