CS605/CS635 - Modeling and Simulation
Course Sheets

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Contents

Chapter 1

Introduction to Discrete-Event System Simulation

1. Name several *entities*, *attributes*, *activities*, *events* and *state variables* for the following systems :
   (a) A cafeteria
   (b) A grocery store
   (c) A laundromat
   (d) A fast-food restaurant
   (e) A hospital emergency room
   (f) A taxicab company with 10 taxis
   (g) An automobile assembly line
2. Consider the following systems. Identify whether each is : *stochastic* or *detirministic*, *static* or *dynamic* and *discrete-time* or *continuous-time*. Explain your answers:
   (a) Ideal pendulum with fixed length, fixed mass, fixed gravitational field, operating in a vacuum.
   (b) Internet traffic (packets per second) flowing in/out of Stevens Network.
   (c) Velocity distribution of water flowing in water pipe at very low speed ($Re < 10$).
   (d) Number of raindrops per second hitting a 12" horizontal metal plate during a hurricane.
   (e) Number of dots showing on the face of a pair of dice during a game of dice.

3. Describe what you think would be the most effective way to study each of the following systems, in terms of the possibilities in Figure 1.1 below, and discuss why.
   (a) A small section of an existing factory.
   (b) A freeway interchange that has experienced severe congestion
   (c) An emergency room in an existing hospital
   (d) A pizza-delivery operation
   (e) The shuttle-bus operation for a rental-car agency at an airport
   (f) A battlefield communications network

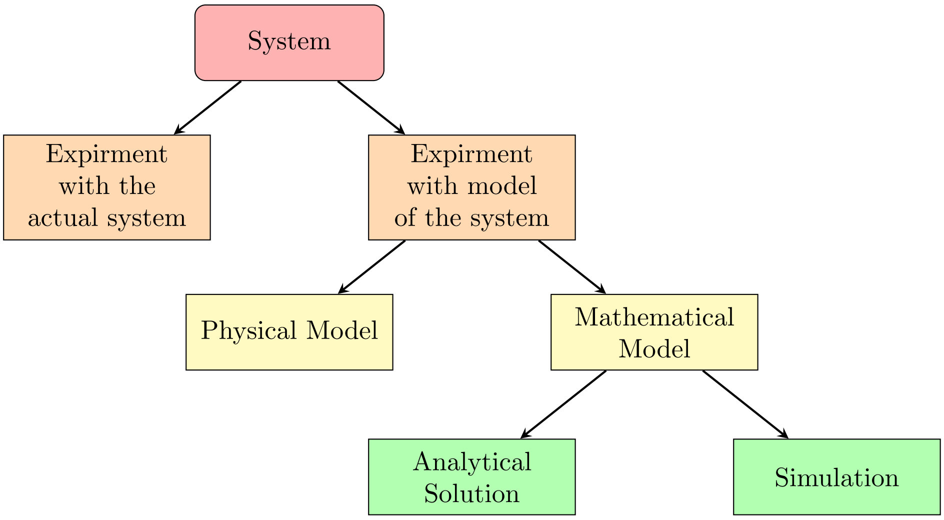

Figure 1.1: System Study Techniques

Chapter 2

Hand Simulation Examples

Refer to the following textbook solved examples :

i. The Grocery Checkout, a Single-Server Queue [Example 2.5 Banks et al., 2009, p. 63] [Example 2.1 Banks et al., 2004, p. 25]

ii. Replacing Bearings in a Milling Machine(A Reliability Problem) [Example 2.9 Banks et al., 2009, p. 83] [Example 2.5 Banks et al., 2004, p. 43]

Exercises :

1. Consider a batch manufacturing process in which a machine processes jobs in batches of three units. The process starts only when there are three or more jobs in the buffer in front of the machine. Otherwise, the machine stays idle until the batch is completed. Assume that job interarrival times are uniformly distributed between 2 and 8 hours, and batch service times are uniformly distributed between 5 and 15 hours.
   Assuming the system is initially empty, simulate the system manually for three batch service completions and calculate the following statistics:

   - Average number of jobs in the buffer(excluding the batch being served)
   - Probability distribution of number of jobs in the buffer (excluding the batch being served)
   - Machine utilization
   - Average job waiting time (time in buffer)
   - Average job system time (total time in the system, including processing time)
   - System throughput (number of departing jobs per unit time)

   Approach :

   - After each arrival, schedule the next interarrival time, and when each batch goes into service, schedule it's service completion time. To obtain these quantities, use random numbers obtained from Table A.1 [See Banks et al., 2009, p. 610] [Banks et al., 2004, p. 501] (these are equally likely between 0 and 1, and statistically independent of each other).
   - Then transform these numbers as follows :
     (a) To generate the next random interarrival time (A), get the next random number R from Table A.1 and set $A = 2 + 6R$
     (b) To generate the next random batch service time (B), get the next random number R from Table A.1 and set $B = 5 + 10R$

2. For a job shop the interarrival times of jobs are distributed as follows :

Time Between Arrivals (Hours)	*Probability*
0	.23
1	.37
2	.28
3	.12

Following random values are uniformly distributed between 0 and 999. Use these values to generate the first 10 values of the interarrival times.

1	2	3	4	5	6	7	8	9	10
786	903	240	874	308	237	490	128	787	364

3. Pizza delivery time is distributed as follows:

Delivery Time (Minutes)	20	25	30	35	40
Probability	0.15	0.25	0.35	0.20	0.05

Use the following random number generator to generate the first 10 values of the delivery times.

$$R_i = (37 \times R_{i-1}) \mod 107, \qquad \text{where } R_0 = 35$$

4. For a job shop with single queue and single server the interarrival times of the jobs and their service time is provided as follows:

Job	1	2	3	4	5	6	7	8	9	10
Interarrival Time	0	4	2	3	2	3	3	4	2	1
Service Time	3	4	2	3	4	5	2	2	3	4

Construct the simulation table, then calculate the following statistics :

(a) Average time in queue.
(b) Average time in system.
(c) Average queue length.
(d) Server utilization.

5. For *The Able-Baker Call Center Problem* [Example 2.6 Banks et al., 2009, p. 69] [Example 2.2 Banks et al., 2004, p. 32] draw the flowcharts of both arrival and departure events.

6. A bank branch has two tellers, the customers arrive in the main hall and wait for the empty teller, if all tellers are occupied the customer may balk. For 10 customers construct the simulation table and calculate the average queue length and the average waiting time in the system.
Use the following random values for *interarrival time*, *service time* and *balk decision*:

Interarrival time	3	2	2	3	1	1	3	2	1	3
Service Time	4	3	5	6	5	4	3	3	3	4
Balk Decision	No	Yes	No	No	No	No	Yes			

7. A news dealer receive 100 newspaper every day, the daily demand is subjected to day type (good, fair, poor). Simulate the system in 10 days, given the following random numbers for *day type* and *daily demand*:

Day Type	Good	Fair	Fair	Good	Poor	Fair	Fair	Poor	Poor	Good
Good Day	90	80	110	105	95	70	85	90	90	60
Fair Day	70	85	90	80	65	88	100	75	95	75
Poor Day	45	75	50	60	65	40	50	55	65	90

Given that:

- Newspaper cost : 40¢.
- Newspaper price : 60¢.
- Scrap paper price : 10¢.

Calculate the net profit.

Chapter 5

Probability Distributions

Refer to the following textbook solved examples :

- **Discrete Probability Distributions :**
  i. Binomial Distribution : [Example 5.10 Banks et al., 2009, p. 203] [Banks et al., 2004, p. 142]
  ii. Negative Binomial Distribution : [Example 5.11 Banks et al., 2009, p. 205] [Banks et al., 2004, p. 144]
  iii. Poisson Distribution : [Example 5.12 and Example 5.13 Banks et al., 2009, p. 206] [Banks et al., 2004, p. 144,145]

- **Continuous Probability Distributions :**
  i. Uniform Distribution : [Example 5.16 Banks et al., 2009, p. 209] [Banks et al., 2004, p. 147]
  ii. Exponential Distribution : [Example 5.17 and Example 5.18 Banks et al., 2009, p. 210,211] [Banks et al., 2004, p. 149]
  iii. Normal Distribution :
    (a) [Example 5.21 Banks et al., 2009, p. 217] [Banks et al., 2004, p. 154]
    (b) [Example 5.22 Banks et al., 2009, p. 218] [Banks et al., 2004, p. 155]
    (c) [Example 5.23 Banks et al., 2009, p. 220] [Banks et al., 2004, p. 157]
    (d) [Example 5.24 Banks et al., 2009, p. 221] [Banks et al., 2004, p. 157]

Exercises :

5.1 Discrete Probability Distributions

1. (The Poisson distribution can be used to approximate the binomial distribution when n is large and p is small – say, p less than 0.1. In utilizing the Poisson approximation, let $\lambda = np$). In the production of ball bearings, bubbles or depressions occur, rendering the ball bearing unfit for sale. It has been noted that, on the average, one in every 800 of the ball bearings has one or more of these defects. What is the probability that a random sample of 4000 will yield fewer than three ball bearings with bubbles or depressions ?

2. Accidents at an industrial site occur one at a time, independently, and completely at random, at a mean rate of one per week. What is the probability that no accidents occur in the next three weeks ?

3. A production process manufactures alternators for outboard engines used in recreational boating. On the average, 1% of the alternators will not perform up to the required standards when tested at the engine assembly plant. when a large shipment of althernators is received at the plant, 100 are tested, and if more than two are nonconforming; the shipment is returned to the alternator manufacturer. What is the probability of returning a shipment ?

4. An industrial chemical that will retard the spread of fire in paint has been developed. The local representative has estimated, from past experience that 48% of the sales calls will result in an order.
   (a) What is the probability that the first order will come on the fourth sales call of the day ?
   (b) If eight sales calls are made in a day, what is the probability of receiving exactly six orders ?
   (c) If four sales calls are made before lunch, what is the probability that one or fweer results in an order?

5. The number of hurricanes hitting the coast of Florida annually has a Poisson distribution with mean of 0.8.
   (a) What is the probability that more than two hurricanes will hit the Florida coast in a year?
   (b) What is the probability that exactly one hurricane will hit the coast of Florida in a year ?

6. Suppose that an average of 30 customers per hour arrive at the Sticky Donut Shop in accordance with a Poisson process. What is the probability that more than 5 minutes will elapse before 2 customers arrive?

5.2 Continuous Probability Distributions

1. Let X be a random variable that is normally distributed, with mean 10 and variance 4. Find the values a and b such that $p(a < X < b) = 0.90$ and $|\mu - a| = |\mu - b|$

2. Batteries of a certain manufacturer have a time-to-failure following a Weibull distribution $\alpha = \dfrac{1}{2}, \beta = \dfrac{1}{4}, \gamma = 0$
   (a) What fraction of batteries are expected to fail prior to 1.5 years ?
   (b) What fraction of batteries are expected to last longer than the mean life ?
   (c) What fraction of batteries are expected to last between 1.5 and 2.5 years ?

3. The lifetime, in years of a satellite placed in orbit is given by the following pdf:
$$f(x) = \begin{cases} 0.4e^{-0.4x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$
   (a) What is the probability that the satellite is still "alive" after 5 years?
   (b) What is the probability that the satellite dies between 3 and 6 years from the time it is placed in orbit ?

4. Determine the variance $V(X)$ of the triangular distribution.

5. A mainframe computer crashes in accordance with a Poisson processs, with a mean rate of one crash every 36 hours. Determine the probability that the next crash will occur between 24 and 48 hours after the last crash.

Markov Chains

1. A car rental has three locations A, B and C. A car rented from any location can be returned to any of the three locations according to the following probabilities :

		Returned to		
		A	B	C
Rented from	A	0.3	0.4	0.3
	B	0.2	0.3	0.5
	C	0.4	0.5	0.1

   (a) If a car is rented from location A, returned and then rented again, what is the probability that it is returned to location C the second time?
   (b) If the agency has a total of 1000 cars, what rates of space should they use for locations A, B and C?

2. A petrol station owner is considering the effect on his business *(Superpet)* of a new petrol station *(Global)* which has opened just down the road. Currently (of total market shared between *Superpet* and *Global*) *Superpet* has 80% of the market and *Global* has 20%.
   Analysis over the last week has indicated the following probabilities for customers switching the station they stop at each week :

		Returned to	
		Superpet	*Global*
Rented from	*Superpet*	0.75	0.25
	Global	0.55	0.45

   (a) What will be the expected market share for *Superpet* and *Global* after another two weeks have passed?
   (b) What would be the long-run prediction for the expected market share for *Superpet* and *Global*?

3. The population of trees in a forest is divided as follows:

   - Baby trees (age : 0-15 years).
   - Young trees (age : 15-30 years).
   - Middle trees (age : 30-45 years).
   - Old trees (age > 45 years).

   Assume the following :

   - The forest is newly planted.
   - A certain percentage dies from each group (10% of the baby population, 20% of the young, 30% of the middle and 40% of the old).
   - Surviving trees enter the next phase.
   - Dead trees are replaced by baby trees.
   - The total population of trees is constant and equals 5000 trees.

   (a) Draw the state diagram.
   (b) Calculate the steady-state probabilities.
   (c) Show the distribution of the trees in 15 years, 30 years, 60 years given that the forest is newly planted.

Chapter 6

Queueing Theory

Refer to the following textbook solved examples :

i. [Example 6.10 Banks et al., 2009, p. 268] [Banks et al., 2004, p. 197]
ii. [Example 6.12 Banks et al., 2009, p. 270] [Banks et al., 2004, p. 199]

Exercises :

1. Consider a situation where we have exponentially distributed packet lengths with mean 1250 bytes and a 10Mbit/s link. Assuming Poisson arrivals, for what values of λ (the packet arrival rate) will the average total time in the system be less than or equal to 10ms.

2. A printing $M/M/1$ facility have a mean arrival rate of 5 jobs/min and a mean service time of 10 seconds.

   (a) Draw the state space diagram for the first 4 states.

   (b) State the equivalent transition rate matrix.

3. Consider an $M/M/1$ system in which customers arrive according to a Poisson process of rate λ. Service rate is $\mu = 20$ customers/minute. The average number of customers is $N = 3$. Calculate λ and the *average waiting time*.

4. Consider an $M/M/1$ queuing system with arrival rate λ and service rate μ.

   (a) Derive the formula for the average number of customers in the system.

   (b) Calculate the average total time T if the service rate is $\mu = 50$ customers/minute and the average number of customers is $N = 4$.

5. An $M/M/1$ system has the service rate $\mu = 10$ customers per minute. Average time in the system for one customer is $T = 3$ minutes.

   (a) Derive the formula for the average time in the system T.

   (b) Evaluate *arrival rate*, *average number of customers*, and *service time*.

6. A communication channel is operating at a transmission rate of 1,000,000 bps. To the channel arrive packets according to a Poisson process with rate 100 packets per second.The packets have an exponentially distributed length with a mean of 5,000 bits. We assume that the channel can be modeled as an $M/M/1$ system with queuing discipline FCFS(First-Come First-Served).
   Calculate the following :

   (a) Utilization

   (b) Average service time.

   (c) Average number of packets.

   (d) Average waiting time.

7. Data packets arrive to a communication node according to a Poisson process with an average rate of $\lambda = 2400$ packets per minute. The packets have exponentially distributed lengths with a mean of $v = 1000$ bits. A single outgoing communication link is operating at a transmission rate of K bits/second. We assume that the link has a very large buffer so that it can be modeled as an $M/M/1$ system with queuing discipline FCFS(First-Come First-Served).

   (a) Evaluate K if the average system time is 1 s.

   For that value of K determine the following :

   (b) Departure rate.

   (c) Average packet count.

   (d) Average service time.

   (e) Average waiting time.

8. Jobs(customers) arriving at an $M/M/1$ system according to a Poisson process with an average rate of 8 jobs per second. The Service rate is $\mu = 10$ jobs per second.
   Find :

   (a) The probability that the system is idle (no customers in the system).

   (b) The probability that there are exact 2 customers in the system.

   (c) Average number of customers in the system.

   (d) Average number of customers in the queue.

9. Consider an $M/M/1$ system in which customers arrive according to a Poisson process of rate λ . Service rate is $\mu = 10$ customers/second. The average system time is $T = 0.2$ s.

   (a) Calculate λ.

   (b) Find T if we replace server with a faster one, which has a service rate of $\mu = 40$ customers/second (but the arrival rate remains the same, λ customers/second).

Chapter 7

Random Number Generation

Refer to the following textbook solved examples :

- **Linear Congruential Method :**
  - i. [Example 7.1 Banks et al., 2009, p. 298] [Banks et al., 2004, p. 224]
  - ii. [Example 7.2 Banks et al., 2009, p. 299] [Banks et al., 2004, p. 225]
  - iii. [Example 7.3 Banks et al., 2009, p. 300] [Banks et al., 2004, p. 226]
- **Tests for Random Numbers :**
  - **Frequency Tests**
    - i. *The Kolmogorov-Smirnov test* : [Example 7.6 Banks et al., 2009, p. 306] [Banks et al., 2004, p. 231]
    - ii. *The chi-square test* : [Example 7.7 Banks et al., 2009, p. 307] [Banks et al., 2004, p. 232]
  - **Tests for Autocorrelation** : [Example 7.8 Banks et al., 2009, p. 311] [Banks et al., 2004, p. 234]

Exercises :

1. Use the linear congruential method to generate a sequence of three two-digit random integers and corresponding random numbers. Let $X_0 = 27$, $a = 8$, $c = 47$, and $m = 100$.

2. Consider the multiplicative congruential generator under the following circumstances :

   (a) $X_0 = 7$, $a = 11$, $m = 16$

   (b) $X_0 = 8$, $a = 11$, $m = 16$

   (c) $X_0 = 7$, $a = 7$, $m = 16$

   (d) $X_0 = 8$, $a = 7$, $m = 16$

   Generate enough values in each case to complete a cycle. What inferences can be drawn? Is maximum period achieved?

3. In some applications, it is useful to be able to quickly skip ahead in a pseudo-random number sequence without actually generating all of the intermediate values.

   (a) For a linear congruential generator with $c = 0$, show that $X_{i+n} = (a^n X_i) \mod m$.

   (b) Next, show that $(a^n X_i) \mod m = (a^n \mod m) X_i \mod m$ (this result is useful because $a^n \mod m$ can be precomputed, making it easy to skip ahead n random numbers from any point in the sequence).

4. Use the multiplicative congruential method to generate a sequence of four three-digit random integers and corresponding random numbers. Let $X_0 = 117$, $a = 43$, and $m = 1000$.

5. The sequence of numbers 0.54, 0.73, 0.98, 0.11 and 0.68 has been generated. Use the Kolmogorov-Smirnov test with $\alpha = 0.05$ to learn whether the hypothesis that the numbers are uniformly distributed on the interval $[0, 1]$ can be rejected.

6. Test the following sequence of numbers for uniformity and independence, using procedures you learned in this chapter :

0.594	0.928	0.515	0.055	0.507	0.351	0.262	0.797	0.788	0.442
0.097	0.798	0.227	0.127	0.474	0.825	0.007	0.182	0.929	0.852

7. The following is the set of single-digit numbers from a random number generator.

6	7	0	6	9	9	0	6	4	6
4	0	8	2	6	6	1	2	6	8
5	6	0	4	7	1	3	5	0	7
1	4	9	8	6	0	9	6	6	7
1	0	4	7	9	2	0	1	4	8
6	9	7	7	5	4	2	3	3	3
6	0	5	8	2	5	8	8	3	1
4	0	8	1	7	0	0	6	2	8
5	6	0	8	0	6	9	7	0	0
3	1	5	4	3	8	3	3	2	4

Using the appropriate test, check whether the numbers are uniformly distributed.

Chapter 8

Random-Variate Generation

Refer to the following textbook solved examples :

i. Inverse-Transform Technique - Exponential Distribution : [Example 8.1 Banks et al., 2009, p. 320] [Banks et al., 2004, p. 241]

ii. Inverse-Transform - Empirical Discrete Distribution : [Example 8.4 Banks et al., 2009, p. 330] [Banks et al., 2004, p. 250]

iii. Inverse-Transform - The Geometric Distribution : [Example 8.5 Banks et al., 2009, p. 332] [Example 8.6 Banks et al., 2004, p. 253]

Exercises :

1. Develop a random-variate generator for a random variable X with the pdf

$$f(x) = \begin{cases} e^{2x}, & -\infty < x \leq 0 \\ e^{-2x}, & 0 < x < \infty \end{cases}$$

2. Develop a generator for a triangular distribution with range (1, 10) and mode at $x = 4$. Generate 1000 values of the random variate, compute the sample mean, and compare it to the true mean of the distribution.
3. Given the cdf $F(x) = x^4/16$ on $0 \leq x \leq 2$, develop a generator for this distribution. Generate 1000 values of the random variate, compute the sample mean, and compare it to the true mean of the distribution.
4. Lead times have been found to be exponentially distributed with mean 3.7 days. Generate five random lead times from this distribution.
   Use random numbers obtained from Table A.1 [See Banks et al., 2009, p. 610] [Banks et al., 2004, p. 501]

Chapter 9

Input Modeling

Refer to the following textbook solved examples :

- **Parameter Estimation :**
  - **Preliminary Statistics: Sample Mean and Sample Variance**
    i. *Grouped Data:*
    [Example 9.8 Banks et al., 2009, p. 368] [Example 9.5 Banks et al., 2004, p. 280]
    ii. *Continuous Data in Class Intervals:*
    [Example 9.9 Banks et al., 2009, p. 369] [Example 9.6 Banks et al., 2004, p. 281]
  - **Suggested Maximum Likelihood Estimators (MLE)**
    i. *Exponential Distribution:*
    [Example 9.10 Banks et al., 2009, p. 371] [Example 9.11 Banks et al., 2004, p. 284]
    ii. *Poisson Distribution:*
    [Example 9.12 Banks et al., 2009, p. 374] [Example 9.7 Banks et al., 2004, p. 283]
    iii. *Normal Distribution:*
    [Example 9.14 Banks et al., 2009, p. 374] [Example 9.9 Banks et al., 2004, p. 283]
- **Goodness-of-Fit Tests:**
  - Chi-Square Test
    i. *Chi-Square Test Applied to Poisson Assumption:*
    [Example 9.17 Banks et al., 2009, p. 378] [Example 9.14 Banks et al., 2004, p. 289]
    ii. *Chi-Square Test for Exponential Distribution:*
    [Example 9.18 Banks et al., 2009, p. 380] [Example 9.15 Banks et al., 2004, p. 290]
  - Kolmogorov-Smirnov Goodness-of-Fit Test
    i. *Kolmogorov-Smirnov Test for Exponential Distribution:*
    [Example 9.19 Banks et al., 2009, p. 382] [Example 9.16 Banks et al., 2004, p. 292]

Exercises :

1. The highway between Atlanta, Georgia and Athens, Georgia has a high incedence of accidents along its 100 kilometers. Public safety officers say that the occurrence of accidents along the highway is randomly (uniformly) distributed, but the news media say otherwise. The Georgia Department of Public Safety published records for the month of September. These records indicated the point at which 30 accidents involving an injury or death occurred, as follows (the data points representing the distance from the city limits of Atlanta):

88.3	40.7	36.3	27.3	36.8
91.7	67.3	7.0	45.2	23.3
98.8	90.1	17.2	23.7	97.4
32.4	87.8	69.8	62.6	99.7
20.6	73.1	21.6	6.0	45.3
76.6	73.2	27.3	87.6	87.2

   Use the Kolmogorov-Smirnov test to discover whether the distribution of location of accidents is uniformly distributed for the month of September.

2. Records pertaining to the monthly number of job-related injuries at an underground coal mine were being studied by a federal agency. The values for the past 100 months were as follows:

Injuries per Month	*Frequency of Occurrence*
0	35
1	40
2	13
3	6
4	4
5	1
6	1

   (a) Apply the chi-square test to these data to test the hypothesis that the underlying distribution is Poisson. Use the level of significance $\alpha = 0.05$

   (b) Apply the chi-square test to these data to test the hypothesis that the distribution is Poisson with mean 1.0. Again let $\alpha = 0.05$.

   (c) Whate are the differences between parts (a) and (b), and when might each case arise?

3. Suppose one wishes to determine just how biased an *unfair coin* is.
Call the probability of tossing a HEAD p. The goal then becomes to determine p.
Suppose the coin is tossed 80 times, i.e. the sample might be something like $x_1 = H, x_2 = T, \ldots, x_80 = T$, and the count of the number of HEADS "H" is observed.
The probability of tossing TAILS is $1 - p$ (so here p is the value abbove).
Suppose the outcome is 49 HEADS and 31 TAILS, and suppose the coin was taken from a box containing three coins:
one which gives HEADS with probability $p = \dfrac{1}{3}$, one which gives HEADS with probability $p = \dfrac{1}{2}$ and another which gives HEADS with probability $p = \dfrac{2}{3}$.
The coins have lost their labels, so which one it was is unknown.
Using *maximum likelihood estimation*, find the coin.

4. Using MLE method, estimate the parameters of an exponential distribution fitted to the readings of Question 1

5. According to the records of the National Safety Council, accidental deaths in the United States during 2002 had the following distribution according to the principal types of accidents :

Motor Vehicle	*Falls*	*Drowning*	*Fire*	*Poison*	*Other*
45%	15%	4%	3%	16%	17%

Suppose that an accidental death dataset from a particular geographical region yielded the following frequency distribution for the principal types of accidents :

Motor Vehicle	*Falls*	*Drowning*	*Fire*	*Poison*	*Other*
442	161	42	33	162	150

Perform a Chi-square goodness of fit test.

6. Consider the following data on 12 northern red oaks from an unthinned stand in Southwestern Wisconsin:

Age(Years)	97	93	88	81	75	57	52	45	28	15	12	11
DBH(inch)	12.5	12.5	8.0	9.5	16.5	11.0	10.5	9.0	6.0	1.5	1.0	1.0

   (a) Estimate the linear model that fits these data.

   (b) Comment on the correlation between the two variables

   (c) Assume that a new value of the yield is observed after the regression model is fit to the data. This new observation is independent of the observations used to obtain the regression model. If 93 is the level of the age at which the new observation was taken, estimate the new value of DBH.

   (d) If the actual value of the DBH is 14.1. Using confidence interval technique, comment on your hypothesis.

7. The time required for 50 different employees to compute and record the number of hours worked during the week was measured, with the following results in minutes:

Employee	*Time (minutes)*	*Employee*	*Time (minutes)*	*Employee*	*Time (minutes)*	*Employee*	*Time (minutes)*
1	1.88	14	0.79	26	0.04	39	0.17
2	0.54	15	0.21	27	1.49	40	4.29
3	1.90	16	0.80	28	0.66	41	0.80
4	0.15	17	0.26	29	2.03	42	5.50
5	0.02	18	0.63	30	1.00	43	4.91
6	2.81	19	0.36	31	0.39	44	0.35
7	1.50	20	2.03	32	0.34	45	0.36
8	0.53	21	1.42	33	0.01	46	0.90
9	2.62	22	1.28	34	0.10	47	1.03
10	2.67	23	0.82	35	1.10	48	1.73
11	3.53	24	2.16	36	0.24	49	0.38
12	0.53	25	0.05	37	0.26	50	0.48
13	1.80			38	0.45		

   Use the chi-square test(as in Example 9.18) to test the hypothesis that these service times are exponentially distributed. Let the number of class intervals be $k = 6$. Use the level of significance $\alpha = 0.05$.

8. The time (in minutes) between requests for the hookup of electric service was accurately recorded at the Gotwatts Flash and Flicker Company, with the following results for the last 50 requests:

0.661	4.910	8.989	12.801	20.249
5.124	15.033	58.091	1.543	3.624
13.509	5.745	0.651	0.965	62.146
15.512	2.758	17.602	6.675	11.209
2.731	6.892	16.713	5.692	6.636
2.420	2.984	10.613	3.827	10.244
6.255	27.969	12.107	4.636	7.093
6.892	13.243	12.711	3.411	7.897
12.413	2.169	0.921	1.900	0.315
4.370	0.377	9.063	1.875	0.790

   How are the times between reuests for service distributed? Develop and test a suitable model.

Chapter 10

Estimation of Absolute Performance(Output Analysis)

10.1 Confidence-Interval Estimation

1. A simulation model of a job shop was developed to investigate different scheduling rules.To validate the model, the scheduling rule currently used was incorporated into the model and the resulting output was compared against observed system behaior. By searching the previous year's database records, it was estimated that the average number of jobs in the shop was 22.5 on a given day.Seven independent replications of the model were run, each of 30 days' duration, with the following results for average number of jobs in the shop :

   18.9 22.0 19.4 22.1 19.8 21.9 20.2

   (a) Develop and conduct a statistical test to evaluate whether model output is consistent with system behavior. Use the level of significance $\alpha = 0.05$.

2. System data for the job shop of Exercise 1 revealed that the average time spent by a job in the shop was approximately 4 working days. The model made the following predictions, on seven independent replications, for average time spent in the shop :

   3.70 4.21 4.35 4.13 3.83 4.32 4.05

   (a) Is model output consistent with system behavior? Conduct a statistical test, using the level of significance $\alpha = 0.01$

3. For the job shop of Exercise 1, four sets of input data were collected over four different 10-day periods, together with the average number of jobs in the shop Z_i for each period. The input data were used to drive the simulation model for four runs of 10 days each, and model predictions of average number of jobs in the shop Y_i were collected, with these results :

i	1	2	3	4
Z_i	21.7	19.2	22.8	19.4
Y_i	24.6	21.1	19.7	24.9

   (a) Conduct a statistical test to check the consistency of system output and model output. Use the level of significance $\alpha = 0.05$.

   (b) If a difference of two jobs is viewed as important to detect, what sample size is required to guarantee a probability of at least 0.80 of detecting this difference, if it indeed exists? (Use $\alpha = 0.05$.)

4. Use the *confidence-interval* approach to assess the validity of the revised bank model [refer to Banks et al., 2009, p. 423] [Banks et al., 2004, p. 324] , $\varepsilon = 1$ minute.

Replication	*Average waiting time (in minutes)*
1	5.46
2	7.54
3	3.45
4	3.98
5	4.89
6	6.87

Use $\alpha = 0.05$.

5. A sample of size $n = 100$ produced the sample mean of $X = 16$. Assuming the population standard deviation $\sigma = 3$, compute 95% confidence interval for the population mean μ.

6. Assuming the population standard deviation $\sigma = 3$, how large should a sample be to estimate the population mean μ with a margin of error not exceeding 0.5?

7. We observed 28 successes in 70 independent Bernoulli trials. Compute a 90% confidence interval for the population proportion p.

8. The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3.6 minutes.
   (a) after observing 120 workers assembling similar devices, the manager noticed that their average time was 16.2 minutes. Construct a 92% confidence interval for the mean assembly time.
   (b) How many workers should be involved in this study in order to have the mean assembly time estimated up to ± 15 seconds with 92% confidence?

9. Suppose a consumer advocacy group would like to conduct a survey to find the proportion p of consumers who bought the newest generation of an MP3 player were happy with their purchase.
   (a) How large a sample n should they take to estimate p with 2% margin of error and 90% confidence?
   (b) The advocacy group took a random sample of 1000 consumers who recently purchased this MP3 player and found that 400 were happy with their purchase. Find a 95% confidence interval for p.

10. In order to ensure efficient usage of a server, it is necessary to estimate the mean number of concurrent users. According to records, the sample mean and sample standard deviation of number of concurrent users at 100 randomly selected times is 37.7 and 9.2, respectively.
    (a) Construct a 90% confidence interval for the mean number of concurrent users.
    (b) Do these data provide significant evidence, at 1% significance level, that the mean number of concurrent users is greater than 35?

11. To assess the accuracy of a laboratory scale, a standard weight that is known to weigh 1 gram is repeatedly weighed 4 times. The resulting measurements (in grams) are:
    0.95 1.02 1.01 0.98
    Assume that the weighings by the scale when the true weight is 1 gram are normally distributed with mean μ.
    (a) Use these data to compute a 95% confidence interval for μ.
    (b) Do these data give evidence at 5% significance level that the scale is not accurate? Answer this question by performing an appropriate test of hypothesis.

12. In their advertisements, a new diet program would like to claim that their program results in a mean weight loss (μ) of more than 10 pounds in two weeks. To determine if this is a valid claim, the makers of the diet should test the null hypothesis $H_0 : \mu = 10$ against the alternative hypothesis :

    (a) $H_1 : \mu < 10$
    (b) $H_1 : \mu > 10$
    (c) $H_1 : \mu \neq 10$
    (d) $H_1 : \mu \neq 0$
    (e) None of the above.

13. Suppose we would like to estimate the mean amount of money (μ) spent on books by CS student in a semester. We have the following data from 10 randomly selected CS students: $\bar{X} = \$249$ and $S = \$30$. Assume that the amount spent on books by CS students is normally distributed. To compute a 95% confidence for μ, we will use the following critical point :

    (a) $z_{0.025} = 1.96$
    (b) $z_{0.05} = 1.645$
    (c) $t_{9,0.025} = 2.262$
    (d) $t_{10,0.025} = 2.228$
    (e) $t_{9,0.05} = 1.833$

14. Installation of a certain hardware takes a random amount of time with a standard deviation of 5 minutes. A computer technician installs this hardware on 64 different computers, with the average installation time of 42 minutes. Compute a 95% confidence interval for the mean installation time.

15. Bags of a certain brand of tortilla chips claim to have a net weight of 14 ounces. The net weight actually vary slightly from bag to bag and are normally distributed with mean μ. A representative of a consumer advocacy group wishes to see if there is any evidence that the mean net weight is less than advertised. For this, the representative randomly selects 16 bags of this brand and determines the net weight of each. He finds the sample mean to be $\bar{X} = 13.82$ and the sample standard deviation to be $S = 0.24$. Use these data to perform an appropriate test of hypothesis at 5% significance level.

16. The time needed for college student to complete a certain maze follows a normal distribution with a mean of 45 seconds. To see if the mean time μ (in seconds) is changed by vigorous exercise, we have a group of nine college students exercise vigorously for 30 minutes and then complete the maze. The sample mean and standard deviation of the collected data is 49.2 seconds and 3.5 seconds respectively. Use these data to perform an appropriate test of hypothesis at 5% level of significance.

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