

Key characteristics of three-phase oil relative permeability formulations for improved oil recovery predictions

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ABSTRACT: Significant three-phase regions can occur in a range of reservoir development strategies and oil relative permeability may then critically affect ultimate oil recovery. Unfortunately, three-phase oil relative permeabilities are not generally well characterized. In this paper we focus on theoretical methods of estimating three-phase oil relative permeabilities, as typically applied in reservoir simulation. In the absence of good physical models, we propose applying a mathematical filter to the many existing methods before fitting to measured data. First, the key characteristics of methods for predicting three-phase oil relative permeabilities are discussed, including choice of variables, behaviour at low oil saturations and three-phase residual oil saturations. Second, a numerical comparison of both predicted oil relative permeabilities and predicted incremental oil recoveries for immiscible WAG over waterflood is presented.

None of the four most commonly used formulations assessed passed the mathematical filter successfully. Shortcomings were found in both of Stone's commonly used formulations for estimating expected recoveries. A wide range of incremental oil recoveries for immiscible WAG was found from choosing different formulations or different three-phase residual oil saturations. Some recommendations for best practice have been made.

KEYWORDS: *three-phase flow, relative permeability, enhanced recovery, combination flooding, immiscible displacement*

INTRODUCTION

Significant regions containing oil, water and gas may occur in various oil recovery situations. For example, such regions of three-phase flow may occur both when gas is injected for improved oil recovery, or gas disposal, or when gas is evolved, if pressures fall below the bubble point. There are several gas injection projects in the North Sea where gas may not be fully miscible with oil, so modelling three-phase oil mobility is potentially important. WAG (water alternating gas) has been applied to several fields including Gullfaks, Snorre, Veslefrikk and South Brae. An important example of a reservoir where three-phase regions will occur through gas being released from solution is Brent, for which a project to significantly depressurize the reservoir is underway. In such situations, oil mobility, conventionally expressed in terms of oil relative permeability, may critically affect ultimate oil recovery. The relative permeability is a factor which allows for a reduction in phase mobility due to the presence of other phases. Unfortunately, compared with two-phase oil/water relative permeabilities, laboratory measured three-phase oil relative permeability data are rare, sparse and prone to error. There may, therefore, be considerable uncertainty in the estimation of oil recoveries in such situations. Some of the reasons for this are due to the

following difficulties in obtaining good three-phase laboratory data:

- much higher mobility of gas compared with liquids (also relevant to gas/oil);
- the difficulty of controlling experiments to obtain a suitable range of results;
- the lack of an appropriate analysis method;
- the need for *in situ* saturation measurements.

There are many ways of estimating three-phase oil relative permeabilities, but none of these fit measured data outstandingly over the whole range (Baker 1988). Agreement is unfortunately worst in the low oil saturation regime, which is the most important for prediction of ultimate oil recoveries. In this paper we shall focus on theoretical methods of estimating three-phase oil relative permeabilities, as typically applied in reservoir simulation. These are generally not derived from sound physical arguments, but are effectively mathematical interpolation between two-phase oil–water and oil–gas regimes. In the absence of a good physical model, we propose applying a mathematical filter to the many existing methods before fitting to measured data. Methods which may fit sparse measured data acceptably, but which inevitably give unphysical or unlikely predictions elsewhere should not be accepted.

The paper is divided into two main parts. The first is a theoretical discussion of the key characteristics of methods for predicting three-phase oil relative permeabilities including

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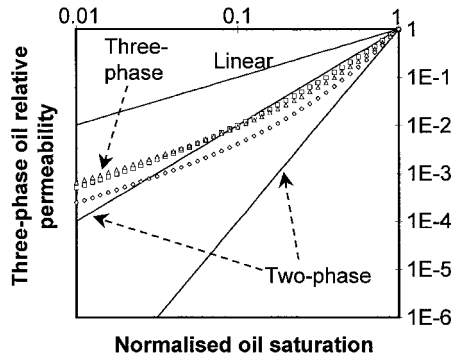


Fig. 2. Three-phase oil relative permeabilities using standard Stone I, assuming S_{om} linear with α and Corey two-phase relative permeabilities with $n_{ow}=2$ and $n_{og}=4$. Dotted lines are three-phase with $\alpha=0.25, 0.5$ and 0.75 . Full lines are two-phase with $\alpha=0$ and 1 .

likely to be mobilized by lower residual saturations in the presence of gas. However, as can be seen from Table 1, for two of the most commonly used methods, Stone I & II, the functional form at low oil saturations is **independent** of the two-phase oil relative permeabilities. For both of Stone's methods, as long as there are three phases present, the functional form at low oil saturations is **linear** in terms of the normalized oil saturation, see equations A1, A2, A10. For Stone I this is simply a consequence of the linear S_{om} factor in the numerator of the interpolating function (A2), which appears to have been chosen arbitrarily. For Stone II the linearity is due more generally to the formulation using unnormalized variables, see Appendix C. For these methods there is likely to be a step change in the functional form of the oil relative permeability as soon as a third phase is present, whatever its saturation, because of the change in functional form. For example, the standard choice of S_{om} for Stone I is affected by this step change in character, see later section on 'Problems in specifying S_{om} '. For the other formulations, the low oil saturation functional form does depend on the two-phase function, although the precise manner depends on the formulation.

The linearity of Stone I is illustrated in Fig. 2, where the logarithm of relative permeability is plotted against S_{on} for various values of α for two-phase functions of the Corey type (power law functions of S_{on}). There does not appear to be any physical basis for choosing an interpolation method which incorporates such an unnecessary constraint. Linear relative permeabilities are characteristic of miscible, rather than immiscible fluids. A more likely assumption is that the three-phase function is similar in form to the two-phase functions, and that it varies smoothly from two-phase to three-phase. For immiscible fluids there may therefore be a tendency for both the Stone formulations to overestimate oil mobility at lower oil saturations.

Three-phase residual oil saturation

Another important characteristic of the chosen interpolation method is the freedom to specify S_{om} , since this allows the formulation to fit measured data. Not all formulations possess this characteristic, some implicitly determine the values of the three-phase residual, see Table 1. This freedom always exists if normalized variables are used.

No freedom to specify S_{om}

The unnormalized saturation-weighted method and Stone II are both examples of formulations which imply S_{om} . Setting the oil

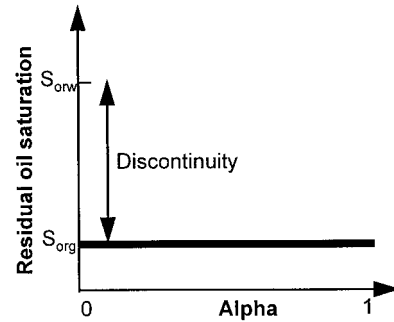


Fig. 3. Jump in implied S_{om} at $\alpha=0$ for unnormalized saturation-weighted formulation.

relative permeability to zero and inverting the formulation definition allows S_{om} to be calculated because the formulations are defined in terms of unnormalized variables. For example, considering equation A11 which defines the saturation-weighted formulation, it can be concluded that, because both functions multiplying the two-phase functions are non-negative, then S_{om} must be equal to the minimum of the two-phase residual oil saturations. Note that this means that if the two-phase residuals are unequal, S_{om} undergoes a sudden jump on one of the two-phase axes (Fig. 3). This lack of smoothness in S_{om} is inherent in the way the formulation is defined.

There are some similarities for Stone II to the saturation-weighted case, though the detailed analysis is much more complicated. For Stone II, inversion of equation A10 to determine S_{om} gives a complicated non-linear function, which must be solved numerically, and which may have more than one solution. To calculate S_{om} for a particular value of α , we therefore need to choose the **highest** oil saturation for which the relative permeability is zero on a line of constant α . This is necessary to avoid encountering multiple residual oil saturations. For Stone II, S_{om} has been calculated numerically using a Newton-Raphson iterative method for a number of different relative permeability functions, all of Corey form, and plotted against α as shown in Fig. 4. For this example S_{orw} has been set to 30% and S_{org} to zero, but there is no loss of generality in choosing these values. Generally S_{om} is predicted to be significantly more than the two-phase values, most clearly seen near $\alpha=1$. Note also that the variation is not monotonic, detailed analysis shows that the reason for this is that the behaviour of the function is generally singular in the vicinity of both the two-phase residual oil saturations. This is shown quite generally without restriction to Corey two-phase functions in Appendix B. However, to illustrate the result, consider the expansion of $S_{om}(\alpha)$ about the two-phase values for two-phase functions of Corey form:

$$\text{Near } \alpha=0, (S_{om}(\alpha) - S_{orw}) \text{ is proportional to } (\alpha)^{1/n_{ow}}$$

$$\text{Near } \alpha=1, (S_{om}(\alpha) - S_{org}) \text{ is proportional to } (1 - \alpha)^{1/n_{og}}$$

If the Corey exponent is greater than one, the derivative of S_{om} with respect to α is unbounded approaching two-phase conditions. This is why S_{om} increases rapidly away from the two-phase value in Fig. 4, except for the unit Corey exponent example. Note that the increase is greatest for larger Corey exponents, as the derivative varies more rapidly. Evidence of such behaviour can also be seen on published Stone II ternary plots which do not use Corey forms, for example Guzman & Fayers (1995). S_{om} tends to have a similar form to Fig. 4, though the rapid change from the value on the two-phase axis is obscured on ternary plots. Note that this singular behaviour,

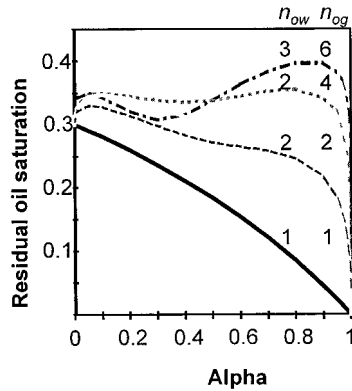


Fig. 4. S_{om} implied by Stone II for Corey type two-phase relative permeabilities with $S_{orw}=0.3$, $S_{org}=0$.

which is inherent in the Stone II formulation (Appendix B), generally results in the predicted S_{om} being much higher than the two-phase values. This will tend to make Stone II predict lower oil recoveries. On consideration of the derivation of Stone II (Stone 1973), the authors consider this singular behaviour to be a mathematical artefact, rather than a physical prediction. Since there is a proliferation of available formulations, there is no need to use a formulation with such unverified singular behaviour. Another problem with this formulation is that it is relatively difficult to calculate the implied S_{om} , which is not usually output by reservoir simulators. Reservoir engineers do not therefore calculate S_{om} and so do not generally realize the assumptions that Stone II implies. Use of Stone II is therefore not recommended.

Limited freedom to specify S_{om}

Stone I does allow some freedom to specify S_{om} , although this is not total. The freedom arises because of the S_{on} factor in the numerator of the interpolating function – equations A1, A2 – which can be used to force K_{ro} to zero. However, this specification needs to be consistent with the two-phase curves and it is possible to specify an S_{om} curve and then find on evaluating the two-phase functions for a point in the interior of the three-phase region, that one of them is zero. If this is inconsistent with the specified S_{om} , modification is then necessary. This is illustrated in Fig. 5 where the chosen S_{om} curve is ABC. Evaluating K_{ro} at point D, which has an oil saturation greater than S_{om} , gives k_{row} zero. From the definition of Stone I – equation A1 – it follows that K_{ro} must also be zero. Note that this inconsistency only occurs with formulations of the Stone I type which mix normalized and unnormalized variables.

Problems in specifying S_{om}

There are some other potential problems in specifying S_{om} for Stone I. In the absence of measured data, the standard assumption for S_{om} is one of the simplest – that S_{om} varies linearly with gas saturation between the two-phase saturations (Fayers & Matthews 1984). However, as noted earlier, this choice suffers from a technical defect in that there may be a discontinuity in S_{om} in going from two-phase oil and gas to three-phase. Referring to Fig. 6a, consider evaluating S_{om} at a point P close to the oil/gas axis. If S_{om} is a function of gas saturation, the result is A, which is very different from the two-phase value at point C. If, alternatively, S_{om} is chosen to be a function of the angular variable α , B is obtained, which is close to C, as is desirable since P is close to the oil/gas axis. This is also illustrated in Fig. 6b, which is a schematic of the

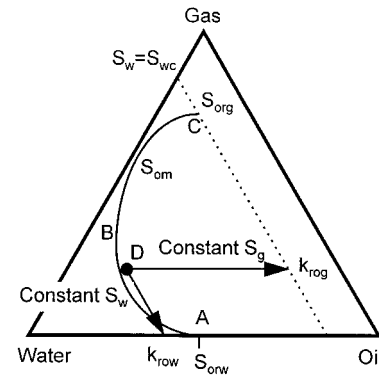


Fig. 5. Schematic ternary diagram showing potential inconsistency of chosen residual oil saturation with two-phase functions for Stone I.

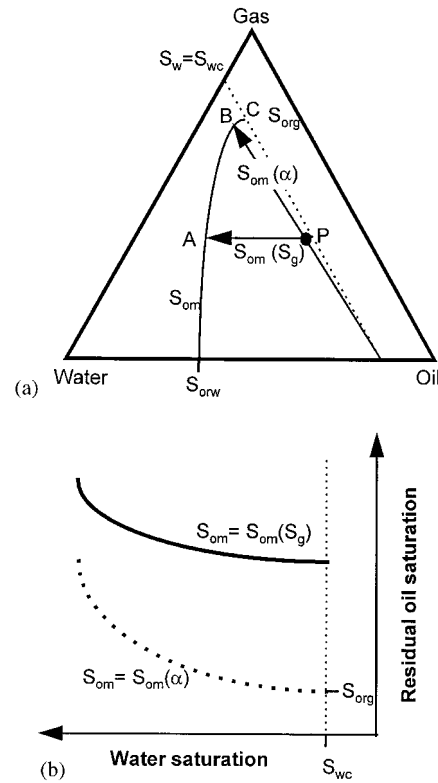


Fig. 6. (a) Ternary diagram showing schematic of discontinuity in S_{om} at $S_w=S_{wc}$ for Stone I if it is assumed that $S_{om}=S_{om}(S_g)$. (b) Schematic of discontinuity in S_{om} at $S_w=S_{wc}$ for Stone I as a function of water saturation if it is assumed that $S_{om}=S_{om}(S_g)$.

variation of S_{om} comparing both choices for different water saturations. This plot shows that for points in the interior of the ternary diagram with the Fayers & Matthews' assumption, as the gas/oil axis is approached, S_{om} does not tend to S_{org} . The predicted values of oil relative permeability are therefore biased by this assumption. It is, therefore, recommended that S_{om} be specified in terms of α .

A more general problem may occur if an arbitrary S_{om} is specified in terms of either the gas or water saturations. Depending on the shape of the curve, S_{om} may have multiple values which could cause simulator implementation problems. This is illustrated in Fig. 7, where the curve S_{om} has been chosen as the curve ABCD. Points B and C have the same gas saturation, but different values of S_{om} . This problem can be averted if again S_{om} is specified in terms of the angular variable

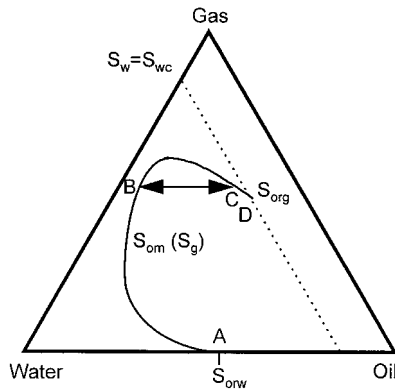


Fig. 7. Schematic ternary diagram of potential multiple values for S_{om} for Stone I if specified as a function of gas saturation.

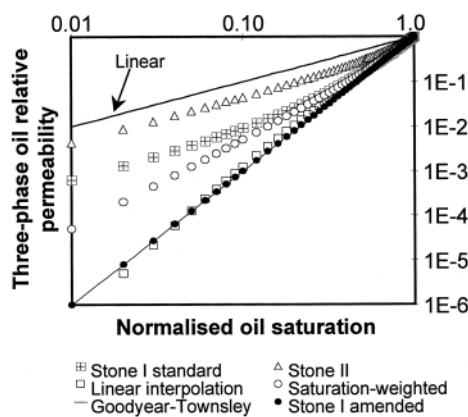


Fig. 8. A comparison of three-phase oil relative permeability formulae assuming Corey two-phase functions with $n_{ow}=2$, $n_{og}=4$, $n_w=4$, $n_g=2$ and $\alpha=0.5$.

α . However, we note that if a sufficiently complicated S_{om} were to be specified, then it could be multiple-valued even in terms of α . This consideration is currently somewhat academic though, since the sparsity of measured data does not justify complicated assumptions about the variation of S_{om} .

NUMERICAL COMPARISON OF FORMULATIONS

Comparison of predicted oil relative permeabilities

A better quantitative appreciation of the differences between the formulations can be obtained from comparative line plots for particular values of α . In Fig. 8 the logarithm of the oil relative permeability for several formulations is plotted against the logarithm of normalized oil saturation for $\alpha=0.5$. This has also been studied for other values of α . Two-phase functions of pure Corey form have been assumed and a line showing linear dependence has also been included for comparison. Stone II, when considered in terms of normalized oil saturation, gives the highest relative permeabilities, as it is linear over the longest range. This is typical of other values of α and partly balances the tendency for lower predictions by Stone II due to its high S_{om} . The next highest values come from the Stone I standard and renormalized forms which are very similar for $\alpha=0.5$, so only the standard values have been plotted. However, if α is lower, the renormalized form gives noticeably higher values, whereas if α is higher, the standard form does. Note that the linear region is noticeably shorter for Stone I compared to Stone II. The next highest values are obtained from the

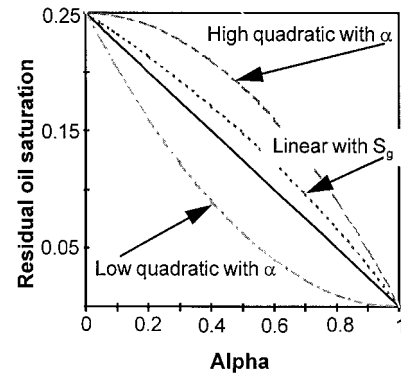


Fig. 9. Specified S_{om} as a function of α .

Table 2. Three-phase oil relative permeability sensitivities

Formulation	Incremental oil recovery over waterflood (%STOIP)
Stone I	5.9
Stone I renormalized	5.3
GT	1.6
Stone I amended	1.5
Saturation-weighted	1.2
Linear isoperms	0.2
Stone II	-0.2

saturation-weighted formulation. This, and the remaining formulations, do not necessarily encounter a linear regime at low oil saturations. The remaining formulations – linear interpolation, GT and Stone I amended – exhibit similar, though slightly different behaviour. For $\alpha=0.5$, the low oil saturation dependence of the Stone I amended formulation is the same as the GT formulation if Corey two-phase functions are assumed. Note that these plots do not necessarily predict incremental oil recoveries, since they do not show the effect of different residual oil saturations.

Comparison of predicted incremental oil recoveries

A detailed two-dimensional generic black-oil model of an injector–producer pair based on a single realization of a heterogeneous North Sea geology was constructed. It was assumed throughout that gas is strictly immiscible with the oil, so no oil swelling effects were included. WAG was implemented from the start of field life. Note that this study focused on relative permeability sensitivities rather than the effectiveness of WAG, so that incremental recovery predictions might be increased by different parameter choices. This study should not therefore be used to assess the value of immiscible WAG in general. S_{om} was modelled as varying linearly in α between the two-phase saturations for all the methods except linear isoperms and Stone II (Fig. 9). This required the saturation-weighted formulation to be applied using normalized variables. The linear isoperms formulation was applied assuming that S_{om} varies linearly in gas saturation between the two-phase residual oil saturations. This gives similar values to assuming a linear variation with α (Fig. 9).

Table 2 shows the predicted incremental recoveries over waterflooding for the different formulations. Appropriate rate and pressure limits were set to determine the end of production. The results may be divided into three groups depending on increment size. Stone I and Stone I renormalized both

Table 3. Sensitivities to residual oil saturation

Model (%STOIP)	S_{om}	Increment
Stone I	Linear in α	5.9
Stone I	Linear in S_g	5.1
Stone I	$\min(S_{org}, S_{orw})$	6.9
Stone I	Quadratic, convex	6.3
Stone I	Quadratic, concave	4.8
GT	Linear in α	1.6
GT	$\min(S_{org}, S_{orw})$	6.9
GT	Quadratic, convex	3.5
GT	Quadratic, concave	-1.1

produced increments in excess of 5% STOIP, significantly above any of the other models. This is due to the linear mobile oil dependence for low oil saturations. This is illustrated by the amended Stone I formulation, which has a Corey type dependence at low oil saturations. It forms another group with the GT and saturation-weighted formulations which have much lower increments between 1.2 and 1.6% STOIP. These models both exhibit more likely behaviour at low oil saturations, with a higher power dependence on mobile oil saturation. The last two formulations, which both defined their own three-phase residual oil saturation, gave near zero increments. Stone II gave the lowest prediction because of its very high implied S_{om} . Linear isoperms gave a low prediction because the other formulations showed a curvature of isoperms benefitting oil recovery.

Sensitivity to three-phase residual oil saturation

Two relative permeability formulations have been chosen – Stone I and GT – to exemplify the two groups with linear and unconstrained mobile oil saturation dependence at low oil saturations. S_{om} was set in various ways including to the minimum of the two-phase residuals, and high and low quadratic interpolations between the two-phase residuals (Fig. 9). The minimum of the two-phase residuals is S_{org} for the data assumed. There is, therefore, a discontinuity in the assumed S_{om} on the oil/water line. The GT formulation is sensitive to this artificial discontinuity, although Stone I is not. In order to obtain a useful comparison, the GT S_{om} was therefore artificially modified locally to obtain a smooth transition from the oil/water residual to the three-phase in this case. Note that the need to modify the GT formulation here is not due to a deficiency in the formulation, but a lack of smoothness in the entered data.

Table 3 lists the results of the sensitivity cases performed and shows a significant effect on incremental recoveries due to varying S_{om} . The effect of S_{om} varying linearly with gas saturation, rather than with α , for Stone I was to reduce the increment by just 0.8% STOIP. Both formulations achieved the same maximum increment for the case with a uniform S_{om} . Overall Stone I was less affected by changes in S_{om} dependence than GT. This insensitivity is another deficiency of Stone I. To understand this, consider comparing three-phase oil relative permeabilities with two values of S_{om} . For GT and other methods using normalized variables, the difference between the curves, determined by the difference in S_{om} , is roughly maintained initially as the oil saturation is increased from S_{om} . However, for Stone I, the difference quickly reduces to significantly less than the difference in S_{om} . Analysis shows that this reduction in the difference for Stone I is principally due to the use of unnormalized variables in the two-phase relative permeabilities. GT and other formulations using normalized variables

are not susceptible to this problem. Reservoir engineers should be aware of such considerations when choosing formulations and sensitivity cases.

CONCLUSIONS

Relative permeability formulations which directly rely on two-phase relative permeability data have been reviewed and key factors such as choice of variables, behaviour at low oil saturations and ability to specify S_{om} investigated. A mathematical filter has been proposed to exclude unacceptable formulations before matching to measured data.

Shortcomings were found in both of Stone's commonly used formulations. They are both constrained to have a linear dependence for low mobile oil saturations, irrespective of the two-phase data and, therefore, tending to overestimate relative permeabilities. There could also be a problem of consistency in some choices of S_{om} for Stone I. Although Stone II implies S_{om} , this is mathematically singular near the two-phase axis, which generally leads to predicted S_{om} being much larger than two-phase values.

The choice of three-phase oil relative permeability formulation affected incremental recovery from immiscible WAG significantly. A range from about zero to 6% of STOIP was found for the chosen set of data. The highest incremental recoveries came from Stone I and the lowest from Stone II and the linear isoperms model. The saturation-weighted, GT and amended Stone I formulations gave fairly similar intermediate recoveries. Stone I gave high recoveries mainly because of its linearity at low oil saturations. This was removed for the amended Stone I formulation, which gave intermediate recoveries. Stone II gave low recoveries because of its implied high S_{om} .

The choice of S_{om} could make a significant difference to predicted incremental recoveries. However, Stone I is relatively insensitive to this choice compared with GT and other methods using normalized variables. This is due to the specific functional form of Stone I. It is recommended that predictions are made for a range of assumed S_{om} curves using a method based on normalized variables (not Stone I).

Since the choice of formulation can have a significant effect on predictions, it is recommended that a prediction range be calculated using different formulations. Of the methods considered here an upper estimate could be obtained from the saturation-weighted or GT formulations. Linear isoperms might be used for a lower estimate. It is recommended that Stone II is not used because of its generally high implied S_{om} and lack of transparency, and that Stone I is not used because of its linearity at low oil saturations.

It is suggested that future interpolation methods should be derived using normalized variables to avoid technical problems and to facilitate entry of alternative S_{om} curves. It is also suggested that S_{om} is specified in terms of an angular variable, such as α used here, in order to avoid inconsistencies. Spatial plots of α are also useful for identifying three-phase regions in simulation models.

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NOMENCLATURE

S_o	oil saturation
S_w	water saturation
S_g	gas saturation
S_{wc}	connate water saturation

S_{on}	normalized oil saturation
S_{orw}	residual oil saturation in the presence of water
S_{org}	residual oil saturation in the presence of gas
S_{om}	three-phase residual oil saturation
K_{ro}	three-phase oil relative permeability
k_{ro}^0	endpoint three-phase oil relative permeability
k_{row}	oil relative permeability in presence of oil and water
k_{rog}	oil relative permeability in presence of oil, gas and connate water
α	normalized variable defining balance of gas and water
$I(\alpha, S_{on})$	interpolating function for Stone I
n_{ow}	oil relative permeability Corey exponent in presence of oil and water
n_{og}	oil relative permeability Corey exponent in presence of oil and gas
p	assumed exponent
$f(\alpha)$	exponent in GT formulation
STOIP	Stock tank oil initially in place
IOR	improved oil recovery
WAG	water alternating gas

APPENDIX A: DEFINITIONS OF THREE-PHASE OIL RELATIVE PERMEABILITY FORMULATIONS

The two-phase relative permeabilities are assumed to be normalized to unity at the maximum oil saturation.

Stone I and associated formulations

We express the original Stone I formulation, Stone (1970), in the following non-standard way:

$$K_{ro} = I(\alpha, S_{on}) k_{row}(S_w; S_{orw}) k_{rog}(S_g; S_{org}) \quad (A1)$$

where $I(\alpha, S_{on})$ is the following interpolating function with range $[0, 1]$ and $k_{row}(S_w; S_{orw})$ indicates that the two-phase relative permeability is to be taken as a function of S_w , and evaluated using the two-phase residual oil saturation in the standard way.

$$I(\alpha, S_{on}) = \frac{S_{on}}{[\alpha + (1 - \alpha) S_{on}][1 - \alpha + \alpha S_{on}]} \quad (A2)$$

An alternative non-standard interpretation is to evaluate the two-phase functions using S_{om} , rather than the two-phase residuals:

$$K_{ro} = I(\alpha, S_{on}) k_{row}(S_w; S_{om}) k_{rog}(S_g; S_{om}) \quad (A3)$$

This is done by renormalizing the two-phase functions to take account of the different range of oil saturations given by evaluating using S_{om} instead of S_{orw} . We shall refer to this formulation as Stone I renormalized. The meaning of this is best understood by an example. Assuming two-phase functions of normalized Corey form, k_{row} is given by

$$k_{row} = \left[\frac{1 - S_w - \xi}{1 - S_{wc} - \xi} \right]^{n_{ow}} \quad (A4)$$

Evaluating k_{row} in the conventional way, $k_{row} = k_{row}(S_w; S_{orw})$, the function ξ is given by

$$\xi = S_{orw} \quad (A5)$$

which is a constant.

Alternatively, renormalizing so, $k_{row} = k_{row}(S_w; S_{om})$, ξ is given by

$$\xi = S_{om}(S_w; S_g) \quad (A6)$$

so that ξ may vary with saturation.

The other two-phase function k_{rog} is evaluated similarly.

Note that for two-phase functions of Corey form, renormalization gives the following simplified expression for the three-phase relative permeability:

$$K_{ro} = S_{on} [\alpha + (1 - \alpha) S_{on}]^{n_{ow} - 1} [1 - \alpha + \alpha S_{on}]^{n_{og} - 1} \quad (A7)$$

The following is the Stone I formulation amended to have a Corey-type power law dependence, $(S_{on})^p$, at low oil saturations in the three-phase region:

$$K_{ro} = (I(\alpha, S_{on}))^p k_{row}(S_w; S_{orw}) k_{rog}(S_g; S_{org}) \quad (A8)$$

Here the power, p , can be chosen arbitrarily, but a simple choice for two-phase functions of Corey form at low oil saturations is to interpolate linearly between the two phase powers in α :

$$p = (1 - \alpha) n_{ow} + \alpha n_{og} \quad (A9)$$

We refer throughout to the formulation specified by equations A8 & A9 as Stone I amended.

Stone II

This is defined by Stone (1973) as:

$$K_{ro} = (k_{row}(S_w) + k_{rw}(S_w))(k_{rog}(S_g) + k_{rg}(S_g)) - k_{rw}(S_w) - k_{rg}(S_g) \quad (A10)$$

Saturation-weighted interpolation

The standard definition in terms of unnormalized variables is (Baker 1988):

$$K_{ro} = (1 - \alpha) k_{row}(S_o) + \alpha k_{rog}(S_o) \quad (A11)$$

in which case S_{om} must be the minimum of the two-phase residuals.

Alternatively, in terms of normalized variables,

$$k_{ro} = (1 - \alpha) k_{row}(S_{on}) + \alpha k_{rog}(S_{on}) \quad (A12)$$

in which case S_{om} is arbitrary.

GT formulation

This is (S. G. Goodyear & P. H. Townsley, pers. comm, 1995),

$$K_{ro} = \{k_{rog}(S_{on})\}^{f(\alpha)} \{k_{row}(S_{on})\}^{(1-f(\alpha))} \quad (A13)$$

The function $f(\alpha)$ may be chosen arbitrarily, providing $f(0)=0$ and $f(1)=1$. However, for definiteness and simplicity, here we have chosen

$$f(\alpha) = \alpha \quad (A14)$$

Note that the arguments of the two-phase functions are functions of the normalized oil saturation. These are chosen to be linear in the normalized oil saturation:

$$k_{row}(S_{on}) = k_{row}(S_{orw} + S_{on}[1 - S_{wc} - S_{orw}]) \quad (A15)$$

and

$$k_{rog}(S_{on}) = k_{rog}(S_{org} + S_{on}[1 - S_{wc} - S_{org}]) \quad (A16)$$

Note if we consider two-phase functions of Corey form, then equation A13 simplifies to:

$$K_{ro} = k_{ro}^0 S_{on}^{[n_{og}f(\alpha) + n_{ow}\{1-f(\alpha)\}]}, \quad (A17)$$

which is also of Corey form.

Linear isoperms

Three-phase relative permeabilities are obtained by interpolating linearly between oil/water and oil/gas points with the same values. Note that this also implies that S_{om} varies linearly. A linear interpolation method for which S_{om} may be set arbitrarily is obtained by using normalized variables.

APPENDIX B: SINGULAR NATURE OF S_{OM} FOR STONE II

Following Slack & Ehrlich (1981), consider isoperms defined by

$$dK_{ro} = \left[\frac{\partial K_{ro}}{\partial S_w} dS_w + \frac{\partial K_{ro}}{\partial S_g} dS_g \right] = 0 \quad (B1)$$

Solving for dS_g/dS_w from equation B1, the isoperms are defined by

$$\frac{dS_g}{dS_w} = - \frac{\partial K_{ro}}{\partial S_w} \bigg/ \frac{\partial K_{ro}}{\partial S_g} \quad (B2)$$

Consider the trajectory of the isperm at $S_o = S_{orw}$ and zero gas saturation. The following argument also applies to the same isperm at the other end, due to the symmetry between gas and water in the definition of Stone II (equation A10). Evaluating the derivatives in equation B2 at this point using equation A10 gives

$$\frac{\partial K_{ro}}{\partial S_g} = k_{rw} \frac{\partial k_{rog}}{\partial S_g} + \frac{\partial k_{rg}}{\partial S_g} (k_{rw} - 1) \quad (B3)$$

$$\frac{\partial K_{ro}}{\partial S_w} = \frac{\partial k_{row}}{\partial S_w} \quad (B4)$$

and so the trajectory of the zero isperm at zero gas saturation is given by

$$\frac{dS_g}{dS_w} = - \frac{\partial k_{row}}{\partial S_w} \bigg/ \left\{ k_{rw} \frac{\partial k_{rog}}{\partial S_g} + \frac{\partial k_{rg}}{\partial S_g} (k_{rw} - 1) \right\} \quad (B5)$$

Now the denominator of the right-hand side of equation B5 is generally finite for immiscible fluids. For example, for traditional Corey fits to data, the second term in the denominator is zero due to the derivative term, but the first term is finite. However, the numerator in equation B5 is typically zero for immiscible fluids. Measured two-phase oil relative permeabilities are generally non-linear with very low gradients at residual oil. For example, traditional Corey fits to data give a zero gradient at this point. The trajectory of the zero isperm near zero gas saturation is therefore predicted by Stone II to be initially very close to the water/oil axis on a ternary diagram. A similar argument applies to the other end of the zero isperm near connate water saturation by symmetry. For Stone II there is therefore an intimate connection between low initial oil gradients in the two-phase oil relative permeabilities and sharp changes in the implied S_{om} near two-phase residual oil. The more slowly varying the initial two-phase oil relative permeability, the sharper the initial change in S_{om} . The analysis above explains the numerical calculations for the zero isperm presented in the main part of the paper.

APPENDIX C: LINEARITY OF STONE II FOR LOW OIL SATURATIONS

We wish to show that dK_{ro}/dS_o is generally non-zero on $S_{on}=0$ (i.e. $S_o=S_{om}$), for Stone II, so the leading order terms in an expansion of K_{ro} about $S_{on}=0$ for $S_{on} \ll 1$ are $O(S_{on})$. Note that we specifically exclude the points at which $\alpha=0$ and $\alpha=1$, where S_{om} may be singular (see Appendix B). K_{ro} is specified as a function of gas and water saturation – equation A10 – so taking the derivative normal to the curve $S_o=S_{om}$ defined by equation B2 gives

$$\frac{dK_{ro}}{dS_o} = \frac{- \left\{ \left(\frac{\partial K_{ro}}{\partial S_w} \right)^2 + \left(\frac{\partial K_{ro}}{\partial S_g} \right)^2 \right\}}{\left(\frac{\partial K_{ro}}{\partial S_w} + \frac{\partial K_{ro}}{\partial S_g} \right)} \quad (C1)$$

For the derivative to be generally zero, we therefore require

$$\frac{\partial K_{ro}}{\partial S_w} = \frac{\partial K_{ro}}{\partial S_g} = 0 \quad (C2)$$

Evaluating the derivatives in (C2) for Stone II from equation A10 gives

$$\begin{aligned} \frac{\partial K_{ro}}{\partial S_w} = & (k_{rog}(S_g) + k_{rg}(S_g)) \left(\frac{\partial k_{row}(S_w)}{\partial S_w} + \frac{\partial k_{rw}(S_w)}{\partial S_w} \right) \\ & - \frac{\partial k_{rw}(S_w)}{\partial S_w} \end{aligned} \quad (C3)$$

On S_{on} we have from equation A10

$$0 = (k_{row}(S_w) + k_{rw}(S_w))(k_{rog}(S_g) + k_{rg}(S_g)) - k_{rw}(S_w) - k_{rg}(S_g) \quad (C4)$$

Substituting equation C4 into C3 and imposing C2 gives

$$\begin{aligned} k_{rg}(S_g) = & \frac{\partial k_{rw}(S_w)}{\partial S_w} \left\{ \frac{(k_{row}(S_w) + k_{rw}(S_w))}{\left(\frac{\partial k_{row}(S_w)}{\partial S_w} + \frac{\partial k_{rw}(S_w)}{\partial S_w} \right)} \right\} \\ & - k_{rw}(S_w), \end{aligned} \quad (C5)$$

that is

$$k_{rg} = k_{rg}(k_{row}, k_{rw}) \quad (C6)$$

and similarly for $\frac{\partial K_{ro}}{\partial S_g}$

$$k_{rw} = k_{rw}(k_{rog}, k_{rg}) \quad (C7)$$

Now equations C6 & C7 are relations between single and two-phase relative permeabilities which are not satisfied in general. Therefore, for Stone II, dK_{ro}/dS_o is generally non-zero on $S_{on}=0$ and so the leading order terms in an expansion of K_{ro} about $S_{on}=0$ are $O(S_{on})$.

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