

COSC 417

Assignment 1

Instructions:

- (a) Due Feb 11.
- (b) Work in teams of 3-4 students (hand in one assignment with all the names on it).
- (c) For editing your homework. I recommend that you use Latex, see the template posted on the course website.
<http://orion.towson.edu/~mzimand/athcomp/assignment-template.tex> and
<http://orion.towson.edu/~mzimand/athcomp/assignment-template.pdf>
- (d) If a problem has more questions, write down your answers in the same order as the order of questions. In principle, this should help you.

Problem 1. Let $A = \{x, y, z\}$ and $B = \{x, y\}$. (a) Is A a subset of B ?

(b) Is B a subset of A ?

(c) What is $A \cup B$?

$=\{x,y,z\}$

(d) What is $A \cap B$?

$=\{x,y\}$

(e) What is $A \times B$?

$=\{(x,x),(x,y),(y,x),(y,y),(z,x),(z,y)\}$

(f) What is $P(B)$? ($P(B)$ is the powerset of B).

$=\{\emptyset, \{x\}, \{y\}, \{x,y\}\}$

(g) What is $P(P(B))$? (Be careful with the parenthesis syntax).

$\{\{\emptyset\},$
 $\{\{x\},$
 $\{\{y\},$
 $\{\{x,y\}$
 $\{\{\emptyset\},\{x\},$
 $\{\{\emptyset\},\{y\},$
 $\{\{\emptyset\},\{x,y\},$
 $\{\{x\},\{y\},$
 $\{\{x\},\{x,y\},$
 $\{\{y\},\{x,y\},$
 $\{\{\emptyset\},\{x\},\{y\},$
 $\{\{\emptyset\},\{x,y\},$

Problem 2. Show that the set $\text{EVEN} = \{0, 2, 4, \dots\}$, of even positive integers, is countably infinite by giving an explicit bijective function $f : \mathbb{N} \rightarrow \text{EVEN}$. Prove that your function f is a bijection.

- $\text{Even} = \{0, 2, 4\}$
- $f: \mathbb{N} \rightarrow \text{Even}$
- $f(n)$ (defined function)

- Prove $f \rightarrow \text{Even}$, $f(n) = 2n$ is a bijective function:
 - One-to-one (injective)
 - If x_1 and $x_2 \in \mathbb{N}$ so $x_1 \neq x_2$
- Now,
 - $f: \mathbb{N} \rightarrow \text{Even}$, $f(n) = 2n$
 - Let $x_1, x_2 \in \mathbb{N}$ and $x_1 \neq x_2$
 - $f(x_1) = 2x_1$
 - $f(x_2) = 2x_2$
 - $2x_1 = 2x_2$
 - $x_1 = x_2$

Therefore $x_1 \neq x_2$

- $f: \mathbb{N} \rightarrow \text{Even}$ $f(n)$ is 2, one – one

If $f: X \rightarrow Y$ be a surjective function, then every element of Y has at least one element of $x \in X$ such that $f(x) = y$

Now, $f: \mathbb{N} \rightarrow \text{Even}$, $f(n) = 2n$

$y \in \text{Even}$, there exists $x \in \mathbb{N}$ such that $f(x) = y$

So, $f: \mathbb{N} \rightarrow \text{Even}$, $f(n) = 2n$ is onto

Therefore, the function $f: \mathbb{N} \rightarrow \text{Even}$ $f(n) = 2n$ is both one to one and onto. So it is bijective

Problem 3. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ be two sets. Recall that $A \times B$ is the set of pairs (u, v) with $u \in A$, $v \in B$.

(a) List all the elements of $A \times B$.

$\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

(b) Write down one element (whichever you want) of $P(A) \times P(B)$. (Be careful with the parenthesis syntax).

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$P(B) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

(c) What is the size of $P(A) \times P(B)$?

- $P(A)$ has 8 elements.
- $P(B)$ has 4 elements
- $8 \times 4 = \mathbf{32}$

Problem 4. - EXTRA CREDIT. Let A be countably infinite set and $B = \{0, 1\}$. Show that $A \times B$ is countably infinite. You need to show how to obtain an enumeration g of $A \times B$ from an enumeration f of A . (Recall that an enumeration of a set C is a bijective function mapping \mathbb{N} to C).