In His Name

Quiz 4 - Discrete Mathematics
Date : 28th of Mehr - 1398
Time: 2 hours

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1)

Show that this implication is a tautology, by using a table of truth: $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$.

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2)

Show that $[(p \vee q) \wedge (\neg p \vee r) \to (q \vee r)$ is a tautology

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3)

Determine whether these are valid arguments:

a) "If x^2 is irrational, then x is irrational. Therefore, if x is irrational, it follows that x^2 is irrational."

It is not a valid argument. The two statements can be simplified as "$p \to q$, therefore $q \to p$". The second statement is the converse of the first statement: they are not equivalent.

b) "If x^2 is irrational, then x is irrational. The number π^2 is irrational, it follows that π is irrational."

It is a valid argument: we are just applying the result to the specific case $x = \pi$

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4)

Prove that a square of an integer ends with a 0, 1, 4, 5 6 or 9. (Hint: let $n = 10k + l$, where $l = 0$, 1, ,9)

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5)

Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

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6)

Prove thatthese statements are equivalent:

- $p : n^2$ is odd
- $q : 1 - n$ is even
- $s : n^2 + 1$ is even

$q \leftrightarrow p$

$q \leftrightarrow s$

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7)

Prove that at least one of the real numbers $a_1, a_2, \ldots a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

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8)

8-1)

(p ∧ q) ∨ r
r → s

a) p ∨ r
b) p ∨ s
c) q ∨ s
d) q ∨ r

8-2)

$p \rightarrow (q \vee r)$
$s \rightarrow \neg r$
$p \wedge s$

Good

Luck !