Question #1

(a)

$$\begin{bmatrix} -13 & 18 \\ -6 & 8 \end{bmatrix} = \det\left(\begin{pmatrix} -13 & 18 \\ -6 & 8 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \begin{pmatrix} -13 & 18 \\ -6 & 8 \end{pmatrix} - \begin{pmatrix} \lambda 1 & \lambda 0 \\ \lambda 0 & \lambda 1 \end{pmatrix}$$

$$= \begin{pmatrix} -13-\lambda & 18 \\ -6 & 8-\lambda \end{pmatrix} = \det\begin{pmatrix} -13-\lambda & 18 \\ -6 & 8-\lambda \end{pmatrix}$$

$$= (13-\lambda)(8-\lambda) - 18(-6)$$

$$= 13 \cdot 8 + 13\lambda - 8\lambda + \lambda\lambda$$

$$= \lambda^2 + 5\lambda + 4$$

$$= \lambda = -1,\ \lambda = -4$$

b

eigenvectors $= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} : \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

C.

$$P = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \quad P^{-1}\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = \frac{1}{\det\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}}\begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix}$$

PDP^{-1}

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -4 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -13 & 18 \\ -6 & 8 \end{pmatrix}$$

$$P = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix},\ D = \begin{pmatrix} -1 & 0 \\ 0 & -4 \end{pmatrix},\ P^{-1} = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$$

2.

(a) = Incorrect dimensions columns do not match number of rows

(b) $\begin{pmatrix} 1 & -3 & -4 \\ 4 & 2 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 1\cdot 2 + (-3)\cdot 3 + (-4)\cdot 2 \\ 4\cdot 2 + 2\cdot 3 + 4\cdot 2 \end{pmatrix} =$

$\begin{pmatrix} -15 \\ 22 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -3 & -4 \\ 4 & 2 & 4 \end{pmatrix}\begin{pmatrix} -1 & -1 \\ -4 & -2 \\ -1 & 4 \end{pmatrix} =$

$\begin{pmatrix} 1\cdot(-1) + (-3)(-4) + (-4)(-1) & 1\cdot(1) + (-3)(-2) + (-4)\cdot 4 \\ 4(-1) + 2(-4) + 4(-1) & 4(-1) + 2(-2) + 4\cdot 4 \end{pmatrix}$

$= \begin{pmatrix} 15 & -11 \\ -16 & 8 \end{pmatrix}$

(d) $\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} = 2\cdot 2 + 3\cdot 3 + 2\cdot 2 = 17$

(e) $\begin{vmatrix} 4 & 1 & -1 \\ 3 & 3 & 1 \\ 0 & 0 & -4 \end{vmatrix} = 4\times 3\times(-4) + 1\times 1\times 0 + (-1)\times 3\times 0 - 0\times 3\times(-1) - 0\times 1\times 4 (-1) -$
$-(4)\times 3\times 1 = -36$

(g) $\sqrt{17} = a = \frac{17}{\sqrt{17}^2}\cdot(2\ 3\ 2) = (2\ 3\ 2)$

3

(a)

$$\begin{bmatrix} -1 & 2 & 3 \\ -1 & 2 & 3 \\ -1 & 4 & 7 \end{bmatrix} \begin{matrix} R_1 \leftarrow R_1 \cdot (-1) \\ R_2 \leftarrow 1 \times R_1 \end{matrix} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 0 & 0 \\ -1 & 4 & 7 \end{bmatrix} R_3 \leftarrow 1 \times R_1$$

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 0 & 0 \\ 0 & 2 & 4 \end{bmatrix} R_2 \longleftrightarrow R_3 \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} R_2 \longleftarrow R_2 \times 1/2$$

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$1x_1 + 1x_3 = 0$

$1x_2 + 2x_3 = 0$

$0 = 0$

basis $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} \right\}$

c

The matrix is not invertiable from A because it is singular must be non singular to be invertiable

d.

$$\left[\begin{array}{ccc|ccc} -3 & 0 & 0 & 0 & 0 & 0 \\ -1 & -3 & 3 & 0 & 1 & \\ -2 & 0 & 1 & 0 & 0 & 9 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 0 & 0 \\ 0 & 1 & 0 & 7/4 & -1/3 & -1 \\ 0 & 0 & 1 & 2/3 & 0 & -1 \end{array}\right]$$

$1c_1 = (-1/3)d_1$

$1c_2 = (7/4)d_1 + (-1/3)d_2 - 1d_3$

$1c_3 = 2/3 - 1d_3$

e

4

(a) $\left[\begin{array}{ccccc|c} 2 & 6 & -1 & 3 & 1 & 7 \\ -2 & -6 & 2 & 2 & -2 & -8 \\ 1 & 3 & -1 & -1 & 1 & 4 \end{array}\right] \sim$

$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 4 & 0 & 3 \\ 0 & 0 & 1 & 5 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 = 3 - 3x_2 - 4x$

$x_3 = -1 - 5x_4 + x_5$

(b)

(c) $\begin{bmatrix} 2 & 6 & -1 & 3 & 1 & 7 \\ -2 & -6 & 2 & 2 & -2 & -8 \\ 1 & 3 & -1 & -1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 & -1 & 3 & 1 & 7 \\ 0 & 0 & 1 & 5 & -1 & -1 \\ 0 & 0 & -1/2 & -5/2 & 1/2 & 1/2 \end{bmatrix} =$

$\begin{bmatrix} 2 & 6 & -1 & 3 & 1 & 7 \\ 0 & 0 & 1 & 5 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ = Rank = 2

Bonus

(a)

given that $\{w_1, w_2, w_3, \dots\}$ is orthogona basis for W and $\{v_1, v_2 \dots v_q\}$ is also orthogonal basis for $W^{\perp}$

since $\{w_1, w_2, w_3, \dots w_p\}$ orthogonal

$\rightarrow \langle w_i, w_j \rangle = 0$ for $i \neq j$

since $\langle v_i, v_j \rangle = 0$ for $i = j$

verify $S \{w_1, w_2, w_3, \dots w_p, v_1, v_2 \dots, v_q\}$ is orthogonal

$\langle w_i, w_j \rangle = 0$ for $i \neq j$

since $\langle v_i, v_j \rangle = 0$ for $i \neq j$

and $\langle w_i, v_j \rangle = 0$ for $i \neq j$ hence the given is orthogonal

(b)

from point (a) the set B is orthogonal

$\langle w_i, w_j \rangle = 0$ for $i \neq j$

since $\langle v_i, v_j \rangle = 0$ for $i \neq j$

and $\langle w_i, v_j \rangle = 0$ for $i \neq j$

for each $i = 1, 2, \dots n$

$\langle \beta, a_i \rangle = a_1 w_1 + a_2 w_2 + \dots + a_n w_n + b_1 v_1 + b_2 v_2 + \dots b_n v_n$

therefore S spans R^n

(c)

$\{w_1, w_2, w_3, \dots w_p\}, \{v_1 v_2 \dots v_q\}$ is orthogonal

but every ~~orthogonal~~ set is linearly independent

hence it is basis for W so $\dim W = p$

$\therefore \{w_1, w_2, v_3 \dots v_p\}$ is basis for where dim

since $\{w_1, v_2 \dots v_q\}$ is basis for $W^{\perp}$

hence the set $\{w_1, w_2, w_3 \dots W_p, v_1, v_2 \dots v_q\}$ is basis for $W + W^{\perp}$

$\therefore W + \dim W^{\perp} = p + q$