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Cost-Effective Parallel Reservoir Simulation on Shared Memory

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Abstract

Reservoir simulation is an important tool for petroleum engineers to predict oil production and optimize the management of oil fields. Nowadays, large-scaled reservoir simulations are required by oil industry to simulate complex geological models in order to obtain high resolution results. When the reservoir engineers design new production processes, it is necessary to run dozens of simulations to find optimal solutions. The speed of serial reservoir simulators can be a big challenge. Parallel reservoir simulators with fast computational methods should be invested.

In this paper, we developed cost-effective parallel reservoir simulation techniques, which include mathematic fluid models, FIM discretization method, multilevel preconditioners and its implementations on shared memory. Based on the mathematical characteristics of the pressure and saturation in black-oil model, a multilevel preconditioner is set up which includes algebraic multigrid method (AMG), incomplete LU factorization, et al. The preconditioner is implemented on share memory to speed up the efficiency of numerical simulation. Several numerical experiments such as the benchmark test two-phase and three-phase SPE10 simulation and a field-scale mature waterflooding reservoir simulation were performed to test the efficiency, robustness of the proposed simulation technique and its parallel speedup.

In the numerical experiments, the proposed reservoir simulation techniques can successfully simulate numerical cases with different reservoir properties on a desktop computer. Numerical results show that the proposed reservoir simulation techniques are quite efficient in solving the nonlinear PDEs and robust for highly heterogeneous field-scale problems.

Introduction

Reservoir numerical simulation, the necessary technique in oil field development, has played a key role in improving the oil production and enhancing the oil recovery [1-5]. In China, most oil reservoir are located in continental basins and many are characterized by serious heterogeneity, low permeability, high oil viscosity, and most of the developed oilfields have been into mature development period with high water-cut and recovery percent, thus there are lots of challenges in redeveloping those mature fields [6-7].

In order to improve oil yield economically, redevelopment is carried out in the mature reservoir, which includes fine geology description, fine and complex fluid flow model, and well pattern adjustment. As a result, more fine reservoir numerical models are set up, which have more grids, longer production history, more wells and measures, more complex oil and gas distribution than before. It greatly increases the workload and difficulty of reservoir numerical simulation and could be more time-consuming [8-9].

As a main part of reservoir simulation, the mathematical model is composed of several nonlinear PDEs with strong nonlinear, discontinuity and coupling, such as pressure PDEs and saturation PDEs. In general, especially for black-oil simulation model, the time of solving these PDEs accounts for more than 70%-80% of the simulation time. Thus, how to solve these nonlinear PDEs efficiently is the key to numerical simulation and many research achievements on this respect is obtained which can accelerate the computation [10-16].

In this paper, we develop a multilevel preconditioner for solving large-scale fully-implicit simulation of the black oil model and this effort has result in the new-generation simulator called HiSim (an in-house reservoir simulator, developed by RIPED). First, we review the mathematical model and its fully implicit discretization method. Then, we discuss a multilevel preconditioner based on a successive subspace correction framework and its implementation on shared memory. Finally, we perform numerical experiments to test efficiency and robustness of the multilevel preconditioner.

Reservoir Models

Reservoir models or mathematical models of fluid flows through porous media play an important role in developing oil/gas reservoirs. In China, most oil/gas reservoir are located in continental basins characterized by serious heterogeneity, and are developed by primary manner and secondary manner such as waterflooding [8, 17-18]. The development is under isothermal condition and so, the black-oil model is available to simulate the development.

In the black-oil model, there are three phases in the flow, which are water, oil and gas. We assume that the water phase do not exchange mass with other phases, and the hydrocarbon phases such as oil and gas exchange mass between them. Let o, g and w denote the oil phase, gas phase and water phase. Let p, S, ρ, K, kr, and μ denote the pressure, saturation, density, formation permeability, relatively permeability and viscosity, respectively. Ø is the porosity, Z is the depth and g is the acceleration due to gravity. Under isothermal condition, the black-oil model is composed of three mass conservation equations for oil, water, and gas components [7], which are,

Oil component:

$$\nabla\left[K\frac{K_{\mathrm{ro}}}{\mu_{\mathrm{o}}}\rho_{\mathrm{o,o}}\nabla\left(p_{\mathrm{o}}-g\rho_{\mathrm{o}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{o,o}}S_{\mathrm{o}}\right) \tag{1}$$

Water component:

$$\nabla\left[K\frac{K_{\mathrm{rw}}}{\mu_{\mathrm{w}}}\rho_{\mathrm{w}}\nabla\left(p_{\mathrm{w}}-g\rho_{\mathrm{w}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{w}}S_{\mathrm{w}}\right) \tag{2}$$

Gas component:

$$\nabla\left[K\frac{K_{\mathrm{ro}}}{\mu_{\mathrm{o}}}\rho_{\mathrm{g,o}}\nabla\left(p_{\mathrm{o}}-g\rho_{\mathrm{o}}Z\right)\right]+\nabla\left[K\frac{K_{\mathrm{rg}}}{\mu_{\mathrm{g}}}\rho_{\mathrm{g}}\nabla\left(p_{\mathrm{g}}-g\rho_{\mathrm{g}}Z\right)\right]=\frac{\partial}{\partial t}\left(\phi\rho_{\mathrm{g,o}}S_{\mathrm{o}}+\phi\rho_{\mathrm{g}}S_{\mathrm{g}}\right) \tag{3}$$

The phase saturations appear to satisfy the condition

$$S_o+S_g+S_w=1. \tag{4}$$

The pressure differences between phases can be characterized by the capillary pressures:

$$P_o-P_w=P_{cow},\quad P_g-P_o=P_{cgo}. \tag{5}$$

The properties of fluids in the conservation equations are functions of pressure and saturation. The density and viscosity of oil are functions of oil pressure and bubble pressure. The relatively permeability K_{rw}, K_{ro} and K_{rg} are functions of water and oil saturation S_w and S_o.

For the wells, there are other constraints such as the injection rate and the production rate for injectors and producers, respectively. When the bottom hole pressure P_{bh} cannot meet the constraint of flow rate, the well constraint automatically switches to the bottom hole pressure case.

For the reservoir, when pressure drops below bubble-point pressure (undersaturated state), the hydrocarbon phase splits into an oil phase and a gaseous phase at the thermodynamical equilibrium. In this case, P_o, S_w, and S_g are chosen as the primary variables, with the rest of the unknowns represented by the primary variables using the relations (4) and (5). On the other hand, if the gas phase does not exist (saturated state), R_s is used instead of S_g as a primary variable.

In this paper, conservative finite difference schemes with fully implicit method (FIM) are employed to discretize the black-oil model. The standard Newton linearization method and upstream-weighting finite difference spatial discretization (for details, see [9, Chapter 8]) are employed to solve the non-linear equations.

When combined with the cell-center finite difference method, in each Newton iteration of FIM, a fully coupled linear algebraic system must be solved. These linear systems (the Jacobian systems) often take the following form

$$Au=f \tag{6}$$

Preconditioner for FIM and Its Shared Memory Paradigm

Preconditioner

The Jacobian systems A in (6) are usually ‘large, sparse, nonsymmetric, and ill-conditioned’ [4]. It is composed of both reservoir variables and well variables. Therefore, A could be reformed as,

$$A = \begin{pmatrix} A_{\mathrm{R}} & A_{\mathrm{Rw}} \\ A_{\mathrm{wR}} & A_{\mathrm{w}} \end{pmatrix}$$

where subscripts R and W denote the reservoir and implicit well parts of the main variables, respectively; WR or RW denotes the variables connected with both the reservoir and the well.

The variables in A are related to the pressure or the saturation, so A_R can be decoupled as,

$$A_R = \begin{pmatrix} A_{pp} & A_{pS} \\ A_{Sp} & A_{SS} \end{pmatrix}$$

in which p denotes the pressure variables (oil pressure or well bottom whole pressure) and S denotes the saturation variables.

It is well-known that the equations describing the mass balance in terms of pressure unknowns P are mainly elliptic and the equations describing the mass balance in terms of saturation unknowns S are mainly hyperbolic [4, 19-21]. Therefore, in this paper, different preconditioners are employed for different variables.

For pressure variables, we use the algebraic multigrid (AMG) methods [7, 22-23] to solve the pressure block A_{PP}. In order to improve the performance of the AMG solver, we developed an approach that combines an iterative method with AMG and incompleted LU factorization (ILU) as preconditioner, which is designed as follow:

$$
\begin{aligned}
&① \; u^{k,1} = u^{k,0} + \mathrm{AMG}\,(f\text{-}Au^{k,0}),\\
&② \; u^{k,2} = u^{k,1} + \mathrm{ILU}(k)\,(f\text{-}Au^{k,1}),\\
&③ \; u^{k+1} = u^{k,2} + \mathrm{AMG}\,(f\text{-}Au^{k,2}),
\end{aligned}
\tag{7}
$$

where $u^{k,0}=u^{k}$.

In this paper, we use V-cycle multigrid which consists of setup stage and the solve stage, as shown,

Setup Stage:

$$A_l = P_l^T A_{l+1} P_l, \; l = L\text{-}1, \ldots, 0, \text{ and } A_L = \mathrm{A},$$

where P_l is the intergrid operator and A_l is the coarse grid matrix;

Solve Stage: ① Pre-smoothing: $u_l^1 = u_l^0 + S_l(f_l - A_l u_l^0)$

where S_l is the smoother, u_l^0 is the an initial guess;

② Coarse-grid correction:

a. $f_{l-1} = P_{l-1}^T(f_l - A_l u_l^1)$
b. If $l = 1$, $e_0 = A_0^{-1} f_0$; else, apply V-cycle multigrid for $A_{l-1}e_{l-1} = f_{l-1}$ with zero initial guess
c. $u_l^2 = u_l^1 + P_{l-1}e_{l-1}$

③ Post-smoothing: $u_l^3 = u_l^2 + S_l^T(f_l - A_l u_l^2)$

For saturation variables, we use block Gauss-Seidel method to solve the saturation block as variable $S = (S_w, S_g)$ has hyperbolic characteristics. In order to obtain better parallel speed-up, we use a block Gauss-Seidel method with multi-color ordering.

For well variables, the reservoir and well variables have strong coupling due to the high permeability around the perforations and different type of wells, the well equation and the reservoir part should be solved together. Compared with reservoir grids, the number of perforated well grid is very small and Integrating A_{ww} and the perforated grid-blocks in A_{rr} forms implicit well blocks are solved by direct method.

As shown above, a multilevel preconditioner to linear solver in reservoir numerical simulation is defined as

$$I - B_m A = (I - RA)(I - \Pi_p B_p \Pi_p^T A)(I - \Pi_s B_s \Pi_s^T A)(I - \Pi_w B_w \Pi_w^T A) \tag{8}$$

Shared Memory Paradigm

In order to improve the simulation efficiency, the multilevel preconditioner is implemented on share memory. Compared to message-passing implementations, the shared memory paradigm can greatly simplify the programming task in a multicore environment. In this paper, share memory paradigm is mainly implemented in AMG preconditioner, especially at the setup stage of AMG [4].

For the matrix A_{PP} as $A \in R^{n\times n}$, let $G_A\,(V, E)$ be the graph of the matrix A, where V is the set of vertices (i.e., unknowns) and E is the set of edges (i.e., connections that correspond to nonzero off-diagonal entries of A). Let C is coarse-level vertices and F is fine-level vertices, the index set of vertices V is split into two sets:

$$V = C \cup F \quad \text{and} \quad C \cap F = \varnothing$$

Let F^c is the map from F-vertices to C-vertices. The set of neighboring variables of i is defined as

$$N_i := \{ j \in V : A_{ij} \neq 0, j \neq i \}$$

The strong-connected variables is defined as

$$S_i(\theta) := \left\{ j \in N_i : -A_{ij} \geq \theta \max_{k \neq i}(-A_{ik}) \right\} \tag{9}$$

Let $P = (P_{ij_c}) \in R^{n \times n_c}$ is the standard prolongation (or interpolation) matrix, where its entries can be determined as follows:

$$P_{ij_c} = \begin{cases} -(A_{ij} + \sum\limits_{k \in D_i^{F,s} \setminus F_i} \dfrac{A_{ik}\hat{A}_{kj}}{\sum\limits_{m \in D_i^{C,s}} \hat{A}_{km}}) / (A_{ii} + \sum\limits_{k \in D_i^{w} \cup F_i} A_{ik}), \\ \qquad\qquad\qquad\qquad i \in F,\ j \in D_i^{C,s},\ j_c = F^C[j] \\ 1.0, \quad i \in C, j_c = F^C[i] \\ 0.0, \quad \text{otherwise.} \end{cases}$$

where P is sparse matrix. Let M_p is an auxiliary integer marker to quickly locate the column index of each non-zero entry, which is shown as,

$$M_P[j] := \begin{cases} J_{j_c},\ j \in D_i^{C,s}, \quad j_c = F^C[j] \\ -2 - i,\ j \in D_i^{F,s} \setminus F_i \\ -1, \qquad \text{otherwise} \end{cases} \tag{10}$$

where J_{jc} is the position of P_{ij_c} entry in the column index array of the storage of P. In shared memory paradigm, an integer array is allocated for the marker M_P for each thread. The length of each M_P is n, and the total length of M_P of all threads is then $N_T \times n$ where N_T is the total number of threads.

Benchmark Test

The parallel reservoir simulation was tested on a suite of 4 test problems, three from SPE benchmark test and one from field test.

SPE 10 Test

This benchmark test uses the model from the Tenth SPE Comparative Solution Project (SPE 10) [4, 7, 25], which was designed to predict the performance of water flooding in a highly heterogeneous black-oil reservoir with simple geometry described by a fine-scale (60×220×85=1,122,000) regular Cartesian geological model. The reservoir in SPE 10 test is 3,657.6 m in depth with the pressure of 41.37 MPa. The permeability is from 0.00066×10^{-3} μm^2 to 20000×10^{-3} μm^2 with the average of 364×10^{-3}μm^2(see Fig.1). The porosity is from 0.0001 to 0.5 with the average of 0.1749 (see Fig.1).

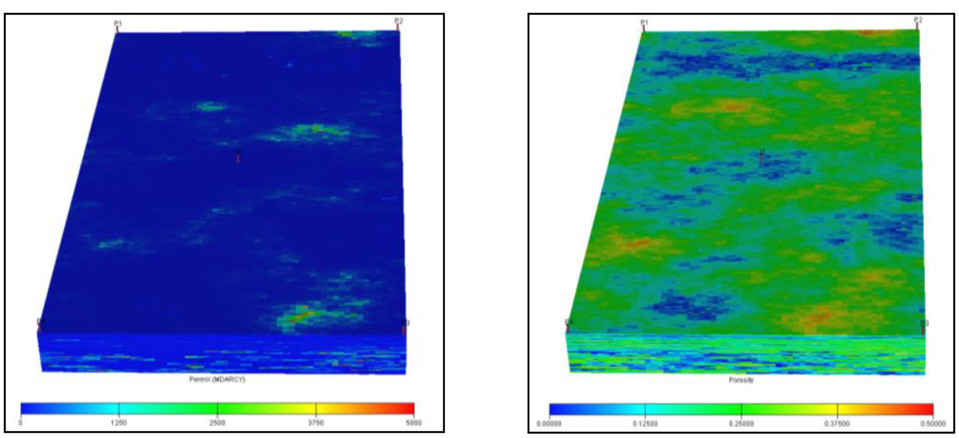

Figure 1—Permeability (Left) and Porosity (Right) of SPE 10 Benchmark

In this paper, three models of SPE 10 and its variation were tested, which are two-phase model with 1,122,000 grid cells (Case 1), three-phase model with 1,122,000 grid cells (Case 2) and two-phase model with 8,976,000 grid cells (Case 3). In the each of the three models, there is one injector at the center of the field and four producers, one at each of the four corners. The total simulation time in each model is 2,000 days.

For Case 1 two-phase problem, the simulation was performed on a Dell desktop PC with Intel Core i7 3.33 GHz CPU (4 cores) and 8 GB DDR3 RAM by an in-house reservoir simulator HiSim with preconditioner algorithm discussed in this paper. The problem has 1.1M grid cells, 2.2M degrees of freedom. The total wall time for one single thread is about 42 minutes, and the linear solver takes more than 80% of the total simulation time. The numerical results was compared with the benchmark results by Landmark, Geoquestand Streamsim [4, 25], see Figure 2 and 3 which show that the oil rate, reservoir pressure, and water-cut are good agreement with the reported results using other simulators.

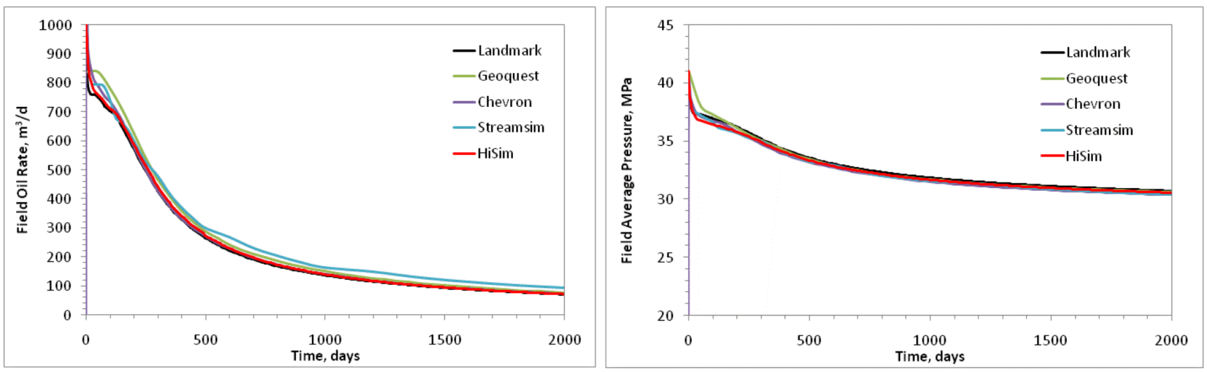

Figure 2—Comparison of field oil rate (Left) and reservoir pressure (Right) by different simulators

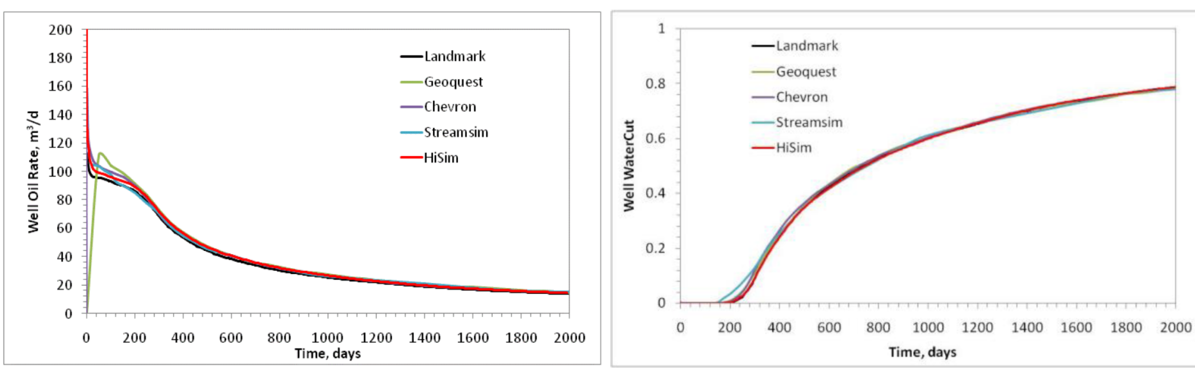

Figure 3—Comparison of oil rate (Left) and water-cut (Right) of Producer 1 by different simulators

Table 1 reported the algorithm efficiency and parallel speed-up, which included Newton steps, linear iterations and total wall time. For 2 threads on the 4-core i7 CPU, the wall time is about 32 minutes with parallel speed-up of 1.29 and speed-up in linear solver of 1.38. For 4 threads on the 4-core i7 CPU, the wall time is about 30 minutes with parallel speed-up of 1.37 and speed-up in linear solver of 1.5.

Table 1—Performance of the parallel algorithm for SPE10 (Case1: 2-phase, 1.1M)

Number of threads N_T	Total Newton steps	Average linear iterations	Wall time minutes	Linear solver time minutes	Parallel speed-up (Linear solver)
1	254	9.87	42.17	35.36	1.00
2	262	10.19	32.60	25.61	1.38
4	260	10.00	30.45	23.23	1.52

For Case 2 three-phase problem, the simulation was performed on HP Z800 server with two Intel Xeon X5590 CPU (4 cores) and 24GB DDR3 RAM also by HiSim. The problem has 1.1M grid cells, 3.3M degrees of freedom (geology model is the same as that of Case 1 and there are oil, water and gas phases). The total wall time for one single thread is about 7.33 hours and linear solvers take 5.2 hours. The parallel speed-up in linear solver is 1.31 and 1.66 for 2 and 4 threads, respectively, see Table 2.

Table 2—Performance of the parallel algorithm for SPE10 (Case2: 3-phase, 1.1M)

Number of threads N_T	Total Newton steps	Average linear iterations	Wall time hours	Linear solver time hours	Parallel speed-up (Linear solver)
1	1510	9.25	7.33	5.20	1.00
2	1532	9.44	5.46	3.98	1.31
4	1391	8.67	4.50	3.14	1.66

For Case 3 two-phase large simulation problem, the simulation was performed on HP Z800 server but with 96GB DDR3 RAM also by HiSim. The simulation has 8.9 M grid cells and 27 M degrees of freedom, which is 8 copies of Case 1. In Case 3, the parallel speed-up in linear solver is 1.22 and 1.34 for 2 and 4 threads, respectively, see Table 3.

Table 3—Performance of the parallel algorithm for SPE10 (Case3: 2-phase, 8.9M)

Number of threads N_T	Total Newton steps	Average linear iterations	Wall time hours	Linear solver time hours	Parallel speed-up (Linear solver)
1	759	2.45	10.36	6.98	1.00
2	765	2.49	9.55	5.73	1.22
4	764	2.46	9.38	5.22	1.34

Field Test

The field test uses the model from a matured waterflooding reservoir located in Baohai Bay Basin with 6 fault-blocks which is designed to predict the performance of water flooding in a highly heterogeneous three-phase black-oil reservoir described by a fine-scale (71×140× 42=417,480) regular Cartesian geological model. There are 242 wells in the model and the total simulation time is 30 years.

The model has 6 faults with 6 initial water-oil interfaces (see Fig.4). The depth of the reservoir is 1750 m with reservoir pressure of 19.5MPa and saturation pressure of 5.98MPa. Oil density at the standard condition is 0.88 g/cm^3 and oil viscosity at the reservoir condition is 24.47 mPa·s. During the development, in the most part of area there are only two phases (oil and water) while nearby the wellbore of producer there is gas phase due to low bottomhole pressure. Therefore, in this test the three-phase model is used. The average permeability is $140\times10^{-3}\mu m^2$ and the average porosity is 22%. Figure 5 gives the oil saturation after 30-year's waterflooding.

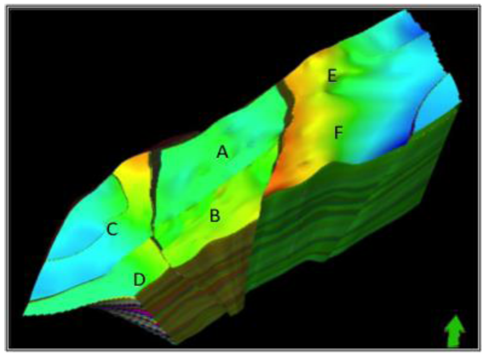

Figure 4—Geological Structure Map

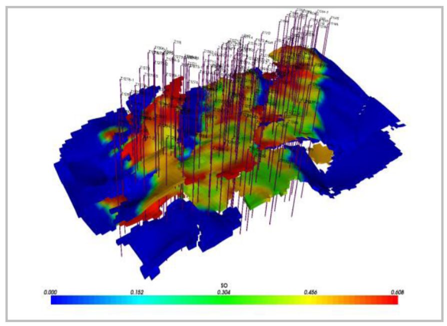

Figure 5—Oil Saturation after 30-year's Waterflooding

The wall time for one simulation run takes 153 minutes (1,425 time steps and 5,839 Newton iterations in total) on the same computer platform used in the previous numerical test (still using only one CPU core as Case 2). Figure 6 shows comparisons between the simulation results (field oil rate, liquid rate, water cut, and injection rate) of HiSim and the field observed data. The curves of field oil rate, field fluid rate, field injection rate, and well water-cut in the figures show good agreement with the field observed data.

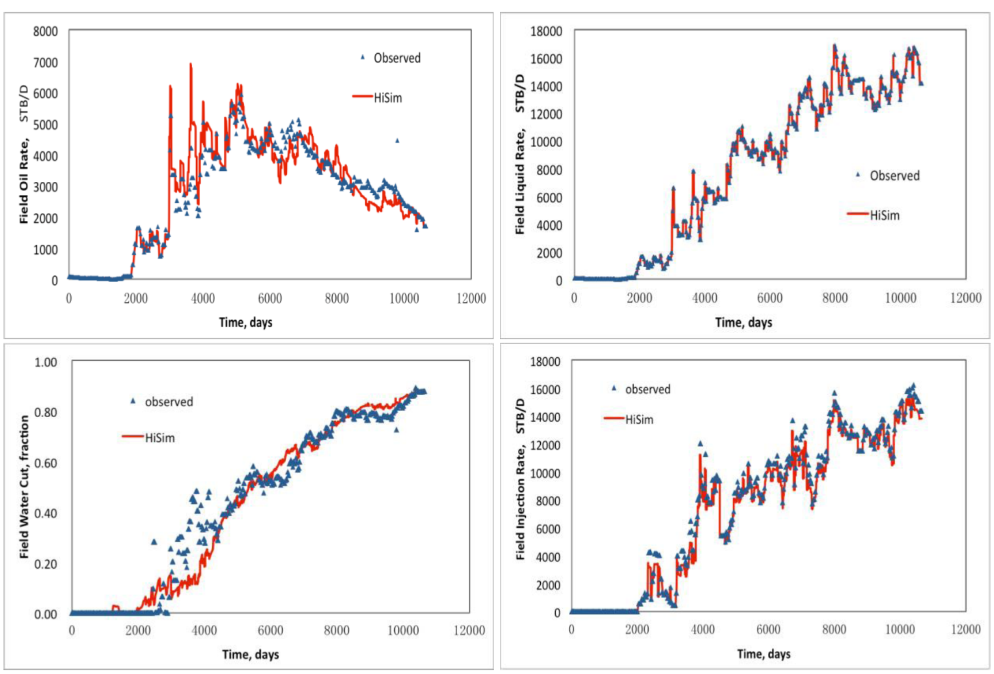

Figure 6—Comparisons of simulation results and field observed data

To compare the parallel speed-up of the proposed preconditioner, we run the simulation with 2, 4 and 8 threads, repectively. The numerical results (total wall time in minutes, linear solver time in minutes and parallel speed-up) are reported in Table 4. The parallel speed-up is 1.53 when using 8 threads. And speed-up of the solver part is about 2.1 folds.

Table 4—Performance of the parallel algorithm for Field Test (3-phase, 0.4M)

Number of OpenMP threads NT	Wall time minutes	Linear solver time minutes	Linear solver speed-up (Linear solver)
1	153	104	1.00
2	109	73	1.43
4	101	52	2.01
8	100	49	2.10

Summary and Conclusions

Reservoir simulation is an important tool to guide oil production and it has played a key role in improving the oil production and enhancing the oil recovery in petroleum industry. Nowadays, large-scaled reservoir simulations are required to simulate complex geological models in order to obtain high resolution results.

Cost-efficient parallel reservoir simulation techniques on shared memory were developed in the paper, in which a practical and efficient multilevel preconditioner was set up to solve the large sparse linear systems arising from black oil model discretized by fully implicit method. The multilevel preconditioner is composed of algebraic multigrid method (AMG), incomplete LU factorization, and et al.

Numerical experiments of benchmarks and field-scale mature waterflooding reservoir simulation performed show that the proposed reservoir simulation techniques in the paper are quite efficient in solving the nonlinear PDEs and robust for highly heterogeneous field-scale problems. The preconditioner and its implementation on shared memory could speed up the efficiency of numuerical simulation. The performance of the proposed techniques is reasonably good for relatively large reservoir simulation models.

Acknowledgements

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