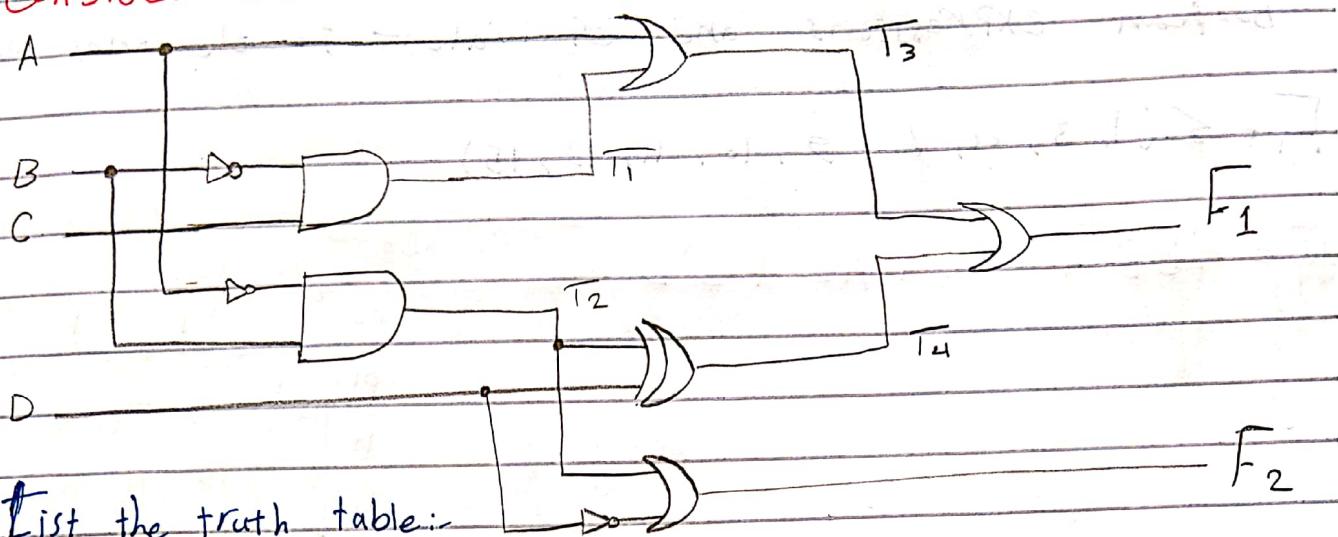


## Exercise Five - P1

1- Consider the following Combinational Circuit:-



(b) List the truth table:-

A	B	C	D	$T_1$	$T_2$	$T_3$	$T_4$	$F_1$	$F_2$	$T_1 = B'C$
0	0	0	0	0	0	0	0	0	10	$T_2 = A'B$
0	0	0	1	0	0	0	1	11	0	$T_3 = A + T_1$
0	0	1	0	1	0	0	0	0	13	$= A + B'C$
0	0	1	1	1	0	0	1	13	0	$T_4 = D + T_2$
0	1	0	0	0	1	0	1	14	14	$= D + A'B$
0	1	0	1	0	1	0	0	0	15	$F_1 = T_3 + T_4$
0	1	1	0	0	1	0	1	16	16	
0	1	1	1	0	1	0	0	0	17	$F_2 = D + T_2$
1	0	0	0	0	0	0	0	0	18	$= A'B + D$
1	0	0	1	0	0	0	1	19	0	
1	0	1	0	1	0	1	0	110	110	
1	0	1	1	1	0	1	1	111	0	
1	1	0	0	0	0	0	0	0	112	
1	1	0	1	0	0	0	1	113	0	
1	1	1	0	0	0	0	0	0	114	
1	1	1	1	0	0	0	1	115	0	

(C) Plot the output boolean expressions functions obtained in part (b) on maps and show that the simplified boolean expressions are equivalent to the ones ---?

$$F_1 = \sum(1, 3, 4, 6, 9, 10, 11, 13, 15)$$

$$F_1 = A'B'D' + AB'C + AD + B'D$$

AB	CD	00	01	11	10
00	0	1	1	1	2
01	4	1	5	7	6
11	12	13	1	15	14
10	8	9	1	11	10

$$F_2 = \sum(0, 2, 4, 5, 6, 7, 8, 10, 12, 14)$$

$$F_2 = \sum(0, 2, 4, 5, 6, 7, 8, 10, 12, 14)$$

$$F_2 = D' + A'B$$

$$= A'B + D' \quad \#$$

AB	CD	00	01	11	10
00	0	1	1	3	2
01	4	1	5	7	6
11	12	13	15	14	1
10	8	9	11	10	1

3. For the circuit shown in Fig. 2.26 (Section 2.11)

(a) Write the Boolean functions for the four outputs in terms of the inputs variables.

$$Y_0 = A_0 S'E' + B_0 S'E'$$

$$Y_1 = A_1 S'E' + B_1 S'E'$$

$$Y_2 = A_2 S'E' + B_2 S'E'$$

$$Y_3 = A_3 S'E' + B_3 S'E'$$

4. Design a Combinational Circuit with three inputs and one output.

(b) The output is 1 when the binary value of the inputs is an even number?

Inputs			Output	
X	Y	Z	F	
0	0	0	0	$F = \Sigma(2, 4, 6)$
0	0	1	0	
0	1	0	1 $\leftarrow 2$	$F = XZ' + YZ'$
0	1	1	0	
1	0	0	1 $\leftarrow 4$	
1	0	1	0	
1	1	0	1 $\leftarrow 6$	
1	1	1	0	

6. A majority Circuit is Combinational Circuit whose output is equal to 1 if the input variables have more 1's than 0's. The output is zero otherwise.

(b) Write and verify a Verilog gate-level model?

8. Design a Code Converter that converts a decimal digit from:

(b) The 8,4,-2,-1 Code to Gray Code?

Inputs  $\rightarrow a \ b \ c \ d$

Outputs  $\rightarrow w \ x \ y \ z$

### 1) Truth Table

D	a	b	c	d	w	x	y	z
0	0	0	0	0	0	0	0	0
1	0	1	1	1	1	0	1	0
2	0	1	1	0	0	1	0	0
3	0	1	0	1	0	1	1	1
4	0	1	0	0	0	1	1	0
5	1	0	1	1	1	1	1	0
6	1	0	1	0	1	1	1	1
7	1	0	0	1	1	1	0	1
8	1	0	0	0	1	1	0	0
9	1	1	1	1	1	0	0	0

don't care's =  $\Sigma(1, 2, 3, 12, 13, 14)$

2

$$w = \Sigma(8, 9, 10, 11, 15) + d$$

ab	cd	00	01	11	10
00	0	0	1	X	X
01	1	4	5	7	6
11	1	12	13	X	14
10	0	8	9	11	10

$$w = a$$

$$z = \Sigma(5, 9, 10) + d$$

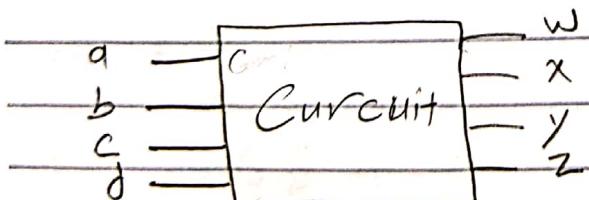
$$x = \Sigma(4, 5, 6, 7, 8, 9, 10, 11, 15) + d$$

ab	cd	00	01	11	10
00	0	1	X	3	X
01	1	4	5	7	6
11	1	12	X	13	15
10	0	8	9	11	10

$$z = acd' + c'd$$

$$x = b + a$$

Block diagram



$$y = \Sigma(4, 5, 10, 11) + d$$

ab	cd	00	01	11	10
00	0	0	1	X	X
01	1	4	5	7	6
11	1	12	X	13	15
10	0	8	9	11	10

$$y = bc' + b'c$$

$$= b \oplus c$$

9) An ABCD-to-seven-segment decoder — Converts a ~~digital~~ decimal digit in BCD to an appropriate code .....

The seven outputs of decoder (a, b, c, d, e, f, g)

1- Using Truth table .

2- K-map.

3- To design the BCD-to-Seven-Segment decoder using a minimum number of gates.

Dec	Inputs				outputs							$d = \sum (10, 11, 12, 13, 14, 15)$
	A	B	C	D	a	b	c	d	e	f	g	
0	0	0	0	0	1 <sup>0</sup>	0						
1	0	0	0	1	0	1 <sup>1</sup>	1 <sup>1</sup>	0	0	0	0	
2	0	0	1	0	1 <sup>2</sup>	1 <sup>2</sup>	0	1 <sup>2</sup>	1 <sup>2</sup>	0	1 <sup>2</sup>	
3	0	0	1	1	1 <sup>3</sup>	1 <sup>3</sup>	1 <sup>3</sup>	1 <sup>3</sup>	0	0	1 <sup>3</sup>	
4	0	1	0	0	0	1 <sup>4</sup>	1 <sup>4</sup>	0	0	1 <sup>4</sup>	1 <sup>4</sup>	
5	0	1	0	1	1 <sup>5</sup>	0	1 <sup>5</sup>	1 <sup>5</sup>	0	1 <sup>5</sup>	1 <sup>5</sup>	
6	0	1	1	0	1 <sup>6</sup>	0	1 <sup>6</sup>					
7	0	1	1	1	1 <sup>7</sup>	1 <sup>7</sup>	1 <sup>7</sup>	0	0	0	0	
8	1	0	0	0	1 <sup>8</sup>							
9	1	0	0	1	1 <sup>9</sup>	1 <sup>9</sup>	1 <sup>9</sup>	0	1 <sup>9</sup>	1 <sup>9</sup>	1 <sup>9</sup>	

$$a = \sum (0, 2, 3, 5, 6, 7, 8, 9) + d$$

ABCD			
00	01	11	10
00	1	3	2 <sup>D</sup>
01	5	7	6
11	X	X	X
10	8	9	X

$$a = C + BD + AB' + B'D' \\ = AB' + BD + B'D + C \\ (B \oplus D)$$

$$b = \sum (0, 1, 2, 3, 4, 7, 8, 9) + d \quad C = \sum (0, 1, 3, 4, 5, 6, 7, 8, 9) + d \quad d = \sum (0, 2, 3, 5, 6, 8, 9) + d$$

AB		CD					
00	01	11	10	00	01	11	10
00	1	3	2 <sup>D</sup>	0	1	3	2
01	5	7	6	4	5	7	6
11	X	X	X	12	13	15	14
10	8	9	X	18	19	17	16

$$b = AB' + C'D' + CD \\ (C \oplus D)$$

AB		CD					
00	01	11	10	00	01	11	10
00	1	3	2	0	1	3	2
01	5	7	6	4	5	7	6
11	X	X	X	12	13	15	14
10	8	9	X	18	19	17	16

$$C = C' + D + B + A \\ - A + B + D + C'$$

AB		CD					
00	01	11	10	00	01	11	10
00	1	3	2	0	1	3	2
01	5	7	6	4	5	7	6
11	X	X	X	12	13	15	14
10	8	9	X	18	19	17	16

$$d = BC'D + A + CD' \\ + B'D \\ = A + BC'D + CD' + B'D$$

$$e = \Sigma(0, 2, 6, 8) + d$$

$$f = \Sigma(0, 4, 5, 6, 8, 9) + d$$

$$g = \Sigma(2, 3, 4, 5, 6, 8, 9) + d$$

AB		CD	00	01	11	10
00	1	3	2	1		
01	4	5	7	6	1	
11	X	X	X	X	X	
10	8	9	X	X	X	

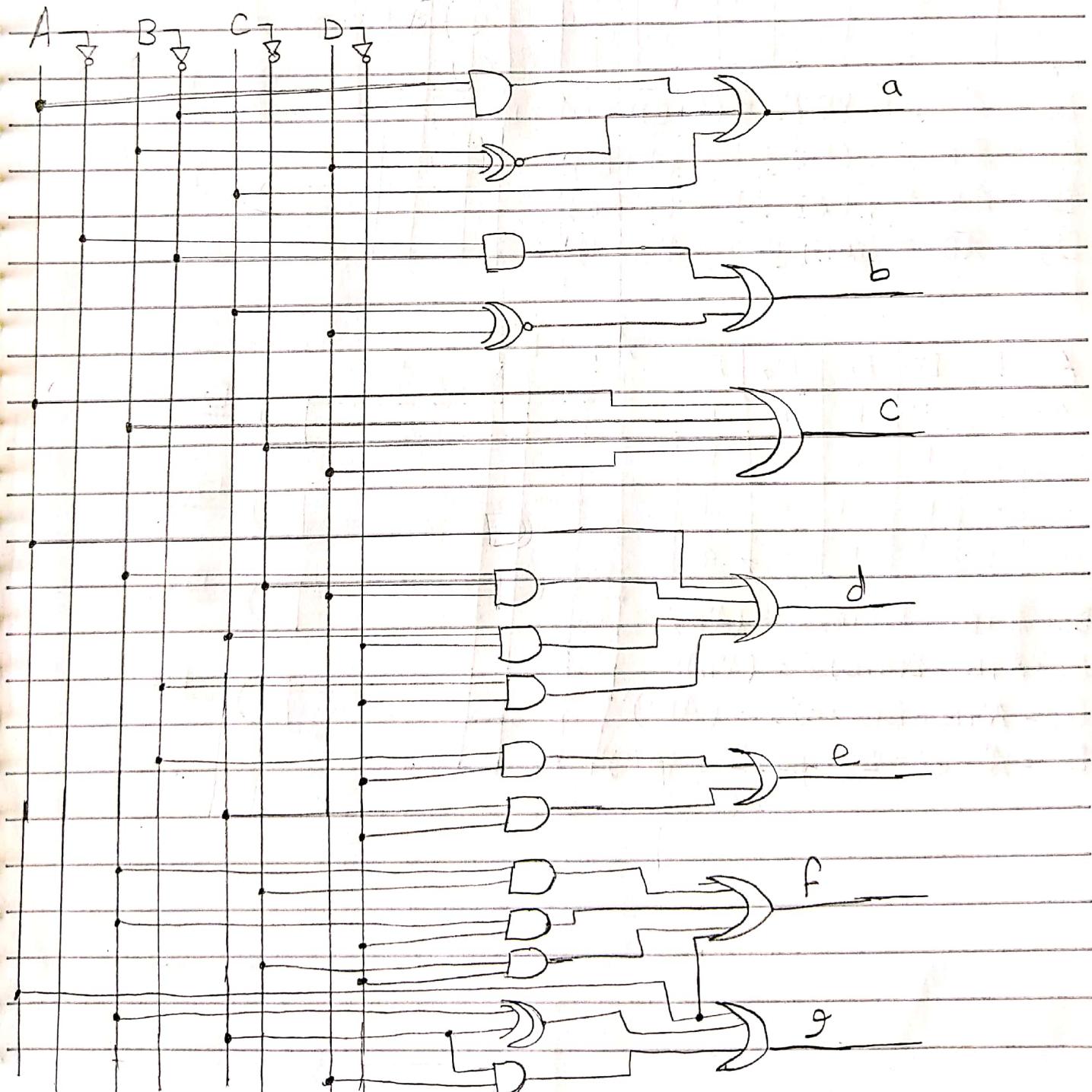
$$e = B'D' + CD'$$

AB		CD	00	01	11	10
00	1	3	2	1		
01	4	5	7	6	1	
11	X	X	X	X	X	
10	8	9	X	X	X	

$$\begin{aligned} f &= BD' + BC' + C'D' \\ &\quad + A \\ &= A + BC' + BD' + C'D' \end{aligned}$$

AB		CD	00	01	11	10
00	1	3	2	1		
01	4	5	7	6	1	
11	X	X	X	X	X	
10	8	9	X	X	X	

$$\begin{aligned} g &= BC' + A + B'C + CD \\ &= A + \underbrace{B'C + BC'}_{(B \oplus C)} + CD \end{aligned}$$



## 11. Using four half-adders:

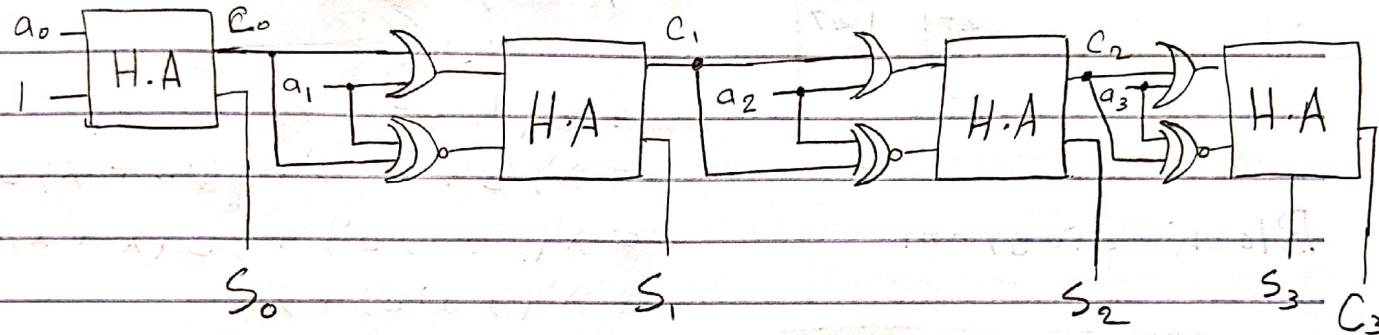
(b) Design a four-bit Combinational decrementer?

$a_3$	$a_2$	$a_1$	$a_0$	$+1$	$a_0$	$C_0$	$1$	$A$	$B$
1	1	1	1	+	0	0	1	0	1
$C_3$	$S_3$	$S_2$	$S_1$	$S_0$	0	1	1	1	0
					1	0	1	1	0

$$A = a + C$$

$$B = a'c' + ac$$

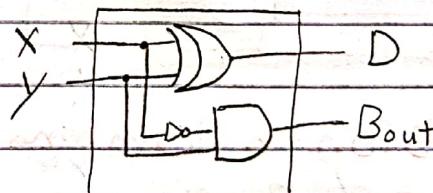
$$= (a \oplus c)$$



12- Design a ~~half~~ Full-Subtractor circuit with inputs  $X$  and  $Y$  and outputs Diff &  $B_{out}$  :-

$$X \quad Y \quad D \quad B_{out} \quad D = x'y + xy' = X \oplus Y$$

$$B_{out} = x'y$$



(b) Design a full-Subtractor Circuit with inputs  $X, Y, B_{in}$  and two outputs Diff and  $B_{out}$ .

## Truth table

## Functions

X	Y	Bin	Diff	B <sub>out</sub>
0	0	0	0	0
0	0	1	1 ↪ 1	1 ↪ 1
0	1	0	1 ↪ 2	1 ↪ 2
0	1	1	0	1 ↪ 3
1	0	0	1 ↪ 4	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1 ↪ 7	1 ↪ 7

$$\text{Diff} = \Sigma(1, 2, 4, 7)$$

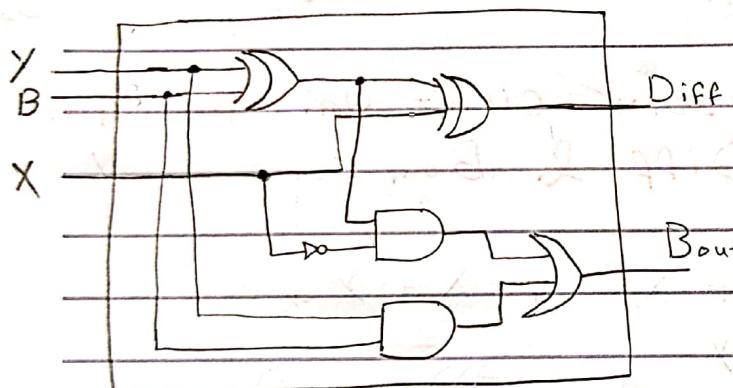
X	Y	00	01	11	10
0	0	0	1	1	1
1	0	1	1	0	0
0	1	1	0	1	1
1	1	0	0	0	0

$$\begin{aligned}
 \text{Diff} &= XY'B' + X'Y'B + XYB + X'YB' \\
 &= y'(XB' + X'B) + y(XB + X'B') \\
 &= y'(X \oplus B) + y(X \oplus B') \\
 &= X \oplus Y \oplus B
 \end{aligned}$$

$$B_{out} = \Sigma(1, 2, 3, 7)$$

$$\begin{aligned}
 &= X'Y'B + X'YB' + XYB + XYB' \\
 &= X'(Y'B + YB') + YB(X' + X) \\
 &= X'(Y \oplus B) + YB
 \end{aligned}$$

## Block diagram



18- Design a Combinational Circuit that generates

the 9's Complement of (a) BCD digit

Dec	BCD	9's Complement
0	0001	8
1	0010	7
2	0011	6
3	0100	5
4	0101	4
5	0110	3
6	0111	2
7	1000	1
8	1001	0
9		

18. Design a Combinational Circuit that generates the 9's Compl of a (a) BCD digit

Dec	BCD	9's Compl
0	0000	1001
1	0001	1000
2	0010	0111
3	0011	0110
4	0100	0101
5	0101	0100
6	0110	0011
7	0111	0010
8	1000	0001
9	1001	0000

(w, x, y, z)

$$F_1 = \sum (0, 1) + d = \sum (10, 11, 12, 13, 14, 15)$$

w	x	y	z	00	01	11	10
0	1	1	0	1	0	0	1
1	0	1	1	0	1	1	0
2	1	0	1	1	1	0	0
3	0	0	1	0	0	1	1
4	1	1	0	1	0	1	0
5	0	1	0	0	1	0	1
6	1	0	0	0	1	1	1
7	0	0	1	1	0	0	0
8	1	0	0	0	0	1	1
9	0	0	0	0	0	0	0

$$F_1 = XY' + X'Y \\ = X \oplus Y$$

w	x	y	z	00	01	11	10
0	1	1	0	1	0	0	1
1	0	1	1	0	1	1	0
2	1	0	1	1	1	0	0
3	0	0	1	0	0	1	1
4	1	1	0	1	0	1	0
5	0	1	0	0	1	0	1
6	1	0	0	0	1	1	1
7	0	0	1	1	0	0	0
8	1	0	0	0	0	1	1
9	0	0	0	0	0	0	0

$$F_2 = \sum (2, 3, 4, 5) + d$$

w	x	y	z	00	01	11	10
0	1	1	0	1	0	1	1
1	0	1	1	0	1	1	0
2	1	0	1	1	1	0	0
3	0	0	1	0	0	1	1
4	1	1	0	1	0	1	0
5	0	1	0	0	1	0	1
6	1	0	0	0	1	1	1
7	0	0	1	1	0	0	0
8	1	0	0	0	0	1	1
9	0	0	0	0	0	0	0

$$F_3 = \sum (2, 3, 6, 7) + d$$

w	x	y	z	00	01	11	10
0	1	1	0	1	0	1	1
1	0	1	1	0	1	1	0
2	1	0	1	1	1	0	0
3	0	0	1	0	0	1	1
4	1	1	0	1	0	1	0
5	0	1	0	0	1	0	1
6	1	0	0	0	1	1	1
7	0	0	1	1	0	0	0
8	1	0	0	0	0	1	1
9	0	0	0	0	0	0	0

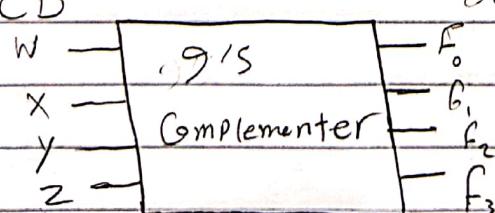
$$F_4 = \sum (0, 2, 4, 6, 8) + d$$

w	x	y	z	00	01	11	10
0	1	1	0	1	0	1	1
1	0	1	1	0	1	1	0
2	1	0	1	1	1	0	0
3	0	0	1	0	0	1	1
4	1	1	0	1	0	1	0
5	0	1	0	0	1	0	1
6	1	0	0	0	1	1	1
7	0	0	1	1	0	0	0
8	1	0	0	0	0	1	1
9	0	0	0	0	0	0	0

Combinational Circuit

input BCD

output



21. Design a Combinational Circuit that Compares two 4-bit numbers to check if they are equal.

Output = 1 if the two numbers are equal  
= 0 otherwise.

1st No  $\Rightarrow$  A  $a_3 \ a_2 \ a_1 \ a_0$

2nd No  $\Rightarrow$  B  $b_3 \ b_2 \ b_1 \ b_0$

Truth Table of 2-bit Com

$a_3$	$b_3$	Equal
0	0	1
0	1	0
1	0	0
1	1	1

$$E = a_3 b_3' + a_3' b_3$$
$$= (a_3 \oplus b_3)'$$

