

Estimation of primary drainage three-phase relative permeability for organic liquid transport in the vadose zone

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Received 19 November 2001; accepted 10 February 2003

Abstract

The modeling of transport of organic liquid contaminants through the vadose zone often requires three-phase relative permeabilities. Since these are difficult to measure, predictive models are usually used. The objective of this study is to assess the ability of eight common models to predict the drainage relative permeability to oil in a three-phase system (water–oil–air). A comparison of the models' estimates using data set from Oak [Oak, M.J., 1990. Three-phase relative permeability of water-wet Berea. In: Seventh Symposium on Enhanced Oil Recovery, Paper SPE/Doe 20183. Tulsa, OK, April 22–25] showed that they provide very different predictions for the same system. The goodness of the models does not increase with the amount of data or computation that the models require. Also, the calculations showed how different interpretations of the models and of the terminology associated with them can significantly impact the predictions. Thus, considerable error may be introduced into the simulations of organic liquid transport in the vadose zone depending on the selection and interpretation of the three-phase relative permeability model.

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Keywords: Relative permeability; NAPL; Multiphase flow; Hydraulic properties

1. Introduction

In the case of infiltration of an organic liquid from a surface spill or from a leaking underground storage tank, the subsurface contains three phases which may be flowing

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simultaneously: air, water and an organic liquid. To model the movement of the contaminant in such a situation requires constitutive relationships to form a closed system of equations, among them the relative permeability as a function of saturation.

The definition of relative permeability is derived from the extension of Darcy's law to multiphase flow:

$$q_i = -[k_i(S)/\mu_i](\nabla P_i - \rho_i g \nabla h) \quad (1)$$

where q_i is the volumetric flow of the fluid phase i per unit total area perpendicular to the flow; $k_i(S)$ is the effective permeability to fluid phase i as a function of saturation; μ_i is that phase's viscosity; P is its pressure; ρ is its density; g is the gravitational acceleration constant; and h is the elevation above a datum. Relative permeability to a particular fluid is usually defined as the ratio of the effective permeability, k_{ri} , to that fluid to the intrinsic permeability of the porous medium:

$$k_{ri}(S) = k_i(S)/k \quad (2)$$

where k_{ri} is the relative permeability to the i th phase, and k is the intrinsic permeability.

Based on the measurements of relative permeability to oil in the presence of water and air in unconsolidated sands, [Leverett and Lewis \(1941\)](#) came to some fundamental conclusions about water-wet three-phase systems:

- the relative permeability to water is determined by the water saturation only and is not affected by whether the remaining void space is filled by air, an organic liquid or both;
- similarly, the relative permeability to gas is a function of its own saturation only;
- the relative permeability to oil arises in a more complex manner, being dependent on both water and gas saturations.

These conclusions, however, are not unilaterally supported. Some dependence of water and the gas relative permeabilities on other phase saturations is shown in the results of [Snell \(1962\)](#), [Donaldson and Dean \(1966\)](#) and [van Spronsen \(1982\)](#) (only for water). Other investigators have suggested that all three—oil, water, and air—relative permeabilities are functions only of their own saturations ([Sarem, 1966](#); [DiCarlo et al., 2000](#)). Despite these deviations, the preponderance of the studies performed over the years suggest that water and gas relative permeabilities are, in fact, dependent only on their respective saturations ([Leverett and Lewis, 1941](#); [Corey et al., 1956](#); [Saraf and Fatt, 1967](#); [Oak, 1990](#)), and consequently their permeabilities in a three-phase system are equal to their permeabilities in a two-phase system. Because only the oil relative permeability is considered to behave differently in a three-phase system, most work has emphasized the relative permeability to the organic liquid.

The difficulties in measuring the relative permeability to oil in three-phase systems have led to the use of models for the estimation of this property. Compilations of such models have been published ([Saraf and McCaffery, 1985](#); [Honarpour et al., 1986](#); [Baker, 1988](#); [Delshad and Pope, 1989](#)). In addition, a number of studies have assessed the goodness of various models by comparing the models' predictions with published

data (Manj Nath and Honarpour, 1984; Honarpour et al., 1986; Baker, 1988; Delshad and Pope, 1989) and have come to conflicting conclusions about the performance of the models.

Part of the inconsistency of the models' evaluations may be attributable to the data used. Data measured in the 1950s and 1960s have been used most commonly. Some of these data sets are not uniform in the direction of saturation change. Thus, the data may reflect hysteretic effects. Furthermore, these data sets are not complete in that they do not include both two-phase and three-phase relative permeability measurements as well as capillary pressure–saturation measurements. Oak (1990) produced a comprehensive set of measurements of two- and three-phase relative permeabilities that provides a new basis for comparing the available models. Also, additional models have been developed since the last comparative studies were undertaken. Thus, the purpose of this study is to compare eight different predictive models for the relative permeability to an organic liquid using the more complete set of data produced by Oak.

2. Background

Many three-phase models require data from two-phase relative permeability curves, sometimes using saturations of different phases as the reference point. For instance, a model may require the value of the relative permeability to oil in the oil–water system, often written as k_{row} . However, a second important piece of information is frequently missing in that notation, namely which saturation should k_{row} be a function of oil or water. To refer to the two-phase relative permeability value needed in a three-phase model, as well as to the saturation it should be a function of, the designation used in this study is $k_{\text{ri}(j)}$. This should be interpreted as the two-phase relative permeability to phase i in the two-phase system of i and j , taken as a function of the saturation of j . The notation k_{ro} is reserved for the three-phase relative permeability to oil at some (S_{w} , S_{o} , S_{g}) where $S_{\text{w}} + S_{\text{o}} + S_{\text{g}} = 1.0$. So two saturations must be designated to specify a value of k_{ro} .

Furthermore, the description of the models often employs inconsistent or inexact terminology, particularly in regard to the residual, minimal, or irreducible saturation of a phase. These parameters are meant to represent the minimum amount of a phase that is necessary for flow; below this saturation, the phase is considered immobile because it is discontinuous. In petroleum engineering, *irreducible* and *connate* are the terms most frequently used to refer to this saturation threshold for the water phase, while *residual* is commonly used for both the oil and gas phases. Although occasionally used, the term *connate* seems improperly applied when used to refer to this minimum (Morrow and Melrose, 1991). According to Muskat (1981), connate water is the water present in an oil formation, presumably indigenous to the rock, and can range from about 2% to more than 50% of the total pore space. Once the formation is tapped, this saturation can be reduced; thus, the often interchangeable use of the terms connate and irreducible does not seem appropriate.

Both in soil science and contaminant hydrology, the term *residual* is used to refer to the minimum saturation of water in particular reached in experiments for the determination of

capillary pressure–saturation curves. Yet, the minimum saturation reached in a capillary pressure–saturation experiment and that reached in a relative permeability experiment are often not the same. In the latter, the residual saturation is defined when a phase is no longer flowing as a continuous phase. Whether the phase saturation has reached the lowest value possible or not is immaterial, for a zero flux is sufficient to define the irreducible saturation (Aziz and Settari, 1979).

In this paper, the terms irreducible, residual, and connate are used as follows:

- *connate* denotes the amount of water indigenous to an oil reservoir or the water present when making measurements of “two-phase” oil–gas relative permeabilities;
- *irreducible* saturation refers to the lowest saturation reached in a primary drainage event, after high capillary pressures are applied to the soil core and no additional fluid leaves the soil (defined by the capillary pressure–saturation relationship);
- *residual* saturation refers to the saturation greater than or equal to the irreducible, below which there is a cessation of bulk flow (therefore, determined by relative permeability measurements).

Eight models for prediction of the relative permeability to oil are presented in the next section. The models selected for examination in this study were those that provided a closed equation for the relative permeability of oil when water is draining, whose implementation required a minimum number of assumptions, and whose parameters could be determined independently. In addition, consideration was given to models commonly used in the literature and recently proposed models that have yet to be evaluated. These models are presented with the goal of preserving the original authors’ intentions yet, at the same time, being consistent in the use of terminology.

2.1. Stone I

The model often referred to as Stone I (Baker, 1988; Delshad and Pope, 1989; Oak, 1990) was originally suggested as an interpolation between 2 two-phase flow conditions (Stone, 1970):

$$k_{ro} = S_o^* \beta_w \beta_g \quad (3)$$

where β_w and β_g are factors that account for oil blockage by water and gas, respectively, and are obtained by:

$$\beta_w = \frac{k_{ro(w)}}{1 - S_w^*} \quad (4)$$

$$\beta_g = \frac{k_{ro(g)}}{1 - S_g^*} \quad (5)$$

and S_o^* , S_w^* and S_g^* are normalized saturations defined as:

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om}} \quad (\text{for } S_o \geq S_{om}) \quad (6)$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om}} \quad (\text{for } S_w \geq S_{wc}) \quad (7)$$

$$S_g^* = \frac{S_g}{1 - S_{wc} - S_{om}} \quad (8)$$

where S_o , S_w and S_g are the saturations of oil, water and gas, respectively; S_{wc} is the water connate saturation; and S_{om} is an adjustable parameter that represents a minimum value of residual oil saturation in the three-phase system. Stone (1970) suggested that the value of S_{om} should be about $1/2(S_{wc})$.

According to Stone (1970), when the normalized oil saturation S_o^* is equal to 1.0, k_{ro} is also equal to 1.0, which led to the conclusion that his model reduces to the two-phase curves only if (Aziz and Settari, 1979; Fayers and Matthews, 1984; Delshad and Pope, 1989):

$$k_{row}(S_{wr}) = 1 \text{ and } k_{rog}(S_g = 0) = 1$$

These conditions are unrealistic for two reasons: (1) the relative permeability to oil in the presence of a residual saturation of water should be less than 1 because the void space is not fully occupied by oil; (2) k_{rog} is usually measured in the presence of connate water (whose saturation is denoted by S_{wc}) to simulate oil reservoir conditions and, therefore, the relative permeability to oil will never equal 1 because the void space is always shared with at least water.

Because the end point relative permeabilities are not, in fact, equal to unity, Aziz and Settari (1979) introduced a normalization of the Stone I model by assuming that the end point relative permeabilities for the oil–water and oil–gas systems are equal (a condition not normally achieved experimentally) but have a value other than unity:

$$k_{row}(S_{wr}) = k_{rog}(S_g = 0) = k_{ro}^o$$

They proposed that Eqs. (4) and (5) be divided by k_{ro}^o , and Eq. (3) be multiplied by k_{ro}^o , so that the normalized form of the Stone I model is:

$$k_{ro} = \frac{S_o^* \beta_w \beta_{g(L)}}{k_{ro}^o} \quad (9)$$

where

$$\beta_{g(L)} = \frac{k_{rog(L)}}{1 - S_g^*} \quad (10)$$

and k_{rog} should be read as a function of the total liquid saturation, S_L . Aziz and Settari (1979), however, did not specify how to obtain the appropriate value of S_L . Conceivably, it

may be calculated from: (1) the value of S_g in the three-phase system, $S_L = 1 - S_g$, in which case $\beta_{g(L)} = \beta_g$ (Eq. (5)); or (2) from $S_L = S_o + S_{wc}$, where S_o is the oil saturation in the three-phase system and S_{wc} is obtained from two-phase oil–gas data.

To determine S_{om} in the Stone I model, [Fayers and Matthews \(1984\)](#) proposed that this parameter varies linearly between the two limiting residual saturations:

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org} \quad (11)$$

and

$$\alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}} \quad (12)$$

where S_{orw} is the oil residual saturation in the oil–water system, and S_{org} is the oil residual saturation in the oil–gas system. They suggested that these equations be used in conjunction with Aziz and Settari's modification (Eq. (9)).

2.2. Stone II

Dissatisfied with the need of the adjustable parameter S_{om} in his first model, [Stone \(1973\)](#) proposed a second model, this time using 4 two-phase relative permeability curves (two for oil–water and two for oil–gas):

$$k_{ro} = (k_{ro(w)} + k_{r(w)o})(k_{ro(g)} + k_{r(g)o}) - k_{r(w)o} - k_{r(g)o} \quad (13)$$

subject to the constraint that $k_{ro} \geq 0$ because negative values are mathematically possible but have no meaning. This formulation suffers from the same unrealistic assumption made in the Stone I model, that the oil relative permeability at residual water saturation is equal to 1. Thus, [Aziz and Settari \(1979\)](#) extended their modification to the Stone II model, introducing a normalized form of this model:

$$k_{ro} = k_{ro}^o \left[\left(\frac{k_{ro(w)}}{k_{ro}^o} + k_{r(w)o} \right) \left(\frac{k_{ro(g)}}{k_{ro}^o} + k_{r(g)o} \right) - k_{r(w)o} - k_{r(g)o} \right] \quad (14)$$

Whether the Stone II model represents an improvement over the Stone I is unclear. [Baker \(1988\)](#) analyzed the Stone I and II formulations and found that Stone II gave predictions for k_{ro} that were too low, whereas Stone I gave predictions that were too high, a complaint echoed by [Grader and O'Meara \(1988\)](#). [Delshad and Pope \(1989\)](#) found that both the Stone I and II models performed well for some sets of data but not so well for others. Simulation results from [Guzman et al. \(1994\)](#) showed good agreement between the predictions of the Stone I and II models in the high oil saturation region but considerable discrepancies at the low saturations. [Oak \(1990, 1991\)](#) found both models predictions unsatisfactory and [Balbinsky et al. \(1999\)](#) went one step further and suggested that both Stone models should be avoided due to problems with their mathematical formulations.

Some of the discrepancies in the assessments may be attributable to how the model is interpreted: how the saturations are obtained for use in Eqs. (9) and (14) and how the

parameter S_{wc} is interpreted and determined. Because these issues may lead to significant discrepancies in the performance of the Stone models, they need to be examined systematically.

2.3. Baker

Reasoning that the intermediate-wetting phase relative permeability should depend on the other phases' saturations, Baker (1988) presented a saturation-weighted interpolation model using oil–water and oil–gas data:

$$k_{ro} = \frac{(S_w - S_{wc})k_{r(o)w} + (S_g - S_{gr})k_{r(o)g}}{(S_w - S_{wc}) + (S_g - S_{gr})} \quad (15)$$

where S_{gr} and S_{wc} are the “expected residual values of water and gas saturation”. The weighting factors, $(S_w - S_{wc})$ and $(S_g - S_{gr})$, must be greater than zero, or taken as zero if negative values are obtained. For the specific, indeterminate case where $S_w = S_{wc}$ and $S_g = S_{gr}$, Baker (1988) suggested that the value of either $k_{r(o)w}$ or $k_{r(o)g}$ should give the appropriate value of k_{ro} since both systems are similar at the limit (if the differences between a wetting and a nonwetting fluid are ignored). He also suggested that the appropriate value of $k_{r(o)g}$ could be obtained from the oil–gas two-phase curve measured at “irreducible water saturation” (which he labeled S_{wc}) and interpolated at the actual gas saturation. Despite this stipulation by Baker (1988), Balbinsky et al. (1999) and Blunt (2000) presented Baker's model with k_{rog} as a function of the oil saturation, i.e.:

$$k_{ro} = \frac{(S_w - S_{wc})k_{r(o)w} + (S_g - S_{gr})k_{r(o)g}}{(S_w - S_{wc}) + (S_g - S_{gr})} \quad (16)$$

This function, according to Blunt (2002), is now the industry standard use of Baker's model. Another source of confusion in Baker's model is that S_{wc} is called both the “residual” and the “irreducible” water saturation although Baker (1988) states that the value of this parameter could be adjusted to force agreement with oil–gas data measured at a constant water saturation (usually the connate saturation). Blunt (2000) applied Eq. (16) using the connate water saturation while Delshad and Pope (1989) used Eq. (15), using S_{wc} as S_{wr} , the residual water saturation in the oil–water system.

Conceivably, these differences in interpretation may have a significant impact on the goodness of the models' predictions. Yet the experience reported in the literature does not present the same range of conflict as with Stone's models. Comparing his predictions with measured data, Baker (1988) found that his proposed model could fit experimental data as well as other models did (including Stone's models and Parker and Lenhard's model). Delshad and Pope (1989) also found good agreement between Baker's model's predictions and three different sets of data. Because how the model is interpreted could conceivably have a large impact on its performance, the alternative interpretations need to be examined.

2.4. Hustad–Hansen

Reasoning that data from all three sets of two-phase relative permeability curves, and in particular, all six residual saturations need to be honored to construct a consistent formulation for three-phase flow, [Hustad and Hansen \(1995\)](#) proposed:

$$k_{ro} = \frac{S_w}{S_g + S_w} k_{row}^* + \frac{S_g}{S_g + S_w} k_{rog}^* \quad (17)$$

where k_{row}^* and k_{rog}^* are representative two-phase oil relative permeabilities obtained from normalized two-phase relative permeability curves taken at a normalized three-phase oil saturation:

$$k_{row}^* = k_{row}(S_o^{**}) \quad (18)$$

$$k_{rog}^* = k_{rog}(S_o^{**}) \quad (19)$$

$$S_o^{**} = \frac{S_o - S_{omn}}{S_{omx} - S_{omn}} \quad (20)$$

$$S_{omn} = \frac{S_w S_{orw} + S_g S_{org} + S_{org} S_{orw} (S_o - 1)}{S_g (1 - S_{orw}) + S_w (1 - S_{org})} \quad (21)$$

$$S_{omx} = \frac{S_w S_{gro} + S_g S_{wro} + S_{gro} S_{wro} (S_o - 1)}{S_g S_{wro} + S_w S_{gro}} \quad (22)$$

where S_{orw} and S_{org} are oil residual saturations in the oil–water and in the oil–gas systems, respectively; S_{wro} is the water residual in the oil–water system; and S_{gro} is the gas residual saturation in the oil–gas system.

Depending on the values chosen for the parameters in this model, some problems may arise. For instance, if $S_{gro} = 0$, then Eq. (22) reduces to $S_{omx} = S_g S_{wro} / S_g S_{wro} = 1$ and S_{wro} , the water residual saturation in the oil–water system, has no impact on the model's predictions. Because this is a fairly recent model, it has not been subjected to the same level of scrutiny as Stone's and Baker's models. Thus the performance of this model relative to those that require fewer data needs to be evaluated.

2.5. Delshad–Pope

[Delshad and Pope \(1989\)](#) proposed the following as being a more flexible model in that it can take advantage of measured three-phase data if available:

$$k_{ro} = k_{row}^o [a \bar{S}_o^\alpha (1 - \bar{S}_w)^\beta + (1 - a) \bar{S}_o^\gamma (1 - \bar{S}_g)^\delta] \quad (23)$$

where

$$\bar{S}_o = \frac{1 - S_w - S_g - S_{or}}{1 - S_{wr} - S_{gr} - S_{or}} \quad (24)$$

$$\bar{S}_w = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{gr} - S_{or}} \quad (25)$$

$$\bar{S}_g = \frac{S_g - S_{gr}}{1 - S_{wr} - S_{gr} - S_{or}} \quad (26)$$

where S_{wr} , S_{gr} and S_{or} are the residual saturations of the water, gas and oil phases, respectively, in the three-phase system; k_{row}^o is the end-point oil relative permeability in the oil–water system [$k_{row}^o = k_{r(o)w}(S_o = 1 - S_{wr})$]; and α , β , γ , δ , and a are fitting parameters to experimental three-phase data.

In the absence of three-phase data, α , β , γ , and δ may be calculated by assuming that the three-phase relationships are equivalent to the two-phase (oil–water and oil–gas) at the endpoints so that $\gamma = \alpha + \beta = eow$, a parameter obtained by fitting the following equation to the data for relative permeability to oil in the oil–water system:

$$k_{row} = k_{row}^o \left(\frac{S_o - S_{orw}^*}{1 - S_{wr}^* - S_{orw}^*} \right)^{eow} \quad (27a)$$

where S_{wr}^* , and S_{orw}^* and eow are fitting parameters. In turn, $\alpha = \gamma + \delta = eog$, and eog can be obtained by fitting the following equation to the relative permeability to gas in the oil–gas system:

$$k_{rog} = k_{rog}^o \left(\frac{S_L - S_{Lrg}^*}{1 - S_{Lrg}^* - S_{gr}^*} \right)^{eog} \quad (28a)$$

where S_L is the total liquid saturation and, similar to the previous equation, S_{Lrg}^* , S_{gr}^* , and eog are fitting parameters. The parameter a is assumed to be equal to 1/2. S_{or} can be set equal to S_{om} (Eqs. (11) and (12)); yet, no recommendation is given as to how to obtain S_{gr} and S_{rw} .

If three-phase data are available, Delshad and Pope found that their model offers superior predictions. However, in the absence of such data, it is not clear whether the model still offers better predictions.

2.6. Parker–Lenhard

Parker et al. (1987) developed a parametric model based on van Genuchten's (1980) capillary pressure–saturation relationship and Mualem's (1976) model for predicting the hydraulic conductivity in unsaturated porous media. This model is distinct from the previous models presented here in that it does not use two-phase relative permeability data.

Rather, it is based on the capillary pressure–saturation relationship and has the following form:

$$k_{ro} = (\bar{S}_t - \bar{S}_w)^{1/2} \left\{ [1 - \bar{S}_w^{1/m}]^m - [1 - \bar{S}_t^{1/m}]^m \right\}^2 \quad (29)$$

where

$$\bar{S}_t = \frac{S_w + S_o - S_m}{1 - S_m} \quad (30)$$

$$\bar{S}_w = \frac{S_w - S_m}{1 - S_m} \quad (31)$$

and S_m is the irreducible wetting phase (water) saturation. The parameter m is obtained from the fitting of the [van Genuchten \(1980\)](#) equation to the scaled capillary pressure–saturation relationship:

$$S^* = \begin{cases} [1 + (\alpha h^*)^n]^{-m} & h^* > 0 \\ 1 & h^* \leq 0 \end{cases} \quad (32)$$

where S^* is the scaled effective saturation, h^* is the scaled capillary pressure head, α and n are fitting parameters, and $m = 1 - 1/n$. To obtain the scaled function $S^*(h^*)$, the saturation of the wetting phase of each two-phase system is first normalized by its irreducible saturation to obtain the effective saturation:

$$\bar{S}_j^{ij} = \frac{S_j^{ij} - S_m}{1 - S_m} \quad (33)$$

where \bar{S}_j^{ij} is the effective wetting saturation of phase j in the i – j two-phase system ($i, j = o, w, g$; $i \neq j$). Then the capillary pressure head in each two-phase relationship (air–water, oil–water and air–oil [denoted by the superscripts aw, ow and ao, respectively]) may be scaled such that:

$$\bar{S}_w^{aw}(\beta_{aw} h_{aw}) = \bar{S}_w^{ow}(\beta_{ow} h_{ow}) = \bar{S}_o^{ao}(\beta_{ao} h_{ao}) = S^*(h^*) \quad (34)$$

where β_{aw} , β_{ow} , β_{ao} are fluid-pair-dependent scaling factors, and h_{ij} is the unscaled capillary pressure head in the i – j system. According to [Parker et al. \(1987\)](#), $\beta_{aw} = 1$; hence $\alpha = \alpha_{aw}$ and $S^*(h^*) = \bar{S}_w^{aw}(h)$.

A major advantage of this model is that it depends solely on capillary pressure relationships, data that are far more readily available than two-phase relative permeability data. Because the [van Genuchten \(1980\)](#) equation is commonly used by hydrologists to describe the capillary pressure relationship, the use of this model to obtain the relative permeability relationship requires little additional work. Therefore, it is often preferred by hydrologists; [van Geel and Sykes \(1994\)](#), for instance, used an extended Parker–Lenhard model to describe the spill of a light nonaqueous phase liquid (LNAPL) in a variably saturated sand. [Delshad and Pope \(1989\)](#) found that the Parker–

Lenhard model's predictions for oil three-phase relative permeabilities were reasonably good for a specific set of data but not so good overall. Baker (1988) also found the Parker–Lenhard model's performance inferior to saturation-interpolation models. Consequently, whether the model often utilized by contaminant hydrologists is the superior model is open to question.

2.7. Brooks–Corey–Burdine

Oostrom and Lenhard (1998) presented a model for oil relative permeability in three-phase systems based on the model of Burdine (1953) for relative permeability and on the model of Brooks and Corey (1964) for the capillary pressure–saturation relationship:

$$k_{ro} = (\bar{S}_t - \bar{S}_w)^2 \left(\bar{S}_t^{\frac{2+\lambda}{\lambda}} - \bar{S}_w^{\frac{2+\lambda}{\lambda}} \right) \quad (35)$$

where \bar{S}_t and \bar{S}_w are defined by Eqs. (30) and (31), respectively, and λ is the Brooks–Corey pore-size distribution index, obtained by fitting the capillary pressure–saturation relationship for the air–water system with the following relationship:

$$\bar{S}_w = \left(\frac{h_d}{h_{aw}} \right)^\lambda \quad \text{for } h_{aw} > h_d \quad (36)$$

where h_d is the air-entry pressure head.

Like the Parker–Lenhard model, this model does not use any two-phase relative permeability data. The developers of the model found that its performance was superior to the Parker–Lenhard model for the simulation of LNAPL infiltration. Yet, because of its relatively recent introduction, this assessment has not been validated by independent investigations.

2.8. DiCarlo–Sahni–Blunt

DiCarlo et al. (2000) measured drainage three-phase oil relative permeabilities in sandpacks for water-, oil- and fractionally wet porous media. Their results showed that, for a water-wet medium, the oil relative permeability could be described by simple power relationships much like Wyllie (1962) and others found for two-phase relationships:

$$k_{ro} = \begin{cases} S_o^4, & S_o \geq S_{orw} \\ S_o^2 S_{orw}^2, & S_o < S_{orw} \end{cases} \quad (37)$$

depending on the saturation range. S_{orw} , the so-called “waterflood residual” was determined by first saturating the porous medium with the most wetting phase (water for water-wet media) followed by flooding with five pore volumes of the oil phase, followed by flooding with another five pore volumes of water. In the three-phase system, saturations lower than the two-phase “waterflood residual” can be achieved by

oil layer drainage. In this process, oil flows due to thinning of the connected layer of oil that exists in the corners and crevices of the porous medium resulting in a saturation of oil of only 0.01 measured after 7 weeks of drainage.

The most important aspect of this formulation is that it represents a departure from the currently accepted notion that the oil relative permeability is dependent on the saturations of water and gas. In fact, this model requires no two-phase data other than S_{orw} for the prediction of three-phase relative permeabilities. Because of its simplicity and limited data requirements this model is attractive. Yet its predictive capability has still to be assessed using additional data sets, independently.

3. Methods

In order to assess the best model for estimating drainage relative permeability to oil in three-phase systems, calculations were made using the eight models described above and compared to relative permeability measurements reported by Oak (1990) in Berea sandstone cores. The data for core #6 were selected since a comprehensive set of experiments were obtained for this core including capillary pressure–saturation measurements (given in Oak et al., 1990) and all three sets of two-phase relative permeability relationships (gas–water, oil–water, and oil–gas), in addition to drainage three-phase measurements. A total of 29 measured values of drainage oil three-phase relative permeability were compared with the values predicted by the eight models.

Table 1 lists the data requirements for the eight models examined in this study. To use those models requiring capillary pressure data (Parker–Lenhard and Brooks–Corey–Burdine), the air–water curve was taken from Oak et al. (1990) and fitted with Eq. (32) to obtain the parameter m , and with Eq. (36) to obtain λ , both by minimizing the sum of the squared errors. Most of the models are based on two-phase relative permeability data, specifically the curves for the oil–water and oil–gas systems. To use these data, they must be fit with some functional form. One set of fitting equations for

Table 1
Data requirements for the three-phase oil relative permeability models examined

Model	Data required	
<i>Based on two-phase relative permeability data</i>		
Stone I	$k_{\text{row}}, k_{\text{rog}}, S_{\text{wc}}, S_{\text{om}}$	Eqs. (3)–(8)
Stone II	$k_{\text{row}}, k_{\text{two}}, k_{\text{rog}}, k_{\text{rgo}}$	Eqs. (13) and (14)
Baker	$k_{\text{row}}, k_{\text{rog}}, S_{\text{wt}}, S_{\text{gr}}$	Eqs. (15) and (16)
Hustad–Hansen	$k_{\text{row}}, k_{\text{rog}}, S_{\text{orw}}, S_{\text{gro}}, S_{\text{wro}}$	Eqs. (17)–(22)
Delshad–Pope	$k_{\text{ro}}^o, S_{\text{wt}}, S_{\text{or}}, S_{\text{gr}}$	Eqs. (23)–(28a)
DiCarlo et al.	S_{orw}	Eq. (37)
<i>Based on soil characteristic curve</i>		
Parker–Lenhard	m, S_m	Eqs. (29)–(34)
Brooks–Corey–Burdine	λ, S_m	Eqs. (35) and (36)

the relative permeability curves is the Corey-type equations presented by [Delshad and Pope \(1989\)](#):

$$k_{\text{rwo}} = \left(\frac{S_{\text{w}} - S_{\text{wro}}^*}{1 - S_{\text{wro}}^*} \right)^{\text{ew}} \quad (38)$$

$$k_{\text{row}} = k_{\text{row}}^o \left(\frac{S_{\text{o}} - S_{\text{orw}}^*}{1 - S_{\text{wro}}^* - S_{\text{orw}}^*} \right)^{\text{eow}} \quad (27\text{b})$$

where S_{wro}^* , and S_{wro}^* are parameters that account for residual saturations while ew and eow are fitting parameters. As noted before, relative permeability curves in oil–gas two-phase systems are usually measured in the presence of some constant water saturation and are plotted as a function of total liquid saturation, $S_{\text{L}} = 1 - S_{\text{g}}$. Thus, the corresponding set of fitting equations for this case would be:

$$k_{\text{rog}} = k_{\text{rog}}^o \left(\frac{S_{\text{L}} - S_{\text{Lrg}}^*}{1 - S_{\text{Lrg}}^* - S_{\text{gro}}^*} \right)^{\text{eog}} \quad (28\text{b})$$

$$k_{\text{rgo}} = k_{\text{rg}}^o \left(\frac{S_{\text{g}} - S_{\text{gr}}^*}{1 - S_{\text{Lrg}}^* - S_{\text{gro}}^*} \right)^{\text{eg}} \quad (39)$$

where S_{Lrg}^* and S_{gro}^* account for residual saturations and eog and eg are fitting parameters.

Another set of fitting equations is the Mualem-type introduced by [Demond and Roberts \(1993\)](#) and modified here by the introduction of k_{rwet}^o to account for the presence of connate water:

$$k_{\text{rwet}} = k_{\text{rwet}}^o S_{\text{e}}^a [1 - (1 - S_{\text{e}}^{1/b})^b] \quad (40)$$

$$k_{\text{rnonwet}} = c(1 - S_{\text{e}})^d (1 - S_{\text{e}}^{1/e})^e \quad (41)$$

Eq. (40) is appropriate for the water curve in the oil–water system and the oil curve in the oil–gas system, and Eq. (41) is appropriate for the oil curve in the oil–water system and the gas curve in the oil–gas system. For the oil–water system, S_{e} is defined as:

$$S_{\text{e}} = \frac{S_{\text{w}} - S_{\text{wro}}^*}{1 - S_{\text{wro}}^*} \quad (42)$$

and for the oil–gas system, it is defined as:

$$S_{\text{e}} = \frac{S_{\text{L}} - S_{\text{Lrg}}^*}{1 - S_{\text{Lrg}}^*} \quad (43)$$

where S_{wro}^* and S_{Lrg}^* account for water residual and total liquid residual saturations, respectively. The fitting parameters a , b , c , d , and e are different for the two systems.

To compare the estimates given by the different predictive models, ternary diagrams with isoperms of oil relative permeability for primary drainage were produced. The

isoperms were calculated by curve tracing, not by contour interpolation, thus giving an exact representation of the model. Three-phase oil isoperms were calculated for the saturation ranges that represented the intersection of saturations covered in the two-phase and in the three-phase data sets. The residual saturations S_{wr} , S_{wto} , S_{orw} , and S_{org} were taken directly from two-phase data. Since Oak (1990) reported different values for the residual gas saturation, the value of 0.10 was assigned to S_{gr} . The connate water saturation was fixed at 0.30, the value used in the measurement of the oil–gas two-phase relative permeability curves. To use the Hustad–Hansen model, the two-phase data sets were normalized and the resulting curves fitted with a power function, $k_{ro} = aS_o^{**b}$, where S_o^{**} is the normalized saturation defined by Eq. (20), and a and b are fitting parameters. To use the Delshad–Pope model, the parameters α , β , γ , and δ were calculated from the two-phase fitting parameters eow and eog , as suggested by Delshad and Pope (1989).

The impact of which saturation to honor was investigated for the Stone I, Stone II, and Baker models. The Stone I and Stone II models as modified by Aziz and Settari (1979) can be interpreted in two ways: (1) the total liquid saturation is taken as the complement of the three-phase gas saturation: $S_L = 1 - S_g$, thus honoring the gas saturation as suggested originally by Stone (1970, 1973); or (2) the total liquid saturation is taken as $S_L = S_o + S_{wc}$, thus honoring the oil three-phase saturation and obtaining the corresponding two-phase value by adding the value of S_{wc} present in the oil–gas two-phase system as suggested by the notation in Aziz and Settari (1979). With respect to Baker, two options were investigated: (1) using $k_{ro(g)}$ as proposed by Baker (1988); and (2) using $k_{r(o)g}$ as suggested by Blunt (2000). The predictions based on each of these formulations were calculated and compared with the measured data.

To assess the impact of confusion over the definition of the minimum water saturation, estimates of three-phase relative permeabilities were calculated and compared using the values of connate water saturation (S_{wc}) and residual water saturation (S_{wr}) interchangeably: first, S_{wr} was taken as equal to 0.30, the value of the connate water saturation or the saturation of water present in the “two-phase” measurements of oil–gas relative

Table 2

Parameter values obtained from fitting the two-phase relative permeability data for drainage in Berea sandstone from Oak (1990)

Oil–water												
	Corey-type (Eqs. (27a), (28a), (38) and (39))					Mualem-type (Eqs. (40) and (41))						
	$S_{wr}^{*\dagger}$	S_{orw}^*	k_{ro}^o	ew	eow	a	b	c	d	e	k_{rwet}^o	
k_{rw}	0.26	–	–	3.925	–	3.925	4.999	–	–	–	1	
k_{row}	0.26	0	0.9	–	1.823	–	–	0.897	1.816	0.100	–	
Oil–gas												
	Corey-type					Mualem-type						
	$S_{Lrg}^{*\dagger}$	S_{gr}^*	k_{rog}^o	k_{rg}^o	eog	eg	a	b	c	d	e	k_{rwet}^o
k_{rog}	0.40	0	0.9	–	4.020	–	2.479	0.733	–	–	–	0.9
k_{rg}	0.40	0	–	0.51	–	1.748	–	–	0.498	1.477	0.537	–

[†] The same value was used in the Mualem-type equations.

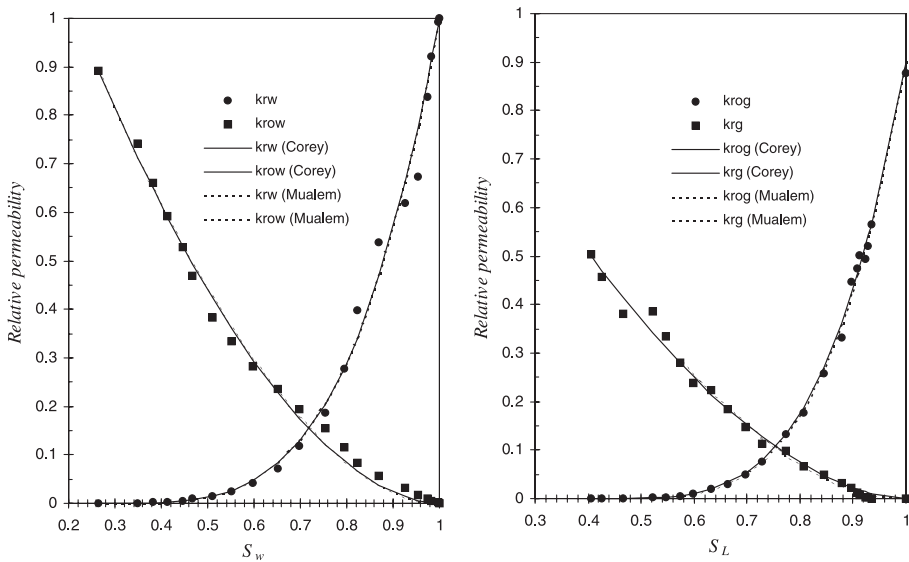


Fig. 1. Curve fitting of two-phase relative permeability data: oil–water system (left) and oil–gas system with $S_{wc}=0.30$ (right).

permeabilities; then, S_{wc} was taken as equal to 0.26, the minimum water saturation achieved in the oil–water two-phase relative permeability measurements.

The average errors of the predictive models relative to the measured data were calculated using both linear and logarithmic forms, similar to the approach of [Delshad and Pope \(1989\)](#):

$$SD = \sqrt{\frac{\sum (k_{ro}^* - k_{ro})^2}{n - 1}} \quad (44)$$

$$LE = \sqrt{\frac{\sum (\log k_{ro}^* - \log k_{ro})^2}{n - 1}} \quad (45)$$

where k_{ro}^* and k_{ro} are the estimated and measured values, respectively; and n is the number of data points. Eq. (44) is appropriate for expressing the error at high values of relative permeability, whereas Eq. (45) better represents the errors at low relative permeability values, especially if the error is of the same order of magnitude as the prediction itself. The

Table 3
Fitting errors for two-phase relative permeability data using Corey- and Mualem-type functions

Function	k_{rwo}	k_{row}	k_{rog}	k_{rgo}
Corey-type (Eqs. (27a), (28a), (38) and (39))	3.30×10^{-2}	1.99×10^{-2}	1.44×10^{-2}	1.53×10^{-2}
Mualem-type (Eqs. (40) and (41))	3.30×10^{-2}	2.01×10^{-2}	3.01×10^{-2}	1.44×10^{-2}

Errors computed using Eq. (44).

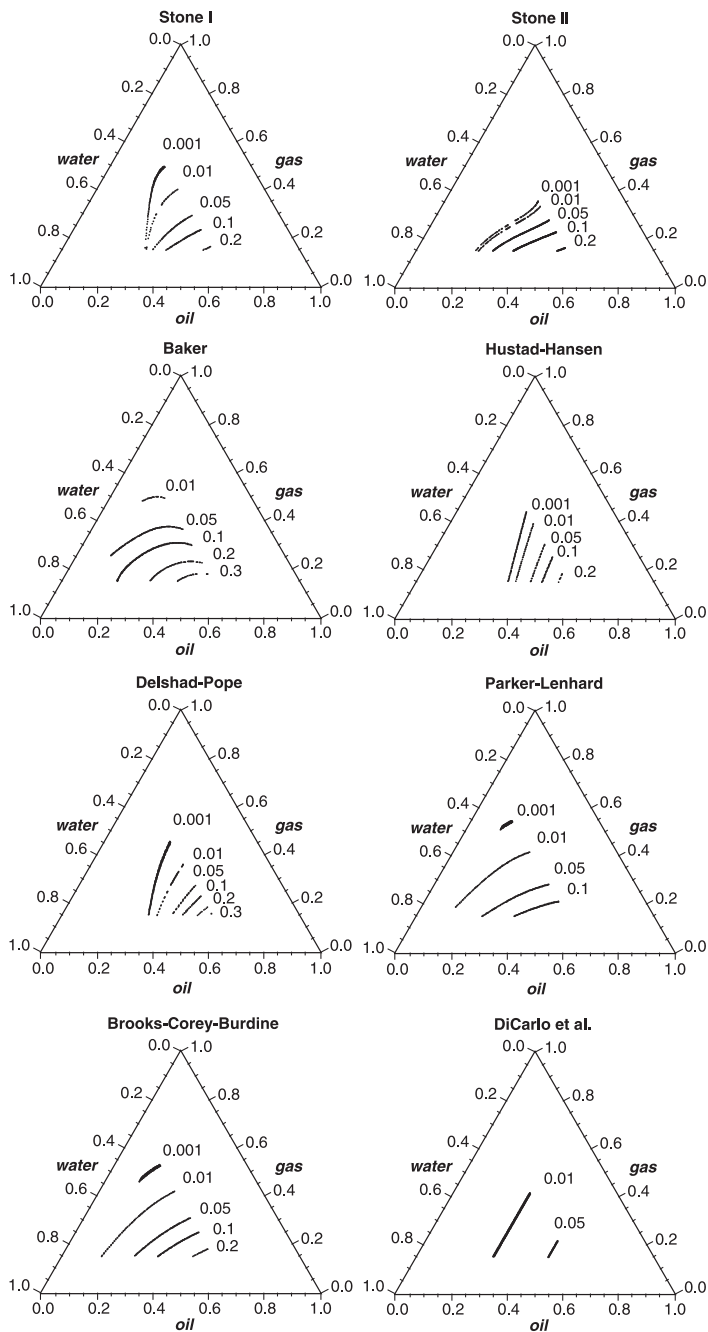


Fig. 2. Predicted oil three-phase relative permeability isoperms for primary drainage.

mean, median, and standard deviation of the sets of predicted values and of the measured ones were also calculated so that one can compare how the estimates produced by each model differ from each other and from the measured data.

4. Results and discussion

The first issue investigated was whether the use of the Corey-type (Eqs. (27a), (28a), (38) and (39)) or the Mualem-type (Eqs. (40) and (41)) for fitting two-phase relative permeability data would make a difference in the goodness of prediction of the three-phase relative permeability to oil. The values for S_{wr}^* , S_{orw}^* , S_{Lrg}^* , and S_{gr}^* were taken directly from the two-phase data whereas k_{ro}^o , ew , eow , k_{rog}^o , k_{rg}^o , eog , eg , a , b , c , d , e , and k_{rwt}^o were treated as adjustable parameters. The values for the fitting parameters are given in Table 2. Fig. 1 shows a comparison of the fitted curves and measured data. For the two-phase data sets analyzed here, both models fit the experimental points almost identically, as shown by the fitting errors (Table 3). Therefore, for this data set, no significant impact will be evident in relative permeability predictions based on the selection of either Corey- or Mualem-type equations to interpolate two-phase data. Since Mualem-type models appear to be the norm in environmental simulations (e.g., Abriola et al., 1997), this model was used for all predictions using models that call for two-phase relative permeability data.

Fig. 2 shows the predicted isoperms for oil relative permeability and Table 4 lists the parameters used. For the Stone I, Stone II, and Baker models, the gas saturation was honored (as suggested by the originators of these models). A preliminary inspection reveals that the general shape of the isoperms varies considerably. Not all models were able to provide an estimate over the same saturation regions; in particular the Stone I, Stone II, Hustad–Hansen, and Delshad–Pope models were limited for some saturation combinations which either resulted in effective saturation values outside the range [0, 1] or in negative values of the oil three-phase relative permeability. Furthermore, the estimates obtained with the Stone I and Stone II models are almost the same for relative permeabilities higher than 0.05 but differ significantly for lower ones. The isoperms obtained with Baker's model show more concavity and yield higher estimates of relative permeability than those generated by any of the other models. The Hustad–Hansen and Delshad–Pope models show similar patterns of saturation dependency, but the latter predicts higher values than the former, especially in the high oil saturation range. The Parker–Lenhard model's estimates are similar to those from Brooks–Corey–Burdine, with the latter predicting somewhat higher relative permeabilities than the former in the higher oil saturation range. The predictions obtained with the DiCarlo et al. model are markedly different from the others, and its isoperms are the only ones which run parallel to the water axis, indicative of the dependence of the predictions only on the oil saturation.

Table 4
Parameters used in three-phase oil relative permeability predictions

S_{wc}	S_{wr}	S_{wto}	S_{gr}	S_{orw}	S_{org}	S_{gro}	S_m	λ	m
0.30	0.26	0.26	0.10	0.36	0.10	0.36	0.26	0.97	0.557

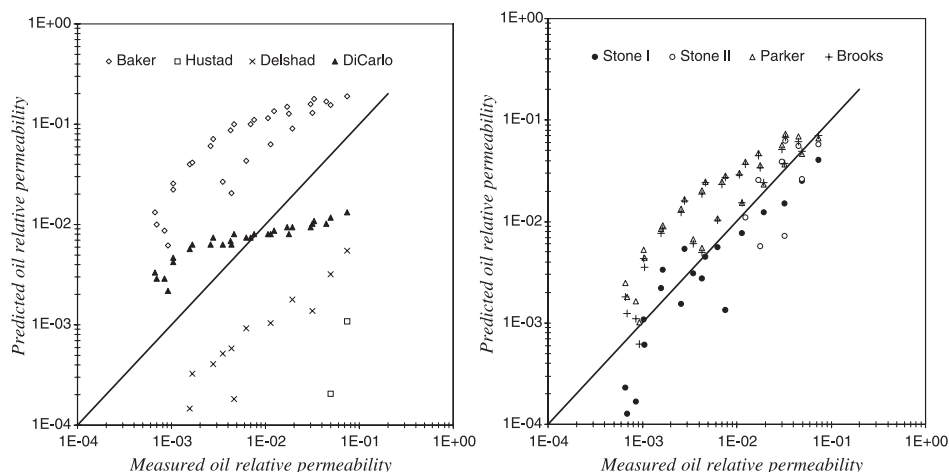


Fig. 3. Comparison of measured data and predicted oil three-phase relative permeabilities. (When the predicted value coincides with the measured one, the mark on the plot falls on the solid line.)

Thus, the predictions of three-phase relative permeability to oil may differ significantly, up to 150%, depending on the selected model.

Fig. 3 shows two plots of measured relative permeability values versus their respective estimated values, obtained using the eight models as outlined by their originators. While the measured values spanned almost three orders of magnitude, the estimated values generally spanned only one. Two groups of models can be identified: one group consisting of the Baker, Hustad–Hansen, and Delshad–Pope models whose predictions were often off by more than one order of magnitude. The Baker model consistently overestimated the measured values whereas the Delshad–Pope model and the Hustad–Hansen model underestimated those values. Unfortunately in the data set used here, only two points could be obtained using the Hustad–Hansen model because negative effective saturation values were obtained for the remaining saturation combinations. The other group,

Table 5
Comparison of models' statistics

Model	SD (Eq. (44))	LE (Eq. (45))	Mean	Median	Standard deviation	n^*
Stone I	0.0109	0.3726	0.0107	0.0053	0.0126	19
Stone II	0.0184	0.3325	0.0132	0.0056	0.0181	9
Baker	0.0844	1.0562	0.0843	0.0866	0.0580	29
Hustad–Hansen	0.0877	2.7460	0.0097	0.0043	0.0118	2
Delshad–Pope	0.0215	1.4734	0.0067	0.0009	0.0114	19
Parker–Lenhard	0.0160	0.4774	0.0249	0.0200	0.0217	29
Brooks–Corey–Burdine	0.0141	0.4407	0.0239	0.0187	0.0209	29
DiCarlo et al.	0.0170	0.4510	0.0074	0.0075	0.0027	29
Measured	0.0000	0.0000	0.0139	0.0063	0.0179	29

* The smaller value of n is due to the discarding of negative values of k_{ro} and values of effective saturation outside of the range $[0, 1]$.

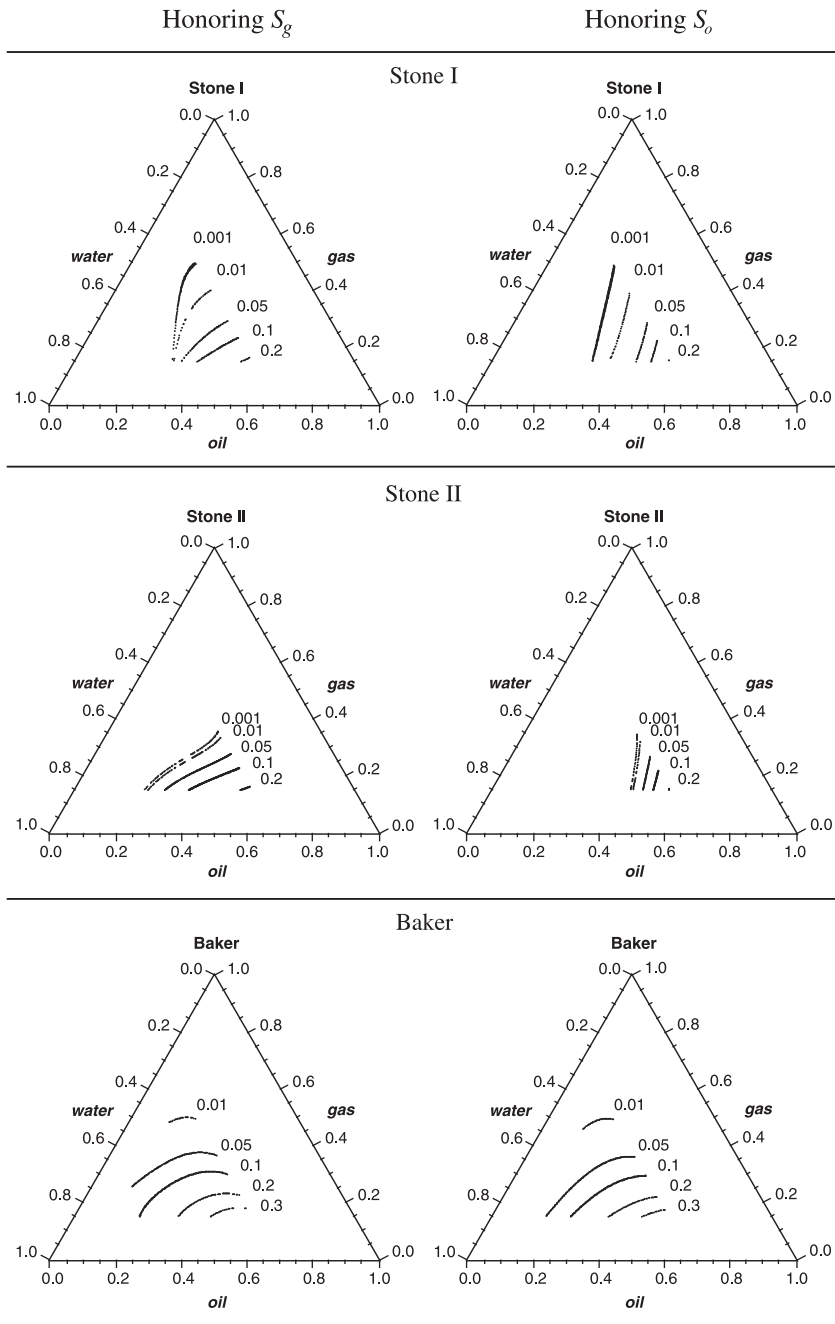


Fig. 4. k_{ro} isoperms honoring: the S_g three-phase saturation (left graphs) and the S_o three-phase saturation adding S_{wc} (right graphs).

consisting of the Stone I, Stone II, Parker–Lenhard, Brooks–Corey–Burdine and DiCarlo et al. models produced estimates whose values were generally within the same order of magnitude of the measured relative permeability values. The Stone models mostly underpredicted the measured values whereas the Parker–Lenhard and the Brooks–Corey–Burdine models consistently overestimated them. Finally, the DiCarlo et al. model underpredicted high values of relative permeability and overpredicted the low ones.

The calculated errors and statistics associated with Fig. 3 are listed in Table 5. These statistics confirm that the predictions obtained using the Hustad–Hansen, Baker and Delshad–Pope models were the poorest for this data set (the fact that the Hustad–Hansen model could provide only two estimates out of the 29 points available may render the statistics meaningless; nevertheless, this further confirms this model's inability to predict these data). The large log errors given by Baker's and Delshad–Pope's models of 1.06 and 1.47, respectively, are indicative of differences of an order of magnitude or more between predictions and measurements. The smallest log errors were obtained with the Stone models whereas the smallest standard errors were given by Stone I model, followed by the models based on capillary pressure data, the Brooks–Corey–Burdine and the Parker–Lenhard models. The fact that the Stone I model was the best predictor for the three-phase oil relative permeabilities in this data set must be tempered by the realization that it was unable to give an estimate for 10 out of 29 points. From those models, which were able to predict the relative permeability for all the points, the Brooks–Corey–Burdine model was the best predictor.

The issue of which saturation to employ proved to be a critical factor in the performance of the Stone models and had a lower impact on that of Baker's model. Fig. 4 shows the isoperms obtained using $S_L = 1 - S_g$ (honoring S_g) and $S_L = S_o + S_{wc}$ (honoring S_o) in the Stone models, and using $k_{ro(g)}$ and $k_{r(o)g}$ in Baker's model. When the oil saturation was honored (right graphs), both Stone isoperms assumed very different shapes and patterns of saturation dependency. Baker's isoperms, however, changed slightly, showing less concavity and lower values. A comparison of the errors generated with the alternative interpretation of the models is given in Table 6. By honoring the oil saturation, the predictions obtained with the Stone I model worsened while those obtained with Baker's model improved. The Stone II model, however, was not able to give a prediction for any of the 29 values: all calculations resulted in the prediction of negative relative permeability values. Therefore, which saturation is honored can have significant impact on a model's

Table 6
Comparison of model performance depending upon saturation honoring scheme

Model	SD (Eq. (44))	LE (Eq. (45))	n^*
Stone I (S_g)	0.0109	0.3726	19
Stone I (S_o)	0.0219	1.2065	19
Stone II (S_g)	0.0184	0.3325	9
Stone II (S_o)	–	–	0
Baker ($k_{ro(g)}$)	0.0844	1.0562	29
Baker ($k_{r(o)g}$)	0.0573	0.9465	29

* The smaller values of n are due to the discarding of negative value of k_{ro} and values of effective saturation outside of the range [0, 1].

Table 7

Comparison of model performance depending upon the values used for residual and connate water saturation

Model	$S_{\text{wT}}=0.26$		$S_{\text{wT}}=0.30$		$S_{\text{wT}}=0.26$		n
	$S_{\text{wC}}=0.30$		$S_{\text{wC}}=0.30$		$S_{\text{wC}}=0.26$		
	SD	LE	SD	LE	SD	LE	
Stone I	0.0109 ⁽¹⁹⁾	0.3726	0.0109 ⁽¹⁹⁾	0.3726	0.0155 ⁽¹³⁾	0.3650	a
Stone II	0.0184	0.3325	0.0184	0.3325	0.0184	0.3325	9
Baker ^b	0.0844	1.0562	0.0599	0.8907	0.0562	0.9369	29
Hustad–Hansen	0.0877	2.7460	0.0874	2.6567	0.0877	2.7460	2
Delshad–Pope	0.0215	1.4734	0.0214	1.5964	0.0268	1.5380	19
Parker–Lenhard	0.0160	0.4774	0.0166	0.4564	0.0160	0.4774	29
Brooks–Corey–Burdine	0.0141	0.4407	0.0160	0.4389	0.0141	0.4407	29
DiCarlo et al.	0.0170	0.4510	0.0170	0.4510	0.0170	0.4510	29

^a The values of n for Stone I are given in the parentheses. All values of n less than 29 are due to the discarding of negative values of relative permeability and values of effective saturation outside of the range [0, 1].

^b Calculated honoring the oil saturation.

behavior and may even prevent the model's implementation. Thus, this issue should not be overlooked in numerical simulations utilizing these models.

To determine the impact of the selection of the value of the minimum saturation of water, first S_{wr} was set equal to the connate water saturation reported by Oak (1990) i.e., $S_{wr}=0.30$. Next, S_{wc} was set equal to the minimum water saturation reported in the relative permeability measurements, that is $S_{wc}=0.26$. The oil saturation was honored in the calculations with the Baker model, since this interpretation yielded better results. Table 7 shows the impact of these changes on the standard deviation and log errors of the estimates. Setting S_{wr} equal to 0.30 rather than 0.26 had no impact on the estimates obtained with Stone I, Stone II and DiCarlo et al.'s models since they do not include this parameter in their formulations. The error for other models' predictions did not change significantly, except in the case of Baker's model, where the standard error decreased by 29%. Next, the connate water saturation was set equal to the residual water saturation in the relative permeability measurements, i.e., $S_{wc}=0.26$. This time, the errors in the predictions obtained with the Stone I and Delshad–Pope increased considerably (42% and 25% increase in the standard error, respectively) while the error in Baker's predictions decreased even more, by another 33%. The remaining models predictions were unchanged because S_{wc} was not a parameter employed by these models. These calculations show that the often interchangeable use of the terms connate and residual (and even irreducible for that matter) saturation may affect a model's performance. The difference here between the values was small, only 0.04, yet, in some cases, this difference was enough to affect the predictions significantly. In other data sets, this difference may be considerably larger, with perhaps even more dramatic consequences for the confusing use of these terms.

5. Conclusions

This study showed that models available for oil three-phase relative permeability yield significantly different estimates that, in turn, could have a significant impact on the

simulations of transport of organic liquids in the vadose zone. In addition, for the data set used here, the errors associated with these estimates were quite high. The goodness of the models does not increase with the amount of data or computation that the models require. For instance, the Brooks–Corey–Burdine and Lenhard–Parker models, which are based only on the capillary pressure–saturation curve, gave predictions close to the ones by Stone I and Stone II, which require two-phase relative permeability measurements. Furthermore, the Brooks–Corey–Burdine and Lenhard–Parker models were able to provide predictions over the entire range of saturations covered by the data set while the Stone models were not. Negative values may occur for some ternary saturation combinations, which may prevent the implementation of the model in numerical simulators. For this reason, it is suggested that a preliminary analysis of the generated isoperms be undertaken prior to the model use.

When dealing with three-phase relative permeability models that use two-phase data, the issue of which saturation to honor is an important detail, that is often overlooked in the description and implementation of such models. This study showed that very different results can be obtained by the selection of which phase's saturation to honor in the reading of two-phase data. While the predictions of Stone I worsened, Baker's predictions improved and Stone II was unable give any predictions for the saturations covered here.

This paper examined only oil relative permeabilities during primary drainage, focusing on a set of data obtained in water-wet porous medium. To fully assess the ability of models to predict the relative permeabilities pertinent to transport in the vadose zone, other aspects need to be included in the analysis such as hysteresis and altered wettabilities. Several attempts have been made, especially in the recent years, in order to incorporate hysteretic behavior in the prediction of oil relative permeability in three-phase systems (Land, 1968; Killough, 1976; Lenhard and Parker, 1987; Parker and Lenhard, 1987; Parker et al., 1987; Larsen and Skauge, 1998; Blunt, 2000; Fayers et al., 2000; Hustad, 2000; Lerdahl et al., 2000). In addition, the impact of interfacial tension and wettability has also been investigated (Oak, 1991; Øren and Pinczewski, 1995; Sahni et al., 1998; DiCarlo et al., 2000; Hui and Blunt, 2000; Djike et al., 2001). The phenomena near residual saturation when oil is often assumed to be immobile have begun to receive attention (Blunt et al., 1995; Keller et al., 1997; Fenwick and Blunt, 1998; Firincioglu et al., 1999). Because of the additional complexity of these models, they often call for parameters such as trapped saturations and the history of saturation changes that are not readily available. Consequently, more work is needed to make these models more user accessible such as those examined here.

Acknowledgements

The authors thank David DiCarlo and the reviewers for their helpful comments, and Martin J. Blunt and Mojdeh Delshad for their comments and for providing spreadsheets of Oak's data. One of the authors (L. Oliveira) would like to acknowledge the support received from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq)-Brazil.

References

- Abriola, L.M., Lang, J., Rathfelder, K., 1997. Michigan Soil Vapor Extraction Remediation (MISER) model: a computer program to model soil vapor extraction and bioventing of organic chemicals in unsaturated geological material. Tech. Rep. EPA/600/R-97/099.
- Aziz, K., Settari, A., 1979. *Petroleum Reservoir Simulation*. Elsevier Applied Science, London.
- Baker, L.E., 1988. Three-phase relative permeability correlations. Enhanced Oil Recovery Symposium (Paper SPE/DOE 17369), April 17–20. SPE, Tulsa, OK, pp. 539–554.
- Balbinsky, E.F., Fishlock, T.P., Goodyear, S., Jones, P.I.R., 1999. Key characteristics of three-phase oil relative permeability formulations for improved oil recovery predictions. *Petroleum Geoscience* 5, 339–346.
- Blunt, M.J., 2000. An empirical model for three-phase relative permeability. *SPE Journal* 5 (4), 435–445.
- Blunt, M.J., 2002. Personal communication. Imperial College, UK.
- Blunt, M.J., Zhou, D., Fenwick, D., 1995. Three-phase flow and gravity drainage in porous media. *Transport in Porous Media* 20, 77–103.
- Brooks, R.H., Corey, A.T., 1964. Hydraulic properties of porous media. *Hydrology Papers* 3, 27 (Colorado State University, Fort Collins, CO).
- Burdine, N.T., 1953. Relative permeability calculations from pore size distribution data. *Transactions of AIME* 198, 71–78.
- Corey, A.T., Rathjens, C.H., Henderson, J.H., Wyllie, M.R.J., 1956. Three-phase relative permeability. *Transactions of AIME* 207, 349–351.
- Delshad, M., Pope, G.A., 1989. Comparison of the three-phase oil relative permeability models. *Transport in Porous Media* 4, 59–83.
- Demond, A.H., Roberts, P.V., 1993. Estimation of two-phase relative permeability relationships for organic liquid contaminants. *Water Resources Research* 29 (4), 1081–1090.
- DiCarlo, D.A., Sahni, A., Blunt, M.J., 2000. The effect of wettability on three-phase relative permeability. *Transport in Porous Media* 39, 347–366.
- Djike, M.I.J., Sorbie, K.S., McDougall, S.R., 2001. Saturation-dependencies of three-phase relative permeabilities in mixed-wet and fractionally wet systems. *Advances in Water Resources* 24, 365–384.
- Donaldson, E.C., Dean, G.W., 1966. Two- and three-phase relative permeability studies. Tech. Rep. Report of Investigations 6826, United States Department of Interior—Bureau of Mines, Washington, DC.
- Fayers, F.J., Matthews, J.D., 1984. Evaluation of normalized Stone's methods for estimating three-phase relative permeabilities. *SPE Journal* 24 (2), 224–232.
- Fayers, F.J., Foakes, A.P., Lin, C.Y., Puckett, D.A., 2000. An improved three phase flow model incorporating compositional variance. SPE/DOE Symposium on Improved Oil Recovery (SPE Paper 59313), Tulsa, OK, April 2–5.
- Fenwick, D.H., Blunt, M.J., 1998. Three-dimensional modeling of three phase imbibition and drainage. *Advances in Water Resources* 21 (2), 121–143.
- Firincioglu, T., Blunt, M.J., Zhou, D., 1999. Three-phase flow and wettability effects in triangular capillaries. *Colloids and Surfaces. A: Physicochemical and Engineering Aspects* 155, 259–276.
- Grader, A.S., O'Meara, D.J., 1988. Dynamic displacement measurements of three-phase relative permeabilities using three immiscible liquids. 63rd Annual Technical Conference and Exhibition of the SPE (Paper SPE/DOE 18293), October 2–5. SPE, Houston, TX, pp. 325–338.
- Guzman, R.E., Giordano, D., Fayers, F.J., Aziz, K., Godi, A., 1994. Three-phase flow in field-scale simulations of gas and WAG. European Petroleum Conference (Paper SPE 28897), October 25–27. SPE, London, UK, pp. 311–324.
- Honarpour, M., Koederitz, L., Harvey, A.H., 1986. *Relative Permeability of Petroleum Reservoirs*. CRC Press, Boca Raton, FL.
- Hui, M., Blunt, M.J., 2000. Pore-scale modeling of three-phase flow and the effects of wettability. 2000 SPE/DOE Improved Oil Recovery Symposium (Paper SPE 59309), Tulsa, OK, April 3–5.
- Hustad, O.S., 2000. A coupled model for three-phase capillary pressure and relative permeability. 2000 SPE Annual Technical Conference and Exhibition (Paper SPE 63150), Dallas, TX, Oct. 1–4.
- Hustad, O.S., Hansen, A.G., 1995. A consistent correlation for three-phase relative permeabilities and phase

- pressures based on three sets of two-phase data. Eighth European Symposium on Improved Oil Recovery, Vienna, Austria, May 15–17.
- Keller, A.A., Blunt, M.J., Roberts, P.V., 1997. Micromodel observation of the role of oil layers in three-phase flow. *Transport in Porous Media* 26 (1–2), 277–297.
- Killough, J.E., 1976. Reservoir simulation with history-dependent saturation functions. *SPE Journal* 16 (1), 37–48.
- Land, C.S., 1968. Calculation of imbibition relative permeability for two- and three-phase flow from rock properties. *SPE Journal* 243, 149–156.
- Larsen, J.A., Skauge, A., 1998. Methodology for numerical simulation with cycle-dependent relative permeabilities. *SPE Journal* 3, 163–173.
- Lenhard, R.J., Parker, J.C., 1987. A model for hysteretic constitutive relations governing multiphase flow: 2. Permeability–saturation relations. *Water Resources Research* 23 (12), 2197–2206.
- Lerdahl, T.R., Øren, P.E., Bakke, S., 2000. A predictive network model for three-phase flow in porous media. SPE/DOE Symposium on Improved Oil Recovery (SPE Paper 59311), Tulsa, OK, April 2–5.
- Leverett, M.C., Lewis, W.B., 1941. Steady flow of gas–oil–water mixtures through unconsolidated sands. *Transactions of AIME* 142, 107–116.
- Manj Nath, A., Honarpour, M.M., 1984. An investigation of three-phase relative permeability. 1984 Rocky Mountain Regional Meeting (SPE Paper 12915), May 21–23. SPE, Casper, WY, pp. 205–214.
- Morrow, N.M., Melrose, J.C., 1991. Application of capillary pressure measurements to the determination of connate water saturation. *Interfacial Phenomena in Petroleum Recovery*. Marcel Dekker, New York, pp. 257–287.
- Mualem, Y., 1976. A new model for predicting the hydraulic conductivity of unsaturated porous media. *Water Resources Research* 12 (3), 513–522.
- Muskat, M., 1981. *Physical Principles of Oil Production*, 2nd ed. International Human Resources Development, Boston, MA.
- Oak, M.J., 1990. Three-phase relative permeability of water-wet Berea. Seventh Symposium on Enhanced Oil Recovery (Paper SPE/DOE 20183), Tulsa, OK, April 22–25.
- Oak, M.J., 1991. Three-phase relative permeability of intermediate-wet Berea sandstone. 66th Annual Technical Conference and Exhibition of the SPE (Paper SPE 22599), October 6–9. SPE, Dallas, TX, pp. 137–144.
- Oak, M.J., Baker, L.E., Thomas, D., 1990. Three-phase relative permeability of Berea sandstone. *Journal of Petroleum Technology* 42 (8), 1054–1061.
- Oostrom, M., Lenhard, R.J., 1998. Comparison of relative permeability–saturation–pressure parametric models for infiltration and redistribution of a light nonaqueous-phase liquid in sandy porous media. *Advances in Water Resources* 21 (2), 145–157.
- Øren, P.E., Pinczewski, W.V., 1995. Fluid distribution and pore scale displacement mechanisms in drainage dominated three-phase flow. *Transport in Porous Media* 20, 105–133.
- Parker, J.C., Lenhard, R.J., 1987. A model for hysteretic constitutive relations governing multiphase flow: 1. Saturation–pressure relations. *Water Resources Research* 23 (12), 2187–2196.
- Parker, J.C., Lenhard, R.J., Kuppusamy, T., 1987. A parametric model for constitutive properties governing multiphase flow in porous media. *Water Resources Research* 23 (4), 618–624.
- Sahni, A., Burger, J., Blunt, M., 1998. Measurement of three phase relative permeability during gravity drainage using CT scanning. SPE/DOE Improved Oil Recovery Symposium (SPE Paper 39655), April 19–22. SPE, Tulsa, OK, pp. 11–24.
- Saraf, D.N., Fatt, I., 1967. Three-phase relative permeability measurement using a nuclear magnetic resonance technique for estimating fluid saturation. *Society of Petroleum Engineers of AIME Journal* 7, 235–242.
- Saraf, D.N., McCaffery, F.G., 1985. *Relative Permeabilities*. Elsevier, New York, pp. 75–118.
- Sarem, A.M., 1966. Three-phase relative permeability measurements by unsteady-state method. *Society of Petroleum Engineers of AIME Journal* 9, 199–203.
- Snell, R.W., 1962. Three-phase relative permeability in an unconsolidated sand. *Institute of Petroleum Journal* 48, 80–88.
- Stone, H.L., 1970. Probability model for estimating three-phase relative permeability. *Journal of Petroleum Technology* 22, 214–218.
- Stone, H.L., 1973. Estimation of three-phase relative permeability and residual oil data. *Journal of Canadian Petroleum Technology* 12, 53–61.

- van Geel, P.J., Sykes, J., 1994. Laboratory and model simulations of a NAPL spill in a variably-saturated sand: 2. Comparison of laboratory and model results. *Journal of Contaminant Hydrology* 17, 27–53.
- van Genuchten, M.T., 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America* 44, 892–898.
- van Spronsen, E., 1982. Three-phase relative permeability measurements using the centrifuge method. Third Joint Symposium on Enhanced Oil Recovery of the SPE (Paper SPE/DOE 10688), Tulsa, OK, April 4–7.
- Wyllie, M.R.J., 1962. Relative permeability. In: Frick, T., Taylor, R. (Eds.), *Petroleum Production Handbook*. Reservoir Engineering, vol. II. McGraw-Hill, New York, NY, pp. 25.1–25.14.