

Evaluation of Normalized Stone's Methods for Estimating Three-Phase Relative Permeabilities

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Abstract

This paper examines normalized forms of Stone's two methods for predicting three-phase relative permeabilities. Recommendations are made on selection of the residual oil parameter, S_{om} , in Method I. The methods are tested against selected published three-phase experimental data, using the plotting program called CPS-1 to infer improved data fitting. It is concluded that the normalized Method I with the recommended form for S_{om} is superior to Method II.

Introduction

Stone has produced two methods^{1,2} for estimating three-phase relative permeability from two-phase data. Both models assume a dominant wetting phase (usually water), a dominant nonwetting phase (gas), and an intermediate wetting phase (usually oil). The relative permeabilities for the water and gas are assumed to depend entirely on their individual saturations because they occupy the smallest and largest pores, respectively. The oil occupies the intermediate-size pores so that the oil relative permeability is an unknown function of water and gas saturation.

For his first method,¹ Stone proposed a formula for oil relative permeability that was a product of oil relative permeability in the absence of gas, oil relative permeability in the absence of mobile water, and some variable scaling factors. He compared this formula with the experimental results of Corey *et al.*,³ Dalton *et al.*,** and Saraf and Fatt.⁴ The formula is likely to be most in error at low oil relative permeability where more data are needed that show the behavior of residual oil saturation as a function of mixed gas and water saturations. In particular, the best value for the parameter S_{om} that occurs in the model is not well resolved.

In his second method,² Stone developed a new formula and compared it against the data of Corey *et al.*, Dalton *et al.*, Saraf and Fatt, and some residual oil data from Holmgren and Morse.⁵ Stone suggested that his second method gave reasonable agreement with experiments without the need to include the parameter S_{om} . If in the absence of residual oil data, $S_{om}=0$ is used in the first method, the second method is then better than the first method, although it tends to underpredict relative permeability.

Dietrich and Bondor⁶ later showed that Stone's second

method did not adequately approximate the two-phase data unless the oil relative permeability at connate water saturation, k_{rocw} , was close to unity. Dietrich and Bondor suggested a normalization that achieved consistency with the two-phase data when k_{rocw} was not unity. This normalization can be unsatisfactory because k_{ro} can exceed unity in some saturation ranges if k_{rocw} is small. More recently this objection has been overcome by the normalization of Method II introduced by Aziz and Settari.⁷ Aziz and Settari also pointed out a similar normalization problem with Stone's first method and suggested an alternative to overcome the deficiency. However, no attempt was made to investigate the accuracy of these normalized formulas with respect to experimental data.

In the next section of the paper we review the principal forms of Stone's formulas, and introduce some new ideas on the use and choice of the parameter S_{om} . Later we examine the first of Stone's assumptions—that water and gas relative permeabilities are functions only of their respective saturations for a water-wet system. This involves a critical review of all the published experimental measurements. Earlier reviews^{6,8} did not take into account some of the available data. Last, we examine the predictions of normalized Stone's methods for oil relative permeability against the more reliable experimental results. It is concluded that the normalized Stone's Method I with the improved definition of S_{om} is more accurate than the normalized Method II.

Mathematical Definition of Three-Phase Relative Permeabilities

We briefly review the principal forms of the Stone's formulas that use the two-phase relative permeabilities defined by water/oil displacement in the absence of gas,

$$k_{rw} = k_{rw}(S_w)$$

and

$$k_{row} = k_{row}(S_w)$$

and gas/oil displacement in the presence of connate water,

$$k_{rg} = k_{rg}(S_g)$$

and

$$k_{rog} = k_{rog}(S_g).$$

In each method the water is assumed to be the dominant wetting phase so that the three-phase water relative

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permeability depends only on water saturation—i.e.,

$$k_{rw}(S_w, S_g) = k_{rw}(S_w).$$

Also, each method assumed that the gas was the least wetting phase so that the three-phase gas relative permeability depends only on gas saturation—i.e.,

$$k_{rg}(S_w, S_g) = k_{rg}(S_g).$$

The three-phase oil relative permeability was assumed to be a function of water and gas saturation with the forms described next. Hysteresis was assumed to be accounted for by use of the appropriate two-phase data. This puts some limitations on the methods because there are six hysteresis classes with three flowing phases and only two classes with each set of two-phase data. There are six additional hysteresis classes associated with one of the three phases being immobile or trapped.

1. Stone's Method I unnormalized is given by

$$k_{ro}(S_w, S_g) = \frac{S_o^* k_{row} k_{rog}}{(1 - S_w^*)(1 - S_g^*)}, \quad \dots \quad (1)$$

where

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om}},$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om}},$$

$$S_g^* = \frac{S_g}{1 - S_{wc} - S_{om}},$$

and S_{om} is a residual oil saturation parameter to be optimized in some manner.

Stone originally proposed Method I on the basis of gas/oil displacement data at zero water saturation, in which case no normalization is necessary to achieve the correct two-phase limits. However, zero water saturation is generally not relevant to reservoir problems and it seems more reasonable to base the formula on gas/oil displacements at connate water saturation. This then introduces the need for the normalized form.

2. Stone's Method II unnormalized is given by

$$k_{ro}(S_w, S_g) = (k_{row} + k_{rw})(k_{rog} + k_{rg}) - (k_{rw} + k_{rg}). \quad \dots \quad (2)$$

3. Stone's Method I normalized by Aziz and Settari becomes

$$k_{ro}(S_w, S_g) = \frac{S_o^* k_{row} k_{rog}}{k_{rocw}(1 - S_w^*)(1 - S_g^*)}, \quad \dots \quad (3)$$

where k_{rocw} is the two-phase oil relative permeability at connate water saturation.

4. Stone's Method II normalized by Aziz and Settari becomes

$$k_{ro}(S_w, S_g) = k_{rocw} \left[\left(\frac{k_{row}}{k_{rocw}} + k_{rw} \right) \left(\frac{k_{rog}}{k_{rocw}} + k_{rg} \right) - (k_{rw} + k_{rg}) \right]. \quad \dots \quad (4)$$

Eqs. 3 and 4 are used later to establish their characteristics as implied by comparisons with experimental data. We continue to refer to the normalized equations as Stone's Methods I and II.

Factors Governing Choice of Parameter S_{om} . Stone's Method I has the potential advantage of containing the free parameter S_{om} , which could be adjusted to match residual oil measurements. In practice, $S_{or}(S_w, S_g)$ is generally not measured. The standard measurements of k_{row} provide an estimate of S_{orw} , and the necessary measurements of k_{rog} for the Stone models should also provide S_{org} (in the presence of connate water). Thus, a simple choice for S_{om} is given by

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org}, \quad \dots \quad (5)$$

where we require that $\alpha = 1$ when $S_g = 0$ and $\alpha = 0$ when $S_w = S_{wc}$. This implies that α is given by the relation:

$$\alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}}. \quad \dots \quad (6)$$

In practice it is not clear that S_{om} should behave linearly between the two limiting residual oil saturations, although this should certainly be better than an arbitrary choice of S_{om} in the absence of residual oil data.

Previous authors (e.g., Ref. 9) have reported that small amounts of gas can sometimes improve the oil relative permeability and sometimes worsen it. At oil saturations somewhat in excess of residual oil, it is argued that the nonwetting gas phase will compete with the oil for the opportunity to move through the larger pores, so that the oil relative permeability will be reduced in the presence of small levels of gas saturation. However, near the limiting residual oil condition it can then be argued that water is bypassing two nonwetting hydrocarbon phases, and that the total residual hydrocarbon saturation would be essentially unchanged from S_{orw} . This argument implies that the residual oil component must have decreased and the oil relative permeability should therefore be increased near to the residual oil saturation. The practically important aspect of these apparently contradictory arguments is likely to be dominated by the region where lower oil relative permeability occurs since, in the field application, economic cutoff usually will not allow absolute residual oil saturation to be attained. Also, in many instances residual oil saturation is not reached completely in laboratory experiments.

At the opposite extreme of high gas saturation, the magnitude of residual oil saturation, S_{org} , will usually be significantly smaller than S_{orw} . This happens because the nonwetting gas phase invades the larger pores and displaces the oil from them, resulting in a lower residual oil saturation. Thus, S_{om} as defined in Eq. 5 will be a decreasing function of gas saturation. This may be closer to the theoretical limiting behavior for residual oil at low gas saturations as previously discussed, but may be inconsistent with the more practical effect on low values of k_{ro} . In view of the uncertainties involved, we suggest that Eqs. 5 and 6 are a reasonable compromise. Some of the experimental data such as those of Corey *et al.*³ and

Saraf and Fatt⁴ apparently imply that $S_{org} > S_{orw}$, but this is probably because a drainage process rather than an imbibition measurement has been used to infer S_{orw} .

Three-Phase Relative Permeabilities for Gas and Water

Before comparing Stone's models of three-phase oil relative permeability with experiments, we review the other two assumptions, namely that gas and water relative permeabilities are only functions of their respective saturations. The experimental evidence is examined critically in chronological order.

Leverett and Lewis¹⁰ made the first measurements of three-phase relative permeabilities in 1941. Although these authors claimed to have observed that water relative permeability behaved as a function only of water saturation, the reliability of their conclusions must be questioned. The experiments were undertaken at low pressures with rudimentary saturation measurements (by electrical resistance for brine saturations and by a pressure/volume technique for gas saturations). The interpretation ignored hysteresis and it is not certain that end effects had been eliminated. Many of the experiments were beset by lack of repeatability, which the authors attributed to equilibrium conditions not having been established. Later attempts by Snell¹¹ in 1963 to correct the Leverett and Lewis results for hysteresis are probably unsatisfactory because of the issues of experimental reliability. The Leverett and Lewis work must be regarded as pioneering but probably inadequate in accuracy for developing quantitative conclusions on three-phase relative permeability models.

Caudle *et al.*¹² were the next group of workers to report on three-phase relative permeabilities in 1951. The authors described their work as incomplete and it appears that an oil-wet sandstone was used. Their work therefore does not contribute to the present review.

Later that year, Holmgren and Morse⁵ reported measurements of oil and water relative permeabilities in the presence of trapped gas in Nellie Bly sandstone. Saturation measurements were obtained by a combination of brine electrical resistivities and material balance. Corrections for end effects were made and pressures were high enough to avoid variable gas saturations along the test section. Although the core was 12 cm [4.7 in.] in diameter, the occurrence of gas gravity segregation would have been minimized by working only with trapped gas. Holmgren and Morse found variations of about 0.01 in measured water relative permeabilities in the range 0 to 0.15 because of changes in gas saturation from 0 to 25%. This behavior therefore suggests that water relative permeability is essentially dependent on water saturation alone, but the effects of a flowing gas phase were not included in the experiments.

Corey *et al.*³ reported measurements of three-phase oil and gas relative permeabilities on Berea sandstone in 1956. The water was rendered immobile by semipermeable barriers so that water relative permeability could not be measured. Note that this procedure does not eliminate end effects. Different cores were used at different water saturations to avoid hysteresis effects. Two of the nine cores showed gas relative permeabilities very different from those on the other cores. In two of

the other cores only low gas saturations were used, but in the remaining five cores Corey *et al.* considered that gas relative permeability was a function of gas saturation alone. We regard the evidence as dubious because of the uncertainties associated with nonuniform trapped water and the limited number of consistent results.

Reid¹³ in 1956 reported results from a sandpack for his three-phase relative permeability measurements. Brine saturations were measured by electrical resistance and gas saturations by a technique Reid pioneered, namely gamma adsorption. He ignored hysteresis but eliminated end effects. Reid was the first to draw attention to the difficulty in drawing ternary diagrams of the isoperms using large numbers of data points, each with its separate experimental scatter. We have therefore replotted all of Reid's data using the CPS-1 contour plotting package,¹⁴ since use of this approach should be reasonably free from subjective bias. Reid reported gas isoperms concave toward the 100% gas point (i.e., gas relative permeability is higher in the presence of mixed oil and water saturations than in the presence of oil or water alone). The CPS-derived isoperms are slightly more concave than those produced by Reid. At a gas saturation of 35%, gas relative permeability varied between 0.05 and 0.07, with a similar fractional variation at lower gas saturations. The water isoperms are also concave toward the 100% water point, with relative permeabilities up to 0.05 higher at mixed gas and oil saturations than in the presence of gas or oil alone, for a wide range of relative permeabilities. Following Snell's suggestion¹¹ that experimental data points should be divided into different hysteresis classes, we replotted the Reid isoperms for a number of different hysteresis classes. None of these attempts showed water or gas relative permeabilities to be functions only of their respective saturations. Although these results imply that water and gas relative permeabilities are not functions of their respective saturations alone, it is possible that Reid's measurements suffered from poor brine saturation measurements as a result of electrode corrosion. Subsequent workers have either avoided or improved this technique of saturation measurement.

Snell¹⁵ in 1961 reported results using a sandpack for his three-phase relative permeability measurements and produced the most extensive set of results to date. Water saturation was determined by resonance behavior of an inductance circuit at radio frequency. By irradiating the test section with fast neutrons, he obtained the total liquid saturation by measuring the number of neutrons thermalized by hydrogen in the oil and water. End effects were eliminated and hysteresis was recorded. The long wide test section (120×10 cm OD [47.2×3.9 in. OD]) may have allowed some gravity segregation of gas to occur. Snell reported gas isoperms that were convex toward the 100% gas point. However, a replot of Snell's data using CPS to reduce subjective bias in filtering out experimental scatter showed that linear gas isoperms are equally realistic. Snell also reported water isoperms that were concave toward the 100% water point. Again, we find that replotting Snell's data using CPS produces near-linear water isoperms. An example of this is shown in Fig. 1. We conclude, therefore, that Snell's work tends to substantiate the basic hypothesis required by Stone's models that water and gas relative permeabilities

are functions of their respective saturations only. The experimental scatter in this large set of experimental results makes rigorous interpretation rather difficult.

Hosain¹⁶ reported more results in 1961 using Snell's experimental apparatus, but in this case a highly refined nonpolar oil was used, so that changes in wettability would be avoided, which may have contributed to the hysteresis effects in Snell's work. His results contained far less scatter than those of Snell. The results apparently demonstrated quite clearly that water and gas relative permeabilities were functions of their respective saturations only. The reduction in experimental scatter achieved by Hosain may have been a consequence of achieving equilibrium distribution of the phases that Snell did not always get, even after steady flow for 6 hours. Normally one might expect equilibrium to be established more quickly when nonpolar oil was used, because there is very little competition between the nonpolar oil and the water for wetting the sandstone surfaces, so that snap-off and reconnection of the oil ganglia are less random than with polar oil.

Sarem¹⁷ in 1966 used an unsteady-state method to obtain three-phase relative permeabilities. To analyze the experiments, he assumed that the fractional flow and relative permeability of each phase were a function only of the respective saturation. These assumptions invoke the hypothesis we wish to demonstrate, and thus this work does not contribute significantly to the overall conclusions. However, Sarem's interpretation of his experiments is consistent with the assumptions of the analysis.

Donaldson and Dean¹⁸ reported measurements on Berea sandstone and Arbuckle limestone in 1966. Their measurements were also undertaken with an unsteady-state method in which air displaced water and oil. End effects were reduced by use of high flow rates, and hysteresis was ignored, but the high flow rates may have caused nonuniform air density variations. There are again difficulties in interpreting unsteady experiments where an approach somewhat different from the Sarem method was used. They found that gas relative permeability was not a function of gas saturation alone. In the sandstone there was a gas relative-permeability variation of up to 0.05 at a fixed gas saturation. We suggest that the difficulties of using the unsteady-state method under three-phase conditions again render these results inconclusive.

Saraf and Fatt⁴ reported steady-state results for Boise sandstone in 1967. Hysteresis effects were overcome by use of several different cores, and end effects were eliminated by measurement over the central region of the cores. The oil saturation was measured by a nuclear magnetic resonance technique, and the water saturation inferred from the water flow rate on the assumption that water relative permeability was only a function of water saturation. Saraf and Fatt observed that gas relative permeabilities were low (~ 0.04), and there was considerable experimental scatter. Nevertheless, the authors claimed that gas relative permeability was a function of gas saturation only. A CPS plot of the data also produced the same conclusion, but the high degree of experimental scatter among low absolute values precludes a firm conclusion.

The unpublished data of Dalton *et al.* were quoted by

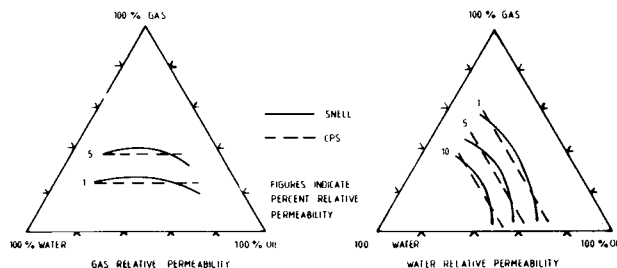


Fig. 1—Comparison of Snell's interpretation and CPS-1 fitting of results.

Stone in 1970 as providing evidence that water and gas relative permeabilities are functions of their respective saturations only. The unprocessed experimental data are not reported by Stone, so it is difficult to check his conclusions.

In 1970 Schneider and Owens⁹ reported measurements of three-phase relative permeabilities in reservoir cores (limestone and sandstone) having various degrees of wettability. Measurements were undertaken both with two flowing phases (i.e., one phase trapped) and with three flowing phases, mostly using the Penn steady-state technique. Gas saturations were measured by X-ray absorption and water saturations by brine resistivity. The use of four electrodes instead of two eliminates some of the consequences of electrode corrosion noted earlier, since the inner pair of electrodes can be used for voltage measurements. We restrict ourselves to comments on the water-wet sandstone cores. Measurements of water relative permeability in the presence of trapped oil, or under conditions of full three-phase flow, showed water relative permeability to be a function of water saturation alone. The water relative permeability in the presence of trapped gas showed variations of 0.04, for which an adequate explanation could not be advanced. Gas relative permeability was shown to be a function of gas saturation alone under full three-phase flow conditions. However, when the oil was trapped, the gas relative permeability fell dramatically from values with a flowing oil phase. At a gas saturation of 30% with mobile oil, the gas relative permeability was 0.30, whereas with the same gas saturation and a trapped oil saturation of 24%, the relative permeability was reduced to 0.10. This behavior may be explained by trapped oil blocking the large pores through which the gas would normally pass. Differences in gas relative permeability were also observed according to the direction of changes in the gas saturation. These results seemed to follow the normal two-phase drainage and imbibition behavior. These experiments apparently support the hypothesis that water and gas relative permeabilities are functions of their respective saturations when all three phases are flowing, but if either oil or gas is trapped at various saturations, the simple behavior breaks down.

In 1982 Saraf *et al.*¹⁹ reported measurements of three-phase relative permeabilities in Berea sandstone using a nonpolar oil. Saturations were measured by material balance. Some steady-state tests were made and also some unsteady-state tests, where the analysis was carried out by Sarem's procedures, with the associated difficulties in assumptions. The authors concluded that

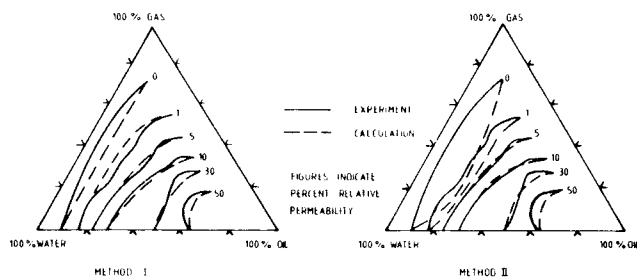


Fig. 2—Comparison of predicted and measured oil isoperms from data of Corey *et al.*

water relative permeability was a function of water saturation alone based on their steady-state measurements. Steady-state gas relative permeabilities were also found to be dependent on gas saturation alone. The observed hysteresis effects were small, although the authors reported larger hysteresis in the unsteady-state measurements. As with the Schneider and Owens results, gas relative permeabilities were found to decrease with increasing trapped oil saturations. The Saraf *et al.* results for water and gas relative permeabilities therefore broadly support the Stone's hypothesis.

Summarizing the experimental evidence we find that the majority of experimentalists claim that water relative permeability is a function of water saturation alone. The exceptions are Reid, who shows variations in water relative permeability up to 0.05 for a fixed water saturation, Schneider and Owens who report unexplained variations with trapped-gas saturation, and Donaldson and Dean (the latter report variations of up to 0.12, but we have suggested reservations on the interpretation).

For the gas relative permeability the majority of experimentalists claim that it is a function of gas saturation alone, but Schneider and Owens show variations with trapped-oil saturations and hysteresis.

Comparison of Predicted and Measured Oil Relative Permeability

Stone compared his unnormalized methods against just four sets of experimental results (Corey *et al.*, Dalton *et al.*, Saraf and Fatt, and Holmgren and Morse). We shall compare the predictions of the normalized methods against the original four sets of results, and on the basis of the review of the previous section, we will also examine the experimental data published by Hosain, Schneider and Owens, and Saraf *et al.*

The Data of Corey *et al.* To test Stone's Method I against the experiments of Corey *et al.*,³ data for k_{rog} and k_{row} are needed. We have obtained these by plotting the oil relative permeability data on a ternary diagram using CPS. Residual oil saturation was not measured by Corey *et al.*, but was estimated by an extrapolation process derived by Corey in an earlier paper.²⁰ Fig. 2 shows the oil isoperms produced by CPS using the measured oil relative permeability data. The functions of k_{rog} and k_{row} were then read off from the plots at connate-water and zero gas saturation, respectively.

To test Method II, data for k_{rw} and k_{rg} were assumed

to be those values recommended by Corey *et al.* He obtained smoothed values of k_{rg} from experiments by omitting some inconsistent results. Values of k_{rw} were presented by Corey *et al.* on the basis that gas/oil relative permeabilities would be the same as oil/water relative permeabilities for a water-wet system. This interchangeability is not likely to be valid (see Holmgren and Morse results later), but in the absence of measured data for k_{rw} this is the best assumption that can be made. The resulting predictions for the two methods are shown in Fig. 2. Stone's Method I with the definition of S_{om} in Eq. 5 appears to give the best overall fit. A slightly better fit of the zero oil isperm might have been obtained using a fixed value of 0.08 for S_{om} . However, because the experimental techniques were limited, direct measurements of essential input (k_{rw} and k_{row}) were lacking, and only a drainage process was considered, an unambiguous recommendation for Stone's Method I from these experiments is precluded. We note that the accuracy of the fit for both methods is much the same at high relative permeabilities. This is a result of the fact that both formulas tend to the same limit as the oil saturation increases to its maximum value of $(1 - S_{wc})$. This feature therefore appears in all the comparisons.

The Data of Dalton *et al.* The results of Dalton *et al.* are not published outside Stone's paper. Very little is known about the experiments other than that unconsolidated Miocene sand of permeability 10.8 darcies was used. The residual oil saturation to water displacement was 22% and is typical for an unconsolidated sand. However, the residual oil saturation to gas displacement (35%) was unusually high. Different connate-water saturations were quoted by Stone. In Ref. 1, Stone quotes 15% with a corresponding value of oil relative permeability at connate water, k_{rowc} , of 0.70. In Ref. 2 connate water is 25% and k_{rowc} is 0.58. In unconsolidated sands, connate-water saturation is usually low (e.g., see Refs. 10 and 15) because there are few trapping sites, and in consequence k_{rowc} should be near unity; thus, the value quoted in Ref. 1 appears to be more appropriate. The functions k_{row} , k_{rw} , k_{rog} , and k_{rg} are all provided by Stone.

Table 1 shows the predictions for the two methods. Stone's Method I with Eq. 5 used for the parameter S_{om} shows the best fit, but only by a small margin over Stone's Method II. Fig. 3 illustrates these results graphically. The very wavy nature of the experimental oil isoperms prevents a clear distinction being drawn from the two methods. However, the fit of the data to Stone's Method I (as with Corey) is encouraging.

The Data of Saraf and Fatt. Saraf and Fatt⁴ used separate high-permeability Boise sandstone cores to obtain results free of hysteresis but, like Corey, obtained results only on oil displacing water. The minimum water saturation investigated was 30%, although connate-water saturation was considered to be about 25%.

Since experiments were not carried out at connate-water saturation, triaxial oil isoperms were produced using CPS for obtaining the necessary two-phase data. An apparent inconsistency was evident in the results obtained on Core 4, which had a water saturation of 41%. Accordingly, we ignored this core in comparing predictions. For calculations using Method I the relevant data,

k_{rog} , k_{row} , were taken directly from the CPS plots. For Method II the additional data, k_{rg} , k_{rw} , were taken directly from the work of Saraf and Fatt. The results of Methods I and II compared with the experiment are shown in Fig. 4. At low relative permeability, neither model gives good agreement and this is because of the variable nature of the residual oil isoperm. For relative permeabilities in excess of 0.05, Method I gives marginally the better fit. As with the Corey data, the use of a drainage displacement process, rather than more typical reservoir displacement conditions, limits the value of the comparison of the relative permeability models.

The Data of Holmgren and Morse. Holmgren and Morse measured oil relative permeability as a function of water saturation at zero gas saturation and then with a trapped gas saturation of 25%. They also obtained residual oil values at intermediate values of trapped-gas saturation. Stone considered that the data of Holmgren and Morse substantiated his Method II, but this was based on the unnormalized method. Stone had also derived the essential data, k_{rog} , on the basis of interchangeability with k_{rw} (see Ref. 20), but Holmgren and Morse had made measurements of k_{rog} at two values of gas saturation, which are at variance with Stone's assumption. Since k_{rog} and k_{rg} were not measured at high gas saturations, there is no information on S_{org} needed for Method I, so that a comparison of the relative performance of the two predictive methods is not easy for these experiments.

In comparing the predictions of oil relative permeability at a trapped-gas saturation of 25%, we note that $k_{rg}=0$ (because the gas is trapped) and $k_{rog} \approx 0.13$ (Fig. 5 of Ref. 5). The values of k_{rw} and k_{row} are also given. For Method I we choose the value $S_{om}=0.17$ as measured by Holmgren and Morse. The predicted and experimental values of oil relative permeability as a function of water saturation are given in Table 2 and illustrated graphically in our Fig. 5A. The fit on Method I is good, but Method II overpredicts relative permeability for the middle saturation range and underpredicts it at high saturation. This is equivalent to an overprediction of residual oil. The good fit on Method I is partly attributable to the use of the measured value of S_{om} . Note that Stone does suggest a procedure whereby his unnormalized Method II may be adjusted to take account of residual oil data.² However, the procedure is somewhat arbitrary and requires nonlinear regression analysis. The effect on the Holmgren and Morse predictions is to raise the oil relative-permeability values so that the known residual oil condition is satisfied, but in consequence the fit on oil relative permeability is thereby worsened because the original predictions were too high over the middle saturation range.

It is possible to use the measurements of the residual oil isoperm for intermediate trapped-gas saturation if some questionable assumptions are made about the values of k_{rog} at different gas saturations. Using the Corey approximation²⁰ that the oil relative permeability to gas drive (drainage) is a quartic function in effective oil saturation, we have

$$k_{rog} = S_{oe}^4 \quad (7)$$

TABLE 1—OIL RELATIVE PERMEABILITIES FROM DALTON'S DATA

Saturation (%)		k_{ro}		
S_g	S_w	Experimental	Method I	Method II
10	30	0.15	0.23	0.24
20	30	0.05	0.10	0.12
10	40	0.14	0.17	0.18
20	40	0.03	0.05	0.06
10	50	0.08	0.06	0.09
20	50	0.02	0.01	0.01

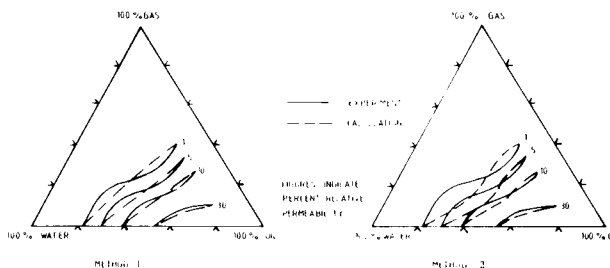


Fig. 3—Comparison of predicted and measured oil isoperms from data of Dalton *et al.*

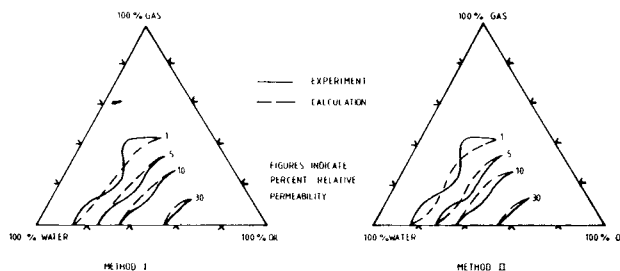


Fig. 4—Comparison of predicted and measured oil isoperms from data of Saraf and Fatt.

If we make the further assumption that this form for k_{rog} also applies to imbibition and substitute the two measured values of k_{rog} , 0.85 and 0.13, at gas saturations of 0% and 25%, respectively, then Eq. 7 becomes

$$k_{rog} = 0.85(1 - 1.5S_g)^4 \quad (8)$$

The extrapolated value of S_{org} is thus 11%.

The resulting predictions of residual oil saturation for Methods I (using Eq. 5) and II are shown in Fig. 5B. It is seen that both methods seriously overpredict the measured residual oil saturation. We conclude that there are difficulties in predicting residual oil behavior in the presence of trapped gas and direct measurements of S_{om} are desirable if this type of hysteresis in gas saturation is needed for reservoir studies. In the absence of measured values of S_{om} , we find in reviewing experiments with trapped gas (Refs. 5, 9, and 21) that a reasonable approximation for residual oil for a wide range of water-wet sandstones is

$$S_{om} = S_{orw} - \frac{1}{2}S_g \quad (9)$$

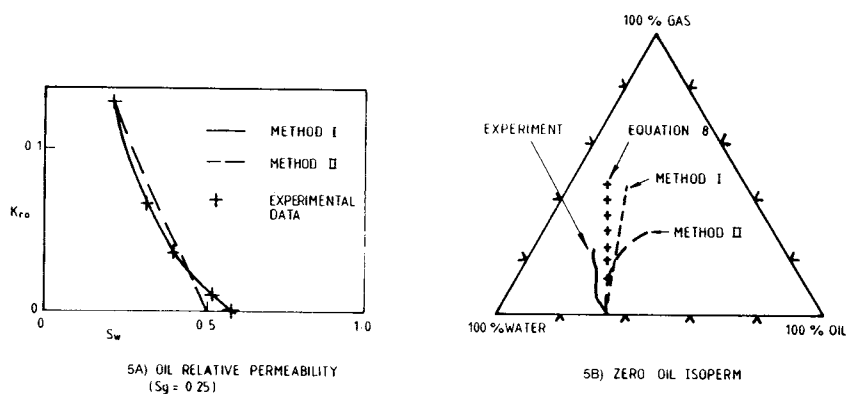


Fig. 5—Comparison of predicted and measured oil relative permeability from data of Holmgren and Morse.

TABLE 2—HOLMGREN AND MORSE OIL RELATIVE PERMEABILITY AT 25% TRAPPED-GAS SATURATION

S_w (%)	k_{ro}		
	Experimental	Method I	Method II
22	0.13	0.13	0.13
25	0.11	0.10	0.11
32	0.07	0.07	0.08
42	0.03	0.03	0.04
52	0.01	0.01	0
58	0	0	0

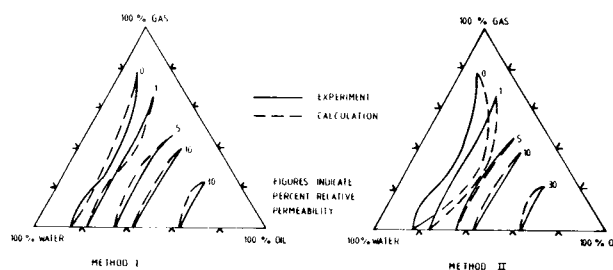


Fig. 6—Comparison of predicted and measured oil isoperms from data of Hosain.

This result is illustrated in Fig. 5B. Use of this approximation, as an alternative to the known value of $S_{om}=0.17$, would still give a better fit for Method I than Method II in the comparison in Fig. 5A for the Holmgren and Morse data.

The Data of Hosain. The data of Hosain¹⁶ contain very little scatter, and it is easy to prepare plots of the isoperms for all three phases. A plot of the oil isoperms is shown in Fig. 6. The essential data, k_{rog} and k_{row} , for use in Stone's methods may be read off this figure. A similar procedure may be used to obtain k_{rw} and k_{rg} . The predicted values of k_{ro} are shown in Fig. 6 for both methods. At high oil relative permeability (0.05 to 0.10), Stone's Method II is slightly superior to Method I. However, at low relative permeability, Method I is clearly preferable.

The Data of Schneider and Owens. The measurements of oil relative permeability by Schneider and Owens⁹ fall into two parts. In the first part they measured oil relative permeability in the presence of trapped gas at four different gas saturations. We may compare the predictions of the two methods with the experiments by using the same procedure as adopted for the Holmgren and Morse data. Fig. 7 shows the results at gas saturations of 8.5%, 11.6%, 15.3%, and 18.9%. Both methods perform satisfactorily, though Method I has a slight advantage over Method II except at a gas saturation of 8.5%. The quality of the fit obtained by Method I is again partly attributable to the use of values of S_{om}

measured by Schneider and Owens, although, as with the Holmgren and Morse data, the use of Eq. 9 is a good approximation if S_{om} had not been measured. This is illustrated in Table 3, where $S_{orw}=32\%$.

In the second part of their measurements, where all three phases were flowing, the oil relative-permeability values were all ~ 0.01 , and lay substantially on a single curve that was only a function of oil saturation. This is equivalent to having straight-line oil isoperms on the ternary diagram parallel to the axis of zero oil saturation. The measurements thus support the linear approximation for S_{om} in Eq. 5, but in view of the limited amount of data, no further valid comparisons can be made with Method I or Method II.

The Data of Saraf *et al.* Only a few of the measurements of oil relative permeability made by Saraf *et al.*¹⁹ used steady-state techniques that we believe are appropriate for the present comparison. The individual data points are not reported in detail, therefore precluding a definitive comparison against predictions of Method I and Method II. However, the authors reported that the results followed closely those obtained by unsteady-state experiments. The oil isoperms shown by Saraf *et al.* for unsteady-state measurements are illustrated in Fig. 8. We can make a limited comparison of the predictions of Methods I and II with these data by making a number of assumptions. First the values of k_{rog} and k_{row} have been taken directly from this ternary diagram. We note that the unsteady-state measurements of two-phase permeabilities plotted separately in their

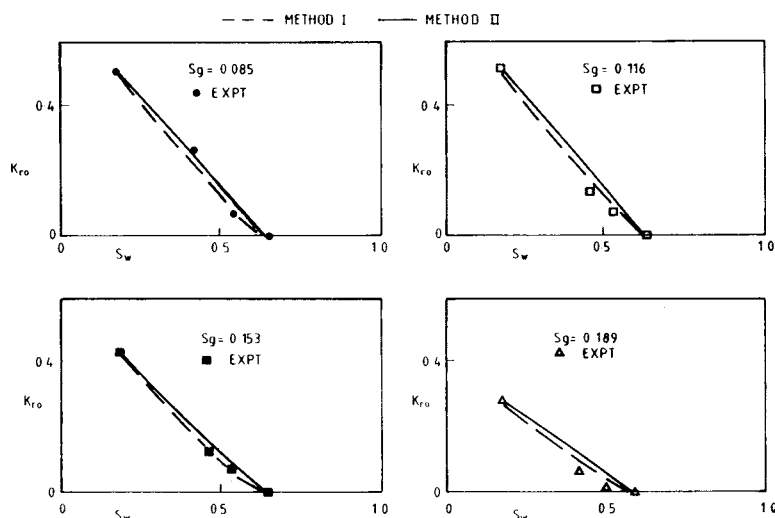


Fig. 7—Comparison of predicted and measured oil relative permeability from data of Schneider and Owens.

TABLE 3—COMPARISON OF MEASURED AND ESTIMATED VALUES OF S_{om}

Trapped-Gas Saturation S_g (%)	Measured S_{om} (Ref. 10)	S_{om} Calculated From Eq. 9
8.5	0.27	0.28
11.6	0.24	0.26
15.3	0.23	0.24
18.9	0.22	0.22

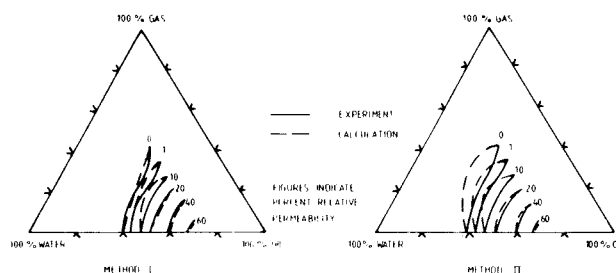


Fig. 8—Comparison of predicted and measured oil isoperms from data of Saraf *et al.*

paper are not consistent with the three-phase ternary diagram. The values of S_{orw} and S_{org} were not specifically reported, but have been taken to be 40% and 32%, respectively, based on the plotted isoperm with a value of 0.001. Actual values of S_{orw} and S_{org} may have been somewhat smaller. Three-phase data for k_{rg} and k_{rw} using steady-state measurements were reported. Smoothed values of k_{rg} and k_{rw} have been used in the saturation range of interest ($S_o < 35\%$). These do not have a strong influence on Method II in this range.

The predictions of Method I and Method II are shown against the reported experimental curves in Fig. 8. The fit is somewhat better for Method I than Method II. Some slight improvement in the fit of Method I could be obtained if lower values for S_{orw} and S_{org} are assumed. Contrary to the conclusions of Saraf *et al.*, the Stone predictions are quite good, although this may be influenced by the choice of the basic two-phase data.

Conclusions

Estimates of three-phase relative permeabilities are needed for a variety of oil-recovery applications, such as water drive of reservoirs at pressures below the bubble-point, alternating gas/water drives in EOR, hot gas/oil/water systems in thermal recovery, and low-pressure gas recycling in condensate fields with aquifers. The essential part of all these calculations concerns the

behavior of residual oil saturation and relative oil permeability at low oil saturation in the presence of mixed-gas and water saturations. The values of oil relative permeabilities when oil saturations are large are not particularly crucial. The normalized forms of Stone's Methods I and II, as suggested by Aziz and Settari, give very similar results in this range, but differ considerably as residual oil saturation is approached. Method I, which incorporates the residual parameter, S_{om} , has been shown in this study to be superior to Method II, if Eq. 5 is used to estimate S_{om} when both gas and water are mobile. Gas relative permeabilities are subject to strong hysteresis effects, and in the presence of trapped gas, Eq. 9 should be used to calculate S_{om} . Although the superiority of the improved Method I is not dramatic, it appears to be substantiated for at least seven sets of experiments.

The basic assumptions of the Stone methods that water and gas relative permeabilities are dependent on their respective saturations alone are reasonably well supported by the majority of experiments. However, there are some exceptions, and hysteresis effects in gas displacement may well invalidate this hypothesis for gas relative permeability.

We suggest that experimental work is particularly required to investigate systematic behavior of the variation of residual oil saturation in the presence of gas and water saturations. A direct fit to S_{om} would then firmly establish the improved Stone Method I as more reliable. For example, a second-order equation with the correct end points could be fitted in the form:

$$S_{om} = S_{orw} - \left(\frac{S_{orw} - S_{org}}{1 - S_{wc} - S_{org}} \right) S_g - \beta [(1 - S_{wc} - S_{org}) S_g - S_g^2]. \quad (10)$$

This contains the free parameter, β , which could be used to fit measurements of S_{om} , but there is no evidence that β should have a universal value for all sandstones. The lack of a systematic trend in the results obtained in this paper and the limited accuracy of the available measurements suggest that adoption of the linear form is a reasonable compromise in the absence of further data.

Acknowledgment

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Nomenclature

- k_r = relative permeability
- k_{ro} = oil relative permeability in the presence of three phases
- k_{rog} = oil relative permeability in the presence of gas (with connate water)
- k_{row} = oil relative permeability in the presence of water
- k_{rocw} = oil relative permeability in the presence of connate water
- S = saturation, %
- S_{or} = residual oil saturation, %
- S_{wc} = connate-water saturation, %
- S_{om} = residual oil parameter (Method I)
- S_{org} = residual oil in the presence of gas, %
- S_{orw} = residual oil in the presence of water, %
- α = parameter in linear form for S_{om}
- β = parameter in second-order form for S_{om}

Subscripts

- e = effective
- g = gas
- o = oil
- w = water

References

1. Stone, H.L.: "Probability Model for Estimating Three-Phase Relative Permeability," *J. Pet. Tech.* (Feb. 1970) 214-18; *Trans.*, AIME, **249**.
2. Stone, H.L.: "Estimation of Three-Phase Relative Permeability and Residual Oil Data," *J. Cdn. Pet. Tech.* (Oct.-Dec. 1973) **12**, No. 4, 53-61.
3. Corey, A.T. *et al.*: "Three-Phase Relative Permeability," *Trans.*, AIME (1956) **207**, 349-51.
4. Saraf, D.N. and Fatt, I.: "Three-Phase Relative Permeability Measurement Using a Nuclear Magnetic Resonance Technique for Estimating Fluid Saturation," *Soc. Pet. Eng. J.* (Sept. 1967) 235-42; *Trans.*, AIME, **240**.
5. Holmgren, C.R. and Morse, R.A.: "Effect of Free Gas Saturation on Oil Recovery by Waterflooding," *Trans.*, AIME (1951) **192**, 135-40.
6. Dietrich, J.K. and Bondor, P.L.: "Three-Phase Oil Relative Permeability Models," paper SPE 6044 presented at the 1976 SPE Annual Technical Conference and Exhibition, New Orleans, Oct. 3-6.
7. Aziz, K. and Settari, T.: *Petroleum Reservoir Simulation*, Applied Science Publishers, London (1979) 30-38.
8. Donaldson, E.C. and Kayser, M.B.: "Three-Phase Fluid Flow in Porous Media," Bartlesville Energy Technology Centre (1981) DOE/BETC/IC-80/4.
9. Schneider, F.N. and Owens, W.W.: "Sandstone and Carbonate Two- and Three-Phase Relative Permeability Characteristics," *Soc. Pet. Eng. J.* (March 1970) 75-84; *Trans.*, AIME, **249**.
10. Leverett, M.C. and Lewis, W.B.: "Steady Flow of Gas-Oil-Water Mixtures Through Unconsolidated Sands," *Trans.*, AIME (1941) **142**, 107-16.
11. Snell, R.W.: "The Saturation History Dependence of Three-Phase Oil Relative Permeability," *J. Inst. Pet.* (1963) **49**, 81-84.
12. Caudle, B.H. *et al.*: "Further Developments in the Laboratory Determination of Relative Permeability," *Trans.*, AIME (1951) **192**, 145-50.
13. Reid, S.: "The Flow of Three Immiscible Fluids in Porous Media," PhD dissertation, U. of Birmingham, UK (1956).
14. Contour Plotting System (CPS-1), Radian Corp., Austin, TX.
15. Snell, R.W.: "Three-Phase Relative Permeability in an Unconsolidated Sand," *J. Inst. Petr.* (1962) **84**, 80-88.
16. Hosain, A.: "Three-Phase Relative Permeability Measurements," MS thesis, U. of Birmingham, UK (1961).
17. Sarem, A.M.: "Three-Phase Relative Permeability Measurements by Unsteady-State Method," *Soc. Pet. Eng. J.* (Sept. 1966) 199-205; *Trans.*, AIME, **237**.
18. Donaldson, E.C. and Dean, G.W.: "Two- and Three-Phase Relative Permeability Studies," RI6826, USBM (1966).
19. Saraf, D.N. *et al.*: "An Experimental Investigation of Three-Phase Flow to Water/Oil/Gas Mixtures Through Water-Wet Sandstones," paper SPE 10761 presented at the 1982 SPE California Regional Meeting, San Francisco, March 24-26.
20. Corey, A.T.: "The Interrelation Between Gas and Oil Relative Permeabilities," *Prod. Monthly* (Nov. 1954) **19**, No. 1, 38-41.
21. Kyte, J.R. *et al.*: "Mechanism of Water Flooding in the Presence of Free Gas," *Trans.*, AIME (1956) **207**, 215-21.

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