Chapter 03
Digital Logic System

Boolean Algebra

Inventor : George Boole

Book : An Investigation of the Laws of Thought

Theory : Logic>>>>>> Mathematical Expression

Basic : ***conjunction*** **$\wedge$,**

disjunction **$\vee$,**

negation **$\neg$**

Constants/ Variable : 0 or 1

Logical Addition (OR, *conjunction* ∧)

0	+	**0**	=	**0**
0	+	**1**	=	**1**
1	+	**0**	=	**1**
1	+	**1**	=	**1**

Logical Multiplication (AND, *disjunction* ∨)

0	×	**0**	=	**0**
0	×	**1**	=	**0**
1	×	**0**	=	**0**
1	×	**1**	=	**1**

Logical Conversion (NOT, *negation* ¬)

A	**NOT A =** $\overline{A}$
0	**1**
1	**0**

Commutative Law

(a) $A + B = B + A$

(b) $A\,B = B\,A$

Associate Law

(a) $(A + B) + C = A + (B + C)$

(b) $(A\,B)\,C = A\,(B\,C)$

Distributive Law

(a) $A\,(B + C) = A\,B + A\,C$

(b) $A + (B\,C) = (A + B)\,(A + C)$

Identity Law

(a) $A + A = A$

(b) $A\,A = A$

(c) $AB + A\overline{B} = A$

(d) $(A+B)(A+\overline{B}) = A$

Redundance Law

(a) $A + A\,B = A$

(b) $A\,(A + B) = A$

(c) $A + \overline{A}\,B = A + B$

(d) $A\,(\overline{A} + B) = A\,B$

A	+	A	=	A
A	+	1	=	1
A	+	0	=	A
A	+	$\overline{A}$	=	1
$\overline{A}$	+	1	=	1
$\overline{A}$	+	0	=	0

A	*	A	=	A
A	*	1	=	1
A	*	0	=	A
A	*	$\overline{A}$	=	1
$\overline{A}$	*	1	=	1
$\overline{A}$	*	0	=	0

$$\overline{\overline{A}} = A$$

De Morgan's Law

Two Variables	i)	$\overline{A+B}$	=	$\overline{A}*\overline{B}$
	ii)	$\overline{A*B}$	=	$\overline{A}+\overline{B}$
Three Variables	i)	$\overline{A+B+C}$	=	$\overline{A}*\overline{B}*\overline{C}$
	ii)	$\overline{A*B*C}$	=	$\overline{A}+\overline{B}+\overline{C}$

Proof of $\overline{A+B}$ **=** $\overline{A} * \overline{B}$

L.H.S = $\overline{A+B}$ =

A	B	A+B	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

R.H.S = $\overline{A} * \overline{B}$ =

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} * \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

Proof of $\overline{A*B}$ = $\overline{A}+\overline{B}$

L.H.S = $\overline{A*B}$ =

A	B	A*B	$\overline{A*B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

R.H.S = $\overline{A}+\overline{B}$ =

A	B	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Proof of $\overline{A+B+C} = \overline{A} * \overline{B} * \overline{C}$

L.H.S $= \overline{A+B+C} =$

A	B	C	A+B+C	$\overline{A+B+C}$
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

R.H.S $= \overline{A} * \overline{B} * \overline{C} =$

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A} * \overline{B} * \overline{C}$
0	0	0	1	1	1	1
0	0	1	1	1	0	0
0	1	0	1	0	1	0
0	1	1	1	0	0	0
1	0	0	0	1	1	0
1	0	1	0	1	0	0
1	1	0	0	0	1	0
1	1	1	0	0	0	0

Proof of $\overline{A*B*C} = \overline{A}+\overline{B}+\overline{C}$

L.H.S = $\overline{A*B*C}$ =

A	B	C	A*B*C	$\overline{A*B*C}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	0

R.H.S= $\overline{A}+\overline{B}+\overline{C}$ =

A	B	C	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A}+\overline{B}+\overline{C}$
0	0	0	1	1	1	1
0	0	1	1	1	0	1
0	1	0	1	0	1	1
0	1	1	1	0	0	1
1	0	0	0	1	1	1
1	0	1	0	1	0	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

Logic Gate

1. Electronic Circuit

2. Implement Boolean functions practically

3.

Value	Voltage Range
0	+0V to +0.8 V
1	+2V to +5V
Not Assigned	0.8V to +2V

Types of Logic gates

Logic Gates are classified in two categories. These are

1. Basic Logic Gates

a) NOT Logic Gate (Logical Conversion)

b) AND Logic Gate (Logical Multiplication)

c) OR Logic Gate (Logical addition)

2. Combined Logic Gates

a) NAND Logic Gate (AND + NOT)

b) NOR Logic Gate (OR + NOT)

c) XOR Logic Gate (AND +OR+ NOT)

d) XNOR Logic Gate (AND +OR+ NOT)

Basic Logic Gates

NOT gate

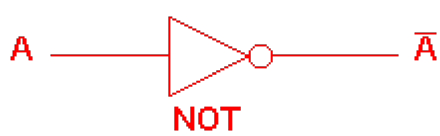
ইহা একটি Basic Logic গেইট।এর একটিমাত্র Input এবং একটিমাত্র Output আছে। ইহার দ্বারা Logical Conversion করা হয়।

যদি A Input হয়, তবে Output হবে X=NOT A= $\overline{A}$ ।

Truth Table

INPUT	OUTPUT
A	X= $\overline{A}$
0	1
1	0

Symbol



Result discussion:

Input 1 হলে Output 0 হবে। অন্যথায় Input 0 হলে Output 1 হবে।

AND gate

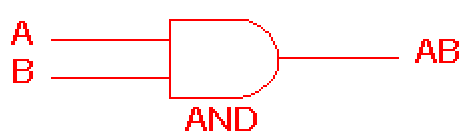
ইহা একটি Basic Logic গেইট ।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহার দ্বারা Logical Multiplication করা হয়।

যদি A, B Input হয়, তবে Output হবে X=A AND B = A.B ।

Truth Table

INPUT		OUTPUT
A	B	X=A.B
0	0	0
0	1	0
1	0	0
1	1	1

Symbol



Result discussion:
সকল Input 1 হলে Output 1 হবে। অন্যথায় Output 0 হবে।

OR gate

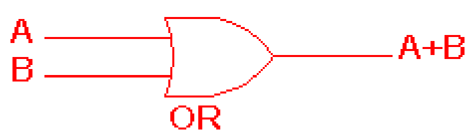
ইহা একটি Basic Logic গেইট ।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহার দ্বারা Logical Addition করা হয়।

যদি A, B Input হয়, তবে Output হবে X=A OR B = A+B।

Truth Table

INPUT		OUTPUT
A	B	X=A+B
0	0	0
0	1	1
1	0	1
1	1	1

Symbol



Result discussion:
এক বা একাধিক Input 1 হলে Output 1 হবে। অন্যথায় Output 0 হবে।

Combined Logic Gates

NAND gate

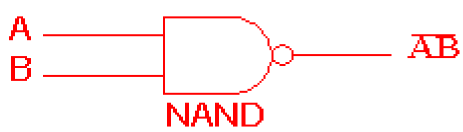
ইহা একটি Combined Logic গেইট।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহা AND ও NOT Logic গেইট দ্বারা গঠিত।

যদি A, B Input হয়, তবে Output হবে X=NOT (A AND B) = $\overline{AB}$ ।

Truth Table

INPUT		OUTPUT
A	B	X= $\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

Symbol



Result discussion:
এক বা একাধিক Input 0 হলে Output 1 হবে। অন্যথায় Output 0 হবে।

NOR gate

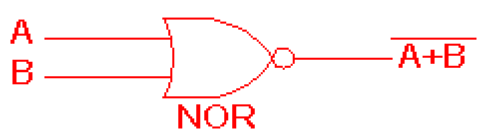
ইহা একটি Combined Logic গেইট ।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহা OR ও NOT Logic গেইট দ্বারা গঠিত।

যদি A, B Input হয়, তবে Output হবে X=NOT (A OR B) = $\overline{A+B}$ ।

Truth Table

INPUT		OUTPUT
A	B	X= $\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Symbol



Result discussion:

সকল Input 0 হলে Output 1 হবে। অন্যথায় Output 0 হবে।

X-OR gate

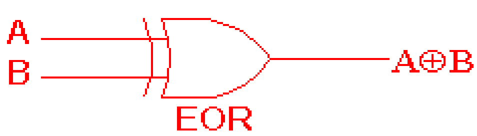
ইহা একটি Combined Logic গেইট।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহা AND, OR ও NOT Logic গেইট দ্বারা গঠিত। একে Excusive OR বলা হয।

যদি A, B Input হয়, তবে Output হবে X=A XOR B = $A \oplus B = \overline{A}B + A\overline{B}$ ।

Truth Table

INPUT		OUTPUT
A	B	X= $A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Symbol



Result discussion:
Input একই না (আলাদা) হলে Output 1 হবে। অন্যথায় Output 0 হবে।

X-NOR gate

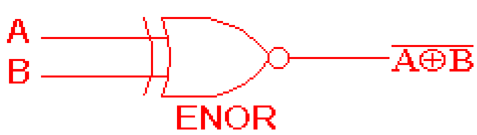
ইহা একটি Combined Logic গেইট।এর দু ইবা ততোধিক Input এবং একটিমাত্র Output আছে। ইহা AND, OR ও NOT Logic গেইট দ্বারা গঠিত। একে Excusive NOR বলা হয।

যদি A, B Input হয়, তবে Output হবে X=NOT (A XOR B) = $\overline{A \oplus B} = AB + \overline{A}\,\overline{B}$ ।

Truth Table

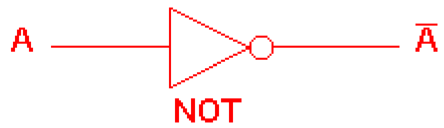
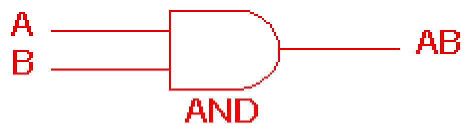
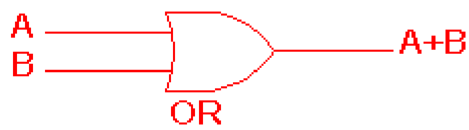
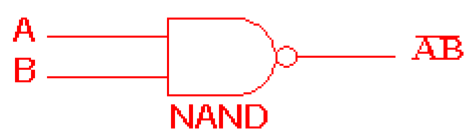
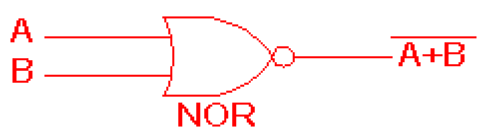
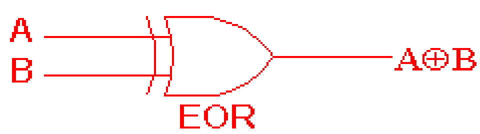
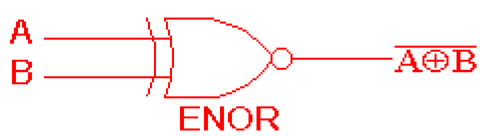
INPUT		OUTPUT
A	B	X= $A \oplus B$
0	0	1
0	1	0
1	0	0
1	1	1

Symbol



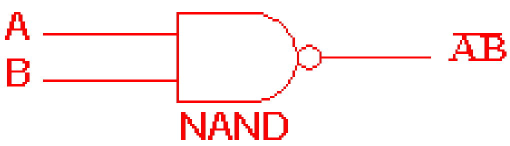
Result discussion:
সকল Input একই হলে Output 1 হবে। অন্যথায় Output 0 হবে।

Logic Gate Summary

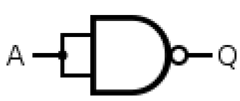
SL#	Logic Gate	Input	Symbol	Output	Remark
1	NOT	A		X= $\overline{A}$	Only 01 Input
2	AND	A,B		X= A+B	2 or more Input
3	OR	A,B		X= A.B	
4	NAND	A,B		X= $\overline{AB}$	
5	NOR	A,B		X= $\overline{A+B}$	
6	XOR	A,B		X = $A\oplus B = \overline{A}B + A\overline{B}$	
7	XNOR	A,B		X= $\overline{A\oplus B} = AB + \overline{A}\,\overline{B}$	

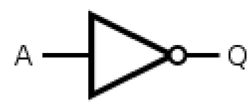
Universality NAND Logic

NAND gates are so-called "universal gates" that can be combined to form any other kind of logic gate.



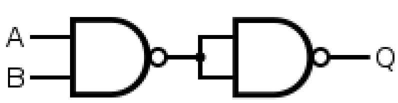
NAND gate দ্বারা NOT gate বাস্তবায়ন

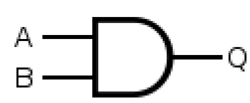




Q= $\overline{A*A} = \overline{A}$

NAND gate দ্বারা AND gate বাস্তবায়ন



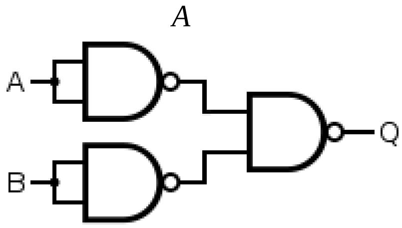


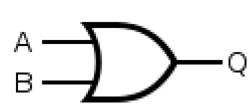
$\overline{A*B}$

Q= $\overline{\overline{(A*B)}*\overline{(A*B)}} = \overline{A*B}$

NAND gate দ্বারা OR gate বাস্তবায়ন

$\overline{A}$



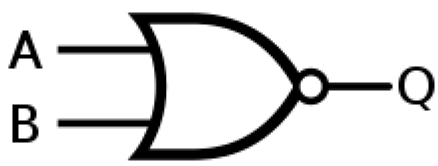


$\overline{B}$

Q= $\overline{\overline{A}*\overline{B}} = \overline{\overline{A}}+\overline{\overline{B}} = A+B$

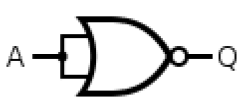
Universality Of NOR logic

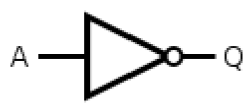
NOR gates are so-called "universal gates" that can be combined to form any other kind of logic gate.



$Q=\overline{A+B}$

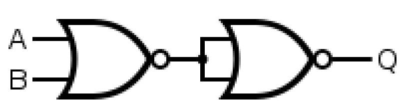
NOR gate দ্বারা NOT gate বাস্তবায়ন

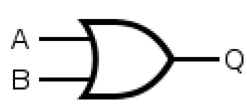




$Q=\overline{A+A}=\overline{A}$

NOR gate দ্বারা OR gate বাস্তবায়ন



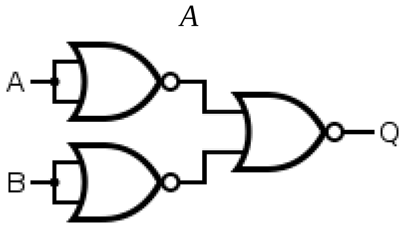


$\overline{A+B}$

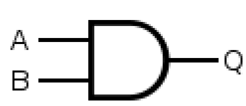
Q= $\overline{\overline{A+B}+\overline{A+B}}=\overline{\overline{A+B}}$=A+B

NOR gate দ্বারা AND gate বাস্তবায়ন

$\overline{A}$



$\overline{B}$



Q=$\overline{\overline{A}+\overline{B}}$**=**$\overline{\overline{A}}*\overline{\overline{B}}$**=AB**

Half Adder

ইহা Binary যোগের বর্তনী ।এর দুইটি Input এবং দুইটি Output আছে। ইহা AND, OR ও NOT Logic গেইট দ্বারা গঠিত।

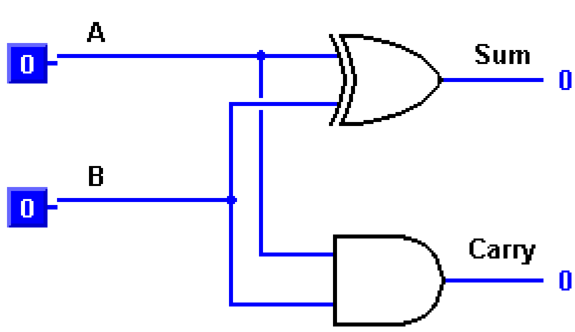
যদি A, B Input হয়, তবে Output হবে Sum(S) ও Carry(C) ।

Truth Table

INPUTS		OUTPUTS	
A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

SUM = $\overline{A}B + A\overline{B} = A \oplus B$

CARRY = AB



Full Adder

ইহা Binary যোগের বর্তনী।এর তিনটি Input এবং দুইটি Output আছে। ইহা AND, OR ও NOT Logic গেইট দ্বারা গঠিত।

যদি A, B,C Input হয়, তবে Output হবে Sum(S) ও Carry(C_{out}) ।

Truth Table

Input			Output	
A	B	C	SUM	CARRY (C_{OUT})
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

SUM $= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

$= \bar{A}(\bar{B}C + B\bar{C}) + A(\bar{B}\bar{C} + BC)$

$= \bar{A}(B \oplus C) + A(\overline{B \oplus C})$

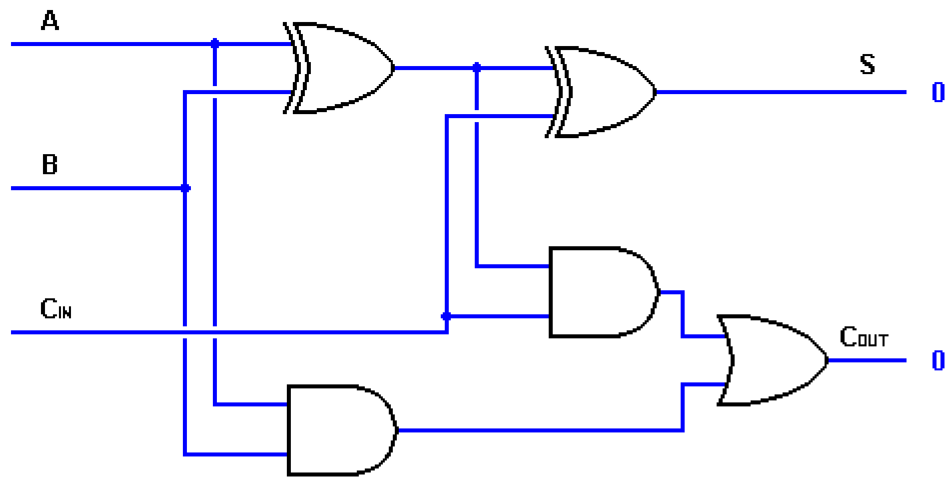
$= A \oplus (B \oplus C)$

$= A \oplus B \oplus C$

CARRY (C_{OUT}) $= \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= C(\bar{A}B + A\bar{B}) + AB(\bar{C} + C)$

$= C(A \oplus B) + AB$


A
B
CIN
S
0
COUT
0

Decoder

Decode (কোডমুক্ত) করে

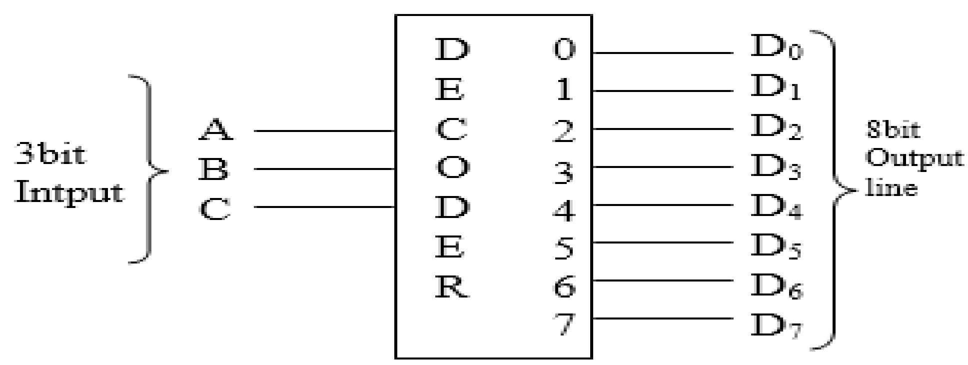
Binary (B_2) to Decimal (B_{10})

Used in Output Device

Equation $n \times 2^n$ [Input n, Output 2^n]

Example : 2×4, 3×8, 4×16,

3×8 Line Decoder



ব্লক চিত্র

Code			Decode Output							
A	B	C	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

3 to 8 line Decoder Truth table

$D_0 = \overline{A}\,\overline{B}\,\overline{C}$

$D_1 = \overline{A}\,\overline{B}C$

$D_2 = \overline{A}B\overline{C}$

$D_3 = \overline{A}BC$

$D_4 = A\overline{B}\,\overline{C}$

$D_5 = A\overline{B}C$

$D_6 = AB\overline{C}$

$D_7 = ABC$

Encoder

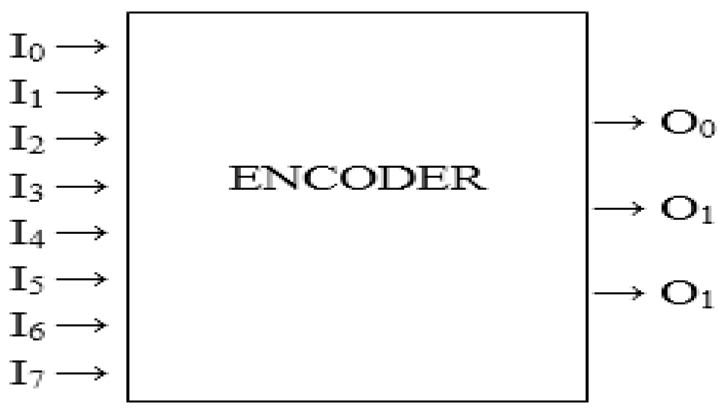
\# Encode/ Code (কোডযুক্ত) করে

\# Decimal (B_{10}) to Binary (B_2)

\# Used in Input Device

\# Equation $2^n \times n$ [Input 2^n, Output n]

\# Example : 4×2, 8×3, 16×4,



চিত্র : ৮ টু ৩ লাইন অকট্যাল এনকোডার

সত্যক সারণি :

ইনপুট সংকেত								আউটপুট সংকেত		
I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_0	I_1	I_2
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

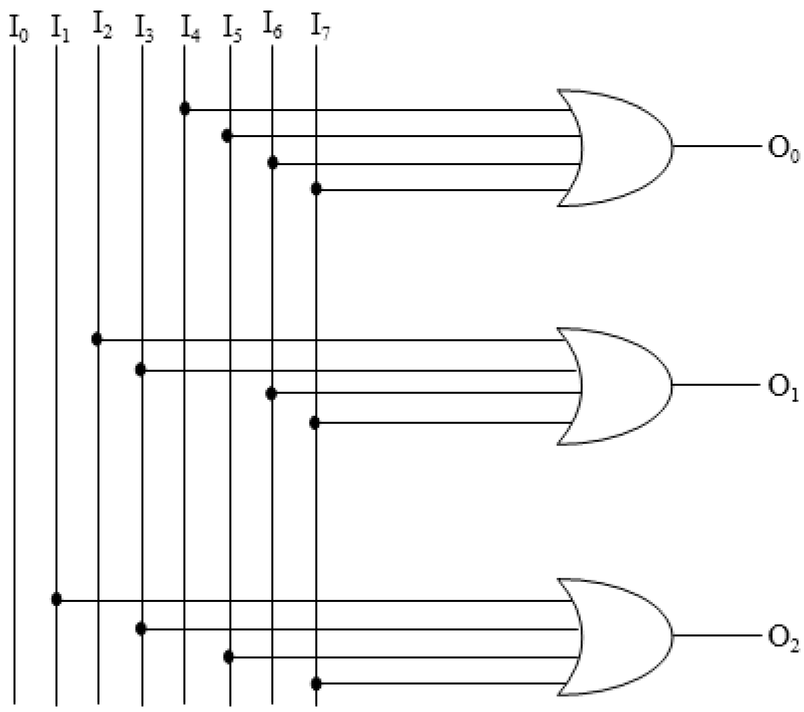
বুলিয়ান সমীকরণ :

$O_0 = I_4 + I_5 + I_6 + I_7$

$O_1 = I_2 + I_3 + I_6 + I_7$

$O_2 = I_1 + I_3 + I_5 + I_7$

যুক্তি বর্তনী :

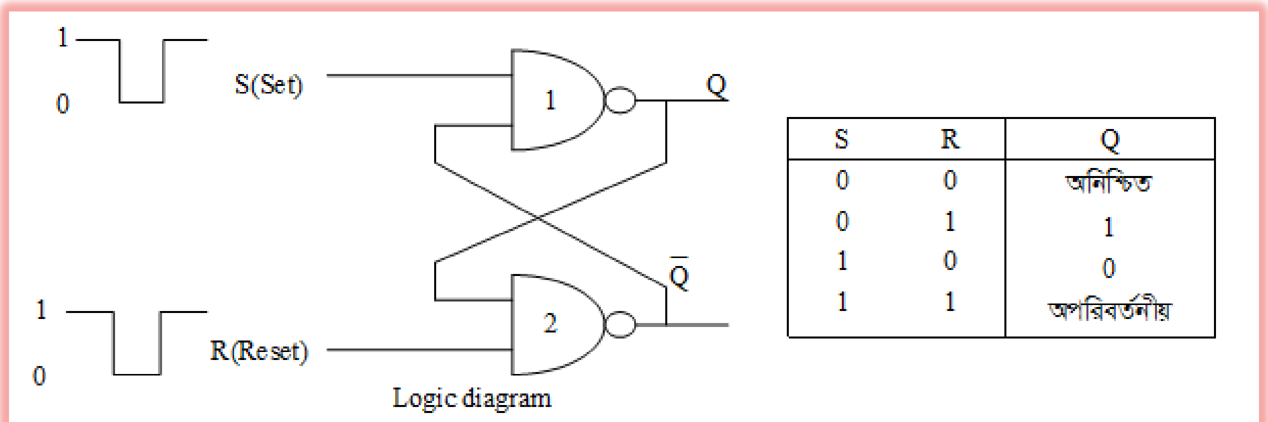


q

Flip-Flop

- # Basic Memory Element
- # Unit of memory
- # It can store 1 or 0 (1 bit)
- # Implemented by NAND or NOR Gate
- # Output $Q, \overline{Q}$
- # Ex : S-R, J-K, T, D
- # Crossed Coupled, feedback

S-R (SET- RESET)



চিত্র : Nand গেইট দ্বারা তৈরি সাধারণ ফ্লিপ-ফ্লপ সার্কিট

Truth Table

Input		Output
S	**R**	$Q or \overline{Q}$
0	0	$Q = \overline{Q} = 1$, Toggle
0	1	$Q = 1, \overline{Q} = 0$
1	0	$Q = 0, \overline{Q} = 1$
1	1	Unchanged