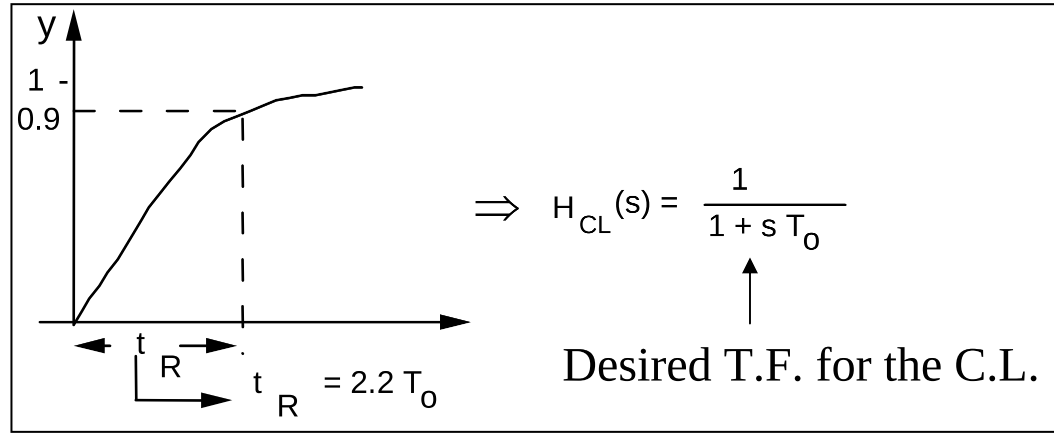
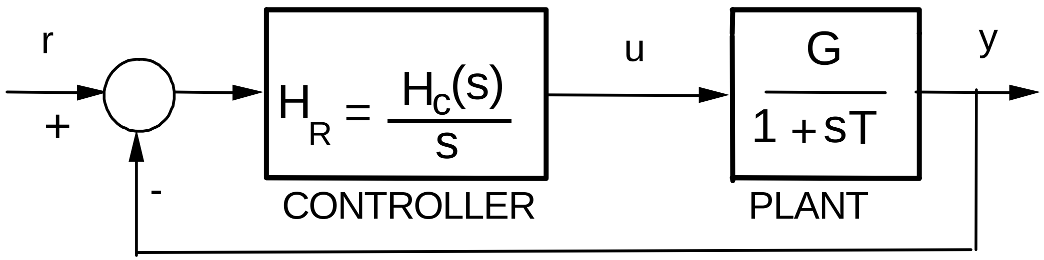
# PI Controller

Plant : $\boxed{G/(1+sT)}$

Objectives :

1) Null steady state error
2) Rising time $t_R$




$$H_{CL}(s) = \frac{H_c(s)G}{G\,H_c(s)+s+s^2T} = \frac{1}{1+sT_0} = \frac{H_c(s)G}{H_c(s)G(1+sT_0)}$$

$H_c(s)\,G\,s\,T_0 = s^2\,T + s$ $\quad \rightarrow \quad H_c(s) = \dfrac{1}{G\,T_0}(1+sT)$

# PI Controller

$$H_R(s) = \frac{\frac{1}{GT_0}(1+sT)}{s} = \frac{T}{GT_0}\left[1+\frac{1}{Ts}\right] = K\left[1+\frac{1}{T_i s}\right]$$

**Proportional Gain** **Integral Action**

*Remark:*

The controller parameters depend on the desired prformances ($T_0$) and on the plant transfer function parameters ($G,T$)

# PID Controller

Several structures of the PID controller are possible.

For example, consider the structure :

proportional gain

deravative action

$$H_{PID}(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + \frac{T_d}{N} s}\right) \qquad (*)$$

integral action

filtering on the deravative action

Plant :

$$H(s) = \frac{G}{(1 + sT_1)(1 + sT_2)} = \frac{b_0}{1 + a_1 s + a_2 s^2}$$

Objectives:

1) $t_R$, M

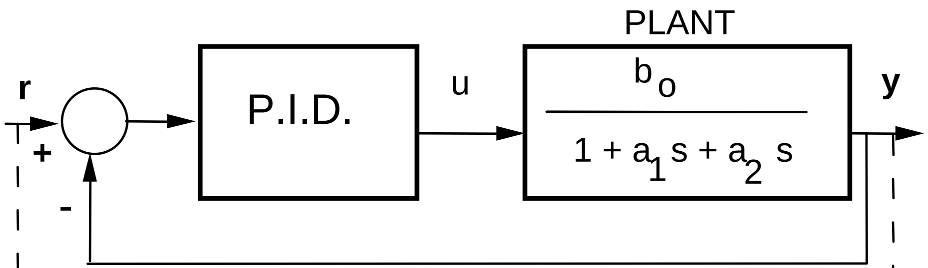
2) Null steady state error

See slide #18

$$H_{CL}(s) = \frac{\omega_0^2}{\omega_0^2 + 2\zeta\,\omega_0 s + s^2}$$

Desired Closed Loop transfer function

# PID Controller




$$H_{CL}(s) = \frac{\omega_o^2}{\omega_o^2 + 2\,\zeta\,\omega_o s + s^2}$$

$$H_{PID}(s) = \frac{K\left[1 + s\left(T_i + \frac{T_d}{N}\right) + s^2\left(T_iT_d + \frac{T_iT_d}{N}\right)\right]}{T_i s\left(1 + \frac{T_d}{N}s\right)} \qquad (*)$$

PID T.F. Numerator = Plant T.F. Denominator

# PID Controller

$$H_{OL}(s) = H(s)\cdot H_{PID}(s) = \frac{Kb_0}{T_i s\left(1+\frac{T_d}{N}s\right)} \quad \longleftrightarrow \quad \begin{aligned} a_1 &= T_i + \frac{T_d}{N}; \\ a_2 &= T_i T_d\left(1+\frac{1}{N}\right). \end{aligned}$$

$$\Big\downarrow$$

$$H_{CL}(s) = \frac{Kb_0}{Kb_0 + T_i s + \frac{T_i T_d}{N}s^2} = \frac{\frac{Kb_0 N}{T_i T_d}}{\frac{Kb_0 N}{T_i T_d} + \frac{N}{T_d}s + s^2} = \frac{\omega_0^2}{\omega_0^2 + 2\zeta\omega_0 s + s^2}$$

$$T_i = a_1 - \frac{T_d}{N} = a_1 - \frac{1}{2\zeta\omega_0} \qquad T_d = \frac{a_2}{T_i} - \frac{T_d}{N} = \frac{a_2}{T_i} - \frac{1}{2\zeta\,\omega_0} \qquad K = \frac{\omega_0 T_i}{2\zeta\, b_0} \qquad \frac{T_d}{N} = \frac{1}{2\zeta\omega_0}$$

The controller parameters depend on the desired prformances ($\omega_0$ ,$\zeta$) and on the plant transfer function parameters ($a_1$, $a_2$, $b_0$)

## Concluding Rermarks

- The dynamics of a plant running around a specific operative point can be often described by a *linear dynamic model*.
- The linear dynamic systems are described by *linear differential equations* in the time domain and by *transfer functions* in the frequency domain.
- The control systems are closed loop systems containing: a controller, the plant (which contains the actuator and the sensor) and the *feedback loop*.
- The desired closed loop performances can be expressed by the desired (frequency) characteristics of the closed loop system.
- The Nyquist plot (frequency domain) plays a fundamental role for the closed system stability analysis and its robustness with respect to plant parameters variations.