Correspondence of the Gardner and van Genuchten–Mualem relative permeability function parameters

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[1] The Gardner and van Genuchten–Mualem models of relative permeability are widely used in analytical and numerical solutions to flow problems, respectively. Comparison of analytical and numerical solutions therefore requires defining some correspondence between the Gardner and van Genuchten–Mualem models. In this paper, we introduce generalized conversion formulae that reconcile these two models in the midrange of saturation. In general, we find that the Gardner parameter α_G is related to the van Genuchten parameters α_{vG} and n as $\alpha_G \approx 1.3\alpha_{\mathrm{vG}}\, n$. The performance of the proposed conversion formulae is poor when n is much smaller than 2.

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1. Introduction

[2] The relation of relative permeability to matric potential is an essential component of mathematical description of multiphase flow in porous media. This relation is often described using empirical equations. Two of the most widely used functions are the Gardner equation [*Gardner*, 1958] and the van Genuchten–Mualem equation [*van Genuchten*, 1980; *Mualem*, 1976]. The simple exponential equation of Gardner is mostly used to derive analytical solutions of flow problems. Whereas the more flexible van Genuchten–Mualem equation is used in numerical models to provide more accurate descriptions of real-world problems. Consequently, comparison of analytical and numerical solutions of flow problems requires correspondence between the parameters of their respective relative permeability vs. matric potential relations. In this paper, we introduce generalized conversion formulae between the parameters of the Gardner and van Genuchten–Mualem functions, which provide excellent matches in the midrange of matric potential.

2. Theoretical Considerations

[3] Unsaturated flow in porous media can mathematically be described by Richards' equation

$$\frac{1}{K_S}\frac{\partial\theta}{\partial t} = \nabla\cdot[k_r(\psi)\nabla\psi] + \frac{\partial k_r}{\partial z} \tag{1}$$

where K_S [L T^{-1}] is saturated hydraulic conductivity, ψ [L] is matric potential, θ [L^3/L^{-3}] is water content and k_r is relative permeability. Using the Kirchhoff integral transformation $\phi = \int_{-\infty}^{\psi} k_r d\psi$ the Richards equation (1) can be rewritten as

$$\frac{1}{K_S}\frac{\partial\theta}{\partial t} = \nabla^2\phi + \alpha_*\frac{\partial\phi}{\partial z} \tag{2}$$

where ϕ [L] is the Kirchhoff potential and α_* [L^{-1}] = $(1/k_r)(dk_r/d\psi)$ is the sorptive number. Many analytical and semianalytical solutions are based on the linearization of the right-hand side of equation (2) by assuming that α_* is constant (for detailed reviews, see *Pullan* [1990] and *Raats* [2001]). This condition is equivalent to requiring $k_r \propto \exp(\alpha_*\psi)$. *Gardner* [1958] introduced the earliest and most widely used exponential relationship. A modified version of the Gardner model—with finite air entry pressure—is given by [e.g., *Gardner and Mayhugh*, 1958]

$$k_r = \begin{cases} 1 & \psi \ge \psi_b \\ \exp[\alpha_G(\psi - \psi_b)] & \psi < \psi_b \end{cases} \tag{3}$$

where α_G is a constant sorptive number (a measure of the pore size distribution of the medium) and ψ_b [L] is air entry pressure.

[4] Quasi-linearization of the flow equation (2) has been used to solve numerous multidimensional steady flow problems, such as surface and subsurface drip sources, flow to sinks, and flow around solid and air-filled obstructions [e.g., *Raats*, 2001]. However, the application of the quasi-linear analyses is often limited to relatively simple geometries and initial/boundary conditions.

[5] Numerical solutions, on the other hand, are not constrained by the need for linearization and often utilize more flexible Θ-ψ and k_r-ψ functions such as the van Genuchten–Mualem (vGM) model [*van Genuchten*, 1980; *Mualem*, 1976]

$$\Theta = [1 + (|\alpha_{\mathrm{vG}}\,\psi|)^n]^{-m} \tag{4}$$

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$$k_r = \sqrt{\Theta}\left[1-\left(1-\Theta^{1/m}\right)^m\right]^2 \tag{5}$$

where $\alpha_{vG}[L^{-1}]$ is a parameter related to the modal pore size, n is a function of the spread of pore size distribution, and $m = 1 - 1/n$. The effective saturation (Θ) is defined as a function of volumetric water content θ, satiated water content (θ_s), and residual water content (θ_r) by $\Theta = (\theta - \theta_r)/(\theta_s - \theta_r)$.

[6] Typically, van Genuchten parameters are obtained by fitting equation (4) to empirical Θ-ψ data that are less difficult to measure than k_r-ψ data.

[7] Several methods for translating the van Genuchten parameters to Gardner parameters (and vice versa), have been proposed [e.g., *Birkholzer et al.*, 1999; *Furman and Warrick*, 2005; *Morel-Seytoux et al.*, 1996; *Rucker et al.*, 2005]. Brief summaries of these approaches are given in the next subsection.

2.1. Previous Approaches

2.1.1. Least Squares Optimization

[8] The most straightforward approach for estimating the Gardner parameters ψ_b and α_G using the known vGM k_r-ψ function involves minimizing the differences between the Gardner and van Genuchten relative permeability curves [e.g., *Birkholzer et al.*, 1999; *Furman and Warrick*, 2005]. The number and distribution of matching points for least squares fitting can be chosen to provide the best match in the regime of interest. However, the method does not lend itself to derivation of a general correspondence between the Gardner and van Genuchten parameters.

2.1.2. Capillary Drive Approach

[9] In this approach, the Gardner parameters are estimated by matching the effective capillary drive of the Gardner function to that of the vGM function. The effective capillary drive (H_c) is given by [*Morel-Seytoux et al.*, 1996]

$$H_c = \int_{-\infty}^{0} k_r d\psi \tag{6}$$

[10] The H_c of the Gardner and vGM functions are obtained by substituting equations (3) and (5) in equation (6), respectively

$$H_{c(G)} = \psi_b + 1/\alpha_G \tag{7}$$

$$H_{c(vG)} = \frac{1}{\alpha_{vG}}\frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2} \tag{8}$$

The latter is a polynomial fit to numerical evaluation of equation (6) with equation (5) evaluated over a wide range of m values [*Morel-Seytoux et al.*, 1996]. An approximate air entry pressure (ψ_b) for vGM function was defined by *Rucker et al.* [2005] as the matric potential that gives some critical relative permeability. For a critical relative permeability of 0.9, the best fit polynomial for the vGM ψ_b is [*Rucker et al.*, 2005]

$$\psi_b = \frac{1}{\alpha_{vG}}\left(-2.0692m^3 + 4.4099m^2 - 1.5366m + 0.1504\right)^2 \tag{9}$$

Then, α_G can be estimated by requiring that the capillary drive of the Gardner function (equation (7)) match the capillary drive of the vGM function (equation (8)), and using equation (9):

$$\left(\frac{\alpha_G}{\alpha_{vG}}\right)^{-1} = \frac{0.046m + 2.07m^2 + 19.5m^3}{1 + 4.7m + 16m^2} - (-2.0692m^3 + 4.4099m^2 - 1.5366m + 0.1504)^2 \tag{10}$$

[11] Note that the capillary drive–based conversion approach attempts to match the Gardner and vGM relative permeability functions over the entire range of matric potentials, $-\infty \leq \psi \leq 0$. As a result, the agreement between the two functions is poor, especially in wet and very dry regimes. Moreover, as will be shown using illustrative examples (section 3), the performance of the capillary drive–based conversion approach deteriorates when the vGM m parameter significantly deviates from a value of 0.5.

2.1.3. Capillary Length Approach

[12] The capillary length approach partly alleviates the problems of the capillary drive approach by restricting the range of matric potential over which matching is sought. Capillary length is defined as [*Philip*, 1985; *Warrick*, 1995]

$$\lambda = \frac{1}{k_{wet} - k_{dry}}\int_{\psi_{dry}}^{\psi_{wet}} k_r d\psi \tag{11}$$

The capillary length approach can be conditioned to give an excellent match between the two functions by limiting the range of matric potential ($\psi_{dry} \leq \psi \leq \psi_{wet}$) and relative permeability ($k_{dry} \leq k_r \leq k_{wet}$) to the problem at hand. In order to derive generalized correspondence between the Gardner and vGM parameters, however, it is necessary to define ψ_{dry} and ψ_{wet} values that are applicable to a wide range of porous media. In section 2.3, we present a special case of the capillary length method that uses general purpose capillary pressure bounds.

2.2. Proposed Conversion Formulae

[13] The *Gardner* [1958] relative permeability function can be uniquely defined using two (ψ, k_r) points. The main task of this paper is to define two characteristic ψ values at which the *Gardner* [1958] and *van Genuchten* [1980] should be matched, thereby enabling derivation of algebraic conversion equations.

[14] The Gardner and vGM k_r-ψ curves are schematically shown in Figure 1a. On a semilog chart, the Gardner model is represented by two straight segments intersecting at $\psi = \psi_b$ (marked as point i in Figure 1a). On the wet side of the intersection, $k_r = 1$; whereas on the dry side, k_r is on a decreasing straight segment of slope α_G. In contrast, the vGM model is a smooth and continuous function for all matric potential values.

2.2.1. First Matching Point

[15] We set the air entry matric potential ($\psi = \psi_b$ in the Gardner function) to be the first characteristic matching point. The concept of air entry pressure stems from the observation that, for many porous media, a finite quantity of matric potential $\psi < \psi_b$ is needed to create an unsaturated state with relative permeability less than unity. Although the

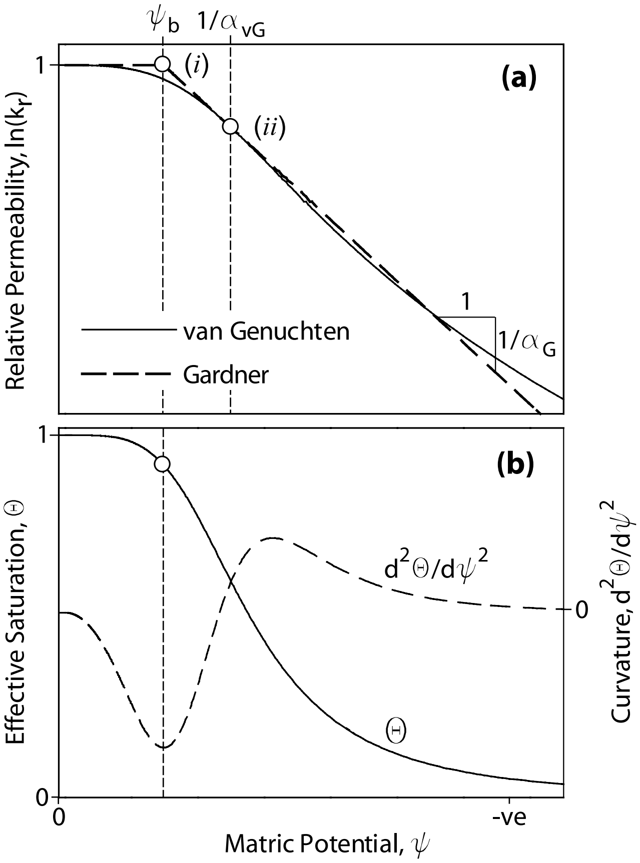

Figure 1. Schematic diagrams showing (a) Gardner and van Genuchten–Mualem relative permeability curves and (b) a van Genuchten retention curve along with its curvature. For this example, vGM parameter $n = 4$.

vGM k_r-ψ curve does not have such a distinct transition point, careful inspection of typical van Genuchten Θ-ψ curves reveals that deviation from $\Theta = 1$ occurs at matric potential (ψ) values significantly below zero. In Figure 1b, a typical vGM Θ-ψ curve and its curvature ($d^2\Theta/d\psi^2$) are plotted. Negative curvature denotes downward concavity. Figure 1b shows that the curvature has two local extreme values. At the point of maximum downward concavity of the Θ-ψ curve (denoted by a circular symbol in Figure 1b), the effective saturation Θ begins to rapidly drop below unity. For the purpose of matching with the Gardner k_r-ψ function, we define this point as the air entry matric potential of the vGM k_r-ψ function. Accordingly, the point of maximum curvature should satisfy

$$d^3\Theta/d\psi^3 = 0 \text{ and } d^2\Theta/d\psi^2 < 0 \tag{12}$$

[16] After substituting equation (4) in equation (12) and performing elementary algebraic manipulations, we arrive at

$$\psi_b = \frac{1}{\alpha_{vG}}\left\{\frac{5m - m^2 + \sqrt{8m + 5m^2 - 2m^3 + m^4}}{4m^2 - 2m}\right\}^{m-1} \tag{13}$$

[17] Note that the denominator of equation (12) has real values only for $m > 0.5$; hence the effective air entry pressure of vGM k_r-ψ function (ψ_b) is defined only for $m > 0.5$ (which is equivalent to $n > 2$).

[18] For $n < 2$, it is interesting to note that the classical vGM model yields unrealistic relative permeability values (see section 3). On the wet end of the k_r-ψ relation, the vGM models predicts much steeper decline in relative permeability compared to measurements. *Ippisch et al.*, [2006] attributed this poor performance to lack of air entry parameter in the classical vGM formulation and showed that introducing such parameter for $n < 2$ improved the ability of Richards' equation to fit experimental data. Limitations of the proposed conversion rules for $n < 2$ will be further discussed in section 3.

2.2.2. Second Matching Point

[19] The second point needed to uniquely define the Gardner k_r-ψ function can be placed anywhere along the sloping segment. Here, we seek a ψ value in the vGM k_r-ψ function that represents the most nonlinear regime to be the second matching point. Note that the right-hand side of Richards' equation (2) becomes nonlinear when the sorptive number α_* varies with ψ (i.e., $\alpha_*(\psi) = d\ln[K(\psi)]/d\psi$ is not constant) and the strongest nonlinearity should occur in the regimes that result in the largest $|d\alpha_*(\psi)/d\psi|$. Thus the special value of matric potential $\psi = \psi_*$ representing the maximum nonlinearity satisfies

$$\frac{d^2\alpha_*(\psi)}{d\psi^2} = 0 \tag{14}$$

For the vGM relative permeability function (5), the values of ψ_* obtained by numerically evaluating equation (14) in the range $0.5 < m < 1$ are $\psi_* \ \alpha_{vG} = 1 \pm 0.3$. For mathematical simplicity, we set $\psi_* = 1/\alpha_{vG}$, marked as point ii in Figure 1a, as the second characteristic matching point, so that both the vGM and Gardner functions comparably describe this highly nonlinear portion of the k_r-ψ relationship.

2.2.3. Approximation of Sorptive Number

[20] In the preceding subsections, we defined two characteristic matric potential values in the vGM relative permeability function (5) that represent the onset of unsaturated conditions and the maximum nonlinearity. Using these characteristic points, the slope of the Gardner k_r-ψ function on a semilog plot is expressed by

$$\alpha_G = -\frac{\ln(1) - \ln[k_r(1/\alpha_{vG})]}{\psi_b - 1/\alpha_{vG}} \tag{15}$$

[21] Substituting equation (5) in equation (15) and carrying out algebraic manipulation gives

$$\alpha_G = -\frac{\ln\left[\left(2^{-5m/4}(1 - 2^m)\right)^2\right]}{\psi_b - 1/\alpha_{vG}} \tag{16}$$

where ψ_b is as given by equation (13). The equivalence of the Gardner and vGM parameters is visualized best when plotted in dimensionless form. Notice that the parameters $1/\alpha_G$, ψ_b, and $1/\alpha_{vG}$ have the same unit as the matric potential ψ, whereas n and m are dimensionless. In Figure 2, the ratios α_G/α_{vG} and ψ_b/α_{vG}^{-1} are plotted against the vGM parameter n. From Figure 2, it is evident that α_G/α_{vG} can be

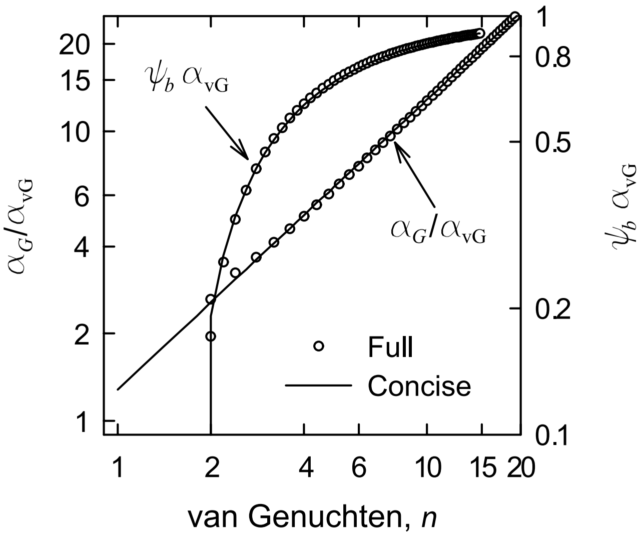

Figure 2. Correspondence of Gardner and van Genuchten–Mualem parameters.

linearly related to n by the following fitting curve (with r^2 = 0.99)

$$\frac{\alpha_G}{\alpha_{vG}} \approx 1.3\,n \tag{17}$$

[22] Equation (17) provides a concise conversion formula that can also be extrapolated to $n < 2$ as shown in Figure 2. Similarly, a concise conversion rule for the air entry pressure is given by the following power law fit (with r^2 = 0.99)

$$\psi_b \cdot \alpha_{vG} \approx 1 - (n/2)^{-1.163} \qquad \text{for } n > 2 \tag{18}$$

[23] However, equation (18) cannot be extrapolated to $n \leq 2$ because it results in unphysical positive air entry pressure. Thus we restrict the air entry pressure of the vGM function in the $n \leq 2$ range to $\psi_b = 0$.

2.3. Special Case of the Capillary Length Approach

[24] As discussed in subsection 2.1.3, for the capillary length approach [*Warrick*, 1995] to provide generalized correspondence between the Gardner and vGM models, it is necessary to define limits of integration (ψ_{dry} and ψ_{wet}) that are applicable to a wide range of porous media. We present here a special case of the capillary length approach that uses the first matching point introduced in subsection 2.2.1 as the upper limit of integration ($\psi_{wet} = \psi_b$) and the lower limit set at $\psi_{dry} = -\infty$. Use of $\psi_{dry} = -1/\alpha_{vG}$ as the lower limit resulted in poorer match between the Gardner and vGM k_r-ψ functions (not shown) and will not be further discussed.

[25] To facilitate derivation of the capillary length–based conversion formulae, We rewrite the Gardner (3) and vGM (5) k_r-ψ functions as

$$k_r = \begin{cases} 1 & \Psi \geq \Psi_b \\ \exp\left[\frac{\alpha_G}{\alpha_{vG}}(\Psi - \Psi_b)\right] & \Psi < \Psi_b \end{cases} \tag{19}$$

and

$$k_r = \frac{\left(1 - \left[1 - \left(1 + \Psi^{1/(1-m)}\right)^{-1}\right]^m\right)^2}{\left(1 + \Psi^{1/(1-m)}\right)^{-m/2}} \tag{20}$$

where $\Psi = \psi\ \alpha_{vG}$ and $\Psi_b = \psi_b\alpha_{vG}$. Then, the corresponding capillary lengths are obtained by substituting equations (19) and (20) into equation (11) and using the integration limits $-\infty$ and Ψ_b, resulting in

$$\lambda_G = \frac{\alpha_{vG}}{\alpha_G} \tag{21}$$

and

$$\begin{aligned}\lambda_{vGM} &= \frac{1}{k_r(\Psi_b)}\int_{-\infty}^{\Psi_b} k_r d\Psi \\ &= -1.674m^4 + 7.192m^3 - 11.4m^2 + 6.852m - 0.970 \end{aligned} \tag{22}$$

The right-hand side in equation (22) is a best fit to numerical evaluation of the capillary length. Then, matching the capillary lengths of the Gardner and vGM models ($\lambda_G = \lambda_{vGM}$) leads to

$$\frac{\alpha_G}{\alpha_{vG}} = \frac{1}{-1.674m^4 + 7.192m^3 - 11.4m^2 + 6.852m - 0.970} \tag{23}$$

[26] This special case of capillary length approach uses the same expressions for ψ_b as the new method proposed in this paper (equation (13) or (18)).

3. Illustrative Examples

[27] To test the applicability of the conversion formulae introduced in this paper (equations (17) and (18)) we compare the Gardner k_r-ψ function with vGM function and measured data. For this purpose, we selected 12 Θ-ψ and k_r-ψ data sets that were used by *Brooks and Corey* [1964], *van Genuchten* [1980], and *Tuller and Or* [2001]. The measured data points were digitally extracted from figures in the respective publications. These data sets cover a broad range of porous media, ranging from loose, fragmented mixtures to clay soil. The vGM parameters of these media were obtained by fitting equation (4) to the measured Θ-ψ data. Corresponding Gardner parameters were calculated using four different approaches. The first approach is direct fitting of equation (3) to the k_r-ψ data sets. Best fit Gardner parameters were obtained by minimizing the sum of squared differences between measured and predicted log-transformed relative permeabilities

$$\text{SSE} = \sum\left(\log\left(k_r^P\right) - \log\left(k_r^M\right)\right)^2 \tag{24}$$

where k_r^P and k_r^M are predicted and measured relative permeabilities. In addition, Gardner parameters were estimated on the basis of the corresponding vGM parameters, using the three methods discussed in section 2: capillary drive approach (equations (9) and (10)), the special case of capillary length method (equations (23) and (13)) and the conversion formulae introduced in this paper

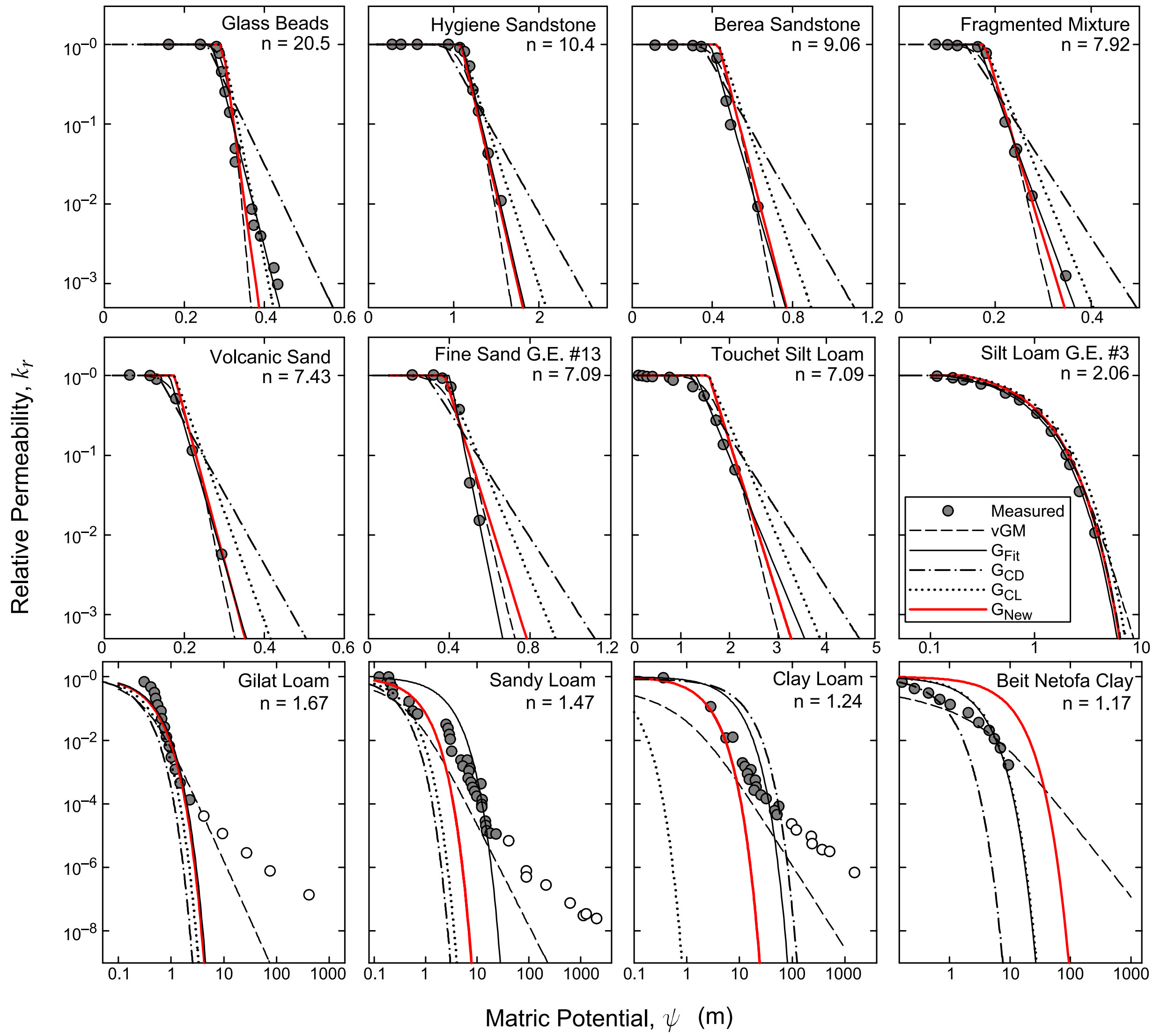

Figure 3. Illustrative examples comparing relative permeability functions predicted using van Genuchten–Mualem (vGM) and Gardner equations with measured data. Notation: vGM, van Genuchten–Mualem; G_{Fit}, Gardner curve fitted to measured k_r-ψ data; G_{CD}, Gardner curve using capillary drive method; G_{CL}, Gardner curve using special case of the capillary length method; and G_{New}, Gardner curve using conversion formulae introduced in this paper. Open circles denote data not used in direct fitting of the Gardner equation.

(equations (17) and (18)). The predicted van Genuchten and Gardner k_r-ψ curves, along with the measured data, of all the example porous media are shown in Figure 3. Table 1 provides the vGM and Gardner parameters and the root mean squared errors (RMSE) between the measured data and the models:

$$\mathrm{RMSE} = \sqrt{\frac{\sum\left(\log\left(k_r^P\right) - \log\left(k_r^M\right)\right)^2}{N}} \tag{25}$$

The porous media are listed in Figure 3 and Table 1 in decreasing order of the van Genuchten parameter n.

[28] From Figure 3, it is apparent that the predicted Gardner curves based on the formulae introduced in this paper are in good agreement with the vGM curves and the fitted Gardner curves for the porous media with $n \geq 2$. In contrast, the Gardner curves that are based on the effective capillary drive and capillary length approaches poorly match both the vGM and the measured values, with the exception of Silt Loam GE 3 ($m \approx 0.5$).

[29] For the porous media with $n < 2$, the vGM model fails to fit the experimental data at both the wet and dry ends. *Ippisch et al.* [2006] attribute this problem to the steep form of the vGM k_r-ψ function at the wet end. They argue this mathematical artifact arises because of the absence of an air entry pressure in the van Genuchten model. In the dry end of the k_r-ψ data, the major deviation of the van Genuchten prediction from the measured values occurs at a relative permeability of about 10^{-5}, which could be attributed to significantly different modes of flow at very low saturations versus moderate to high saturations. *Tuller and Or* [2001] argue that at very low saturations, the predominant flow mechanism is not through capillary pools

Table 1. Fitted van Genuchten–Mualem Parameters and Gardner Parameters[a]

	vGM			G_{FIT}			G_{CD}			G_{NEW}			G_{CL}	
	α_{vG}, m^{-1}	n	RMSE	α_G, m^{-1}	ψ_b, m	RMSE	α_G, m^{-1}	ψ_b, m	RMSE	α_G, m^{-1}	ψ_b, m	RMSE	α_G, m^{-1}	RMSE
Glass Beads[b]	3.15	20.5	1.58	45.0	0.27	0.16	24.0	0.26	0.70	83.5	0.30	0.79	60.3	0.52
Hygiene[c]	0.79	10.4	0.20	10.3	1.11	0.03	4.36	0.89	0.46	10.5	1.09	0.08	7.65	0.32
Berea Sandstone[b]	1.94	9.06	0.18	20.3	0.39	0.07	9.89	0.34	0.45	22.4	0.43	0.21	16.4	0.49
Fragmented Mixture[b]	4.56	7.92	0.37	37.9	0.17	0.07	21.3	0.14	0.55	45.7	0.18	0.16	33.6	0.46
Volcanic Sand[b]	4.57	7.43	0.12	39.5	0.16	0.03	20.5	0.13	0.44	42.9	0.17	0.11	31.6	0.46
Fine Sand GE 13[b]	2.09	7.09	0.22	29.0	0.40	0.08	9.10	0.29	0.57	18.7	0.38	0.22	13.8	0.46
Touchet Silt Loam[c]	0.50	7.09	0.11	3.39	1.31	0.05	2.18	1.19	0.27	4.48	1.58	0.15	3.31	0.40
Silt Loam GE 3[c]	0.42	2.06	0.09	1.26	0.13	0.02	1.17	0.14	0.23	1.20	0.21	0.08	1.06	0.32
Gilat Loam[d]	2.29	1.67	0.57	4.58	0.00	0.39	8.08	0.01	1.15	6.79	0.00	1.03	6.26	0.66
Sandy Loam[d]	1.45	1.47	0.91	0.74	0.00	0.82	6.87	0.00	4.26	4.75	0.00	16.0	5.21	0.69
Clay Loam[d]	0.55	1.24	0.89	0.25	0.00	1.23	5.94	0.00	7.17	2.28	0.00	23.3	25.6	9.20
Biet Netofa Clay[d]	0.15	1.17	0.31	0.78	0.00	0.32	2.71	0.00	1.42	0.70	0.00	0.35	0.76	1.07

[a]Notation: vGM, van Genuchten–Mualem; G_{Fit}, Gardner curve fitted to measured k_r-ψ data; G_{CD}, Gardner curve using capillary drive approach; G_{NEW}, Gardner curve using conversion formulae introduced in this paper; and G_{CL}, Gardner curve using special case of the capillary length approach.
[b]*Brooks and Corey* [1964].
[c]*van Genuchten* [1980].
[d]*Tuller and Or* [2001].

(the underlying assumption behind the van Genuchten model) but that of adsorbed films. For this reason, the data points that are believed to be inconsistent with the bundle of capillaries concept (shown as open circular points in the plots of Gilat Loam, Sandy Loam, and Clay Loam of Figure 3) were not used in fitting the Gardner equation to the measured data.

[30] For these porous media therefore we focus on the comparisons between the vGM and Gardner models (which is the main objective of this paper). When n is not far below 2 (Gilat Loam and Sandy Loam), the Gardner models based on the both the special form of the capillary length method (subsection 2.3) and the new formulae proposed in this paper provide good match with the vGM model. For the remaining two porous media (Clay Loam and Biet Netofa Clay with very low n values of 1.24 and 1.17, respectively) none of the three approaches provides acceptable agreement with the vGM model.

4. Conclusions

[31] In this paper we introduced new formulae that relate the parameters of the Gardner relative permeability (k_r-ψ) function with that of van Genuchten–Mualem. In addition, we also introduced a special case of the capillary length method. We compared Gardner k_r-ψ curves that are based on parameters derived from vGM curves using the capillary drive, the special case of capillary length, and the new formulae with vGM k_r-ψ curves and measured data. These comparisons show that the proposed formulae perform well when the van Genuchten parameter $n \geq 1.5$. The special case of the capillary length method also performs well when the values of the parameter n are not too far off $n = 2$. The capillary drive method was found to perform well only when $n = 2$. However, the capillary drive and capillary length approaches that match the sorptivities of the Gardner and vGM models will probably do well for early time infiltration.

[32] In addition, we also provided concise forms of the proposed formulae. The conversion formula $\alpha_G/\alpha_{vG} = 1.3n$ can be particularly useful in recasting analytical solutions of the Richards equation derived using the Gardner model in terms of the van Genuchten parameters. In many of these analytical solutions, the characteristic quantity $2/\alpha_G$ is a useful measure of macroscopic capillary length [*Pullan*, 1990] that enters the dimensionless quantity s (sorptive length), a measure of the relative importance of gravity and capillarity in determining flow,

$$s = \frac{1}{2}\,\alpha_G\,\ell \tag{26}$$

where ℓ is a problem-specific characteristic physical length. This quantity appears in many analytical solutions [e.g., *Philip et al.*, 1989] and semianalytical solutions. Using the proposed conversion equation (17) s can be rewritten in terms of van Genuchten parameters as

$$s \approx \frac{1.3}{2}\,n\,\alpha_{vG}\,\ell \tag{27}$$

[33] Equation (27) combines the effects of the van Genuchten parameters α_{vG} and n (hydrologic properties) with the physical characteristic length ℓ in a single parameter.

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