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Multiobjective invasive weed optimization: Application to analysis of Pareto improvement models in electricity markets

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ABSTRACT

This paper presents a proposal for multiobjective Invasive Weed Optimization (IWO) based on nondominated sorting of the solutions. IWO is an ecologically inspired stochastic optimization algorithm which has shown successful results for global optimization. In the present work, performance of the proposed nondominated sorting IWO (NSIWO) algorithm is evaluated through a number of well-known benchmarks for multiobjective optimization. The simulation results of the test problems show that this algorithm is comparable with other multiobjective evolutionary algorithms and is also capable of finding better spread of solutions in some cases. Next, the proposed algorithm is employed to study the Pareto improvement model in two complex electricity markets. First, the Pareto improvement solution set is obtained for a three-player oligopolistic electricity market with a nonlinear demand function. Then, the IEEE 30-bus power system with transmission constraints is considered, and the Pareto improvement solutions are found for the model with deterministic cost functions. In addition, NSIWO algorithm is used to analyze this system with stochastic cost data in a risk management problem which maximizes the expected total profit but minimizes the profit risk in the market.

1. Introduction

Multiobjective optimization is to determine the best Pareto-optimal solution set or a representative subset which is defined as a set of solutions that are nondominated with respect to each other. Note that vector $\boldsymbol{\sigma} = (\sigma, \ldots, \sigma_n)$ dominate vector $\boldsymbol{\tau} = (\tau, \ldots, \tau_n)$ (denoted by $\boldsymbol{\sigma} \succ \boldsymbol{\tau}$) if and only if:

$$\sigma_i \geq \tau_i \quad i = 1, \ldots, n \tag{1}$$

And there is at least one $i (1 \leq i \leq n)$ such that,

$$\sigma_i > \tau_i \tag{2}$$

In this regard, evolutionary computing has been one of the main approaches for multiobjective optimization. In fact, due to population-based nature of evolutionary algorithms, there is an intuitive hope to find the Pareto-optimal solutions in a single run. During the last two decades, many techniques have been proposed for multiobjective optimization based on evolutionary algorithms. Vector evaluated genetic algorithm (VEGA) was the first implementation of multiobjective evolutionary algorithms (MOEA) in the mid-1980s [1]. Afterwards, several MOEAs were developed including multiobjective genetic algorithm (MOGA) [2], niched Pareto genetic algorithm (NPGA) [3], weight-based genetic algorithm (WBGA) [4], random weighted genetic algorithm (RWGA) [5], nondominated sorting genetic algorithm (NSGA) [6–8], strength Pareto evolutionary algorithm (SPEA) [9], improved SPEA (SPEA2) [10], Pareto-archived evolution strategy (PAES) [11,12], Pareto envelope-based selection algorithm (PESA) [13], region-based selection in evolutionary multiobjective optimization (PESA-II) [14], multiobjective evolutionary algorithm (MOEA) [15], micro-GA [16], rank-density based genetic algorithm (RDGA) [17], dynamic multiobjective evolutionary algorithm (DMOEA) [18], and a real-coding jumping gene genetic algorithm (RJGGA) [19].

Some of the most well-known and efficient MOEAs will be shortly discussed here. PESA is an algorithm which works on cell-based density. In this approach, the objective space is divided into a number of cells or hyper-cubes, and the number of solutions in each cell defines its density. This density metric is used to select more diverse solutions out of the nondominated solutions. PESA is easy to implement and computationally efficient, but it has two disadvantages: the performance depends on the cell sizes and the prior information is needed for the objective space. PAES is another efficient algorithm which is equipped with a random mutation hill climbing strategy. There are two main disadvantages associated with PAES. First, it is not a population-based approach, and also its performance depends on the cell sizes. SPEA is a well-known and popular MOEA, but it has a complex clustering algorithm. SPEA2 improves SPEA by making sure that the extreme points in Pareto

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front are preserved, but it has a computationally expensive fitness and density calculation. NSGA is a fast algorithm for multiobjective optimization, but it has some problems related to niche size parameter. NSGA-II is an efficient and well-tested algorithm with only one single parameter (without GA parameters) to be tuned, however, the crowding distance described in this algorithm works in objective space only [20].

Invasive Weed Optimization (IWO) is a novel ecologically inspired algorithm that mimics the process of weeds colonization and distribution. Despite its recent development, it has shown successful results in a variety of practical applications like optimization and tuning of a robust controller [21], optimal positioning of piezoelectric actuators [22], developing a recommender system [23], antenna configuration [24], cooperative identification and adaptive control of a surge tank [25], analysis of electricity markets dynamics [26,27], cooperative multiple task assignment of the UAVs [28], etc. Due to its wide range applicability and relative fast convergence rate, we are motivated to introduce a multiobjective form of IWO based on the fast nondominated sorting approach proposed in NSGA-II.

The first part of this paper is dedicated to Pareto-optimal solution search for benchmark problems in multiobjective optimization using nondominated sorting IWO (NSIWO). In the second part, application of NSIWO algorithm for investigating the Pareto improvement model [30] in electricity markets is studied. The Pareto improvement model is a newly developed model for analysis of pool-based electricity markets. Actually, this model approximates infinitely repeated games under tacit collusion among the firms [30]. In our experimental simulations, we study two types of electricity market: an unconstrained electricity market with nonlinear system demand function and a transmission-constrained electricity market with linear demand curves. In these problems, we are facing some challenging issues like nonconvexity (because of the nonlinearity in the first problem) and discontinuity (because of the constraints in the second problem), which lead us to the games with local optima. As a result, the capability of the proposed evolutionary algorithm for global search could help us escape from local traps. In addition, we study a constrained form of collusion model (as a special case of Pareto improvement model) for the second market with stochastic cost data. In this problem, we use the proposed NSIWO algorithm to find the Pareto-optimal solutions which maximize the expected total profit but minimize the total risk of the market players. Consequently, after finding the Pareto front, the decision makers may choose the solution that satisfies their needs.

The organization of this paper is as follows. Section 2 presents NSIWO algorithm accompanied by a quick review of IWO and NSGA-II algorithms. In Section 3, the simulation results for multiobjective optimization of a well-known test suit are provided, and the performance of NSIWO algorithm is compared with some other MOEAs. Section 4 explains the Pareto improvement model in electricity markets and shows the application of NSIWO to find the Pareto improvement solution set for two different test cases: a three-player unconstrained electricity market with a nonlinear demand function, and the transmission-constrained IEEE 30-bus power system with deterministic and stochastic cost data. Finally, the conclusions are drawn in Section 5.

2. NSIWO algorithm design

2.1. NSGA-II

NSGA-II is an elitist algorithm for multiobjective optimization proposed by Deb et al. in [7]. Firstly, the current archive is determined based on the combination of the current population and the previous archive. To do this, NSGA-II uses dominance ranking to classify the population into a number of layers (fronts), such that the first layer is the best in the population. Next, the archive is created based on the order of ranking layers, i.e., the best rank is selected first. If the number of individuals in the archive is smaller than the population size, the next layer will be taken into account and so on. This procedure is called fast nondominated sorting and elaborated in the following pseudocode [7]:

Pseudocode for fast nondominated sorting

```
1.      Calculate N_p and S_p
        1.1. For each p ∈ P
          1.1.1. S_p = ∅; N_p = 0
        1.2. For each q ∈ P
          1.2.1. If(p ≺ q) Then S_p = S_p ∪ {q}
          1.2.2. Else if(q ≺ p) Then N_p = N_p + 1
        1.3. If N_p = 0 Then F_1 = F_1 ∪ {p}
2.      Divide solutions into different fronts
        2.1. i = 1; While F_i ≠ φ
          2.1.1. Q = φ
          2.1.2. For each p ∈ F_i
            2.1.2.1 For each q ∈ S_p
              2.1.2.1.1. N_q = N_q − 1
              2.1.2.1.2. If (N_q = 0) Then Q = Q ∪ {q}
          2.1.3. i = i + 1
          2.1.4. F_i = Q
```

Note that in this pseudocode, N_p indicates the number of solutions that dominate the solution p, S_p shows the solution set dominated by the solution p, and F_i represents the ith front in the archive. It is verified in [7] that the computational complexity of this sorting is $O(MN^2)$ where M is the number of objectives and N is the population size. Thus, this approach outperforms the nondominated sorting in the other MOEAs which usually have computational complexity of $O(MN^3)$.

If adding a layer would increase the number of individuals in the archive to exceed the initial population size, a truncation operator is applied to that layer based on the crowding distance (CD). The crowding distance of a solution x is the average of the objective-value differences between the two adjacent solutions of x, where the population is sorted according to each objective to find the adjacent solutions, and the boundary solutions have infinite values. The pseudocode to assign the crowding distance of each solution is as follows:

Pseudocode for crowding distance assignment

```
1.      For each front F_i
        1.1. For each individual x ∈ F_i
          1.1.1. CD(x) = 0
        1.2. For each objective m
          1.2.1. Sort individuals in F_i according to objective m, I = sort(F_i,m)
          1.2.2. Assign infinite distance to boundary solutions,
        CD(I[1]) = CD(I[|F_i|]) = ∞
          1.2.3. For j = 2 to |F_i| − 1
            1.2.3.1.
        CD(I[j]) = CD(I[j] + (f_m(I[j + 1]) − f_m(I[j − 1]))/(f_m^max − f_m^min)
```

Note that in this pseudocode, f_m indicates the mth objective function. It is also shown in [7] that the computational complexity of crowding distance assignment is $O(M(2N)\log(2N))$

After crowding distance assignment, a truncation operator removes the individuals with the smallest crowding distance. Next, an offspring population of the same size is created from the archive by using crowded tournament selection, crossover, and mutation operators and makes a new population for the next iteration. The crowded tournament selection rule is that the winner of two same-rank solutions is the one that has the greater crowding distance value.

2.2. IWO

The Invasive Weed Optimization was developed by Mehrabian and Lucas in 2006 [21]. IWO algorithm is a bio-inspired numerical optimization algorithm that simply simulates natural behavior of weeds in colonizing and finding suitable places for growth

Table 1
Invasive weed optimization parameters.

Symbol	Definition
N_0	Number of initial population
$iter_{\max}$	Maximum number of iterations
dim	Problem dimension
$p_{\max}$	Maximum number of plant population
$s_{\max}$	Maximum number of seeds
$s_{\min}$	Minimum number of seeds
n	Nonlinear modulation index
σ_{initial}	Initial value of standard deviation
σ_{final}	Final value of standard deviation

and reproduction. Some of the distinctive properties of IWO in comparison with other evolutionary algorithms are the ways of reproduction, spatial dispersal, and competitive exclusion [21].

In IWO algorithm, the process begins with initializing a population. It means that a population of initial solutions is randomly generated over the problem space. Then members of the population produce seeds depending on their relative fitness in the population. In other words, the number of seeds for each member begins with the value of $S_{\min}$ for the worst member and increases linearly to $S_{\max}$ for the best member. For the third step, these seeds are randomly scattered over the search space by normally distributed random numbers (with zero mean and an adaptive standard deviation):

$$w_s[k] = w[k] + \mathfrak{N}(0, \sigma_{\text{iter}}^2), \tag{3}$$

where, $w[k]$ indicates the kth variable of a solution vector in the current iteration, and $w_s[k]$ shows the kth variable of its sth seeds. The standard deviation for the normal distribution variable is computed adaptively according to (4):

$$\sigma_{\text{iter}} - \frac{(iter_{\max} - iter)^n}{(iter_{\max})^n}(\sigma_{\text{initial}} - \sigma_{\text{final}}) + \sigma_{\text{final}}, \tag{4}$$

where, $iter_{\max}$ is the maximum number of iterations, σ_{iter} is the standard deviation in the current iteration and n is the nonlinear modulation index. The produced seeds, accompanied by their parents are considered as the potential solutions for the next generation. Finally, a competitive exclusion is conducted in the algorithm. In fact, after a number of iterations the population size reaches its maximum, and an elimination mechanism should be employed. To this end, the seeds and their parents are ranked together and those with better fitness survive and become reproductive. The set of parameters for IWO algorithm is provided in Table 1, and the pseudocode for this algorithm is given as follows:

Pseudocode for IWO algorithm

1. Generate a random population of N_0 solutions (W)
2. For iter = 1 to the maximum number of generations ($iter_{\max}$)
   2.1. Evaluate the objective function for each individual in W
   2.2. Find the maximum and minimum fitness in the colony
   2.3. For each individual $w \in W$
      2.3.1. Compute the number of seeds of w, corresponding to its fitness
      2.3.2. Randomly distribute the generated seeds over the search space with normal distribution around the parent plant (w)
      2.3.3. Add the generated seeds to the solution set, W
   2.4. If $(|W| = N) > \mathfrak{p}_{\max}$
      2.4.1. Sort the population W in descending order of their fitness
      2.4.2. Truncate population of weeds with smaller fitness until $N = \mathfrak{p}_{\max}$

2.3. NSIWO

The proposed NSIWO algorithm is in the large part similar to NSGA-II except for the process of making a new population. In fact, the genetic operators in NSGA-II are replaced by IWO reproduction and colonization approach of making offspring population. The whole procedure for NSIWO algorithm is summarized as follows. In each generation, a binary tournament selection is used to select the candidate parents from the current population (archive). Next, the offspring population is generated by a process of seed reproduction (cf. Fig. 1) and seed dispersal (cf. Eq. (3)). The offspring solution set is added to the previous population, and the fronts are derived for this combined population through fast nondominated sorting algorithm. The crowding distance is also assigned for each individual in the population. Then, the weakness (opposite of fitness) of each individual w is calculated according to the following formula:

$$\text{weakness }(w) = \text{rank}(w) + \frac{1}{\text{CD}(w) + 2}, \tag{5}$$

where, rank(w) is the front number and CD(w) is the crowding distance for w. In this equation, the fitness (opposite of weakness) is proportionate to the crowding distance, but it is disproportionate to the rank. As a result, the individuals in the lower fronts and with better density have higher fitness. The term $1/CD(w) + 2$ is always less than 1, which means no individual with a worse rank can obtain a better fitness value. Actually, this term only sorts the individuals in the same front according to their crowding distances. Finally, the individuals with lower fitness are eliminated from the combined population explained above, and a new population is formed for the next iteration.

Note that the parameters for NSIWO are exactly the same as those in Table 1. To have a better understanding of the algorithm, the overall scheme of NSIWO algorithm is illustrated in Fig. 2, and the pseudocode for this algorithm is summarized as follows:

Pseudocode for NSIWO algorithm

1. Genearte a random population of N_0 solutions (W)
2. Evaluate objective functions for all the individuals in W
3. For each individual $w \in W$
   3.1. Assign the rank based on fast nondominated sorting
   3.2. Assign the crowding distance
   3.3. Compute the weakness of each individual according to its rank and crowding distance
4 For $iter$ = 1 to the maximum number of generations ($iter_{\max}$)
   4.1. Use the binary tournoment selection to obtain a selected parent population (P)
   4.2. Find the maximum and minimum weakness in P
   4.3. For each individual $p \in P$
      4.3.1. Compute the number of seeds of w, corresponding to its weakness
      4.3.2. Randomly distribute the generated seeds over the search space with normal distribution around the parent plant ($\mathfrak{p}$)
      4.3.3. Evaluate the objective functions for the seeds
      4.3.4. Add the generated seeds to the previous solution archive W
   4.4. For each individual $w \in W$
      4.4.1. Assign the rank based on the fast nondominated sorting
      4.4.2. Assign the crowding distance
      4.4.3. Compute the weakness of each individual according to its rank and crowding distance
   4.5 If $(|W| = N) > \mathfrak{p}_{\max}$
      4.5.1. Sort the population W in descending order of their fitness (opposite of weakness)
      4.5.2. Truncate the population of weeds with smaller fitness until $N = \mathfrak{p}_{\max}$

2.4. Parameter settings for NSIWO

Comparing with genetic algorithm, IWO algorithm has more parameters. Although our investigations have shown moderate robustness for parameter selection in NSIWO, it seems somehow a challenging process for a practitioner to set the parameters effectively. In this section, we present some guidelines for tuning the parameters in NSIWO.

First, similar to IWO algorithm, n or $iter_{\max}$ can control the speed of convergence in the algorithm. However, although the increase of n or decrease of $iter_{\max}$ makes convergence faster, but the possibility of trapping into local optima also increases. According to our experiences and the extensive experiments in [21], it is suggested to set n equal to 3 in order to have a fairly good

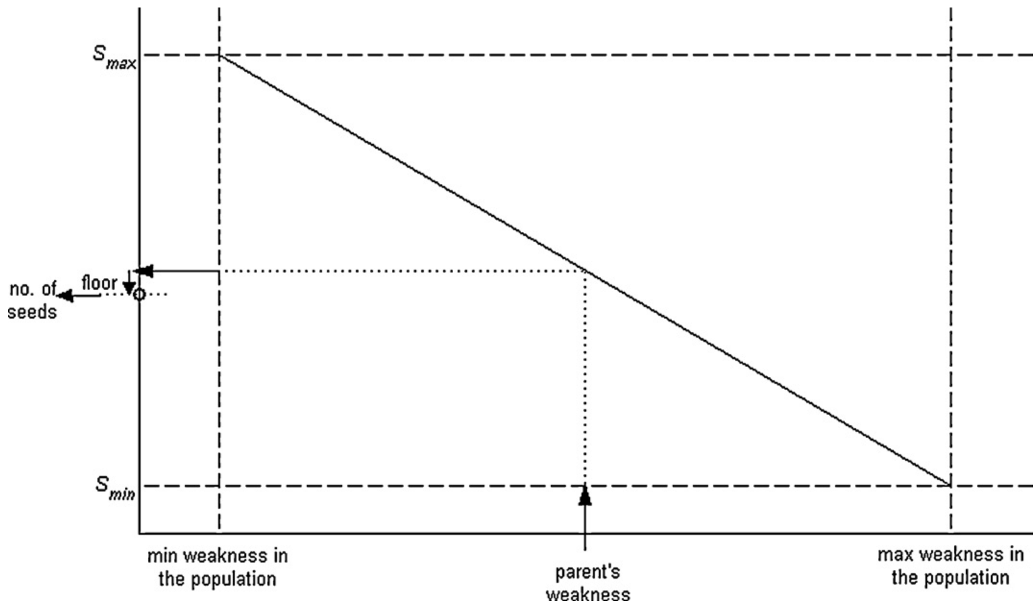

Fig. 1. Process of seed reproduction in a population of parent weeds.

convergence. It is also preferable to set $iter_{max}$ as large as possible to improve optimality of solutions, but it is not always true, and better results might be achieved by less iterations in some cases. Thus, usually some trial-and-error is needed. In addition, the experiments in [21,27] show that a good guess for minimum and maximum seed numbers for IWO algorithm are $S_{min} = 0$ or 1 and $S_{max} = 3$. Our experiments for these parameters suggest that the best choices are $S_{min} = 1$ and $S_{max} = 3$. Moreover, our investigations show that N_0 should be simply set to $\mathfrak{p}_{max}$. However, the maximum population size (i.e., $\mathfrak{p}_{max}$) should be chosen for each experiment in a trade-off between the computational cost and the performance on Pareto optimality and distribution diversity. In fact, a large population can better represent a Pareto-optimal front, but it increases the computational time and cost.

Finally, $\sigma_{initial}$ and σ_{final} are parameters that control the exploration in the search space during evolution of solutions. Actually, $\sigma_{initial}$ is usually set much larger than σ_{final} to make the algorithm have bigger steps at first iterations, which lead to more exploration of the search space. But, the standard deviation is getting decreased to σ_{final} in the last iterations, where we are willing to search the vicinity of the best solutions found (exploitation). The large values of these variables reinforce the exploration which is needed for global search, but decrease the exploitation which is required for fine convergence. According to our investigations in numerous experiments, $\sigma_{initial}$ should be set around 1/5 to 1/10 of the variable bounds in the search space. It is also suggested to set σ_{final} less than 1/10 of $\sigma_{initial}$, but the best value varies according to the shape of functions and distribution of solutions. For example, when there are many Pareto-optimal

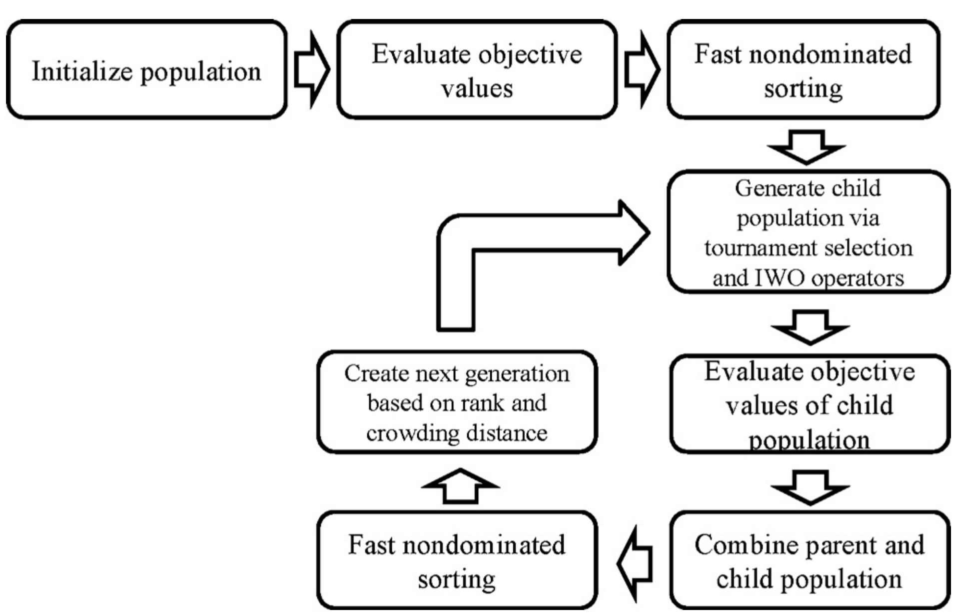

Fig. 2. An overall scheme of NSIWO algorithm.

Table 2
Description of the test problems.

Problem	n	Variable bounds	Objective functions	Characteristics
ZDT1	30	[0 1]	$f_1(x)=x_1$ $f_2(x)=g(x)[1-\sqrt{x_1/g(x)}]$ $g(x)=1+9(\sum_{i=2}^{n}x_i)/(n-1)$	Convex
ZDT2	30	[0 1]	$f_1(x)=x_1$ $f_2(x)=g(x)[1-(x_1/g(x))^2]$ $g(x)=1+9(\sum_{i=2}^{n}x_i)/(n-1)$	Nonconvex
ZDT3	30	[0 1]	$f_1(x)=x_1$ $f_2(x)=g(x)[1-\sqrt{x_1/g(x)}-x_1/g(x)\sin(10\pi x_1)]$ $g(x)=1+9(\sum_{i=2}^{n}x_i)/(n-1)$	Convex, disconnected
ZDT4	10	$x_1\in[0\,1]$ $x_i\in[-5+5]$ $i=2,\ \ldots,\ n$	$f_1(x)=x_1$ $f_2(x)=g(x)[1-\sqrt{x_1/g(x)}]$ $g(x)=1+10(n-1)+$ $(\sum_{i=2}^{n}[x_i^2-10\cos(4\pi x_i)])$	Nonconvex, many local Pareto fronts
ZDT6	10	[0 1]	$f_1(x)=1-\exp(-4x_1)\sin^6(6\pi x_1)$ $f_2(x)=g(x)[1-(f_1(x)/g(x))^2]$ $g(x)=1+[9(\sum_{i=2}^{n}x_i)/(n-1)]^{0.25}$	Nonconvex, nonuniformly spaced

solutions close to each other, a lower value of σ_{final} should be chosen.

3. Simulation studies

3.1. Test problems

This paper considers five well-known test problems suggested in [29] to evaluate the performance of NSIWO algorithm for multiobjective optimization. Each of these functions illustrates a different class of problems. All problems have two objectives, $f_1(x)$ and $f_2(x)$ that must be minimized. The specifications of these benchmark problems (ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6) are provided in Table 2.

To evaluate the efficiency of NSIWO, we use two performance metrics defined in [7] to measure: (1) convergence to the Pareto-optimal set and (2) extent of spread or diversity in the Pareto-optimal set. The first metric is the convergence metric Υ which measures the extent of convergence to a known set of Pareto-optimal solutions. To this end, the distance between the obtained Pareto front W and the set of uniformly spaced solutions of the true Pareto-optimal front F^* is measured according to:

$$\Upsilon=\frac{\sum_{i=1}^{|W|}d_i}{|W|}, \tag{6}$$

where, d_i is the minimum Euclidian distance (in the objective space) between the solution $w_i\in W$ and any of the solutions in F^*.

The second metric is the diversity metric Δ shown in Eq. (7), which measures the extent of spread achieved among the obtained solutions. Here, the objective is to achieve a set of solutions that spans the entire Pareto-optimal front.

$$\Delta=\frac{d_f+d_l+\sum_{i=1}^{|W|-1}\left|d_i-\bar{d}\right|}{d_f+d_l+(|W|-1)\bar{d}} \tag{7}$$

Note that in Eq. (7), d_i is the Euclidean distance (in the objective space) between consecutive solutions in the obtained Pareto-optimal set of solutions (W), $\bar{d}$ is the average of these distances, and finally d_f and d_l are Euclidean distances between the boundary solutions in W and the extreme solutions in F^*.

3.2. Comparing NSIWO with other MOEAs

In this section an experiment is performed to evaluate the performance of NSIWO with respect to some other MOEAs. To have a concise and fair comparison, the results of NSIWO algorithm are compared with the results provided in [7] for the following MOEAs: real coded NSGA-II, SPEA, and PAES. All the tests were repeated 10 times with random initial populations.

Table 3 presents the mean (and variance in parenthesis) of the convergence metric and the diversity metric (in ten runs) for all the algorithms. The number of function evaluations is roughly the same for all methods (about 25,000 function evaluations). NSIWO is the best algorithm in the convergence metric for ZDT3 test problem, while it is the second best for ZDT2 and ZDT6. However, it can be observed that the convergence of NSIWO is poor for ZDT4. For the diversity metric, NSIWO is better than other algorithms for ZDT1 and ZDT3. It is the second best algorithm for ZDT2 and ZDT4, and the third best for ZDT6. In overall, it can be concluded that NSIWO has an acceptable performance for multiobjective optimization and is comparable with the best previously proposed algorithms. Figs. 2–7 show nondominated fronts obtained by a single run of NSIWO algorithm during this experiment. In these figures the resulting nondominated solutions are compared with the true Pareto-optimal fronts.

Parameter settings for NSIWO algorithm in this experiment are shown in Table 4. We chose the parameters' values mostly based on the guidelines provided in Section 2.4. For example, the value of σ_{final} is very low for ZDT6 test problem because of the nonuniform spread of solutions along the Pareto front, i.e., there are many Pareto-optimal solutions close to each other.

4. NSIWO for analysis of Pareto improvement model in electricity markets

4.1. Games, Nash equilibrium, and Pareto improvement model

A general multi-player game consists of an index set $\mathbb{N}=\{1,2,3,\ldots N\}$ called the player's set and an index set $K=\{1, 2, 3, \ldots K\}$ showing the stages of the game. At each stage, the players take strategies from a set of strategy spaces $U=\{U^i{}_k\}$, and receive a payoff $\pi_i(u^i,u^{-1})$, where $u^i\in U^i$ is the pure strategy for player i, given the pure strategy set of others $u^{-1}=\{u^1,\ldots,u^{i-1}u^{i+1},\ldots u^N\}\in U^{-i}$. The pure strategy Nash Equilibrium (NE) is a point where no player can obtain a higher profit by unilateral movement. The satisfying NE condition for the combined strategy $\{u^{i*}, u^{-i*}\}$ is characterized in Eq. (8).

$$\forall i,\ \forall u^i\in U^i,\qquad \pi_i(u^{i*},u^{-i*})\geq\pi_i(u^i,u^{-i*}) \tag{8}$$

Table 3
The mean (variance) of the convergence metric and the diversity metric.

Metric	Alg.	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
Convergence	NSIWO	0.0377 (0.0015)	0.0517 (0.0025)	0.0207 (0.0032)	2.1884 (0.7360)	0.1512 (0.0498)
	NSGA-II	0.0334 (0.0048)	0.0723 (0.0317)	0.1145 (0.0079)	0.5130 (0.1185)	0.2965 (0.0131)
	SPEA	0.0018 (0.0000)	0.0013 (0.0000)	0.0475 (0.0000)	7.3402 (6.5725)	0.2211 (0.0004)
	PAES	0.0820 (0.0087)	0.1267 (0.0369)	0.0238 (0.0000)	0.8548 (0.5272)	0.0854 (0.0067)
Diversity	NSIWO	0.3148 (0.0239)	0.4485 (0.0515)	0.5647 (0.0188)	0.7909 (0.0491)	1.1120 (0.0770)
	NSGA-II	0.3907 (0.0019)	0.4307 (0.0047)	0.7385 (0.0197)	0.7026 (0.0646)	0.6680 (0.0099)
	SPEA	0.7845 (0.0044)	0.7551 (0.0045)	0.6729 (0.0036)	0.7984 (0.0146)	0.8493 (0.0027)
	PAES	1.2297 (0.0048)	1.1659 (0.0077)	0.7899 (0.0017)	0.8704 (0.1014)	1.1530 (0.0039)

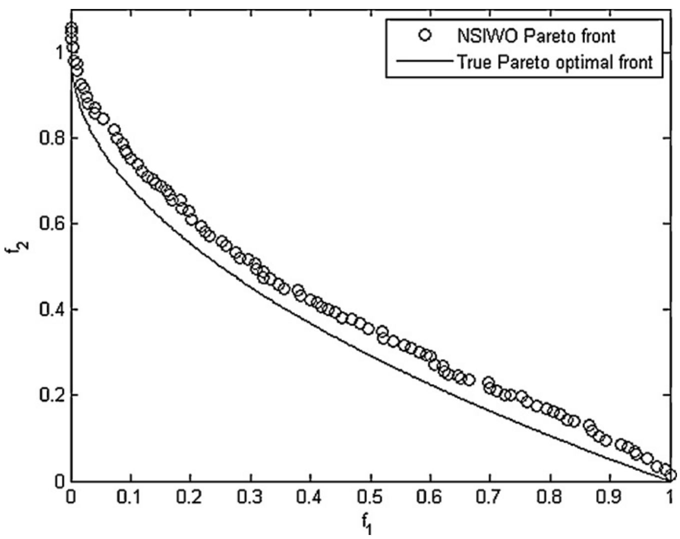

Fig. 3. Nondominated solutions with NSIWO for ZDT1 test problem.

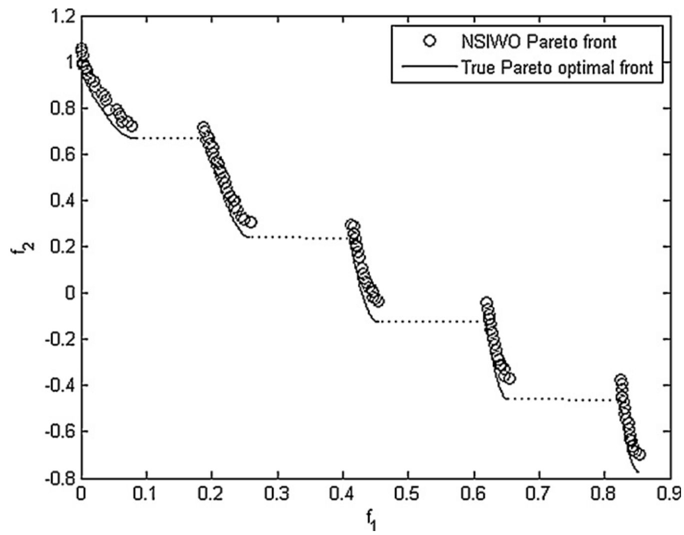

Fig. 5. Nondominated solutions with NSIWO for ZDT3 test problem.

Here, the Pareto improvement model in electricity markets with the Cournot oligopoly is briefly introduced. The Cournot oligopoly model is extensively used to analyze pool-based electricity markets [30]. In this model, the strategy of each firm (player) is the quantity it produces. The Cournot–Nash equilibrium is the result of a one-shot non-cooperative game. However, in the real electricity market, we deal with the games repeated in a pool-based electricity market. Under some assumptions, it can be shown that if the non-cooperative games are infinitely repeated, they will reach to the Pareto-optimal solutions [30–32]. So, we can say that Pareto-optimal solutions are non-cooperative Equilibria for infinitely repeated games. To come up with this situation, a new Pareto-optimal model is presented in [30]. In this model, the general Pareto improvement solutions of the Cournot oligopoly model are searched. It is defined that Pareto improvement solutions are the Pareto-optimal solutions with better payoffs than the Cournot-Nash equilibrium.

First, Pareto optimality conditions are mathematically represented as follows:

A point $\boldsymbol{u}^* = (u_1{}^*, \ldots, u_N{}^*)$ is Pareto-optimal if for any possible $= (u_1, \ldots, u_N)$,

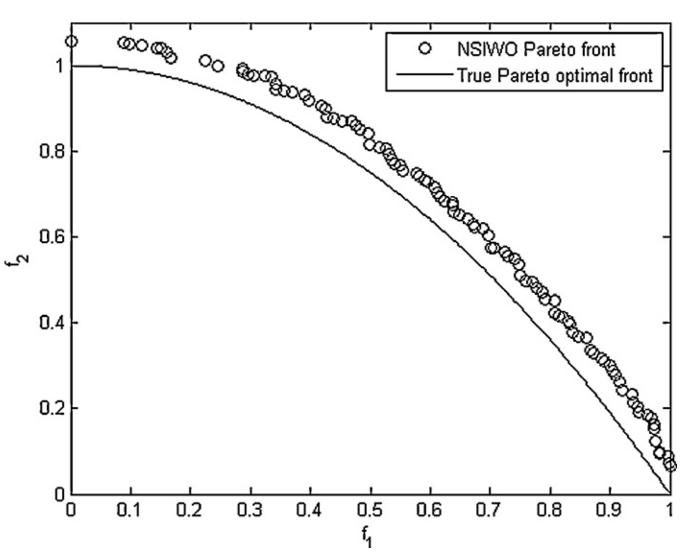

Fig. 4. Nondominated solutions with NSIWO for ZDT2 test problem.

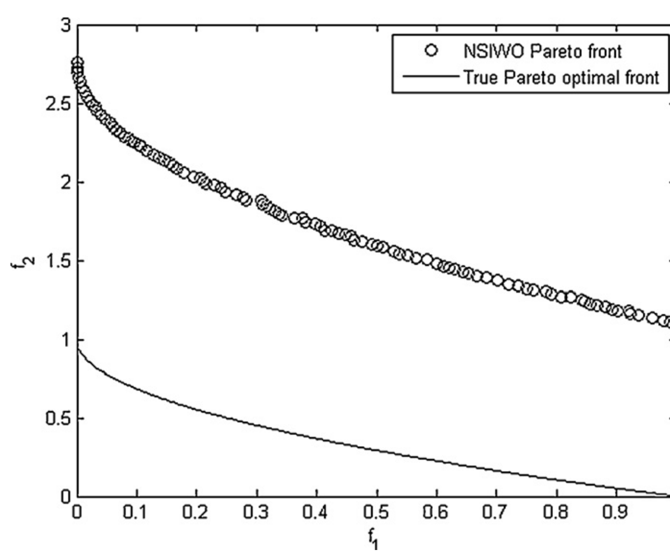

Fig. 6. Nondominated solutions with NSIWO for ZDT4 test problem.

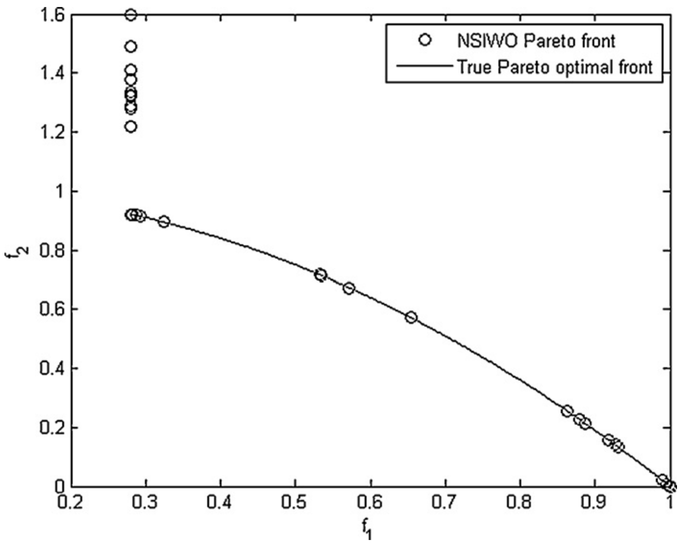

Fig. 7. Nondominated solutions with NSIWO for ZDT6 test problem.

Either,

$$\pi(\boldsymbol{u}) = \pi_i(\boldsymbol{u}^*) \quad i = 1, \ldots, N \tag{9}$$

or, there is at least one $i(1 \le i \le N)$ such that,

$$\pi_i(\boldsymbol{u}) < \pi_i(\boldsymbol{u}^*) \tag{10}$$

Let us denote the NE of the Cournot game in electricity markets with: $\boldsymbol{q}^{*(n)} = (q_1^{*(n)}, \ldots, q_N^{*(n)})$, where q_i denotes the quantity generated by producer (firm or player) i. Following the definition of Pareto dominance in (1) and (2), the Pareto improvement set (Ω) in Pareto improvement model is defined as follows:

$$\begin{aligned} &\Omega = \left\{ \boldsymbol{q}^{*(p)} \left| \pi_i(\boldsymbol{q}^{*(p)}) \right| \ge \pi_i(\boldsymbol{q}^{*(n)}), \right. \\ &\left. \boldsymbol{q} = (q_1, \ldots, q_N) : \pi(\boldsymbol{q}) \succ \pi(\boldsymbol{q}^{*(p)}) \quad 1 \le i \le N \right\} \end{aligned} \tag{11}$$

The aim of the Pareto improvement model is to find the Ω for the Cournot oligopoly model [30]. Indeed, this is a constrained multiobjective optimization which can be solved using any MOEA. In this paper, the proposed NSIWO is employed with the constraint-handling approach presented in [30] to find the Pareto improvement solutions.

4.2. Simulation of a three-player unconstrained electricity market with a nonlinear demand function

The problem which is analyzed in this part is an unconstrained Cournot model from [30,36] with three producers and a constant elasticity demand function characterized in Eq. (12). In this unconstrained electricity market model, P denotes the price of market, and Q denotes the total amount of generation which is also equal to the total demand. The profit maximization problem (i.e., Pareto improvement model formulation) for this game is presented in Eq. (13), and the producers' cost data are provided in Table 5. In fact, Eq. (13) shows that there are three objective functions in this problem, each one demonstrates one of the profits (π_i).

Table 4
NSIWO parameters for the test problems.

Parameters	ZDT1	ZDT2	ZDT3	ZDT4[a]	ZDT6
N_0	100	100	100	100	100
$p_{\max}$	100	100	100	100	100
$s_{\max}$	1	1	1	1	1
$s_{\min}$	3	3	3	3	3
$iter_{\max}$	350	350	350	300	300
σ_{initial}	0.1	0.1	0.2	1	0.1
σ_{final}	0.01	0.01	0.01	0.001	0.00001
n	3	3	3	3	3

[a] For the first variable in ZDT4, $\sigma_{\text{initial}} = 0.2$ and $\sigma_{\text{final}} = 0.05$.

Table 5
Producers' cost data [30].

Cost function	No.	a_i	b_i	c_i
$Cost_i(q_i) = a_i q_i^2 + b_i q_i + c_i \; 0 \le q_i \le 2000$	1	0.007859	1.360575	9490.366
	2	0.010526	2.07807	11128.95
	3	0.006478	8.105354	6821.482

$$P = 9969.7Q^{-2/3}, \quad Q = q_1 + q_2 + q_3 \tag{12}$$

$$\begin{aligned} &\max_{\boldsymbol{q}} \pi_i(\boldsymbol{q}) = P \cdot q_i - Cost_i(q_i) \quad i = 1, 2, 3 \\ &S.T. \pi_i(\boldsymbol{q}) \ge \pi_i(\boldsymbol{q}^{*(n)}) \end{aligned} \tag{13}$$

This problem has a pure NE at $\boldsymbol{q}^{*(n)} = (1652.9, 1447.7, 1532.6)$ with $\pi(\boldsymbol{q}^*(n)) = (26{,}083, 21{,}751, 20{,}518)$. Here, the proposed NSIWO is applied to find the Pareto improvement set for this problem and the outcomes are compared with those obtained by NSGA-II. The Pareto improvement results for NSIWO are summarized in Table 6. As the Pareto improvement solutions are not unique, the average and standard deviation (in parenthesis) values of the market aggregated demand, market price, generators' outputs and their market profits are calculated and shown in Table 6. Moreover, the resulting data for three boundary solutions (i.e., S1 with the highest π_1, S2 with the highest π_2, and S3 with the highest π_3) in addition to the solution (i.e., S4) with the highest uniformly weighted sum of profits are shown in this table. It can be seen that the average values of the generators' outputs on the Pareto set are lower than those at the Cournot–Nash equilibrium while the market profits for the players are higher than those at the Cournot–Nash equilibrium. This outcome was also concluded in [30]. Note that all the Pareto improvement solutions obtained in this experiment satisfy all the constraints of the problem. In addition, although we do not mention it in the next sections, however, NSIWO algorithm successfully finds the constraint-feasible solutions in all the experiments of this paper.

To compare NSIWO and NSGA-II, the Pareto improvement solution sets obtained by these two algorithms are illustrated in Fig. 8. Although it may be not apparent from Fig. 8 (due to the 3d representation), the Pareto improvement set of NSIWO algorithm dominates 28 individual points of NSGA-II set. For better visualization of this comparison, the Pareto solutions of each algorithm are interpolated with a surface fitting tool, and the results are demonstrated in Fig. 9. As shown in this figure, the Pareto front of NSIWO algorithm outperforms the Pareto front of NSGA-II. Note that the parameter settings for NSIWO algorithm in this experiment are provided in Table 7. For parameters of NSGA-II, we adopted the same settings used in [7] for real-coded NSGA-II. In fact, the original NSGA-II uses simulated binary crossover (SBX) operator and polynomial mutation. The crossover rate, mutation rate, and distribution indexes for crossover and mutation operators are $p_c = 0.9$, $p_m = 1/n$ (where n is the number of variables), $\eta_c = 20$, $\eta_m = 20$, respectively. In this experiment, we set the population size equal to 100 and the number of generations equal to 1000.

4.3. The IEEE 30-bus power system with transmission constraints

To better evaluate the performance of NSIWO, a more practical and complicated problem is studied in this section. The IEEE 30-bus power system (cf. Fig. 10) is composed of six generators (firms) and 20 consumers. In this case study, three of the transmission lines (7, 25, and 33) have 5 MW capacity limits (cf. Table 8). Since the transmission lines are

Table 6
The Pareto improvement results for the three-player unconstrained electricity market.

Solution	P [$/MW]	Q [MW]	Producer no.	Producers' results	
				q_i [MW]	π_i [$]
Average (SD)	52.994 (1.147)	2582.7 (84.0)	1	924.43 (129.3)	31,359 (4654)
			2	836.12 (120.6)	27,421 (4647)
			3	822.13 (138.9)	25,501 (4266)
S1	54.012	2507.7	1	1180.79	41,724
			2	670.59	21,752
			3	656.33	20,518
S2	54.009	2508.0	1	762.45	26,083
			2	1089.14	37,471
			3	656.39	20,518
S3	52.741	2599.0	1	787.12	26,083
			2	691.66	21,751
			3	1120.22	35,051
Highest aggregated payoff (S4)	52.939	2584.4	1	976.23	33,373
			2	865.11	28,589
			3	743.02	22,915

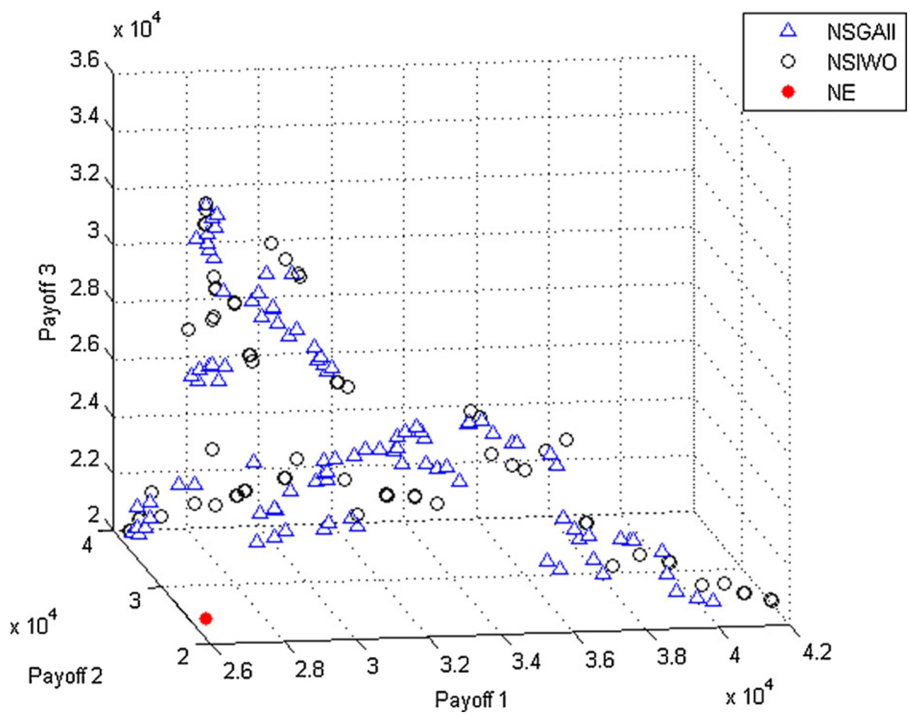

Fig. 8. The Pareto improvement solutions obtained by NSIWO and NSGA-II for a three-player unconstrained electricity market.

constrained in this system, first we briefly introduce transmission-constrained electricity markets in the following section, and next we use NSIWO to find Pareto improvement solutions for this system.

4.3.1. Transmission-constrained electricity markets

Transmission-constrained electricity market is an important model for market power analysis in the restructured electricity industry. Trading in this electricity market can be represented by the maximization of total welfare subject to the constraints on the system as shown in Eq. (14),

$$\max\left(\sum_{j=1}^{n_d} Benefit_j - \left(\sum_{i=1}^{n_p} Cost_i\right)\right)$$
$$S.T.\begin{cases}\text{Transmission Limits}\\ \text{Total Supply} = \text{Total Demand}\\ \text{Kirchoff's Laws}\end{cases} \tag{14}$$

Table 7
NSIWO parameters for the three-player unconstrained electricity market.

Symbol	Value	Symbol	Value
N_0	100	S_{max}	3
$iter_{max}$	1000	n	3
p_{max}	100	σ_{init}	200
S_{min}	1	σ_{final}	0.01

Table 8
The lines with capacity constraints.

Line	From bus	To bus	Flow limit [MW]
7	4	6	5
25	10	20	5
33	24	25	5

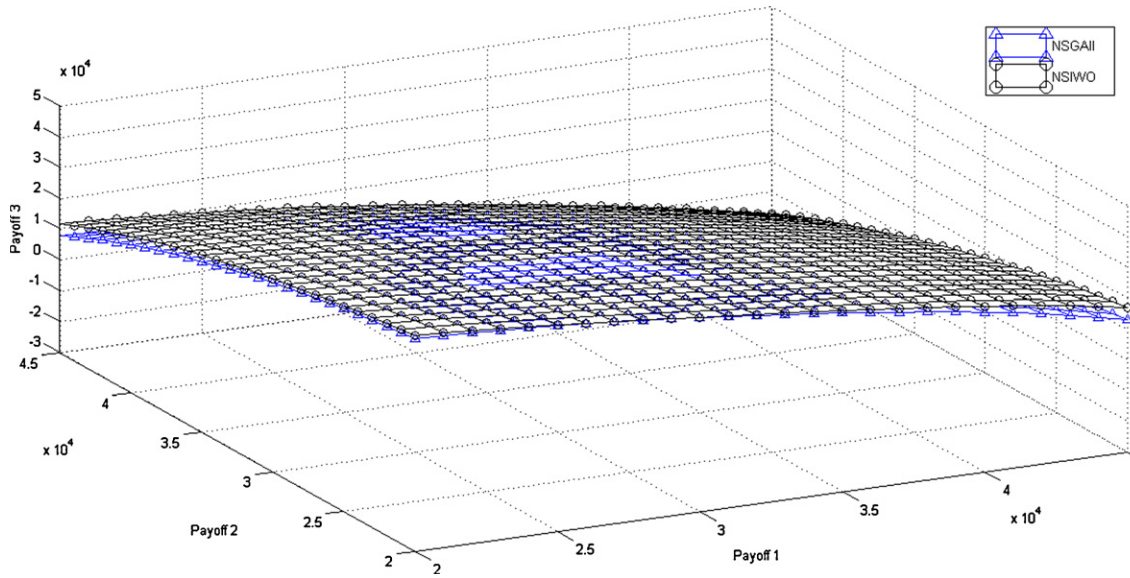

Fig. 9. Comparison between the interpolated Pareto improvement fronts obtained by NSIWO and NSGA-II for a three-player unconstrained electricity market.

where, n_d and n_p show the number of demanders (or consumers) and producers of electricity, respectively. This problem is also known as market clearing problem. When the transmission constraints bind, the competitive electricity market will break into zonal markets with price differences. This would increase the difficulty of finding the profit maximizing bidding strategies for the market players. In this case, the profit function of each player has an embedded transmission-constrained welfare maximization problem (i.e., market clearing problem) within its major formulation. In fact, the generation and transmission line constraints are included in the market clearing sub-problem. Thus, the profit function of each power producer is given by Eq. (15).

$$\pi_i(\boldsymbol{q}) = P_i \cdot q_i - Cost_i \left| \begin{array}{c} \max \sum_j Benefit_j - \sum_i Cost_i \\ \text{S.T.Constraints} \end{array} \right. \tag{15}$$

The bus prices (p_i), are determined by the Lagrange multipliers of the bus energy balance constraints (i.e., power flow equations) in the market clearing sub-problem [33,34]. Eq. (15) can be considered as a complex oligopolistic game for which the Pareto improvement model can be studied with the proposed NSIWO algorithm. The challenges for analysis of this game are the transmission constraints that may cause profit functions to have local optima [35]. Thus, exploration of the search space to find the global Pareto-optimal solutions is not simple.

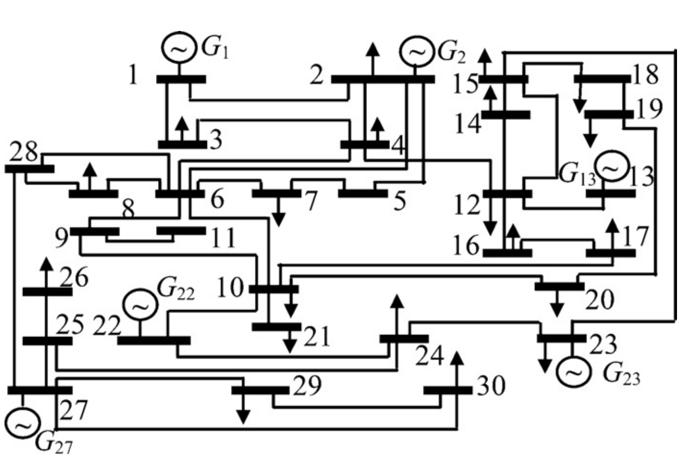

Fig. 10. The IEEE 30-bus power system [33].

Throughout this paper, a quadratic programming algorithm is used to solve the embedded market clearing sub-problem, since the generation cost and demand utility functions in the electricity markets are modelled quadratic. For more explanations about the usage of EAs in solving profit maximization problems in constrained electricity market models, the interested reader is referred to [35].

4.3.2. Simulation of the IEEE 30-bus power system with Pareto Improvement Model

The generators and load data for this case study are provided in Tables 9 and 10, respectively. Note that the demand utility functions (benefit functions) are assumed quadratic (w.r.t the bus load, i.e., d_j) as shown in Eq. (16). With these settings, the system has one pure Cournot–Nash equilibrium at $\boldsymbol{q}^*(n)$ = (26.6, 45.4, 36.8, 24.2, 43.4, 27.9) with $\pi\boldsymbol{q}^*(n)$ = (447, 896, 731, 505, 1054, 800).

$$Benefit_j(d_j) = e_j \cdot d_j + \frac{1}{2} f_j \cdot d_j^2 \tag{16}$$

In this section, we use NSIWO to find Pareto-optimal solutions for the IEEE 30-bus power system. The multiobjective optimization problem to simulate this system with Pareto improvement model is presented by Eq. (17). In this equation, there are six objective functions which characterize the profits of the six producers. In addition, prices (i.e. P_i) are corresponding to the Lagrange multipliers of the power flow equations obtained through solving the market clearing problem, given the offered quantities by the producers (cf. Section 4.3.1).

$$\begin{array}{l} \max_q \pi_i(\boldsymbol{q}) = P_i \cdot q_i - Cost_i(q_i) \left| \begin{array}{c} \max \sum_j Benefits_j - \sum_i Cost_i, \quad i = 1, 2, \ldots, 6 \\ \text{S.T.Constraints} \end{array} \right. \\ S.T. \pi_i(\boldsymbol{q}) \geq \pi_i(\boldsymbol{q}^{*(n)}) \end{array} \tag{17}$$

NSIWO Pareto improvement solution set for this case study is summarized in Table 11. In the first data row, the average and standard deviation (in parenthesis) values of the market aggregated demand, generators' power outputs, their bus prices and profits are calculated and demonstrated. The rest of the table shows the resulting data for six boundary solutions (i.e., S1 with the highest π_1, S2 with the highest π_2, and S3 with the highest π_3, S4 with the highest π_4, S5 with the highest π_5, and S6 with the highest π_6) in addition to the Pareto-optimal solution (i.e., S7) with the highest uniformly weighted sum of the profits. Note that the parameter settings for NSIWO algorithm in this case study are provided in Table 12.

Table 9
The generators' cost function data for the IEEE 30-bus power system.

Cost function	Bus	a_i [\$/MW]	b_i [\$/MW^2]	$q_i^{\min}$ [MW]	$q_i^{\max}$ [MW]
$Cost_i(q_i) = a_i q_i + \frac{1}{2} b_i q_i^2$	#1	25	0.15	5	80
	#2	20	0.25	5	60
	#13	23	0.2	5	60
	#22	22	0.25	5	60
	#23	20	0.2	5	80
	#27	22	0.15	5	70

Table 10
The load data for the IEEE 30-bus power system.

Bus	e_j [\$/MW]	f_j [\$/MW^2]	Bus	e_j [\$/MW]	f_j [\$/MW^2]
2	125	−5	17	100	−4.5
3	80	−4	18	80	−4
4	100	−4	19	100	−5
7	150	−5	20	100	−5
8	120	−4.5	21	75	−3.5
10	100	−4	23	70	−3
12	120	−5	24	80	−4.5
14	80	−3.5	26	80	−4
15	80	−3	29	75	−4
16	80	−4	30	100	−5

4.3.3. Risk analysis for the IEEE 30-bus power system using NSIWO

In this section, we focus on a special case of the Pareto improvement model where all the generating firms are equally weighted. This problem may also be considered as the constrained form of the so-called collusion model [37] in electricity markets, where the firms try to maximize their total profits via cooperation. Indeed, the collusion model is an ordinary single objective optimization problem described by Eq. (18).

$$\max_q \sum_i \pi_i(\boldsymbol{q} = [q_1, q_2, \ldots, q_{n_p}]) \tag{18}$$

However, unlike the deterministic model in the previous section, a stochastic electricity market model is studied in this section. The uncertainties are considered in the generators' cost data as given in Table 13 with normal distributions. Note that the transmission line constraints and the load data are the same as those in Tables 8 and 10.

The problem which is investigated in this section is summarized as a multiobjective (two-objective) optimization problem

Table 11
The pareto improvement results for the IEEE 30-bus power system.

Solution	Q [MW]	Producers' results			
		Bus	q_i [MW]	π_i [\$]	p_i [\$/MW]
Average (SD)	145.2 (6.7)	#1	16.89 (4.87)	581 (164)	60.70 (1.55)
		#2	28.63 (6.56)	1059 (221)	60.74 (1.54)
		#13	24.41 (7.33)	881 (210)	62.06 (1.44)
		#22	17.93 (5.93)	640 (176)	60.45 (1.81)
		#23	31.01 (5.79)	1214 (195)	62.40 (1.44)
		#27	26.34 (6.39)	978 (203)	61.42 (1.85)
Highest aggregated payoff (S7)	143.2	#1	18.16	645	61.84
		#2	24.03	932	61.76
		#13	19.93	754	62.81
		#22	15.49	582	61.51
		#23	41.54	1613	62.97
		#27	24.03	914	61.81

Q [MW]			Producers' results									
S1	S2	S3	Bus	q_i [MW]			π_i [\$]			p_i [\$/MW]		
				S1	S2	S3	S1	S2	S3	S1	S2	S3
142.1	135.1	141.1	#1	32.19	12.04	13.03	1058	448	447	60.27	63.11	60.30
			#2	23.59	44.27	23.13	896	1659	896	60.92	63.00	61.62
			#13	19.86	18.48	44.21	731	732	1491	61.80	64.45	61.14
			#22	14.51	13.09	12.98	551	511	520	61.79	62.65	63.69
			#23	29.79	26.89	25.85	1199	1129	1054	63.20	64.66	63.35
			#27	22.12	20.26	21.90	864	801	901	62.72	63.06	64.77

Q [MW]			Producers' results									
S4	S5	S6	Bus	q_i [MW]			π_i [\$]			p_i [\$/MW]		
				S4	S5	S6	S4	S5	S6	S4	S5	S6
143.6	151.4	141.8	#1	13.11	13.08	12.56	455	448	447	60.68	60.23	61.53
			#2	23.92	24.53	23.26	899	911	896	60.54	60.19	61.41
			#13	19.58	20.53	19.99	732	732	761	62.32	60.68	63.04
			#22	35.66	14.42	13.57	1170	505	507	59.25	58.80	61.01
			#23	26.74	52.46	27.45	1054	1825	1113	62.08	60.02	63.28
			#27	24.54	26.34	44.93	940	1013	1623	62.11	62.42	61.48

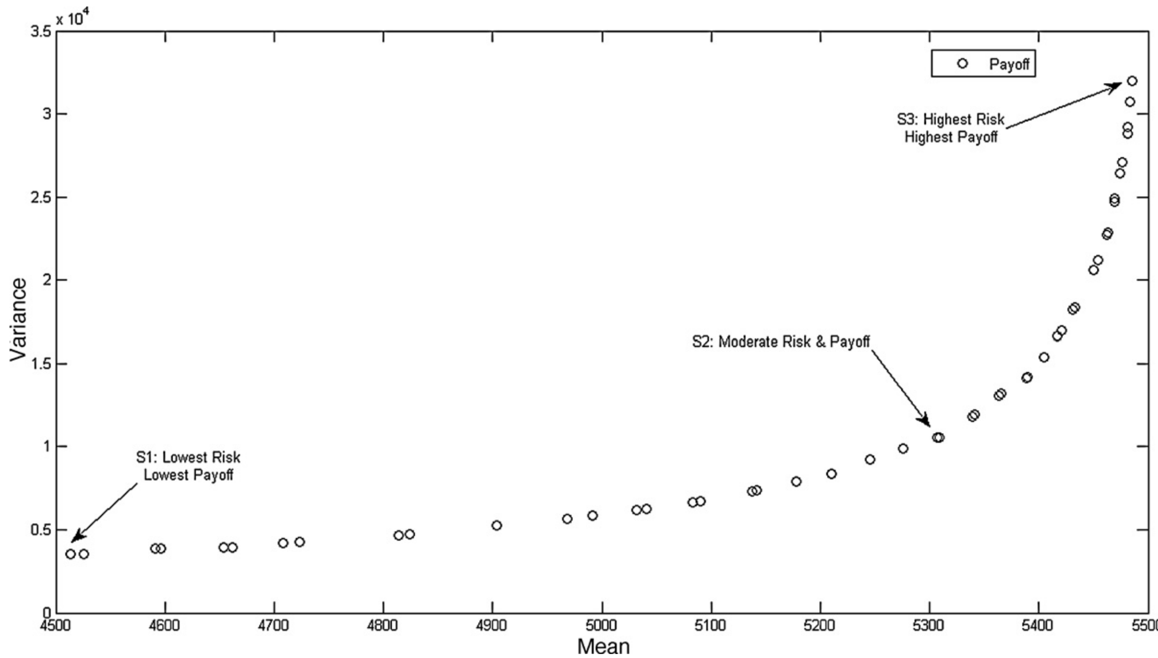

Fig. 11. The nondominated solutions found by NSIWO for the risk management problem.

Table 12
NSIWO parameters for the IEEE 30-bus power system.

Symbol	Value	Symbol	Value
N_0	50	$S_{\max}$	3
$iter_{\max}$	500	n	3
$P_{\max}$	50	σ_{init}	20
$S_{\min}$	1	σ_{final}	0.01

that maximizes the expected total profit of the players and minimizes the total risk (modelled as the total variance of the profits):

$$\begin{aligned} &\max_q E\left\{\sum_i \pi_i(\boldsymbol{q}=[q_1,q_2,\ldots q_{n_p}])\right\} = E\left\{\sum_i P_i\cdot q_i - Cost_i(q_i)\right\}\Bigg|_{\substack{\max \sum_j Benefit_j - \sum_i Cost_i \\ \text{S.T.Constraints}}} \\ &\min_q Var\left\{\sum_i \pi_i(\boldsymbol{q}=[q_1,q_2,\ldots q_{n_p}])\right\} = Var\left\{\sum_i P_i\cdot q_i - Cost_i(q_i)\right\}\Bigg|_{\substack{\max \sum_j Benefit_j - \sum_i Cost_i \\ \text{S.T.Constraints}}} \\ &S.T.\ \pi_i(\boldsymbol{q}) \geq \pi_i(\boldsymbol{q}^{*(n)}) \end{aligned} \tag{19}$$

This is a risk management problem that tries to find an optimal trade-off between the profit and risk of the players in an electricity market. It is also possible to formulate this problem with a single objective function as follows:

$$\begin{aligned} &\max\ E\left\{\sum_i \pi_i(\boldsymbol{q}=[q_1,q_2,\ldots q_{n_p}])\right\} - \rho Var\left\{\sum_i \pi_i(\boldsymbol{q}=[q_1,q_2,\ldots q_{n_p}])\right\}, \\ &S.T.\ \pi_i(\boldsymbol{q}) \geq \pi_i(\boldsymbol{q}^{*(n)}) \end{aligned} \tag{20}$$

However, selection of the appropriate risk factor ρ is not an easy task. Therefore, there is a common preference to sketch the Pareto-optimal curve and then choose the best solution matching the decision makers' desired criteria.

Note that within our framework, the market prices in Eq. (19) will be deterministic. In fact, at each generation of any evolutionary algorithm, a number of solutions are produced and entered into the objective functions, where the objectives are evaluated for the given solutions. Also in our problem, the solutions (i.e., bidding quantities) are provided to the objective functions. In the market clearing sub-problem, since the cost functions are defined with respect to the bidding quantities, the cost functions become constant when the quantities are available. Then, because the constants have no effect in optimization of the market clearing sub-problem (due to zero derivatives), this problem would be independent of the cost functions and the market clearing prices could be obtained deterministically for the given bidding quantities. Consequently, calculation of the expectation and variance for the profit functions in Eq. (19) would be simple (as $E\{P_i{\cdot}q_i\}=P_i{\cdot}qi$ and $\text{Var}\{P_i{\cdot}q_i\}=0$) and limited to the expectation and variance of the cost functions. Since the cost function is linear with respect to the stochastic parameters of Table 13, its expectation and variance could be easily derived analytically as follows:

Table 13
The Stochastic generators' cost data for the IEEE 30-bus power system.

Cost function	Bus	a_i [\$/MW]		b_i [\$/MW^2]		$q_i^{\min}$ [MW]	$q_i^{\max}$ [MW]
		Mean	Var.	Mean	Var.		
$Cost_i(q_i)=a_iq_i+\frac{1}{2}b_iq_i^2$	#1	25	1.5	0.15	0.0075	5	80
	#2	20	1	0.25	0.0125	5	60
	#13	23	1.5	0.20	0.0100	5	60
	#22	22	1	0.25	0.1250	5	60
	#23	20	1	0.20	0.0100	5	80
	#27	22	1	0.15	0.0075	5	70

Table 14
The Pareto improvement results for the risk analysis test problem.

Q [MW]			Bus	q_i [MW]			$E\Sigma_i\pi_i$ [$] (Var.)		
S1	S2	S3		S1	S2	S3	S1	S2	S3
88.7	150.6	129.9	#1	10.6	12.9	18.9	4513 (3524)	5486 (31,966)	5309 (10,565)
			#2	17.2	23.9	21.9			
			#13	14.7	29.4	23.1			
			#22	9.9	13.8	17.2			
			#23	20.2	44.2	26.4			
			#27	15.9	26.3	22.5			

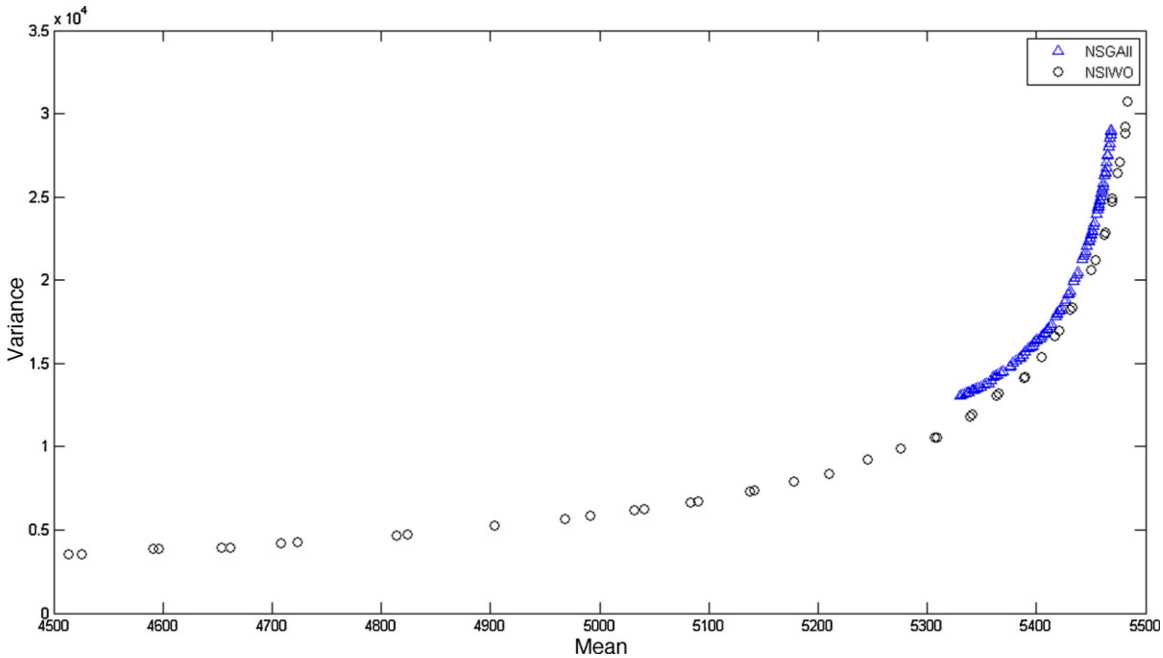

Fig. 12. Comparison between the Pareto fronts obtained by NSIWO and NSGA-II for the risk management problem.

$$E\{\sum_i Cost_i(q_i)\} = \sum_i E\{Cost_i(q_i)\} = \sum_i E\{a_i\}q_i + \frac{1}{2}E\{b_i\}q_i^2, \quad (21)$$

$$\begin{aligned} Var\{\sum_i Cost_i(q_i)\} &= \sum_i Var\{Cost_i(q_i)\} \\ &= \sum_i Var\{a_i\}q_i^2 + \frac{1}{4}Var\{b_i\}q_i^4, \quad (22) \end{aligned}$$

And consequently, the expectation and variance of the total profit will be:

$$\begin{aligned} E\{\sum_i \pi_i(q_i)\} &= \sum_i E\{P_i \cdot q_i\} = E\{Cost_i(q_i)\} \\ &= \sum_i P_i \cdot q_i - E\{a_i\}q_i + \frac{1}{2}E\{b_i\}q_i^2, \quad (23) \end{aligned}$$

$$\begin{aligned} Var\{\sum_i Cost_i(q_i)\} &= \sum_i Var\{P_i \cdot q_i\} = Var\{Cost_i(q_i)\} \\ &= \sum_i Var\{a_i\}q_i^2 + \frac{1}{4}Var\{b_i\}q_i^4. \quad (24) \end{aligned}$$

We used NSIWO with the same parameter settings as those in Table 12 to solve this problem. The Pareto front obtained for this case study is shown in Fig. 11. Three points on this curve are marked as interesting points. The lower left point (S1) yields the worst payoff but endows the lowest risk. Thus, it is favoured by the conservative players. On the contrary, the upper right point (S2) has the highest payoff and risk together, which represents a greedy policy for the players. Eventually, the point in the middle (S3) has a good trade-off between the profit and risk. Thus, it provides a moderate policy for the players. The detailed information about these three points is given in Table 14. To compare the results with NSGA-II results, the Pareto front found by NSIWO algorithm is compared with the front found by NSGA-II (with the same parameters setting of Section 4.2) in Fig. 12. It is clear that NSIWO is able to find better spread of solutions in this case study.

5. Conclusions

In this paper, a novel multiobjective evolutionary algorithm was introduced. This algorithm is an integration of the fast nondominated sorting approach in NSGA-II and Invasive Weed Optimization algorithm. The efficiency of the proposed algorithm (compared to other MOEAs) was shown through a set of well-known benchmarks in multiobjective optimization. It was shown that NSIWO algorithm is comparable with other state-of-the-art MOEAs and can provide better results in some cases. Finally, application of NSIWO to find the Pareto improvement solutions for nonlinear, complex (transmission-constrained), and stochastic (with risk management) electricity market models was studied. Our simulation results showed the performance of this algorithm to obtain successful and promising results for multiobjective decision making problems. Since the infinitely repeated non-cooperative games have shown to reach the Pareto-optimal solutions, NSIWO algorithm is capable of finding the steady state Nash equilibriums for the stochastic nonlinear competitive games.

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