Ann, Bob, Charlie (*replace with your names*)
COSC 417
3/19/2020

Assignment 8

Instructions.

1. Due April 23.

2. This is a team assignment. Work in teams of 3-4 students. Submit one assignment per team, with the names of all students making the team.

3. You will submit on **Blackboard** one single pdf file with the solutions to all exercises. For this you'll take the .tex file for this assignment and modify it. In the box above replace Ann, Bob, Charlie with your names. Write down your anwers for each question after **Answer:**.

   For editing the above document with Latex, see the template posted on the course website.

   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.tex` and

   `http://orion.towson.edu/~mzimand/adatastruct/assignment-template.pdf`

   To append in the latex file a .jpg file (for a photo; for example, in case you draw a picture by hand and take a photo of it with your phone camera), use

   ```
   \includegraphics[angle=270,origin=c,width=\linewidth]{file.jpg}
   ```

   The parameter angle=270 is for rotating the photo, and you may have to change 270 to whatever angle works for your photo.

Exercise 1. Show that the language

$$L = \{< M_1 > |\ M_1 \text{ is a Turing machine that accepts } 0\}.$$

is Turing-recognizable. (You need to give an informal description of a Turing machine V that accepts $< M_1 >$ if and only if M_1 is a TM that accepts 0. Your description should start with "V on input $< M_1 >$ ...." and describe what V does. For a similar example see the description of a machine U that recognizes A_{TM} on page 202 in the textbook).

Answer:.

Exercise 2. (a) Give an informal description of a computable function f that on input a Turing machine M and an input string w, outputs a Turing machine M_1 (in other words $f(\langle M, w\rangle) = \langle M_1\rangle$) with the property

- if M accepts w, then M_1 accepts 0, and
- if M does not accept w, then M_1 does not accept 0.

Answer:.

(Recall that to describe M_1 you need to consider an arbitrary input string x, and say how M_1 operates on x. Thus your description of M_1 should start with: "M_1 on input x: .... ", and next you explain in English what M_1 does Of course, M_1 has to simulate M on input w and do certain things depending on the outcome of the simulation.)

(b) Interpret part (a) as a reduction from a certain language X and explain what it implies about the language

$$L = \{< M_1 > |\ M_1 \text{ is a Turing machine that accepts } 0\}.$$

Answer:. (You need to say what the problem X mentioned above is, and whether you can conclude whether L is decidable/undecidable/Turing-recognizable/ not-Turing recognizable)

Exercise 3. Let

$$A = \{< M_1 > |\ M_1 \text{ is a Turing machine that does not accept } 0\}.$$

Explain what is wrong in the following alleged reduction $A_{TM} \leq_m A$.

Transform $< M, w >$ into the following Turing machine M_1:

M_1 on input x:

> Simulate M on w and
> if M accepts w, then M_1 enters the state q_{reject}.
> if M does not accept w, then M_1 enters the state q_{accept}.

Answer: (The error can be that either the transformation $< M, w > \mapsto < M_1 >$ is not computable, or the transformation does not map yes-instances of A_{TM} into yes-instances of A, or the transformation does not map no-instances of A_{TM} into no-instances of A.)