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A Multilevel Preconditioner and Its Shared Memory Implementation for New Generation Reservoir Simulator

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Abstract

The mathematical models in reservoir simulation are usually discretized into large linear equations, and solving them needs lots of time. Taking into account the mathematical characteristics of the black oil model, a multilevel preconditioning solution method is designed to deal with the algebraic equations in reservoir numerical simulation. Takes into account some of the properties of pressure, saturation, and implicit well variables in flow model, the multilevel preconditioner is comprised of several different iterative methods, such as algebraic multigrid method, Incomplete LU factorization, Gauss-Seidel iteration with downwind ordering and crosswind blocks and et al. The efficiency and robustness of multilevel preconditioner is proved by a million-cell benchmark problem and a real-world matured reservoir with high heterogeneity, high water-cut, geological faults, and complex well scheduling. The numerical results indicate that the proposed method is not only robust with respect to the heterogeneity, anisotropy, and number of wells but also efficient method that can solve large Jacobian system in reservoir simulation quickly and precisely.

Introduction

Reservoir numerical simulation is to use available reservoir geological parameters and dynamic development data to reproduce reservoir development history, predict reservoir development trend, formulate or modify development program, which is a necessary technique in oilfield development study [1-3]. It plays a key role both in effecting an efficient development process and in improving recovery rate [4-5]. Furthermore, most of China’s oil fields are located in continental basins and many are characterized by serious heterogeneity, low permeability, high oil viscosity, and most of the developed main oilfields have been in late development period with high water-cut and recovery percent, thus it is more difficult to further develop the reservoir [6-7]. In order to improve oil yield economically and efficiently, it is needed to deepen the reservoir description and build more fine reservoir numerical models, which have more grids, longer production history, more wells and measures, more complex oil and gas distribution, than the existing geological models, greatly increasing the workload and difficulty of reservoir numerical simulation history match and development prediction of adjustment plan and being more time-consuming [8-9].

The mathematical model in reservoir numerical simulation are composed of several nonlinear PDEs with strong linear, discontinuity and coupling, thus the time of solving these PDES accounts for about 70%-80% of the simulation time, which would increase as the scale and complexity of model increases. Thus, how to solve these equations efficiently is the key to numerical simulation and many research achievements on this respect is obtained which can accelerate the computation [10-16].

In the present paper, a framework of multilevel preconditioner is developed for reservoir numerical simulation which is made up of AMG, ILU factorization, Gauss-Seidel with downwind ordering and other iterative method. For the pressure system that produced in reservoir simulation with highly heterogeneity, anisotropy media, the method AMG combined with ILU is more efficient than AMG used lonely. Block Gauss-Seidel along from higher pressure to lower pressure can solved the saturation system quickly. The multilevel preconditioner is formed when the above operators integrated multiplicatively. Finally, this preconditioning method is applied in numerical simulation of a million-cell benchmark problem and a real-world matured reservoir with high heterogeneity, and the results show that the multilevel preconditioning solution method is accurate and rapid.

Basic Model

Black oil simulation is adopted as an example in this article to illustrating the multilevel preconditioning methods. Assuming that the flow in the porous media is isothermal and the phases in fluid flow are satisfying the Darcy's Law. Supposing there are three components i.e. oil, gas and water in the fluid, gas component is included in oil and gas phases, while oil and water components exist in oil and phases respectively. There is no mass exchanging between water and other two phases. The three-phase equations of the black oil model are as follows:

$$\nabla\left[K\frac{K_{\mathrm{rw}}}{\mu_{\mathrm{w}}}\rho_{\mathrm{w}}\nabla\left(p_{\mathrm{w}}-g\rho_{\mathrm{w}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{w}}S_{\mathrm{w}}\right) \tag{1}$$

$$\nabla\left[K\frac{K_{\mathrm{ro}}}{\mu_{\mathrm{o}}}\rho_{\mathrm{o,o}}\nabla\left(p_{\mathrm{o}}-g\rho_{\mathrm{o}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{o,o}}S_{\mathrm{o}}\right) \tag{2}$$

$$\nabla\left[K\frac{K_{\mathrm{ro}}}{\mu_{\mathrm{o}}}\rho_{\mathrm{g,o}}\nabla\left(p_{\mathrm{o}}-g\rho_{\mathrm{o}}Z\right)\right]+\nabla\left[K\frac{K_{\mathrm{rg}}}{\mu_{\mathrm{g}}}\rho_{\mathrm{g}}\nabla\left(p_{\mathrm{g}}-g\rho_{\mathrm{g}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{g,o}}S_{\mathrm{o}}+\phi\rho_{\mathrm{g}}S_{\mathrm{g}}\right) \tag{3}$$

The pressure of each phase related with the capillarity pressure is characterized:

$$p_{\mathrm{cow}}(S_{\mathrm{w}})=p_{\mathrm{o}}-p_{\mathrm{w}}$$

$$p_{\mathrm{cgo}}(S_{\mathrm{g}})=p_{\mathrm{g}}-p_{\mathrm{o}}\,.$$

And the phase saturations satisfy the condition:

$$S_{\mathrm{o}}+S_{\mathrm{g}}+S_{\mathrm{w}}=1\,.$$

Where, K is permeability, 10^{-3} μm^2; φ is porosity, %; K_{rw}, K_{ro}, K_{rg} are the phase relative permeability of water, oil, gas respectively, f; μ_w, μ_o, μ_g are the phase viscosities of water, oil, gas respectively, mPa·s; ρ_w, ρ_o, ρ_g are the phase densities of water, oil, gas respectively, g/cm^3; $\rho_{o,o}$ is the density of the oil component in oil phase, g/cm^3, and $\rho_{g,o}$ is the density of the gas component in oil phase, g/cm^3; S_w, S_o, S_g are the phase saturations of water, oil, gas respectively, %; t is time, s; p_w, p_o, p_g are the phase pressures of water, oil, gas respectively, MPa; g is the acceleration of gravity, m/s^2; Z is the depth, m; p_{cow}, p_{cgo} are the interface capillary pressures of oil-water, gas-oil respectively, Mpa.

Assuming that there are n cell-center grids in three dimensional model, the equations (1) - (3) can be discrete as equation (4) using fully implicit finite difference method:

$$A\mathrm{x}=\mathrm{b}, \tag{4}$$

where coefficient matrix

$$\boldsymbol{A}=\begin{pmatrix} \boldsymbol{a}_{1,1} & \boldsymbol{a}_{1,2} & & \boldsymbol{a}_{1,n_x} & & \boldsymbol{a}_{1,n_{xy}} & & & & & \\ \boldsymbol{a}_{2,1} & \boldsymbol{a}_{2,2} & \boldsymbol{a}_{2,3} & & \boldsymbol{a}_{2,n_x+1} & & \boldsymbol{a}_{2,n_{xy}+1} & & & & \\ & \ddots & \ddots & \ddots & & \ddots & & \ddots & & & \\ \boldsymbol{a}_{n_x,1} & & \boldsymbol{a}_{n_x,n_x-1} & \boldsymbol{a}_{n_x,n_x} & \boldsymbol{a}_{n_x,n_x+1} & & \boldsymbol{a}_{n_x,2n_x-1} & & \boldsymbol{a}_{n_x,n_{xy}+n_x-1} & & \\ & \ddots & & \ddots & \ddots & \ddots & & \ddots & & \ddots & \\ \boldsymbol{a}_{n_{xy},1} & & \boldsymbol{a}_{n_{xy},n_{xy}-n_x} & & \boldsymbol{a}_{n_{xy},n_{xy}-1} & \boldsymbol{a}_{n_{xy},n_{xy}} & \boldsymbol{a}_{n_{xy},n_{xy}+1} & & \boldsymbol{a}_{n_{xy},n_{xy}+n_x-1} & & \boldsymbol{a}_{n_{xy},2n_{xy}-1} \\ & \ddots & & \ddots & & \ddots & \ddots & \ddots & & \ddots & \end{pmatrix}_{n\times n},$$

in which n_x, n_y and n_z grids are distributed in x, y, z directions separately, namely, $n=n_xn_yn_z$, $n_{xy}=n_xn_y$. The form of element a_{ij} in the coefficient matrix is

$$\boldsymbol{a}_{i,j}=\begin{bmatrix} B_\mathrm{w} & C_\mathrm{w} & D_\mathrm{w} \\ B_\mathrm{o} & C_\mathrm{o} & D_\mathrm{o} \\ B_\mathrm{g} & C_\mathrm{g} & D_\mathrm{g} \end{bmatrix}_{i,j}, (1 \le \mathrm{i} \le \mathrm{n};\ 1 \le \mathrm{j} \le \mathrm{n}) \tag{5}$$

where, B, C, D are the coefficients of water saturation, oil saturation and oil phase pressure on the relevant active nodes in the composition differential equations, respectively. The subscript w, o, g indicate water, oil and gas composition respectively.

Multilevel preconditioning solution for reservoir simulation

Owing to the highly heterogeneous of A and the strong coupling of reservoir and wells, direct solution of (4) is too expensive and many iterative solvers converged too slowly or even fail to converge. Considering the oil phase pressure equation in the black oil model has the properties of parabolic equation and the saturation equation has the properties of hyperbolic equation, it may be efficient to solve system (4) with different methods. Firstly the coefficient matrix A can be reformed as

$$A=\begin{pmatrix} A_\mathrm{R} & A_\mathrm{Rw} \\ A_\mathrm{wR} & A_\mathrm{w} \end{pmatrix}, \tag{6}$$

where the subscripts R and w denote the reservoir and implicit well parts of the main variables, respectively. Furthermore, the sub-matrix A_R are composed of coefficients of pressure and saturation unknowns, so AR can be decoupled as

$$A_R=\begin{pmatrix} A_{pp} & A_{pS} \\ A_{Sp} & A_{SS} \end{pmatrix}, \tag{7}$$

in which p denotes the pressure variables (oil pressure and the well bottom whole pressure) and S denotes the saturation variables (including physical water and oil saturations for the reservoir blocks and artificial saturations for the implicit well blocks).

In order to solve equation (4) quickly and accurately, a multilevel preconditioner is adopted in this paper, which deals pressure, saturation and well equations with different methods.

(1) Solution method of pressure system

Owing to the elliptical character of the pressure equations in flow model (1)-(3), algebraic multigrid (AMG) methods can be used to solve the corresponding discrete algebraic system efficiently. But in practice, the performance and efficiency of AMG may degenerate as the physical and geometric properties of the problems become more complex. In order to solve the pressure system quickly, a combined preconditioner is adopted in the present article which is comprised of AMG and ILU(k) method. The preconditioning operator is designed as follow:

① $u^{k,1} = u^{k,0} + \text{AMG}\,(f - Au^{k,0})$,
② $u^{k,2} = u^{k,1} + \text{ILU}\,(k)\,(f - Au^{k,1})$,
③ $u^{k+1} = u^{k,2} + \text{AMG}\,(f - Au^{k,2})$,

where $u^{k,0} = u^{\mathrm{k}}$.

In this method, a classic AMG method are chosen which consists of setup stage and the solve stage. In the setup stage, the intergrid operator P_l is constructed, and the coarse grid matrix is defined as

$$A_l = P_l^T A_{l+1} P_l, \quad l = L\text{-}1, \ldots, 0,$$

and $A_L = \mathrm{A}$. In the solve stage, the smoother S_l and coarse-grid correction are applied recursively as shown in the following general V-cycle multigrid.

V-cycle multigrid for solving $A_l u_l = f_l$, with an initial guess u_l^0:

① Pre-smoothing: $u_l^1 = u_l^0 + S_l(f_l - A_l u_l^0)$
② Coarse-grid correction:

a. $f_{l-1} = P_{l-1}^T (f_l - A_l u_l^1)$
b. If $l = 1$, $e_0 = A_0^{-1} f_0$; else, apply V-cycle multigrid for $A_{l-1} e_{l-1} = f_{l\text{-}1}$ with zero initial guess
c. $u_l^2 = u_l^1 + P_{l-1} e_{l-1}$

③ Post-smoothing: $u_l^3 = u_l^2 + S_l^T (f_l - A_l u_l^2)$

ILU(k) selected in this paper is based on the *level of fill* to determine the off-diagonal positions in the L and U factors where the entries fill-in will not be introduced. The *level of fill* is defined as Def.1.

Def.1. (*Level of fill*) The initial level of fill of the elements of a sparse matrix A defined as

$$L_{ij} = 0 \text{ if } a_{ij} \neq 0 \text{ or } i = j, \text{ otherwise } L_{ij} = \infty.$$

When an entry a_{ij} is updated in the factorization procedure $a_{ij} := a_{ij} - a_{ik} * a_{kj}$, its level of fill is also updated by $L_{ij} = \min\{ L_{ij}, L_{ik} + L_{\mathrm{kj}} + 1\}$.

The ILU(k) algorithm is given as follow:

① For all nonzero elements a_{ij}, define $L_{ij} = 0$
② For $i = 1, \ldots, \mathrm{n}$, Do
  For $m = 1, \ldots, i\text{-}1$ and $L_{im} \leq k$, Do
    Compute $a_{im} := a_{im}/a_{mm}$
    Compute $a_i* := a_i* - a_{im} a_m*$, where a_i* is the i-th row of A
    Update the levels of fill of non-zero by $L_{ij} = \min\{ L_{ij}, L_{ik} + L_{\mathrm{kj}} + 1\}$
  End
  Replace any element in row i with $L_{ij} > k$ by zero
End

This combing method can efficiently overcome the degeneration of AMG in some complicated physical and geometric problems such as heterogeneous media and anisotropic permeability, which is proved and tested in [10].

(2) Solution method of saturation system

Because the saturation variables $S = (S_w, S_g)$ have hyperbolic characteristics, the saturation equations can be solved accurately quickly if Gauss-Seidel iteration with downwind ordering and crosswind blocks is employed. Because the fluid flows according to the negative pressure gradient, the flux between two

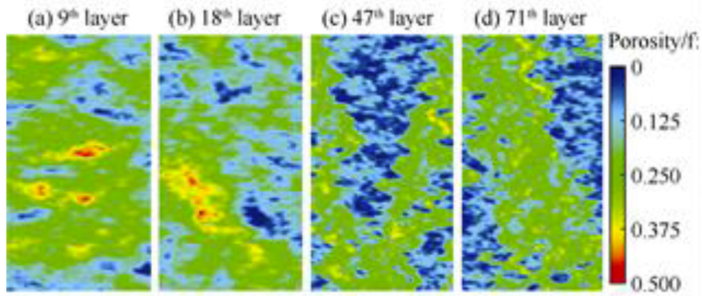

Figure 1—Porosities of some horizontal layers in SPE10

interfacing cells depends only on the saturation of the upstream cell. Reordering the cells on the basis of pressure in descending order, the saturation block will turn into a lower triangular matrix that can be solved by Gauss-Seidel method very efficiently.

(3) Solution method of implicit well system

As for the well block, the coupling between the reservoir and well might be very strong due to the high permeability around the perforations and different type of wells, which means that the well equations and the reservoir part should be solved together. Integrating A_{ww} and the perforated grid-blocks in A_{rr} forms implicit well blocks, which can be solved by direct method because the number of perforated grid-blocks is very small relative to the reservoir.

At last, to applying the above methods to linear solver in reservoir numerical simulation, a multiplication preconditioner are defined as

$$I - B_m A = (I - RA)(I - \Pi_p B_p \Pi_p^T A)(I - \Pi_s B_s \Pi_s^T A)(I - \Pi_w B_w \Pi_w^T A) \tag{8}$$

where Π_p: $V_p \rightarrow V$, Π_s: $V_s \rightarrow V$ and Π_w: $V_w \rightarrow V$ are the inclusion operators, and R is smoother. V_p, V_s, and V_w are the subspaces of the pressure, saturation variables and well separately.

This multilevel preconditioner coordinates with *Krylov* subspace methods such as GMRes and BICGstab can simulate complicated reservoir, especially for simulations of enhancement oil recovery using both water flooding and polymer flooding.

Benchmark Test

(1) SPE10 Benchmark Test

SPE10 is an international standard test, which describes the performance of large scale water flooding in a highly heterogeneous black oil reservoir. This model is an important example for testing accuracy, computational speed and capability of the simulator.

The problem is a two phase (water and oil) model with a reservoir depth of 3657.6 m, an initial oilfield pressure of 41.37 MPa, a surface oil density, a gas density, and a surface water density being 0.849 g/cm^3, 1.000×10^{-4} g/cm^3 and 1.024 g/cm^3 respectively, volume factor of water and underground oil both of 1.01, the compressibility coefficient of water and rock being 4.41×10^{-5} MPa^{-1} and 1.47×10^{-5} MPa^{-1} respectively, a water viscosity and an underground oil viscosity of 0.3 mPa·s and 3.0 mPa·s respectively.

In SPE10 case, the permeability is between 0.00066×10^{-3} and 20000×10^{-3} μm^2, averaging 364.52×10^{-3} μm^2. The oil layer has an extremely poor permeability of 23.3×10^{-3} μm^2. The ratio between the vertical and horizontal permeability varies from 0.001 in basement to 0.3 in channel. The average and maximum porosities of the field are 0.1749 and 0.5, the porosities of some single layers are shown in Figure 1.

In the case of SPE10, an area of 366×671×51.85 (m) with one water injector, four producers and 1122000 grids was simulated.

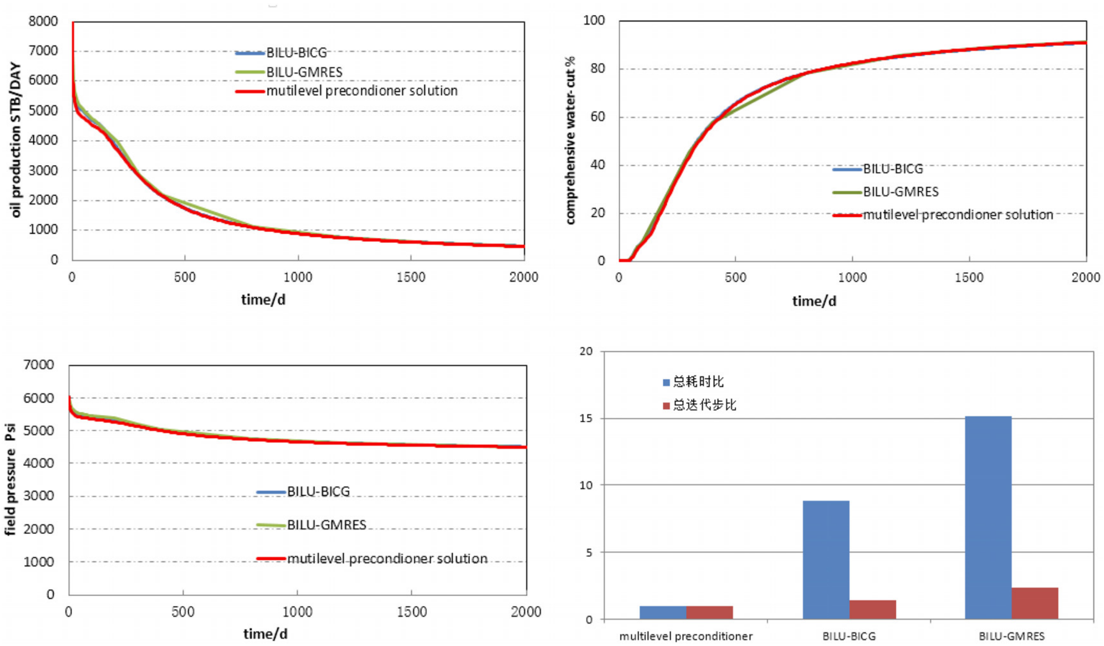

Figure 2—Comparison of the multilevel preconditioning solution method and the traditional algorithm

The comparison of the multilevel preconditioning solution method and the traditional algorithm was accomplished on the PC with Intel(R) Core(TM) i7-3520M CPU @ 2.90 GHz 2.90 GHz. The simulation of the production index curve such as daily oil production, comprehensive water cut and field pressure are shown in Fig.2. The results in Fig.2 indicate that the accuracies of the multilevel preconditioning solution method and the traditional algorithm are same, but the former accomplishes the calculation within an hour, which is much less than the later. All these illustrate that the multilevel preconditioning solution method is efficient for reservoir numerical simulation, especially for large-scale reservoir simulation.

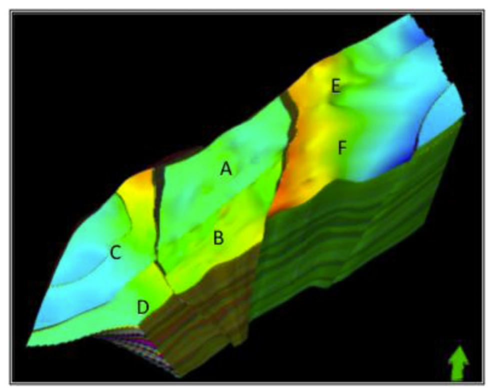

Figure 3—Geological structure map.

(2) Application in mature water-flooding oilfield

Another example is a real oilfield simulation which is a matured heterogeneous oil reservoir with six fault-blocks and each fault block has its own independent oil-water system (see Fig.3). The major production beds are divided into 42 single sand layers, which belong to two oil formations separated by a stable mudstone interlayer. The field is a medium porosity, medium permeability and high viscosity stanstone reservoir, in which the porosity is 22%, the average air permeability is 140 md and the crude oil viscosity is 36.1 mPa.s. Due to lack of natural energy, the main driving force is water injection during development. There are 242 wells in total in the over 30 years' development period.

In order to give an accurate description of the reservoir, we constructed a simulation model with 417,480 grid cells, the planar mesh size is 30m and there are 71 grids in X direction, 140 grids in Y direction and 42 grids in Z direction (according to the layer division). Non-equilibrium option is used to initialize and simulate the multi-contacts. At the same time, different relative permeability curves and PVT tables are applied to different blocks.

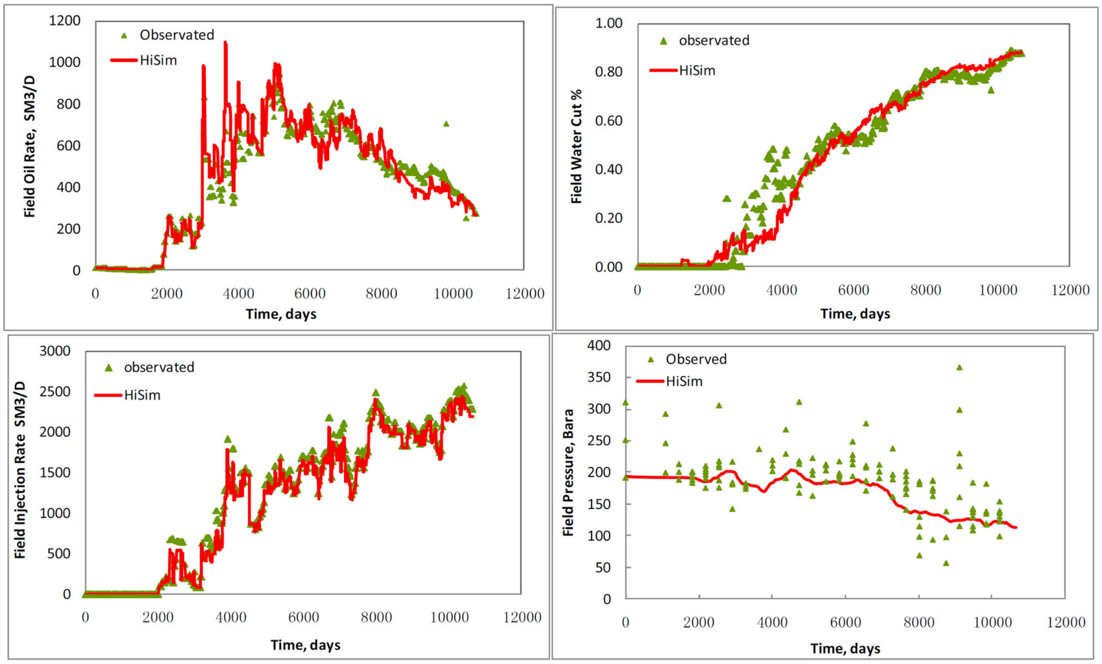

Figure 4—Comparisons of simulation results and observed data.

The simulation results of the multilevel preconditioning solution method (which is used in reservoir simulation software HiSim) are shown in Fig.4. By comparison with the field observed data, it can be obtained that the simulation results are exact.

Summary and Conclusions

A multilevel preconditioner for black oil simulation is developed in this paper which is the reasonable combination of AMG, ILU, Gauss-Seidel with downwind, and other iterative methods. The application results in a large-scale example and a real oilfield showed that the proposed preconditioning solution method is efficient and robust for highly heterogeneous and field-scale problems.

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