Nash Equilibrium Search in Oligopolistic Electricity Markets Using Coevolutioanry Invasive Weed Optimization

Hossein Hajimirsadeghi, Ashkan Rahimi–Kian, Caro Lucas

***Abstract*—Finding Nash Equilibrium (NE) for nonlinear games is a challenging work due to existence of local Nash Equilibrium traps. So, devising algorithms that are capable of escaping from local optima and finding global solutions is needed for analysis of nonlinear games. Evolutionary Algorithms as the popular stochastic global search algorithms can be exploited for this purpose. This paper presents a cooperative coevolutionary invasive weed optimization (CIWO) for Nash equilibrium search in games with numerous local NEs. Transmission-constrained electricity markets with linear and nonlinear demand functions and unconstrained electricity markets with a nonlinear total demand function are studied in this paper. Cournot model is considered in all the cases and the global NE is obtained for each problem. The results show that the proposed coevoltuionary algorithm is a very promising technique to come up with complex theoretical and practical games.**

***Index Terms*—Evolutionary Algorithms, Invasive Weed Optimization, Particle Swarm Optimization, Biomimicry.**

I. Introduction

MANY techniques have been developed for searching Nash Equilibrium (NE) in game theory problems. All the approaches are inspired by NE definition which is maximizing the payoff, given other players' strategies. The simplest method which can be applied to two or three player games is finding the intersection of best response curves (reaction curves) by drawing or Algebra. For graphical approach, some geometric techniques have been also proposed to come up with more than two player problems [1]. Algebra can improve the method to solve games with several players, but it can be applied to problems with simple mathematical manipulations. This algorithm is commonly used in Cournot or Bertrand models of electricity markets with linear demand functions, using the first-order condition for maximizing each player's payoff [2], [3], [4].

Iterative NE search in which players repeatedly maximize their payoff by turn is another method that is applied to more complex problems. The profit maximization problem which is embedded in this method can be solved by local or global optimization algorithms. In literature, local search is more popular and have been employed in [5], [6] and [7], however in [8], a GA-based algorithm is also presented for profit maximization.

In recent years, with development of Soft Computing [9], and increasing growth of Biomimicry [10], and Bioinspired Computing in a variety of applications, there has been a considerable attention to evolutionary game theory and computational intelligence for game learning and simulation of electricity markets [5], [8], [11]-[18]. Coevolutionary programming is the most popular technique for this purpose. In [5], a novel Hybrid Coevolutionary is applied to solve constrained-transmission electricity markets, and in [13], a GA-based coevolutionary algorithm is exploited to simulate a simple electricity pool. Besides coevolutionary algorithms, learning methods in agent-based approach have also been used to study imperfect competition in electricity markets [19]-[21]. In fact, these days, agent-based economics is a rigorous opponent of game theory to simulate electricity markets.

Another approach for searching NE is characterization of NEs in terms of minima of a function and then minimizing this objective function. This method was firstly employed in finding mixed strategy NEs [14], [15], but recently a similar technique has been introduced in [12] to identify pure NE in games with a large number of players. It seems that more investigations are needed to understand the efficiency of this model.

In this study, Invasive Weed Optimization (IWO) algorithm as an efficient evolutionary algorithm for fast and global search is employed for analysis of the electricity market models. In fact, cooperative Coevolutionary Invasive Weed Optimization (CIWO) proposed in the previous work for global function optimization [38], is modified for Nash equilibrium search in this paper. Invasive Weed Optimization is a novel ecologically inspired algorithm that mimics the process of weeds colonization and distribution. Despite its recent development, it has shown successful results in a number of practical applications like optimization and tuning of a robust controller [22], optimal positioning of piezoelectric actuators [23], developing a recommender system [24], antenna configuration [25], distributed identification and adaptive control of a surge tank [26], analysis of electricity markets dynamics [27], cooperative task assignment of UAVs

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[28], etc.

Section II provides a short definition for games and Nash Equilibrium. In section III, coevolutionary programming is explained with a quick review of cooperative approach for coevolution, and also IWO algorithm for global optimization is introduced. Section IV comprises the simulation studies for NE search in Cournot model of electricity markets, and finally, the conclusions are drawn in section V.

II. Games and Nash Equilibrium

A general multi-player game consists of an index set $N = \{1, 2, 3, \dots, \mathrm{N}\}$ called player's set and an index set $K = \{1, 2, 3, \dots, \mathrm{K}\}$ as the stages of the game, showing the allowable number of moves for each player. In each stage, players take strategies from a set of strategy spaces $U = \{U_k^i\}$, and receive a payoff of $\pi_i(u^i, u^{-i})$, where $u^i \in U^i$ is the pure strategy for player i, given pure strategy set of others $u^{-i} = \{u^1, \dots, u^{i-1}u^{i+1}, \dots, u^N\} \in U^{-i}$. Pure strategy Nash Equilibrium (NE) is a point where no player can obtain a higher profit by unilateral movement. The satisfying NE condition for the combined strategy $\{u^{i*}, u^{-i*}\}$ is characterized in (1).

$$\forall i, \forall u^i \in U^i, \qquad \pi_i(u^{i*}, u^{-i*}) \geq \pi_i(u^i, u^{-i*}) \tag{1}$$

As we will use the term ***local NE*** in this dissertation, here a definition of that from [5] is also provided.

$$\begin{aligned} &\exists \varepsilon > 0 \textit{ such that } \forall i, \forall u^i \in B^{i,\varepsilon}(u^{i*}), \\ &\qquad \pi_i(u^{i*}, u^{-i*}) \geq \pi_i(u^i, u^{-i*}) \end{aligned} \tag{2}$$

where $B^{i,\varepsilon}(\hat{u}^i) = \{u^i \epsilon U^i \ \ \|u^i - \hat{u}^i\| < \epsilon\}$

III. Coevolutionary Programming

In [29], coevlolutioanry algorithm (CEA) is defined as "an evolutionary algorithm that employs a subjective internal measure for fitness assessment." The term ***subjective internal measure*** means that fitness for the individuals are measured based on their interaction with each other and this fitness value influences their evolution in some way. This is a general definition for coevolutionary algorithm which most the coevolutionary computation researchers agree, and it is widely employed in a variety of applications. Neglecting the controversies on definition of CEA, in this study, we focus on multi-population models in which the fitness for individuals is measured by their interaction with individuals in other populations. CAEs are categorized in two distinct types: cooperative and competitive. In cooperative algorithms, solutions are evaluated based on quality of cooperation between individuals while, in competitive coevolution, fitness is calculated based on direct competition among individuals in different populations. The rest of this section presents our proposed cooperative coevolutionary algorithm to find Nash Equilibrium in games.

A. *Cooperative Coevolutionary Algorithm*

In Cooperative CEA, each population represents a piece of a larger problem and the populations evolve their own pieces in interaction with each other to solve the larger problem. A general framework for cooperative coevolutionary algorithms is explained in Fig. 1.

1. For population $p_s \, \epsilon \, P$, all populations
   (a) Initialize population p_s
2. For population $p_s \, \epsilon \, P$, all populations
   (a) Evaluate population p_s with collaborators
3. t:=0
4. do
   (a) For population $p_s \, \epsilon \, P$, all populations
      i. Evolutionary Process to make the next generation
      ii. Evaluate next generation with collaborators
   (b) t:= t+1
5. Repeat 4 until terminating criteria is met

Fig. 1. General framework for Cooperative CEA

For evaluation part, each individual is combined with its collaborators from other populations to form a complete solution and the objective function is evaluated. Terminating criteria can be satisfied by falling short of the acceptable tolerance for changes in strategies or exceeding the maximum number of iterations. In evolutionary process, any evolutionary algorithm (EA) can be exploited, like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Mimetic Algorithm (MA), Simulated Annealing (SA). We employ Invasive Weed Optimization (IWO), a novel EA proposed by Mehrabian and Lucas [22], which is explained in part B.

To put NE search algorithms in the framework characterized in Fig. 1, each player is represented by a population of strategies. The fitness of each strategy is evaluated by selecting the collaborating players from other populations and payoff calculation for that strategy. Selecting the collaborators is very important in coevolutionary programming to have the best performance and find the optimal solutions. In [29], a number of attributes for this purpose are named: sample size, selective pressure and credit assignment. Sample size determines the number of collaborators, while selective pressure is the bias we impose on the selection procedure, and credit assignment deals with the fact how to assign one fitness value to each individual from the results of multiple objective function evaluation.

In our proposed coevolutionary programming, for the purpose of NE search, we set the sample size for each player to 1, i.e., each player takes one collaborator and for the selective pressure, we consider two cases:

1) Collaborators are selected at random
2) The best strategies from the last evaluation are taken as the collaborators.

The former was studied in [5] and [13], while the latter was applied in part of the proposed Hybrid Coevolutionary Algorithm with GA and Hill Climbing in [5] for the goal of NE search. In part C, we have a comparison between these two cases to find NE for a numerical example of a nonlinear static game.

B. *Invasive Weed Optimization*

Invasive weed optimization was developed by Mehrabian and Lucas [22] in 2006. IWO algorithm is a bio-inspired numerical optimization algorithm that simply simulates natural behavior of weeds in colonizing and finding suitable place for growth and reproduction. To model and simulate colonizing behavior of weeds for introducing a novel optimization algorithm, some basic properties of the process is discussed in [22]:

1) *Initialization* with a number of seeds dispread over the search area;
2) Seeds, after growing to a fruity plant, start ***reproduction*** based on their fitness;
3) The newly produced seeds are being randomly dispread around the plants (*spatial dispersal*);
4) Only a limited number of plants can survive in the area and the plants with lower fitness are eliminated until maximum number of plants is remained (*competitive exclusion*). The process repeats for the maximum number of iterations;

Some of the distinctive properties of IWO in comparison with other evolutionary algorithms are way of reproduction, spatial dispersal, and competitive exclusion [22].

In Invasive Weed Optimization algorithm the process begins with initializing a population. It means that a population of initial solutions is randomly generated over the problem space. Then members of the population produce seeds depending on their relative fitness in the population. In other words, the number of seeds for each member is beginning with the value of S_{min} for the worst member and increases linearly to S_{max} for the best member. For the third step, these seeds are randomly distributed over the search space by normally distributed random numbers with mean equal to zero and an adaptive standard deviation. The equation for determining the standard deviation (SD) for each generation is presented in (3).

$$\sigma_{iter} = \frac{(iter_{max} - iter)^n}{(iter_{max})^n}\left(\sigma_{initial} - \sigma_{final}\right) + \sigma_{final} \qquad (3)$$

where $iter_{max}$ is the maximum number of iterations, σ_{iter} is the SD at the current iteration and n is the nonlinear modulation index. The produced seeds, accompanied by their parents are considered as the potential solutions for the next generation. Finally, a competitive exclusion is conducted in the algorithm. It means that after a number of iterations the population reaches its maximum, and an elimination mechanism should be employed. To this end, the seeds and their parents are ranked together and the ones with better fitness survive and are allowed to reproduce. Pseudocode for IWO algorithm is summarized in Fig. 2, and set of parameters for IWO is provided in Table I.

1. Genearte random population of N_0 solutions;
2. For $iter$ = 1 to the maximum number of generations;
   - (a) Calculate fitness for each individual;
   - (b) Compute maximum and minimum fitness in the colony;
   - (c) Set P_g as the best position of all individuals;
   - (d) For each individual $w \in W$;
     - i. Set P_i as the best position of individual w in comparison with its predecessors;
     - ii. Compute number of seeds of w, corresponding to its fitness;
     - iii. For each seed s;
       - 1) Calculate the velocity according to (4);
       - 2) Update the position according to (5);
     - iv. Randomly distribute generated seeds over the search space with normal distribution around the parent plant (w);
     - v. Add the generated seeds to the solution set, W;
   - (e) If $(|W| = N) > p_{max}$;
     - i. Sort the population W in descending order of their fitness;
     - ii. Truncate population of weeds with smaller fitness until $N = p_{max}$;
3. Next $iter$;

Fig. 2. Psuedocode for IWO/PSO algorithm

TABLE I
INVASIVE WEED OPTIMIZATION PARAMETERS

Symbol	Definition
N_0	Number of initial population
it_{max}	Maximum number of iterations
dim	Problem dimension
p_{max}	Maximum number of plant population
s_{max}	Maximum number of seeds
s_{min}	Minimum number of seeds
n	Nonlinear modulation index
$\sigma_{initial}$	Initial value of standard deviation
σ_{final}	Final value of standard deviation

C. *Simulation with a Numerical Example to Find the Best Selective Pressure*

This is a nonlinear static game with *local NE traps* [5], which is also analyzed in [5] and [8], and we can consider it as a good benchmark for nonlinear games. The profit function for this game is characterized in (4), and the global best responses and the local best responses for this game are illustrated in Fig. 3.

$$\begin{aligned} \pi_1(x_1, x_2) &= 21 + x_1 \sin(\pi\, x_1) + x_1\, x_2 \sin(\pi\, x_2) \\ \pi_2(x_1, x_2) &= 21 + x_2 \sin(\pi\, x_2) + x_1\, x_2 \sin(\pi\, x_1) \end{aligned} \qquad (4)$$

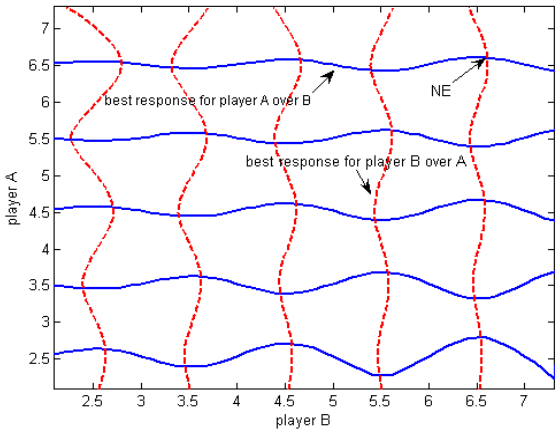


Fig. 3. Local and global best responses for the numerical example

We use IWO for evolutionary process and apply the proposed coevolutionary framework, explained in Fig. 1 with the both cases of selective pressure described in part A. The coevolution process for the both cases is presented in Fig. 4.

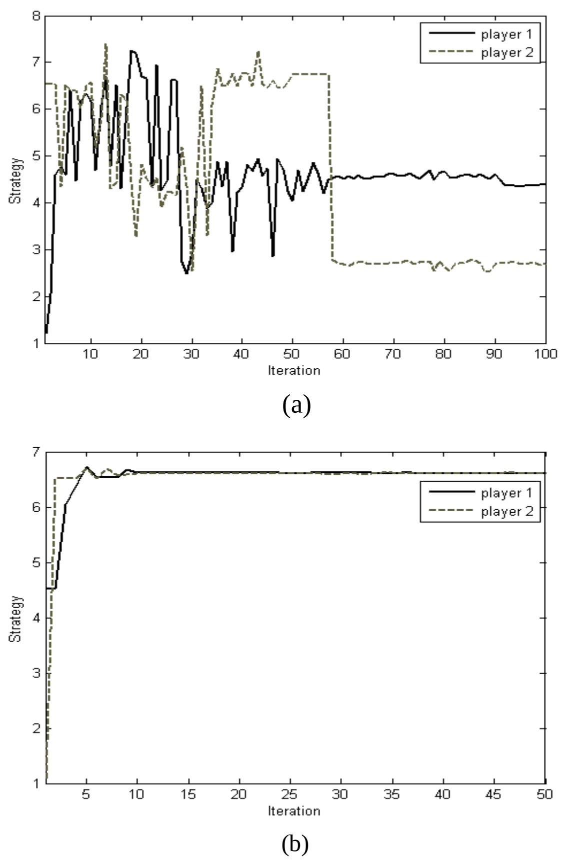


Fig. 4. Comparison between two cases for selective pressure.
a) Cooperative CIWO with random collaborators

It is shown that the coevolutionary approach with random collaborators fail to find the global NE while in the second case, the strategies quickly converge to the global NE. Note that the both cases have the same number of fitness evaluation and are the same in computational complexity, but the selective pressure we adopt for the second case improves the algorithm. Moreover, for the purpose of comparison with the previously proposed coevolutionary algorithm in [5], we can say that our algorithm is better than simple coevolutionary genetic algorithm in finding the global Nash Equilibrium and also outperforms the hybrid coevolutionary genetic algorithm in number of function evaluations and computational complexity.

IV. CIWO FOR NE SEARCH IN COURNOT MODEL

In this section, we simulate a number of electricity market games, and try to fine NE using cooperative CIWO. Indeed, two general systems are considered for study: 1) transmission-constrained electricity market with linear demand functions and 2) electricity market with a nonlinear total demand function. These are both known as complicated games because of the possibility of existence of *local NE traps* [5] in their Cournot model. In addition, we study a transmission-constrained electricity market with nonlinear demand functions which has both difficulties of constrained and nonlinear games in this section.

Although transmission-constrained electricity markets with linear demand functions have linear demand curves, but the transmission constraints can cause individual profit functions to have local optima [7]. Actually, reaction curves in this model are discontinuous piecewise linear functions that might make local NE traps or even disrupt existence of pure strategy equilibrium for the game [5], [30], [31]. Besides the fact that transmission-constrained electricity market model is a good mathematical example with a complex game structure and local optima, it is an important model for market power analysis in the restructured electricity industry [31], [32]. Hence, transmission-constrained electricity market is a good example of complex game for our purpose of *Soft Computing*. Shortly, trading in electricity markets can be represented by the maximization of total welfare subject to the constraints on the system (5).

$$\max \left(\sum_j Benefit_j - \sum_i Cost_i \right)$$

$$S.T. \begin{cases} Transmission\ thermal\ limits \\ Total\ supply\ =\ total\ demand \\ \quad Kirchoff's\ laws \end{cases} \tag{5}$$

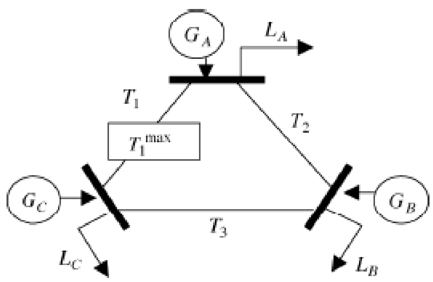
When transmission constraints are binding in the imperfectly competitive market, cournot behavior will produce locational price differences similar to a competitive market with constraints present. This increases the difficulty of computing the profit maximizing condition of the strategic players. The profit maximizing function of each strategic player has an embedded transmission-constrained welfare maximization problem within its major problem. The generation and transmission line constraints are included in the welfare maximization subproblem. The profit function maximization of each utility is given in (3.2).

$$\max\left\{P_i\, q_i - Cost_i \;\middle|\; \begin{matrix} \max \sum_j Benefit_j \quad , \\ Transmission\ Constraints \end{matrix} \right\} \tag{6}$$

Locational prices (P_i), are determined by the Lagrange multipliers of the locational energy balance equality condition for Kirchoff's laws in the welfare maximization problem which is also the market-clearing problem, here [33], [34].

A. Three-Bus Transmission-constrained Cournot Model with Linear Demand Function

This is a complex model of a three-bus transmission-constrained electricity market with a generator and a load at each bus and a pure NE at $s_1 = 1106$, $s_2 = 1046$, $s_3 = 995$ which is solved in [5] and [30] with hybrid coevolutionary programming and graphic representation, respectively. This three-bus network is depicted in Fig. 5



$B_1(d) = -.0555 d_1^2 + 108.4096 d_1$ $C_1(s) = 0.00786 s_1^2 + 1.3606 s_1$
$B_2(d) = -.0669 d_2^2 + 103.8238 d_2$ $C_2(s) = .010526 s_2^2 - 2.07807 s_2$
$B_3(d) = -.0637 d_3^2 + 105.6709 d_3$ $C_3(s) = .006478 s_3^2 + 8.105354 s_3$

Fig. 5. Three-bus Cournot model [3]

Here, we use our proposed Cooperative CIWO to find NE in the case of $T_1^{max} = 100$. The coevolution process for Cooperative CIWO is shown in Fig. 6. Despite poor performance of the simple coevolutionary genetic algorithm in [5], Fig. 6 shows that our coevolutionary algorithm converges to the optimal solution after a limited number of iterations.

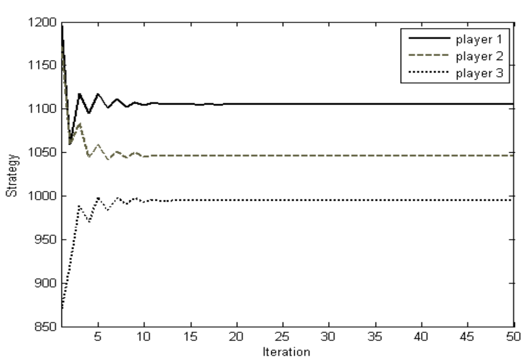


Fig. 6. cooperative CIWO for the three-bus model with $T_1^{max} = 100$

B. Four-Bus Unconstrained Cournot Model with a Total Nonlinear Demand Function

The model which is studied in this part is a modified oligopoly simulation of a restructured ERCOT market, introduced in [35]. The cournot model with four firms (players) and one total nonlinear function is considered and the cost data is listed in Table II. The inverse demand function for this system is characterized in (7). This is a constant elasticity demand function which can be used to remove the problem of sensitivity to elasticity of market demand in Cournot model.

$$P = 1.658 * 10^{14}\, Q^{-2.581} \tag{7}$$

TABLE II
COST DATA FOR FOUR-BUS COURNOT MODEL

Players		TXU	Reliant & CPSB	AEP	Others
Total Cost	ϕ	0.002255	0.00212	0.00573	0.00478
$C = \frac{1}{2}\phi q^2 + \gamma q$	γ	-11.346	-8.751	3.641	-7.226

The solution for this system is summarized in Table III, and the coevoltion process using Cooperative CIWO for finding NEs is illustrated in Fig. 7.

TABLE III
SOLUTION FOR FOUR-BUS COURNOT MODEL

p (\$/MWh)	Q (MHh)	Producers' Results		
		No.	q_i (MWh)	π_i (\$)
111.56	52003	#1	15198.1	1341508
		#2	15713.8	1628728
		#3	9578.5	722079
		#4	11514.8	1050850

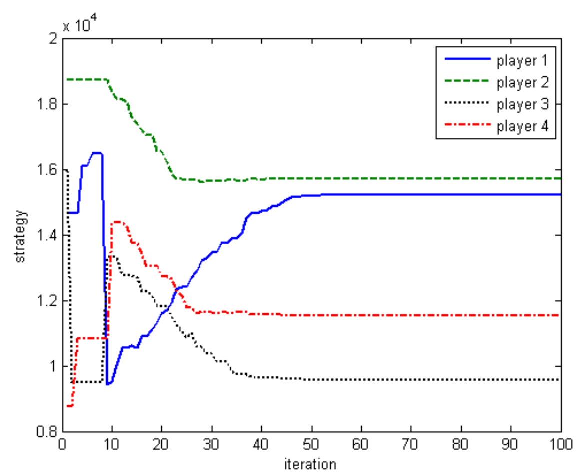
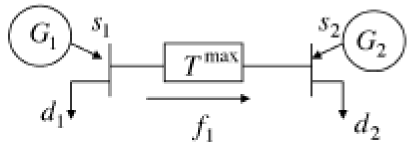


Fig. 7. Cooperative CIWO for the four-bus Cournot model with a constant elasticity demand function

C. Two-Bus Transmission-constrained Cournot Model with Non-Linear Demand Function

The model for this network is depicted in Fig. 8. Utility functions are in the form of power functions which have the required properties of general utility functions like positive slope and convexity. Two cases are studied for this game with $\mathrm{T}^{\max} = 300$ and $\mathrm{T}^{\max} = 30$. In the first case, there is a NE at [323, 323] with a uniform price across the market, while in the second case, NE is located at [303, 173], and locational price differences occur.

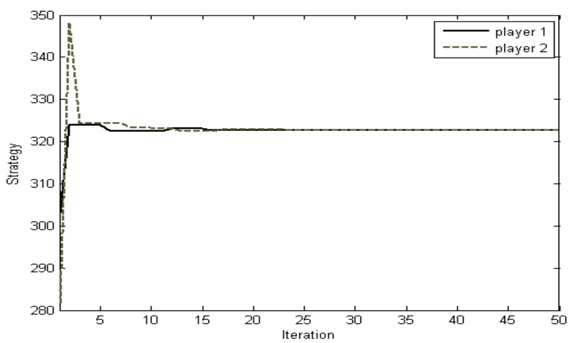


$B_1(d) = 143.8\, d_1^{0.7268}$ $\quad$ $B_2(d) = 70.41\, d_2^{0.7795}$

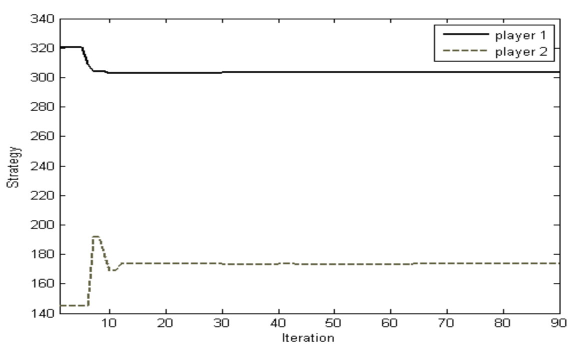
$C_1(s) = .01s_1^2 + 10s_1$ $\quad$ $C_2(s) = .01s_2^2 + 10s_2$

Fig. 8. Two-bus Cournot model with nonlinear demand

The coevolution process to solve these two problems, using Cooperative CIWO is illustrated in Fig. 9, and also the results for $\mathrm{T}^{\max} = 30$ are summarized in Table IV.



(a)



(b)

Fig. 9. Cooperative CIWO for two-bus Cournot model with nonlinear demand. (a) $T^{max} = 300$ (b) $T^{max} = 30$

TABLE IV
SIMULATION RESULTS FOR TWO-BUS NONLINEAR MODEL WITH $\boldsymbol{T^{max} = 30}$

s_1 (MW)	s_2 (MW)	d_1 (MW)	d_2 (MW)	f_1 (MW)	p_1 (MW)	p_2 (MW)
303	173	333	143	-30	21	18

Fig. 9 shows that our proposed algorithm is able to come up with the nonlinearity of demand functions in transmission constrained market problems through a few steps of coevolution.

D. Three-Bus Transmission-constrained Cournot Model with No Pure NE

In this part, a three node problem which was studied in [19] and [36] is presented. The main purpose is to show the performance of our proposed algorithms when there is no pure NE. The system is presented in Fig. 10.

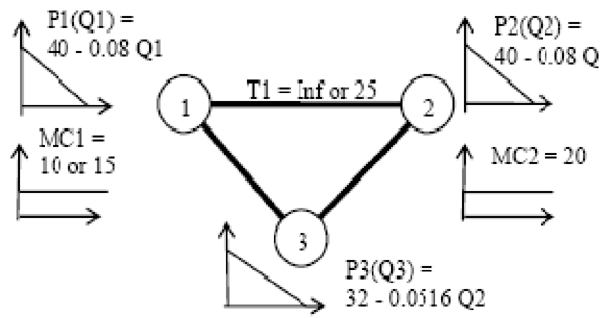


Fig. 10. Three-bus cournot model [5]. Generators are in node 1 and node 2.

When MC1=10, there exists one pure NE at $s_1 = 256$ and $s_2 = 144$, however in the case of MC1=15 there is no pure NE. We are to know whether our coevolutionary algorithm is capable to show nonexistence of pure NE or not. In [19], a coevolutionary algorithm was posed which doesn't converge when there is no pure equilibrium. Although, this feature was declared as an advantage of the proposed algorithm, in [19], one of the issues for comparison between Agent-Based approach and Game Theory was their behavior in absence of NE. In Agent-Based approach, with each run agents converge to a plausible equilibrium and by averaging the results a fairly acceptable equilibrium is taken.

The coevolution process for the both cases (MC1=10 and MC1=15), using Cooprative CIWO with the same parameters is depicted in Fig. 11. It is shown that the proposed coevolutionary algorithm doesn't converge when no NE exists (MC1=15). But, in Fig. 11, it is illustrated that when σ_{final} in CIWO is set to an adequate small value, CIWO converges to an equilibrium, i.e., our proposed coevolutionary algorithm can fulfill the both sides by appropriate tuning of parameters.

V. CONCLUSION

In this paper, cooperative CIWO was employed for Nash equilibrium search in electricity market games with local NE traps. Transmission-constrained electricity markets with linear and nonlinear demand functions and unconstrained electricity markets with a nonlinear total demand function were the main case studies in this work. Results showed that cooperative CIWO has an outstanding performance for the purpose of global NE search within optimal precision of the solutions and high speed of convergence.

For future work, we are to study the proposed algorithm for NE search in mixed strategic games with numerous equilibria.

In addition, analysis of Pareto improvement model for electricity markets with a Multiobjective IWO Algorithm (e.g., NSIWO [37]) is the focus of current research.

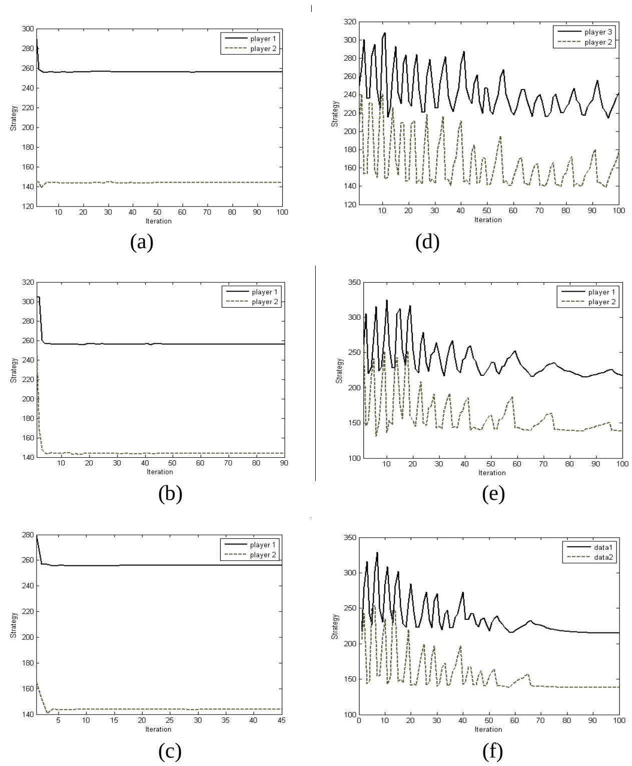


Fig. 11. Cooperative CIWO for three-bus Cournot in Fig. 9
(a) MC1=10 and $\sigma_{final} = 5$ (d) MC1=15 and $\sigma_{final} = 5$
(b) MC1=10 and $\sigma_{final} = 1$ (e) MC1=15 and $\sigma_{final} = 1$
(c) MC1=10 and $\sigma_{final} = 0.1$ (f) MC1=15 and $\sigma_{final} = 0.1$

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