1)

a) $[p \wedge (p \to q)] \to q$

p	q	$[p \wedge (p \to q)]$	$\to$	q
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	T	F

b) $q \leftrightarrow (\sim p \vee \sim q)$

p	q	$q \leftrightarrow (\sim p \vee \sim q)$
T	T	F
T	F	T
F	T	T
F	F	T

c) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$[(p \to q) \wedge (q \to r)]$	$\to$	$(p \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	T	T

2)

a) $[p \to (q \to r)] \to [(p \to q) \wedge (p \to r)]$

$\equiv \sim[\sim p \vee (\sim q \vee r)] \vee [(\sim p \vee q) \wedge (\sim p \vee r)]$

$\equiv [p \wedge q \wedge \sim r] \vee [\sim p \vee (q \wedge r)]$

. (not tautology) ✗

b) $p \to [q \to (p \wedge q)] \equiv$

$\sim p \vee [\sim q \vee (p \wedge q)] \equiv$

$\sim p \vee [(\sim q \vee p) \wedge \underbrace{(\sim q \vee q)}_{T}] \equiv$

$\sim p \vee [(\sim q \vee p) \wedge T] \equiv$

$\sim p \vee (\sim q \vee p) \equiv (\sim p \vee p) \vee \sim q$

$\equiv T \vee q \equiv T$ ✓ (tautology)

c)

$(p \vee q) \to [q \to (p \wedge q)]$

$\equiv \sim(p \vee q) \vee [\sim q \vee (p \wedge q)]$

$\equiv \sim(p \vee q) \vee [(\sim q \vee p) \wedge \underbrace{(\sim q \vee q)}_{T}]$

$\equiv \sim(p \vee q) \vee [(\sim q \vee p) \wedge T]$

$\equiv \sim(p \vee q) \vee (\sim q \vee p)$

$\equiv (\sim p \wedge \sim q) \vee (\sim q \vee p)$

$\equiv \underbrace{((\sim q \wedge \sim p) \vee \sim q)}_{\sim q} \vee p$

$\equiv \sim q \vee p$ ✗ (not tautology)

3)

a) $p \wedge (q \vee r) \wedge (\sim p \vee \sim q \vee r) \xrightarrow{\text{negate}} \sim p \vee \sim(q \vee r) \vee \sim(\sim p \vee \sim q \vee r)$

$\equiv \sim p \vee (\sim q \wedge \sim r) \vee ((p \wedge q) \wedge \sim r) \equiv \sim p \vee [\sim r \wedge (\sim q \vee (p \wedge q))]$

$\equiv \sim p \vee [\sim r \wedge [(\sim q \vee p) \wedge (\underbrace{\sim q \vee q}_{\top})]] \equiv \sim p \vee [\sim r \wedge (\sim q \vee p)] \equiv$

$(\sim p \vee \sim r) \wedge [\underbrace{\sim p \vee (\sim q \vee p)}_{\sim p \vee p \vee \sim q \equiv \top}] \equiv \sim p \vee \sim r \equiv \sim(p \wedge r)$

b) $p \rightarrow (\sim q \wedge r) \xrightarrow{\text{negate}} \sim [p \rightarrow (\sim q \wedge r)] \equiv$

$\sim [\sim p \vee (\sim q \wedge r)] \equiv p \wedge \sim(\sim q \wedge r) \equiv p \wedge (q \vee \sim r) \equiv$

$(p \wedge q) \vee (p \wedge \sim r)$

4)

a) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

steps: ① $p \wedge (p \vee q) \equiv (p \vee F) \wedge (p \vee q) \equiv p \vee (F \wedge q) \equiv p \vee F \equiv p$

② $p \vee p \equiv p$ (Idempotent law) (absorption law قانون الامتصاص)

b) $p \vee q \vee (\sim p \wedge \sim q \wedge r) \Leftrightarrow (p \vee q \vee r)$

steps: $\sim p \wedge \sim q \equiv \sim(p \vee q)$ De Morgan's law

$(p \vee q) \vee (\sim(p \vee q) \wedge r) \equiv$ Distributive law $[(p \vee q) \vee \sim(p \vee q)] \wedge [(p \vee q) \vee r] \equiv$

$\top$ ← negation law

$T \wedge (p \vee q \vee r) \equiv p \vee q \vee r$ (Identity law)

c) $[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] \leftrightarrow (p \wedge q)$

$(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r) \equiv \sim(\sim p \vee \sim q) \vee (p \wedge q \wedge r) \equiv$

$\sim(\sim(p \wedge q)) \vee (p \wedge q \wedge r) \equiv (p \wedge q) \vee ((p \wedge q) \wedge r) \equiv p \wedge q$ (absorption laws) قانون جذب

$p \wedge q$ ← double negation laws

5)

a) $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

$\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv (\sim(\sim p \wedge q) \wedge \sim(\sim(p \vee q)))$

$\vee (p \wedge q) \equiv ((p \vee \sim q) \wedge (p \vee q)) \vee (p \wedge q) \equiv$

$(p \vee (q \wedge \sim q)) \vee (p \wedge q) \equiv (p \vee F) \vee (p \wedge q) \equiv p \vee (p \wedge q) \equiv p \vee$

b) $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) \equiv \sim p \wedge (q \vee \sim q)$

$\equiv \sim p \wedge T \equiv \sim p$ X