

$$\ddot{\vartheta}_1 = -\frac{m_2}{m_1 + m_2} \frac{l_2}{l_1} \left(\ddot{\vartheta}_2 \cos(\vartheta_1 - \vartheta_2) + \dot{\vartheta}_2^2 \sin(\vartheta_1 - \vartheta_2) \right) - \frac{g}{l_1} \sin(\vartheta_1) \quad (1)$$

$$\ddot{\vartheta}_2 = -\frac{l_1}{l_2} \left(\ddot{\vartheta}_1 \cos(\vartheta_1 - \vartheta_2) - \dot{\vartheta}_1^2 \sin(\vartheta_1 - \vartheta_2) \right) - \frac{g}{l_2} \sin(\vartheta_2) \quad (2)$$

(2) in (1):

$$\begin{aligned} \ddot{\vartheta}_1 = & -\frac{m_2}{m_1 + m_2} \frac{l_2}{l_1} \cdot \\ & \left(\cos(\vartheta_1 - \vartheta_2) \left[-\frac{l_1}{l_2} \left(\ddot{\vartheta}_1 \cos(\vartheta_1 - \vartheta_2) - \dot{\vartheta}_1^2 \sin(\vartheta_1 - \vartheta_2) \right) - \frac{g}{l_2} \sin(\vartheta_2) \right] + \dot{\vartheta}_2^2 \sin(\vartheta_1 - \vartheta_2) \right) \\ & - \frac{g}{l_1} \sin(\vartheta_1) \end{aligned}$$

$$\Rightarrow \ddot{\vartheta}_1 = \left[1 - \frac{m_2}{m_1 + m_2} \cos^2(\vartheta_1 - \vartheta_2) \right]^{-1} \cdot$$

$$\begin{aligned} & \left(\frac{m_2}{m_1 + m_2} \frac{l_2}{l_1} \cos(\vartheta_1 - \vartheta_2) \left[\dot{\vartheta}_1^2 \sin(\vartheta_1 - \vartheta_2) + \frac{g}{l_2} \sin(\vartheta_2) \right] \right. \\ & - \frac{m_2}{m_1 + m_2} \frac{l_2}{l_1} \dot{\vartheta}_2^2 \sin(\vartheta_1 - \vartheta_2) \\ & \left. - \frac{g}{l_1} \sin(\vartheta_1) \right) \quad (3) \end{aligned}$$

(1) in (2):

$$\ddot{\vartheta}_2 = -\frac{l_1}{l_2} \cdot$$

$$\begin{aligned} & \left(\cos(\vartheta_1 - \vartheta_2) \left[-\frac{m_2}{m_1 + m_2} \frac{l_2}{l_1} \left(\ddot{\vartheta}_2 \cos(\vartheta_1 - \vartheta_2) + \dot{\vartheta}_2^2 \sin(\vartheta_1 - \vartheta_2) \right) - \frac{g}{l_1} \sin(\vartheta_1) \right] - \dot{\vartheta}_1^2 \sin(\vartheta_1 - \vartheta_2) \right) \\ & - \frac{g}{l_2} \sin(\vartheta_2) \end{aligned}$$

$$\Rightarrow \ddot{\vartheta}_2 = \left[1 - \frac{m_2}{m_1 + m_2} \cos^2(\vartheta_1 - \vartheta_2) \right]^{-1} \cdot$$

$$\begin{aligned} & \left(\cos(\vartheta_1 - \vartheta_2) \left[\frac{m_2}{m_1 + m_2} \dot{\vartheta}_2^2 \sin(\vartheta_1 - \vartheta_2) + \frac{l_1}{l_2} \frac{g}{l_1} \sin(\vartheta_1) \right] + \frac{l_1}{l_2} \dot{\vartheta}_1^2 \sin(\vartheta_1 - \vartheta_2) - \frac{g}{l_2} \sin(\vartheta_2) \right) \quad (4) \end{aligned}$$