Quantum risk and portfolio management in a quantum mechanics framework.

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Owing to the globalization of the economy, the concept of entangled markets started to form, which has smoothed the entrance of quantum mechanics into behavioral finance. In this manuscript, we introduce quantum risk and perform an analysis on portfolio optimization by controlling the quantum potential. We apply this method to eight major indices and construct a portfolio with a minimum quantum risk. The results show quantum risk has a power law behavior with a time-scale just as a standard deviation with different exponents.

Keywords: Finance, Portfolio and Risk management, Quantum Physics

I. INTRODUCTION

Thank you for allowing me to study the paper!

It was Warren Buffet who said that in years from now, cash will be worth less. The growth of the global economy, has made home bound investors to become, almost by default, international investors. Much research has focussed on finding the optimal strategy for allocating wealth among various assets in such a manner to reduce risk rather than maximize returns (which is the main objective of portfolio theory)[1]. It was Harry Markowitz's mean-variance model[2,3] which mobilized portfolio theory in the early 1950's. The mean-variance portfolio optimization model was highly dependent on the estimation errors of sample moments and included negative weights for large portfolios, which required investors to take on short positions. In the case where short positions were prohibited, constraints had to be applied on portfolio weights in the optimization process[4–6]. Enormous amounts of research have been done to develop Markowitz's model in order to reduce its errors. Ledoit, Laloux among others[7–12] used different covariant matrix estimators in their models to get more accurate and diversified portfolios than Markowitz's, and with a lower proportion of negative weights, especially for short time horizons[13–15]. Coelho tried non-equal weighted historical data to distinguish between normal and more risky days in their portfolios[16]. Many researchers have tried to solve the problem by combining the investment horizon with return and risk. Bolgorian et al.[17] found a method to introduce a portfolio with minimized waiting time, for a particular return and known risk. In all of these methods, variance plays an important role as a classical correlation function in the process of optimization.

There is a widely held consensus in the academic community that the historical return probability density function (PDF) is in general non-Gaussian and therefore higher moments are informative. Employing higher moments of a PDF into the estimation of risk would require hard work and in some cases might not even be possible.

However, one can attempt to change perspectives and try to apply a non-classical approach through finding the optimum solution for the portfolio problem. Although on prima facie, it may appear to be far-fetched, but the formalism of quantum mechanics can be a perfect candidate for such a situation, where the PDF is taken as an input of the theory and it gets rid of all the classical problems, including moments. It was the pioneering work of Andrei Khrennikov[18] which established the quantum mechanical approach in finance. Through the works of Segal[19] and Haven[20], amongst others, the usefulness of quantum mechanics in their applications to finance was better understood. It was Choustova[21] who

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first proposed to further analyze financial behavior using Bohmian quantum mechanics. Tahmasebi[22] and Shen et al.[23] used Choustova's idea to show that historical information of an asset could be stored in a quantum potential governing that asset (within a particular period of time). Nasiri et al.[24,25] used empirical methods proposed by Tahmasebi and Shen to analyze the role of trading volume in the quantum potential.

In the next section, we introduce our model and formulate the questions which we shall attempt to answer in section three. There we use a genetic algorithm to optimize the introduced model in order to find the portfolio and the appropriate weights for the minimum risk. In section four we compare two typoes of risk and we conclude in section five.

II. QUANTUM POTENTIAL TO 'QUANTUM' RISK

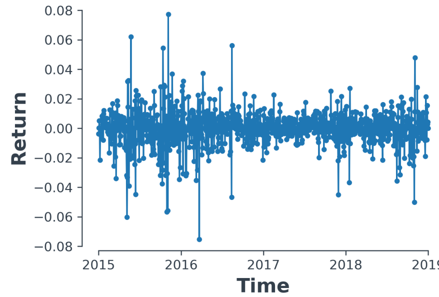
The concept of quantum potential is well known as being part of the edifice of Bohmian mechanics [reference needed] which is also known as the semi-classical approach to quantum mechanics. In this approach, the quantum potential plays a key role in guiding the particle its possible trajectories. Of course no unique particle trajectory exists in quantum mechanics. Introducing the concept of quantum potential into an interdisciplinary context can be ambiguous and may require an innovative interpretation. The potential is easily derived from Schrödinger equation through the substitution of the wave function ψ with its polar form Re^{iS} and resolving the equation. After the separation of real and imaginary parts of the Schrödinger equation, the real part equation will be derived as Eq(1), where the last term in Eq(1) is defined as quantum potential defined in Eq(1)

$$\frac{\partial S}{\partial t} + (\frac{\partial S}{\partial q})^2 + U(q) - \frac{\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2} = 0, \qquad Q(q) = -\frac{\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2}. \tag{1}$$

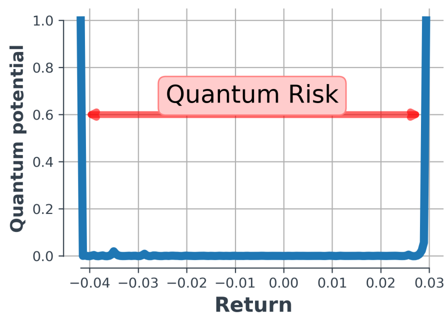
Tahmasebi[22] and Shen[23] showed that there exist quantum potential walls for an arbitrary price return history and Nasiri[24] showed that as market risk increases, the distance between the potential walls also increases.

In this paper, we take the width of the potential walls as an effective measure for introducing the concept of risk. In order to reach a better understanding, we have illustrated the process in Fig(1), in which Fig(1b) depicts the quantum potential governing an arbitrary observed time-series in a particular period of time shown in Fig(1a). The width of the walls shown in Fig(1b) provides for our innovative notion which we term 'quantum risk' in this particular approach and we use this specific notion throughout the paper.

[question: width of walls: but width of walls on real potentials coincide...the idea of quantum risk is interesting - but i think it relates not to the width of the walls of the potential well - since wall width for both real potential and quantum coincide. It may be a idea to look at force - either derived from real or quantum potential - and the gradient of this force. I also think that quantum risk as defined here - is simply measuring maximum spread level of return?]



(a)



(b)

FIG. 1: An schematic process of quantum potential and quantum risk, where a) Log-return time-series plotted for S&P 500 and b) quantum potential corresponding to S&P 500 time-series.

III. PORTFOLIO OPTIMIZATION

Whilst constructing a desired portfolio, it is reasonable to question why one portfolio may be preferred over another. In this paper, we use the notion of quantum risk introduced in Fig(1b), in order to optimize the quantum risk associated to a portfolio, which is constructed by appropriate company shares. The return of the portfolio, or index is defined in a straightforward way as Eq(2):

$$\bar{r}(t) = \sum_{i=1}^{N} \omega_i r_i(t), \tag{2}$$

where ω_i is the weight and $r_i(t)$ is the log-return of the i^{th} security in time t. One can easily construct the quantum potential $\frac{-\hbar^2}{2mR}\frac{\partial^2 R}{\partial q^2}$ where $R(\bar{r})$ is the probability density function of the portfolio return index. The measurable risk to be minimized is the width of the potential's walls. [again real and quantum walls coincide...]

Our method is to find a suitable choice of ω_i's with the help of a genetic algorithm in order to minimize the risk. A graphical illustration of the process is shown in Fig(2) in which an arbitrary set of weights is considered to construct the portfolio index. Each of these signals has been specified with their appropriate quantum potential and their wall width as our new notion of quantum risk. **[Question: weights in the figure should w1, w2,....wn - not w3]**

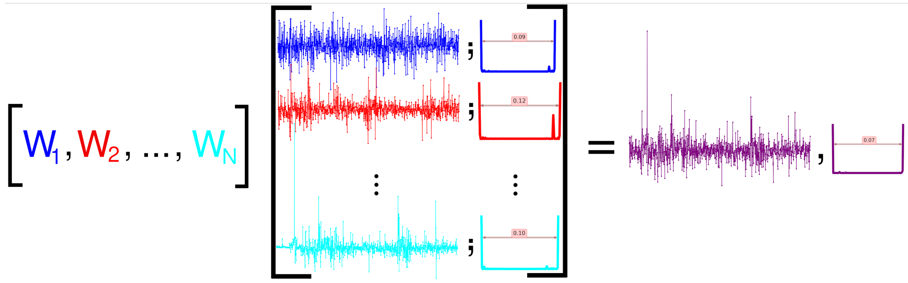


FIG. 2: Schematic portfolio selection with quantum potential.

In the following we demonstrate some case studies and illustrate the optimum values for the weights of the shares composing the appropriate portfolio. A very noticeable fact about an optimal portfolio is that it reduces the risk of loosing money. But not all the feasible combinations of securities promise us to do so. By the method introduced in the previous section, we are going to apply portfolio management to the top major market indices, namely Dow Jones industrial, S&P 500 composite, FTSE 100, TOPIX , DAX 30 performance, NIKKEI 225, Korea SE composite and the Shanghai SE A Share. The scaled quantum risk of these indices has been shown in the Fig(4b) compared with the quantum risk of one arbitrary optimized portfolio. Some of the combinations will show the lower risk among all. In this work we have tried the genetic algorithm to find the suitable combinations which minimizes the quantum risk.[It is pretty obvious that the desired condition is not satisfied with only one solution however lots of answers may give rise to the minimum quantum risk. Five different combinations of indices (forming 5 different portfolios) with minimum desired quantum risk for its portfolio is shown in the Fig(2). [figure 3!!] [Question: **The issue may be here that the return is not juxtaposed next to the risk level. The green portfolio may be useful to have from a risk level but how does it compare to the other portfolios in terms of expected return?**]

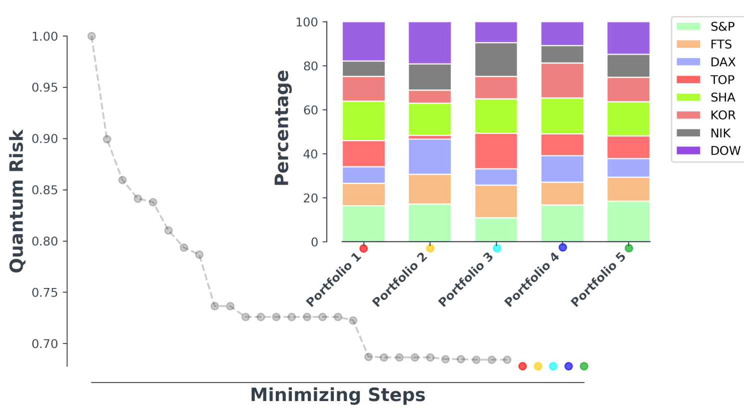


FIG. 3: Portfolio optimization process and selected optimized portfolios.

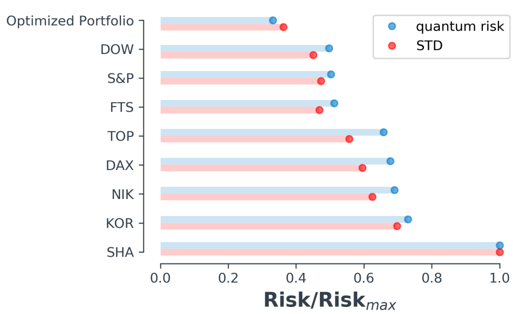
The degeneracy is pretty expected [**question: IT IS UNCLEAR WHAT IS MEANT WITH DEGENERACY**] with the fact that the correlation companies [?], especially the major ones has been risen enormously since last decades [**My apologies - it is a little unclear.**]. Having the degenerate portfolios can come to help in some senses, managing the dynamics of our portfolio in time, walking through various minimum quantum risk portfolios, among others. [**Question: I think this could need a bit more explanation**] ...

IV. COMPARING STANDARD AND QUANTUM RISK

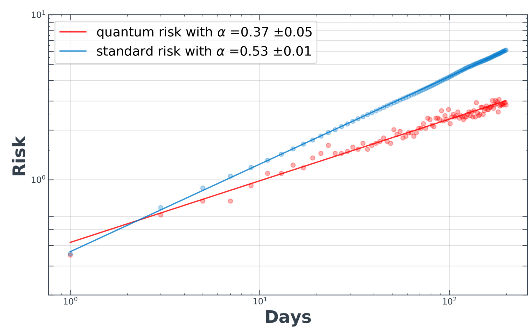
In the previous section, we realized that the optimum risk is highly degenerate [**Question: thanks for indicating what you mean?**] among constructed portfolios. This degeneracy has allowed us to choose different portfolios in each different distinct periods. In this section, we introduce another useful method towards distinguishing among degenerate portfolios. From an arbitrary chosen optimized portfolio, we construct scaled returns for a τ time-scale. We claim that there exists a power-law relation between the $Risk$, and τ as follows:

$$Risk(\tau) \propto \tau^{\alpha}, \tag{3}$$

Where $Risk(\tau)$ is the risk of the scaled log-return of the original series for τ days. One can examine the α exponent for different portfolios and make a comparison between their exponents. In Fig(4b), we have illustrated the amount of α for one of the selected portfolios in Fig(3), where quantum risk and standard risk plays a role for $Risk$.



(a) a



(b) b

FIG. 4: Log-return time-series and its appropriate quantum potential plotted for S&P 500 index.

[colours seem to be reversed in the figure - blue is quantum risk - but not in figure b.?

In Fig(4b), we have demonstrated the scaled risk for different securities, in order that one can get a better feeling towards comparing the risk of each individual security with its standard deviation and also the optimized portfolio, for a determined period of time(From Dec. 1994 to Dec. 2019). Fig(4b) shows the normalized quantum risk of major indices introduced in the previous section, compared with their normalized standard notion existing for risk which is standard deviation. [**Question: SO HERE YOU DISTINGUISH BETWEEN WALL WIDTH AND AVERAGE (MAX SD) ON THE RETURNS. A POSSIBLE QUESTION: WHY WOULD THOSE TWO DIFFER THAT MUCH - KNOWING THAT WALL WIDTH OF REAL AND QUANTUM POTENTIAL COINCIDE (THE PDF ON THE REAL POTENTIAL WILL GIVE INDICATING OF SD LEVEL?)**] As one can follow from Fig(4b) the Dow Jones and S&P 500 have the lower risk of all and Shanghai index itself got the highest risk among these 8 indices. Above all these securities in Fig(4b), the risk and STD of one selected optimized portfolio has been drawn. It is clear that both the quantum risk and STD of the selected portfolio is less than all the securities composing the portfolio. [**question: THIS IS TO BE EXPLAINED BETTER - IT IS NOT CLEAR. BUT IT IS A VERY INTERESTING DEVELOPMENT.**]

V. CONCLUSION

Risk has been the most important variable for investors for a long period of time. In spite of the fact that lots of research has been done in this particular context, we still welcome any innovative idea which formalizes better risk management. Recently, quantum mechanical applications in financial analysis attracted the attention of some researchers. In this work, we showed how quantum mechanics can lead us to perform portfolio and risk management by introducing a method by which, if we control the quantum potential, results in extracting risk information of a security. Since the quantum potential has been shown to analyze the coupling between markets, the bridge between market risk and systematic risk can be connected in further analysis [what do you mean?].

VI. REFRENCES [THERE ARE SPELLING MISTAKES IN THE REFERENCE LIST - I CAN ADJUST]

[1] Jorion, Philippe. "International portfolio diversification with estimation risk." Journal of Business (1985): 259-278.

[2] Markowitz, Harry M. "Foundations of portfolio theory." The journal of finance 46.2 (1991): 469-477.

[3] Markowitz, Harry M. "The early history of portfolio theory: 1600â€"1960." Financial analysts journal 55.4 (1999): 5-16.

[4] Best, Michael J., and Robert R. Grauer. "Positively weighted minimum-variance portfolios and the structure of asset expected returns." Journal of Financial and Quantitative Analysis 27.4 (1992): 513-537.

[5] Green, Richard C., and Burton Hollifield. "When will mean?variance efficient portfolios be well diversified?." The Journal of Finance 47.5 (1992): 1785-1809.

[6] Jagannathan, Ravi, and Tongshu Ma. "Risk reduction in large portfolios: Why imposing the wrong constraints helps." The Journal of Finance 58.4 (2003): 1651-1683.

[7] Ledoit, Olivier, and Michael Wolf. "Improved estimation of the covariance matrix of stock returns with an application to portfolio selection." Journal of empirical finance 10.5 (2003): 603-621.

[8] Laloux, Laurent, et al. "Noise dressing of financial correlation matrices." Physical review letters 83.7 (1999): 1467.

[9] Plerou, Vasiliki, et al. "Universal and nonuniversal properties of cross correlations in financial time series." Physical review letters 83.7 (1999): 1471.

[10] Rosenow, Bernd, et al. "Portfolio optimization and the random magnet problem." EPL (Europhysics Letters) 59.4 (2002): 500.

[11] Potters, Marc, Jean-Philippe Bouchaud, and Laurent Laloux. "Financial applications of random matrix theory: Old laces and new pieces." arXiv preprint physics/0507111 (2005).

[12] Anderberg, Michael R. Cluster analysis for applications: probability and mathematical statistics: a series of monographs and textbooks. Vol. 19. Academic press, 2014.

[13] Kondor, Imre, SzilÃ¡rd Pafka, and GÃ¡bor Nagy. "Noise sensitivity of portfolio selection under various risk measures." Journal of Banking & Finance 31.5 (2007): 1545-1573.

[14] Pafka, SzilÃ¡rd, and Imre Kondor. "Noisy covariance matrices and portfolio optimization II." Physica A: Statistical Mechanics and its Applications 319 (2003): 487-494.

[15] Kondor, Imre, et al. "Estimation Noise in Portfolio Optimization with Absolute Deviation." (2004).

[16] Coelho, Ricardo, et al. "The evolution of interdependence in world equity marketsâ€"Evidence from minimum spanning trees." Physica A: Statistical Mechanics and its Applications 376 (2007): 455-466.

[17] Bolgorian, Meysam, A. H. Shirazi, and G. R. Jafari. "Portfolio Selection Using Level Crossing Analysis." International Journal of Modern Physics C 22.08 (2011): 841-848.

[18] Khrennivov, Andrei. "Classical and quantum mechanics on information spaces with applications to cognitive, psychological, social, and anomalous phenomena." Foundations of Physics 29.7 (1999): 1065-1098.

[19] Segal, Wiliam, and I. E. Segal. "The Blackâ€"Scholes pricing formula in the quantum context." Proceedings of the National Academy of Sciences 95.7 (1998): 4072-4075.

[20] Haven, Emmanuel. "The variation of financial arbitrage via the use of an information wave function." International Journal of Theoretical Physics 47.1 (2008): 193-199.

[21] Choustova, Olga. "Quantum probability and financial market." Information Sciences 179.5 (2009): 478-484.

[22] Tahmasebi F., S. Meskinimood, A. Namaki, S. V. Farahani, S. Jalalzadeh and G. Jafari, Financial market images: a practical approach owing to the secret quantum potential. EPL: Europhysics Letters **109 (3)**, 30001 (2015).

[23] Shen C. and E. Haven, Using empirical data to estimate potential functions in commodity markets: some initial results. International Journal of Theoretical Physics **56 (12)**, 4092–4104 (2017).

[24] Nasiri, Sina, Eralp Bektas, and Gholamreza Jafari. "Risk Information of Stock Market Using Quantum Potential Constraints." Emerging Trends in Banking and Finance. Springer, Cham, 2018. 132-138.

[25] Nasiri, Sina, Eralp Bektas, and G. Reza Jafari. "The impact of trading volume on the stock market credibility: Bohmian quantum potential approach." Physica A: Statistical Mechanics and its Applications 512 (2018): 1104-1112.

[26] Haven, Emmanuel, A. Khrennikov, C. Ma and S. Sozzo, Special section on "Quantum Probability Theory and its Economic Applications". Journal of Mathematical Economics **78**, 127-197 (2018).