Calculus

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This document is based on the contents of the **calculus**.

I started this work to establish a solid foundation for my research activities.

I believe that theoretical understanding is essential in any field, so I studied to build background knowledge alongside basic research.

Since studying alone and leaving it behind leads to nothing remaining, I decided to organize my learning into this document.

This work was done based on the learning attitude taught to me by **Jung Byung-ho** and **Jung Byung-hoon**.

Although the contents may differ, the approach to studying concepts, proofs, and logical reasoning reflects the mindset and methods I learned from them.

I am truly grateful for all they have taught me.

Textbook

Calculus: Early Transcendentals for Scientists and Engineers
Metric Edition
James Stewart

I. Functions and Models

Page 68: Problem 69
Prove that:

$$\cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

Given:

$$\text{Domain of } x \in [-1,1],$$
$$\text{Range of } \sin^{-1}(x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Therefore, since cosine is non-negative on this interval:

$$\begin{aligned}\cos(\sin^{-1}(x)) &= \sqrt{1-\sin^2(\sin^{-1}(x))} \\ &= \sqrt{1-x^2}\end{aligned}$$

Alternative geometric approach (triangle):

$$\sin\theta = x, \quad \cos\theta = y$$
$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow \cos\theta = \sqrt{1-x^2}$$
$$\therefore \cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

Page 68: Problem 70

$$\begin{aligned}\tan(\sin^{-1}(x)) &= \sqrt{\sec^2(\sin^{-1}(x)) - 1} \\ &= \sqrt{\frac{1}{1-\sin^2(\sin^{-1}(x))} - 1} \\ &= \sqrt{\frac{1}{1-x^2} - 1} \\ &= \sqrt{\frac{x^2}{1-x^2}} \\ &= \frac{x}{\sqrt{1-x^2}}\end{aligned}$$

Page 68: Problem 71

$$\begin{aligned}\sin(\tan^{-1}(x)) &= \sqrt{1-\cos^2(\tan^{-1}(x))} \\ &= \sqrt{1-\frac{1}{\sec^2(\tan^{-1}(x))}} \\ &= \sqrt{1-\frac{1}{1+\tan^2(\tan^{-1}(x))}} \\ &= \sqrt{1-\frac{1}{1+x^2}} \\ &= \frac{x}{\sqrt{1+x^2}}\end{aligned}$$

Page 75 : Example 3
Given:

$$f_0(x) = \frac{x}{x+1}, \quad f_{n+1} = f_0(f_n)$$

(1) First Approach:

$$f_n = \frac{x}{nx+1}$$
$$f_{n+1} = f_0(f_n) = \frac{f_n}{f_n+1} = \frac{\frac{x}{nx+1}}{\frac{x}{nx+1}+1} = \frac{x}{(n+1)x+1}$$

(2) Inductive Approach:

$$f_n = \frac{x}{nx+1}$$

Base case $n=1$:

$$f_1 = f_0(f_0) = \frac{f_0}{f_0+1} = \frac{x}{x+2}$$

Inductive step: assume $f_k = \frac{x}{kx+1}$, then

$$f_{k+1} = f_0(f_k) = \frac{f_k}{f_k+1} = \frac{\frac{x}{kx+1}}{\frac{x}{kx+1}+1} = \frac{x}{(k+1)x+1}$$

$$\therefore \text{by induction,} \quad f_n = \frac{x}{nx+1}$$