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COSC 417 Assignment 1

Instructions:

(a) Due Feb 11.

(b) Work in teams of 3-4 students (hand in one assignment with all the names on it).

(c) For editing your homework. I recommend that you use Latex, see the template posted on the course website.
`http://orion.towson.edu/~mzimand/athcomp/assignment-template.tex` and
`http://orion.towson.edu/~mzimand/athcomp/assignment-template.pdf`

(d) If a problem has more questions, write down your answers in the same order as the order of questions. In principle, this should help you.

Problem 1. Let A = {x, y, z} and B = {x, y}. (a) Is
A a subset of B?

(b) Is B a subset of A?

(c) What is A ∪ B?

={x,y,z}

(d) What is A ∩ B?

={x,y}

(e) What is A × B?

={(x,x),(x,y),(y,x),(y,y),(z,x),(z,y)}

(f) What is P(B)? (P(B) is the powerset of B).

=**{{ Ø,**{x},{y},{x,y}}

(g) What is P(P(B))? (Be careful with the parenthesis syntax).

{{ Ø}},
{{x}},
{{y}},
{{x,y}}
{{ Ø},{x}},
{{ Ø},{y}},
{{ Ø},{x,y}},
{{x},{y}},
{{x},{x,y}},
{{y},{x,y}},
{{ Ø},{x},{y}},
{{ Ø},{x,y}},

Problem 2. Show that the set EVEN = {0, 2, 4, . . .}, of even positive integers, is countably infinite by giving an explicit bijective function f : N → EVEN. Prove that your function f is a bijection.

- Even = {0,2,4}
- f:N→ Even
- f(n)(defined function)

- Prove f→ Even, f(n)=2n is a bijective function:
  - One-to-one (injectinve)
    - If $x_1$and $x_2 \in$ so $x_1 \neq x_2$
- Now,
  - F:N→ Even, f(n)=2n
    - Let $x_1,x_2 \in$ and $x_1 \neq x_2$
    - $f(x_1) = 2x_1$
    - $f(x_2) = 2x_2$
    - $2x_1 = 2x_2$
    - $x_1 = x_2$

Therefore $x_1 \neq x_2$

- F: N → Even f(n) is 2, one – one

If f: X→Y be a sunjective function, then every elemt of Y has atleast one element of x ∈ X such that f(x)=y

Now, f N→ Even, f(n)=2n

Y ∈ Even, there exists x ∈ N such that f(x)=y

So, f: N→ Even, f(n)=2n is onto
Therefore, the function f: N→ Even f(n)=2n is both one to one and onto. So it is bijective

Problem 3. Let A = {1, 2, 3} and B = {a, b} be two sets. Recall that A × B is the set of pairs (u, v) with u ∈ A, v ∈ B.

(a) List all the elements of A × B.

{(1,a),(1,b),(2,a),(2,b),(3,a)(3,b)}

(b) Write down one element (whichever you want) of P(A) × P(B). (Be careful with the parenthesis syntax).
P(A)={{ Ø ,{1},{2},{3},{1,2},{1,3}{2,3},{1,2,3}}
P(B)={{ Ø },{a},{b},{a,b}}

(c) What is the size of P(A) × P(B)?

- P(A) has 8 elements.
- P(B) has 4 elements

- 8 x 4 = **32**

Problem 4. **-** EXTRA CREDIT. Let A be countably infinite set and B = {0, 1}. Show that A × B is countably infinite. You need to show how to obtain an enumeration g of A × B from an enumeration f of A. (Recall that an enumeration of a set C is a bijective function mapping N to C).