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COSC 417
3/19/2020

Assignment 10

Instructions.

1. Due May 7.
2. This is a team assignment. Work in teams of 3-4 students. Submit one assignment per team, with the names of all students making the team.
3. You will submit on Blackboard one single pdf file with the solutions to all exercises. For this you'll take the .tex file for this assignment and modify it. In the box above replace Ann, Bob, Charlie with your names. Write down your anwers for each question after Answer:.

   For editing the above document with Latex, see the template posted on the course website.
   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.tex` and
   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.pdf`
   To append in the latex file a .jpg file (for a photo; for example, in case you draw a picture by hand and take a photo of it with your phone camera), use

   ```
   \includegraphics[angle=270,origin=c,width=\linewidth]{file.jpg}
   ```

   The parameter angle=270 is for rotating the photo, and you may have to change 270 to whatever angle works for your photo.

Problem 1. We consider the problem of determining if there is a set of k students (say, that can attend a meeting) such that each student club has at least one representative in the set. In this problem, a club C is a set (of students), and so the formal statement of the problem is as follows.

REPRESENTATIVE SET (RS)

Input: Finite sets (representing student clubs) $C_1, \ldots, C_m$ and a natural number $k \leq m$.

Question: Can we choose a set R with k elements such that for every i the intersection $C_i \cap R$ has size at least one.

(a) Show that RS is in NP. Note: You need to say what a certificate c for a yes-instance is and present a verifier algorithm V that accepts pairs (Yes-instance, certificate). As an example, look at Th 7.24, page 296.

Answer:

Proof Idea: R is the Yes- Certificate
Proof: Verifier for V for RS

V= "On input (C_1, C_2……, Cm),k) where C_i is an finite set and k is a natural number

1.Test whether Is |R| ≤ k. (done in polynomial time)
2. For each i, check if | Ci ∩ R|≥ 1
3. If both pass accept; otherwise reject.

R of size at most k, hence there will be at least one YES-certificate and the verifier will accept this

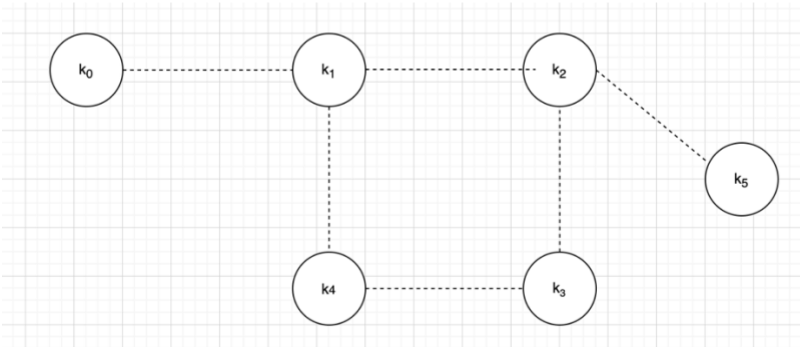
(b) Describe a polynomial-time reduction VC $\leq_p$ RS, where VC is the VERTEX COVER problem. (Notes: The VC (Vertex Cover) problem is presented in the textbook page 312 and in Notes -13, slide 6, where it is shown that this problem is NP-complete. I'll remind you that VC is the following decision problem:

Input: graph G, and positive integer k.

Question: Does G contain a set A of k vertices, so that each edge (u, v) of G has at least one of u or v in A.

You need to describe a polynomial-time computable transformation that maps an arbitrary instance (G, k) of VC into an instance $I_1 = ((C_1, \ldots, C_m), k^0)$ of RS so that yes/no istances of VC are mapped into yes/no instances of RS)

Answer:.



$C1=\{k_1,k_3\}$
$C2=\{k_2,k_4\}$
$C3=\{k_1,k_2,k3\}$
$C1=\{k_1,k_2,k_3,k_4\}$

$F(k_0\ldots\ldots k_m)=C_i$
Polynomial-time computabe

Problem 2. Solve Problem 7.21, (a) textbook, page 324. Hint: You can use algorithms studied in other courses such as COSC 336.
Answer:

"On input 〈G, a, b, k〉 m-node graph G has nodes a and b:

1. Place a mark "0" on node a
2. For each i from 0 to m:
   a. If an edge (s,t) is found connecting s marked "i" to an unmarked nodet, mark node t with i +1
   b. If b is marked with a value of at most k, accept. Otherwise, reject.

Problem 3. Solve Problem 7.21, (b) textbook, page 324. Hint: You can do a reduction from the HAMPATH problem, see Th 7.46, page 314 in the textbook.
Answer:

First, LPATH ∈ NP because we can guess a simple path of length at least K from a to b and verify it in polynomial time. Next, UHAMPATH ≤ p LPATH, because the following ™ F computes the reduction f.

F = "On input 〈G, a, b〉 where graph G has nodes a and b:

1. Let k be the number of nodes of G

2. Output $\langle G, a, b, k \rangle$