Quantum risk and portfolio management in a quantum mechanics framework.

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Owing to the globalization of the economy, the concept of entangled markets started to form, and this occurrence has smoothed the entrance of quantum mechanics into behavioral finance. In this manuscript, we introduce quantum risk and perform an analy**sis** on portfolio optimization by controlling the quantum potential. We apply this method to eight major indices and construct a portfolio with a minimum quantum risk. The results show quantum risk has a power law behavior with a time-scale just as a standard deviation with different exponents.

Keywords: Finance, Portfolio and Risk management, Quantum Physics

I. INTRODUCTION

The growth of the global economy has made home bound investors to become, almost by default, international investors. Clearly, portfolio theory plays a key role in international investments. Much research has focussed on finding the optimal strategy for allocating wealth among various assets in such a manner to reduce risk rather than maximize returns. If we adopt a hedging perspective, we can claim this is the main objective of portfolio theory[1]. Harry Markowitz's mean-variance model[23] has become a gold standard in portfolio theory. The mean-variance portfolio optimization model is highly dependent on the estimation errors of sample moments and includes negative weights for large portfolios, which requires investors to take on short positions. In the case where short positions are prohibited, constraints have to be applied on portfolio weights in the optimization process[456]. A substantial amount of research has been performed so as to develop a reduced error Markowitz model. In[789101112] we observe that the various authors used different covariant matrix estimators in their models in order to achieve more accurate and diversified portfolios with a lower proportion of negative weights, especially for short time horizons (see[1314]). Some researchers tried non-equal weighted historical data to distinguish between normal and more risky days in their portfolios[15]. Many researchers have tried to solve the problem by combining the investment horizon with return and risk. Bolgorian et al.[16] found a method to introduce a portfolio with minimized waiting time, for a particular return and known risk. In all of these methods, variance plays an important role as a classical correlation function in the process of optimization.

There is a widely held consensus in the academic community that the historical return probability density function (PDF) is in general non-Gaussian and therefore higher moments can be informative. Employing higher moments of a PDF into the estimation of risk would require hard work and in some cases might not even be possible.

However, one can attempt to change perspectives and try to apply a non-classical approach through finding the optimum solution for the portfolio problem. Although on prima facie, it may appear to be far-fetched, but the formalism of quantum mechanics can be a perfect candidate for such a situation, where the PDF is taken as an input of the theory and it gets rid of all the classical problems, including moments. It was the pioneering work of Andrei Khrennikov[1718] which established the quantum mechanical approach in finance. Through the works of Segal[19] and Bagarello[2021] and Haven et al.[22], amongst others, the usefulness of quantum mechanics in their applications to finance was better understood. It

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was Choustova[23] who first proposed to further analyze financial behavior using Bohmian quantum mechanics. Tahmasebi[24] and Shen et al.[25] used Choustova's idea to show that historical (public) information of an asset could be stored in a quantum potential governing that asset (within a particular period of time). Nasiri et al.[26,27] used empirical methods also proposed by Tahmasebi and Shen et al.[25] to analyze the role of trading volume in the quantum potential.

In the next section, we introduce our model and formulate the questions which we shall attempt to answer in section three. There we use a genetic algorithm to optimize the introduced model in order to find the portfolio and the appropriate weights for the minimum risk. In section four we compare two types of risk and we conclude in section five.

II. THE QUANTUM POTENTIAL AND A FIRST LOOK AT WHAT WE CALL 'QUANTUM RISK'

The concept of 'quantum potential' is well known as being a core part of the edifice of Bohmian mechanics[28] which is also known as the semi-classical approach to quantum mechanics. In this approach, the quantum potential plays a key role in guiding the particle to its possible trajectories. Of course, no unique particle trajectory exists in quantum mechanics. Introducing the concept of quantum potential into an interdisciplinary context can lead to ambiguity and requires an innovative interpretation. The potential is easily derived from the Schrödinger PDE through the substitution of the wave function ψ in its polar form, Re^{iS}, and resolving the equation. After the separation of real and imaginary parts, the real part equation will be derived as Eq(1), where the last term in Eq(1) is defined as the quantum potential (i.e. $Q(q)$)

$$\frac{\partial S}{\partial t} + (\frac{\partial S}{\partial q})^2 + U(q) - \frac{\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2} = 0, \qquad Q(q) = -\frac{\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2}. \tag{1}$$

Tahmasebi[24] and Shen[25] showed that there exist quantum potential walls (as well as real potential walls) for an arbitrary (commodity) price return history. Nasiri[26] and Shen[25] showed that as market risk increases, the distance between the potential walls also increases.

In this paper, we take the width of the potential walls as an effective measure for introducing the concept of risk. In order to reach a better understanding, we have illustrated the process in Fig(1), in which Fig(1b) depicts the quantum potential governing an arbitrary observed time-series in a particular period of time shown in Fig(1a). The width of the walls shown in Fig(1b) provides for our innovative notion which we term 'quantum risk' in this particular approach and we use this specific notion throughout the paper. We juxtapose standard and quantum risk in the before last section of this paper.

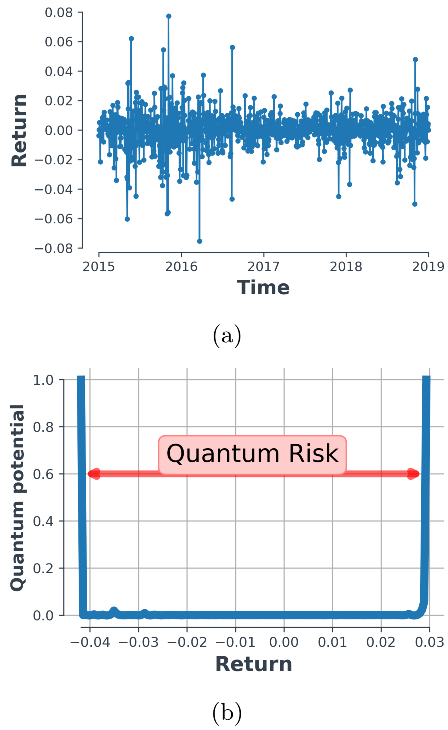


FIG. 1: A schematic process of quantum potential and 'quantum risk', where a) Log-return time-series plotted for S&P 500 and b) quantum potential corresponding to S&P 500 time-series.

III. PORTFOLIO OPTIMIZATION

Whilst constructing a desired portfolio, it is reasonable to question why one portfolio may be preferred over another. In this paper, we use the notion of quantum risk, which we heuristically introduced in Fig(1b), in order to optimize the quantum risk associated to a portfolio. This portfolio is constructed by appropriate company shares. The return of the portfolio, or index, is defined in a straightforward way as Eq(2):

$$\bar{r}(t) = \sum_{i=1}^{N} \omega_i r_i(t), \tag{2}$$

where ω_i is the weight and $r_i(t)$ is the log-return of the i^{th} security at time t. One can easily construct the quantum potential $\frac{-\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2}$ where $R(\bar{r})$ is the probability density function of the portfolio return index. The measurable risk to be minimized is given by the width of the potential's walls.

Our proposed method consists in finding a suitable choice of ω_i's with the help of a genetic algorithm in order to minimize the risk. A graphical illustration of the process is shown in Fig(2) in which an arbitrary set of weights is considered to construct the portfolio index. Each of these signals has been specified with their appropriate quantum potential and their wall width, as our new 'measure' of quantum risk.

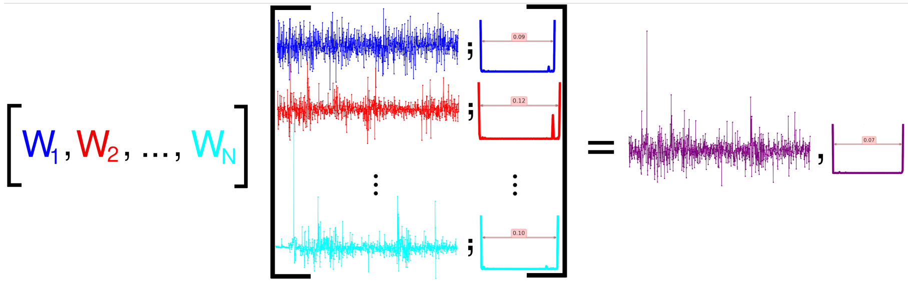


FIG. 2: Schematic portfolio selection with quantum potential.

In what follows, we demonstrate some simulation studies and illustrate the optimum values for the weights of the indices composing the appropriate portfolio. A very noticeable fact about an optimal portfolio is that it reduces the risk of losing money. But not all the feasible combinations of securities promise to do so. By the method introduced in the previous section, we are going to apply portfolio management to the top major market indices (i.e. the Dow Jones industrial, S&P 500 composite, FTSE 100, TOPIX , DAX 30 performance, NIKKEI 225, Korea SE composite and the Shanghai SE A Share). The scaled quantum risk of these indices is shown in Fig(4b) compared with the quantum risk of one arbitrarily optimized portfolio. Some of the combinations (of indices) will show the lower risk amongst all portfolios.

In this paper, we have used the genetic algorithm to find the suitable combinations which minimize the quantum risk. It is quite clear that the desired condition is not satisfied with only one solution. However, multiple answers may give rise to the minimum quantum risk. Five different combinations of indices (forming 5 different portfolios) with minimum desired quantum risk for each portfolio is shown in fig. 3. We note we can expect such levels of low risk given we work with portfolios of indices.

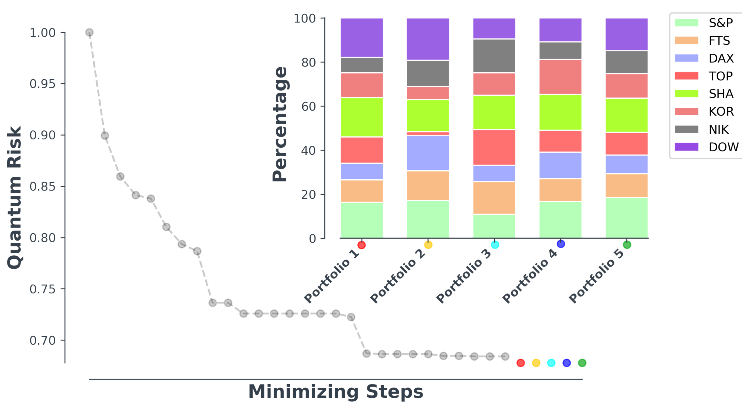


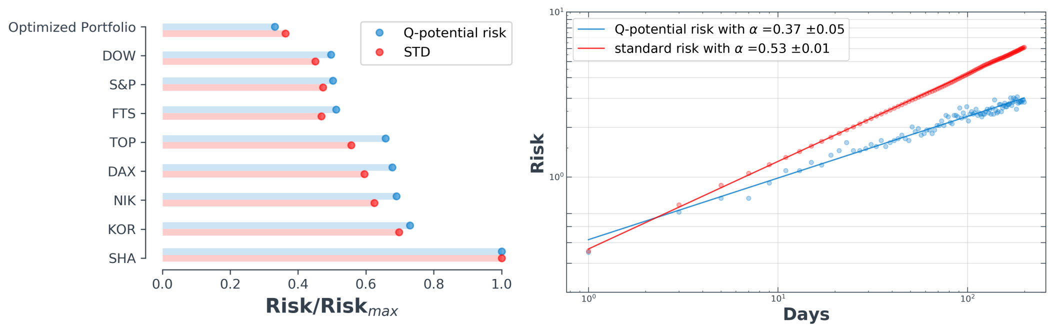
FIG. 3: Portfolio optimization process and selected optimized portfolios.

IV. COMPARING STANDARD AND QUANTUM RISK

From an arbitrary chosen optimized portfolio, we construct scaled returns for a τ time-scale. We claim that there exists a power-law relation between the $Risk$, and τ as follows:

$$Risk(\tau) \propto \tau^{\alpha}, \tag{3}$$

where $Risk(\tau)$ is the risk of the scaled log-return of the original series for τ days. One can examine the α exponent for different portfolios and make a comparison between their exponents. In Fig(4b), we have illustrated the amount of α for one of the selected portfolios in Fig(3), where quantum risk and standard risk plays a role for $Risk$. We make two observations: i) the standard risk and quantum risk have the same trend (i.e. upward); ii) an α close to 0.5 indicates random walk.



(a) a

(b) b

FIG. 4: Log-return time-series and its appropriate quantum potential risk plotted for S&P 500 index.

In Fig(4b), we have demonstrated the scaled risk for different portfolios, in order that one can get a better feeling towards comparing the risk of each individual index with its standard deviation and also the optimized portfolio, for a determined period of time (from Dec. 1994 to Dec. 2019). Fig. 4a shows the normalized quantum risk of major indices introduced in the previous section, compared with their normalized standard notion for risk which is standard deviation. As one can follow from Fig. 4a, the Dow Jones and S&P 500 have the lowest risk amongst all the indices. The Shanghai index itself got the highest risk among these 8 indices. The top bars in Fig. 4a show the risk and standard deviation of one selected optimized portfolio. It is clear that both the quantum risk and standard deviation of the selected portfolio is less than all the indices composing the portfolio.

V. CONCLUSION

In this paper, we showed how a simple concept of the quantum mechanical formalism can begin to aid us in risk management. We introduced a method which, if we control the quantum potential, results in extracting a specific measure of risk information of an index. Since the quantum potential has been shown to help in analyzing the coupling between markets, the bridge between market risk and systematic risk may be worthy of further consideration in future work.

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