

**Part1: Calculations****1(a): build a decision tree by using Gini Index.**

There are possible output variables **Yes** and **No**.

The data has **7 instances of No** and **3 instances of Yes**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Cloudy	Cool	Normal	Weak	No
Sunny	Hot	High	Weak	Yes
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Rainy	Cool	Normal	Strong	No
Cloudy	Mild	High	Weak	Yes
Sunny	Hot	High	Strong	No
Rainy	Cool	Normal	Weak	No
Sunny	Hot	High	Strong	No

**Step 1:** Calculate the Total Gini index using this  $Gini = 1 - \sum_{i=1}^{N_c} (p_i)^2$  formula

$$Gini(S) = 1 - \left( \left( \frac{3}{10} \right)^2 + \left( \frac{7}{10} \right)^2 \right) = 0.42$$

**Step 2:** Calculate the Gini index for feature 1 (Weather).

It has **3 possible outcomes**, **3 instances of Cloudy**, **4 instances of Sunny**, and **3 instances of Rainy**.

-Now we will calculate **Gini(Cloudy)**.

$$Gini(Cloudy) = 1 - \left( \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 \right) = 0.444$$

Weather (F1)	Hiking (Labels)
<del>Cloudy</del>	<del>No</del>
Sunny	Yes
Rainy	Yes
<del>Cloudy</del>	<del>No</del>
Sunny	No
Rainy	No
<del>Cloudy</del>	<del>Yes</del>
Sunny	No
Rainy	No
Sunny	No

-Now we will calculate **Gini(Sunny)**.

$$\text{Gini(Sunny)} = 1 - \left( \left( \frac{3}{4} \right)^2 + \left( \frac{1}{4} \right)^2 \right) = 0.375$$

-Now we will calculate **Gini(Rainy)**.

$$\text{Gini(Rainy)} = 1 - \left( \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right) = 0.444$$

Weather (F1)	Hiking (Labels)
Cloudy	No
<del>Sunny</del>	<del>Yes</del>
Rainy	Yes
Cloudy	No
<del>Sunny</del>	<del>No</del>
Rainy	No
Cloudy	Yes
<del>Sunny</del>	<del>No</del>
Rainy	No
<del>Sunny</del>	<del>No</del>

**Gini(Sunny)**

Weather (F1)	Hiking (Labels)
Cloudy	No
Sunny	Yes
<del>Rainy</del>	<del>Yes</del>
Cloudy	No
Sunny	No
<del>Rainy</del>	<del>No</del>
Cloudy	Yes
Sunny	No
<del>Rainy</del>	<del>No</del>
Sunny	No

**Gini(Rainy)**

-Now we will calculate the total Gini Index score for feature 1 **Gini(Weather)**.

$$\text{Gini(Weather)} = 0.444 * \left( \frac{3}{10} \right) + 0.375 * \left( \frac{4}{10} \right) + 0.444 * \left( \frac{3}{10} \right) = 0.416$$

**Step 3: Calculate the Gini index for feature 2 (Temperature).**

It has **3 possible outcomes**, **3 instances of Cool**, **3 instances of Hot**, and **4 instances of Mild**.

-Now we will calculate **Gini(Cool)**.

$$\text{Gini(Cool)} = 1 - \left( \left( \frac{0}{3} \right)^2 + \left( \frac{3}{3} \right)^2 \right) = 0$$

-Now we will calculate **Gini(Hot)**.

$$\text{Gini(Hot)} = 1 - \left( \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 \right) = 0.444$$

Temperature (F2)	Hiking (Labels)
<del>Cool</del>	<del>No</del>
Hot	Yes
Mild	Yes
Mild	No
Mild	No
<del>Cool</del>	<del>No</del>
Mild	Yes
Hot	No
<del>Cool</del>	<del>No</del>
Hot	No

**Gini(Cool)**

Temperature (F2)	Hiking (Labels)
Cool	No
<del>Hot</del>	<del>Yes</del>
Mild	Yes
Mild	No
Mild	No
Cool	No
Mild	Yes
<del>Hot</del>	<del>No</del>
Cool	No
<del>Hot</del>	<del>No</del>

**Gini(Hot)**

-Now we will calculate **Gini(Mild)**.

$$\text{Gini(Mild)} = 1 - \left( \left( \frac{2}{4} \right)^2 + \left( \frac{2}{4} \right)^2 \right) = 0.5$$

Temperature (F2)	Hiking (Labels)
Cool	No
Hot	Yes
<del>Mild</del>	<del>Yes</del>
<del>Mild</del>	<del>No</del>
<del>Mild</del>	<del>No</del>
Cool	No
<del>Mild</del>	<del>Yes</del>
Hot	No
Cool	No
Hot	No

-Now we will calculate the total Gini Index score for feature 2 **Gini(Temperature)**.

$$\text{Gini(Temperature)} = 0 * \left( \frac{3}{10} \right) + 0.444 * \left( \frac{3}{10} \right) + 0.5 * \left( \frac{4}{10} \right) = 0.333$$

**Step 4:** Calculate the Gini index for feature 3 (Humidity).

It has **2 possible outcomes**, **4 instances of Normal**, **6 instances of High**.

-Now we will calculate **Gini(Normal)**.

$$\text{Gini(Normal)} = 1 - \left( \left( \frac{3}{4} \right)^2 + \left( \frac{1}{4} \right)^2 \right) = 0.375$$

-Now we will calculate **Gini(High)**.

$$\text{Gini(High)} = 1 - \left( \left( \frac{2}{6} \right)^2 + \left( \frac{4}{6} \right)^2 \right) = 0.444$$

Humidity (F3)	Hiking (Labels)
<del>Normal</del>	<del>No</del>
High	Yes
<del>Normal</del>	<del>Yes</del>
High	No
High	No
<del>Normal</del>	<del>No</del>
High	Yes
High	No
<del>Normal</del>	<del>No</del>
High	No

**Gini(Normal)**

Humidity (F3)	Hiking (Labels)
Normal	No
<del>High</del>	<del>Yes</del>
Normal	Yes
<del>High</del>	<del>No</del>
<del>High</del>	<del>No</del>
Normal	No
<del>High</del>	<del>Yes</del>
<del>High</del>	<del>No</del>
Normal	No
<del>High</del>	<del>No</del>

**Gini(High)**

-Now we will calculate the total Gini Index score for feature 3 **Gini(Humidity)**.

$$\text{Gini(Humidity)} = 0.375 * \left( \frac{4}{10} \right) + 0.444 * \left( \frac{6}{10} \right) = 0.416$$

**Step 5:** Calculate the Gini index for feature 4 (Wind).

It has **2 possible outcomes**, **4 instances of Weak**, **6 instances of Strong**.

-Now we will calculate **Gini(Weak)**.

$$\text{Gini(Weak)} = 1 - \left( \left( \frac{2}{4} \right)^2 + \left( \frac{2}{4} \right)^2 \right) = 0.5$$

-Now we will calculate **Gini(Strong)**.

$$\text{Gini(Strong)} = 1 - \left( \left( \frac{1}{6} \right)^2 + \left( \frac{5}{6} \right)^2 \right) = 0.277$$

Wind (F4)	Hiking (Labels)
<del>Weak</del>	<del>No</del>
<del>Weak</del>	<del>Yes</del>
Strong	Yes
Strong	No
Strong	No
Strong	No
<del>Weak</del>	<del>Yes</del>
Strong	No
<del>Weak</del>	<del>No</del>
Strong	No

**Gini(Weak)**

Wind (F4)	Hiking (Labels)
Weak	No
Weak	Yes
<del>Strong</del>	<del>Yes</del>
<del>Strong</del>	<del>No</del>
<del>Strong</del>	<del>No</del>
<del>Strong</del>	<del>No</del>
Weak	Yes
<del>Strong</del>	<del>No</del>
Weak	No
<del>Strong</del>	<del>No</del>

**Gini(Strong)**

-Now we will calculate the total Gini Index score for feature 4 **Gini(Wind)**.

$$\text{Gini(Wind)} = 0.5 * \left( \frac{4}{10} \right) + 0.277 * \left( \frac{6}{10} \right) = 0.366$$

**Step 6:** Now we will choose the root node based on the minimum value of Gini Index.

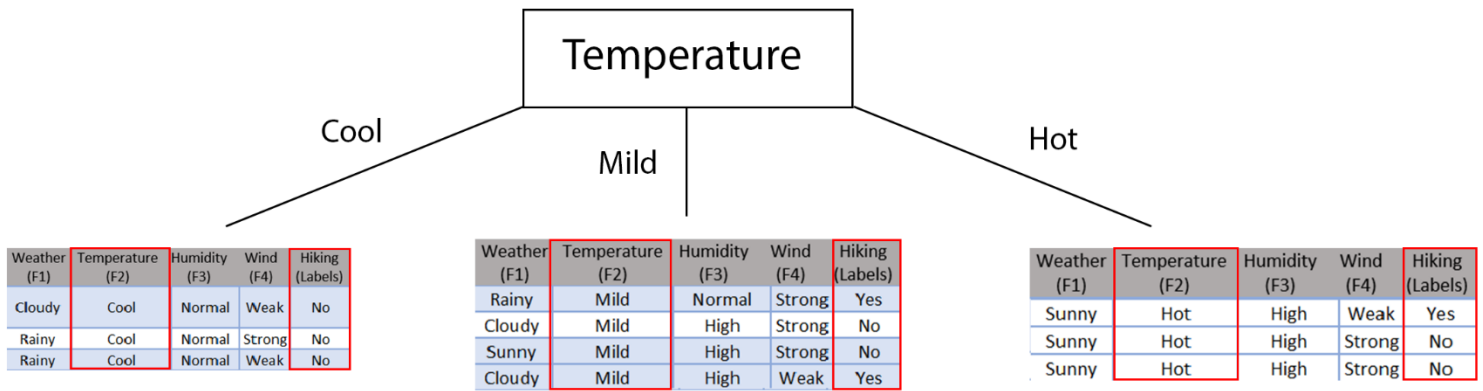
$$\text{Gini(Weather)} = 0.416$$

$$\text{Gini(Temperature)} = 0.333$$

$$\text{Gini(Humidity)} = 0.416$$

$$\text{Gini(Wind)} = 0.366$$

We found that the **Gini(Temperature)** was had the **minimum** value of Gini Index score, so we will take the temperature feature as the root node.



**Step 7:** now we will see the Gini index score with the other features.

-When Temperature = Cool

$$\text{Gini}(\text{Temperature} = \text{Cool} \mid \text{Weather} = \text{Cloudy}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

$$\text{Gini}(\text{Temperature} = \text{Cool} \mid \text{Weather} = \text{Rainy}) = 1 - \left( \left( \frac{2}{2} \right)^2 + \left( \frac{0}{2} \right)^2 \right) = 0$$

- Total = Gini(Temperature = Cool | Weather) =  $0 * \left( \frac{1}{3} \right) + 0 * \left( \frac{2}{3} \right) = 0$**

-When Temperature = Mild

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Rainy}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy}) = 1 - \left( \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \right) = 0.5$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Sunny}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

- Total = Gini(Temperature = Mild | Weather) =  $0 * \left( \frac{1}{4} \right) + 0.5 * \left( \frac{2}{4} \right) + 0 * \left( \frac{1}{4} \right) = 0.25$**

-When Temperature = Hot

$$\text{Gini}(\text{Temperature} = \text{Hot} \mid \text{Weather} = \text{Sunny}) = 1 - \left( \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right) = 0.444$$

- Total = Gini(Temperature = Hot | Weather) =  $0.444 * \left( \frac{3}{3} \right) = 0.444$**

So that's mean the when the **temperature is Cool** the **Decision will be No**.

Now we will compute the temperature with the other feature which is **Humidity**.

-When Temperature = Mild

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Humidity} = \text{Normal}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Humidity} = \text{High}) = 1 - \left( \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right) = 0.444$$

- **Total = Gini(Temperature = Mild | Humidity) =  $0 * \left( \frac{1}{4} \right) + 0.444 * \left( \frac{3}{4} \right) = 0.333$**

-When Temperature = Hot

$$\text{Gini}(\text{Temperature} = \text{Hot} \mid \text{Humidity} = \text{High}) = 1 - \left( \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right) = 0.444$$

- **Total = Gini(Temperature = Hot | Humidity) =  $0.444 * \left( \frac{3}{3} \right) = 0.444$**

Now we will compute the temperature with the other feature which is **Wind**.

-When Temperature = Mild

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Wind} = \text{Strong}) = 1 - \left( \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right) = 0.444$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Wind} = \text{Weak}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

- **Total = Gini(Temperature = Mild | Wind) =  $0.444 * \left( \frac{3}{4} \right) + 0 * \left( \frac{1}{4} \right) = 0.333$**

-When Temperature = Hot

$$\text{Gini}(\text{Temperature} = \text{Hot} \mid \text{Wind} = \text{Strong}) = 1 - \left( \left( \frac{2}{2} \right)^2 + \left( \frac{0}{2} \right)^2 \right) = 0$$

$$\text{Gini}(\text{Temperature} = \text{Hot} \mid \text{Wind} = \text{Weak}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

- **Total = Gini(Temperature = Hot | Wind) =  $0 * \left( \frac{2}{3} \right) + 0 * \left( \frac{1}{3} \right) = 0$**

### Step 8: From these scores we can start to build the tree.

- 1- When the **temperature is Mild** we will see which feature has the minimum value of Gini index score.

**Gini(Temperature = Mild | Weather) = 0.25**

**Gini(Temperature = Mild | Humidity) = 0.333**

**Gini(Temperature = Mild | Wind) = 0.333**

so, we will take feature (**Weather**) with the temperate = **Mild**.

- 2- When the **temperature is Hot** we will see which feature has the minimum value of Gini index score.

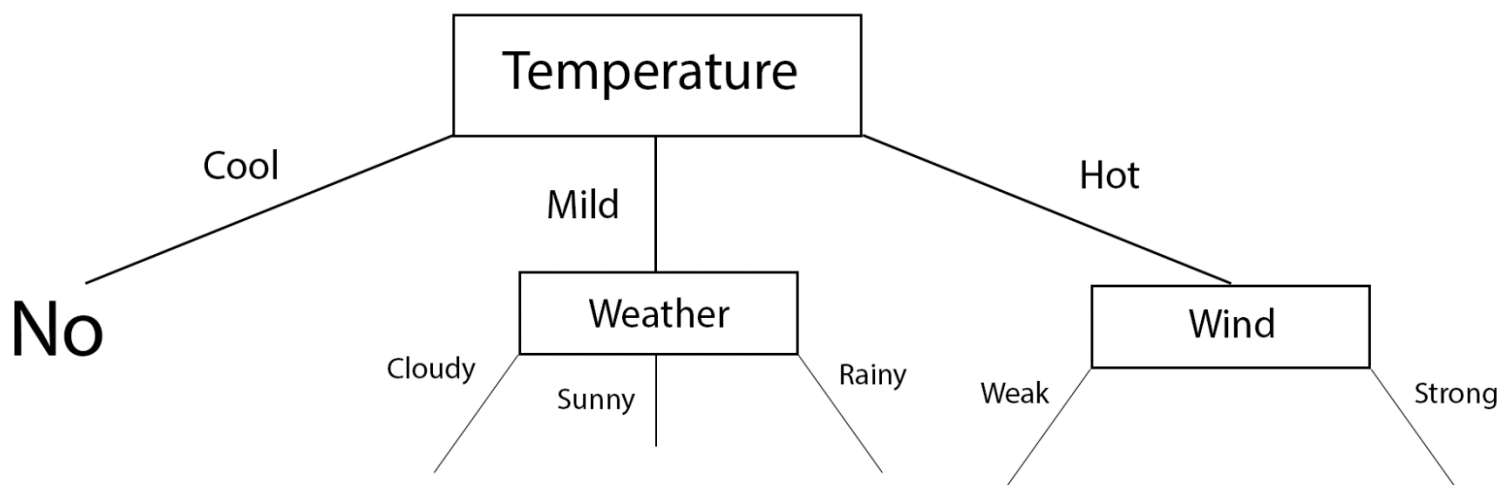
**Gini(Temperature = Hot | Weather) = 0.444**

**Gini(Temperature = Hot | Humidity) = 0.444**

**Gini(Temperature = Hot | Wind) = 0**

so, we will take feature (**Wind**) with the temperate when it's = **Hot**.

So, until now we have something like this.



### Step 9: Complete the Tree.

So now we will see what is the result of each possibility.

We notice from the table, when the **temperature is Mild**,

And the **weather is Rainy** we found that the **only** hiking option is **YES**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

And the same for **weather is Sunny** we found that the **only** hiking option is **NO**.

But we found that there are **and impurity when weather is Cloudy** (one time hiking option was **NO** and the other time it was **Yes**)

So now we will calculate the Gini Index score for...

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy} \mid \text{Humidity}) = 1 - \left( \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^2 \right) = 0.5$$

- **Total = Gini(Temperature = Mild | Weather = Cloudy | Humidity) = 0.5 \*  $\left( \frac{2}{2} \right) = 0.5$**

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy} \mid \text{Wind} = \text{Strong}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy} \mid \text{Wind} = \text{Weak}) = 1 - \left( \left( \frac{1}{1} \right)^2 + \left( \frac{0}{1} \right)^2 \right) = 0$$

- **Total = Gini(Temperature = Mild | Weather = Cloudy | Wind) = 0 \*  $\left( \frac{1}{2} \right) + 0 * \left( \frac{1}{2} \right) = 0$**

We found that **Gini(Temperature = Mild | Weather = Cloudy | Wind)** has the minimum Gini index score.

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy} \mid \text{Humidity}) = 0.5$$

$$\text{Gini}(\text{Temperature} = \text{Mild} \mid \text{Weather} = \text{Cloudy} \mid \text{Wind}) = 0$$

So now when the **Temperature = Mild** and the **Weather = Cloudy**, we will check for **Wind** Value if it was = **strong** it will be **No**, and if it was = **Weak** it will be **Yes**, **based on that table on the below**. **(There is no Impurity)**.

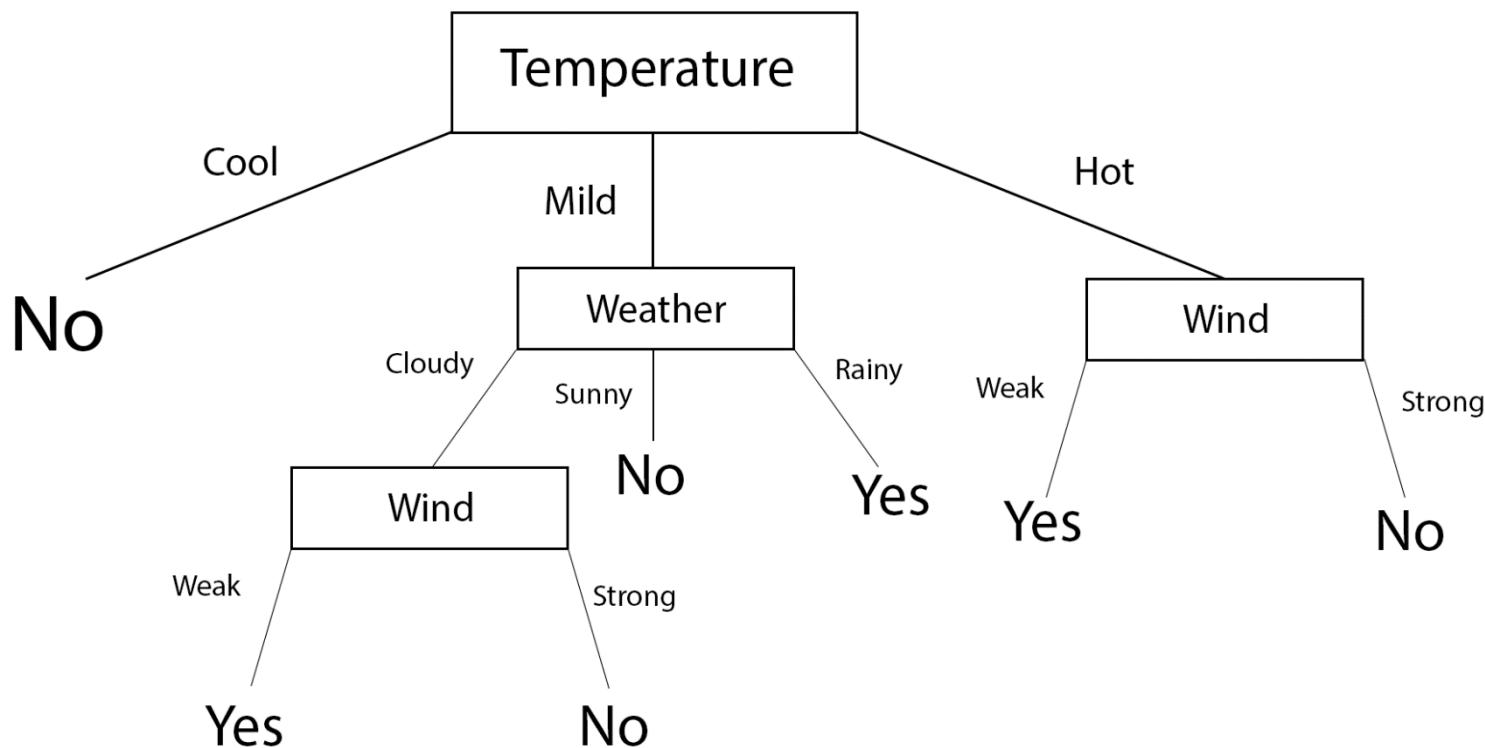
Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

So now when the **Temperature = Hot**, we will check for **Wind** Value if it was = **strong** it will be **No**, and if it was = **Weak** it will be **Yes**, **based on that table on the below (There is no Impurity)**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Sunny	Hot	High	Weak	Yes
Sunny	Hot	High	Strong	No
Sunny	Hot	High	Strong	No



So we have reached to the leaf node (**Final Decision**) on each branch, and we got something like this...



**1(b): build a decision tree by using Information Gain.**

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Cloudy	Cool	Normal	Weak	No
Sunny	Hot	High	Weak	Yes
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Rainy	Cool	Normal	Strong	No
Cloudy	Mild	High	Weak	Yes
Sunny	Hot	High	Strong	No
Rainy	Cool	Normal	Weak	No
Sunny	Hot	High	Strong	No

**Step 1:** Calculate the Entropy(S) using this formula...

$$Entropy(t) = -\sum_j p(j|t) \log_2 p(j|t)$$

$$P(\text{No}) = \frac{7}{10}$$

$$P(\text{Yes}) = \frac{3}{10}$$

$$Entropy(S) = -\frac{7}{10} * \log_2 \frac{7}{10} - \frac{3}{10} * \log_2 \frac{3}{10} = 0.88129$$

**Step 2:** Calculate the Information Gain Score for each feature using this formula...

$$GAIN_{split} = Entropy(p) - \left( \sum_{i=1}^k \frac{n_i}{n} Entropy(i) \right)$$

- Weather**

3 Cloudy -> 2 No, 1 Yes.

4 Sunny -> 3 No, 1 Yes.

3 Rainy -> 2 No, 1 Yes.

$$\begin{aligned} \text{Gain}(S, \text{Weather}) &= 0.88129 - \frac{3}{10} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} * \log_2 \frac{1}{3} \right) - \frac{4}{10} * \left( -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} * \log_2 \frac{1}{4} \right) \\ &\quad - \frac{3}{10} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} * \log_2 \frac{1}{3} \right) = 0.4982 \end{aligned}$$

- Temperature**

3 Cool -> 3 No, 0 Yes.

3 Hot -> 2 No, 1 Yes.

4 Mild -> 2 No, 2 Yes.

$$\begin{aligned} \text{Gain}(S, \text{Temperature}) &= 0.88129 - \frac{3}{10} * \left( -\frac{3}{3} \log_2 \frac{3}{3} - \frac{0}{3} * \log_2 \frac{0}{3} \right) - \frac{3}{10} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} * \right. \\ &\quad \left. \log_2 \frac{1}{3} \right) - \frac{4}{10} * \left( -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} * \log_2 \frac{2}{4} \right) = 0.82279 \end{aligned}$$

- **Humidity**

4 Normal -> 3 No, 1 Yes.

6 High -> 4 No, 2 Yes.

$$\text{Gain}(S, \text{Humidity}) = 0.88129 - \frac{4}{10} * \left( -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} * \log_2 \frac{1}{4} \right) - \frac{6}{10} * \left( -\frac{4}{6} \log_2 \frac{4}{6} - \frac{2}{6} * \log_2 \frac{2}{6} \right) = 0.00580$$

- **Wind**

4 Weak -> 2 No, 2 Yes.

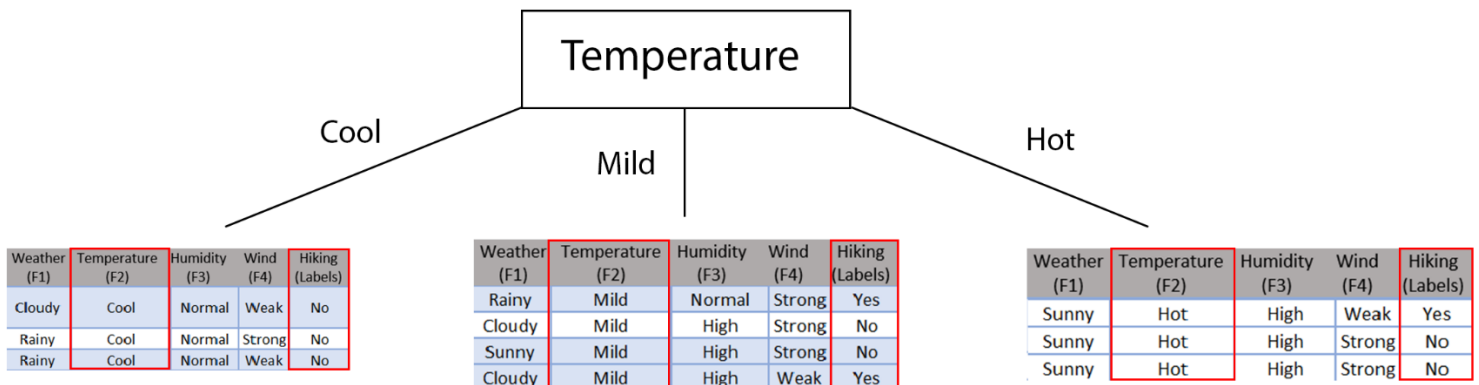
6 Strong -> 5 No, 1 Yes.

$$\text{Gain}(S, \text{Wind}) = 0.88129 - \frac{4}{10} * \left( -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} * \log_2 \frac{2}{4} \right) - \frac{6}{10} * \left( -\frac{5}{6} \log_2 \frac{5}{6} - \frac{1}{6} * \log_2 \frac{1}{6} \right) = 0.09127$$

**Step 3:** Determine which feature will be the root node, and after that calculate the IG Score for the other features.

We found that the **Temperature** has the highest value of information, so it will be the root node.

$$\text{Gain}(S, \text{Temperature}) = 0.82279$$



from the above figure we will found that when **Temperature = Cool**, the **decision will be No**.

now we will calculate the other feature when **Temperature = Mild**, but first we will calculate the **new Entropy**.

$$\text{Entropy}(S) = -\frac{2}{4} * \log_2 \frac{2}{4} - \frac{2}{4} * \log_2 \frac{2}{4} = 1$$

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

- Weather**

1 Rainy -> 0 No, 1 Yes.

2 Cloudy -> 1 No, 1 Yes.

1 Sunny -> 1 No, 0 Yes.

$$\text{Gain}(S, \text{Weather}) = 1 - \frac{1}{4} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) - \frac{2}{4} * \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} * \log_2 \frac{1}{2} \right) - \frac{1}{4} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) = 0.5$$

- Humidity**

1 Normal -> 0 No, 1 Yes.

3 High -> 2 No, 1 Yes.

$$\text{Gain}(S, \text{Humidity}) = 1 - \frac{1}{4} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) - \frac{3}{4} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} * \log_2 \frac{1}{3} \right) = 0.311278$$

- Wind**

3 Strong -> 2 No, 1 Yes.

1 Weak -> 0 No, 1 Yes.

$$\text{Gain}(S, \text{Wind}) = 1 - \frac{3}{4} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} \right) - \frac{1}{4} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) = 0.311278$$

We found that the **Weather** has the highest value of information, When **Temperature = Mild**.

**Gain(S, Weather) = 0.5**

now we will calculate the other feature when **Temperature = Hot**, but first we will calculate the **new Entropy**.

$$\text{Entropy}(S) = -\frac{1}{3} * \log_2 \frac{1}{3} - \frac{2}{3} * \log_2 \frac{2}{3} = 0.9182$$

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Sunny	Hot	High	Weak	Yes
Sunny	Hot	High	Strong	No
Sunny	Hot	High	Strong	No

- Weather**

3 Sunny -> 2 No, 1 Yes.

$$\text{Gain}(S, \text{Weather}) = 0.9182 - \frac{3}{3} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} \right) = 0$$

- Humidity**

3 High -> 2 No, 1 Yes.

$$\text{Gain}(S, \text{Humidity}) = 0.9182 - \frac{3}{3} * \left( -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} \right) = 0$$

- Wind**

2 Strong -> 2 No, 0 Yes.

1 Weak -> 0 No, 1 Yes.

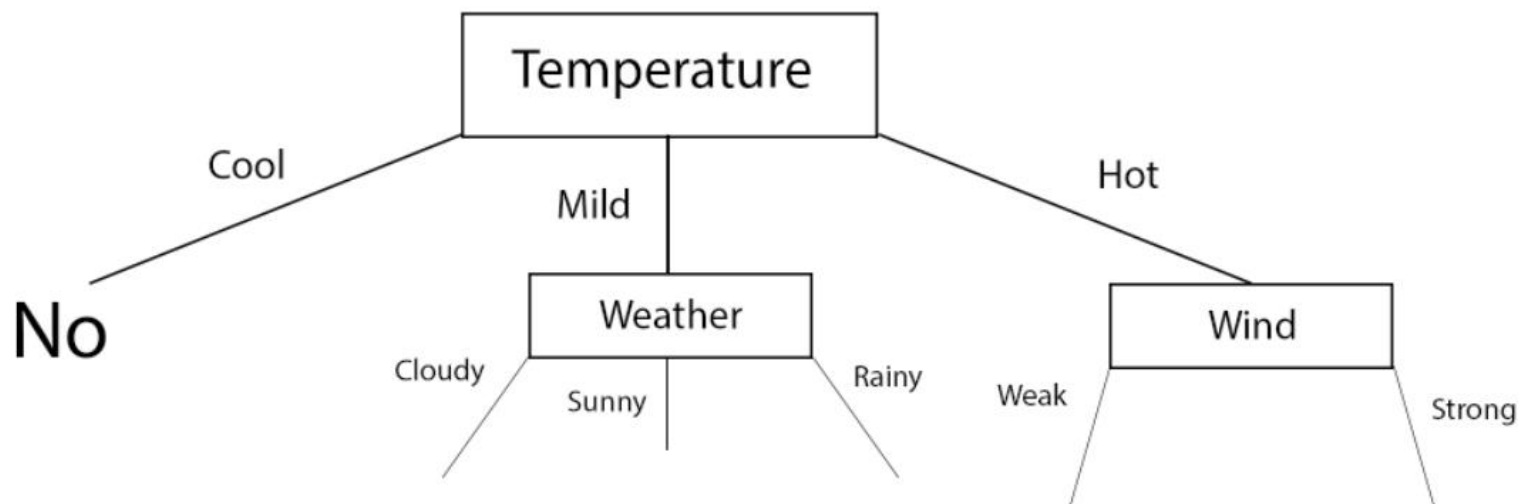
$$\text{Gain}(S, \text{Wind}) = 0.9182 - \frac{2}{3} * \left( -\frac{2}{2} \log_2 \frac{2}{2} \right) - \frac{1}{3} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) = 0.9182$$

We found that the **Wind** has the highest value of information, When **Temperature = Hot**.

$$\text{Gain}(S, \text{Wind}) = 0.9182$$

#### Step 4: continue constructing the tree...

Now we have something like this...



So now we will see what is the result of each possibility.

We notice from the table, when the **temperature is Mild**,

And the **weather is Rainy** we found that the **only** hiking option is **YES**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

And the same for **weather is Sunny** we found that the **only** hiking option is **NO**.

But we found that there are **and impurity when weather is Cloudy** (one time hiking option was **NO** and the other time it was **Yes**)

Now we will calculate the other feature when **Temperature = Mild** and **Weather = Cloudy**, but first we will calculate the **new Entropy**.

$$\text{Entropy}(S) = -\frac{1}{2} * \log_2 \frac{1}{2} - \frac{1}{2} * \log_2 \frac{1}{2} = 1$$

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Cloudy	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

- **Humidity**

2 High -> 1 No, 1 Yes.

$$\text{Gain}(S, \text{Humidity}) = 1 - \frac{2}{2} * \left( -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2} \right) = 0$$

- **Wind**

1 Strong -> 1 No, 0 Yes.

1 Weak -> 0 No, 1 Yes.

$$\text{Gain}(S, \text{Wind}) = 1 - \frac{1}{2} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) - \frac{1}{2} * \left( -\frac{1}{1} \log_2 \frac{1}{1} \right) = 1$$

We found that the **Wind** has the highest value of information, When **Temperature = Mild** and **Weather = Cloudy**.

$$\text{Gain}(S, \text{Wind}) = 1$$

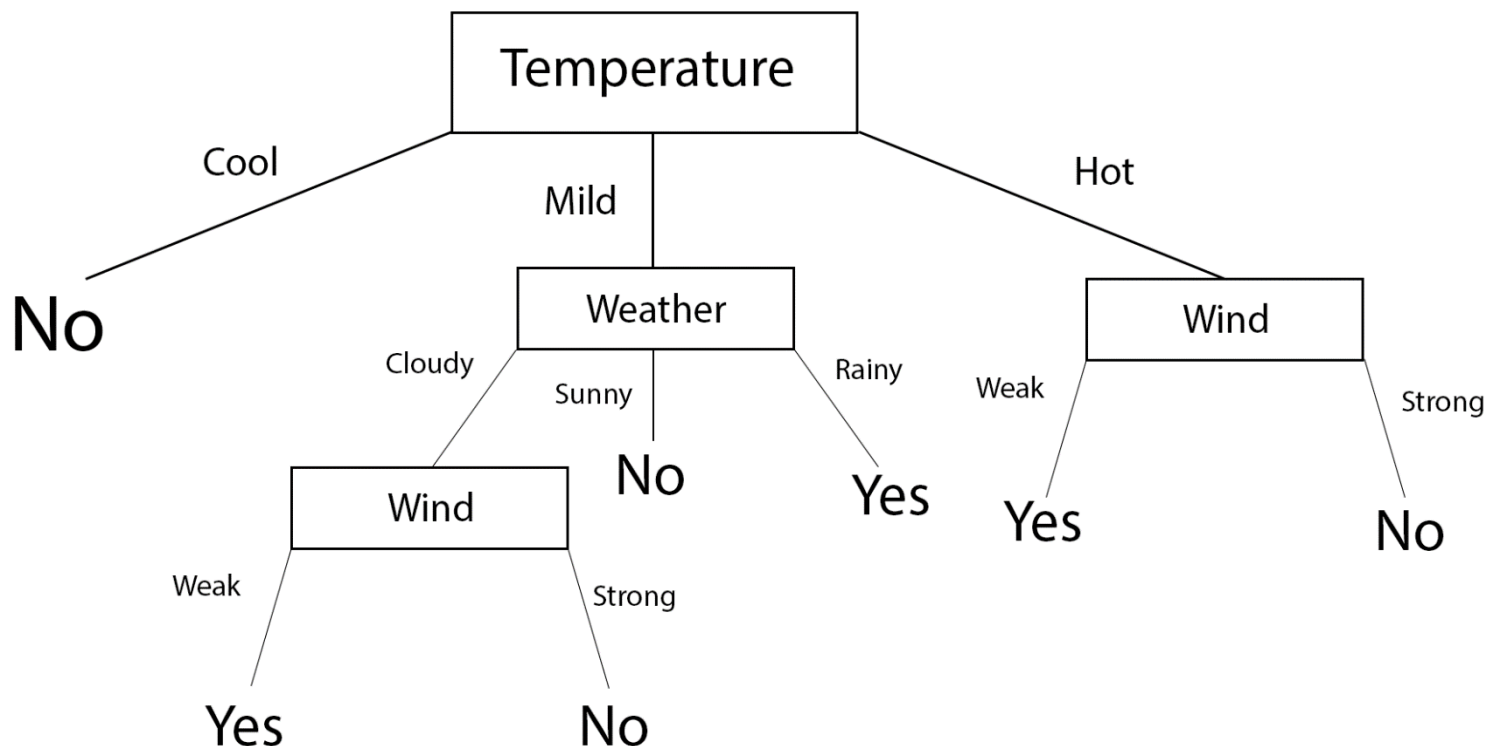
So now when the **Temperature = Mild** and the **Weather = Cloudy**, we will check for **Wind** Value if it was = **strong** it will be **No**, and if it was = **Weak** it will be **Yes**, **based on that table on the below**. **(There is no Impurity)**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Rainy	Mild	Normal	Strong	Yes
Cloudy	Mild	High	Strong	No
Sunny	Mild	High	Strong	No
Cloudy	Mild	High	Weak	Yes

So now when the **Temperature = Hot**, we will check for **Wind** Value if it was = **strong** it will be **No**, and if it was = **Weak** it will be **Yes**, **based on that table on the below (There is no Impurity)**.

Weather (F1)	Temperature (F2)	Humidity (F3)	Wind (F4)	Hiking (Labels)
Sunny	Hot	High	Weak	Yes
Sunny	Hot	High	Strong	No
Sunny	Hot	High	Strong	No

So we have reached to the leaf node (**Final Decision**) on each branch, and we got something like this...



**1(c):** We have seen that both ways gave us the same Tree at the end, but talk about each method from the perspective of computational power, the Gini index will win because...

- Gini index it only goes up to 0.5 and then it starts decreasing, hence it requires less computational power.
- But The range of Entropy (information gain) lies in between 0 to 1 and the range.

**Hence, we can conclude that Gini Impurity is better as compared to entropy (information gain) for selecting the best features.**



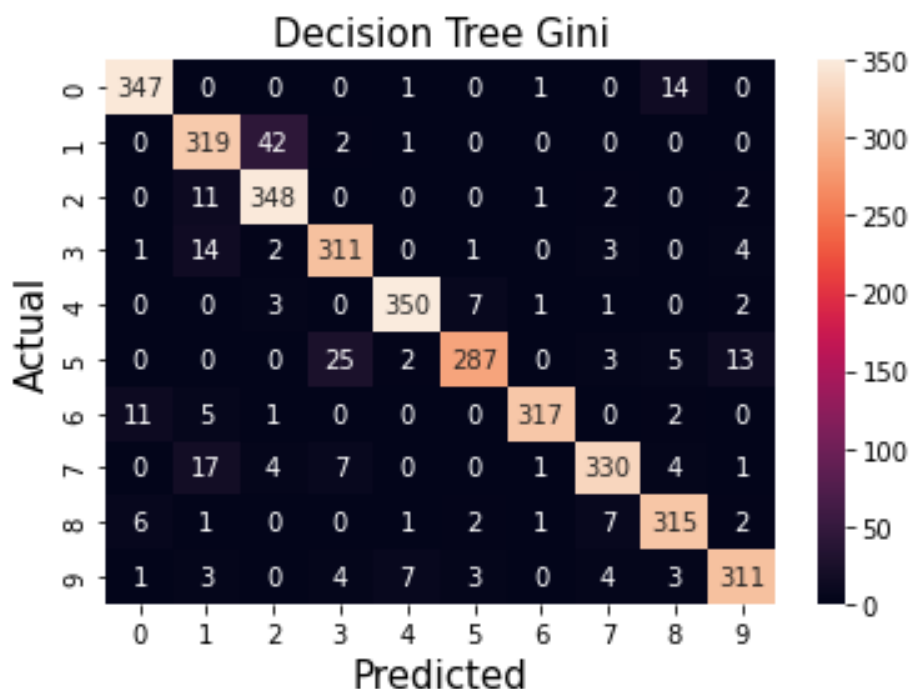
## Part2: Programming

**2(a):** we have tried both Decision tree with **Gini Index** method and **Entropy** method, and we found that Gini Index gave better accuracy...

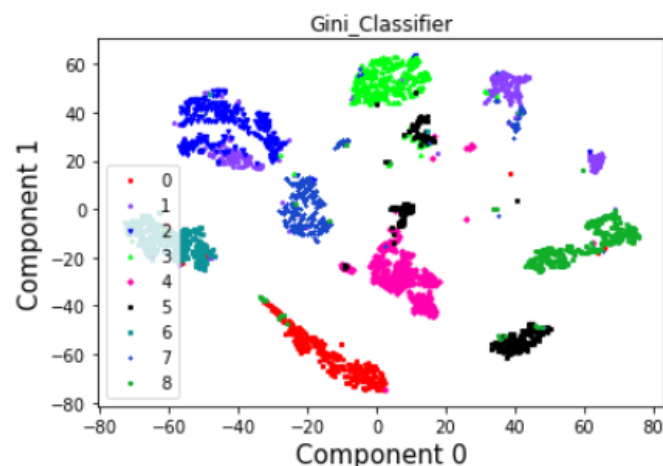
And after that we have plot the different **predicted classes** with different colors.

```
In [33]: print(Gini_report)
```

		precision	recall	f1-score	support
	0	0.95	0.96	0.95	363
	1	0.86	0.88	0.87	364
	2	0.87	0.96	0.91	364
	3	0.89	0.93	0.91	336
	4	0.97	0.96	0.96	364
	5	0.96	0.86	0.90	335
	6	0.98	0.94	0.96	336
	7	0.94	0.91	0.92	364
	8	0.92	0.94	0.93	335
	9	0.93	0.93	0.93	336
	accuracy			0.93	3497
	macro avg	0.93	0.92	0.93	3497
	weighted avg	0.93	0.93	0.93	3497



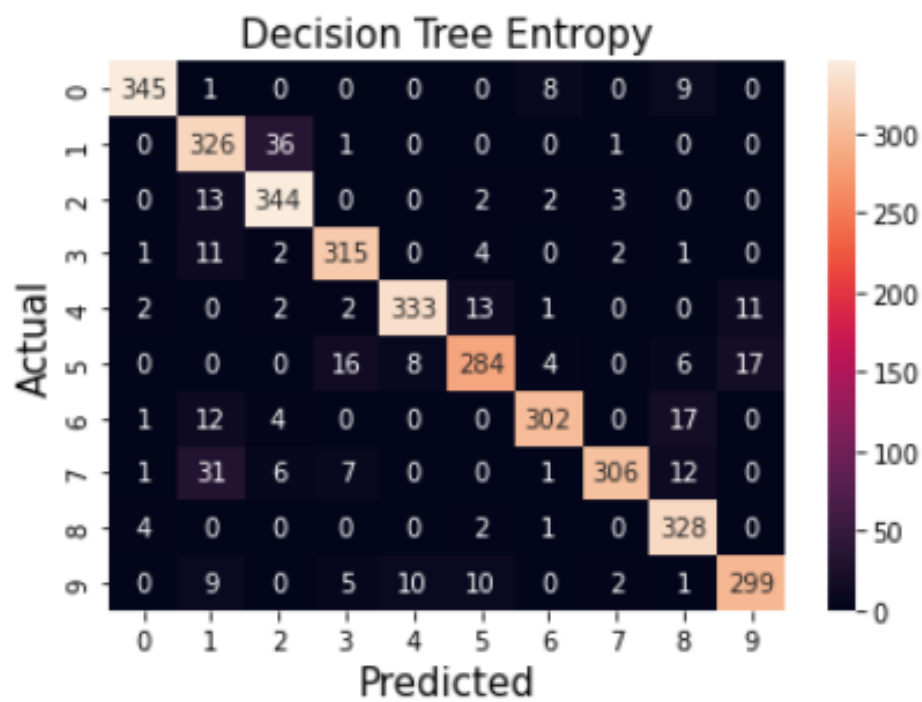
```
In [15]: # Plot Gini
XTsne = pd.concat([pd.DataFrame(XTsne), pd.DataFrame(Gini_Ypred)],axis=1 , ignore_index = True).astype(float)
Ginils = GetListOfClasses(9, XTsne, pd.DataFrame(XTsne).columns[2])
Labels = ['0','1','2','3','4','5','6','7','8']
PlotDataPoints(9, Ginils, 'Component 0', 'Component 1',Labels ,5, 'Gini_Classifier').show()
XTsne = T_SNE(XTest)
```



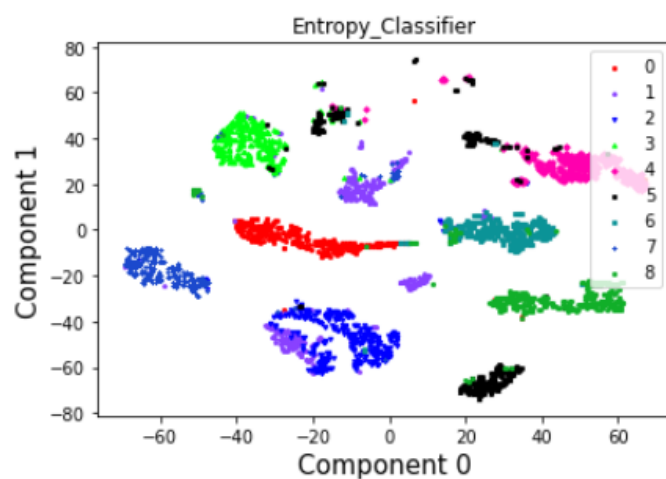
- Entropy

```
In [34]: print(Ent_report)
```

	precision	recall	f1-score	support
0	0.97	0.95	0.96	363
1	0.81	0.90	0.85	364
2	0.87	0.95	0.91	364
3	0.91	0.94	0.92	336
4	0.95	0.91	0.93	364
5	0.90	0.85	0.87	335
6	0.95	0.90	0.92	336
7	0.97	0.84	0.90	364
8	0.88	0.98	0.93	335
9	0.91	0.89	0.90	336
accuracy			0.91	3497
macro avg	0.91	0.91	0.91	3497
weighted avg	0.91	0.91	0.91	3497



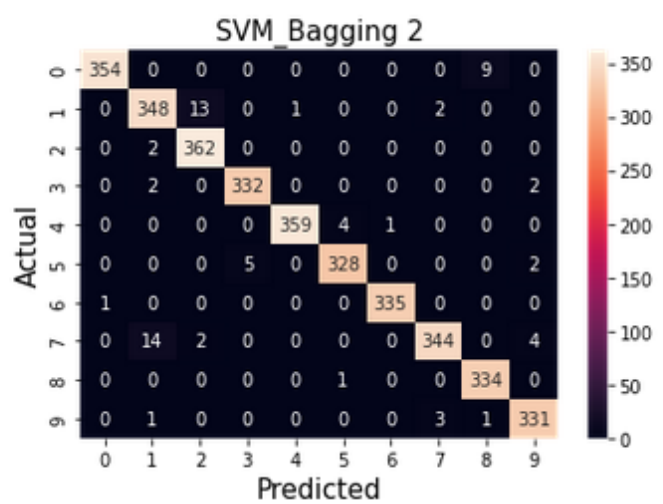
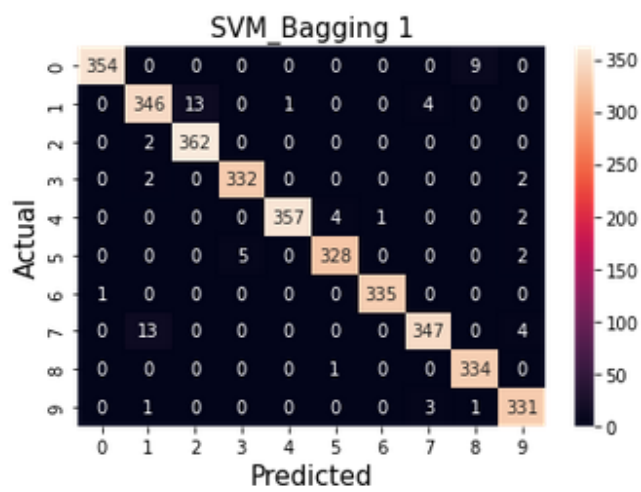
```
In [17]: # Plot Entropy
XTsne = pd.concat([pd.DataFrame(XTsne), pd.DataFrame(Ent_Ypred)],axis=1 , ignore_index = True).astype(float)
EntLs = GetListOfClasses(9, XTsne, pd.DataFrame(XTsne).columns[2])
PlotDataPoints(9, EntLs, 'Component 0', 'Component 1',Labels ,5, 'Entropy_Classifier').show()
```



**3(a):** here we applied `BaggingClassifier(base_estimator=SVC(), n_estimators=(we have tries 1 and 2))`

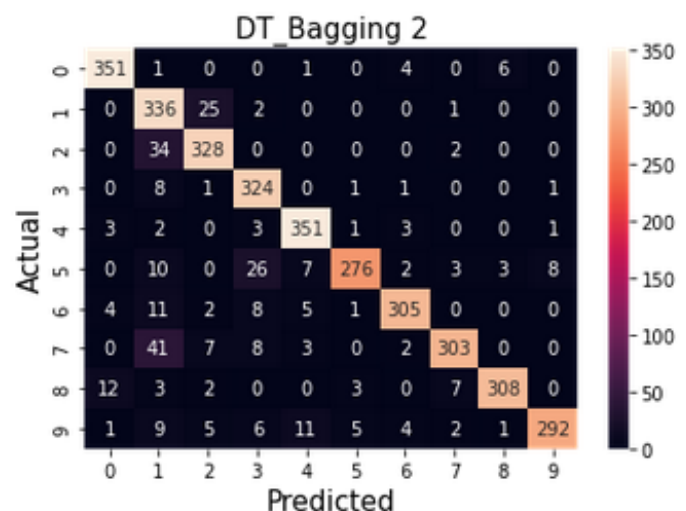
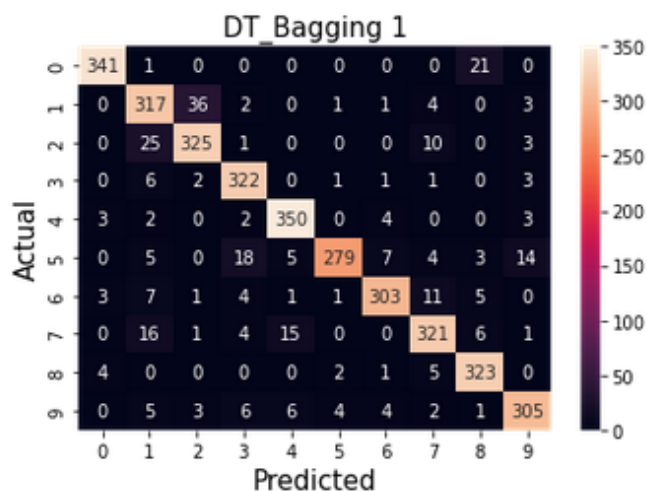
so, the frist time the `n_estimators` was = 1 and the second time, `n_estimators` was = 2.

SVM Bagging: 1					SVM Bagging: 2				
	precision	recall	f1-score	support		precision	recall	f1-score	support
0	1.00	0.98	0.99	363	0	1.00	0.98	0.99	363
1	0.95	0.95	0.95	364	1	0.95	0.96	0.95	364
2	0.97	0.99	0.98	364	2	0.96	0.99	0.98	364
3	0.99	0.99	0.99	336	3	0.99	0.99	0.99	336
4	1.00	0.98	0.99	364	4	1.00	0.99	0.99	364
5	0.98	0.98	0.98	335	5	0.98	0.98	0.98	335
6	1.00	1.00	1.00	336	6	1.00	1.00	1.00	336
7	0.98	0.95	0.97	364	7	0.99	0.95	0.96	364
8	0.97	1.00	0.98	335	8	0.97	1.00	0.98	335
9	0.97	0.99	0.98	336	9	0.98	0.99	0.98	336
accuracy			0.98	3497	accuracy			0.98	3497
macro avg	0.98	0.98	0.98	3497	macro avg	0.98	0.98	0.98	3497
weighted avg	0.98	0.98	0.98	3497	weighted avg	0.98	0.98	0.98	3497



- here we did the same thing but we have changed the base estimator to ...  
**BaggingClassifier(base\_estimator= DecisionTreeClassifier(), n\_estimators=(we have tries 1 and 2))**  
so, the frist time the **n\_estimators was = 1** and the second time, **n\_estimators was = 2**.

Decision Tree Bagging: 1					Decision Tree Bagging: 2				
	precision	recall	f1-score	support		precision	recall	f1-score	support
0	0.97	0.94	0.96	363	0	0.95	0.97	0.96	363
1	0.83	0.87	0.85	364	1	0.74	0.92	0.82	364
2	0.88	0.89	0.89	364	2	0.89	0.90	0.89	364
3	0.90	0.96	0.93	336	3	0.86	0.96	0.91	336
4	0.93	0.96	0.94	364	4	0.93	0.96	0.95	364
5	0.97	0.83	0.90	335	5	0.96	0.82	0.89	335
6	0.94	0.90	0.92	336	6	0.95	0.91	0.93	336
7	0.90	0.88	0.89	364	7	0.95	0.83	0.89	364
8	0.90	0.96	0.93	335	8	0.97	0.92	0.94	335
9	0.92	0.91	0.91	336	9	0.97	0.87	0.92	336
accuracy					accuracy			0.91	3497
macro avg					macro avg	0.92	0.91	0.91	3497
weighted avg					weighted avg	0.91	0.91	0.91	3497



**3(b):** to find the best number of estimators we have used a for loop to iterate on this range [10,200] to try as many options as we can, so we go for range(10,201,10), that's mean that we will have 20 different accuracies, and after that we have sorted these accuracies to know which n\_estimators values will give use the highest accuracies.

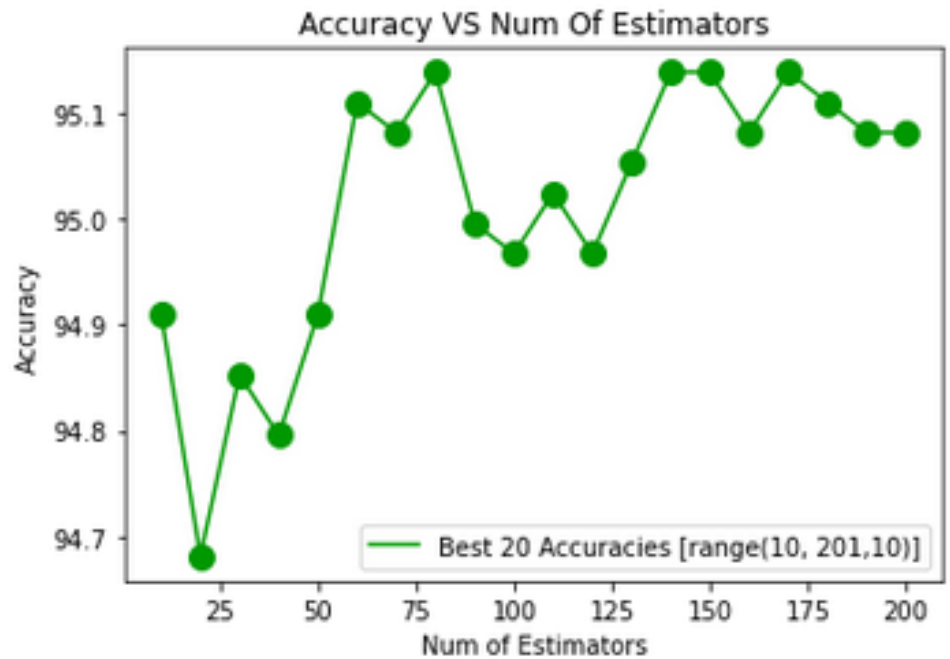
```
In [20]: # 3(b) ----- #Bagging
# Decision Tree
DT_Acc = []
nofEst = []
for numOfEst in range(10, 201,10):
    DT_estimator = BaggingClassifier(base_estimator=DecisionTreeClassifier(), n_estimators=numOfEst, random_state=0).fit(XTrain,
    DT_Ypred = DT_estimator.predict(XTest)
    DT_Acc.append(AccuracyTest(Ytest, DT_Ypred))
    nofEst.append(numOfEst)

EstandAcc = pd.concat([pd.DataFrame(DT_Acc), pd.DataFrame(nofEst)],axis=1 , ignore_index = True).astype(float)
EstandAcc = EstandAcc.sort_values(by=[0],ascending=False)
```

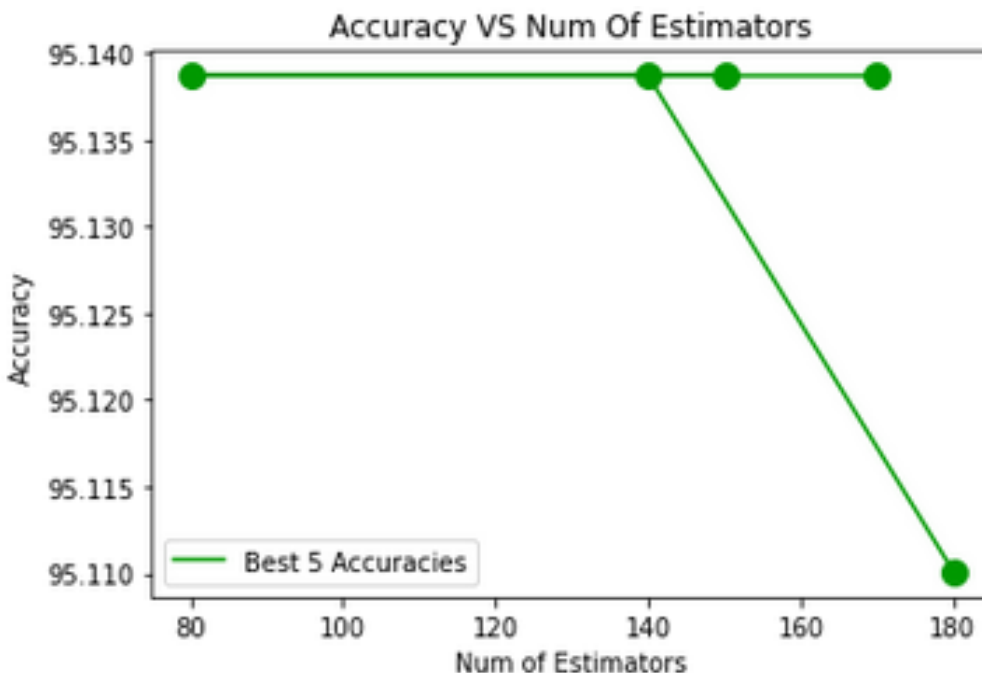
- Best values for n\_estimators that gave us the highest accurices.

In [35]: `print(EstandAcc)`

	0	1
16	95.138690	170.0
7	95.138690	80.0
14	95.138690	150.0
13	95.138690	140.0
17	95.110094	180.0
5	95.110094	60.0
19	95.081498	200.0
18	95.081498	190.0
6	95.081498	70.0
15	95.081498	160.0
12	95.052902	130.0
10	95.024307	110.0
8	94.995711	90.0
11	94.967115	120.0
9	94.967115	100.0
4	94.909923	50.0
0	94.909923	10.0
2	94.852731	30.0
3	94.795539	40.0
1	94.681155	20.0



- Accuracies VS n\_estimators (Full DataFrame)



- Accuracies VS n\_estimators (Best 5 (4 of them are have the same Accuracy))

**4(a):** here we did the same idea like **3(b)** to tune the `n_estimators` values from this range [10,200].

and we found that the best 4 values for `n_estimators` is [200,160,150,140].

```
In [23]: # 4(a) ----- #Boosting
# Tuning the number of estimators Parameter
Boosting_Acc = []
numOfEst = []
for i in range(10,201,10):
    Boosting_estimator = GradientBoostingClassifier(n_estimators=i, random_state=0).fit(XTrain, YTrain)
    Boosting_Ypred = Boosting_estimator.predict(XTest)
    Boosting_Acc.append(AccuracyTest(Ytest, Boosting_Ypred))
    numOfEst.append(i)

Boosting_Est = pd.concat([pd.DataFrame(Boosting_Acc), pd.DataFrame(numOfEst)],axis=1 , ignore_index = True).astype(float)
Boosting_Est = Boosting_Est.sort_values(by=[0],ascending=False)
print(Boosting_Est.iloc[:4,1])

19    200.0
15    160.0
14    150.0
13    140.0
Name: 1, dtype: float64
```

- here we did the same idea like **3(b)** to tune the `learning_rate` values from this range [0.1 -> 0.9].

and we found that the best 4 values for is [0.2,0.3,0.1,0.7].

```
In [24]: # Tuning learning rate parameter
Lr_rate = np.array([0.1,0.2,0.3, 0.4, 0.5, 0.6,0.7,0.8,0.9])
Boosting_Acc = []
Lr = []
for i in Lr_rate:
    Boosting_estimator = GradientBoostingClassifier(learning_rate=i, random_state=0).fit(XTrain, YTrain)
    Boosting_Ypred = Boosting_estimator.predict(XTest)
    Boosting_Acc.append(AccuracyTest(Ytest, Boosting_Ypred))
    Lr.append(i)

Boosting_Lr = pd.concat([pd.DataFrame(Boosting_Acc), pd.DataFrame(Lr)],axis=1 , ignore_index = True).astype(float)
Boosting_Lr = Boosting_Lr.sort_values(by=[0],ascending=False)
print(Boosting_Lr.iloc[:4,1])

1    0.2
2    0.3
0    0.1
6    0.7
Name: 1, dtype: float64
```



- This for loop for finding the best combination between the best values of the parameters.

```
In [25]: # Train GradientBoostingClassifier
Boosting_Acc = []
est = []
lr = []
estLS = list(Boosting_Est.iloc[:4,1])
LrLS = list(Boosting_Lr.iloc[:4,1])

for i in range(len(Boosting_Est.iloc[:4,1])):
    for j in range(len(Boosting_Lr.iloc[:4,1])):
        Boosting_estimator = GradientBoostingClassifier(n_estimators=trunc(estLS[i]),learning_rate=LrLS[j], random_state=0).fit(Xtest, Ytest)
        Boosting_Ypred = Boosting_estimator.predict(XTest)
        Boosting_Acc.append(AccuracyTest(Ytest, Boosting_Ypred))
        print('Done')
        est.append(trunc(estLS[i]))
        lr.append(LrLS[j])

Boosting = pd.concat([pd.DataFrame(Boosting_Acc), pd.DataFrame(est), pd.DataFrame(lr)],axis=1 , ignore_index = True).astype(float)
Boosting = Boosting.sort_values(by=[0],ascending=False)
print(Boosting.iloc[:4])
```

		0	1	2
2	96.568487	200.0	0.1	
1	96.511295	200.0	0.3	
6	96.482699	160.0	0.1	
10	96.482699	150.0	0.1	

- Best combination of the parameters together that gave the highest accuracy.

We found that the best combinations are...

Learning\_rate = [0.1,0.3]

n\_estimators = [160,150,200]

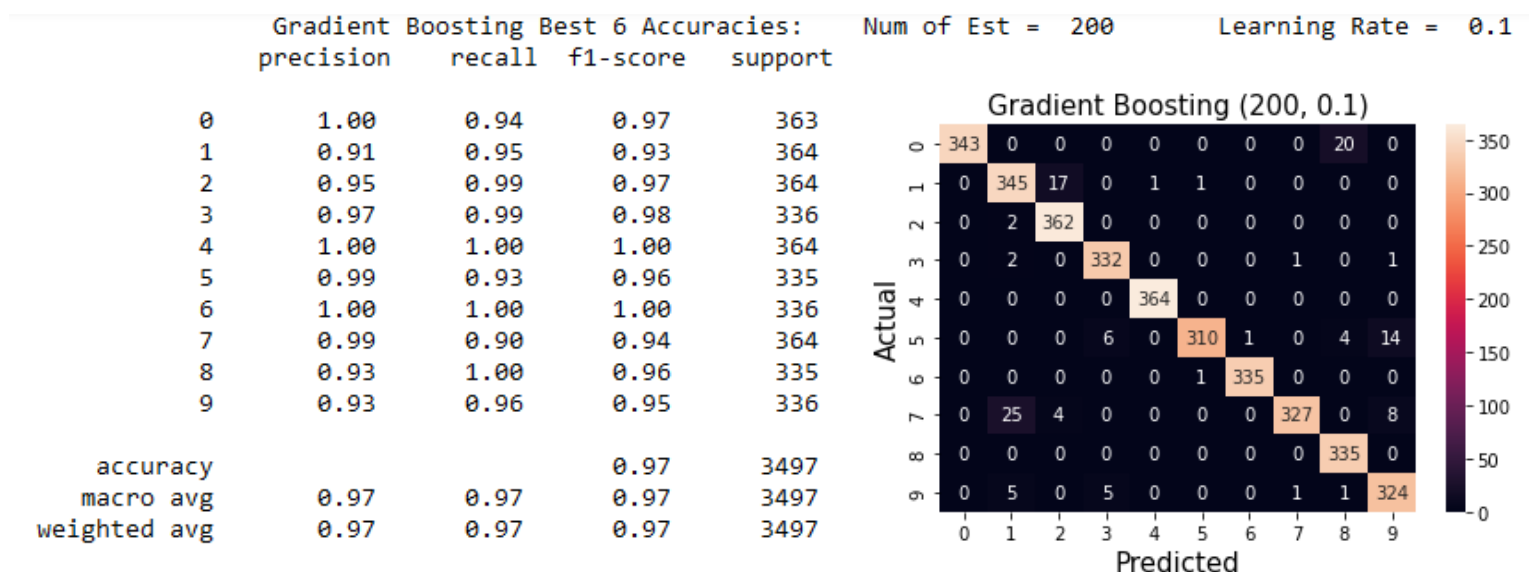
with these combinations we have train gradient boost again and we have to obtain **6 different Confusion matrices**, and **6 different classification\_reports**.

```
for i in range(len(best_estLS)):
    for j in range(len(best_LrLS)):
        Boosting_estimator = GradientBoostingClassifier(n_estimators=trunc(best_estLS[i]),learning_rate=best_LrLS[j], random_state=0).fit(Xtest, Ytest)
        Boosting_Ypred = Boosting_estimator.predict(XTest)

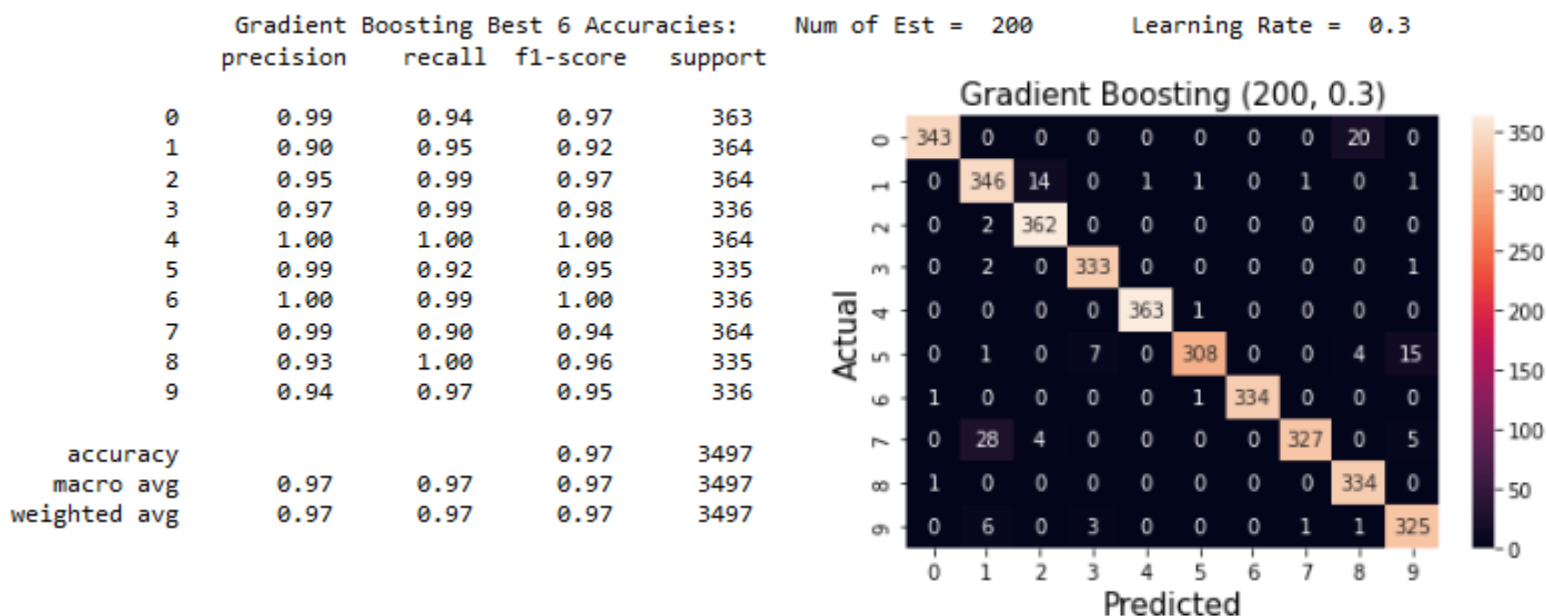
        GB_report = classification_report(Ytest, Boosting_Ypred)
        GB_reports.append(DT_report)
        print('\t\tGradient Boosting Best 6 Accuracies:', '\tNum of Est = ',trunc(best_estLS[i]), '\tLearning Rate = ',best_LrLS[j])
        GB_cf = ConfusionMatrix(Ytest, Boosting_Ypred)
        PLOT_ConfusionMatrix(GB_cf, f'Gradient Boosting {trunc(best_estLS[i]),best_LrLS[j]}')
        GB_Acc.append(AccuracyTest(Ytest, Boosting_Ypred))
        GB_Lr.append(best_LrLS[j])
        GB_Est.append(trunc(best_estLS[i]))
        print('Done')
```



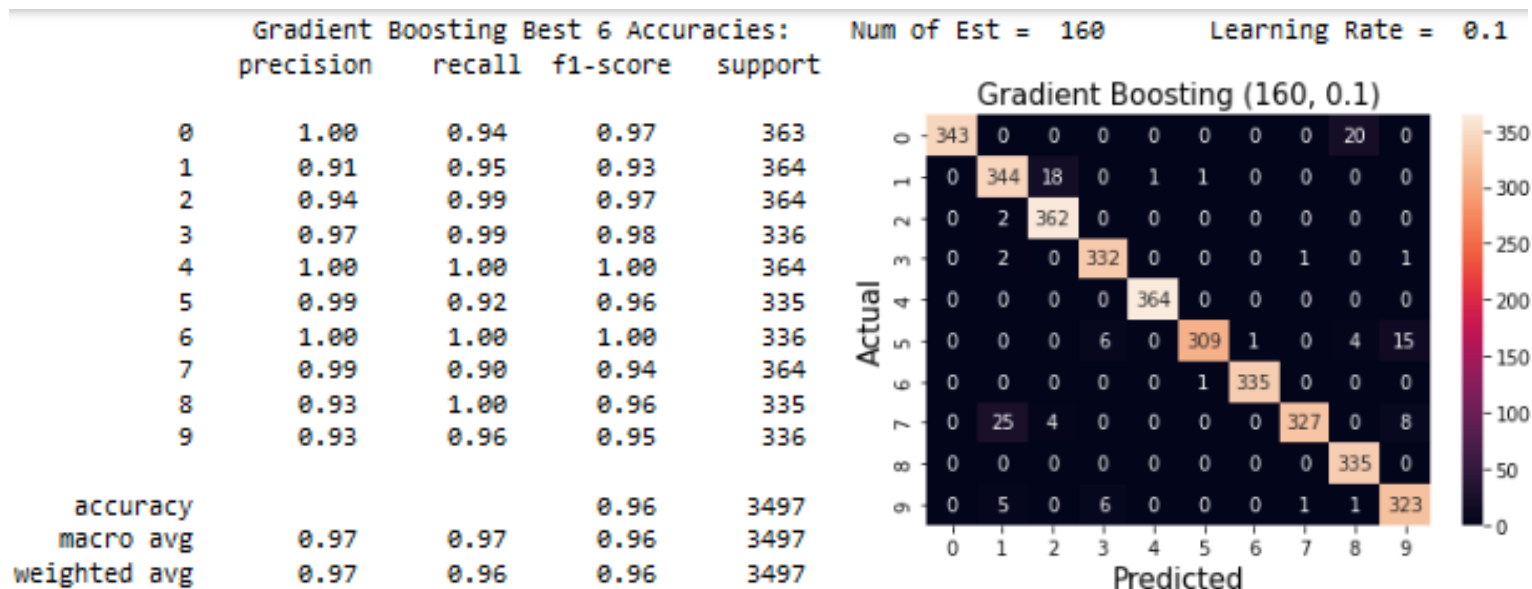
- First model (num of est = 200, lr\_rate = 0.1)



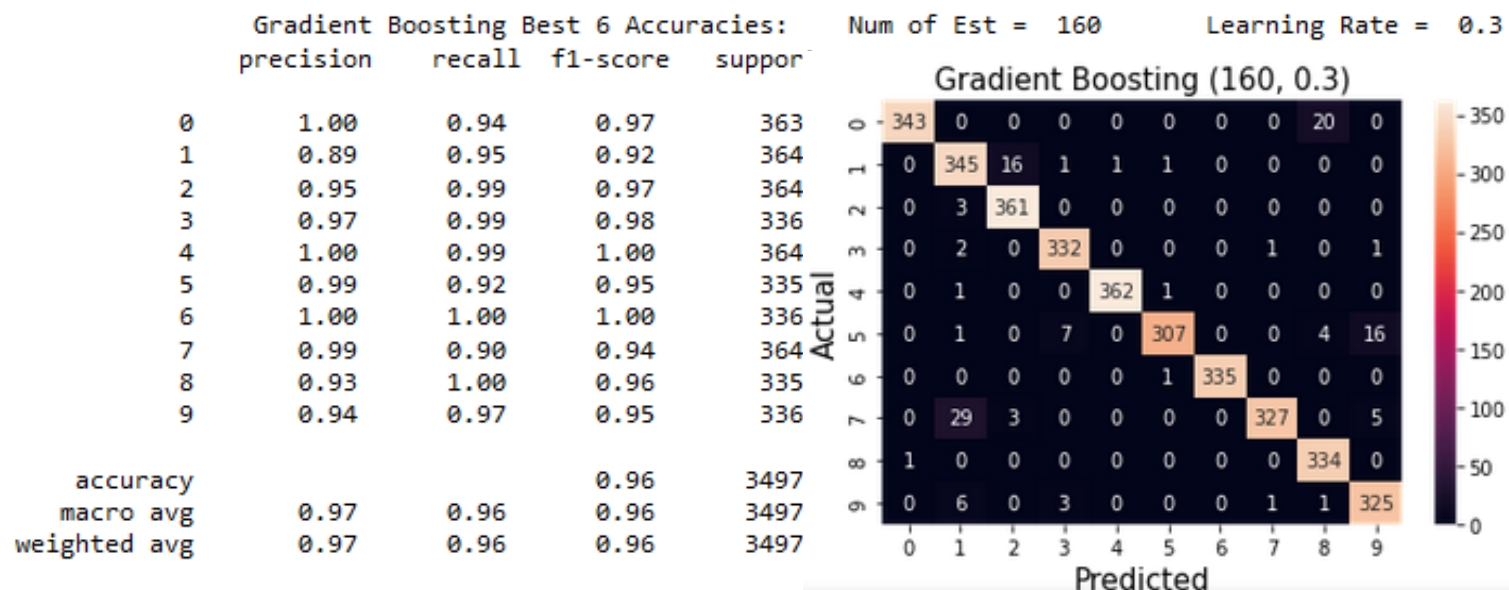
- Second model (num of est = 200, lr\_rate = 0.3)



- Third model (num of est = 160, lr\_rate = 0.1)



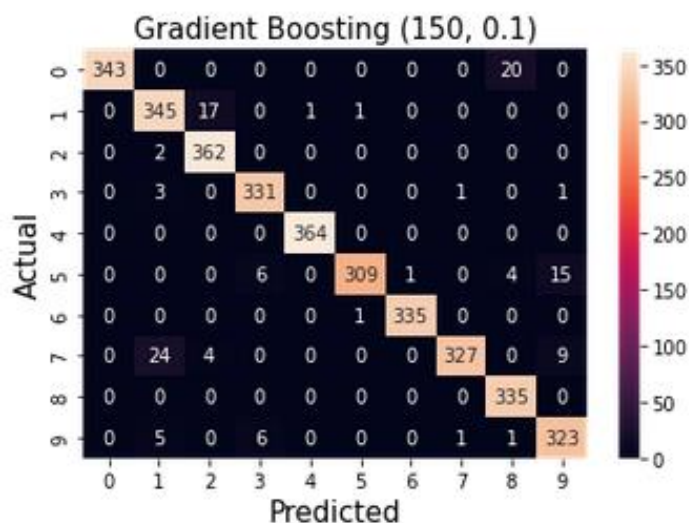
- Fourth model (num of est = 160 , lr\_rate = 0.3)



- Fifth model (num of est = 150 , lr\_rate = 0.1)**

	Gradient Boosting Best 6 Accuracies:			
	precision	recall	f1-score	support
0	1.00	0.94	0.97	363
1	0.91	0.95	0.93	364
2	0.95	0.99	0.97	364
3	0.97	0.99	0.97	336
4	1.00	1.00	1.00	364
5	0.99	0.92	0.96	335
6	1.00	1.00	1.00	336
7	0.99	0.90	0.94	364
8	0.93	1.00	0.96	335
9	0.93	0.96	0.94	336
accuracy			0.96	3497
macro avg	0.97	0.97	0.96	3497
weighted avg	0.97	0.96	0.96	3497

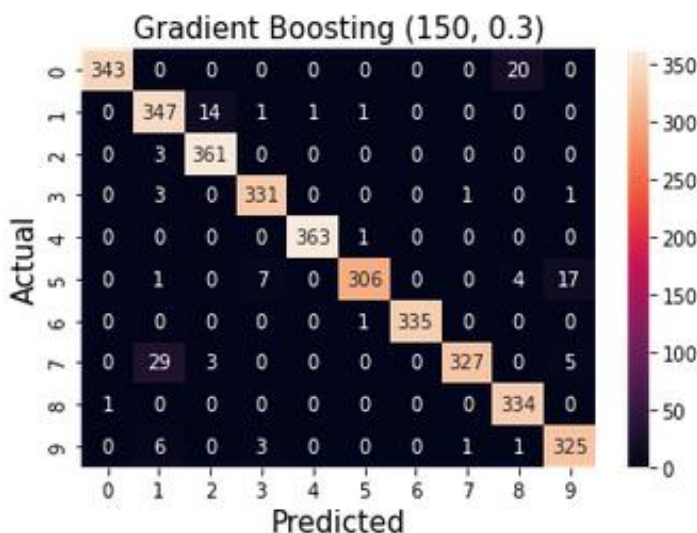
Num of Est = 150      Learning Rate = 0.1



- Sixth model (num of est = 150 , lr\_rate = 0.3)**

	Gradient Boosting Best 6 Accuracies:			
	precision	recall	f1-score	support
0	1.00	0.94	0.97	363
1	0.89	0.95	0.92	364
2	0.96	0.99	0.97	364
3	0.97	0.99	0.98	336
4	1.00	1.00	1.00	364
5	0.99	0.91	0.95	335
6	1.00	1.00	1.00	336
7	0.99	0.90	0.94	364
8	0.93	1.00	0.96	335
9	0.93	0.97	0.95	336
accuracy			0.96	3497
macro avg	0.97	0.96	0.96	3497
weighted avg	0.97	0.96	0.96	3497

Num of Est = 150      Learning Rate = 0.3

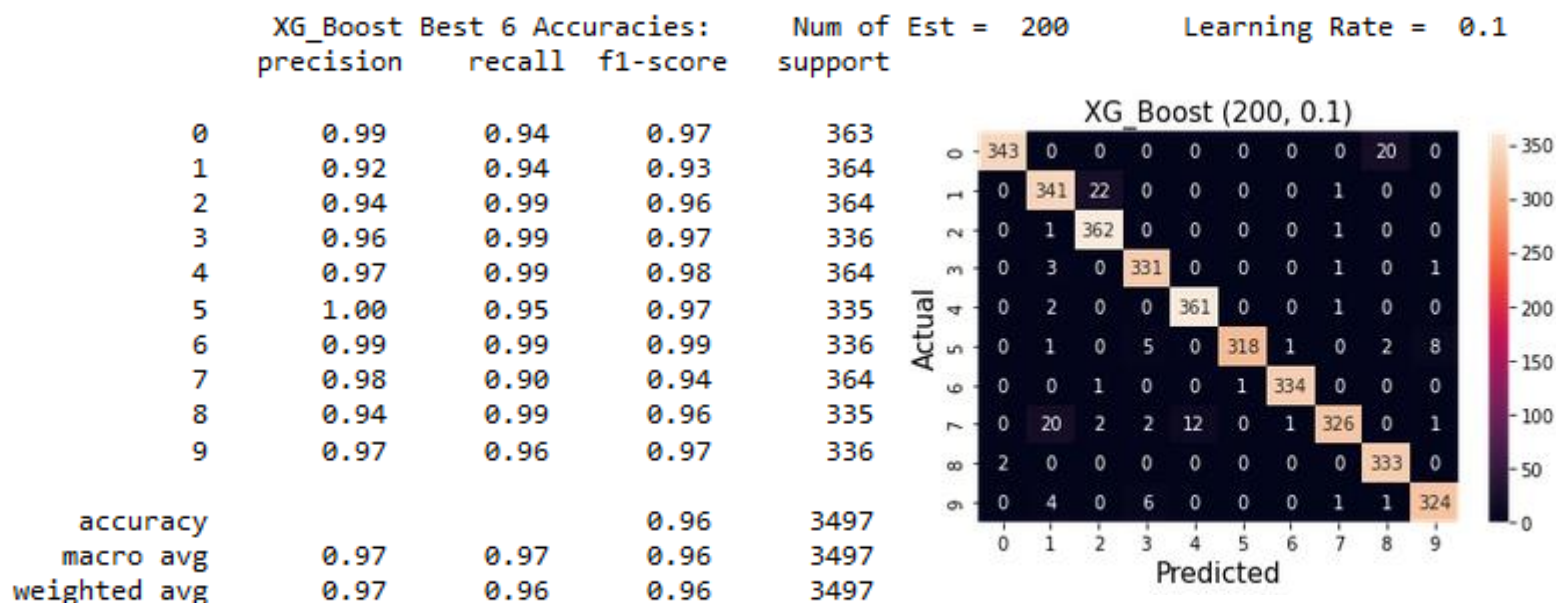


**4(b):** now we will use the same combinations to train XG\_Boost models.

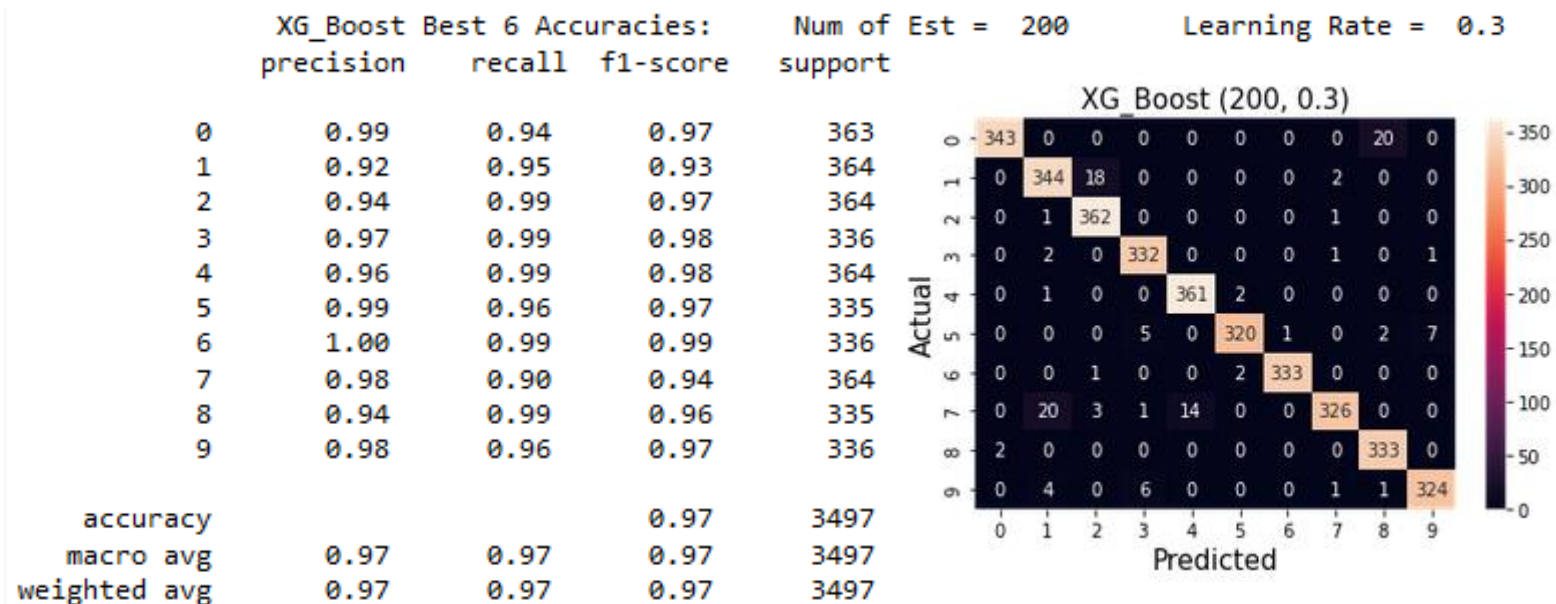
Learning\_rate = [0.1,0.3]

n\_estimators = [160,150,200]

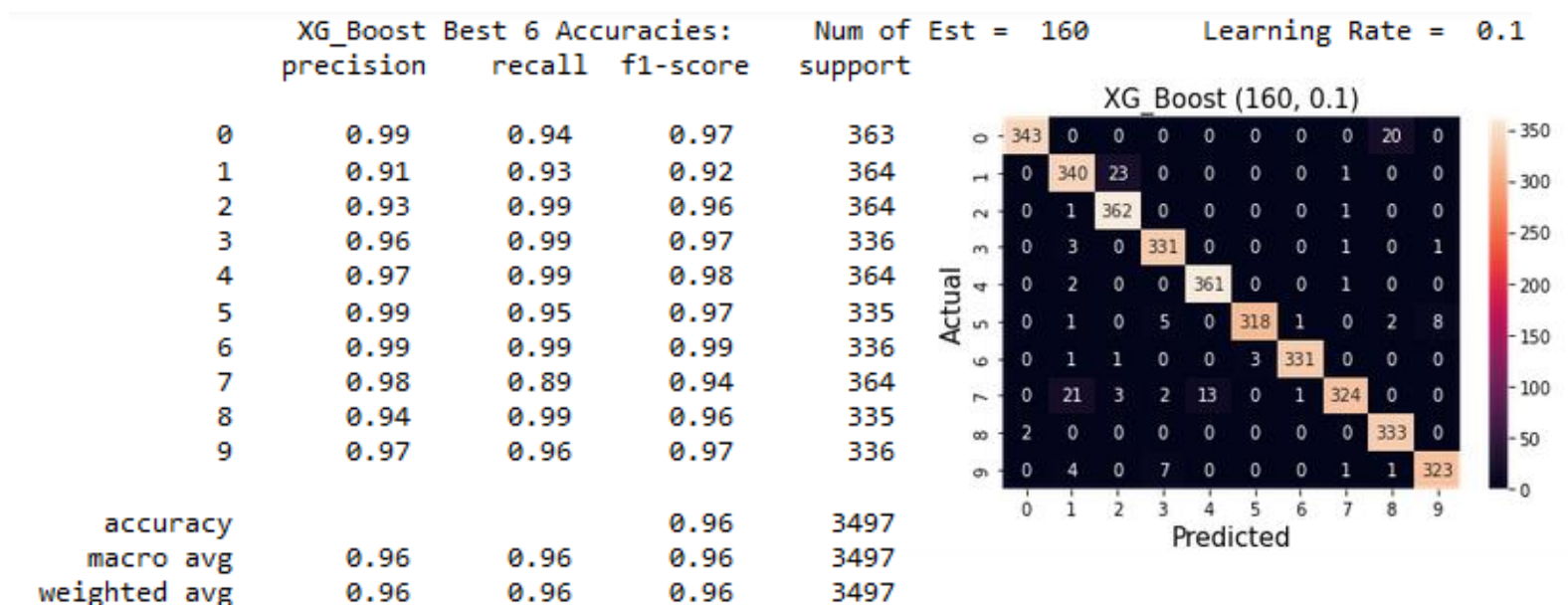
- First XG\_Boost model (num of est = 200, lr\_rate = 0.1)



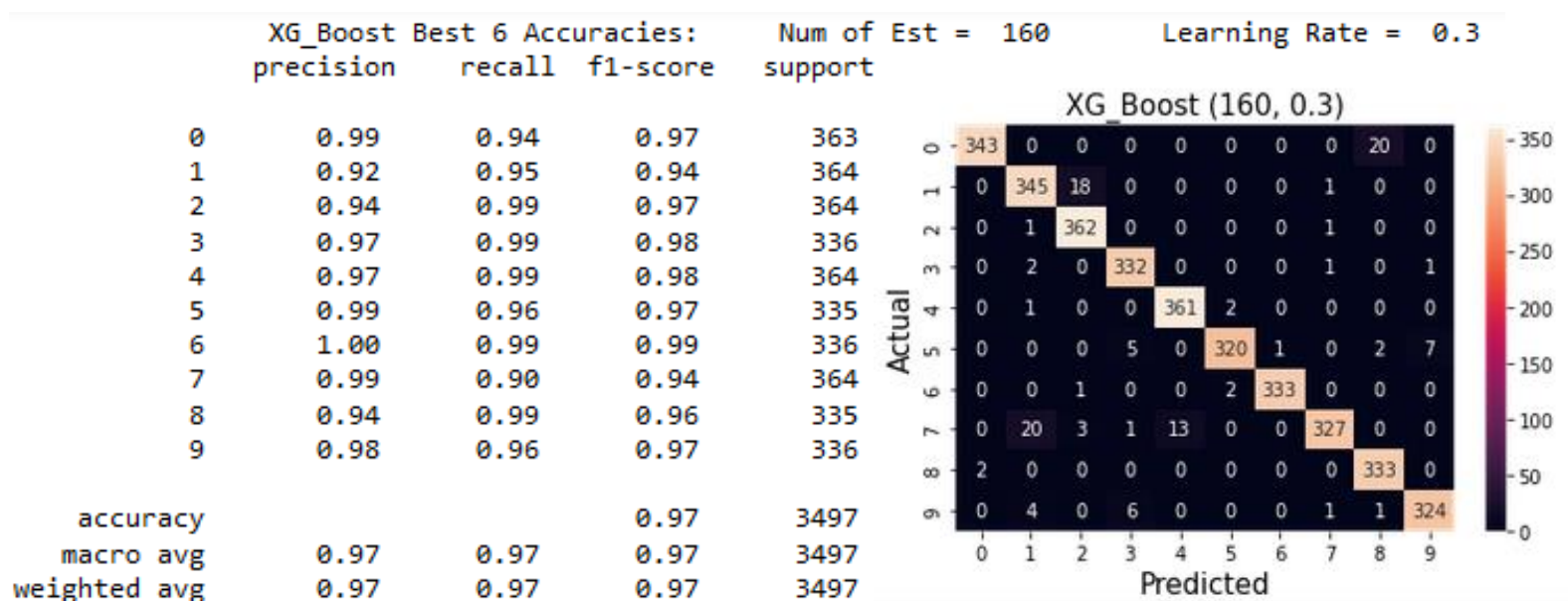
- Second XG\_Boost model (num of est = 200, lr\_rate = 0.3)



- Third XG\_Boost model (num of est = 160, lr\_rate = 0.1)

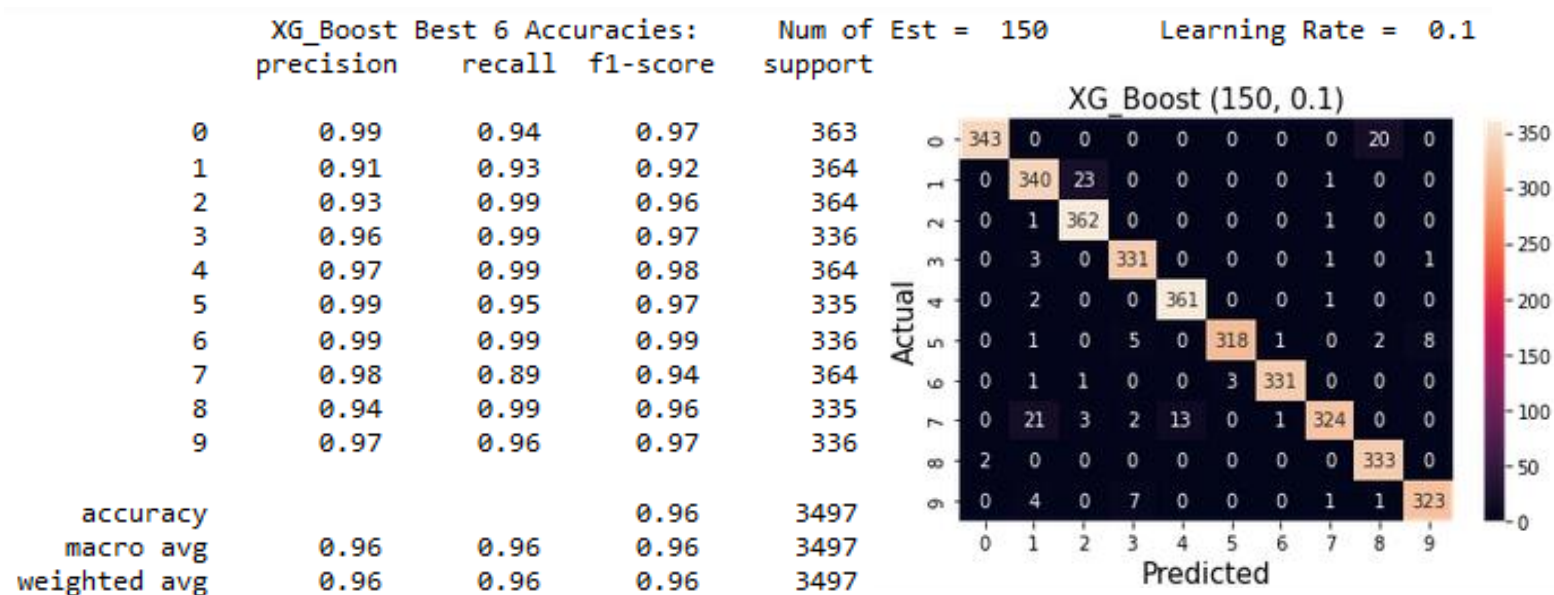


- Fourth XG\_Boost model (num of est = 160, lr\_rate = 0.3)

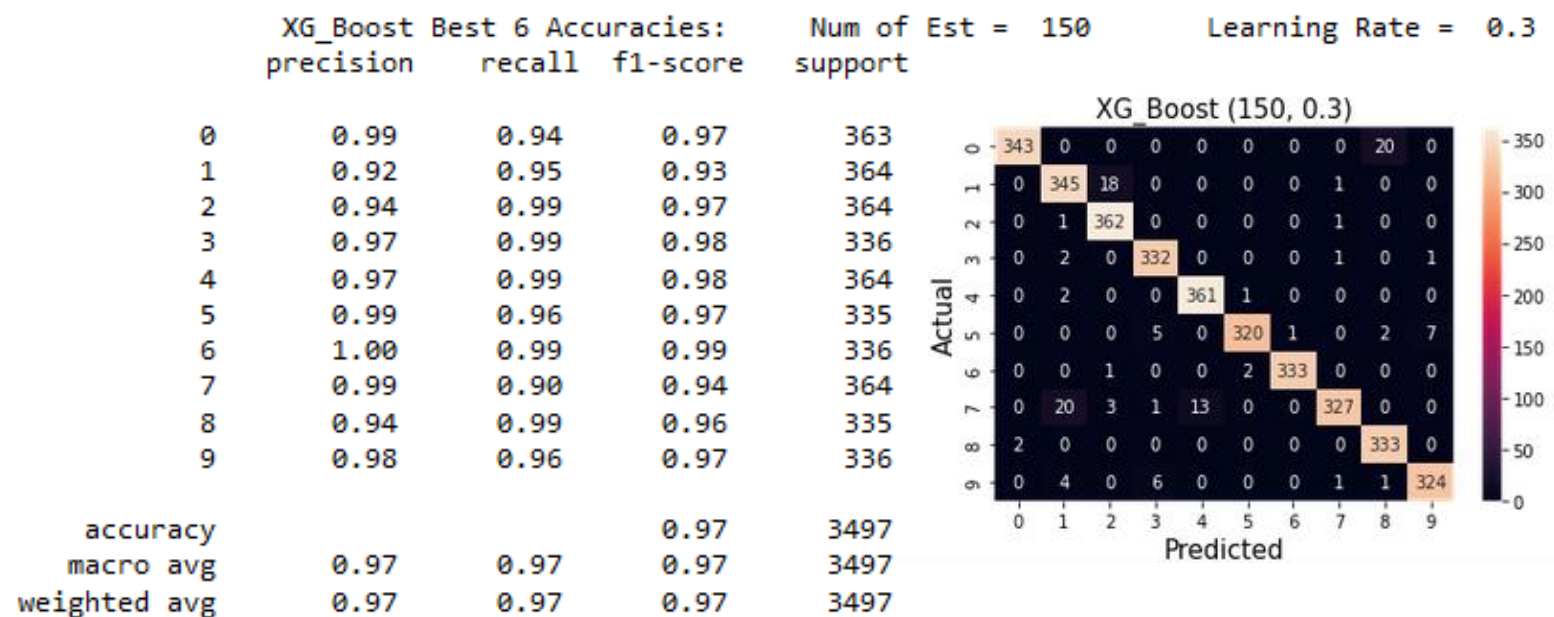




- Fifth XG\_Boost model (num of est = 150, lr\_rate = 0.1)



- Sixth XG\_Boost model (num of est = 150, lr\_rate = 0.3)



**4(c):** here we have measured the time that each method (**XG\_Boost** and **Gradient boost**) takes to train the six model, and we found that **XG\_Boost is faster by Approximately 8 times.**

**and also, XG\_Boost gave better accuracies.**

Done

19.37900400161743

	0	1	2
3	96.654275	160.0	0.3
5	96.654275	150.0	0.3
1	96.597083	200.0	0.3
0	96.454104	200.0	0.1
2	96.253932	160.0	0.1
4	96.253932	150.0	0.1

Done

153.97868871688843

	0	1	2
0	96.568487	200.0	0.1
1	96.511295	200.0	0.3
2	96.482699	160.0	0.1
4	96.482699	150.0	0.1
5	96.425508	150.0	0.3
3	96.396912	160.0	0.3

#### • XG\_Boost Time and accuracies

#### Gradient boost Time and accuracies

- We believe that the both evaluation metrics are important (**accuracy** and **confusion matrix**), but in this case and this dataset (pen digits) we think that **confusion matrix would be more important**, because it will tell us which numbers mislead the model and why...
- for example, when we analyze this confusion matrix, we will find that the model misled the digit '7' and it predicted it as '1' (20 times) and that give us indicator that '7' is kind of similar to '1' in handwritten digits.
- Based on question 3 and 4, we have notice Bagging is the best option to avoid over-fitting.

