In The Name Of God, The Compassionate, The Merciful

Shiraz University

Homework2
Deadline: 19/7/98

1. Prove or disprove the following logical equivalences for quantified statements.
   a) $\exists_x[\boldsymbol{p}(\boldsymbol{x}) \wedge \boldsymbol{q}(\boldsymbol{x})] \Rightarrow [\exists_{\boldsymbol{x}} \boldsymbol{p}(\boldsymbol{x}) \wedge \exists_x \boldsymbol{q}(\boldsymbol{x})]$
   b) $\forall_x[\boldsymbol{p}(\boldsymbol{x}) \wedge \boldsymbol{q}(\boldsymbol{x})] \Leftrightarrow [\forall_{\boldsymbol{x}} \boldsymbol{p}(\boldsymbol{x}) \wedge \forall_{\boldsymbol{x}} \boldsymbol{q}(\boldsymbol{x})]$
   c) $[\forall_{\boldsymbol{x}} \boldsymbol{p}(\boldsymbol{x}) \vee \forall_{\boldsymbol{x}} \boldsymbol{q}(\boldsymbol{x})] \Rightarrow \forall_{\boldsymbol{x}}[\boldsymbol{p}(\boldsymbol{x}) \vee \boldsymbol{q}(\boldsymbol{x})]$
2. Let p(x, y), q(x, y), and r(x, y) represent three quantified statements. What is the negation of the following compound statement?

$$\forall_{\boldsymbol{x}}\exists_{\boldsymbol{y}}[(\boldsymbol{p}(\boldsymbol{x},\boldsymbol{y}) \wedge \boldsymbol{q}(\boldsymbol{x},\boldsymbol{y})) \rightarrow \boldsymbol{r}(\boldsymbol{x},\boldsymbol{y})]$$

3. Prove whether or not the following compound predicates are true for every 2-place predicate p(x, y):
   a) $\exists_{\boldsymbol{x}}\forall_{\boldsymbol{y}} \boldsymbol{p}(\boldsymbol{x},\boldsymbol{y}) \Leftrightarrow \forall_{\boldsymbol{y}}\exists_{\boldsymbol{x}} \boldsymbol{p}(\boldsymbol{x},\boldsymbol{y})$
   b) $\exists_{\boldsymbol{x}}\exists_{\boldsymbol{y}}\boldsymbol{p}(\boldsymbol{x},\boldsymbol{y}) \Leftrightarrow \exists_{\boldsymbol{y}}\exists_{\boldsymbol{x}} \boldsymbol{p}(\boldsymbol{x},\boldsymbol{y})$
4. In calculus, the definition of the limit L of a sequence of real numbers r_1, r_2, r_3, … can be given as

$$\lim_{n\to\infty} r_n = L$$

   if (and only if) for every $\epsilon > 0$ there exists a positive integer k so that for all integers n, if $n > k$ then $|r_n - L| < \epsilon$.
   In symbolic form, it can be expressed as

$$\lim_{n\to\infty} r_n = L \Leftrightarrow \forall_{\epsilon>\mathbf{0}} \exists_{\boldsymbol{k}>\mathbf{0}} \forall_{\boldsymbol{n}} [(\boldsymbol{n} > \boldsymbol{k}) \rightarrow |r_n - L| < \epsilon]$$

   Express $\lim_{n\to\infty} r_n \neq L$ in symbolic form.

Attention: The due date is as specified above, therefore, please try to finish your homework on time as the deadline might not extend. Feel free to ask your graders in case of encountering any problem. Do your best and leave us with the rest.