Extension of Stone's Method 1 and Conditions for Real Characteristics in Three-Phase Flow

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Summary. This paper presents extensions to Stone's Method 1 in the definition of the residual oil saturation (ROS) parameter, S_{orm}, in which linear, quadratic, and cubic forms are compared with measurements. The methods are also examined in terms of the system providing hyperbolic characteristics, and it is shown that small elliptic regions will usually occur in the saturation space. Some of the factors influencing the elliptic regions are analyzed and their significance is discussed.

Introduction

Three-phase relative permeabilities play a central role in most major reservoir simulators, including both black-oil and EOR applications. Thus, oil reservoirs with a free gas phase under production from a waterdrive, condensate fields with liquid dropout under aquifer invasion, such tertiary processes as multicontact-miscible displacement, development of a middle-phase microemulsion in a surfactant system, and steam/hot-water displacement of heavy oil all entail reliance on empiricisms describing three-phase flow. Compared with the extensive research undertaken on the Buckley-Leverett theory and determination of two-phase relative permeabilities, theoretical and experimental foundations for three-phase flow remain remarkably limited. This paper addresses some of the issues involved.

The most commonly used expressions for three-phase relative permeabilities are the normalized forms of Stone's Methods 1 and 2 given by Aziz and Settari.[1] Method 2 is a principal option in many commercial simulators because it does not entail specification of an ROS parameter, S_{orm}, which is required in Method 1. Fayers and Matthews[2] reviewed the normalized forms of Methods 1 and 2 and suggested a simple linear interpolation for S_{orm}, removing the problem of an arbitrary specification. Ref. 2 demonstrates that use of this extension of the normalized form of Stone's Method 1 results in better agreement with the available experimental results than use of Method 2. The results show that the behaviors of both methods are similar for oil relative permeability at larger oil saturations or in the middle of the mobile-oil-saturation range, but more noticeable differences occur as the oil relative permeabilities decrease. It is at these lower oil-saturation values, where the low fractional oil flow rates determine the reservoir's ultimate oil recovery, that the differences in the methods become important. Unfortunately, very few (if any) reliable measurements in this low saturation range exist, nor are the systematics of variation of S_{orm} with changes in gas or water saturation amenable to any simple physical arguments. However, the extension of Stone's Method 1 to embrace the simple linear form for S_{orm} can be taken further to consider quadratic or even cubic forms. We examine some aspects of these extensions in this paper.

In two-phase flow, the general form of two-phase relative permeabilities ensures that the water fractional-flow function will have a monotonically increasing form with a point of inflection. This ensures that, mathematically, the noncapillary displacement equation will have real positive characteristics and gives rise to conditions for formation of the Buckley-Leverett shock front. This result is the unique physical solution to the capillary displacement problem when the capillary pressure tends to zero. In three-phase flow, we do not have established procedures for investigating "method-of-characteristics" solutions to the flow equations, nor do we have any clearly formulated rules for determining the formation and strengths of the shock fronts that may form. The first step is to investigate whether the use of the Stone's relative permeability expressions will necessarily give a hyperbolic system of equations. Arguments on why this mathematical property may be important and tests of its fulfillment are made against a number of experimental data sets in this paper. We find that the three-phase flow problem is not necessarily formally hyperbolic over the complete range of mobile gas and water saturations. The problem becomes nonhyperbolic (i.e., elliptic) in one or more small saturation zones, and the magnitude of the imaginary component of the characteristics is usually quite small in this zone (referred to as an "elliptic region"). Plausible physical arguments are made on why these nonhyperbolic zones may not be significant from the standpoint of unstable solutions. The existence of nonhyperbolic regions, however, presents some mathematical difficulties in obtaining formal solutions to the noncapillary three-phase displacement problem.

This work was undertaken in parallel to more fundamental mathematical studies elsewhere on the characteristics of the three-phase flow problem.[3] We concentrate on practical expressions of two-phase relative permeabilities and their use in various forms of Stone's Method 1. In particular, we numerically examine whether one of the alternatives for S_{orm} is preferable, either from its agreement with measurements or from the standpoint of the extent of nonhyperbolic regions. Other forms of correlations for three-phase relative permeabilities have been evaluated in Ref. 4, in which a simple saturation weighted-interpolation appears to give encouraging comparisons with measurements. This method, however, does not have the flexibility of Method 1 in ensuring that the correct behavior for k_{ro} is obtained in the important region near S_{orm}.

Summary of Previous Position on Evaluation of Stone's Methods

The original statement of Stone's[5] Method 1 was unsatisfactory in some cases because it gave k_{ro} values greater than unity; thus, Aziz and Settari's normalized form, expressed in the following equation, is generally preferred.

$$k_{ro}(S_w,S_g)=\frac{S_o{}^*k_{row}k_{rog}}{k_{rocw}(1-S_w{}^*)(1-S_g{}^*)}, \qquad (1)$$

where $S_o{}^*=\dfrac{S_o-S_{orm}}{1-S_{wc}-S_{orm}}$,

$$S_w{}^*=\frac{S_w-S_{wc}}{1-S_{wc}-S_{orm}},$$

and $S_g{}^*=\dfrac{S_g}{1-S_{wc}-S_{orm}}$.

S_{wc} = connate water saturation, k_{rocw} = two-phase oil permeability at $S_o=1-S_{wc}$, and k_{row}, k_{rog} = two-phase oil permeabilities for the oil/water and oil/gas systems, respectively. Thus, $k_{row}=k_{row}(S_w)$ and $k_{rog}=k_{rog}(S_g)$.

It is usually assumed that the k_{rg} and k_{rog} curves are measured in the presence of interstitial water. S_{orm} is the ROS to simultaneous displacement by gas and water and has to be selected in a manner determined by the user. An example would be to choose $S_{orm}=½(S_{orw}+S_{org})$, where S_{orw} and S_{org} are the ROS values associated with the two-phase curves defined above. The difficulties

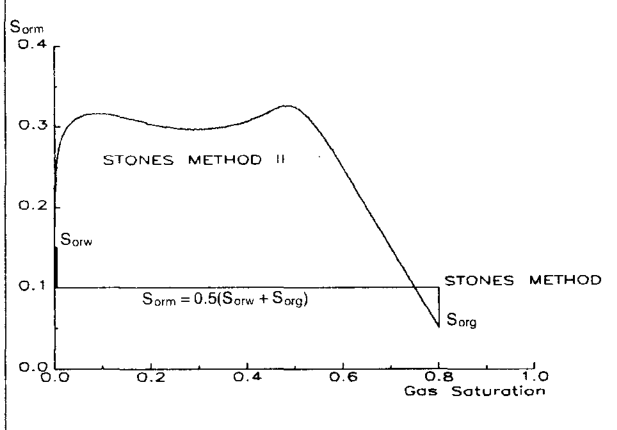

Fig. 1—Comparison of ROS's in basic Stone's methods.

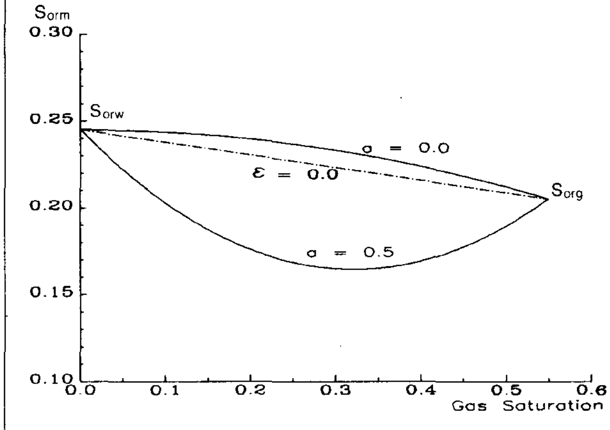

Fig. 2—Comparison of S_{orm} assumptions in Stone's method.

in choosing S_{orm} and the discontinuity in the behavior of the ROS this causes (see Fig. 1) led to the development of Stone's Method 2, expressed in normalized form by

$$k_{ro}(S_w,S_g)=k_{rocw}\left[\left(\frac{k_{row}}{k_{rocw}}+k_{rw}\right)\left(\frac{k_{rog}}{k_{rocw}}+k_{rg}\right)-(k_{rw}+k_{rg})\right]. \qquad (2)$$

Eq. 2 defines its own behavior for S_{orm} without the need for a special parameter. Fig. 1 shows an example of an inferred S_{orm} behavior.

Fayers and Matthews[2] pointed out that a more sensible basis for automatic selection of S_{orm} in Method 1 could be made with

$$S_{orm}=\alpha S_{orw}+(1-\alpha)S_{org}, \qquad (3)$$

where $\alpha=1-[S_g/(1-S_{wc}-S_{org})]$.

This form ensures that S_{orm} behaves smoothly as a function of S_g (i.e., linearly) and that S_{orm} matches S_{orw} and S_{org} when $S_g \rightarrow 0$ or $S_w \rightarrow S_{wc}$, respectively. The extended definition was used in Ref. 2 to compare the predicted performance of the normalized Stone's Methods 1 and 2 with six sets of published data on experimentally determined three-phase relative permeabilities. The results clearly indicate a preference for Method 1 with the use of Eq. 3. In some cases, the shape of the ROS behavior given by Eq. 2 can be unreasonable.

Some experimentalists[7] measured relative oil permeabilities for situations where the gas saturation was initially immobile in the form of a trapped gas saturation. This situation is attractive because the resulting water/oil displacement experiment can be executed and analyzed in a dynamic displacement test; otherwise, the absence of simple solutions to the three-phase flow equations dictates a need for time-consuming steady-state measurements. For trapped gas saturations, however, the resulting ROS from displacement by water is not given by Eq. 3 but appears to be given by the approximate rule

$$S_{orm}=S_{orw}-0.5S_{gtr}. \qquad (4)$$

The evidence for this rule rests largely on measurements in water-wet systems[8]; for intermediate-wettability systems, unreported evidence suggests that $S_{orm}=S_{orw}$ for a range of trapped gas saturations.

The trapped-gas results illustrate that three-phase flow will be influenced by the hysteresis behavior associated with the previous saturation history for each phase. When it is taken into account that each phase can be in a state of increasing or decreasing saturation behavior, or be in a trapped state, there are 12 possible combinations of hysteresis classes. For the present purposes, we are concerned largely with predominantly water-wet systems (i.e., oil is the intermediate phase) in which oil saturation is decreasing in the presence of a small mobile gas saturation with water saturation increasing at one end of the gas/water range and in which oil saturation is decreasing in the presence of a small mobile water saturation with gas saturation increasing at the other end of the gas/water range. At intermediate gas/water saturations, it will be assumed that both of those phases are increasing to reduce oil saturation to its residual value. The Stone's methods cannot cover all 12 hysteresis classes because the basic input of two-phase measurements can give a combination of only four classes.

Further Types of Relation for S_{orm}

Ref. 2 points out that the flexibility to choose the behavior of S_{orm} gave Method 1 a significant advantage because the three-phase relative permeability behavior could be constrained to follow ROS behavior if known. In spite of the importance of the low-oil-permeability region for reservoir recovery studies, a remarkable lack of experimental evidence on this region exists. This is clearly an area where more measurements are needed. The flexibility of the expression in Eq. 4 can be extended by considering the second-order form:

$$S_{orm}=S_{orw}-[(S_{orw}-S_{org})/(1-S_{wc}-S_{org})]S_g$$
$$-\epsilon[(1-S_{wc}-S_{org})S_g-S_g^2]. \qquad (5)$$

Eq. 5 satisfies the two-phase limits S_{orw}, S_{org} when $S_g=0$, or $S_w=S_{wc}$. The parameter ϵ apparently offers a free choice when adequate experimental data are available for fitting to S_{orm}.

A method for choosing ϵ for preferentially water-wet systems can be based on Eq. 4. If we consider very small gas saturations, the gas phase is not mobile until $S_g>S_{gc}$, where S_{gc}=critical gas saturation. Typically S_{gc} is on the order of a few percent, although S_{gc} is not usually identified in a gas/oil dynamic displacement test. When S_g is slightly larger than S_{gc}, k_{rg} remains small, so the overall behavior is still close to a trapped gas saturation. Thus for small values of S_g, we expect that

$$S_{orm}\rightarrow S_{orw}-aS_g, \qquad (6)$$

where $0<a\leqslant 0.5$; i.e., $\partial S_{orm}/\partial S_g \rightarrow -a$ as $S_g\rightarrow 0$. Imposing this constraint on Eq. 5 gives

$$\left.\frac{\partial S_{orm}}{\partial S_g}\right|_{S_g=0}=-\left(\frac{S_{orw}-S_{org}}{1-S_{wc}-S_{org}}\right)-\epsilon(1-S_{wc}-S_{org}).$$

Thus, $$\epsilon=\left(a-\frac{S_{orw}-S_{org}}{1-S_{wc}-S_{org}}\right)/(1-S_{wc}-S_{org}). \qquad (7)$$

With typical values $S_{orw}=0.3$, $S_{org}=0.15$, $S_{wc}=0.25$, and $a=0.5$, we obtain $\epsilon=0.415$. It is suggested that an appropriate choice for ϵ lies in the range $0\leqslant\epsilon\leqslant$the solution of Eq. 7 with $a=0.5$. $a=0$ in Eq. 7 implies that ϵ is negative and gives an S_{orm} function with curvature opposite to that for $a=0.5$, as Fig. 2 shows. The experimental results fitted in Ref. 2 suggest that negative curvatures are not realistic, although the systems investigated would not be tending to intermediate wettability.

Experimental three-phase data for six sets of experiments were reviewed in Ref. 2. Attempts were made to deduce "best-fit" behavior to the oil relative permeability measurements by fitting with the CPS-1 contouring program. Only three sets of these experiments can be judged as perhaps furnishing some evidence about the behavior of the zero oil isoperm (i.e., S_{orm}). These experiments are the data provided by Hosain,[9] Saraf and Fatt,[10] and Saraf *et al.*[11] Examination of the experimental curves suggests that some support exists for inclusion of a curvature term of the form given by Eq. 5 with a positive value for ϵ in the range suggested above. However, the fitted S_{orm} curves for these experiments do not clearly show the desired second-order shape. The departure becomes noticeable as the gas saturation increases. Unfortunately, the reliability of the measurements also becomes worse because measuring ROS behavior for high gas saturations is lengthy and difficult. Two of the sets of experiments suggest that a cubic dependence would be more appropriate (i.e., an S shape), which could be expressed in the form

$$S_{orm}=S_{orw}-\left(\frac{S_{orw}-S_{org}}{1-S_{wc}-S_{org}}\right)S_g-\epsilon[(1-S_{wc}-S_{org})S_g-S_g{}^2]$$
$$+\eta[(1-S_{wc}-S_{org})S_g{}^2-S_g{}^3]. \quad (8)$$

If the gradient of the initial slope of S_{orm} at $S_g=0$ confirms the value of ϵ, then the value of η is determined by the position S_g' at which the cubic form crosses the straight-line approximation; that is, η is obtained from $\eta=\epsilon/S_g'$. We examine this type of fitting later.

Extensions of Two-Phase Flow to Three-Phase Flow System

In two-phase flow, the equation assumed to govern incompressible 1D displacement is

$$\frac{\mathrm{d}G(S)}{\mathrm{d}S_w}\frac{\partial S_w}{\partial x}+\frac{\partial}{\partial x}\left[C(S)\frac{\partial S_w}{\partial x}\right]+\phi\frac{\partial S}{\partial t}=0, \quad (9)$$

where $G(S)=\dfrac{k_{rw}}{\mu_w\lambda_t}\left[q-\dfrac{kk_{ro}(\rho_w-\rho_o)g}{\mu_o}\right]$,

$$C(S)=\frac{k_{rw}}{\mu_w\lambda_t}\frac{kk_{ro}}{\mu_o}\frac{\mathrm{d}P_{cw}}{\mathrm{d}S_w},$$

and $\lambda_t=\dfrac{k_{rw}}{\mu_w}+\dfrac{k_{ro}}{\mu_o}$.

When the $C(S)$ function is not negligible compared with $G(S)$, the capillary pressure term gives a second-order derivative and Eq. 9 is a second-order parabolic equation requiring both upstream and downstream boundary conditions on S_w.[12] When $C(S)$ becomes negligibly small, however, the equation approaches a first-order hyperbolic form (i.e., the well-known Buckley-Leverett equation) in which shock-front solutions develop for which the saturation jump is a function of the relative strength of the viscous and gravity terms in the $G(S)$ function. Fayers and Sheldon[13] demonstrated the interaction of both capillary pressure and gravity terms on the Buckley-Leverett solution and explained how the method of characteristics may be applied for gravity flows. Where gravity does not dominate, the hyperbolic two-phase flow problem requires a single upstream boundary condition corresponding to a single positive real characteristic.[12]

In three-phase flow, the problem becomes significantly more complicated with the appropriate fluid displacement equations given by

$$\frac{\partial G_w}{\partial S_w}\frac{\partial S_w}{\partial x}+\frac{\partial G_w}{\partial S_g}\frac{\partial S_g}{\partial x}+\frac{\partial}{\partial x}\left(C_{ww}\frac{\partial S_w}{\partial x}\right)$$
$$-\frac{\partial}{\partial x}\left(C_{wg}\frac{\partial S_g}{\partial x}\right)+\phi\frac{\partial S_w}{\partial t}=0 \quad (10)$$

and
$$\frac{\partial G_g}{\partial S_w}\frac{\partial S_w}{\partial x}+\frac{\partial G_g}{\partial S_g}\frac{\partial S_g}{\partial x}+\frac{\partial}{\partial x}\left(C_{gw}\frac{\partial S_w}{\partial x}\right)$$
$$-\frac{\partial}{\partial x}\left(C_{gg}\frac{\partial S_g}{\partial x}\right)+\phi\frac{\partial S_g}{\partial t}=0, \quad (11)$$

where $G_w=\dfrac{k_{rw}}{\mu_w\lambda_t}\left[q-\dfrac{kk_{ro}}{\mu_o}(\rho_w-\rho_o)g-\dfrac{kk_{rg}}{\mu_g}(\rho_w-\rho_g)g\right]$,

$$G_g=\frac{k_{rg}}{\mu_g\lambda_t}\left[q+\frac{kk_{ro}}{\mu_o}(\rho_o-\rho_g)g+\frac{kk_{rw}}{\mu_w}(\rho_w-\rho_g)g\right],$$

$$C_{ww}=\frac{kk_{rw}}{\mu_w}\left(1-\frac{k_{rw}}{\mu_w\lambda_t}\right)\frac{\mathrm{d}P_{cw}}{\mathrm{d}S_w},$$

$$C_{wg}=\frac{kk_{rw}}{\mu_w\lambda_t}\frac{k_{rg}}{\mu_g}\frac{\mathrm{d}P_{cp}}{\mathrm{d}S_g},$$

$$C_{gg}=\frac{kk_{rg}}{\mu_g}\left(1-\frac{k_{rg}}{\mu_g\lambda_t}\right)\frac{\mathrm{d}P_{cg}}{\mathrm{d}S_g},$$

$$C_{gw}=\frac{kk_{rg}}{\mu_g\lambda_t}\frac{k_{rw}}{\mu_w}\frac{\mathrm{d}P_{cw}}{\mathrm{d}S_w},$$

and $\lambda_t=\dfrac{k_{rw}}{\mu_w}+\dfrac{k_{ro}}{\mu_o}+\dfrac{k_{rg}}{\mu_g}$.

The above equations are parabolic if the second-order terms involving the C functions are finite, and this leads to the need for two upstream and two downstream boundary conditions to determine solutions for S_w and S_g. The form of the equations in this case is made more difficult by the fact that $k_{ro}=k_{ro}(S_w,S_g)$ in Stone's methods (or in any useful correlation for three-phase relative permeabilities). This implies the need to expand the derivatives of the C functions by use of the chain rule for partial differentiation. Accurate solutions of Eqs. 10 and 11 have not been published, nor are there any three-phase benchmark results with capillary pressure and gravity terms included against which numerical simulators can be tested.

By analogy with two-phase flow, we would expect Eqs. 10 and 11 to converge to a hyperbolic system as the capillary pressures become negligibly small, with shock-front formation involved for both water and gas saturations. The existence of a hyperbolic problem for noncapillary displacement is also a desirable property on physical grounds. The existence of positive real characteristics implies that the solution is determined uniquely by two upstream boundary conditions, one for gas and one for water flow. A reasonable physical expectation is that the results of a three-phase displacement experiment run at high rates (i.e., negligible capillary pressure or gravity effects) should be determined solely by the initial saturation distribution and by control of the fractional flow rates of the phases at the inlet face. A nonhyperbolic problem implies a need for boundary conditions for a problem that is elliptic in some region of space and time (i.e., complex eigenvalues). To make this system stable and well-posed would require a boundary condition to be imposed at later times to control the solution at earlier times, which is not physically plausible. Without such a future time condition, the system will be unstable in the elliptic region. The existence of small second-order terms in Eqs. 10 and 11 would remove the difficulty by introducing a parabolic behavior. By analogy with two-phase flow, it is clearly preferable for the noncapillary three-phase case to be hyperbolic everywhere in the solution space. In the absence of these conditions, we are faced with logical difficulties in establishing the accuracy of numerical solutions for noncapillary three-phase flow and also for the inversion problem associated with deducing three-phase relative permeabilities from dynamic flow tests. As we will show, however, the desired hyperbolic character is not necessarily guaranteed everywhere.

Conditions for Real Characteristics

The displacement equations are simplified by omitting gravity and capillary pressure terms; thus, Eqs. 10 and 11 reduce to

$$F_{ww}\frac{\partial S_w}{\partial x}+F_{wg}\frac{\partial S_g}{\partial x}+\frac{S_w}{\partial t}=0 \quad\text{(12)}$$

$$\text{and } F_{gw}\frac{\partial S_w}{\partial x}+F_{gg}\frac{\partial S_g}{\partial x}+\frac{\partial S_g}{\partial t}=0, \quad\text{(13)}$$

$$\text{where } F_{\ell m}=\frac{\partial}{\partial S_m}(F_\ell),F_\ell=\frac{k_{r\ell}}{\mu_\ell\lambda_t}.$$

For convenience we used dimensionless space and time so that q and ϕ are subsumed in the definition of time.

The characteristic roots of Eqs. 12 and 13 are given by

$$\begin{vmatrix} F_{ww}-v & F_{wg} \\ F_{gw} & F_{gg}-v \end{vmatrix}=0; \quad\text{(14)}$$

$$\text{that is, } (F_{ww}-v)(F_{gg}-v)-F_{gw}F_{wg}=0. \quad\text{(15)}$$

The roots will be real if

$$(F_{ww}-F_{gg})^2+4F_{gw}F_{wg}\geqslant 0. \quad\text{(16)}$$

With the definitions of F_w, F_g, and λ_t and the phase mobility defined as $\lambda_\ell=k_{r\ell}/\mu_\ell$,

$$F_{ww}=\frac{1}{\lambda_t}\frac{d\lambda_w}{dS_w}-\frac{\lambda_w}{\lambda_t^2}\frac{\partial\lambda_t}{\partial S_w},\ F_{wg}=-\frac{\lambda_g}{\lambda_t^2}\frac{\partial\lambda_t}{\partial S_g},$$

$$F_{gg}=\frac{1}{\lambda_t}\frac{d\lambda_g}{dS_g}-\frac{\lambda_g}{\lambda_t^2}\frac{\partial\lambda_t}{\partial S_g},\ F_{gw}=-\frac{\lambda_w}{\lambda_t^2}\frac{\partial\lambda_t}{\partial S_w},$$

$$\frac{\partial\lambda_t}{\partial S_w}=\frac{\partial\lambda_o}{\partial S_w}+\frac{d\lambda_w}{dS_w},\text{ and }\frac{\partial\lambda_t}{\partial S_g}=\frac{\partial\lambda_o}{\partial S_g}+\frac{d\lambda_g}{dS_g}.$$

A sufficient condition to satisfy Eq. 16 is $F_{gw}F_{wg}\geqslant 0$; i.e., the system is hyperbolic if

$$\lambda_g\lambda_w\left(\frac{\partial\lambda_o}{\partial S_w}+\frac{d\lambda_w}{dS_w}\right)\left(\frac{\partial\lambda_g}{\partial S_g}+\frac{d\lambda_g}{dS_g}\right)\geq 0. \quad\text{(17)}$$

$\partial\lambda_o/\partial S_w$ and $\partial\lambda_o/\partial S_g$ will normally be negative, whereas $d\lambda_w/dS_w$ and $d\lambda_g/dS_g$ are positive. If $\mu_o\gg\mu_g$ and $\mu_o\gg\mu_w$ are satisfied, then the terms in parentheses are likely to be positive over most of the saturation space because $\lambda_w\gg\lambda_o$ and $\lambda_g\gg\lambda_o$. For smaller values of oil viscosity so that $\lambda_o\sim\lambda_w$, however, the first term in parentheses in particular can be negative, especially at low water saturations where $\partial\lambda_w/\partial S_w\rightarrow 0$ may occur and $\partial\lambda_o/\partial S_w$ may be large and negative. This situation implies a need to evaluate the complete discriminant in Eq. 16, which may be expressed in the form

$$G\equiv\left[\left(\frac{d\lambda_w}{dS_w}-\frac{d\lambda_g}{dS_g}\right)\lambda_t-\lambda_w\frac{\partial\lambda_t}{\partial S_w}+\lambda_g\frac{\partial\lambda_t}{\partial S_g}\right]^2$$

$$+4\lambda_w\lambda_g\frac{\partial\lambda_t}{\partial S_w},\quad\frac{\partial\lambda_t}{\partial S_g}\geq 0. \quad\text{(18)}$$

The function G is evaluated for a number of data sets for $k_{rw}(S_w)$, $k_{row}(S_w)$, $k_{rg}(S_g)$, $k_{rog}(S_g)$, and with various assumptions on the choice of Stone's method for $k_{ro}(S_g,S_w)$ in a later section.

Conditions for Positive Characteristics

We will assume for the moment that the roots of Eq. 15 are real; i.e., Eq. 18 is satisfied everywhere. We also wish these real roots to be positive because this implies that the hyperbolic problem is determined by two upstream boundary conditions. A negative root would imply the need for a downstream boundary condition[12] on one of the saturations, which is physically implausible. Furthermore, a negative root would suggest a discontinuity in the required boundary conditions as we move from two-phase to three-phase flow with that particular phase.

Returning to Eq. 15, the roots are positive if

$$F_{ww}+F_{gg}\pm\sqrt{[(F_{ww}+F_{gg})^2-4(F_{ww}F_{gg}-F_{wg}F_{gw})]}\geq 0. \quad\text{(19)}$$

The term in brackets is positive under our assumption of real roots. If the definitions of the fractional flows are used, Eq. 19 is satisfied if

$$\left(\lambda_t\frac{d\lambda_w}{dS_w}-\lambda_w\frac{\partial\lambda_t}{\partial S_w}\right)\left(\lambda_t\frac{d\lambda_g}{dS_g}-\lambda_g\frac{\partial\lambda_t}{\partial S_g}\right)-\lambda_w\lambda_g\frac{\partial\lambda_t}{\partial S_w}\frac{\partial\lambda_t}{\partial S_g}\geq 0. \quad\text{(20)}$$

Substituting for the partial derivatives for λ_t, after some algebra, we obtain

$$H\equiv k_{ro}\frac{dk_{rw}}{dS_w}\frac{dk_{rg}}{dS_g}-k_{rg}\frac{dk_{rw}}{dS_w}\frac{\partial k_{ro}}{\partial S_g}-k_{rw}\frac{dk_{rg}}{dS_g}\frac{\partial k_{ro}}{\partial S_w}\geqslant 0. \quad\text{(21)}$$

The expression for H is independent of the viscosities and is a function of only the relative permeabilities. The second two terms will usually be positive because $\partial k_{ro}/\partial S_g$ and $\partial k_{ro}/\partial S_w<0$ will occur over most of the saturation space, while dk_{rg}/dS_g and dk_{rw}/dS_w are always positive. However, near to S_{orm}, we can have $\partial k_{ro}/\partial S_g>0$. An algebraic proof that H is still positive can be readily demonstrated for Larson-type relative permeabilities. The values of H are determined from direct numerical solution by use of selected experimental relative permeabilities in a later section.

Mathematical Representation of Two-Phase Relative Permeabilities

Because we wish to demonstrate mathematical properties of Stone's methods, particularly in the regions close to S_{orm}, it is desirable to represent the two-phase relative permeabilities by a smooth mathematical functional behavior that has continuous derivatives. Although there have been a number of attempts in the petroleum literature to find functional forms that, with their parameters, are applicable to all the shapes that typify measured relative permeabilities, no class of functional representation has been universally acceptable. We have examined the Chierici[14] exponential correlations and, although the success in matching experimental data was not as consistent as Chierici found, it was evident that this type of correlation does perform better than the simple Larson power-law expressions in most cases. We have used modified forms of Chierici's exponential functions as expressed in the following equations ($\ell=w,g$):

$$k_{ro\ell}=k_{rocw}\exp\left[-A_{o\ell}\left(\frac{S_\ell-S_{\ell c}}{S_{\ell t}-S_\ell}\right)^{\alpha_{o\ell}}\right] \quad\text{(22)}$$

$$\text{and } k_{r\ell}=k_{r\ell ro}\exp\left[-A_\ell\left(\frac{S_\ell-S_{\ell c}}{S_{\ell t}-S_\ell}\right)^{-\alpha_\ell}\right], \quad\text{(23)}$$

where $S_{wt}=1-S_{orw}$ and $S_{gt}=1-S_{org}-S_{wc}$. In Eqs. 22 and 23, the parameters k_{rocw}, k_{rgro}, and k_{rwro} define the endpoint relative permeabilities of the oil, gas, and water phases, respectively, and each curve has a shape governed by a two-parameter choice (A_i, α_i).

In comparing the performance of Stone's Methods 1 and 2, we are particularly concerned with the behavior of the two models in the region of low k_{ro} values—i.e., toward the $S_{orm}(S_g)$ limits. The behavior in this region is necessarily an extrapolation of the two sets of two-phase curves from which k_{ro} is derived; i.e., $k_{row}(S_w)$ and $k_{rog}(S_g)$. Thus, in Eqs. 1 and 2, we expect the performance of $k_{ro}(S_w,S_g)$ near S_{orm} to be sensitive to the slopes of the k_{row}, k_{rog} curves near S_{orw} and S_{org}. Unfortunately, the slopes of experimental relative permeability curves near their endpoints (e.g., asymptotic to the limit where no more oil is produced from a core

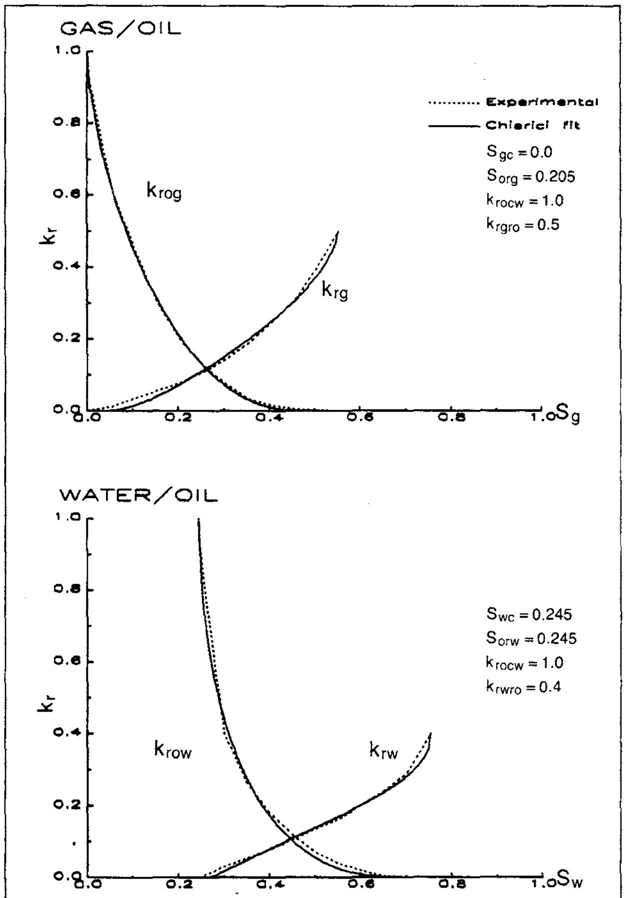

Fig. 3—Two-phase relative permeabilities for NS-1.

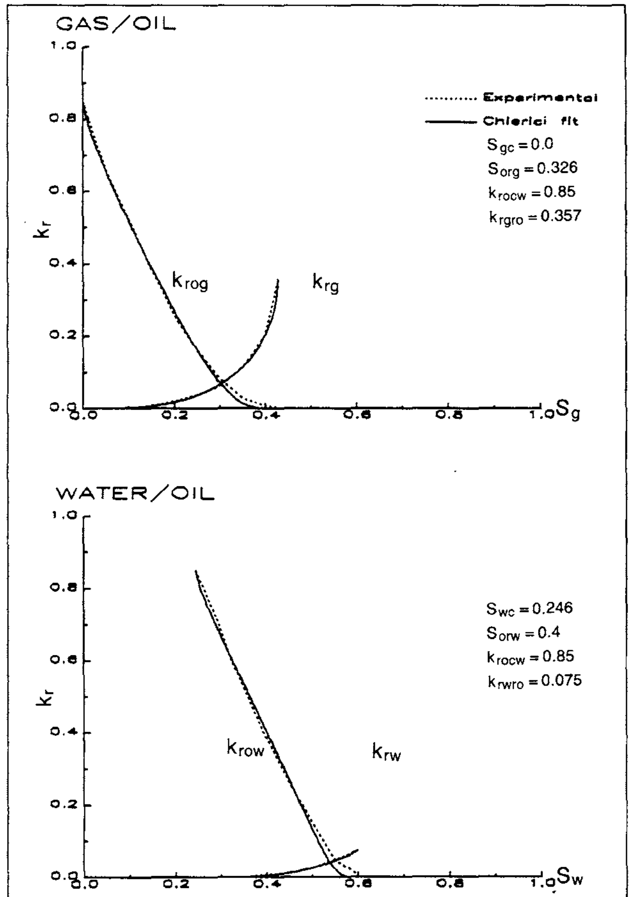

Fig. 4—Two-phase relative permeabilities for Saraf *et al.*[11]

test) are often ill-defined. The Chierici-type expressions defined above cause dk_{row}/dS_w and dk_{rog}/dS_g to tend smoothly to zero as $S_w \rightarrow 1-S_{orw}$ or $S_g \rightarrow 1-S_{org}-S_{wc}$. This is probably reasonable, but may be a factor influencing the results that follow.

We studied a number of sets of "measured" curves for two-phase gas/oil and water/oil relative permeabilities determined from an experimentalist's interpretation obtained by hand curve drawing through the normal scatter of measurements derived from laboratory core tests. The data used refer to a U.K. land-based field and to three North Sea fields where consistent water/oil and gas/oil measurements were available and to three of the preferred sets of measurements dealt with in Ref. 2. In the last cases, inferred three-phase oil relative permeabilities were also adduced from the three-phase measurements by deploying CPS-1 contour fitting to the experimental points. No three-phase measurements were available for North Sea fields, so for the present, the judgment on the validity of Stone's methods rests primarily on the measurements used in Ref. 2, which are the experiments reported by Saraf and Fatt,[9] Hosain,[10] and Saraf *et al.*[11] Typical examples of comparisons of the two-phase results with the Chierici-type fits to these "measured" curves for two sets of experiments are shown in Figs. 3 and 4. The fitted parameters for all the cases are summarized in Table 1. The fitted curves are generally very good, with the possible exception of the Hosain data (Fig. 5), for which the Chierici k_{ro} curves move somewhat too close to the axis in the asymptotic region. It is doubtful, however, whether the error is outside the accuracy range of the measurements. We have taken these results as a sufficiently representative set of analytic fits to "measured" relative permeabilities to allow further analysis of the mathematical and physical characteristics of Stone's methods.

TABLE 1—CHIERICI-TYPE FITS TO EXPERIMENTAL RELATIVE PERMEABILITIES

	UKLB1	NS-1	NS-2	NS-3	Saraf and Fatt[10]	Hosain[9]	Saraf *et al.*[11]
S_{wc}	0.264	0.15	0.1	0.226	0.2	0.15	0.246
S_{orw}	0.215	0.245	0.29	0.161	0.1	0.15	1.4
S_{gc}	0.00	0.0	0.0	0.0	0.05	0.10	0.0
S_{org}	0.18	0.205	0.32	0.236	0.3	0.05	0.326
k_{rocw}	1.0	1.0	1.0	1.0	0.58	0.88	0.85
k_{rwro}	0.105	0.4	0.458	0.50	0.64	0.5	0.075
k_{rgro}	0.625	0.5	0.285	0.355	0.07	0.58	0.357
A_w	2.275	1.047	1.320	1.763	1.647	1.323	2.281
α_w	0.5655	0.512	0.657	0.427	0.585	0.651	0.845
A_{ow}	1.989	2.906	1.407	2.559	1.143	1.644	0.906
α_{ow}	0.754	0.600	0.782	0.469	0.921	0.637	0.773
A_g	2.049	1.349	1.659	4.851	1.190	1.837	2.794
α_g	0.608	0.647	0.627	1.147	1.108	0.617	0.601
A_{og}	2.187	2.314	1.912	2.439	1.557	2.320	1.279
α_{og}	0.775	0.710	0.632	0.401	0.628	0.849	0.792

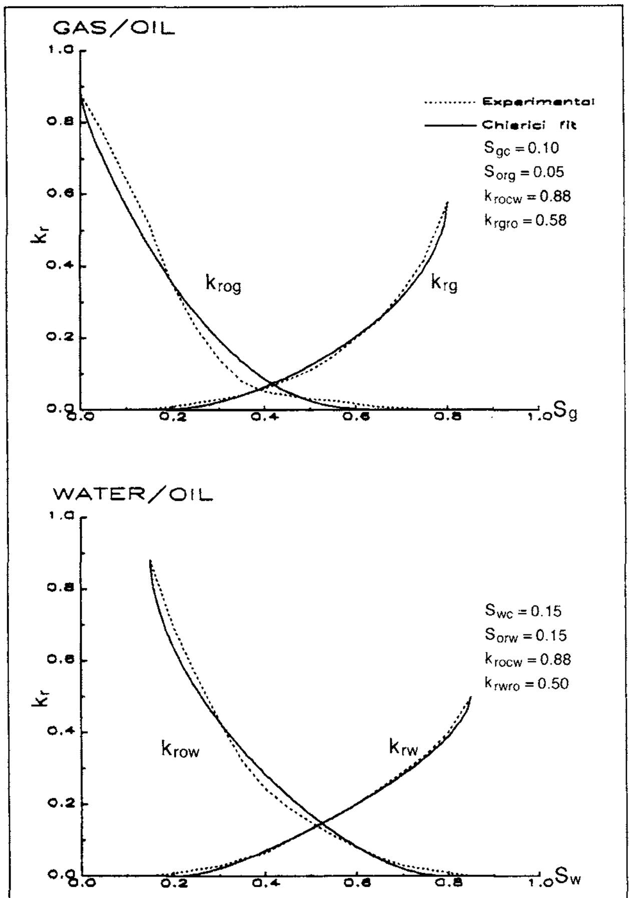

Fig. 5—Two-phase relative permeabilities for Hosain.[9]

Results and Discussion

We used the fits tabulated in Table 1 to seven sets of experimental two-phase curves with the Chierici-type parameters. In the first investigations, we assumed the viscosity ratios $\mu_o/\mu_g=40.0$ and $\mu_o/\mu_w=3.33$. The values of the functions G (Eq. 18) and H (Eq. 21) were evaluated on a 1,000×1,000 grid of S_w and S_g values in the appropriate ranges ($S_{wc}\leq S_w\leq 1-S_{orw}$ and $0\leq S_g\leq 1-S_{org}-S_{wc}$). If necessary, a more detailed search can be made in the locality identified as containing a minimum. The H values are always reasonably smooth and positive. The values of G, however, can show a sharp local minimum, which can easily be missed if the grid is not fine enough. In all cases, the dominant minimum is negative. The negative minima, referred to as elliptic regions, can occur at one or a few places, with the dominant minimum usually occurring at low gas saturation and modest water saturation—i.e., not close to S_{orm}.

The values of the minimum G values for the elliptic points are summarized in Table 2. The results of Stone's Method 1 with linear (Eq. 3) and quadratic (Eq. 5) forms for S_{orm} and Stone's Method 2 are compared. For the Hosain data set, the negative minima are very small and the result for Stone's Method 2 is difficult to find. Fig. 6 illustrates a truncated isometric projection of a typical G-function surface. Fig. 7 shows the behavior of the important term $F_{wg}F_{gw}$ (see Eq. 17) in the discriminant. This has some substantial negative regions, most of which are compensated for by the term $(F_{ww}-F_{gg})^2$ in Eq. 18. Unfortunately, the compensation does not succeed in the nonhyperbolic region of Fig. 6. As may be expected by examining the form of Eq. 17, the dominating negative region of $F_{wg}F_{gw}$ is reduced in magnitude by increasing the oil viscosity. An increase by a factor of five causes the regions of negative G values to shrink in most of the above cases, while an increase by a factor of 100 causes them to become exceptionally small in magnitude. Data Set NS-3 is very difficult and continues to show a substantial negative value.

The sensitivity to the form of parameterization of the two-phase relative permeabilities has been examined. Larson-type power-law fits to Data Set NS-1 gave a somewhat less satisfactory match to the measurements than the Chierici form, but the negative G values still persist. Inspection of Eq. 17 suggests that the slopes of the k_{row} and k_{rw} curves near $S_w=S_{wc}$, or the k_{rog} and k_{rg} curves near $S_g=0$, should be important. In fact, the k_{rg} curve is the most sensitive. Changing the exponent α_g in the Larson fit to k_{rg} from 2.0 to 1.5 gave a positive G function for this data set in the case of Stone's Method 1. It is possible that round-off errors in the double-precision arithmetic could still hide a very small negative value. The effects of changing α_w were less sensitive and required changes outside an acceptable range. Because dynamic two-phase relative permeability tests do not define the slope of the water or gas relative permeability curves near their critical saturations, there is probably freedom to bias curves in this region away from the experimentalists' extrapolation of the measurements. It is interesting that the elliptic behavior tends to occur at relatively high oil saturations where all Stone's variants are in reasonable agreement (with measurements[2]). The sensitivity to the k_{rg} curve suggests that the problem lies with the behavior of the total mobility rather than with the mathematical form for the oil relative permeability.

The usefulness of the cubic form for S_{orm} given in Eq. 8 is not easily evaluated in view of the uncertainty in the inferred behavior of S_{orm} associated with CPS-1 fitting to the limited available measurements discussed in Ref. 2. Reviewing the results presented in Ref. 2 suggests that a cubic form could be better than either linear or quadratic forms, particularly in the case of the Hosain and Saraf *et al.* measurements. Comparisons between inferred and predicted behaviors of S_{orm} with linear, quadratic, and cubic forms for these two sets of measurements are shown in Figs. 8 and 9. The cubic form then gives excellent agreement with the fitted measurements of oil relative permeability at all saturations. For regions not too close to the S_{orm} boundary, however, the linear and cubic forms in Stone's Method 1 give very similar contours. Because these two sets of measurements give negative G values with both linear and quadratic forms, we have not evaluated the properties of the cubic expression from this standpoint.

Although the appearance of a nonhyperbolic region for cases with low oil viscosity might be considered somewhat alarming, it is by no means clear what the practical implications of having a narrow range of unstable saturations are. Inclusion of the capillary pressure terms in Eqs. 10 and 11 should suppress the instability, and the inclusion of gravity-segregation effects in a 3D flow may also be stabilizing. Similarly, in a finite-difference scheme with upstream

TABLE 2—COMPARISON OF MINIMUM G-VALUES FOR VARIOUS METHODS

	UKLB1	NS-1	NS-2	NS-3	Saraf and Fatt[10]	Hosain[9]	Saraf *et al.*[11]
Method 1, linear S_{orm}	−0.00077	−0.117	−0.022	−0.166	0.063	−0.0008	−0.013
Method 1, quadratic S_{orm}	−0.0013	−0.116	−0.021	−0.143	−0.056	−0.0006	−0.044
Method 2	−0.011	−0.195	−0.0099	−0.323	−0.043	−0.0002	−0.073
Position of minimum							
S_g	0.071	0.055	0.083	0.24	0.15	0.17	0.24
S_w	0.42	0.28	0.19	0.25	0.30	0.24	0.34

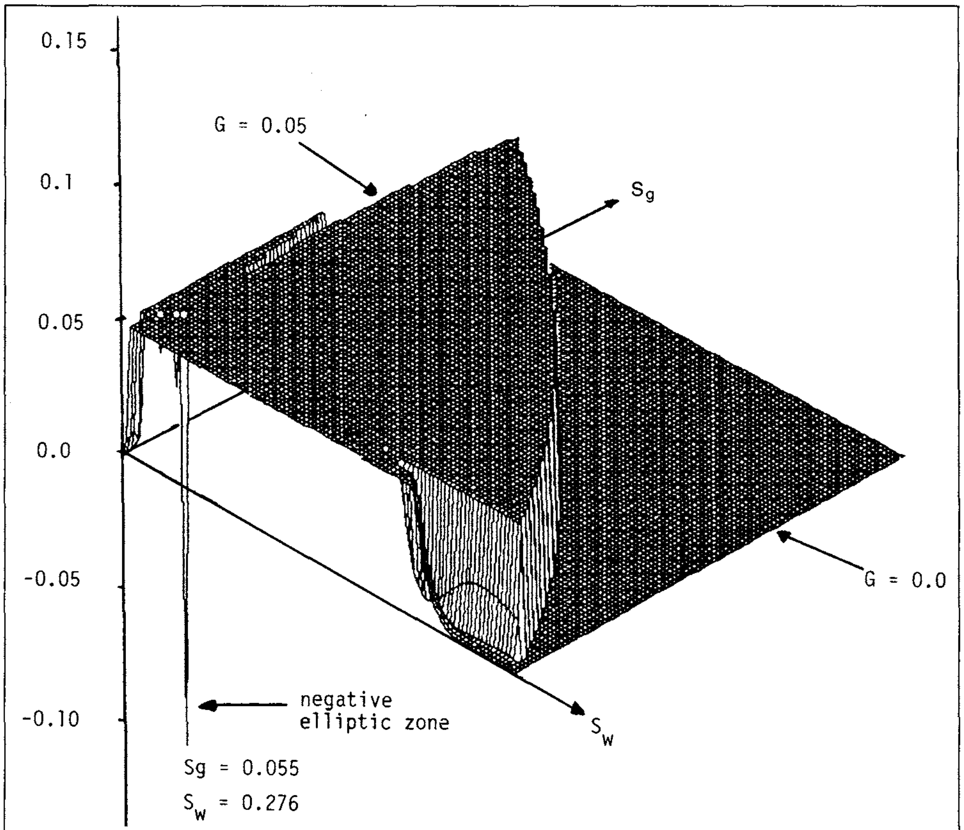

Fig. 6—Example of G behavior.

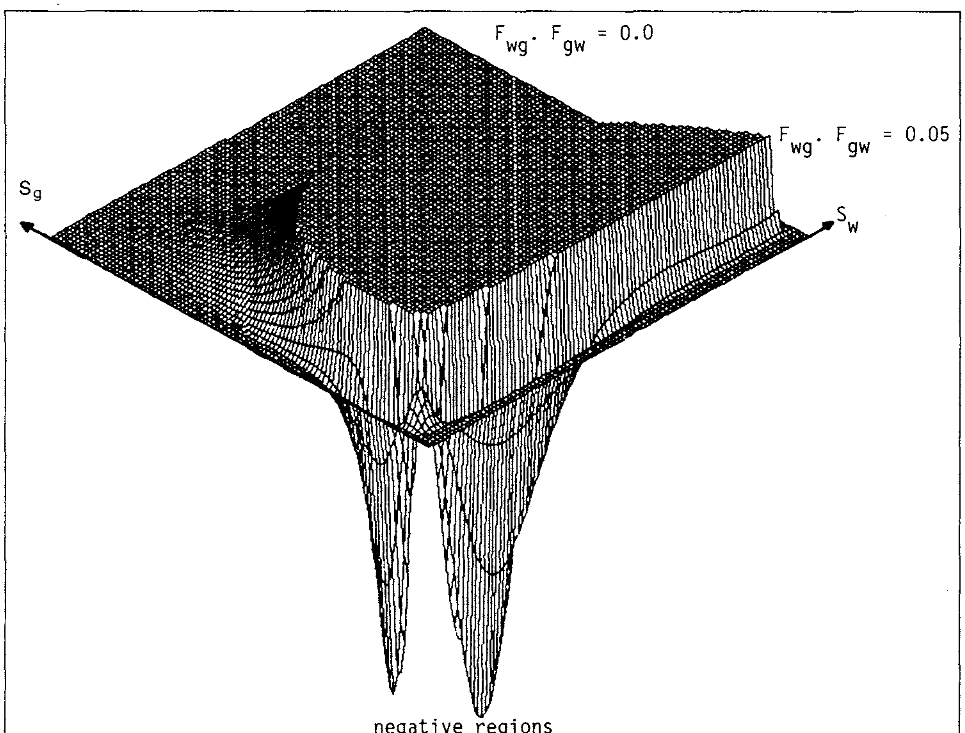

Fig. 7—Example of behavior of $F_{wg}F_{gw}$.

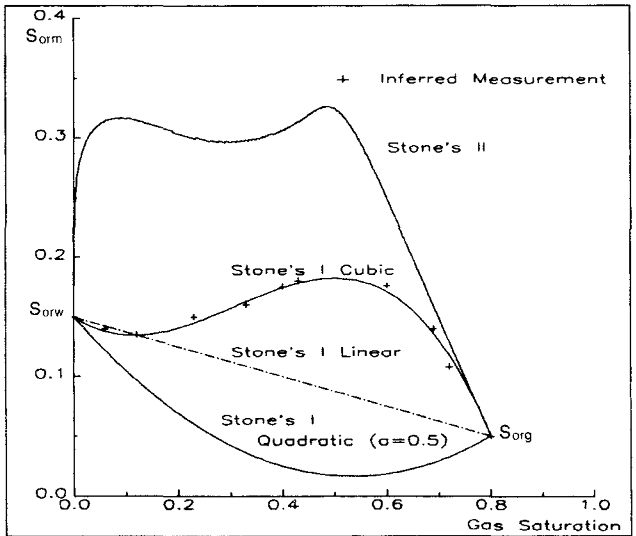

Fig. 8—Comparison of S_{orm} for Hosain's[9] data.

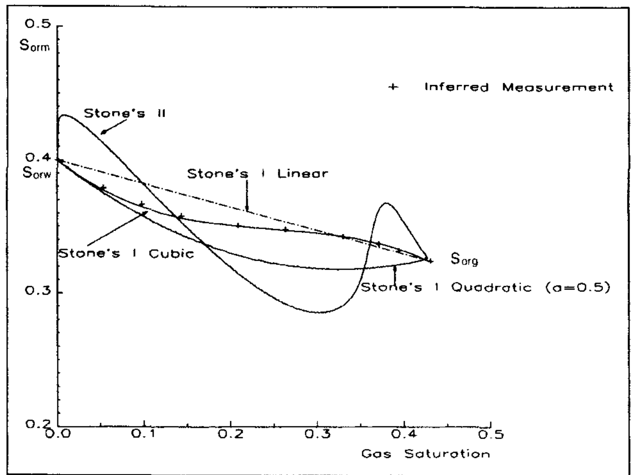

Fig. 9—Comparison of S_{orm} for Saraf *et al.*'s[11] data.

differencing, the introduction of numerical dispersion (i.e., a second-order term in the equivalent partial-differential equation) would also be expected to be stabilizing.[15] For more accurate high-order difference schemes that attempt to preserve steep shock fronts, however, the possibility exists that numerical destabilizing factors could interact with the intrinsic unstable aspect of the relative permeabilities.

The narrowness of the nonhyperbolic region(s) together with the small growth rates associated with the small magnitude of the imaginary term need to be considered. These suggest that the unstable solution variation that can be generated should be quite small before the solution crosses again into a well-resolved hyperbolic region. It is possible that such limited regions of unstable three-phase flow are physically plausible, as in Haines jumps. The circumstances under which the mathematical form of the three-phase relative permeabilities can avoid the occurrence of a nonhyperbolic region are not clear.

Conclusions

1. It would be desirable for the equations of nongravity, noncapillary, three-phase flow to be hyperbolic with positive roots. Neither of the existing choices of variants of Stone's Methods 1 or 2 satisfy the real-roots requirement over the full range of saturations. In all cases with modest oil viscosity, regions of limited extent exist where the roots cross the real axis. The magnitudes of the negative roots generally increase with lower oil viscosities, so condensate systems, for example, could pose difficulties. Stone's Method 1 with a proposed quadratic dependence for $S_{orm}(S_g)$ is marginally better than with the linear dependence, but no strong evidence exists for a preference for any of the methods for avoiding imaginary roots. For a given application, a pragmatic approach is to examine the sensitivity to the k_{rw} and k_{rg} curves near the critical saturations and to choose their gradients to remove or reduce the unwanted imaginary zones.

2. Extended forms of Chierici exponential functions fit a range of measured two-phase relative permeabilities successfully. When these are combined with a cubic representation of the S_{orm} parameter, the agreement with inferred three-phase experimental results from Hosain and Saraf *et al.* is excellent. The linear form for S_{orm} is acceptable within the accuracy of the measurements, but the postulated quadratic form cannot be shown to have a clear advantage within the experimental limitations. Careful steady-state measurements of ROS for simultaneous gas and water flows are needed to confirm the most appropriate representation. Recently reported measurements on Berea sandstone by Oak *et al.*[16] may be useful for this purpose.

3. A number of features of three-phase flow require further theoretical investigations as extensions of the results given in this paper. The mathematical implications of nonhyperbolic regions and how these should be dealt with formally in a rigorous solution are not completely understood (Ref. 15 helps in this regard). The particular qualities of three-phase relative permeabilities needed to avoid such occurrence have not been systematically analyzed. Having a highly compressible gas phase may influence the behavior. In 3D reservoir calculations, it is often necessary to use pseudorelative permeabilities to compensate for unrepresented details of the flow behavior within a coarse gridblock. Partial gravity segregation of the fluids and heterogeneities within the porous medium will influence the pseudorelative permeabilities, which will consequently have directional components. This will also alter the mathematical properties of the equations.

Nomenclature

a = wetting parameter
A = parameter in two-phase correlation
F = fractional flow function
g = component of gravity
G = gravity function or discriminant function
H = positivity function
k = absolute permeability
k_r = relative permeability
k_{rgro} = relative gas permeability at ROS
k_{rocw} = two-phase oil relative permeability at $S_o=1-S_{wc}$
k_{rog} = two-phase oil relative permeability for oil/gas system
k_{row} = two-phase oil relative permeability for oil/water system
k_{rwro} = relative water permeability at ROS
P_c = capillary pressure
q = total flow rate
S = saturation
S_{gtr} = trapped gas saturation
S_{org} = ROS to gas
S_{orm} = ROS in Stone's Method 1
S_{orw} = ROS to water
S_{wc} = connate water saturation
t = time
x = position coordinate
α = exponent in two-phase correlation or variable in S_{orm} relation
ϵ = parameter in S_{orm} relation
η = parameter in S_{orm} relation
λ = phase mobility
λ_t = total mobility
μ = viscosity
ρ = density
v = eigenvalue
ϕ = porosity

Subscripts

c = critical or connate
g = gas
ℓ,m = phase indices
o = oil
r = relative or residual
t = total
w = water

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