Henry

- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$

$= \sum^{\infty} (-1)^{n+1} \frac{x^n}{n} \qquad x \in (-1, 1]$

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad x \in [-1, 1]$

- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

$= \sum_{n=0}^{\infty} \frac{x^k}{k!}$ or $\frac{1}{n!} \qquad x \in \mathbb{R}$

$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$

$= \sum_{n=0}^{\infty} x^n \qquad x \in (-1, 1)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$= \sum_{k=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \qquad x \in \mathbb{R}$

Henry Osei

3.

Yes the sequence can converge because if $\{a_n\}$ was to be divergent aswell as $\{b_n\}$ to be divergent the sequence $\{a_n + b_n\}$ would converge insted of being divergent. Whenever a sequence is divergent it can always converge. That is where the divergent sequence has an oppurtunity to make the sequences added together converge.

1 $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$: <u>diverges</u>

$\lim_{n\to\infty} \left(\frac{\sin(1/n)}{1/n}\right)$

$= \lim_{n\to a} (n\sin(1/n))$

$(\sin(1/n))' = \frac{-\cos(1/n)}{n^2}$

$\frac{d}{dn}(\sin(1/n))$

$= \frac{d}{du}(\sin(u)) \frac{d}{dn}\left(\frac{1}{n}\right) = \frac{d}{du}(\sin(u))' = \cos(u)$

$\lim_{n\to\infty} \left(\frac{\frac{-\cos(\frac{1}{n})}{n^2}}{\frac{-1}{n^2}}\right) \cos\left(\frac{1}{n}\right)$

$= \lim_{n\to\infty} (\cos(1/n))$

$= \lim_{u\to 0} (\cos(u)) = 1$, which correlates to

$\lim_{t\to 0} \frac{\sin t}{t} = 1$

Henry Osei

2.

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{(2n+1)}$$

Ratio Test

If $L<1$ then Σ converges

If $L>1$, then Σ diverges

$$\lim_{\to\infty} \left(\left| \frac{(-1)^{(n+1)}(x-3)^{(n+1)}}{2(n+1)+1} \right| \right) : -\frac{(x-3)(2n+1)}{2n+3}$$

$$\lim_{n\to\infty} \left(\left| -\frac{(x-3)(2n+1)}{2n+3} \right| \right)$$

$$= -|(x-3)| \cdot \lim_{n\to\infty} \left(\left| \frac{2n+1}{2n+3} \right| \right) \Rightarrow |-(x-3)| \cdot 1 = |x-3|$$

$\therefore$ sum converges for $L<1$

$-1 < x-3 < 1$

$x-3 > -1 \therefore x > 2$

$2 < x \leq 4$

$(2, 4]$

Henry Ost

10.

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{-2x}{1-x^2} = \frac{2x}{x^2-1}$$

$$s = \int_0^{1/4} \sqrt{1 + \left(\frac{2x}{x^2-1}\right)^2}\, dx$$

$$s = \int_0^{1/4} \sqrt{\frac{(x^2-1)^2 + (2x)^2}{(x^2-1)^2}}$$

$$s = \int_0^{1/4} \sqrt{\frac{x^4 + 2x^2 + 1}{x^2 - 1}}\, dx$$

$$s = \int_0^{1/4} \frac{x^2+1}{x^2-1} \rightarrow s = \int_0^{1/4} \left(1 + \frac{2}{x^2-1}\right) dx$$

$$\frac{2}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \rightarrow 2 = A(x+1) + B(x-1)$$

$$s = \left[x + \ln\left|\frac{x-1}{x+1}\right| \right]_0^{1/4} = \left(\frac{1}{4} + \ln\left|\frac{1/4 - 1}{1/4 + 1}\right|\right) - \left(0 + \ln\left|\frac{0-1}{0+1}\right|\right)$$

$$\boxed{s = \frac{1}{4} + \ln\left(3/5\right)}$$

Henry Osei

9. $\int \tan^4(x)\,dx$

$\int \frac{\tan^3(x)}{3} - \int \tan^2(x)$

$\int \tan^2(x)\,dx$

$= \int -1 + \sec^2(x)\,dx$

$= -\int 1\,dx + \int \sec^2(x)\,dx$

$\int 1\,dx = x \qquad \int \sec^2(x)\,dx = \tan(x)$

$= -x + \tan(x)$

$= \frac{\tan^3(x)}{3} - (-x + \tan(x))$

$= \frac{\tan^3(x)}{3} + x - \tan(x)$

$= \cancel{\tfrac{1}{3}\tan^3(x) + x - \tan(x)}$

$\boxed{= \tfrac{1}{3}\tan^3(x) + x - \tan(x) + C}$

Henry Gei

8. $\int \sin(3x)\cos(5x)\,dx$

$= \dfrac{\int \sin(3x+5x) + \sin(3x-5x)\,dx}{2}$

$= \frac{1}{2} \cdot \int \sin(3x+5x) + \sin(3x-5x)\,d$

$= \frac{1}{2} \left(\int \sin(3x+5x)\,dx + \int \sin(3x-5x)\,dx \right)$

$\int \sin(3x+5x)\,dx$

$= \sin(u)\, \frac{1}{8}\,du = \frac{1}{8} \cdot \int \sin(u)\,du = \frac{1}{8}(-\cos(u)) =$

$\frac{1}{8}(-\cos(3x+5x)) = \frac{1}{8}\cos(8)$

$\int \sin(3x-5x)\,dx = \int -\frac{1}{2}\sin(u)\,du = \frac{1}{2} \cdot \int \sin(u)\,du = -\frac{1}{2}(-\cos(u))$

$= -\frac{1}{2}(-\cos(3x-5x))$

$= \frac{1}{2}\cos(2x)$

$= \frac{1}{2}\left[-\frac{1}{8}\cos(8x) + \frac{1}{2}\cos(2x) \right]$

$\boxed{= \frac{1}{2}\left(-\frac{1}{8}\cos(8x) + \frac{1}{2}\cos(2x) \right) + C}$

Henry Chu

7. $\int \frac{x-14}{(x+1)(2x-3)} dx$

$$\frac{x-14}{(x+1)(2x-3)} = \frac{a_0}{x+1} + \frac{a_1}{2x-3}$$

$$\frac{(x-14)(x+1)(2x-3)}{(x+1)(2x-3)} = \frac{a_0(x+1)(2x-3)}{x+1} + \frac{a_1(x+1)(2x-3)}{2x-3}$$

$$x-14 = a_0(2x-3) + a_1(x+1)$$

$$-1(-14) = a_0(2(-1)-3) + a_1 1((-1)+1)$$

$a_0 = 3,\ a_1 = -5$

$$\frac{3}{x+1} + \frac{(-5)}{2x-3}$$

$$= \int \frac{3}{x+1} dx - \int \frac{5}{2x-3} dx = \int \frac{3}{x+1} dx = 3\ln|x+1|$$

$$\int \frac{3}{x+1} dx = 3 \cdot \int \frac{1}{x+1} dx = 3 \cdot \int \frac{1}{u} du = 3\ln|u| = 3\ln|x+1|$$

$$\int \frac{5}{2x-3} dx = \frac{5}{2} \ln|2x-3|$$

$$\boxed{= 3\ln|x+1| - \frac{5}{2}\ln|2x-3| + C}$$

Henry Osei

4

$F(x) = 2^x$

Maclaurin series of $f(x)$

$$f(x) = f(0) + \frac{f'(0)}{1!} + \frac{f''(0)}{2!} \cdot (x) + \ldots$$

$f(0) = 2^0 = 1$

$f'(x) = (\ln 2)2^x \rightarrow f'(0) = \ln(2)$

$f''(n) = (\ln^2 2)2^x \rightarrow f''(0) = \ln^2(2)$

$f'''(0) = \ln^3(2)\ ;\ f^4(0)\ \ln^4(2)$

$$\rightarrow f(x) = 1 + \frac{\ln(2)}{1!} x + \frac{\ln^2(2)}{2!} x^2 + \frac{\ln^3(2)}{2} + \ldots$$

$$\sum_{n=0}^{\infty} \frac{[\ln(2)]^n}{n!} \cdot x^n$$

$$L = \lim_{n\to\infty} \left| \frac{\ln(2)^{n+1} \cdot x^{n+1}}{(n+1)!} \times \frac{n!}{[\ln(2)]^n \cdot x^n} \right|$$

$$\left\{ \left| \frac{a_{n+1}}{a_n} \right| \right\}$$

$$= \lim_{n\to\infty} \left| \frac{\ln(2) \cdot x}{n+1} \right| = x \ln(2) \lim_{n\to\infty} \left| \frac{1}{n+1} \right|$$

$= x\ln(2)\ (0) = 0$

$=$ converges

converges for all x $[-\infty, \infty]$

Henry Osei

5. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n^2)}$

Let $f(x) = \frac{1}{x(\ln x)^2}$, ~~then terms of~~ the series and funtion f satisfy the statement that series will converge iff

$$\int_2^\infty f(x)\,dx = \int_2^\infty \frac{1}{x(\ln x)^2}\,dx \text{ is finite}$$

letting $u = \ln$, we have that $du = \frac{1}{x}\,dx$, so I rewrite it as

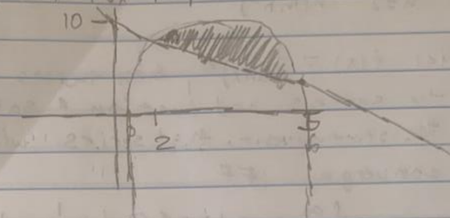
$$\int_{u=\ln 2}^{\infty} \frac{du}{u^2} = -u^{-1}\Big]_{u=\ln 2}^{\infty} = \frac{1}{\ln 2},$$

this is finite the series converges using integral test

Henry Osei

12.

$y = -x^2 + 6x \; ; \; y = -x + 1$



$f(x) = -x^2 + 6x$

$g(x) = -x + 10$

$a = 2, 5$

$$\int_2^5 \pi \left((-x^2 + 6x - 0)^2 - ((-x + 10) - 0)^2 \right) dx$$

$$= \int_2^5 \pi \left[(-x^2+6x)^2 - (-x+10)^2 \right] dx$$

$$= \int_2^5 \left((-x^2 + 6x)^2 - (-x+10)^2 \right) dx = \frac{333}{5} \pi$$

$$\boxed{V = \frac{333}{5} \pi}$$

Henry Osei

11.

$y = \sqrt{1+e^x}$

$$f'(x) = \left(\sqrt{1+e^x}\right)' =$$

$$\frac{d}{du}\left(\sqrt{u}\right)\frac{d}{dx}\left(e^x+1\right) = \frac{d}{du}\left(u^{1/2}\right)\frac{d}{dx}\left(e^x+1\right) =$$

$$\left(\tfrac{1}{2}u^{-1+1/2}\right)\frac{d}{dx}\left(e^x+1\right) = \left(\frac{u^{-1/2}}{2}\right)\frac{d}{dx}\left(e^x+1\right) =$$

$$\frac{\frac{d}{dx}\left(e^x+1\right)}{2\sqrt{u}} = \frac{\frac{d}{dx}\left(e^x\right)+\frac{d}{dx}(1)}{2\sqrt{e^x+1}} = \frac{e^x+\frac{d}{dx}(1)}{2\sqrt{e^x+1}}$$

$$= \frac{e^x}{2\sqrt{e^x+1}}$$

$$S = \int_1^2 2\pi\sqrt{\left(\frac{e^x}{2\sqrt{e^x+1}}\right)^2+1}\,\sqrt{e^x+1}\,dx =$$

$$\int_1^2 \frac{\pi}{\sqrt{e^x+1}}\sqrt{e^{3x}+5e^{2x}+8e^x+4}\,dx$$

$$\boxed{\approx 20.966}$$

Henry Osei

6. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

lert $f(x) = \frac{n^n}{(n+1)^n}$ this will meet the statement ~~that~~ the series will converge iff

$$\int_1^{\infty} f(x)dx \int_1^{\infty} \frac{n^n}{(n+1)^n}$$ is positive when $n \to \infty$

~~other~~ for $\lim_{n\to\infty} \left(\frac{n^n}{(n+1)^n}\right)$

by ratio tes $L<1$ by ratiotest

which means converges