#32
Improved Hamiltonian Simulation

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Problem

- To simulate the dynamics of a physical system, we need exponentiated operators.
- **Want**: A circuit for e^{iHt}
- **Given**: $H = H_1 + H_2$, circuits for $e^{iH_1 t}$ and $e^{iH_2 t}$, $[H_1, H_2] \neq 0$
- Because $[H_1, H_2] \neq 0$, $e^{iH_1 t} e^{iH_2 t} = e^{iHt} + \boldsymbol{\mathcal{O}(t^2)}$
- $\left(e^{\frac{iH_1 t}{\mathrm{M}}} e^{\frac{iH_2 t}{\mathrm{M}}}\right)^{\mathrm{M}} = e^{iHt} + \mathcal{O}\left(\frac{t^2}{M}\right)$
- Another way proposed in arXiv:1907.11679 [quant-ph]:

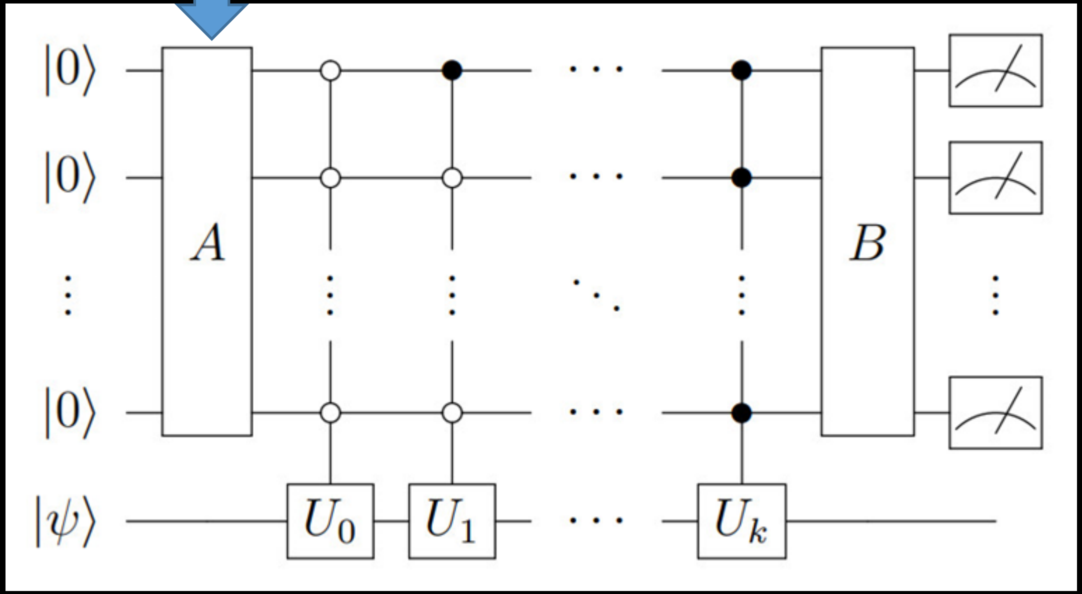
$$U_{\vec{k}}(\Delta) = \sum_{j=1}^{M} a_j U_2^{k_j}\left(\frac{\Delta}{k_j}\right) = e^{-iH\Delta} + \mathcal{O}(\Delta^{2m+1}), \quad (3)$$

We need a way to do operator additions

Our work

- Referring to arXiv:1202.5822 [quant-ph], we implemented a circuit to construct $U = \sum a_j U_j$, given a_j and U_j

$\boldsymbol{a_j}$ Map a_j to amplitudes of ancilla states



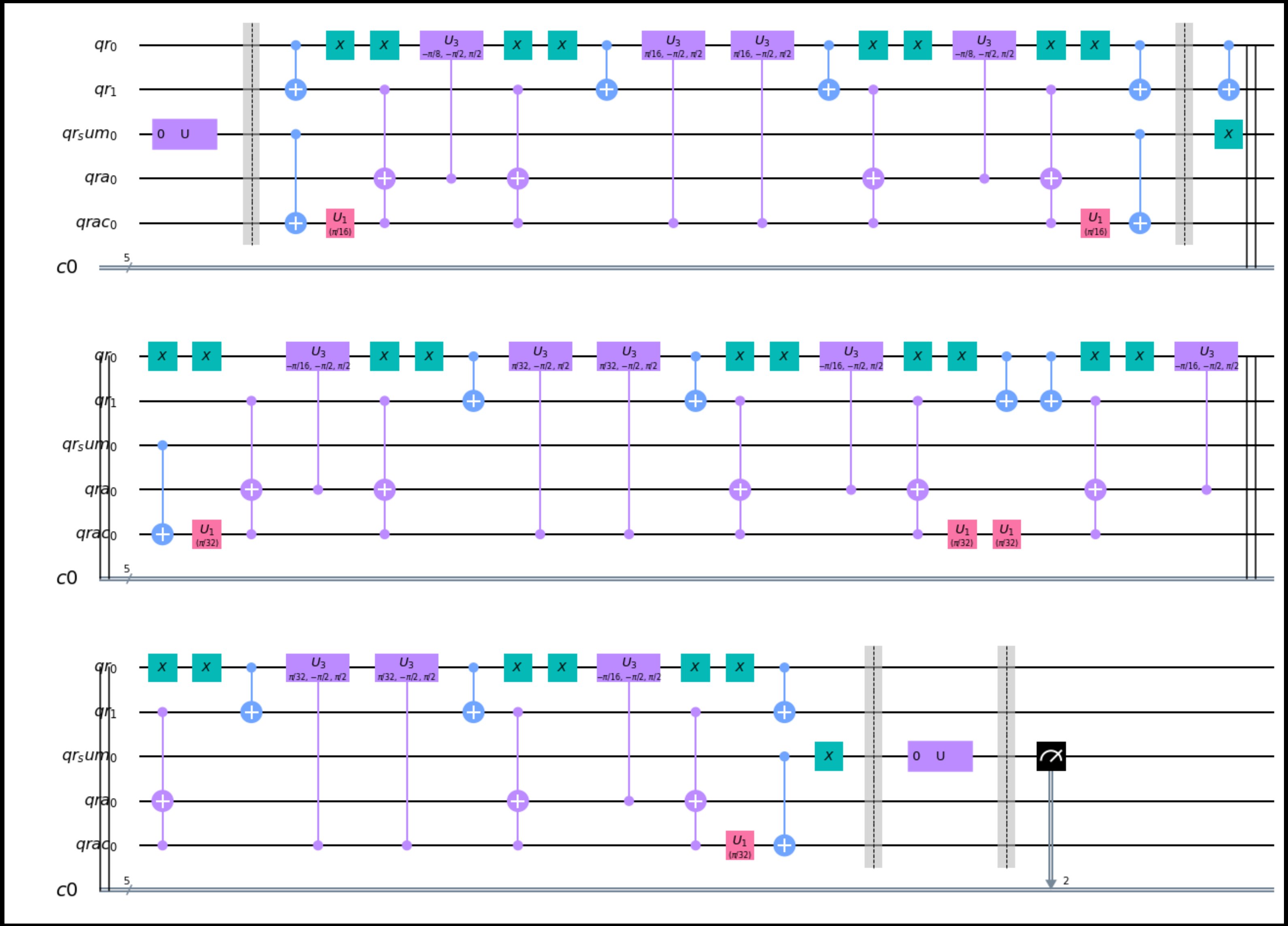
Discard the result if the measurement outcome is not 0 (postselection)

If the measurement outcome is 0,
The circuit $= |00\ldots0\rangle U|\psi\rangle$

Our work

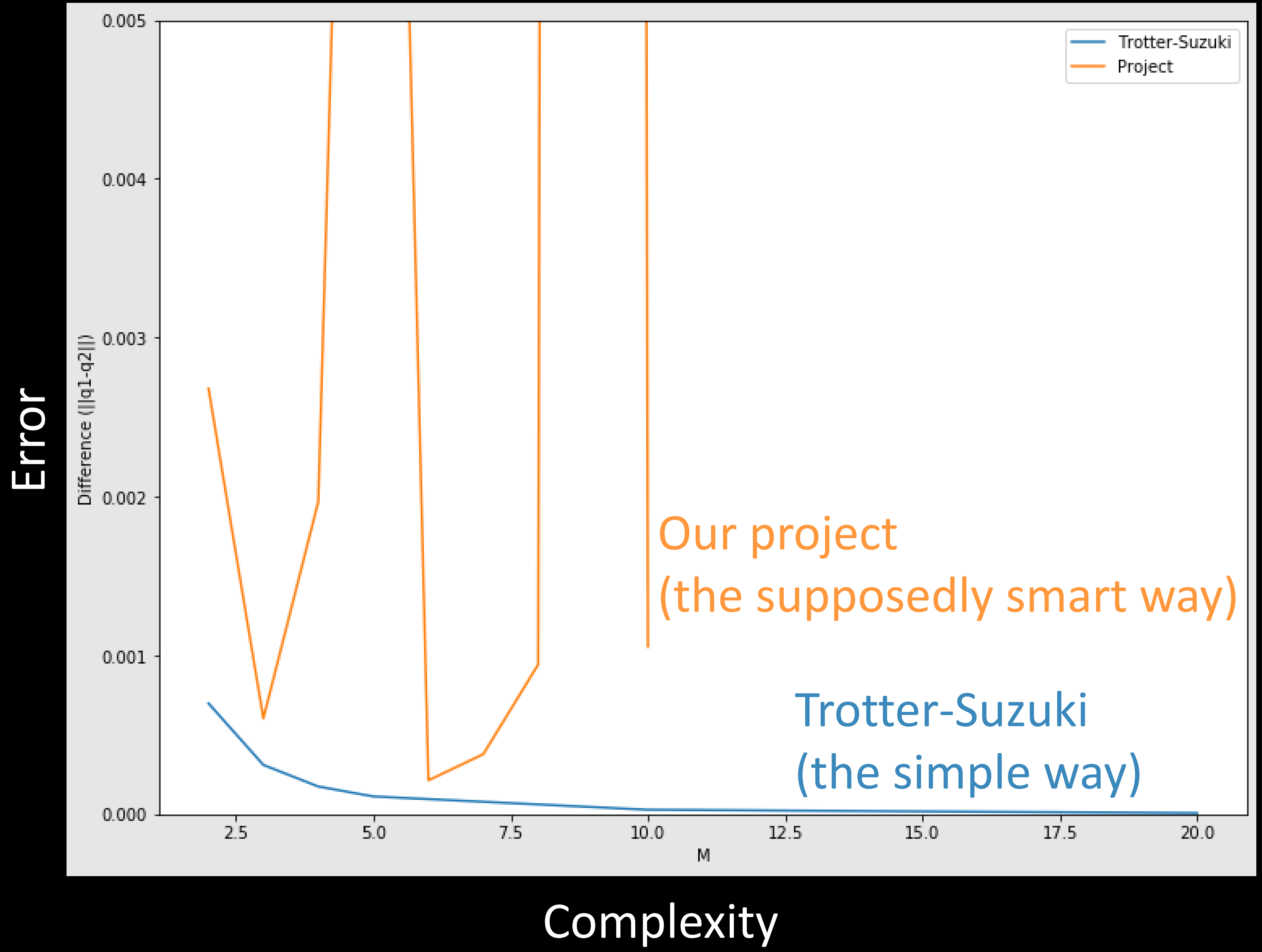
- Circuit implementation of e^{iHt} on qr[0], qr[1]
- $H =$

$$
\begin{pmatrix} \pi/_8 & 0 & 0 & 0 \\ 0 & \pi/_8 & 0 & 0 \\ 0 & 0 & \pi/_8 & 0 \\ 0 & 0 & 0 & \pi/_8 \end{pmatrix} +
\begin{pmatrix} 0 & \pi/_8 & 0 & 0 \\ \pi/_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi/_8 \\ 0 & 0 & \pi/_8 & 0 \end{pmatrix} +
\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \pi/_8 & 0 \\ 0 & \pi/_8 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$



Next step

- Debug



Next next step

- Combine with algorithms to decompose Hamiltonians (#20)
    - Efficient general algorithm for Hamiltonian simulation.

H

(#20)

$$H = H_1 + H_2 + \cdots + H_m$$

(#20)

$$e^{iH_1 t}, e^{iH_2 t}, \ldots, e^{iH_m t}$$

$$U(\Delta) = e^{\frac{iH_1\Delta}{2}} \ldots e^{\frac{iH_{m-1}\Delta}{2}} e^{iH_m\Delta} e^{\frac{iH_{m-1}\Delta}{2}} \ldots e^{\frac{iH_1\Delta}{2}}$$

(#32)

$$e^{iHt} = \sum a_j U^{k_j}\left(\frac{t}{k_j}\right)$$

(#32)

e^{iHt}