

11- Convert the following boolean function from Sum-of-Products form to a Simplified Product-of-Sums Form.

$$F(x, y, z) = \sum(0, 1, 2, 5, 18, 10, 13)$$

$$\begin{aligned} F' &= x'z + y'z' + w'xz' \\ &= w'xz' + x'z + y'z' \end{aligned}$$

$$F' \rightarrow F$$

$$\therefore F = (w'xz' + x'z + y'z')$$

$$= (w+x+z)(x+z')(y+z)$$

| W | X | Y | Z | 00 | 01 | 11 | 10 |
|----|---|---|---|----|----|----|----|
| 0 | 1 | 1 | 0 | 1 | | | |
| 1 | 0 | 1 | 0 | | 0 | | |
| | | | | 12 | 13 | 15 | 14 |
| 11 | 0 | 1 | 0 | | | | |
| | | | | 8 | 9 | 11 | 10 |
| 10 | 1 | 0 | 0 | | | | 1 |

12- Simplify the following boolean function:-

$$(b) F(A, B, C, D) = \sum(1, 3, 6, 9, 11, 12, 14)$$

$$F = ABD' + BCD' + B'D$$

| A | B | C | D | 00 | 01 | 11 | 10 |
|----|---|---|---|----|----|----|----|
| 0 | 1 | 1 | 1 | | | | |
| 1 | 0 | 1 | 0 | | | | |
| | | | | 12 | 13 | 15 | 14 |
| 11 | 0 | 1 | 1 | | | | |
| | | | | 8 | 9 | 11 | 10 |
| 10 | 1 | 0 | 0 | | | | 1 |

13- Simplify the following expression to

(1) Sum-of-Products

(2) Products-of-Sums

$$(b) F = ACD' + C'D + AB' + ABCD$$

(1) Sum-of-Products

$$F = AB' + AC + C'D$$

| | | CD | | AB | | | |
|----|----|----|----|----|----|---|--|
| | | 00 | 01 | 11 | 10 | | |
| AB | CD | 00 | 1 | 3 | 2 | | |
| | | 01 | 0 | 1 | 0 | 0 | |
| 10 | 11 | 4 | 5 | 7 | 6 | | |
| 11 | 10 | 12 | 13 | 15 | 14 | | |
| 10 | 01 | 8 | 9 | 11 | 10 | | |
| 01 | 00 | 1 | 1 | 1 | 1 | | |

(2) Products-of-Sums

$$F' = BC'D' + A'D' + A'C \quad \# \text{ SOP}$$

$$F = (BC'D' + A'D' + A'C)$$

$$= (BC'D')'(A'D')'(A'C)'$$

$$= (B' + C + D)(A + D)(A + C') \quad \# \#$$

$$(c) F = (A' + B + D')(A' + B' + C')(A' + B' + C)(B' + C + D')$$

$$F = POS$$

$$F' = AB'D + ABC + ABC' + BC'D$$

(1) Sum-of-Products

$$F = A'D' + B'D' + A'B' + A'C$$

| | | CD | | AB | | | |
|----|----|----|----|----|----|---|--|
| | | 00 | 01 | 11 | 10 | | |
| AB | CD | 00 | 1 | 3 | 2 | | |
| | | 01 | 1 | 0 | 1 | 0 | |
| 10 | 11 | 4 | 5 | 7 | 6 | | |
| 11 | 10 | 12 | 13 | 15 | 14 | | |
| 10 | 01 | 8 | 9 | 11 | 10 | | |
| 01 | 00 | 1 | 1 | 1 | 1 | | |

(2) Product-of-Sums

$$F' = BC'D + AB + AD$$

$$F = (AB + AD + BC'D)'$$

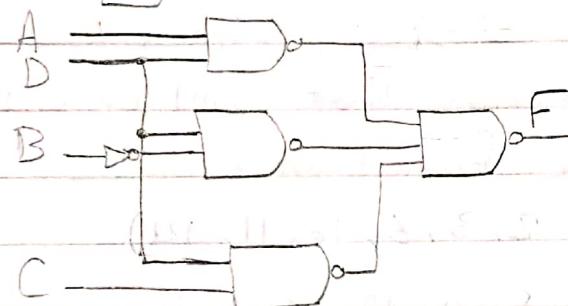
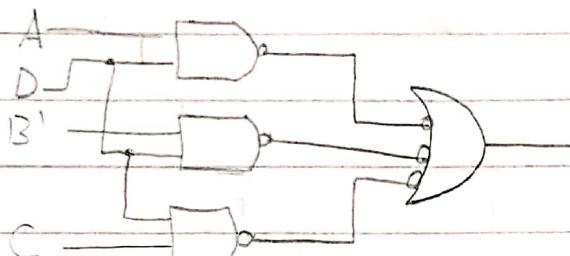
$$= (A' + B')(A' + D')(B' + C + D')$$

| | | CD | | AB | | | |
|----|----|----|----|----|----|---|--|
| | | 00 | 01 | 11 | 10 | | |
| AB | CD | 00 | 1 | 3 | 2 | | |
| | | 01 | 1 | 0 | 1 | 0 | |
| 10 | 11 | 4 | 5 | 7 | 6 | | |
| 11 | 10 | 12 | 13 | 15 | 14 | | |
| 10 | 01 | 8 | 9 | 11 | 10 | | |
| 01 | 00 | 1 | 1 | 1 | 1 | | |

16. Simplify the following functions, and implement them with two-level NAND gate circuits:-

(b) $F(A, B, C, D) = A'B'C'D + C'D + AC'D$

$$F = AD + B'D + CD$$

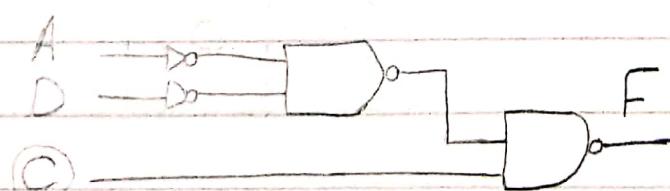
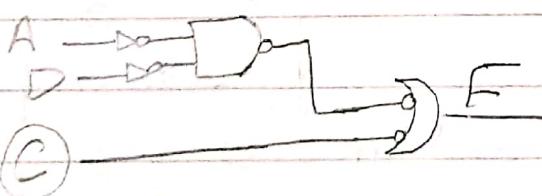


| AB | CD | 00 | 01 | 11 | 10 |
|----|----|----|----|----|----|
| 00 | 00 | 1 | 1 | 1 | 1 |
| 01 | 01 | 1 | 1 | 1 | 1 |
| 11 | 11 | 1 | 1 | 1 | 1 |
| 10 | 10 | 1 | 1 | 1 | 1 |

(c) $F(A, B, C, D) = (A' + C' + D')(A' + C')(C' + D') \rightarrow \text{POS}$

$$F = ACD + AC + CD$$

$$F = C' + A'D'$$



| AB | CD | 00 | 01 | 11 | 10 |
|----|----|----|----|----|----|
| 00 | 00 | 1 | 1 | 0 | 1 |
| 01 | 01 | 1 | 1 | 0 | 1 |
| 11 | 11 | 1 | 1 | 0 | 0 |
| 10 | 10 | 1 | 1 | 1 | 0 |

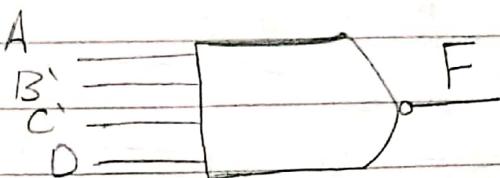
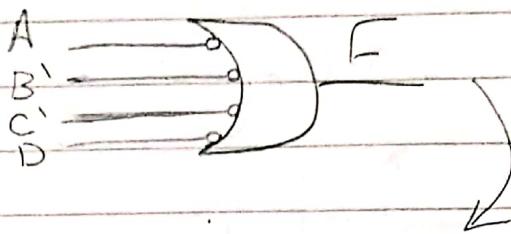
$$(d) F(A, B, C, D) = A' + B + D' + B'C$$

apply absorption rule

$$\therefore F = A' + B + D' + C$$

To draw it with NAND

gate, we should change each input condition.



NAND

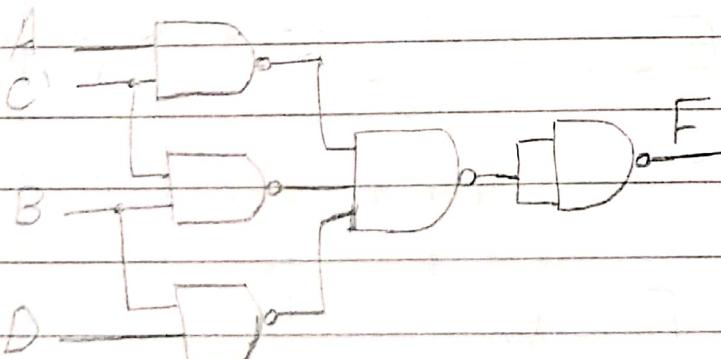
12. Draw a logic diagram that implements the complement of the function:-

$$F(A, B, C, D) = \sum m(0, 1, 2, 3, 6, 10, 11, 12)$$

$$\therefore \text{Then } F' = \sum m(4, 5, 7, 8, 9, 12, 13, 15)$$

$$F' = AC' + BC' + BD$$

$$F = (AC')'(BC')'(BD)'$$



| | | AB | CD | 00 | 01 | 11 | 10 |
|--|--|----|----|----|----|----|----|
| | | 00 | | 1 | 3 | 2 | |
| | | 01 | | 4 | 5 | 7 | 6 |
| | | | 00 | 0 | 0 | 0 | |
| | | | 01 | 12 | 13 | 15 | 14 |
| | | | | 0 | 0 | 0 | |
| | | | 10 | 8 | 9 | 11 | 10 |
| | | | | 0 | 0 | | |

18- Draw a logic diagram using only two-input NOR gates to implement:

$$F(A, B, C, D) = (A \oplus B)' \quad \text{NOR} \quad (C \oplus D) \quad \text{NOR}$$

$$\begin{aligned}
 F &= (AB' + A'B)' (CD' + C'D) \\
 &= (AB')' (A'B)' (CD' + C'D) \\
 &= (A'+B)(A+B') (CD' + C'D) \\
 &= A(A'+B) + B'(A'+B) (CD' + C'D) \\
 &= (AA' + AB + A'B' + BB') (CD' + C'D) \\
 &= (AB + A'B)(CD' + C'D) \\
 &= AB(CD' + C'D) + A'B'(CD' + C'D) \\
 &= AB CD' + AB C'D + A'B' CD' + A'B' C'D \rightarrow \text{SOP}
 \end{aligned}$$

$$F' = AB' + A'B + C'D' + CD$$

| AB | CD | 00 | 01 | 11 | 10 |
|----|----|----|----|----|----|
| 00 | 0 | 1 | | 0 | 1 |
| 01 | 1 | | 1 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 0 | 0 |

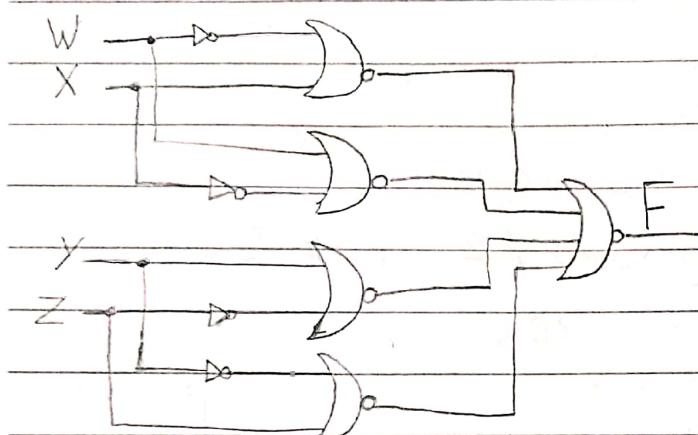
$$\begin{aligned}
 F &= (A'+B)(A+B') (C+D)(C'+D') \\
 &= [(A'+B)(A+B') (C+D)(C'+D')]' \\
 &= [AB' + A'B + C'D' + CD]' \\
 &= [(A+A'B)(B'+A'B) + (C'+CD)(D'+CD)]' \\
 &= (A+A'B)' + (B'+A'B)' + (C'+CD)' + \\
 & \quad (D'+CD)'
 \end{aligned}$$

19. Simplify the following functions, and implement them with two-level NOR gate circuits.

(b) $F(w, x, y, z) = \sum m(0, 3, 12, 15)$

$$F' = w'x + w'x + y'z + yz'$$

$$F = (w' + x)(w + x')(y + z')(y' + z)$$



| w | x | y | z | 00 | 01 | 10 | 11 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|----|----|----|----|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

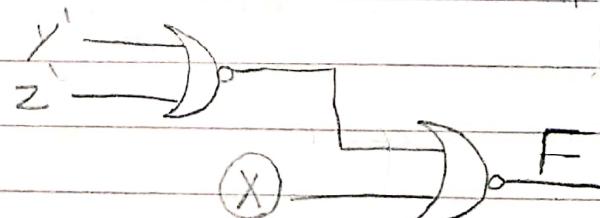
(c) $F(x, y, z) = [(x+y)(x+z)]'$

$$F = (x+y)' + (x+z)'$$

$$= x'y' + x'z' \rightarrow SOP$$

$$F = x + yz$$

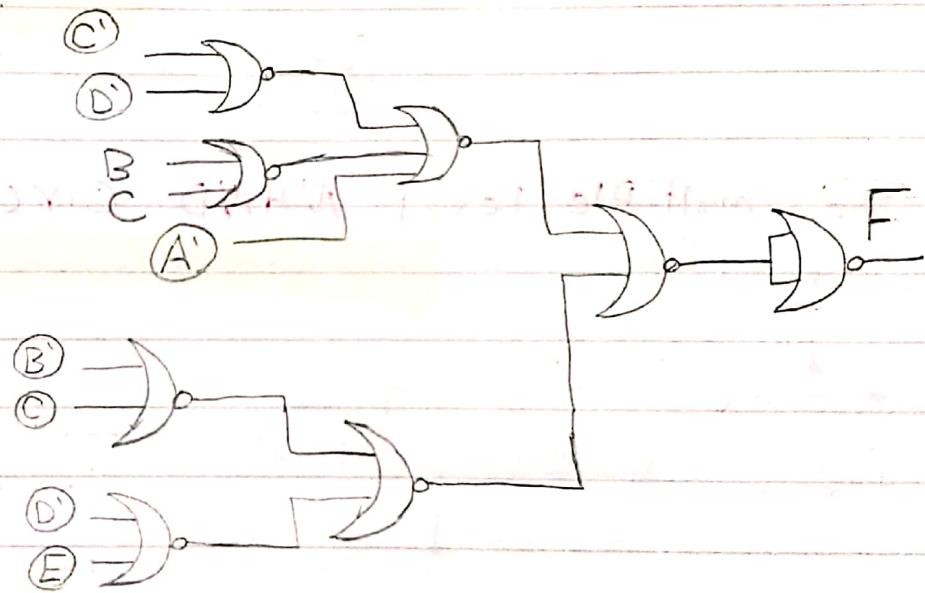
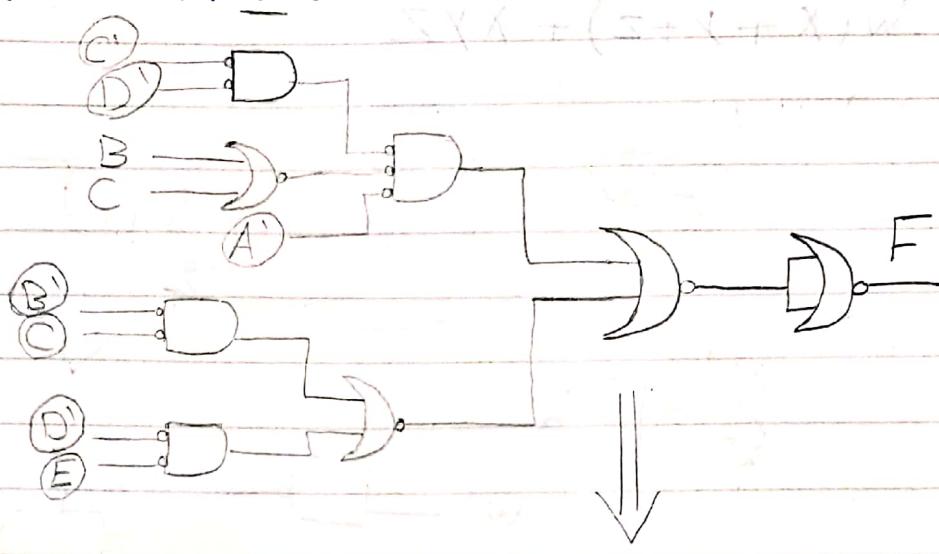
$$F = x'(y' + z')$$



| x | y | z | 00 | 01 | 11 | 10 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|----|----|----|----|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

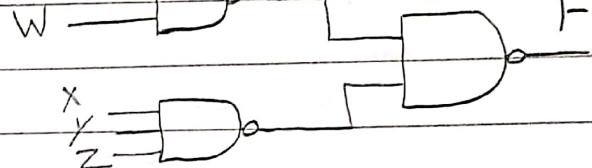
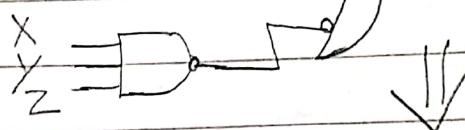
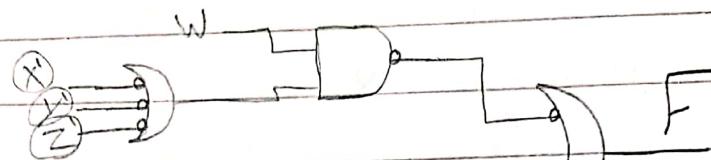
20- Draw the multi-level NOR circuit for the following expression:-

$$F = CD(B+C)A + (BC' + DE')$$

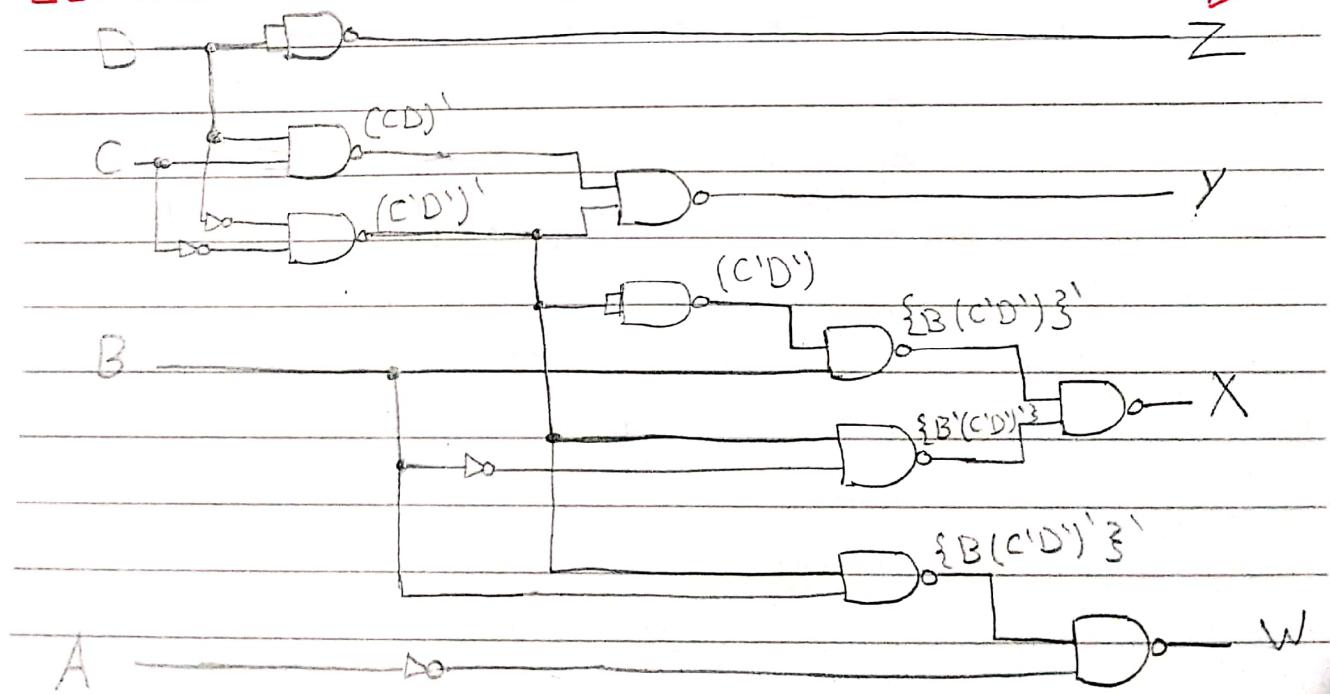


21- Draw the multiple-level NAND Circuits for the following expression:-

$$F = W(X+Y+Z) + XYZ$$



22. The multiple-level NAND Circuit



22- Convert the logic diagram of the following circuit into a multiple-level NAND gates:-

$$Z = D'$$

$$Y = CD + (C+D)'$$

$$X = B(C+D)' + B'(C+D)$$

$$W = A + B(C+D)$$

$$Z = D'$$

$$Y = [\{ CD + (C+D)' \} '] '$$

$$= [(CD)' \cdot (C'D')'] '$$

$$X = [\{ B(C+D)' + B'(C+D) \} '] '$$

$$= [\{ B(C'D') \} ' \cdot \{ B'(C+D) \} '] '$$

$$\xrightarrow{\{ (C+D) \} ' = \{ C'D \} } = (C'D)'$$

$$= [\{ B(C'D') \} ' \cdot \{ B'(C'D') \} '] '$$

$$W = [\{ A + B(C+D) \} '] '$$

$$= [A' \cdot \{ B(C+D) \} '] '$$

$$\xrightarrow{\{ (C+D) \} ' = \{ (C'D') \} } = (C'D)'$$

$$= [A' \cdot \{ B(C'D') \} '] '$$

24- Implement the following Boolean function F , using the two-level forms of logic

(a) NAND-AND

(b) AND-NOR

(c) OR-NAND

(d) NOR-OR

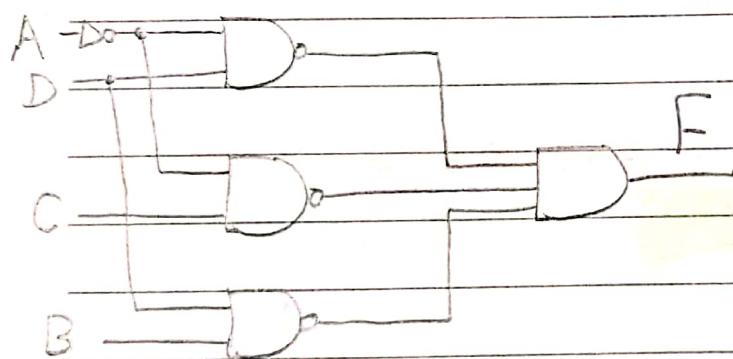
$$F = \sum m(0, 4, 8, 9, 10, 11, 12, 13)$$

(a) NAND-AND

$$F = A'D + A'C + BD$$

$$\begin{aligned} F &= (A'D + A'C + BD)' \\ &= (A'D)' \cdot (A'C)' \cdot (BD)' \end{aligned}$$

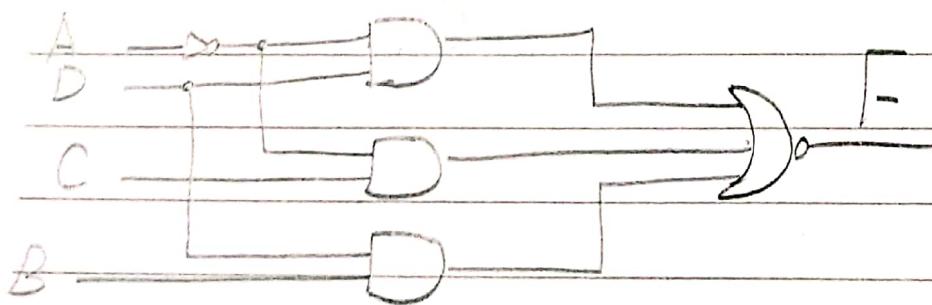
| AB \ CD | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | 1 | 0 | 0 | 0 |
| 01 | 1 | 0 | 0 | 0 |
| 11 | 1 | 0 | 0 | 0 |
| 10 | 1 | 1 | 1 | 1 |



(b) AND-NOR

$$F = A'D + A'C + BD$$

$$F = (A'D + A'C + BD)'$$



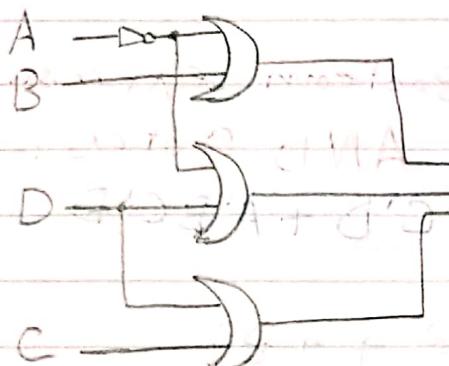
(C) OR-NAND

$$F = AB' + AD' + C'D'$$

$$F = [AB' + AD' + C'D'3']' \quad \text{Demorgan}$$

$$= [(AB')' \cdot (AD')' \cdot (C'D')']'$$

$$= [(A' + B) \cdot (A' + D) \cdot (C + D)]'$$



(d) NOR-OR

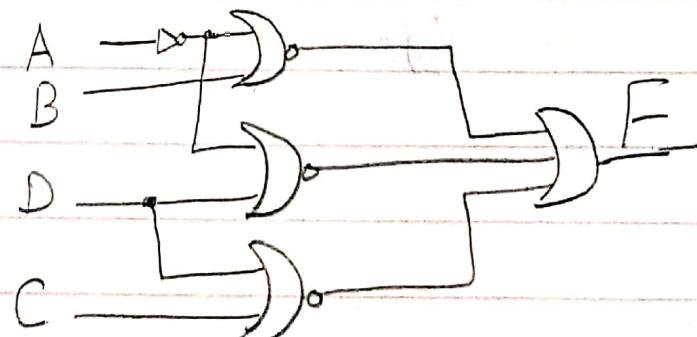
$$F = AB' + AD' + C'D' \quad \text{APPLY Demorgan twice}$$

$$= [AB' + AD' + C'D'3']'$$

$$= [(AB')' \cdot (AD')' \cdot (C'D')']'$$

$$= [(A' + B) \cdot (A' + D) \cdot (C + D)]'$$

$$= (A' + B)' + (A' + D)' + (C + D)'$$



27. Show that the dual of the exclusive-OR is also its complement:-

$$\begin{aligned}
 x \oplus y &= xy' + x'y \\
 \text{dual} &= (x+y')(x'+y) \\
 &= (\underline{xx'} + \underline{xy'}) + (\underline{x'y'} + \underline{y'y}) \\
 &= (xy + x'y) = (x \oplus y)' \quad \#
 \end{aligned}$$

3a. Implement the following Boolean expression with exclusive-OR and AND gates:-

$$F = AB' \underline{CD} + A' \underline{B} \underline{C} D + AB' \underline{C} \underline{D} + A' \underline{B} \underline{C} \underline{D}$$

$$= CD' (AB' + A'B) + C'D (AB' + A'B)$$

$$= (AB' + A'B) + (CD' + C'D) \text{ Ans Q3a (b)}$$

$$= (A \oplus B) (C \oplus D) \quad \#$$

