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Three-Phase Relative Permeability Correlations

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ABSTRACT

Several correlations have been proposed for prediction of three-phase relative permeability from two-phase data or from saturation/capillary-pressure relationships. These include the models of Stone, Hirasaki, Corey et al., Naar and Wygal, Land, Aleman, and Parker et al. This paper compares predictions of these models with predictions of two additional models (saturation-weighted interpolation and true-linear interpolation). The comparison is made using all the published three-phase experimental relative permeability data complete enough for application of the models.

The comparison shows that the models are often not very good predictors of the experimental data. This points out a need for better relative permeability models in cases where three-phase flow may have a significant effect. In most cases, straight-line interpolation or saturation-weighted interpolation between the permeabilities at the two-phase boundaries of the three-phase flow region provided a better fit of the experimental data than did the theoretically-based models.

The paper also demonstrates the utility of comparing the effects of different relative permeability models before using them in a reservoir simulation. Sometimes the result of a simulation can be heavily biased by the choice of relative permeability model used.

INTRODUCTION

Three-phase relative permeability measurements and correlation methods have been reported in the literature since 1941. This paper lists the sources of published three-phase relative permeability data and compares the simple correlation methods which

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References and illustrations at end of paper.

have been proposed for prediction of three-phase relative permeability. Two additional interpolation methods are proposed and compared to the published methods.

DISCUSSION

Data Sources for Three-Phase Relative Permeability

Three-phase (gas/oil/water) relative permeability data for water-wet sandpacks have been reported by Leverett and Lewis[1], Reid[2], Hosain[3], Snell[4] and Ivanov[5]. Three-phase data for water-wet cores have been reported by Caudle, et al.[6], Corey, et al.[7], Sarem[8], Donaldson and Dean[9], Saraf[10,11], Guckert[12], Saraf, et al.[13], van Spronsen[14], Holmgren and Morse[15], Slack and Ehrlich[16] and Schneider and Owens[17]. Very limited three-phase data for oil-wet cores (gas and water relative permeabilities at constant, non-flowing oil saturation) were published by Schneider and Owens[18]. Descriptions of the test procedures and calculation methods are available in the original references. Donaldson and Kayser[19] provide a summary of the test protocols and results for most of the data published prior to 1980.

The data for water-wet cores and for sandpacks show generally consistent behavior; isoperms (contours of constant phase relative permeability) for gas and water primarily depend on the gas or water saturation respectively, and are weak functions of the saturations of other phases present. The permeability to each phase is clearly affected by the saturation history when there is hysteresis between the imbibition and drainage curves for that phase. These data indicate that a strongly wetting phase (water) and a non-wetting phase (gas) are affected little by interactions with other phases (except for the physical obstruction caused by the presence of the other phases). An intermediate wetting phase (oil) appears to be more influenced by interactions with the other phases. The nature of the interaction is not clear, however. In some cases, the

dependence of oil relative permeability on the saturations of the other phases is much less apparent, than in others. In other cases, the scatter of the experimental data obscures the dependencies (for gas and water, as well as for the oil phase). It is instructive (although omitted here for lack of space) to plot the gas, oil and water relative permeabilities versus gas, oil and water saturation respectively (as well as on ternary saturation diagrams) for each of the data sources listed above. These plots show that the data scatter is generally comparable for measurements of gas and of oil relative permeability. This indicates that oil and gas flow are dependent on (or independent of) the saturations of the other phases to about the same extent. With this comparison, some of the effects (such the concave or convex shapes of oil isoperms as compared to gas isoperms) are seen to depend largely on subjective interpretation.

There are insufficient data for oil-wet systems to determine whether the oil, gas and water isoperms are significantly different from isoperms of equally-wetting phases in water-wet porous media.

In this study correlations for prediction of oil relative permeability are compared to each other. Each of the above data sets for which tabular data are available (or could be reasonably interpolated from published figures) is used as a basis for the comparison. Because the data quality is not the same for all data sets, only comparisons of models within each set (not between sets) are meaningful.

Correlation Methods and Relative Permeability Models

Several models have been proposed to fit the observed experimental relative permeability data. These range from simple curve fits to detailed statistical or mechanistic models. At present the simplest models are the most widely used because of the limited experimental data to justify the use of a more complex model. Some of the more complex models such as network or percolation theory models offer the hope of understanding the processes governing the competing flow of fluids in porous media. The emphasis here will be on a comparison of the simple models, with an assessment of their goodness-of-fit to the published data.

Corey, Rathjens, Henderson and Wyllie

The first model of relative permeability in three-phase flow was published by Corey. et al.[7]. The model was based on the concept of approximating the flow paths through a rock by the equivalent hydraulic radius of a bundle of capillary tubes. A tortuosity correction (developed by Burdine[20]) was included to account for the differences in path length of tubes of different sizes. See Brooks and Corey[21] for a complete description of the derivation for wetting and non-wetting phase relative permeability. Corey's model assumed the wetting and non-wetting phase relative permeabilities to be independent of the saturations of the other phases, in agreement with limited experimental data. (This assumption is used in most of the models developed subsequently).

The intermediate-wetting phase occupies the flow channels which are intermediate in size between those occupied by water (wetting) and gas (non-wetting). Its flow may be interfered with by both the wetting and non-wetting fluids (represented by the limits of integration in the following equation). Relative permeability to the intermediate-wetting phase (oil) is proportional to the area of the pores occupied by this phase and to the relative saturation of the phase:

$$k_{ro} = \frac{(S_l - S_w)^2}{(1 - S_{lr})^2} \frac{\int_{S_w}^{1-S_g} \frac{dS}{P_c^2}}{\int_0^1 \frac{dS}{P_c^2}} \quad \ldots\ldots\ldots\ldots(1)$$

The ratio of integrals represents the normalized area of flow channels containing the intermediate-wetting fluid. The saturation function multiplying the integrals is Burdine's tortuosity correction relating connectivity of fluid-filled pores to relative saturation of the phase. Corey solved this equation assuming the relation between saturation and drainage capillary pressure could be represented by a function of the form

$$\left(\frac{P_b}{P_c}\right)^\lambda = \begin{cases} \dfrac{(S - S_{lr})}{(1 - S_{lr})} & \text{for } S \geq S_{lr} \\ 0 & \text{for } S < S_{lr} \end{cases} \quad .. \ (2)$$

with $\lambda = 2$. Brooks and Corey[21] later showed this relation described the saturation:capillary pressure data for several different types of homogeneous, isotropic porous media, with representative λ values in the range 1.8 to 7.3.

The solution to Equation 1, with the assumed saturation:capillary pressure relationship ($\lambda = 2$ in Equation 2) is given by

$$k_{ro} = \frac{(S_l - S_w)^3}{(1 - S_{lr})^4} (S_w + S_l - 2S_{lr}) \quad \ldots\ldots (3)$$

The equation variables are the water and total liquid saturations, with a free parameter (S_{lr}) corresponding to the residual liquid saturation for a two-phase gas/oil displacement. This parameter is obtained by extrapolation of the gas relative permeability curve plotted as a function of normalized saturation (Corey[22]). S_w is implicitly assumed to be greater than or equal to S_{lr} in the derivation of

Equation 3 This assumption was not made in the solution of Equation 1 used for the present study. In addition, Equation 3 predicts the oil permeability will be greater than zero for any case where (1) the oil saturation is positive, and (2) the liquid saturation is greater than S_{lr}. To force the method to predict k_{ro} values consistent with measured two-phase data, the saturation difference $(S_l - S_w)$ in the numerator of the tortuosity function was modified to $(S_o - S_{or})$.

An advantage of Corey's method is that it requires only a single suite of gas/oil relative permeability data (measured at a constant water saturation) to predict gas and oil relative permeabilities for all saturations. However, the model cannot be adjusted to force the end points of the isoperms to correspond to both measured two-phase oil/gas and oil/water relative permeabilities.

The assumed relationship between capillary pressure and saturation has been shown (Brooks and Corey[21]) to be approximately correct; the model can easily be solved for other power-law relationships of saturation to capillary pressure.

Naar and Wygal

Naar and Wygal[23] extended the two-phase imbibition permeability model of Naar and Henderson[24] to the case of three-phase imbibition. This model is also based on the concept of flow in straight capillaries, but with the added complications of random interconnections of pores and of storage capacity which makes possible blocking (trapping) of the non-wetting phase by the invading wetting fluid. Relative permeabilities to the wetting and non-wetting phases are considered to depend on the wetting or non-wetting phase saturations respectively, while the intermediate-wetting phase relative permeability depends on all three phase saturations.

A Corey-Burdine type of equation (as in Equation 1 above) was written relating the intermediate-wetting (oil) phase relative permeability to saturations and capillary pressure.

$$k_{ro} = \frac{(\frac{1-S_{fw}}{1-S_{wc}})^3 S_{of}^{**} \int_0^{S_{of}^{*}} \frac{(S_{of}^{**} - S^{**})\, dS^{**}}{P_c^2}}{\int_0^1 \frac{(1-S^{*})\, dS^{*}}{P_c^2}} \quad ..(4)$$

For Corey's consolidated sandstone saturation:capillary pressure relationship (Equation 2, with $\lambda = 2$) Equation 4 reduces to

$$k_{ro} = S_{of}^{*\,3}(S_{of}^{*} + 3S_{fw}^{*}) \quad ....(5)$$

Because of the assumptions made in deriving the Naar/Wygal model, a single parameter (irreducible water saturation) completely characterizes the isoperms. Thus there is no method for adjusting the curve end points to fit either gas/oil data at an irreducible water saturation or water/oil data at zero gas saturation. In principle these adjustments should be accounted for in the saturation:capillary pressure relationship. Saturation:capillary pressure relationships other than the "consolidated sandstone" one can be used in the model, resulting in different dependencies.

Land

Land[25] developed an improved model for imbibition relative permeability which takes into account the effect of initial saturations. This model assumes (based on published experimental data) the maximum residual hydrocarbon saturation is constant, whether the hydrocarbon is gas or oil, and that the residual hydrocarbon is related to the initial hydrocarbon saturation by

$$\frac{1}{S_{hr}^{*}} - \frac{1}{S_{hi}^{*}} = \frac{1}{S_{hr(max)}^{*}} - 1 = C \quad ....(6)$$

Land's model is similar to the model of Naar and Wygal[23]; however, Land points out that Naar and Wygal erroneously placed the trapped gas saturation in the largest pores rather than the smaller pores more likely to be invaded first by the wetting phase.

Land presented the model as a set of integral equations giving the dependence of relative permeability on saturation, saturation history and the relation between saturation and capillary pressure. For the intermediate-wetting phase (oil) the relationship is

$$k_{ro} = \frac{S_{of}^{*\,2}}{\int_0^1 \frac{dS^{*}}{P_c^2}} \left[\int_{S_w^{*}+S_{ob}^{*}}^{1-S_{gf}^{*}} \frac{dS^{*}}{P_c^2} - S_{hr(max)}^{*\,2} \int_{1-S_{gi}^{*}}^{1-S_{gf}^{*}} \frac{dS^{*}}{(1-(1-S_{hr(max)}^{*})^2)P_c^2} \right] ...(7)$$

The superscript (*) indicates the saturation is normalized with respect to the "effective pore space," $1-S_{wc}$. Similar equations represent the relative permeability to the wetting and non-wetting phases. The first integral in the numerator represents the area of pores available for oil flow. The range of oil saturation excludes oil trapped by water but includes the region of gas trapped by oil. The

second integral represents the reduction in this area available for oil flow due to the trapped gas.

An analytic solution was derived using the saturation:capillary pressure relationship of Equation 2 (with $\lambda = 2$)

$$k_{ro} = S_{of}^{*2} \left[(1-S_{gf}^*)^2 - (S_w^* + S_{ob}^*)^2 \right.$$

$$- \frac{2}{c^2} \{ \ln \left[\frac{S_{hr(max)}^* + (1-S_{hr(max)}^*)S_{gf}^*}{S_{hr(max)}^* + (1-S_{hr(max)}^*)S_{gi}^*} \right]$$

$$+ \frac{1}{S_{hr(max)}^* + (1-S_{hr(max)}^*)S_{gf}^*}$$

$$\left. - \frac{1}{S_{hr(max)}^* + (1-S_{hr(max)}^*)S_{gi}^*} \} \right] \quad \ldots (8)$$

The model qualitatively predicts the increase in wetting phase relative permeability (at a given saturation) caused by trapping of the non-wetting phase. Because it takes into account the fluid saturation history (at least in approximating the trapping of the intermediate- and non-wetting phases) it offers the best approach to date to modeling relative permeability with the bundle-of-capillaries model.

Even if the relationship between saturation and capillary pressure for porous media follows Equation 2, $\lambda = 2$ will not always provide the best fit to experimental data. Analytic solutions of Equation 7 are possible for other values of λ. Land suggested numerical integration of the equations, which would most likely be impractical for use in a reservoir simulator. An alternative method is to calculate the value of the integrals at regularly-spaced values of λ and use a first- or second-order Taylor series method to interpolate the value at the required λ. (A simple recursion relationship can be derived for the values of the integral at half-integer values of $2/\lambda$).

Parker, Lenhard, and Kuppusamy

Parker, et al.[26] proposed a model for intermediate-wetting phase relative permeability based on a model for two-phase relative permeability developed by Mualem[27]:

$$k_{ro} = \left[\frac{S_o - S_{or}}{1 - S_{wc}} \right]^{1/2} \left[\frac{\int_{S_w}^{1-S_g} \frac{dS}{P_c}}{\int_0^1 \frac{dS}{P_c}} \right]^2 \quad \ldots (9)$$

In this model the square-root saturation function takes the place of Burdine's[20] tortuosity correction. The permeability to a particular phase in Mualem's permeability model is assumed to be proportional to the square of the mean hydraulic radius of the pores occupied by that phase; this contrasts to the model of Corey, et al.[7] in which the phase permeability was assumed proportional to the mean hydraulic area, or the mean square hydraulic radius. A comparison of the models with experimental data has been presented by van Genuchten and Nielsen[28], showing that Mualem's model is applicable to a wider variety of soils than is the Corey-Burdine model.

Substituting a saturation:capillary pressure relationship derived by van Genuchten[29]

$$S^* = \{ 1 + (\alpha P_c)^n \}^{-m} \quad \ldots (10)$$

where $m = 1-1/n$

the relationship becomes

$$k_{ro} = S_o^{*1/2} \{ (1-S_w^{*1/m})^m - (1-S_l^{*1/m})^m \}^2 \ldots (11)$$

The saturations should be normalized to the expected ranges. For predicting oil relative permeability in a water-wet medium S_o is normalized to the range of oil saturation measured in the two-phase gas/oil test; S_w^* is normalized to the range of water saturation in the two-phase water/oil test; and S_l is normalized to the range of liquid saturation in the gas/oil test (at irreducible water saturation). These ranges will provide the proper limiting behavior at the two-flowing-phase water/oil and gas/oil conditions at the ends of the oil isoperms. The fitting parameter n (and therefore m) can be obtained by a curve fit of saturation:capillary pressure data (Equation 10) or by fitting two-phase relative permeability:saturation data (Equation 11, and similar relationships for gas and water permeabilities). Use of the model is therefore limited to cases where these equations provide a satisfactory fit to the two-phase data.

Stone's Methods

The methods most commonly used for prediction of three-phase relative permeability are those pro-

posed by Stone (Method I[30] and Method II[31]). Both are probability-based models, and both assume the relative permeabilities of the wetting and non-wetting phases depend only on the saturation of the wetting and non-wetting phases respectively. Thus, the model-dependent permeability applies only to the intermediate-wetting (oil) phase.

Stone's Method I

For the oil phase, Stone's Method I[30] assumes that the flow of oil is impeded by the presence of gas and water, and that the effects of gas and water are mutually independent. Oil relative permeability is treated as a linear function of normalized oil saturation S_o^*, modified by functions β_w and β_g which depend on the water and gas saturations, respectively. Since the effects of the presence of water and gas are assumed to be independent, β_w and β_g can be determined independently. To maintain consistency at the limits of two-phase saturation (oil:water or oil:gas) β_w is chosen to be proportional to the relative permeability to oil in the presence of water at zero gas saturation, and β_g is chosen proportional to the relative permeability to oil in the presence of gas at the connate water saturation. The β's include a saturation weighting function $(1/\{1-S_i\})$, where S_i represents the normalized saturation of water or gas corresponding to β. The base permeability for calculating relative permeability is chosen as k_{ocw}, the permeability to oil at the connate water saturation ("When the normalized oil saturation S^* is 100 percent, k_{ro} also is 100 percent...")[30]. Therefore $k_{row} = k_{rog} = 1$ at zero gas saturation and connate water saturation. Unless otherwise specified, this convention is used throughout the following discussion. Thus,

$$k_{ro} = S_o^* \left\{\frac{k_{row}}{(1-S_w^*)}\right\} \left\{\frac{k_{rog}}{(1-S_g^*)}\right\} = S_o^* \beta_w \beta_g \dots\dots(12)$$

Stone suggested that the normalizing factor (the denominator) for S_i be $1 - S_{wc} - S_{om}$, where S_{wc} is the connate water saturation and S_{om} is the irreducible residual oil saturation. He cited experimental evidence indicating S_{om} should be in the range $0.25*S_{wc}$ to $0.5*S_{wc}$. The effect of using $S_{om} > 0$ is to force the residual oil saturation to be everywhere greater than or equal to S_{om}. Thus, if a fixed value of S_{om} is used it should be less than or equal to the smaller of S_{orw} and S_{org}. Use of a fixed S_{om} larger than either of these will cause a discontinuity in the predicted isoperms at one or both of the two-phase end points, which could lead to difficulty in the use of a reservoir simulator. A non-zero value of S_{om} affects the low-permeability isoperms much more than the higher-permeability isoperms.

Stone's Method II

Stone's Method II[31] assumes the total permeability (sum of oil, gas and water permeabilities) is the product of the total water/oil permeability $(k_{row} + k_{rwo})$ measured at zero gas saturation and the total gas/oil permeability $(k_{rog} + k_{rgo})$ measured at irreducible water saturation. Thus,

$$k_{ro} + k_{rw} + k_{rg} = (k_{row} + k_{rwo})(k_{rog} + k_{rgo})\dots(13)$$

Gas and water relative permeabilities are assumed to be the same in two-phase and three-phase flow, and can be determined from the two-phase data; therefore

$$k_{ro} = (k_{row} + k_{rwo})(k_{rog} + k_{rgo}) - k_{rwo} - k_{rgo}\dots(14)$$

In Equation 14 the subscripts "go" and "wo" have been retained for k_{rg} and k_{rw} to indicate the expected source of the data. Stone specified the basis for relative permeability calculations for Method II should be the permeability to oil at connate water saturation and zero gas saturation, as for Method I. Inspection of Equations 12 and 14 shows that if k_{row} and k_{rog} are not unity at the connate water saturation the equations will not predict the correct two-phase (water/oil and gas/oil) relative permeabilities as the saturation of the third phase approaches zero.

Other Versions of Stone's Methods

Dietrich and Bondor

Dietrich and Bondor[32] noted that the oil permeability at connate water saturation predicted by Stone's Method II would be correct only if k_{row} and k_{rog} "happened to be unity." This is, of course, implicit in the definition of the model, if the basis for relative permeability is k_{ocw} as implied by Stone. Dietrich and Bondor suggested re-writing the model as

$$k_{ro} = \frac{(k_{row} + k_{rwo})(k_{rog} + k_{rgo})}{k_{rocw}} - k_{rwo} - k_{rgo}\dots(15)$$

if some permeability other than k_{ocw} is used as the relative permeability basis. As long as a consistent basis is chosen for all of the relative permeabilities, Equation 15 is mathematically equal to Equation 14 with its implicit basis. Aziz and Settari[33] stated that the Dietrich and Bondor normalization method would predict k_{ro} approaching infinity for small values of k_{rocw}. However, since neither k_{row} nor k_{rog} is expected to be larger than k_{rocw}, this type of pathological behavior cannot occur.

Hirasaki

Dietrich and Bondor[32] also reported Hirasaki's modification of Stone's Method I, which can be represented as

$$k_{ro} = (k_{row} k_{rog}) - S_g (1 - k_{row})(1 - k_{rog}) \quad ..(16)$$

In this representation of the model all permeabilities are calculated relative to k_{ocw}; k_{row} depends only on the water saturation and k_{rog} depends only on the gas saturation.

Aziz and Settari

Aziz and Settari[33] suggested use of the absolute permeability as the basis for calculating relative permeability, and use of k_{ocw} as a partial normalizing factor "in order to preserve the customary definition of permeability". This results in the equation

$$k_{ro} = k_{rocw} S_o^* \{ \frac{k_{row}/k_{rocw}}{(1-S_w^*)} \}$$

$$\{ \frac{k_{rog}/k_{rocw}}{(1-S_g^*)} \}$$

$$= k_{rocw} S_o^* \beta_w \beta_g \quad ....(17)$$

for Method I, and for Method II

$$k_{ro} = [(k_{row} + \alpha k_{rwo})(k_{rog} + \alpha k_{rgo})$$

$$- \alpha k_{rwo} - \alpha k_{rgo}] \quad ....(18)$$

The effect of this normalization on Method I is to restore the implied basis (k_{ocw}) specified by Stone, no matter what relative permeability basis is actually used. Equation 18 relative permeabilities are written with a basis of k_{ocw} to illustrate more clearly the comparison between it and Equation 14. Aziz and Settari suggested $\alpha = k_{ocw}/K$. However, since there is no physical basis for the modification, it is not necessary to limit α to this value. If the value of α is 1.0, Equation 18 is identical to Equation 14. With $\alpha = 0.0$, Equation 18 approaches the form of Stone's Method I. Decreasing the value of α from 1.0 results in the prediction of greater oil permeability in the three-phase region, with no effect on two-phase oil permeability predictions. (Maximum oil relative permeabilities are found for a problem-dependent value of α greater

than zero.) Because Stone's Method II usually predicts too-low oil permeabilities and Method I (unadjusted) usually predicts too-high oil permeabilities, use of α as a free parameter is a convenient way to adjust the predicted permeabilities between these ranges.

The normalization of Stone's Method II given by Aziz and Settari was also presented by Dietrich[34] and credited to J. S. Nolen.

Fayers and Matthews

Fayers and Matthews[35] reviewed Stone's methods (I and II) as normalized by Aziz and Settari[33] and tested them against data from Corey, et al.[7], Saraf and Fatt[11], Holmgren and Morse[15] and Dalton et al. (as reported by Stone[30,31]). They suggested the use of a saturation-dependent value of S_{om} for Stone's Method I, derived from

$$S_{om} = \alpha S_{orw} + (1-\alpha) S_{org} \quad ....(19)$$

and

$$\alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}} \quad ....(20)$$

This choice of S_{om} and α reduces to $S_{om} = S_{orw}$ when gas saturation is zero and to $S_{om} = S_{org}$ at the connate water saturation. The functional description was described as a reasonable estimate, better than an arbitrary choice in the absence of residual oil data. Based on correlation of data from experiments with trapped gas saturation in water-wet sandstone, a second approximation was developed to predict S_{om} in the presence of trapped gas:

$$S_{om} = S_{orw} - 0.5*S_g \quad ....(21)$$

This was shown to provide an improved fit to the residual oil data of Holmgren and Morse[15]. Equation 21 is the mathematical equivalent of saying that the minimum residual oil saturation is given by a straight vertical line through S_{orw} on a ternary saturation diagram. It would fit the residual oil saturation data of Slack and Ehrlich[16] well also.

Fayers

Fayers[36] suggested additional forms for S_{om} (quadratic and cubic dependence on the gas saturation) in Stone's Method I. These forms could allow a better fit of the low-permeability isoperms if sufficiently-accurate experimental data were available to justify it. Examples will be shown

later illustrating the importance of this adjustment.

Aleman

Aleman[37] has suggested another saturation-dependent approximation to S_{om} for Stone's Method I:

$$S_{om} = S_{orw} \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{\alpha}$$

$$+ S_{org} \left[\frac{S_g}{1 - S_{wc} - S_{org}} \right]^{\beta} \quad \ldots\ldots\ldots\ldots(22)$$

The value of the first term must be positive, while the value of the second term must be less than or equal to S_{org}. This expression reduces to $S_{om} = S_{orw}$ at the waterflood end point saturation ($S_g = 0$, $S_o = S_{orw}$) and to $S_{om} = S_{org}$ at the gas flood end point ($S_w = S_{wc}$, $S_o = S_{org}$), thereby honoring the two-phase relative permeability end points for any values of the exponents α and β. The free parameters α and β are used to fit the curvature of the zero-oil-permeability isoperm. A wide range of adjustment for the shape of this isoperm can be obtained with suitable choices of α and β.

Aleman

Aleman, et al.[38] developed a statistical structural model for prediction of two-phase relative permeability based on a local-volume-average approach to a bundle of capillaries model. In this model, pores are randomly oriented in space but pore interconnections within the averaging volume are not recognized. The work was extended by Aleman[39] to three phase flow with the derivation of an equation which reduces to Stone's Method I if either of the two-phase relative permeabilities to the intermediate-wetting phase is a linear function of saturation. It is assumed the saturation change is in the direction of decreasing intermediate-wetting phase saturation, among other assumptions which are either explicitly stated or implied in the development of Stone's methods. Aleman's model for intermediate-wetting phase relative permeability can be written in the form

$$k_{ro} = k_{ro(I)} - \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

where $k_{ro(I)}$ is the relative permeability to oil (intermediate wetting phase) predicted by Stone's Method I and Δ is a correction term:

$$\Delta \approx \frac{S_o^*(k_{row}^*-1)(k_{rog}^*-1)(k_{row}^* \; k_{rog}^* - k_{rwo}^* \; k_{rgo}^*)}{(k_{rgo}^*-1)(k_{row}^*-k_{rwo}^*)-(k_{row}^*-1)(k_{rgo}^*-k_{rog}^*)} . (24)$$

This model is sensitive to the value of S_{om}, and may predict incorrect oil permeabilities for values of S_{om} which are too small. An example isoperm (with $S_{om} = 0$) is shown in Figure 1. The isoperm is reasonable for oil saturations greater than about 25%, but at lower oil saturations it strays from the expected path. This particular isoperm does not exist at low gas saturations (between 2% and 15% gas). Although not shown on Figure 1, a region of higher oil relative permeability is predicted inside the loop of the isoperm. This is not physically realistic. The problem may be due to assumptions made in the model about the capillary pressures between phases and about the distributions of fluids. Larger values of S_{om} give more reasonable predictions, but the value of S_{om} required is problem-dependent. The model should be used only if examination of the predicted isoperms shows "reasonable" behavior (a precaution which applies equally well to all the other models).

Linear Interpolation Methods

Hite

Models of three-phase relative permeability based on linear interpolation have also been used. In Shell Oil's[14] modeling of the Weeks Island CO_2 flood both Stone's methods and Hirasaki's method were found to be inadequate in representing experimentally measured (centrifuge) three phase relative permeabilities at low oil saturation. Hite (reported by Perry[40]) used linear interpolation to represent the laboratory tertiary core flood data in predicting oil recovery for this field. This relative permeability model (not described in detail) allowed the prediction of much higher oil recovery (in accordance with the laboratory results) than did the probability-based (Stone's) models.

Saturation-Weighted Interpolation

Most of the available experimental data indicate the wetting and non-wetting phase relative permeabilities depend primarily on the saturation of the wetting and non-wetting phases, respectively. Evidence for dependence of the intermediate-wetting phase relative permeability on saturations is mixed; in general, there appears to be some dependence on the wetting and non-wetting phase saturations as well as that of the intermediate-wetting phase. However, in light of the uncertainties in the experimental data and the subjective nature of the contouring used to represent isoperms, it may be reasonable to use a linear or near-linear interpolation between the two-phase (e.g., oil/gas and oil/water) data sets.

A simple model of oil relative permeability based on saturation-weighted interpolation between water/oil and gas/oil data is given by

$$k_{ro} = \frac{(S_w - S_{wc}) \, k_{row} + (S_g - S_{gr}) \, k_{rog}}{(S_w - S_{wc}) + (S_g - S_{gr})} \quad ....(25)$$

S_{wc} and S_{gr} are the expected residual values of water and gas saturation. The weighting factors $(S_w - S_{wc})$ and $(S_g - S_{gr})$ in both the numerator and denominator must be non-negative; i.e., if $S_g < S_{gr}$, the weighting factor $(S_g - S_{gr})$ must be zero. In the indeterminate case where $S_g = S_{wc}$ and $S_g = S_{gr}$ and both the numerator and denominator are zero, application of l'Hôpital's rule to either S_w or S_g leads to the proper limit since k_{row} and k_{rog} should be the same at this point. The value of k_{row} to be used is obtained from water/oil relative permeability data, interpolated at the actual oil saturation. The value of k_{rog} is obtained from gas/oil data measured at irreducible water saturation (S_{wc}) and interpolated at the actual gas saturation.

Similar equations could be used for the wetting and non-wetting phases (water and gas):

$$k_{rw} = \frac{(S_o - S_{or}) \, k_{rwo} + (S_g - S_{gr}) \, k_{rwg}}{(S_o - S_{or}) + (S_g - S_{gr})} \quad .....(26)$$

$$k_{rg} = \frac{(S_o - S_{or}) \, k_{rgo} + (S_w - S_{wc}) \, k_{rgw}}{(S_o - S_{or}) + (S_w - S_{wc})} \quad .....(27)$$

The model represented by Equations 25-27 assumes (as do Stone's methods) the end points of the three-phase relative permeability isoperms coincide with the two-phase relative permeability data.

Saturation weighting insures that as the saturation of one phase approaches zero (or an irreducible saturation) the appropriate two-phase data will dominate. The method can be modified to force agreement at any one saturation (for instance, to honor the gas/oil data measured at a specific, fixed water saturation) by changing the saturation weighting functions, so that the appropriate weight approaches zero at the specified saturation. For instance, $(S_w - S_{wc})$ instead of S_w is used to weight the oil permeability contribution from the water/oil data, in order to force a match to the gas/oil data measured at S_{wc} as S_w approaches S_{wc}.

Linear Interpolation

A second interpolation model can be obtained by true linear interpolation between the two-phase relative permeability curves. Straight lines (on a ternary diagram) between corresponding equal permeabilities on the oil/gas and oil/water curves would determine the oil permeability at any given saturation between the two-phase curves. Given a saturation (gas, oil and water) we would seek a straight line passing through the specified saturation and intersecting the oil relative permeability curves (from gas/oil and water/oil data) at equal relative

permeabilities. This is a simple non-linear programming problem, to find a straight line intersecting two other lines (the oil/gas and oil/water curves) at equal permeability values and simultaneously to minimize the distance from the line to a point (the specified saturation $\{x_3\}$ in 3-dimensional saturation-space). The distance D from a point to a line is given by the set of equations

$$D^2 = \sum_{j=1}^{3} \{a_j T + (x_{1j} - x_{3j})^2\} \quad ....(28)$$

$$a_j = x_{1j} - x_{2j}$$

$$T = \sum_{j=1}^{3} a_j (x_{3j} - x_{1j})^2 / \sum a_j$$

where $\{x_1\}$ and $\{x_2\}$ are the vector coordinates (saturations) of any two points on the line and $\{x_3\}$ is the vector coordinate of the point whose distance from the line is to be determined. If $\{x_1\}$ is chosen as a point on the line $S_w = S_{wc}$ then the oil/gas relative permeability at connate water saturation can be written as a function of $\{x_1\}$. Similarly, if $\{x_2\}$ is a point on the line $S_g = 0$ the oil/water relative permeability at zero gas saturation is a function of $\{x_2\}$.

$$k_{rog} = f_1(\{x_1\})$$

$$.......................... (29)$$

$$k_{row} = f_2(\{x_2\})$$

To find the desired relative permeability we need to minimize the value of Equation 28 while minimizing the difference between k_{rog} and k_{row}:

$$\min[\, D^2 = \sum_{j=1}^{3} \{a_j T + (x_{1j} - x_{3j})^2\} \,]$$

$$....... (30)$$

$$\min[\ f_1(\{x_1\}) - f_2(\{x_2\})\ ]$$

The functions f_1 and f_2 describing the two-phase
data may be obtained from a simple linear model
(e.g., straight line interpolation between the
points of the oil/water curve) or by curve-fitting
suitable functional forms to the oil/water and
oil/gas data. Given the functions and their deriva-
tives a Newton iteration method can he used to mini-
mize the two equations simultaneously. The
permeability to be assigned to the given saturation
$\{x_3\}$ is the average of the values of k_{rog} and k_{row}
satisfying the minimization problem. An analogous
procedure may be used to interpolate water and gas
permeabilities.

An advantage of these interpolation methods is
that they do not assume (as do Stone's methods) that
the wetting and non-wetting phase relative permea-
bilities are totally independent of the saturations
of the other phases. Therefore, they are able to
honor both sets of two-phase data for each phase.
For example, Stone's methods assume that in a
water-wet rock the water permeability is a function
of water saturation only; at a fixed water satu-
ration the water relative permeability is constant.
This is more restrictive than necessary; the assump-
tion requires water isoperms (1) to be straight
lines, and (2) to coincide with lines of constant
water saturation. Linear interpolation requires
isoperms to be straight lines, but not necessarily
parallel to lines of constant saturation. Satura-
tion-weighted interpolation requires neither of
these restrictions.

The saturation-weighted interpolation method
may give apparently erroneous results if the two-
phase relative permeability curves being interpo-
lated between are dissimilar. The weighting
functions used cause the predicted isoperms not to
be straight lines across a ternary diagram; the two-
phase curve giving the largest relative permeability
will dominate in the interior of the three-phase
region, even though the two-phase data are honored
at the boundary. This is especially a problem if
the end point saturations (S_{orw} and S_{org}) of the
oil/water and oil/gas curves differ widely.

Parmeswar and Maerefat

Parmeswar and Maerefat[41] proposed the use of a
power-law function relating phase relative permea-
bility to the saturations of each of the phases:

$$k_{rj} = P_{4j}(S_w^*)^{P_{1j}} + P_{5j}(S_o^*)^{P_{2j}}$$
$$+ P_{6j}(S_g^*)^{P_{3j}} + P_{7j} \quad\ldots\ldots\ldots\ldots\ldots(31)$$

where each of the saturations S_i^* is normalized with
respect to a maximum and minimum value (unspeci-

fied). They fit this function to the data of Lever-
ett and Lewis[1], Reid[2], Hosain[3], Snell[4] and Saraf[10].
Plots of isoperms generated from Equation 31 seem to
fit the general trend of the data in each case.
However, no extrapolation of the isoperms out of the
region of measured data was shown, nor were the
derived fitting parameters given so this comparison
could be made. A major use of such a correlation
would be to extrapolate to saturation regions where
data have not been measured. Thus it is important
to examine the extrapolation behavior of the func-
tion for pathological behavior.

Comparison of Prediction Methods

Figure 2 compares the goodness-of-fit of ten of
the above models with the eight data sets which are
complete enough for model application.

For some of the data sets it was necessary to
generate the required two-phase data (water/oil and
gas/oil) by extrapolation of the three-phase data.
In each case smooth curves were fit to the two-phase
data using van Genuchten's[29], Corey's[22] or
Chierici's[42] functions, or other functions as appro-
priate.

The statistic used for comparison is R^2, the
coefficient of determination. A value of $R^2 = 1$
implies a perfect correlation between measured and
predicted values, while $R^2 = 0$ implies a total lack
of correlation. For each data set, oil relative
permeabilities were calculated at each measured sat-
uration. The calculated permeabilities were com-
pared with the measured permeabilities on a
logarithmic and a linear basis. Calculated or meas-
ured values of zero relative permeability were set
equal to 0.00001 for the logarithmic comparison.
The linear comparison gives a larger weight to the
large permeability values, while the logarithmic
comparison weights more heavily the small (larger
negative logarithm) permeability values.

The complete data sets were used for each com-
parison, with the exception of the Saraf, et al.[13]
set from which a subset was extracted. This subset
contained only data for oil-decreasing, water- and
gas-increasing or constant saturations. The other
tests in consolidated media (Corey[7], Saraf[10],
Guckert[12]) also followed this saturation history
during three-phase flow. The sandpack tests (Lever-
ett and Lewis[1], Reid[2], Hosain[3], Snell[4]) showed
either no hysteresis or (because of data scatter or
the testing sequence) no clear distinction between
drainage and imbibition saturation histories.

Stone's Method I is used (1) with a fixed value
of S_{om} equal to the smaller of S_{orw} and S_{org}, and
(2) with Fayers and Matthews'[35] variable S_{rg} corre-
lation (Equations 19-20). Stone's method II was
used (1) with no adjustment and (2) with Aziz and
Settari's[33] adjustment factor α. Values of α were
chosen to optimize the fit to the data for each
problem. Corey's method is used with fixed S_{1r}
values which gave the best fit. Land's method is
used with $S_{br(m-r)}$ equal to the waterflood residual
oil saturation and zero initial gas saturation.
Aleman's method (Equation 23) is also used with
fixed or variable S_{om} (Equation 22), depending on
which gave the better overall fit. Parameters for
Parker's method were obtained by fitting the two-

phase (oil/gas and oil/water) relative permeability data with van Genuchten's[29] relative permeability function. While better choices of parameter values might be made for any given model and data set, these choices represent likely values to use for predicting three-phase relative permeability in the absence of measured three-phase data.

Most of the prediction methods fit the data sets equally well on the basis of the linear R^2 comparison (not tabulated here since there were no very significant differences within data sets). This shows that each of the methods is capable of representing three-phase relative permeabilities in the high-oil-permeability region. The important differences in the models were much more evident in the logarithmic R^2 comparison where the low-permeability values dominate. This comparison is shown in Figure 2.

In Figure 2 the height of each vertical bar denotes the goodness-of-fit of one of the models for one data set. Differences in the level of fit between data sets (especially the Snell and Guckert data sets) are more representative of differences in data scatter than of a failure of the models to approximate the permeability behavior. Comparing the bar heights within data sets, Stone's Method I (with either fixed or variable S_{om}), linear interpolation and saturation-weighted interpolation fit better than Stone's Method II (either adjusted or unadjusted) for most cases. In one case (Snell) Stone's Method II provides a clearly better fit. However, Stone's Method II (unadjusted) provides an extremely poor fit to Hosain's data. Linear interpolation and saturation-weighted interpolation are among the top three models for each of the data sets. Hirasaki's model is consistently one of the worst, except for Snell's data set. Of the remaining models, Parker's is generally equal to or better than Corey's, Land's, Naar and Wygal's and Aleman's. This method predicts nearly straight-line curves (similar to linear interpolation) in the three-phase region. It can be shown mathematically that this is the expected behavior for Parker's model when the difference between S_{orw} and S_{org} is small and the exponent "m" is near unity, as it was in these cases. For other values of "m" the isoperms may show more curvature.

One might question whether the models are equally suited to prediction of relative permeability for the saturation histories represented in these data sets. Since water saturations were held nearly constant in the consolidated core tests, the Naar and Wygal imbibition model would seem ill-suited; and, in general it did not fit the data well. This does not imply the model is unsuitable for use in other cases where the model assumptions are applicable. Land's imbibition model generally fit the data about as well as Corey's drainage model, although neither was consistently among the better models. Even though their fit statistics seem comparable, there are major differences in the shapes of the isoperms predicted by these two models.

The differences in model predictions are most readily apparent in a comparison of oil isoperms. Figures 3-7 show isoperms calculated for Hosain's[3] data set using the linear interpolation method and the methods of Stone (I and II), Hirasaki and Parker. The data points in the figures represent saturations at which non-zero oil relative permeabilities were measured. To illustrate the effect of S_{om} on Stone's Method I, $S_{om} = 0$ was used. A value of S_{om} between 0 and 0.2, or use of Aleman's or Fayers and Matthews' S_{om} correlation would cause the predictions of Stone's Method I to approach the linear interpolation case. Stone's Method II was unadjusted. Application of the adjustment factor α (Equation 18) would cause the isoperms to approach those predicted by linear interpolation.

The isoperms were calculated for each model using a parametric (curve tracing) method, not by contour interpolation. Thus the isoperms exactly represent the model predictions.

In the region of low oil saturation (less than 20%) Stone's Method I (Figure 3) predicts relative permeabilities which are much too high. This is typical of the predictions of Stone's Method I; with $S_{om} = 0$, oil is predicted to be mobile at very low oil saturations. Conversely, Stone's Method II (Figure 4) predicts too-low oil relative permeabilities. The isoperms curve to the right, implying that oil is immobile at less than 40% oil saturation over much of the three-phase region. This type of curvature is typical of the predictions of Stone's Method II. Hirasaki's method (Figure 5) predicts isoperms similar to those of Stone's Method II, while linear interpolation (Figure 6) and Parker's method (Figure 7) follow the experimental data closely.

Implications for Reservoir Simulation

Often one can determine the approximate residual oil saturation which will be predicted by a simulator to exist at the completion of a tertiary recovery project simply by examination of the oil isoperms, without actually running the simulator.

The isoperms of Figures 3-7 show the type of effect the choice of three-phase relative permeability correlation can have on the prediction of oil recovery. If three-phase flow occurs within a significant portion of the reservoir, the oil recovery can be dramatically affected by whether the permeability model predicts oil will flow at low oil saturations. Stone's Method I can be expected to predict too-high oil permeability at low oil saturation, while Stone's Method II predicts too-low oil permeability in the same region. With a 'suitable' choice of S_{om} or α the predictions of Stone's methods can be improved. However, the value of S_{om} or α which should be used is not always easily determined, especially if there are no three-phase-flow data to guide the choice. In general the improvement is in the direction of the predictions of the saturation-weighted or linear interpolation methods.

SUMMARY

A review of three-phase relative permeability data and models available in the literature indicates there are many problems remaining to be solved. From an experimental standpoint the most important problem is obtaining good quality relative permeability and saturation history data, along with sufficient information about the rock to enable

model studies to proceed with a minimum of assumptions. The experimental studies should investigate additional saturation histories not well represented in the published data. From a modeling standpoint the most important is the development of a model incorporating pore-level physics (interactions of interfacial tension, fluid 'aturation and distribution, kinetic effects such as drop coalescence and drop breakup, etc.), saturation history dependence, and realistic pore geometry and interconnections. Until such a pore-level-physics model has been developed which can be economically used in reservoir simulation, other approximate approaches must be used. These include Stone's methods and other correlations, where the correlation can be shown to fit the expected behavior of the three-phase system. This may require the use of different correlations for different systems; no one of the simple correlations may be best for all cases.

The available experimental data indicate that in the three-flowing-phase region much of the intermediate-wetting phase (oil) relative permeability data can be fit as well by straight line interpolation between the gas/oil (at connate water saturation) and water/oil two-phase data as by any of the other proposed correlation models.

A major effect of any correlation used to predict intermediate-wetting phase relative permeability is likely to be in determining the predicted residual saturation (zero-permeability) of that phase. With Stone's Method I (without other adjustment) the residual saturation will be at least S_{om}, the value of the free parameter used to adjust the model. With Stone's Method II (unadjusted) this predicted residual saturation is likely to be larger than with Stone's Method I, based on experience with several data sets. Stone's Method II may also be adjusted, which generally improves the quality of the predictions if the correct value of α is chosen. Both linear interpolation and saturation-weighted interpolation provide good fits to the available three-phase data, and do not require adjustment. While the lack of adjustable parameters may limit the fit of these models in some cases, it also may prevent overly optimistic or pessimistic predictions of oil recovery which may occur if Stone's methods (unadjusted or improperly adjusted) are used.

The best way to determine the effect three-phase relative permeability correlations will have is to examine isoperms (lines of constant relative permeability) predicted by the models of interest. The choice of a suitable oil relative permeability model can then be made with a better appreciation of the effect it will have on predicted oil recovery.

NOMENCLATURE

a   vector (space) coordinate
      defined by Eqn 28

C   constant defined by Eqn 6

D   distance from a point to a line

f_1, f_2 functions defined by Eqn 29

k_r relative permeability

K   absolute permeability

$$k_{rgo}^* = \frac{k_{rgo}}{S_g^*}$$

$$k_{rog}^* = \frac{k_{rog}}{(1 - S_g^*)}$$

$$k_{row}^* = \frac{k_{row}}{(1 - S_w^*)}$$

$$k_{rwo}^* = \frac{k_{rwo}}{S_w^*}$$

$m = 1 - 1/n$ fitting parameter, Eqns 10, 11

n   fitting parameter, Eqns 10, 11

P_b fitting parameter, Eqn 2

P_c capillary pressure

P_{ij} fitting parameters, Eqn 31

S   saturation

$$S^{**} = \frac{S^* - S_{fw}^*}{1 - S_{fw}^*} \quad \text{(integration variable, Equation 4)}$$

$S_{fw} = S_w + S_{ob}$; fictitious irreducible water saturation

$$S_{fw}^* = \frac{S_{fw} - S_{wc}}{1 - S_{wc}}$$

$$S_g^* = \frac{S_g}{1 - S_{wc} - S_{om}}$$

$$S_{hi}^* = \frac{S_{hi}}{1 - S_{wc}} , \text{ normalized initial hydrocarbon saturation}$$

$$S_{hr}^* = \frac{S_{hr}}{1 - S_{wc}} ; \text{ normalized residual hydrocarbon saturation}$$

$S_{hr(max)}^*$ = normalized maximum trapped hydrocarbon saturation

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om}}$$

S_{ob} saturation of oil blocked by invading water

$S_{of} = S_o - S_{ob}$; free oil saturation

$$S_{of}^* = \frac{S_{of}}{1 - S_{wc}}$$

$$S_{of}^{**} = \frac{S_{of}}{1 - S_{fw}} \cdot$$

S_{om} minimum residual oil saturation

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om}}$$

T quantity defined by Eqn 28

$\{x_i\}$ vector (space) coordinate of point i: $\{x_{i1}, x_{i2}, x_{i3}\}$

Subscripts:

c connate or irreducible (water saturation)

f free (phase saturation)

g gas

i initial (saturation)

j phase (o, w, or g); Eqns 28-31

l liquid

o oil

r residual (saturation), or relative (permeability)

w water

Superscripts:

* normalized with respect to saturation

Greek:

α fitting parameter

β permeability function, when subscripted (Equation 12); fitting parameter, Eqn 22

Δ correction term, Eqns 23,24

λ fitting parameter, Eqn 2

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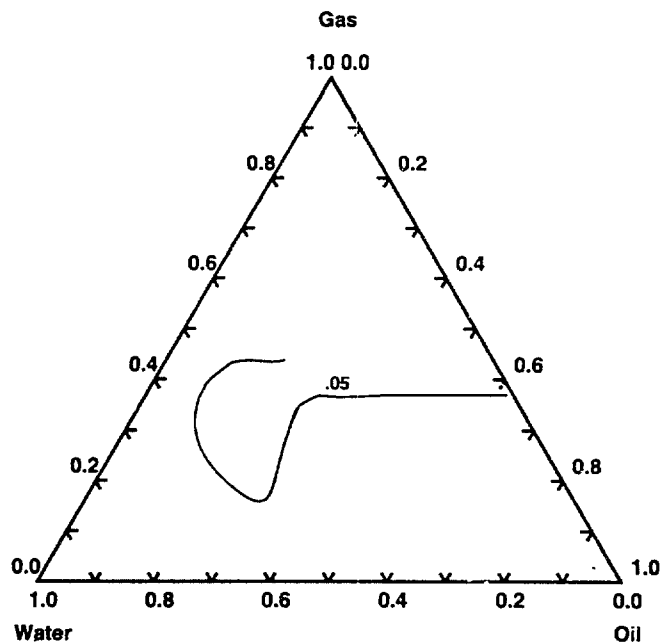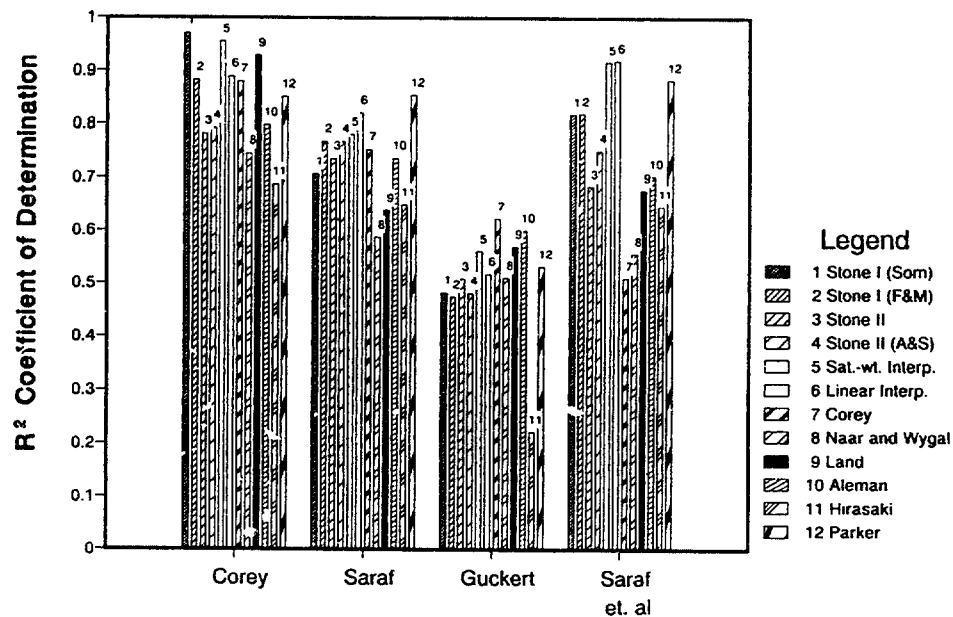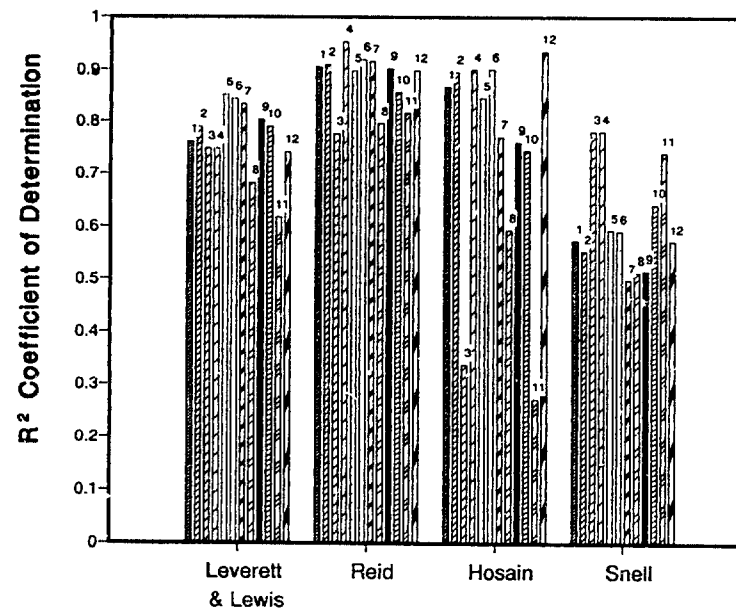
Fig. 1—Isoperm predicted by Aleman model, $S_{om} = 0$.

Fig. 2—Comparison of three-phase relative permeability model fits of experimental data.

Gas

1.0 0.0

0.8   0.2

.001
0.6   .005   0.4
.01

.05

0.4   .1   0.6

.2

0.2   0.8

.5

0.0   1.0
1.0   0.8   0.6   0.4   0.2   0.0
Water   Oil
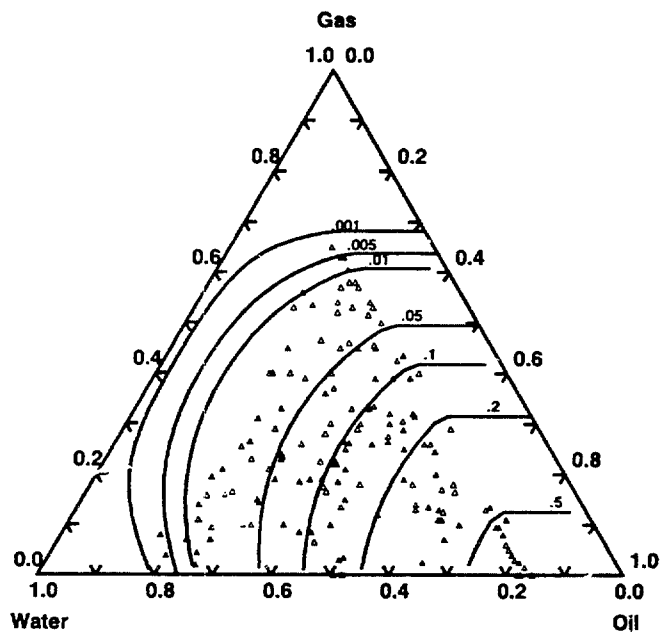
Fig. 3—Stone's Method I (S$_{om}$=0) fit of Hosain's data.

Gas

1.0 0.0

0.8   0.2

.001
0.6   .005   0.4
.01

.05

0.4   .1   0.6

.2

0.2   0.8

.5

0.0   1.0
1.0   0.8   0.6   0.4   0.2   0.0
Water   Oil
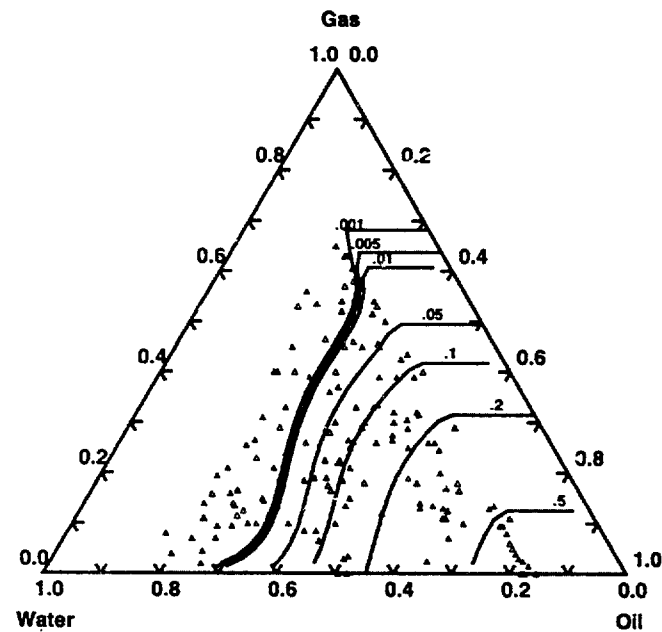
Fig. 4—Stone's Method II (unadjusted) fit of Hosain's data.

Gas

1.0 0.0

0.8   0.2

.001
0.6   .005   0.4
.01

.05

0.4   .1   0.6

.2

0.2   0.8

.5

0.0   1.0
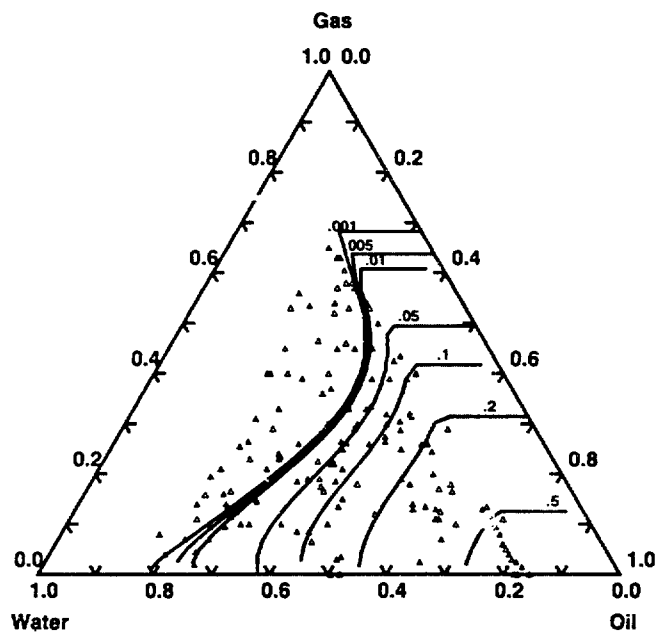1.0   0.8   0.6   0.4   0.2   0.0
Water   Oil

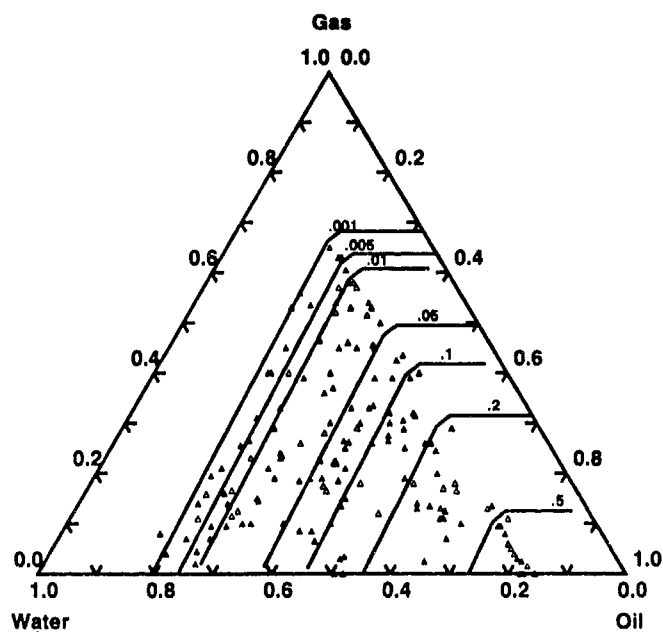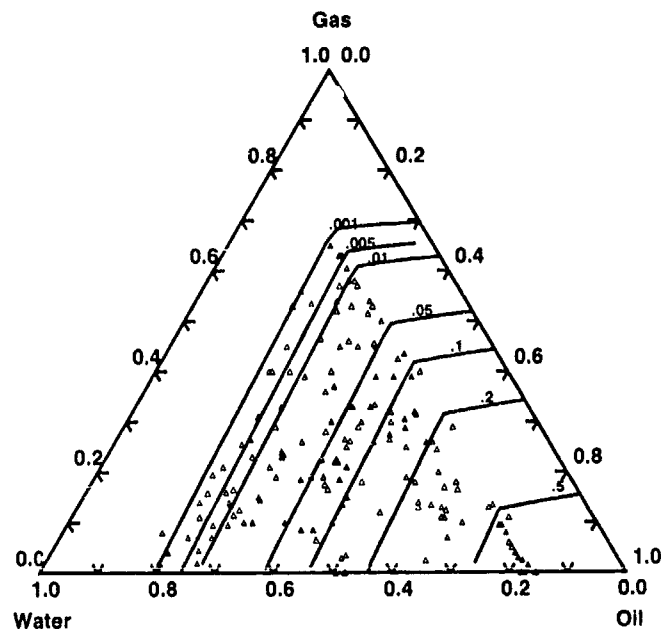Fig. 5—Hirasaki's method fit of Hosain's data.

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Fig. 6—Linear interpolation fit of Hosain's data.

Fig. 7—Parker's method fit of Hosain's data.