Algorithms

Graph Representation Homework 3

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Teaching, Training and Coaching for more than a decade!

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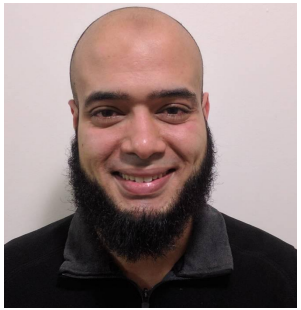
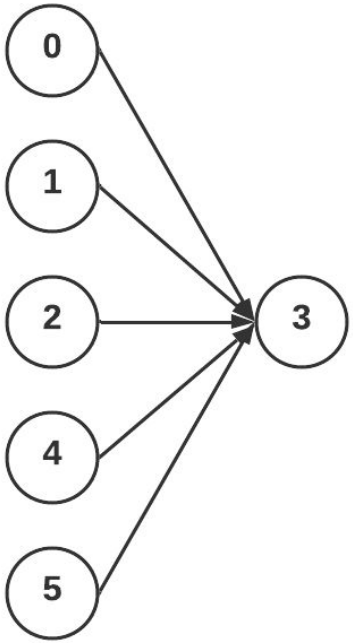


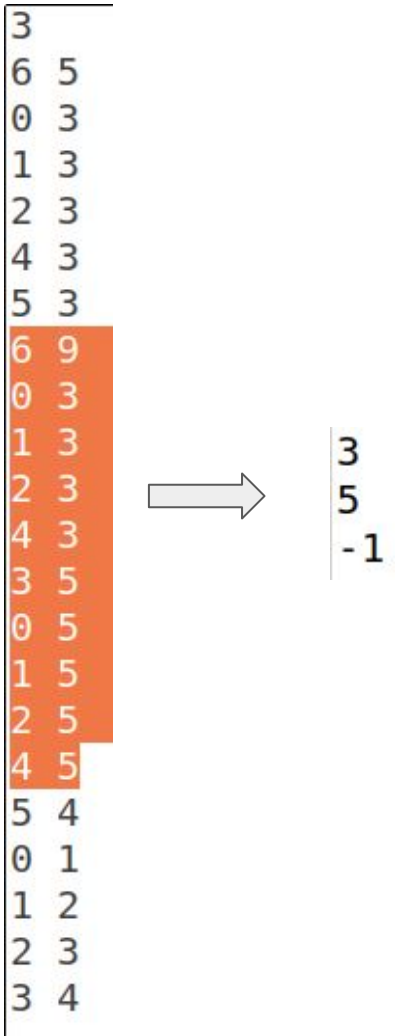
Problem #1: Find a sink

- In a **directed** graph of N nodes, a **universal sink** is a vertex with *in-degree N-1 and out-degree 0.*
- Prove: In a directed graph, there is at **most one** universal sink.
  - I actually give the proof in the quiz
- Write an O(N) algorithm to find the sink. Prove correctness
  - int find_universal_sink_fast(GRAPH &graph)
  - Graph is an **adjacency matrix** for the directed graph (including multiple edges / loops)
  - Return 0-based index for node i that is think or -1 if none
  - Think carefully and develop several cases



Problem #1: Find a sink

- Your program should read multiple graphs as following
  - First read integer T for T test cases
  - Then for each test case read 2 numbers: nodes and edges
  - Then read the edges (0-based)
- There is an **input file** (find_sink-input.txt) and output file (find_sink-output.txt).
  - Copy and test your code (or read from a file).
  - Compare your output with my output

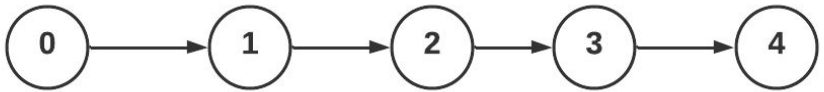


Problem #2: Adjacency matrix power

- Assume we built the **binary** adjacency matrix of a **directed** unweighted graph
  - M[i][j] = 1 IFF there is at least one edge from i to j
- Assume we computed the matrix C = M^2, that is C = MxM
- In this homework, you will think/investigate what is C in terms of the original graph
- Optional challenge: What is M^k: that is raising M to the power K
- You don’t need to write code to compute matrix power.
- Just use any online calculator

Problem #2: Adjacency matrix power

- Tips. How to do some analysis?
- You need a mix of 2 things
  - Logical thinking. With proper thinking skills, you can find observations
  - Finding patterns in concrete examples
    - Another tip. Start with simple examples
    - For example: What is the smallest graph? A chain of N-1 edges



$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

"Acquire knowledge and impart it to the people."

"Seek knowledge from the Cradle to the Grave."