

## Exercise 02

الأسف :

2.1 (a) Prove that  $(X + Y + Z)' = X'Y'Z'$

and  $(XYZ)' = X' + Y' + Z'$

1-  $(X + Y + Z)'$

X	Y	Z	$X+Y+Z$	$(X+Y+Z)'$	$X'$	$Y'$	$Z'$	$X'Y'Z'$
0	0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	0	0
0	1	0	1	0	1	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	1	0	0	1	1	0
1	0	1	1	0	0	1	0	0
1	1	0	1	0	0	0	1	0
1	1	1	1	0	0	0	0	0

Then  $(X + Y + Z)' = X'Y'Z'$  I

X	Y	Z	$XYZ$	$(XYZ)'$	$X'$	$Y'$	$Z'$	$X'+Y'+Z'$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

then  $(XYZ)' = X' + Y' + Z'$  II

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(e) The associative Law  $X(YZ) = (XY)Z$

X	Y	Z	YZ	X(YZ)	XY	(XY)Z
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

then  $X(YZ) = (XY)Z$  #

2.2 : Simplify the following Boolean expression:-

$$\begin{aligned}
 (f) & a'b c + a b c' + a b c + a' b c' \\
 &= a'b (c + c') + ab (c' + c) \quad \text{distributive} \\
 &= a'b \cdot 1 + ab \cdot 1 \quad \text{Complement} \\
 &= a'b + ab \\
 &= b(a' + a) \quad \text{distributive} \\
 &= b \cdot 1 \quad \text{Complement} \\
 &= b \quad \#
 \end{aligned}$$

2.3 - Simplify the following Boolean exp

$$\begin{aligned}
 (f) & (a' + c')(a + b' + c') \\
 &= a'(a + b' + c') + c'(a + b' + c') \\
 &= a'a + a'b' + a'c' + c'(a + b' + c') \\
 &= 0 + a'b' + a'c' + ac' + b'c' + c' \\
 &= a'b' + \underline{c'(a' + a)} + b'c' + c' \quad \text{dis} \\
 &= a'b' + c' \cdot 1 + c' \\
 &= a'b' + c' \quad \#
 \end{aligned}$$



## 2.4 - Reduce the following

(a)  $A'C' + ABC + AC'$  (to three literals)

$$= C'(A' + A) + ABC$$

$$= C' \cdot 1 + ABC$$

$$= C' + (ABC)$$

$$= (A + C')(B + C')(C + C')$$

$$= (A + C')(B + C') \cdot 1$$

$$= C' + AB$$

$$= AB + C' \quad \#$$

(d)  $(A' + C)(A' + C')(A + B + C'D)$  (Four Literals)

$$= A' + CC'(A + B + C'D)$$

$$= A' + 0(A + B + C'D)$$

$$= A'(A + B + C'D)$$

$$= A'A + A'B + A'C'D$$

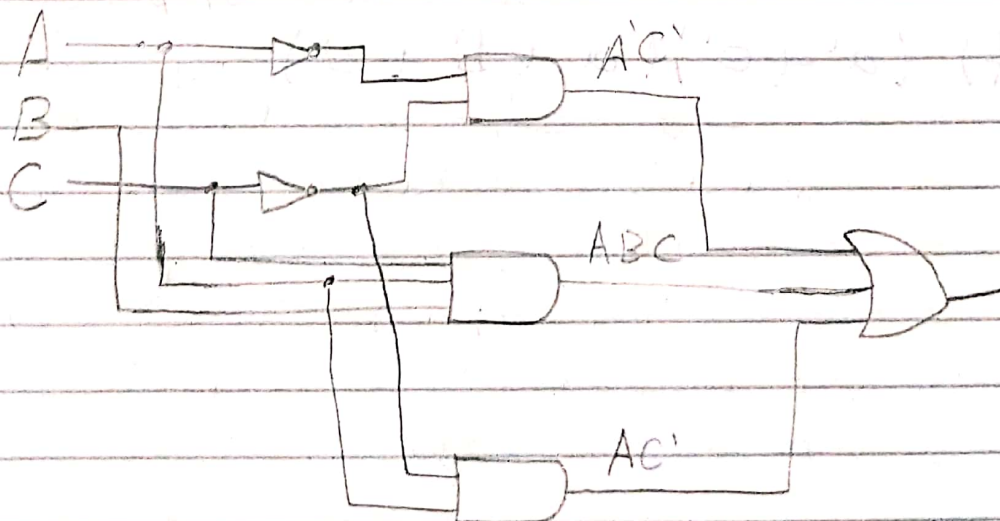
$$= 0 + A'B + A'C'D$$

$$= A'B + A'C'D$$

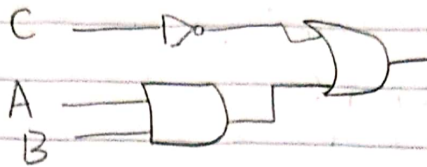
$$= A'(B + C'D) \quad \#$$

2.7 - Draw the logic diagram of the circuits that implement the original & simplified expression in Problem 2.4?

(a) Original =  $A'C' + ABC + AC'$



(b) Simplified =  $AB + C'$



2.8 - Find the Complement of

$$F = WX + YZ;$$

$$F' = (WX + YZ)'$$

$$= (WX)'(YZ)'$$

$$= (W' + X')(Y' + Z') \neq$$

Show that  $FF' = 0$

$$FF' = (WX + YZ)[(W' + X')(Y' + Z')]$$

$$= WX(W' + X')(Y' + Z') + YZ(W' + X')(Y' + Z')$$

$$= (WXW' + WXX')(WXY' + WXZ') + YZ(W' + X')(Y' + Z')$$

$$= 0(WXY' + WXZ') + (YZW' + YZX')(YZY' + YZZ')$$

$$= 0(WXY' + WXZ') + (YZW' + YZX') \cdot 0$$

$$\therefore X \cdot 0 = 0$$

$$\therefore 0 + 0 = 0 \neq \boxed{F \cdot F' = 0}$$

$$F + F' = 1$$

$$F + F' = (WX + YZ) + [(W' + X')(Y' + Z')]$$

$$= [(WX + YZ) + (W' + X')][WX + YZ + (Y' + Z')]$$

$$= [(WX + X') + YZ + W'][YZ + Z' + WX + Y']$$

$$= [W + X' + YZ + W'][Y + Z' + WX + Y']$$

$$= [(W + W') + X' + YZ][Y + Y' + Z' + WX]$$

$$= [1 + X' + YZ][1 + Z' + WX]$$

$$= 1 \cdot 1$$

$$= \boxed{1}$$

$$\therefore F + F' = \boxed{1}$$

2.9. Find the Complement of

$$\begin{aligned} (b) & (a+c)(a+b')(a'+b+c') \\ &= [(a+c)(a+b')(a'+b+c')] \\ &= (a+c)' + (a+b')' + (a'+b+c')' \\ &= a'e' + a'b + ab'C \end{aligned}$$

2.12 - Perform AND operation:

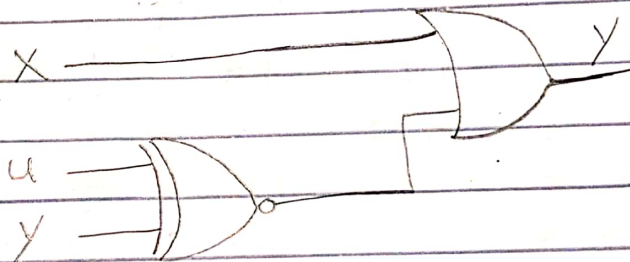
$$A = 10110001$$

$$B = 10101100$$

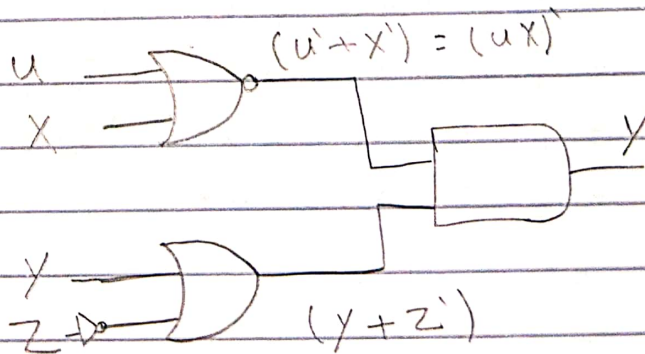
$$10100000$$

2.13. Draw Logical diagram

$$(b) y = (u \oplus y)' + x$$



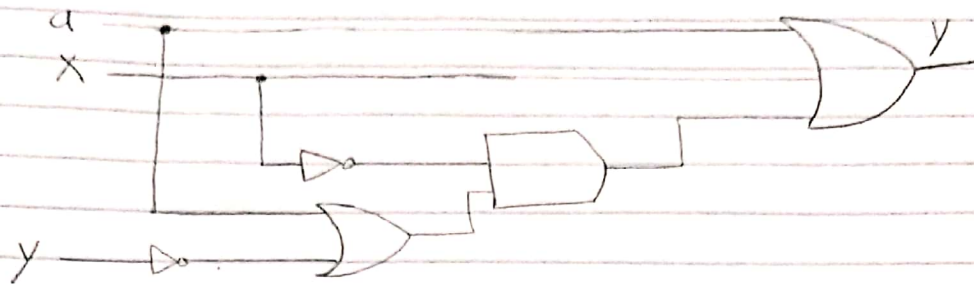
$$(c) y = (u' + x')(y + z')$$





$$(F) y = u + x + x'(u + y')$$

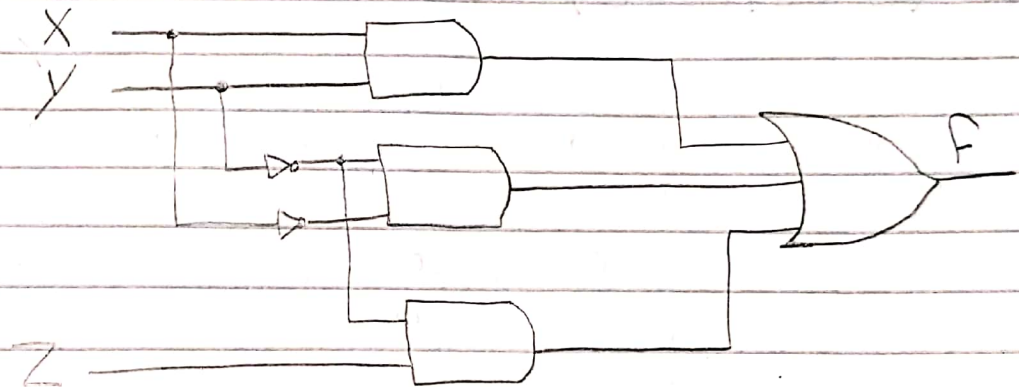
$$= u + x + x'u + x'y'$$



2.14 - Implement the Boolean function

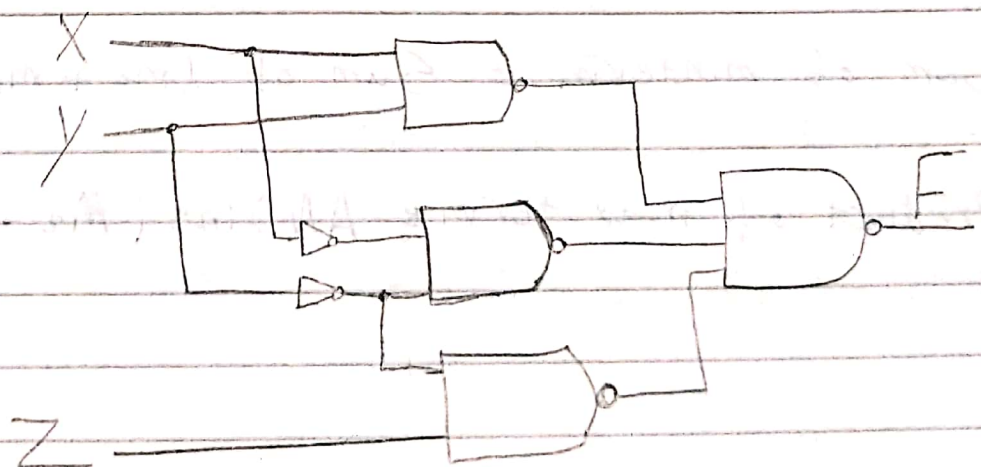
$$F = xy + x'y' + y'z$$

(a) with AND, OR, and inverter gates :-



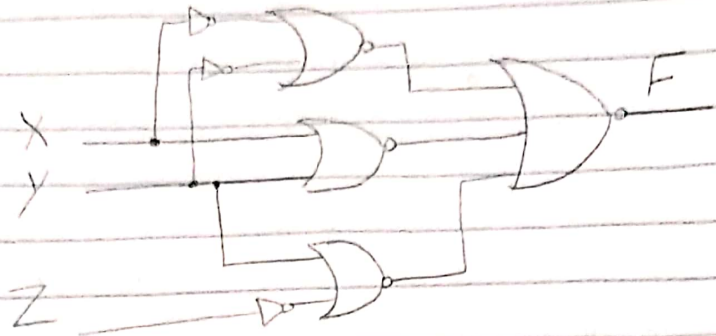
(b) With NAND and inverter gates.

$$F = xy + x'y' + y'z$$



(e) with NOR and inverter gates:-

$$F = XY + X'Y' + Y'Z$$



2.17- Obtain the truth table, and express each function in SOMin and Pmax :-

$$(C) (C' + d)(b + c')$$

b	C	d	C'	C'+d	b+c'	(C'+d)(b+c')	
0	0	0	1	1	1	1	m <sub>0</sub>
0	0	1	1	1	1	1	m <sub>1</sub>
0	1	0	0	0	0	0	m <sub>2</sub>
0	1	1	0	1	0	0	m <sub>3</sub>
1	0	0	1	1	1	1	m <sub>4</sub>
1	0	1	1	1	1	1	m <sub>5</sub>
1	1	0	0	0	1	0	m <sub>6</sub>
1	1	1	0	1	1	1	m <sub>7</sub>

1- Sum of min term = Sum of (m<sub>0</sub> + m<sub>1</sub> + m<sub>4</sub> + m<sub>5</sub> + m<sub>7</sub>)

$$= b'c'd' + b'c'd + b c'd' + b c'd + bcd \quad I$$

2- Product of max terms ANDing (M<sub>2</sub> · M<sub>3</sub> · M<sub>6</sub>)

$$= (b + c + d')(b + c' + d')(b + c + d') \quad II$$

$$(d) \quad bd' + acd' + ab'c + a'c'$$

a	b	c	d	$bd'$	$acd'$	$ab'c$	$a'c'$	ORing =
0	0	0	0	0	0	0	1	$M_0$
0	0	0	1	0	0	0	1	$M_1$
0	0	1	0	0	0	0	0	$M_2$
0	0	1	1	0	0	0	0	$M_3$
0	1	0	0	1	0	0	1	$M_4$
0	1	0	1	0	0	0	1	$M_5$
0	1	1	0	1	0	0	0	$M_6$
0	1	1	1	0	0	0	0	$M_7$
1	0	0	0	0	0	0	0	$M_8$
1	0	0	1	0	0	0	0	$M_9$
1	0	1	0	0	1	1	0	$M_{10}$
1	0	1	1	0	0	1	0	$M_{11}$
1	1	0	0	1	0	0	0	$M_{12}$
1	1	0	1	0	0	0	0	$M_{13}$
1	1	1	0	1	1	0	0	$M_{14}$
1	1	1	1	0	0	0	0	$M_{15}$

$$F = \text{ANDing } (2, 3, 7, 8, 9, 13, 15)$$

$$= M_2 \cdot M_3 \cdot M_7 \cdot M_8 \cdot M_9 \cdot M_{13} \cdot M_{15}$$

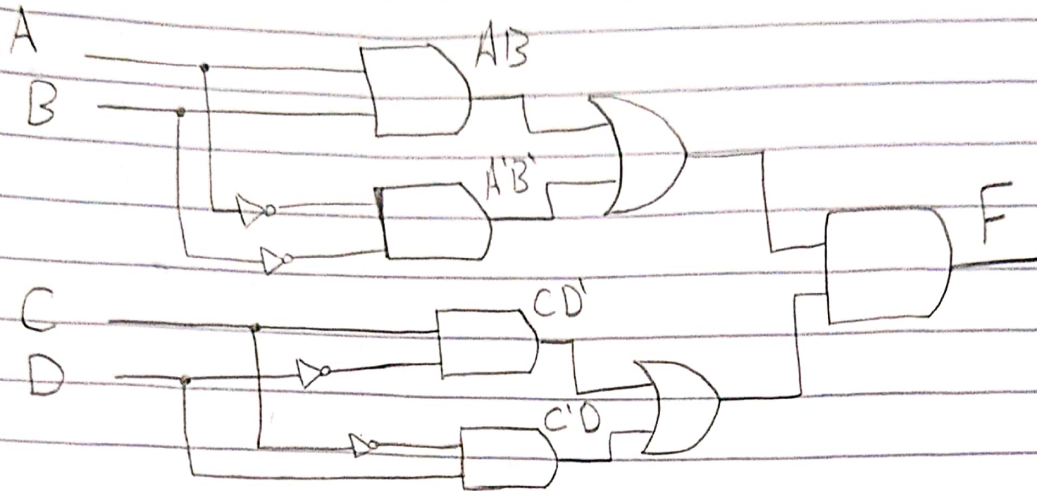
$$= (a+b+c'+d')(a+b+c'+d')(a+b'+c'+d')(a'+b+c+d) \cdot (a'+b+c+d')(a'+b'+c+d')(a'+b'+c'+d')$$

$$F = \text{ORing } (0, 1, 4, 5, 6, 10, 11, 12, 14)$$



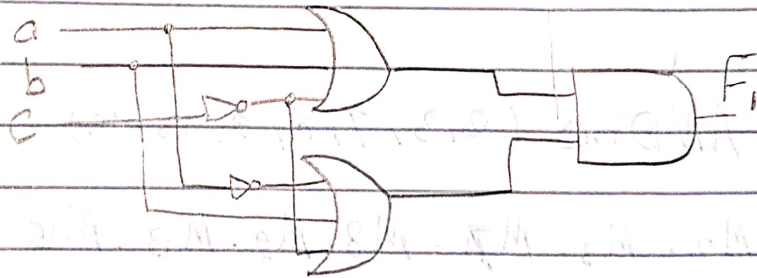
2.23. Draw The logic diagram:-

$$(C) (AB + A'B')(CD' + C'D)$$

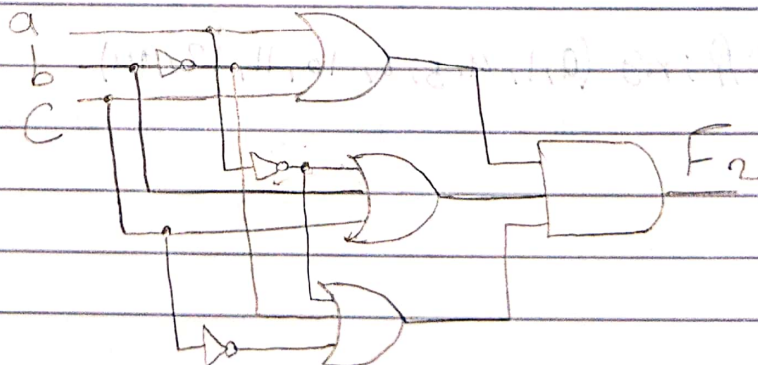


2.27. Write the Boolean equations and Draw the Logic diagram:-

$$F_1 = \text{ANDing of } (M_1 \cdot M_5) \\ = (a+b+c')(a'+b+c')$$



$$F_2 = \text{ANDing of } (M_2 \cdot M_4 \cdot M_6) \\ = (a+b+c)(a'+b+c)(a'+b'+c')$$



2.28. Write the Boolean expression for the truth table:-

$$(a) \quad y = [(a \cdot e (bcd)')']'$$

$$(b) \quad y_1 = (a \oplus (c+d+e))$$

$$y_2 = [b'f(c+d+e)]'$$