1 (a)

$$\begin{bmatrix}3\\-1\\5\end{bmatrix}\begin{bmatrix}3\\-1\\5\end{bmatrix} \overset{w \cdot w}{=} 3\cdot3+(-1)+(-5)(-5)=35$$

$$\overset{x \cdot w}{\begin{bmatrix}6\\-2\\3\end{bmatrix}\begin{bmatrix}3\\-1\\-5\end{bmatrix}} = 6\cdot3+(-2)(-1)+3(-5)=5$$

$$x \cdot w = \frac{5}{25} = \frac{1}{7}$$

(b) $v \cdot v = 5$

$$\left(\frac{1}{5}\right)\begin{bmatrix}-1\\2\end{bmatrix} = \begin{bmatrix}-1/5\\2/5\end{bmatrix}$$

(c) $x \cdot w = 5$

$x \cdot x = 49$

$$\left(\frac{5}{49}\right)\begin{bmatrix}6\\-2\\3\end{bmatrix} = \begin{bmatrix}30/49\\-10/49\\15/49\end{bmatrix}$$

2a) $|(-6\ \ 4\ \ -3)|$

$= \sqrt{(-6)^2+4^2+(-3)^2} = \sqrt{61}$

$= \sqrt{61}$

$\hat{a} = \dfrac{(-6\ \ 4\ -3)}{\sqrt{61}}$

$= \left(\frac{1}{\sqrt{61}}(-6)\ \ \frac{1}{\sqrt{61}}\cdot 4\ \ \frac{1}{\sqrt{61}}(-3)\right)$

$= \left(-\frac{6}{\sqrt{61}}\ \ \frac{4}{\sqrt{61}}\ \ -\frac{3}{\sqrt{61}}\right)$

$= \left(-\frac{6}{\sqrt{61}}\ \ \frac{4}{\sqrt{61}}\ \ -\frac{3}{\sqrt{61}}\right)$

2b) $\left|\left(\frac{8}{3}\ \ 2\right)\right|$

$= \sqrt{\left(\frac{8}{3}\right)^2+2^2}$

$\sqrt{\left(\frac{8}{3}\right)^2+2^2}$

$= \sqrt{\frac{8^2}{3^2}+2^2}$

$\frac{8^2}{3^2} = \frac{64}{3^2} = \frac{64}{9}$

$2^2 = 4 \quad = \sqrt{\frac{64}{9}+4} \quad = \sqrt{\frac{100}{9}} = \frac{\sqrt{100}}{\sqrt{9}} = \frac{\sqrt{100}}{3} = \frac{10}{3}$

$= \dfrac{\left(\frac{8}{3}\ \ 2\right)}{\frac{10}{3}} = \left(\frac{4}{5}\ \ \frac{3}{5}\right.$

3) vector 1: = $0i - 5j + 2k$

vector 2: $-4i - j + 8k$

D = sqrt((0 - (-4))^2 + ((-5) - (-1))^2 + (2 - 8)^2

D = sqrt (16 + 16 + 36)

= sqrt 68

= 8.25 units

4)

a) $\begin{bmatrix}1\\-2\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}, \begin{bmatrix}-5\\-2\\1\end{bmatrix}$

$\begin{bmatrix}1\\-2\\1\end{bmatrix} \cdot \begin{bmatrix}0\\1\\2\end{bmatrix} = 1\times 0 - 2\times 1 + 1\times 2 = 0$

$\begin{bmatrix}0\\1\\2\end{bmatrix} \cdot \begin{bmatrix}-5\\-2\\1\end{bmatrix} = 0 - 2 + 2 = 0$

$\begin{bmatrix}1\\-2\\1\end{bmatrix} \cdot \begin{bmatrix}-5\\-2\\1\end{bmatrix} = -5 + 4 + 1 = 0$

given vectors are orthogonal b/c dot product of all vectors pairwise is zero.

b) $a = \begin{matrix}2\\-5\\-3\end{matrix}$; $b = \begin{bmatrix}0\\0\\0\end{bmatrix}$; $c = \begin{bmatrix}4\\-2\\6\end{bmatrix}$

to be orthogonal, the dot product of 2 vector should be 0

$a \cdot b = \begin{bmatrix}2\\-5\\-3\end{bmatrix} \cdot \begin{bmatrix}0\\0\\0\end{bmatrix} = (2)(0) + (-5)(0) + (-3)(0)$
$= 0 + 0 + 0 = 0$

a and b are orthogonal

$a \cdot c = \begin{bmatrix}2\\-5\\-3\end{bmatrix} \cdot \begin{bmatrix}4\\-2\\6\end{bmatrix} = (2)(4) + (-5)(-2) + (-3)(6)$
$= 8 + 10 - 18 = 0$

a and c is orthogonal

$b \cdot c = \begin{bmatrix}0\\0\\0\end{bmatrix} \cdot \begin{bmatrix}4\\-2\\6\end{bmatrix} = (0)(4) + (0)(-2) + (0)(6)$
$= 0 + 0 + 0 = 0$

b and c are orthogonal

5) $A = [v_1 | v_2 | v_3]$

then $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & -3 \\ 4 & -1 & 0 \end{vmatrix}$

$|A| = 1\{(2\times 0) - (-3\times -1)] - 2[(1\times 0) - (3\times 4)]$
$+ 3[(1\times -1) - (2\times 4)]$

$|A| = 1\{-3] - 2[+12] + 3[-1-8]$

$|A| = -3-24-27 = 54$

$\therefore |A| \neq 0$, it is not an orthogonal basis

6) given $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = y$ (say), $\begin{bmatrix} -1 \\ 3 \end{bmatrix} = u$ (say)

the orthogonal projection of y onto u is

$$\hat{y} = \frac{y \cdot u}{u \cdot u} u$$

$$y \cdot u = \langle (1, -1) \cdot (-1, 3) \rangle = -1 - 3 = -4$$

$$u \cdot u = \langle (-1, 3) \cdot (-1, 3) \rangle = 1 + 9 = 10$$

$$\hat{y} = \frac{y \cdot u}{u \cdot u} u = \frac{-4}{10} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \frac{-2}{5} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ -\frac{6}{5} \end{bmatrix}$$

7)

$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} x1 \\ y1 \end{bmatrix}$ and origin $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} x2 \\ y2 \end{bmatrix}$

$(y-2) = \left(\frac{0-2}{0-1}\right)(x-1)$

$2x - y = 0$

$A = 2,\ B = -1 \ldots\ldots (2)$

$\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix} \ldots\ldots 3$

substituting (2), (3) in (1) we get $d = 3\sqrt{5}$

e)

$$y = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} \quad u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad u_2 \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix}$$

$$\hat{y} = \frac{y \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{y \cdot u_2}{u_2 \cdot u_2} u_2$$

$$= \frac{(-1)\times 1 + 4\times 1 + 3\times 1}{1\times 1 + 1\times 1 + 1\times 1} u_1 + \frac{(-1)\times(-1) + 4\times 3 + 3\times(-2)}{(-1)\times(-1) + 3\times 3 + (-2)\times(-2)} u_2$$

$$= \frac{6}{3} u_1 + \frac{7}{14} u_2$$

$$= 2\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix}$$

1) $y - \hat{y} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$

$$y = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} + \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$$

9)

a) $y = \frac{y \cdot v_1}{v_1 \cdot v_1} v_1 + \frac{y \cdot v_2}{v_2 \cdot v_2} v_2$

$y = \begin{bmatrix} -3 \\ -1 \\ 1 \\ 13 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

$v_1 \cdot v_2 = 4 + 2 - 0 - 6 = 0$

$y \cdot v_1 = -3 - 2 - 1 + 26 = 20$

$y \cdot v_2 = -12 - 1 + 0 - 31 = -52$

$v_1 \cdot v_1 = 1^2 + 2^2 + 1^2 + 2^2 = 9$

$v_2 \cdot v_2 = 16 + 1 + 0 + 9 = 25$

$\hat{y} = \frac{20}{9} \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix} + \frac{-52}{25} \begin{bmatrix} 4 \\ 1 \\ 0 \\ 3 \end{bmatrix}$

$= \begin{bmatrix} 20/9 \\ 40/9 \\ -20/9 \\ -40/9 \end{bmatrix} + \begin{bmatrix} -212/25 \\ -52/25 \\ 0 \\ -156/3 \end{bmatrix} = \begin{bmatrix} -1408/225 \\ 532/225 \\ -20/9 \\ -508/9 \end{bmatrix}$

b) $\hat{y} = \text{proj}_W y = \begin{bmatrix} -1 \\ -5 \\ -3 \\ 9 \end{bmatrix}$

$y - \hat{y} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 13 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \\ -3 \\ 9 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$

$\| y - \hat{y} \|^2 = 4^2 + 4^2 + 4^2 + 4^2$

Distance is $\sqrt{64} = 8$

10) a) $v_1 = (3 \ -4 \ 5)$

$$e_1 = \frac{(3 \ -4 \ 5)}{5\sqrt{2}}$$

$$e_1 = \left(\frac{3}{5\sqrt{2}} \quad -\frac{2\sqrt{2}}{5} \quad \frac{1}{\sqrt{2}}\right)$$

$v_2 = (3 \ 6 \ 3)$

$$e_2 = \frac{(3 \ 6 \ 3)}{3\sqrt{6}}$$

$$e_2 = \left(\frac{1}{\sqrt{6}} \quad \sqrt{\frac{2}{3}} \quad \frac{1}{\sqrt{6}}\right)$$

$$e_1 = \left(\frac{3}{5\sqrt{2}} \quad \frac{-2\sqrt{2}}{5} \quad \frac{1}{\sqrt{2}}\right), \ e_2 = \left(\frac{1}{\sqrt{6}} \quad \sqrt{\frac{2}{3}} \quad \frac{1}{\sqrt{6}}\right)$$

11)

Compute $w_3 \cdot v_1 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = 6$, $w_3 \cdot v_2 = \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 30$,

$v_2 \cdot v_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix} : 12$ and

$$v_3 - w_3 - \frac{w_3 \cdot v_1}{v_1 \cdot v_1} v_1 - \frac{w_3 \cdot v_2}{v_2 \cdot v_2} v_2 - \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} - \frac{6}{12} \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 3 \\ -1 \end{bmatrix}$$

Hence $\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 3 \\ -1 \end{bmatrix} \right\}$ = orthogonal basis for col(A)

$$\left\{ \frac{v_1}{\|v_1\|}, \frac{v_2}{\|v_2\|}, \frac{v_3}{\|v_3\|} \right\} = \left\{ \begin{bmatrix} -\frac{1}{\sqrt{12}} \\ \frac{3}{\sqrt{12}} \\ \frac{1}{\sqrt{12}} \\ \frac{1}{\sqrt{12}} \end{bmatrix}, \begin{bmatrix} \frac{3}{\sqrt{12}} \\ \frac{1}{\sqrt{12}} \\ \frac{1}{\sqrt{12}} \\ -\frac{1}{\sqrt{12}} \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{12}} \\ -\frac{1}{\sqrt{12}} \\ \frac{3}{\sqrt{12}} \\ -\frac{1}{\sqrt{12}} \end{bmatrix} \right\}$$

is an orthonormal basis for col(A).