A Mixed-Wet Hysteretic Relative Permeability and Capillary Pressure Model for Reservoir Simulations

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Summary

A crucial component of all multiphase flow models is the relationship among relative permeabilities, fluid saturations, and capillary pressures. Relative permeability and capillary pressure parametric models can be very useful for predicting fluid behavior in porous media. However, relative permeabilities and capillary pressures used in oil reservoir simulators are commonly determined through interpolation between laboratory measurements. A problem with this approach is that the relations are valid only for the specific saturation path measured. Therefore, simulations of oil production using different saturation paths from those measured are likely to be in error and can limit the investigation of alternative production scenarios. In this paper, saturation-history-dependent relative permeability and capillary pressure functions for two-phase flow in mixed-wet rocks are discussed. Relative permeabilities are predicted by integrating a pore-distribution model between limits that reflect how oil and water are distributed in mixed-wet porous media. The proposed model was tested against mixed-wet capillary pressure data. The model then was incorporated in the U. of Texas Chemical Compositional Simulator (called UTCHEM) to compare waterflood simulations in water- and mixed-wet reservoirs. The simulation results agree qualitatively with laboratory core and field observations. The model and its implementation also were validated against a sandpack experiment.

Introduction

Significant emphasis and efforts have been directed toward better geological reservoir descriptions and numerical solutions of reservoir simulators, but relatively little attention has been paid to better fluid-flow description in the simulation models. Petrophysical properties such as relative permeability and capillary pressure are generally dependent on saturation and saturation history. Reservoir wettability also plays an important role in relative permeability and capillary pressure and their hysteretic behavior.[1–3] The majority of hysteretic relative permeability and/or capillary pressure functions have been developed for strongly water-wet porous media.[4–8] However, it is now generally accepted that many oil reservoirs are mixed-wet.[1,9–11] The definition of mixed wettability is adopted from Salathiel.[11] The oil-wet pores correspond to the largest pore in the rock, and the small pores are water-wet. Despite these findings, there are only a few capillary pressure and relative permeability models developed for mixed-wet porous media,[3,12–14] and only two have actually been incorporated in reservoir simulators.[3,14] In this paper, we briefly describe a hysteretic two-phase oil/water model developed for both capillary pressure and relative permeabilities for mixed-wet rocks.[15] We successfully implemented the model in UTCHEM.[16,17] We have tested and validated the model and its implementation in the simulator against laboratory results.

Model Description

Lenhard and Oostrom[15] developed a hysteretic relative permeability and capillary pressure model for two-phase flow of oil and water in a mixed-wet porous medium based on pore-scale processes. The model described and applied in this paper, however, is a simplified version of the original model. The model requires primary drainage and main imbibition capillary pressure data. By main imbibition capillary pressure, we mean an imbibition curve starting from any point on the primary drainage curve. In addition to the primary curves, a method of developing secondary scanning curves for saturation paths inside the primary curves is also developed. The basic assumption is that the water saturation cannot become less than the water saturation that corresponds to reversal from main drainage to main imbibition paths.

Key features of the capillary pressure-saturation model are that (1) the main drainage is modeled using a Brooks and Corey function[18]; (2) the scanning curves are modeled using an S-shaped function[19,20] that approaches asymptotes at either end; (3) the model is capable of predicting negative capillary pressures observed in mixed-wet rocks; and (4) residual oil saturations are computed using a relation that takes into account the size of the pores that are oil-wet. The relative permeability-saturation function is based on Burdine's pore-size-distribution model[21] using the main drainage capillary pressure parameter. The wettability effects in the relative permeabilities are accounted for by using an index that distinguishes those pore sizes that are water-wet from those that are oil- or mixed-wet.

Main Drainage. The main drainage capillary pressure and relative permeabilities are modeled using a Brooks-Corey function.[18]

$$P_c = p_d \left(S_w^n \right)^{-\frac{1}{\lambda}}. \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

$$k_{rw} = S_w^{n\,(2+3\lambda)/\lambda}. \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

$$k_{ro} = (1 - S_w^n)^2 \left[1 - S_w^{n\,(2+\lambda)/\lambda} \right], \dots\dots\dots\dots\dots\dots\dots (3)$$

where $S_w^n = \dfrac{S_w - S_{wr}}{1 - S_{wr}}.$

Scanning Curves. *Capillary Pressure.* The capillary pressure for any scanning curve is calculated using a modified van Genuchten function[19,20] as follows:

$$P_c = p_{neg} + \frac{1}{\alpha} \left[\frac{1}{(\bar{S}_w)^{1/m}} - 1 \right]^{1/n}, \dots\dots\dots\dots\dots\dots (4)$$

where $\bar{S}_w = \dfrac{S_w - S_{iw}}{1 - S_{iw} - S_{or}}.$

p_{neg} is the maximum negative capillary pressure at which the water saturation reaches a maximum value on the main imbibition path; α and n are model-fitting parameters, and $m = 1 - 1/n$. The irreducible water saturation, S_{iw}, is commonly assumed to be a function of only the pore-size geometry because it is always associated

with the smallest pores in both mixed and strongly water-wet systems (the porous media in these reservoirs possessed only water before the larger pores were filled with oil). However, the residual oil saturation, S_{or}, in mixed-wet media is likely to be a function of both the pore geometry and the sizes of the pores that are oil-wet. The smaller the oil-wet pores, the larger S_{or} is going to be. To index the smallest of the oil-wet pores, Lenhard used saturation, M_{ow}, that characterizes the smallest pores in which oil has displaced water for the required residence time to transform the water-wet pores to oil-wet pores. M_{ow} is likely to be the initial water saturation in the reservoir before oil production. In many reservoirs, this may be equal to the irreducible water saturation.

To develop a relation to calculate the residual oil saturation, it is assumed that S_{or} has a maximum value when $M_{ow} = S_{iw}$ and is equal to zero when $M_{ow} = 1$. The proposed relationship between S_{or} and M_{ow} is

$$S_{or} = S_{or}^{max} (1 - \overline{M}_{ow})^2, \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5)$$

where S_{or}^{max} is the residual oil saturation at $M_{ow} = S_{iw}$ and

$$\overline{M}_{ow} = \left(\frac{M_{ow} - S_{iw}}{1 - S_{iw} - S_{or}} \right). \dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

$\overline{M}_{ow}$ is a saturation that is used to distinguish the pore sizes that are water-wet from those sizes that are oil- or intermediate-wet. The assumption is that the largest pores will be oil- or intermediate-wet in mixed-wet oil reservoirs.

The substitution of Eq. 6 into Eq. 5 and the rearrangement of the resulting equation gives a cubic equation. The implementation in UTCHEM involves the analytical solution to the cubic equation with the root that meets all the imposed constraints to be the residual oil saturation.

Relative Permeabilities. Lenhard obtained analytical expressions for water and oil relative permeabilities using Burdine's relative permeability model and the Brooks-Corey main drainage capillary pressure function.

For $\overline{S}_w \leq \overline{M}_{ow}$:

$$k_{rw} = \overline{S}_w^{(2+3\lambda)/\lambda}. \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

$$k_{ro} = (1 - \overline{S}_w)^2 [1 - \overline{S}_w^{(2+\lambda)/\lambda}]. \dots\dots\dots\dots\dots\dots\dots\dots (8)$$

For $\overline{S}_w > \overline{M}_{ow}$:

$$k_{rw} = S_w^2 [1 + \overline{M}_{ow}^{(2+\lambda)/\lambda} - \Omega^{(2+\lambda)/\lambda}] \dots\dots\dots\dots\dots\dots\dots (9)$$

$$k_{ro} = (1 - S_w)^2 [\Omega^{(2+\lambda)/\lambda} - \overline{M}_{ow}^{(2+\lambda)/\lambda}], \dots\dots\dots\dots\dots (10)$$

where $\Omega = \overline{M}_{ow} + \overline{S}_o$ and $\overline{S}_o = \dfrac{S_o - S_{or}}{1 - S_{iw} - S_{or}}$.

Saturation Path. As stated earlier, the main drainage branch is modeled using the Brooks-Corey function. However, all the scanning paths are modeled with an S-shaped function to capture the capillary pressure asymptotes at the lower and upper saturation limits. Lenhard and Oostrom[15] developed scanning-path saturation relations for any drainage-imbibition and imbibition-drainage reversals. For example, to model an imbibition path with reversal from the main drainage, we used the following equation:

$$\overline{S}_w(P_c) = 1 + \frac{[\overline{S}_w^{I}(P_c) - 1](\overline{S}_w^{DI} - 1)}{\overline{S}_w^{I}(P_c^{DI}) - 1}, \dots\dots\dots\dots\dots (11)$$

where P_c is the capillary pressure of the point being calculated, and P_c^{DI} is the capillary pressure at the reversal from main drainage, as demonstrated in **Fig. 1.** $\overline{S}_w^{I}(P_c)$ and $\overline{S}_w^{I}(P_c^{DI})$ are effective water saturations of the hypothetical main drainage branch at the capillary pressure P_c and the capillary pressure at the reversal point, respectively. $\overline{S}_w^{DI}$ is the effective water saturation at the most recent reversal from main drainage to imbibition.
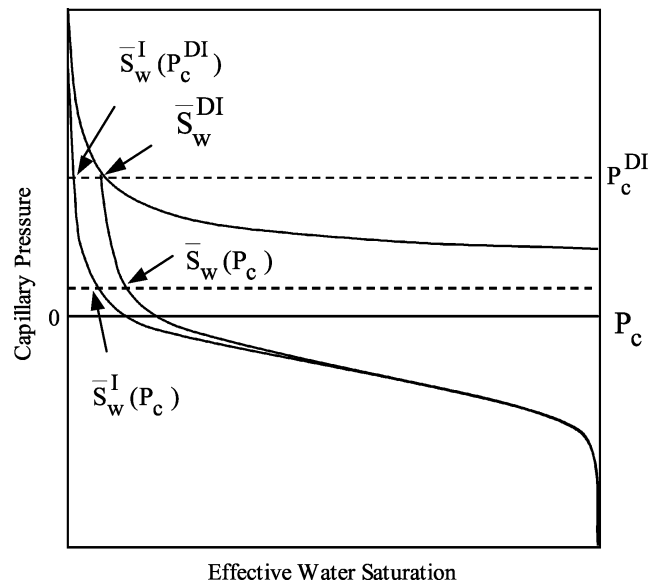
Fig. 1—Schematic of capillary pressure scanning paths describing Eq. 11.

Model Validation Against Static Capillary Pressure Measurements

The model was fit to two sets of published capillary pressure data to evaluate if the model can describe the mixed-wet capillary pressure-saturation relations. The mixed-wet capillary pressure parameters, p_d, λ, S_{iw}, α, p_{neg}, n, and S_{or}^{max}, were fit to the data. The relative permeability curves were obtained with no additional parameters.

The first set of capillary pressure data consisted of a primary drainage and an imbibition curve measured by Killins et al.[22] The measurements were done using a Hassler-type capillary pressure cell on a mixed-wet Berea sandstone core with a permeability of approximately 184.3 md. The core was treated with dri-film to make it oil-wet. The aqueous phase was distilled water, and the oil phase was a 50-50 mixture of mineral oil and Kensol 18[+]. The measured capillary pressure data and the best fit of the data are shown in **Fig. 2.** The model parameters are given in **Table 1.** The parameter M_{ow} was set to 0.4, which was close to the water saturation at the reversal from the water drainage to the water-imbibition path.

The second set of data used for model validation was measured by Lenhard and Oostrom.[23] The experimental cell consisted of alternating treated and untreated porous ceramic plexiglass sleeves.[20,24] The treated and untreated ceramics were connected to a pressure transducer to measure the pressure in both the water and
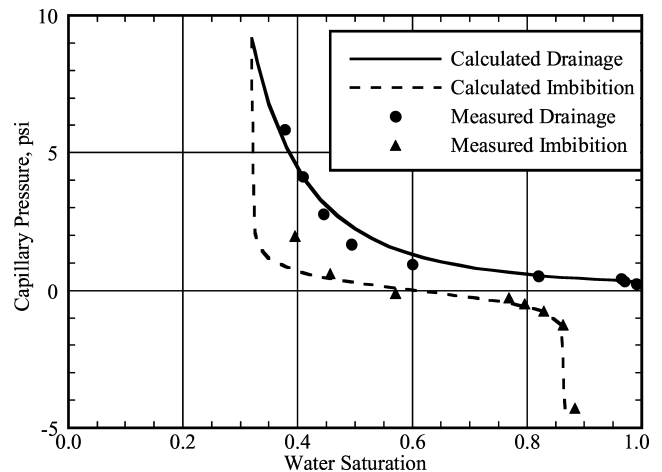

Fig. 2—Comparison of measured capillary pressure data (Killins et al.[22]) and calculated capillary pressure curves.

TABLE 1—MIXED-WET CAPILLARY PRESSURE FITTING PARAMETERS		
Parameters	Data From Killins et al.[22]	Data from Lenhard and Oostrom[23]
Primary Drainage		
Entry pressure (p_d), psi	0.32	0.0603
λ	0.412	3.3855
Irreducible water saturation (S_{iw})	0.092	0.0703
Primary Imbibition		
α, 1/psi	0.234	6.3
n	12.75	7.1829
Irreducible water saturation (S_{iw})	0.092	0.0703
Residual oil saturation $\left(S_{or}^{max}\right)$	0.134	0.10
p_{neg}, psi	−4.227	−0.2116

oil phases. The cell was packed with a mixture of sand of different mesh sizes. The aqueous phase was distilled water, and the oil phase was a 3:1 volumetric mixture of Soltrol 220 and an asphaltic crude oil from Alaska. There were seven measurements involving simultaneous water drainage/oil imbibition yielding to capillary pressure data corresponding to the main water drainage. The system was then left undisturbed for 7 days to allow a mixed-wet porous medium to develop. After the 7-day waiting period, the measurements involved imbibing water and simultaneously draining an equal volume of oil, yielding capillary pressure data corresponding to a primary water-imbibition path. **Fig. 3** shows the comparison between the experimental capillary pressure data and the model results. The best-fit parameters are listed in Table 1.

In general, there is good agreement between the data and the model. The mixed-wet capillary pressure functions are capable of capturing the main features of a mixed-wet capillary pressure curve as well as the saturation-history dependency.

Because of a lack of measured relative permeability data for the same porous medium for which the capillary pressure is measured, there is no validation of the relative permeability model.

Simulation Model

UTCHEM is a 3D, multicomponent, multiphase, compositional chemical flooding finite-difference simulator that accounts for complex phase behavior, chemical and physical properties, temperature variation, and heterogeneous porous media proper-
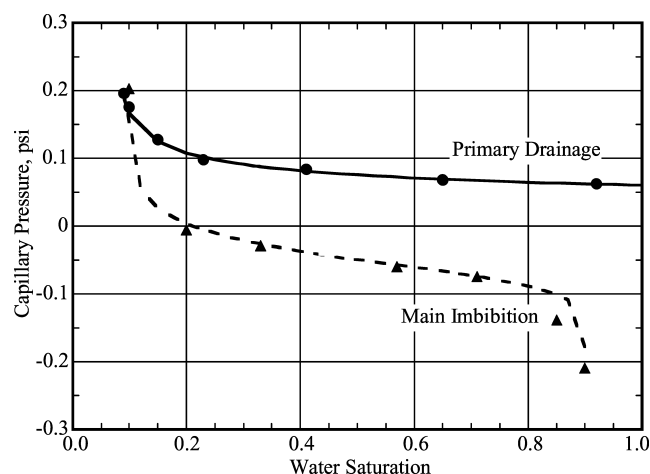

Fig. 3—Measured and calculated capillary pressure curves using Lenhard and Oostrom[23] data.

ties.[16,17] The model uses advanced concepts in high-order numerical accuracy and dispersion control.[25] The temporal discretization in UTCHEM is implicit in pressure and explicit in concentration (IMPES-like). The simulator is capable of modeling several enhanced-oil-recovery and enhanced-aquifer-remediation processes. Capillary pressure and relative permeabilities are modeled using either the Brooks-Corey model[18] or the Parker et al. model.[6] The hysteresis model is based on the work of Kalurachchi and Parker.[4,26] UTCHEM has been used widely to simulate both laboratory and field-scale processes.

The new mixed-wet hysteretic model was implemented in UTCHEM in a modular manner so that it can be imported easily to other IMPES reservoir simulators. The current implementation in terms of the saturation path includes the primary curves and one scanning curve reversed from primary curves. The gridblock saturations are initially assumed to be on the main drainage branch. At each subsequent timestep, once the aqueous phase pressure and saturations are determined by solving the pressure and species mass conservation equations for each gridblock, the capillary pressure and relative permeabilities are computed. At the reversal for each gridblock, the water saturation, the capillary pressure, and a flag for path identification are stored for the subsequent timestep calculations. At the end of each timestep, the water saturation and capillary pressure for each gridblock are also stored.

Sandpack Column Experiment. A transient two-phase oil/brine column experiment, performed at Pacific Northwest Natl. Laboratory, was conducted in a 1-m-long vertical glass column with a 7.5-cm diameter. The sandpack waterflood experiment was designed to provide hysteretic data for further validation of the mixed-wet model and its implementation in the reservoir simulator.

The column was filled under water-saturated conditions with 90 cm of a sandy porous medium. The bottom and top of the sandpack are denoted as $z=0$ and 90 cm, respectively. A dual-energy gamma radiation system was calibrated to measure oil and aqueous phase saturations along the column at 5-cm intervals ranging from $z=15$ to $z=85$ cm. Calibration procedures for two-fluid systems are discussed by Oostrom et al.[27]

Properties of the sand and fluids are listed in **Table 2.** The listed capillary pressure parameters for both primary wetting and imbibition saturation paths were obtained by fitting model equations to pressure-cell experimental data conducted with the same

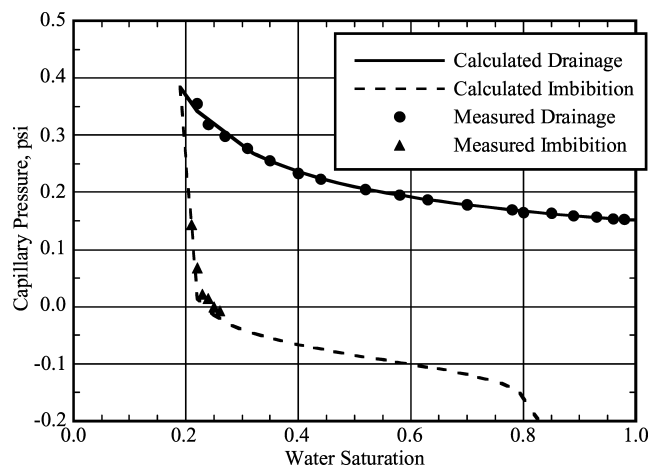| TABLE 2—TRANSIENT COLUMN EXPERIMENT SANDPACK AND FLUID PROPERTIES	
Porosity, fraction	0.316
Permeability, md	22,434
Particle diameter d_{50}, mm	0.604
Uniformity coefficient d_{60}/d_{10}	2.34
Brine viscosity, cp	1.15
Oil viscosity, cp	10.93
Brine solution density, g/cm³	1.0538
Oil density, g/cm³	0.8661
Primary Drainage	
Entry pressure (p_d), psi	0.1507
λ	2.43
Irreducible water saturation (S_{iw})	0.097
Primary Imbibition	
α, 1/psi	5.1392
n	7.99
Irreducible water saturation (S_{iw})	0.097
Residual oil saturation $\left(S_{or}^{max}\right)$	0.174
p_{neg}, psi	−0.285

Fig. 4—Measured and calculated capillary pressure curves for sand used in the transient sandpack-column experiment.
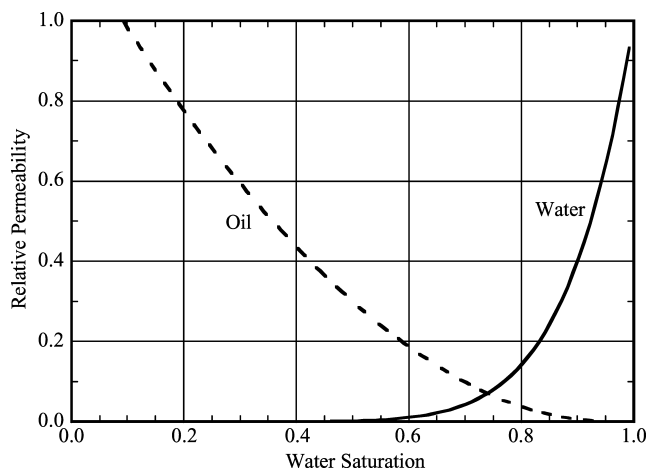

Fig. 5—Calculated main drainage relative permeability curves based on the Killins *et al.*[22] capillary pressure data.

sand and fluids as those used in the transient column experiment. The fitting parameters for the capillary pressure curves are given in Table 2. The pressure-cell apparatus, designed for 100-cm³ samples, is the same as that used by Lenhard and Oostrom.[23] The pressure-cell experimental data and best fits for both saturation paths are shown in **Fig. 4.** The aqueous phase was an 8% NaCl brine, and the oil was a west Texas crude spiked with 10 vol% 1-iodoheptane to obtain sufficiently different attenuation coefficients for the brine and oil. The listed porosity value is the average of 13 gamma measurements along the column. The sandpack permeability was obtained after completion of the packing using a constant-head method. The water level in the column was controlled by manipulating the elevation of an outlet connected through tubing to the bottom of the column.

The actual displacement experiment was initiated after setting the outlet at $z=90$ cm and placing a 5-cm layer of oil on top of the sand. This oil layer was not sufficient to force oil to move into the sandpack. At $t=0$ days, the tube outlet was lowered at a rate of 1 cm/min to $z=80$ cm. As a result of the outlet lowering, the capillary pressure at the top of the sand exceeded the entry pressure, and oil started to infiltrate into the sand. At $t=2$ days, the outlet was lowered to $z=70$ cm to allow more oil to move into the column. The sand then was aged until $t=25$ days, after which the water table was raised to $z=80$ cm to force oil to move out of the column. At $t=27$ days, the water table was raised further to $z=89.5$ cm, and the final step involved lowering the oil level on top of the sand from $z=95$ cm to $z=90.5$ cm. During the displacement process, the gamma system was used frequently to measure fluid saturation at the calibrated locations. A mass-balance comparison between the actual infiltrated oil volume and the volume integrated over the 13 calibrated locations using gamma measurements, obtained at approximately $t=24$ days, yielded a difference of 3.2%. This difference is acceptable given the fact that only 13 locations were calibrated with a spacing of 5 cm.

Numerical Simulations

Water-Wet and Mixed-Wet Waterflood Simulations. One-dimensional simulations were performed to test the implementation of the mixed-wet model in UTCHEM and to compare the results with those of the water-wet model. The simulations are for the case in which the reservoir is assumed to be saturated with water and is then oil injected until the water saturation is close to irreducible water saturation. The capillary pressure curve used in the simulation is shown in Fig. 2 and is labeled as drainage. The computed relative permeability curve used for the main drainage path using the parameters given in Table 1 is shown in **Fig. 5.** The permeability was assumed to be uniform and equal to 2,000 md with a porosity of 0.34. Forty uniform gridblocks of 0.9144 m in the *x*, 0.9144 m in the *y*, and 0.61 m in the *z* directions were used. The fluids were injected at a constant rate of 0.2265 m³/d in the

first gridblock, and the production was at a constant rate of 0.2265 m³/d from the last gridblock.

We then assume that the reservoir wettability changes to mixed-wet (because of its exposure to crude oil) in one case and remains water-wet in another case. Then, we compare the results of waterflooding the water-wet vs. mixed-wet cores (primary imbibition direction), starting from the condition at the end of 1,000 days of oil injection. The capillary pressure function used for primary imbibition in the mixed-wet case is based on the data measured by Killins *et al.*[22] (Fig. 2). Table 1 gives the model parameters for the best fit to the data. The relative permeability curves shown in **Fig. 6** are used in the waterflood simulations and are calculated from Eqs. 7 through 10 assuming a water saturation (M_{ow}) of 0.4. **Figs. 7 and 8** show the corresponding capillary pressure and relative permeability curves used in the waterflooding simulations of the water-wet case. **Table 3** gives the input parameters for the primary drainage and imbibition capillary pressure functions used in the water-wet case. The remaining oil saturation in the swept region for the water-wet case after only 100 days of waterflooding was close to the residual oil saturation of 0.25. The remaining oil saturation of the mixed-wet case ranged from low values near the residual oil saturation of 0.134 to far above the residual oil saturation, even after more than 1 year of waterflooding. **Fig. 9** compares the water-wet and mixed-wet oil recoveries. The oil recovery in the water-wet case quickly reaches the plateau value of 0.55, whereas recovery in the mixed-wet case has a gradual increase and is still on the rise after 2.5 years of water
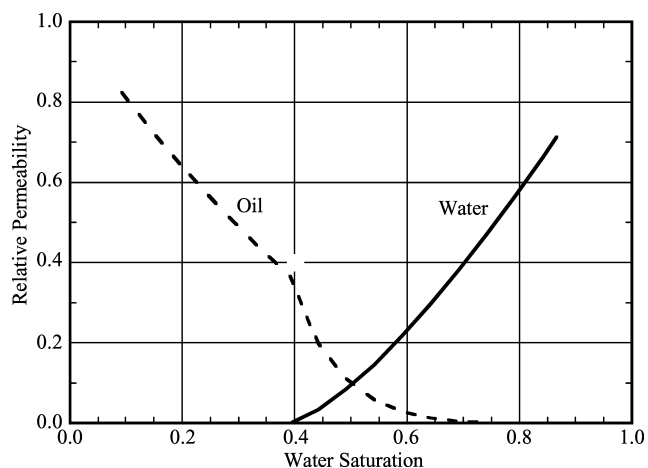

Fig. 6—Calculated imbibition relative permeability curves based on the Killins *et al.*[22] data, assuming an initial water saturation (M_{ow}) of 0.4.
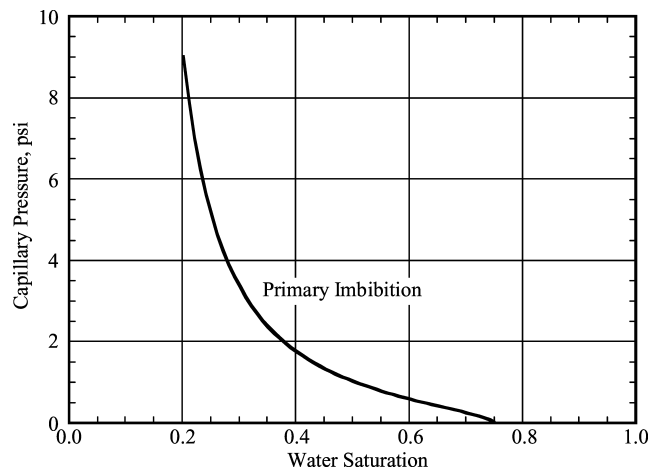
Fig. 7—Calculated imbibition capillary pressure curve used in the water-wet case.

TABLE 3—WATER-WET CAPILLARY PRESSURE PARAMETERS

Primary Drainage	
Entry pressure (p_d), psi	0.32
λ	0.412
Irreducible water saturation (S_{iw})	0.092
Primary Imbibition	
α, 1/psi	1.59
n	1.67
Irreducible water saturation (S_{iw})	0.092
Residual oil saturation (S_{or})	0.25

injection. The oil breakthrough time is approximately the same for the two cases. These results are consistent with the laboratory and field observations of continuing oil recovery after water breakthrough in the mixed-wet rocks.

Simulation of the Sandpack Experiment. A 1×1×91 simulation grid was chosen to simulate the sandpack experiment. The gridblock size was 1 cm, with the exception of the top and bottom gridblock size of 0.5 cm. Sandpack and fluid properties are given in Table 2. Pressure-constrained wells were located in the top and bottom gridblocks to mimic the imposed laboratory boundary conditions at these locations. For example, for the water-drainage period, the top pressure was set at 14.7652 psi, which corresponds to a 5-cm column of crude oil on top of the sand. The pressure at the bottom was linearly reduced from 16.0476 to 15.8976 psi in a period of 10 min., corresponding to a 1 cm/min rate of lowering the water level in the burette from 90 to 80 cm. The pressure remained at 15.8967 psi for 2 days. The pressure again was reduced from 15.8976 to 15.7476 psi in 10 minutes and afterward remained unchanged for 2 days ($t = 4$ days). A shut-in period of 21 days ($t = 25$ days) was then established to allow the wettability alteration of the sand from a water-wet to a mixed-wet condition. Simulated and measured crude-oil-saturation profiles in the column are shown in **Fig. 10.** The simulation results agree well with the measured data.

To simulate the imbibition branch after the aging period, the pressure at the bottom of the block was raised from 15.7476 to 15.8976 psi, which corresponds to raising the water level from 70
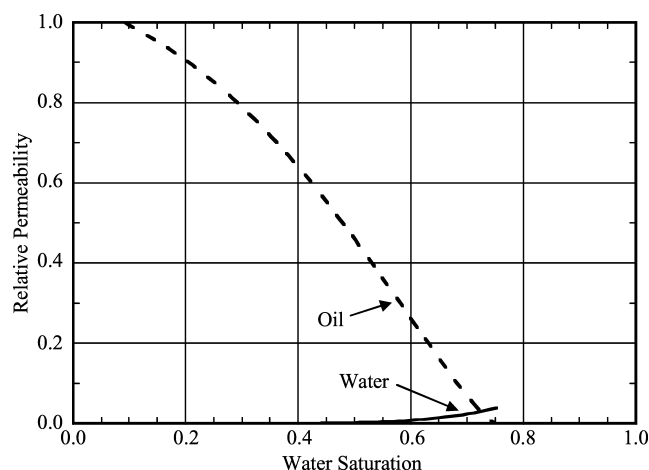
to 80 cm to allow water entering the column from the bottom. The boundary conditions remained unchanged for 2 days ($t = 27$ days). **Fig. 11** compares the simulated oil-saturation profiles with those measured. The agreement is good considering that there is no adjustment in the simulator input parameters. The current mixed-wet model does not allow for additional adjustments in the relative permeability curves. Once the capillary pressure curves are defined, the relative permeabilities are calculated with no additional parameters. Examples of relative permeability curves calculated for several values of M_{ow} on the imbibition path are given in **Fig. 12.** We plan to allow an option in UTCHEM to make the exponents of the relative permeability curves adjustable model parameters.

Conclusions

Relative permeability and capillary pressure are among the most important petrophysical properties for predicting fluid behavior in porous media. These properties are generally dependent on saturation, saturation history, and the wetting preference of the rock. Despite these functional dependencies, relative permeabilities and capillary pressures used in reservoir simulators are commonly determined by interpolation between laboratory measurements or by relationships that are valid only for a certain rock wettability (i.e., strongly water-wet). In this paper, saturation-history-dependent relative permeability and capillary pressure functions for mixed-wet rocks incorporated in UTCHEM are discussed. The experimental data required are the main drainage curve and an intermediate-scanning capillary pressure curve. Relative permeabilities are predicted by integrating a pore-distribution model. The mixed-wet model was tested against mixed-wet capillary pressure data. Results indicate that the model is capable of capturing capillary pressure data in mixed-wet rocks. The simulation results agree qualitatively with laboratory sandpack results. In addition, simulations
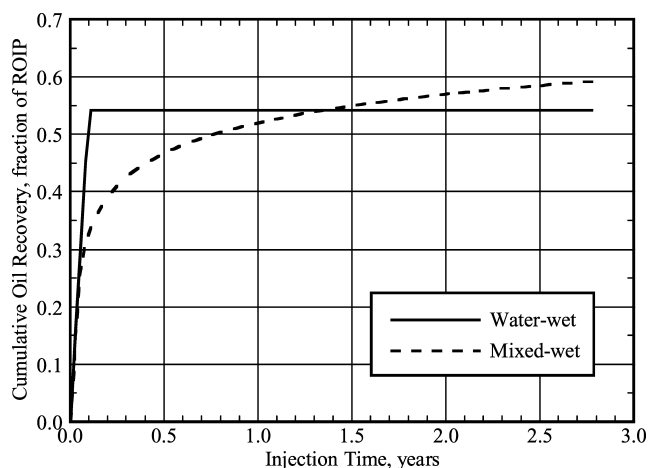

Fig. 8—Calculated imbibition relative permeability curves used in the water-wet case.

Fig. 9—Comparison of waterflood oil recovery for 1D water-wet and mixed-wet simulations.

Fig. 10—Measured and simulated crude-oil-saturation profile for the sandpack during the primary drainage experiment.
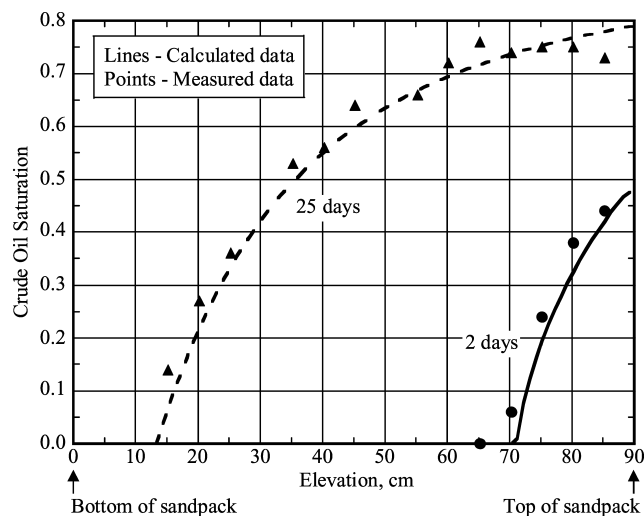

Fig. 11—Measured and simulated crude-oil-saturation profile in the sandpack during the imbibition experiment.

with this proposed model required slightly more storage and no extra computational time compared to simulations without the hysteresis effects.

Nomenclature

k_{ro} = oil relative permeability
k_{rw} = water relative permeability
m = curve-shape parameter ($m = 1 - 1/n$)
M_{ow} = saturation index used to identify the portion of the pore space that is water-wet from the portion that is oil-wet
$\overline{M}_{ow}$ = normalized saturation
n = curve-shape parameter
p_d = displacement (entry) pressure, psi
p_{neg} = maximum negative capillary pressure
P_c = oil/water capillary pressure, psi
S_o = oil saturation
$\overline{S}_o$ = normalized oil saturation
S_{or} = residual oil saturation
S_{or}^{max} = maximum residual oil saturation
S_w = water saturation
$\overline{S}_w$ = normalized water saturation
S_{iw} = irreducible water saturation
t = time
z = depth
α = curve-shape parameter
λ = pore-size distribution index used in the Brooks-Corey model
Ω = term used in relative permeability integrals

Superscripts

DI = reversal from drainage to imbibition
I = imbibition
N = normalized water saturation in main drainage functions

Acknowledgments

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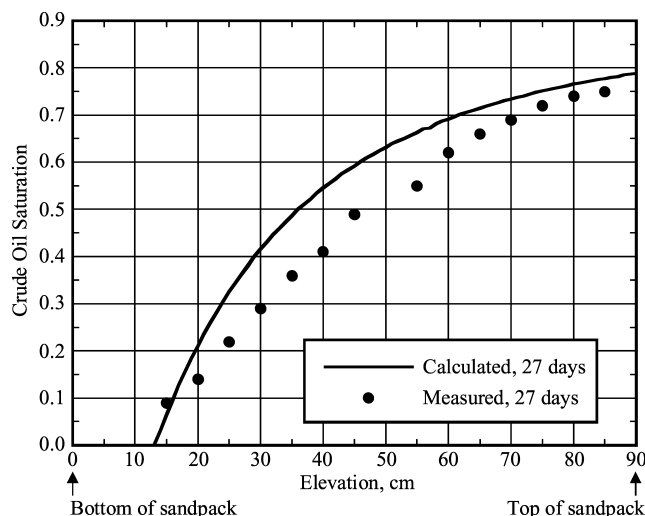2. Rao, D.N., Girard, M.G., and Sayegh, S.G.: "The Influence of Reservoir Wettability on Waterflood and Miscible Flood Performance," *JCPT* (June 1992) **31,** No. 6, 47.

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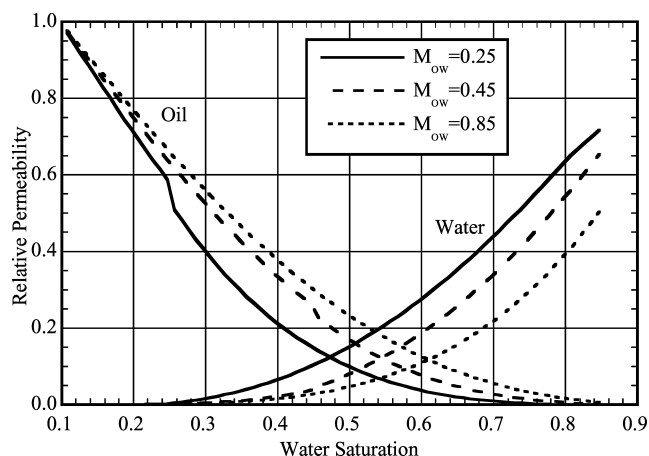

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SI Metric Conversion Factors

cp $\times$ 1.0*	E–03	= Pa·s
ft $\times$ 3.048*	E–01	= m
ft^3 $\times$ 2.831 685	E–02	= m^3
in. $\times$ 2.54*	E+00	= cm
in.^{3} $\times$ 1.638 706	E+01	= cm^3
psi $\times$ 6.894 757	E+00	= kPa

*Conversion factor is exact.

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