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3/14/2020

Assignment 7

Instructions.

1. Due April 16.

2. This is a team assignment. Work in teams of 3-4 students. Submit one assignment per team, with the names of all students making the team.

3. You will submit on Blackboard one single pdf file with the solutions to all exercises. For this you'll take the .tex file for this assignment and modify it. In the box above replace Ann, Bob, Charlie with your names. Write down your anwers for each question after Answer:.

   For editing the above document with Latex, see the template posted on the course website.

   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.tex` and

   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.pdf`

   To append in the latex file a .jpg file (for a photo; for example, in case you draw a picture by hand and take a photo of it with your phone camera), use

   ```
   \includegraphics[angle=270,origin=c,width=\linewidth]{file.jpg}
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   The parameter angle=270 is for rotating the photo, and you may have to change 270 to whatever angle works for your photo.

1 Show that the language A = {hM1i | L(M1) contains the string 0} is not decidable.

Hint: As we did several times in class, you need to reduce A_{TM} to A. You need to transform an input hM, wi for A_{TM} into an input hM1i for the problem A given above, so that hM, wi is in A_{TM} if and only if hM1i is in A (in other words, your transformation is such that yes-instances for A_{TM} become yes-instances for A, and no-instances for A_{TM} become no-instances for A). To present the Turing machine M1, describe in plain English what M1 does on an arbitrary input y.

Present your proof in the style of the proof of Th. 5.3 (page 219, in the 3rd edition, and page 191, in the 2nd edition).

Answer:

Let R be a TM that decides H_{TM} and construct TM K such that it decides A_{TM}. The algorithm is shown below:

-On input <m1, w1>, m1 denotes a TM & w1 is a string.

1. Construct TM m2

M2= on input x

1.If x has form 1_n, accept

2.If c is not in this form, run m1 with input w1 & accept if m1 would accept w1.

2.Run R with input m

3.If R would accept, accept it, if R rejects then reject.

Explanation:

If an algorithm exists, then another algorithm A could be written so that when B is given as input decides if B accepts all strings.

-If B cannot accept all strings, A outputs some program A(B) that doesn't accept all strings.

If B doesn't accept all, A omits some program that would accept all strings.

-There is a TM which doesn't allow any string

-There is a TM that accepts every string

-This means having non trivial property which functions computed by A's elements.

-Hence, A is not decidable

2 Show that the language

B = {hMi | the Turing machine M halts on at least one of the inputs 0,1, or 00}

is Turing recognizable.

Hint: You need to describe an algorithm S that on input < M > has the following properties:

(a) if M accepts at least one of {0, 1, 00}, then S accepts,

(b) if M does not accept any of the three strings, then S rejects or loops.

Of course S will have to run simulations of M on various inputs, and the issue is to organize those simulations in the right way to obtain (a) and (b). The main difficulty is that M may loop on some inputs, and, therefore, the algorithm S must have a way to avoid being stuck on a simulation of M on an input x on which it loops, so it has to use parallelism (see the proof of Theorem 4.22 in the textbook). You need to describe with sufficient details how this parallelism can be achieved by S.

Answer:.
M=input x

1.Rub both M and S simotaneous

1. If<M> is not endcoding some of TM
1.then return reject

2.for i< I to ∞
Do counter:=0
For j < 1 to i
If counter >=3
Return rejject

Running two machines simultaneously means that M has two tapes. Each string is either in M or S.

So M is a decider for A and Btherefore A is decidable.

3 A and B denote languages, and $\overline{A}$ is the complement of A, and $\overline{B}$ is the complement of B. A ∩ B is the intersection of A and B. Consider the following statements:

(a) A is context-free.

(b) B is Turing-

recognizable. (c) A is

context-free.

(d) $\overline{A}$ is decidable.

(e) A ∩ B is decidable.

(f) A ∩ B is Turing

recognizable. (g) B is Turing

recognizable.

Answer the following questions (TRUE or FALSE) and provide short justifications:

(1) Does (a) imply (c)?

- False; complement of context-free language can be context free or not

(2) Does (a) imply (d)?

- True; if language A is context free, then its decidable. Decidable language is closed under complementation. So L' is decidable

(3) Does (a) and (b) imply (e)?

- Fase

(4) Does (a) and (b) imply (f)?

- True

(5) Does (b) imply (g)?

- False; turning recognizable languages are not closed under complement

Answer: