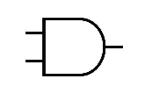
W2022: A Digital Odyssey

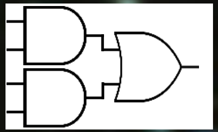
The Ultimate Trip.

Number Systems |$(12)_{10} \rightarrow (1100)_2$

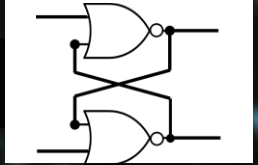
Logic Gates |



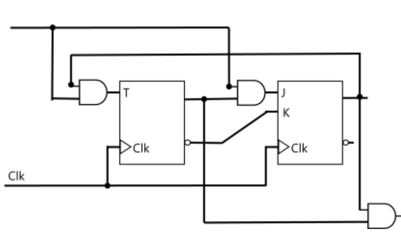
Combinational Logic |

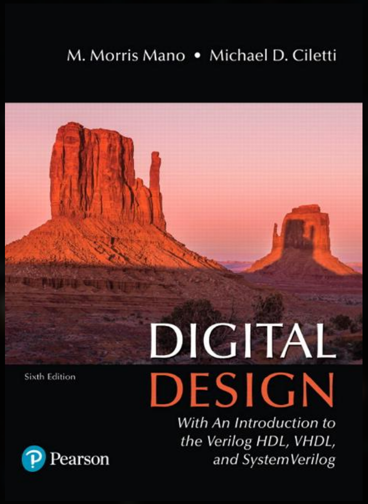


Flip-Flop |



Sequential Logic |





M. Morris Mano • Michael D. Ciletti

DIGITAL DESIGN

Sixth Edition

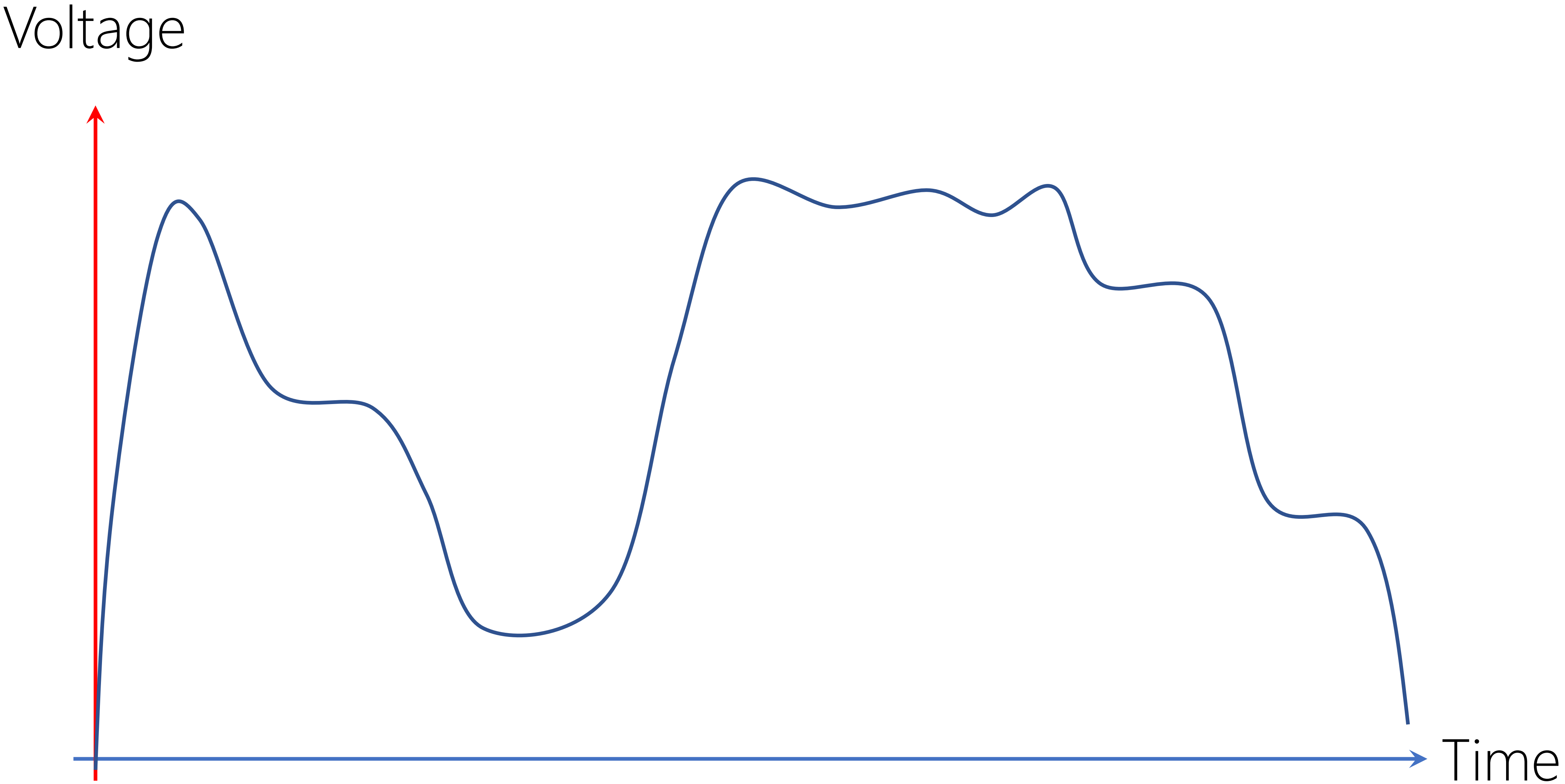
With An Introduction to the Verilog HDL, VHDL, and SystemVerilog

Pearson

Chapter 2
Boolean Algebra and Logic Gates

ANALOG SYSTEMS

Continuous


Voltage
Time

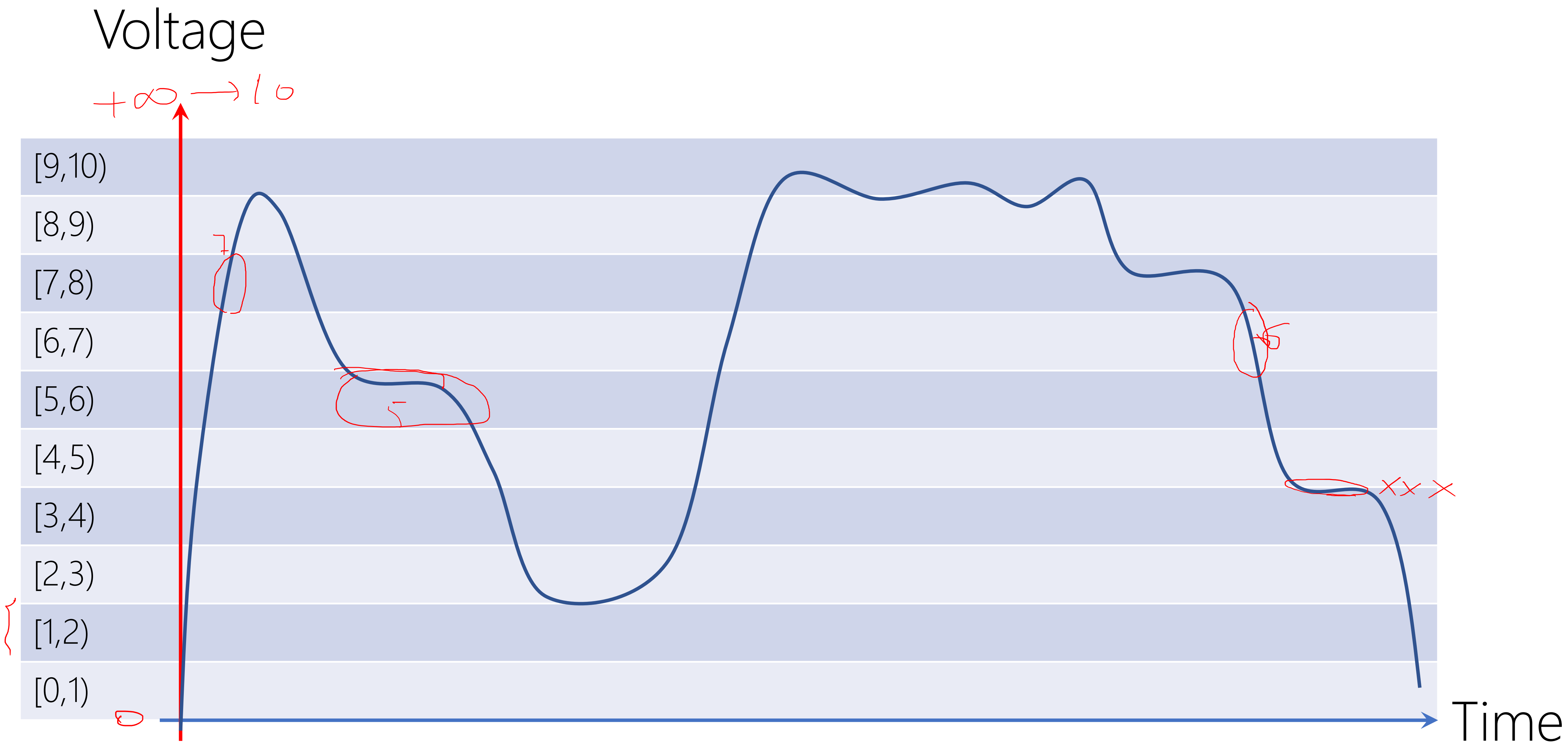
DIGITAL SYSTEMS

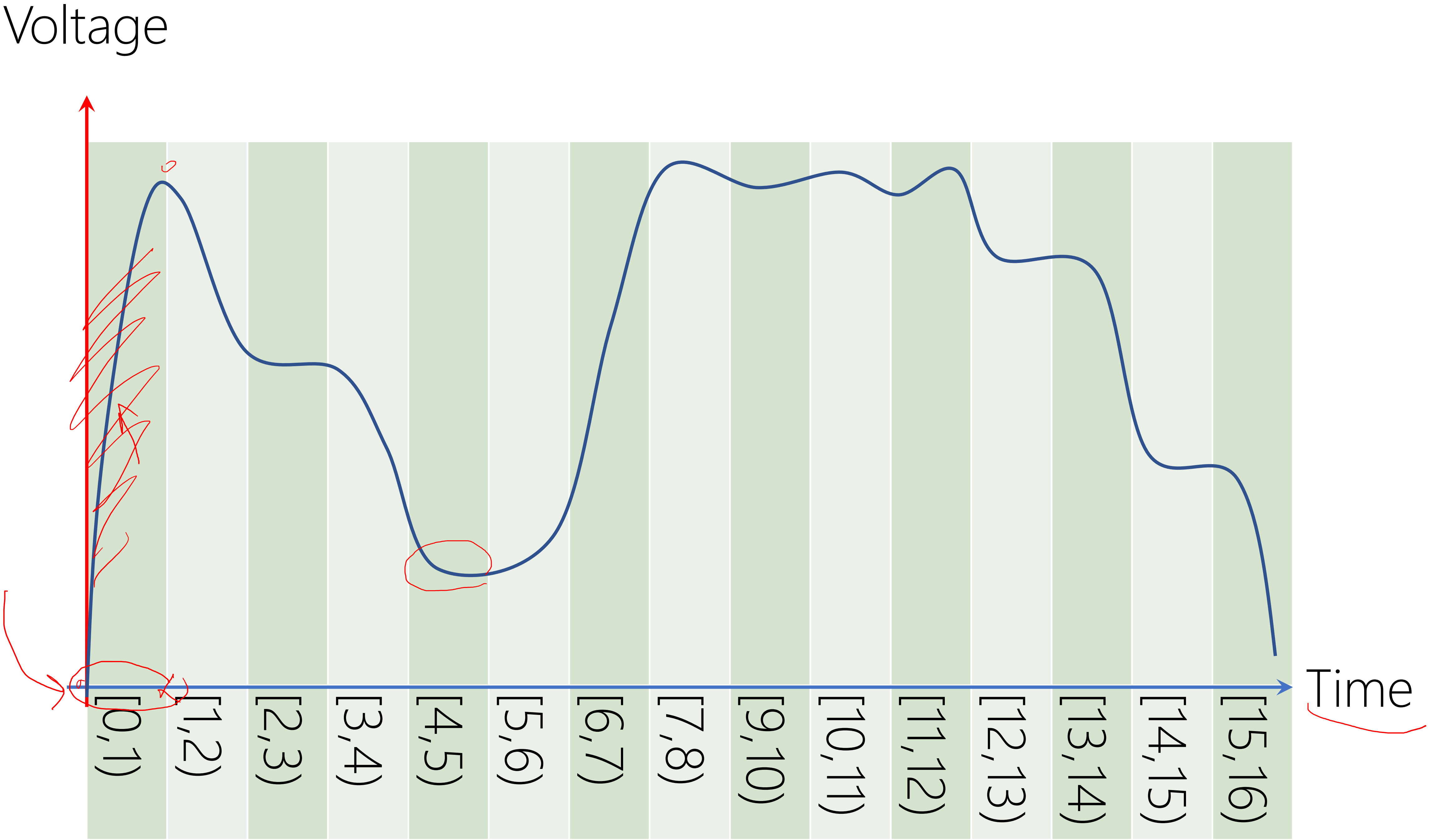
Discrete

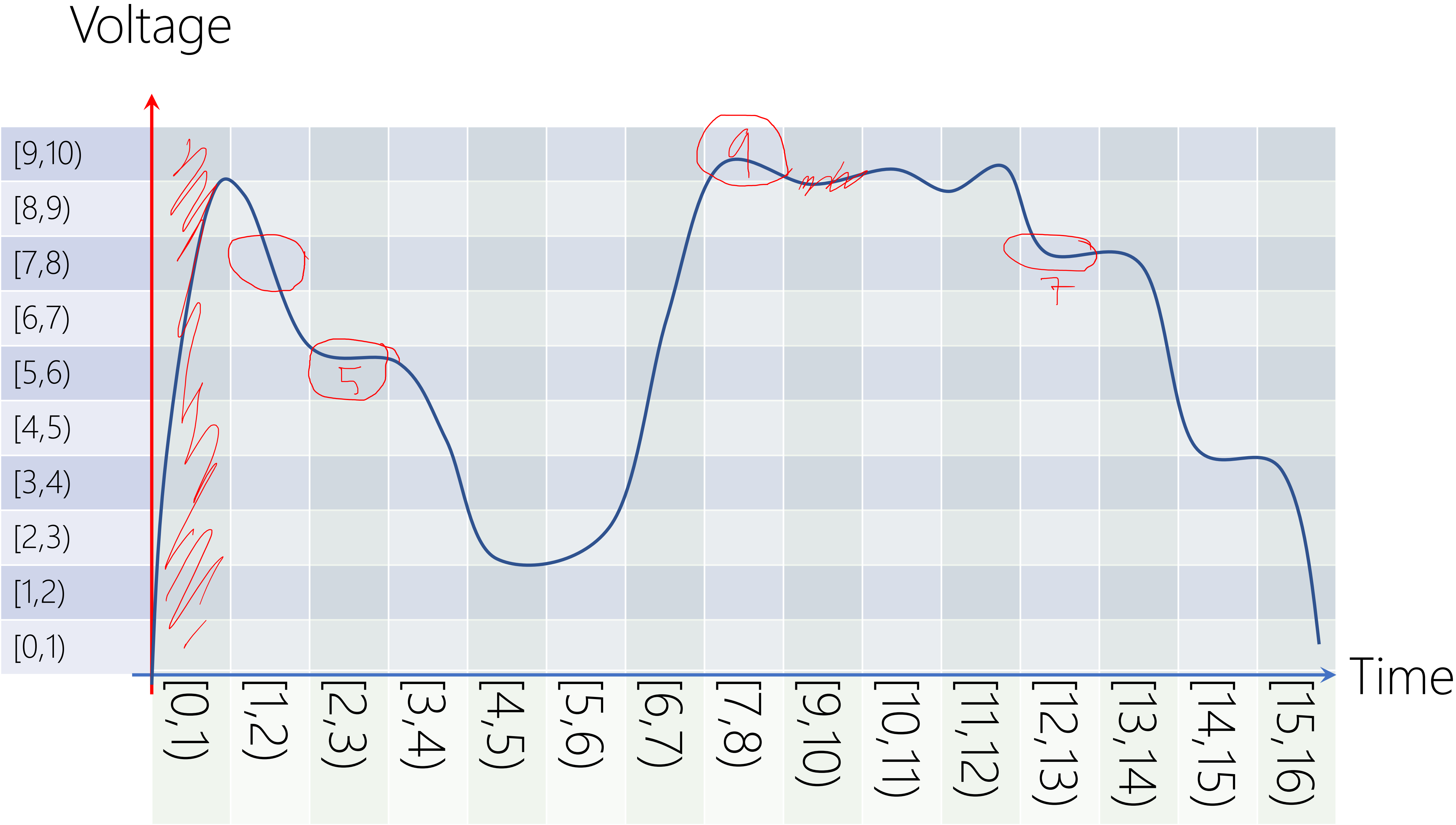
ELECTRICITY
NUMBER SYSTEM

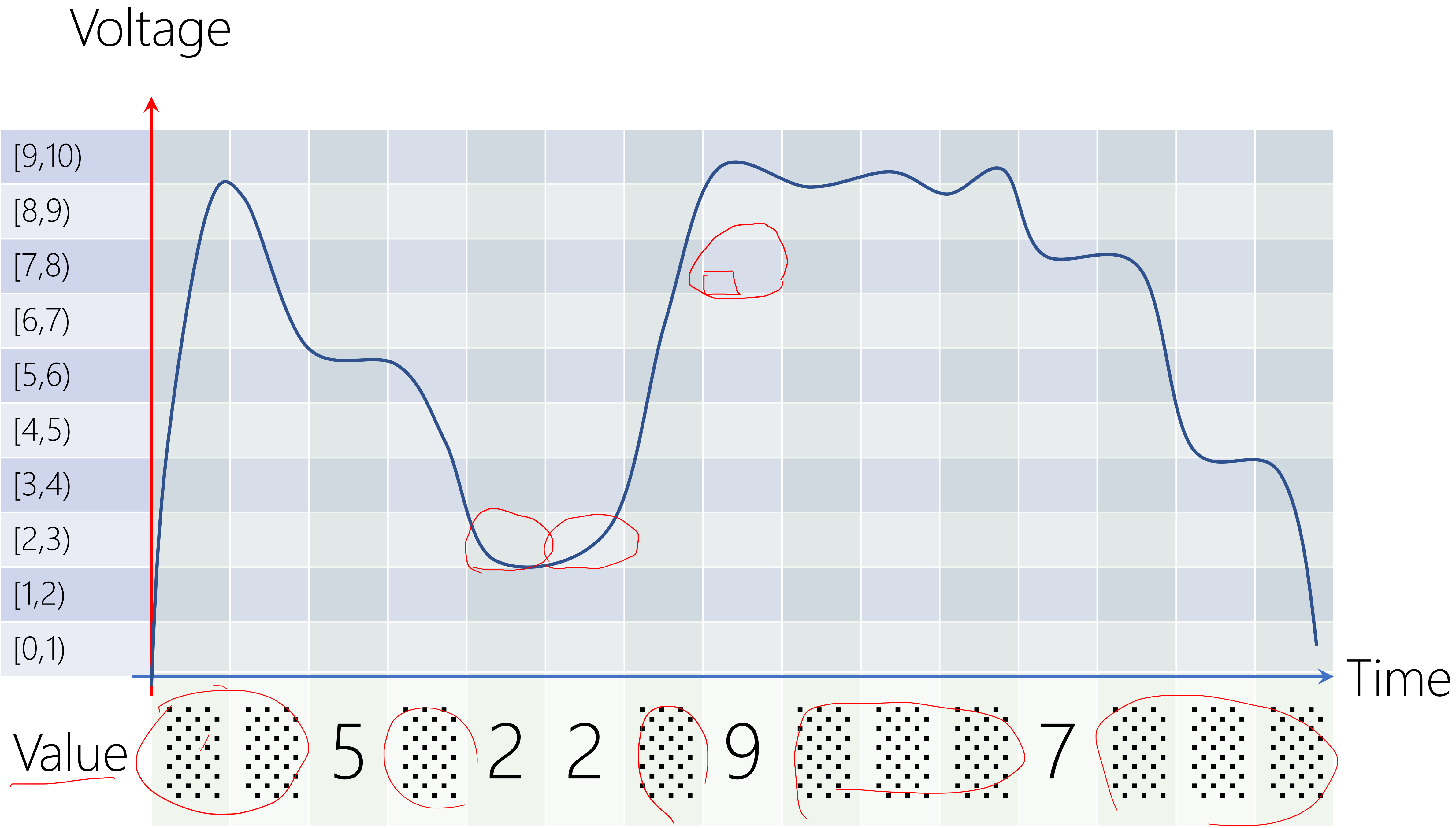
BASE-?

BASE-10

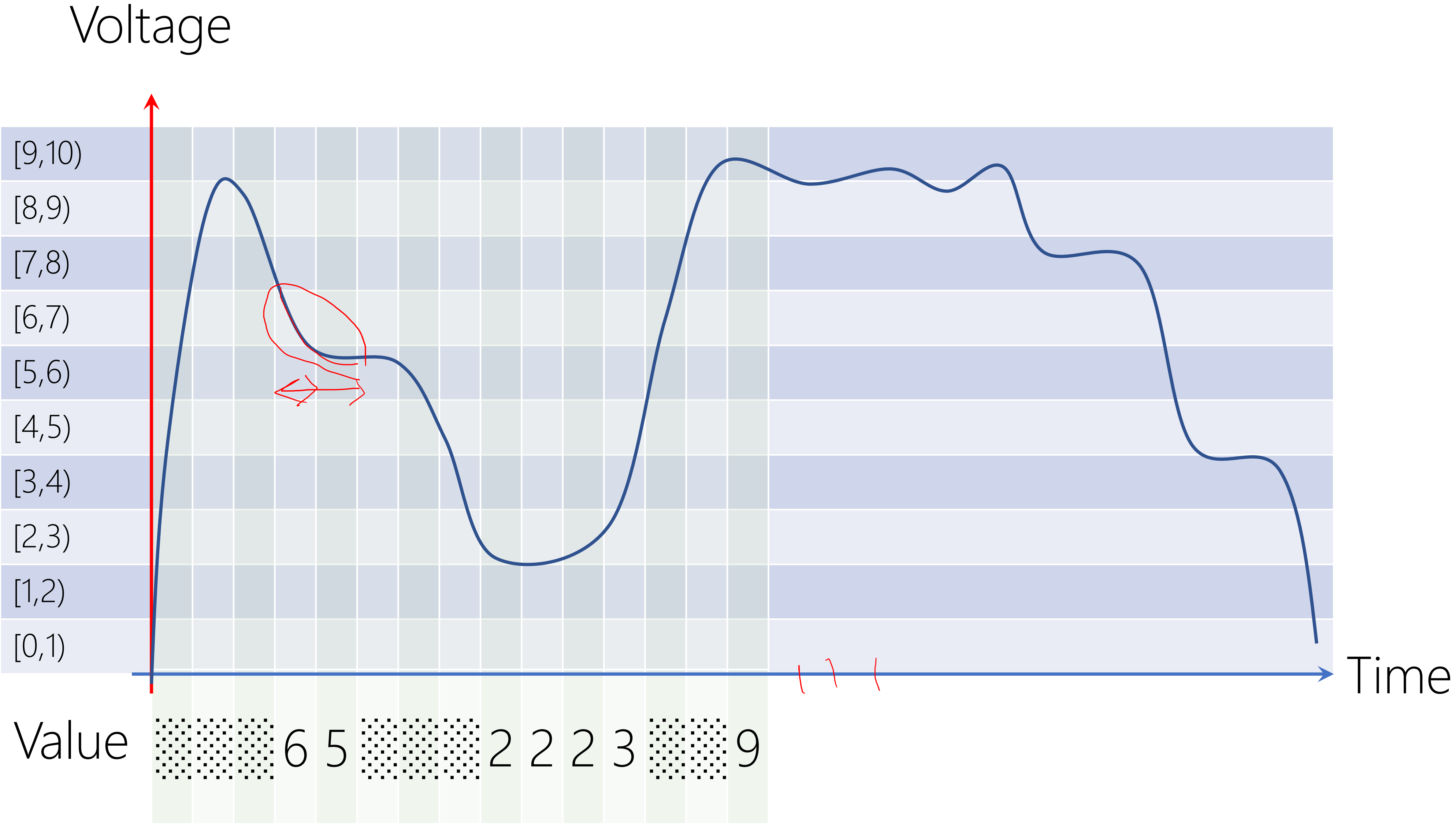

Voltage
+∞ → 10
[9,10)
[8,9)
7
[7,8)
[6,7)
6
[5,6)
5
[4,5)
[3,4)
× × ×
[2,3)
[1,2)
[0,1)
0
Time


Voltage
Time
[0,1)
[1,2)
[2,3)
[3,4)
[4,5)
[5,6)
[6,7)
[7,8)
[9,10)
[10,11)
[11,12)
[12,13)
[13,14)
[14,15)
[15,16)

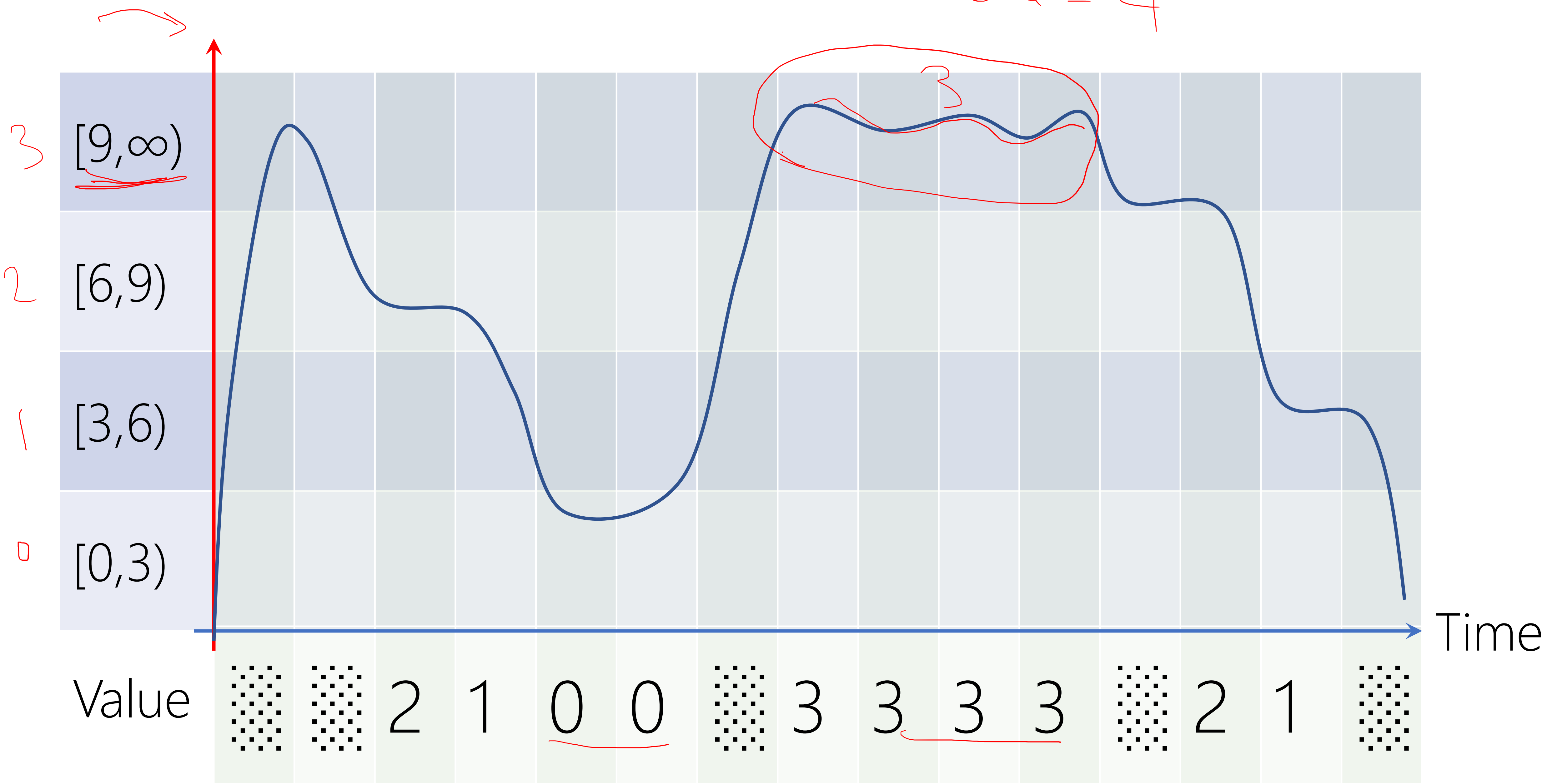

Voltage
[9,10)
[8,9)
[7,8)
[6,7)
[5,6)
[4,5)
[3,4)
[2,3)
[1,2)
[0,1)
Time
[0,1)
[1,2)
[2,3)
[3,4)
[4,5)
[5,6)
[6,7)
[7,8)
[9,10)
[10,11)
[11,12)
[12,13)
[13,14)
[14,15)
[15,16)
9
5
7

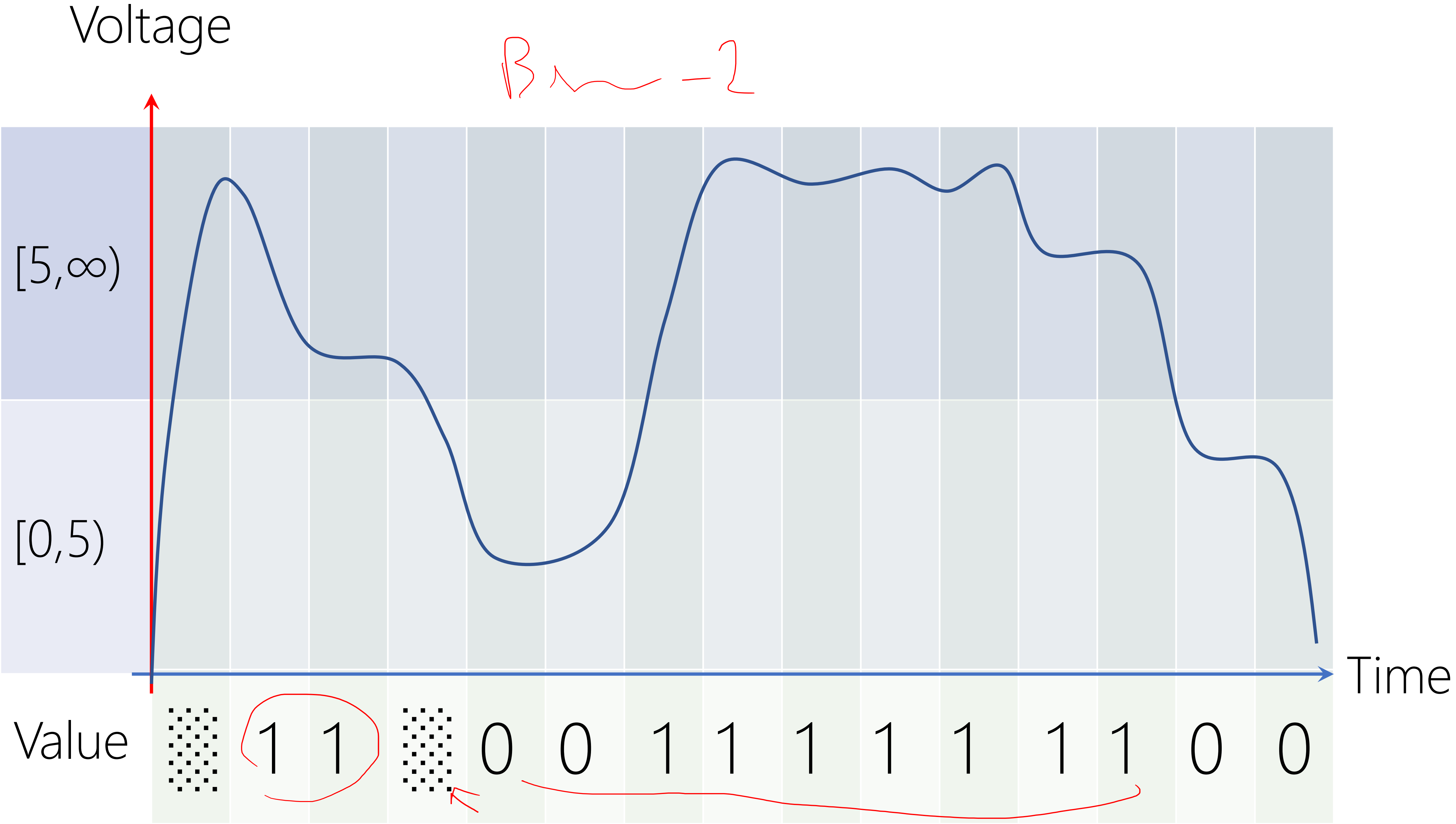

Voltage
[9,10)
[8,9)
[7,8)
[6,7)
[5,6)
[4,5)
[3,4)
[2,3)
[1,2)
[0,1)
Time
Value
5
2
2
9
7

GRANULARITY


Voltage
[9,10)
[8,9)
[7,8)
[6,7)
[5,6)
[4,5)
[3,4)
[2,3)
[1,2)
[0,1)
Time
Value
6 5
2 2 2 3
9

Base-10 → Base_4




Voltage
Bn -1
[5,∞)
[0,5)
Time
Value
▨ 1 1 ▨ 0 0 1 1 1 1 1 1 1 0 0

RELIABILITY

Robust to Noise

Fundamentally Hardware/Engineering Problem

TERNARY COMPUTER

https://en.wikipedia.org/wiki/Ternary_computer

Balanced Trinary {−1,0,1}

Entirely from *Wood*!

Thomas Fowler 1840

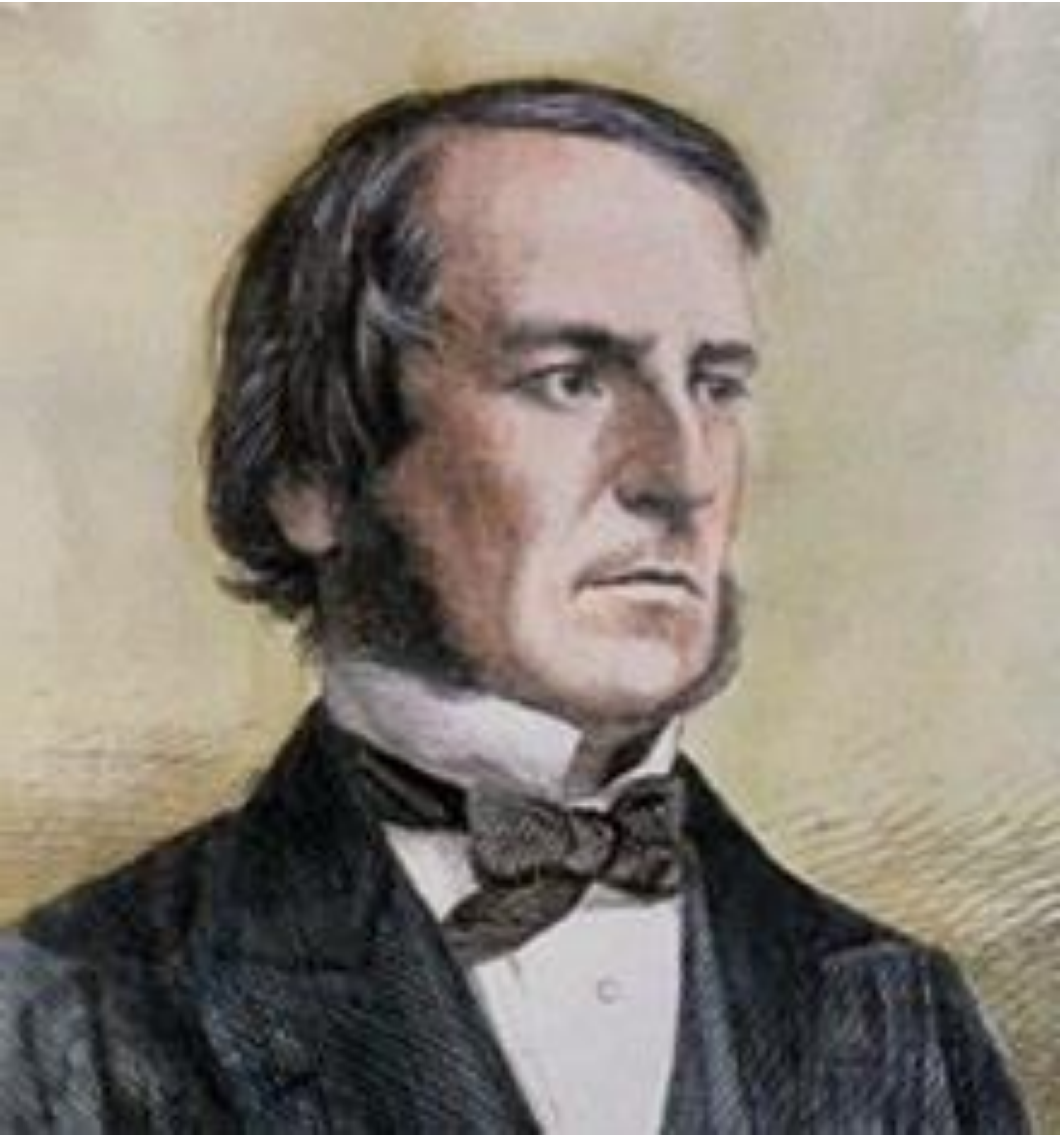
More History and Etymology → https://en.wikipedia.org/wiki/Computer

DECIMAL COMPUTER

https://en.wikipedia.org/wiki/Decimal_computer

They are not actually base-10! We'll cover them later.

TRUE VS. FALSE



George Boole (/bu:l/)

Mathematician

Philosopher

Logician

The Laws of Thought (1854)

Boolean Algebra!

Claude Elwood Shannon

Mathematician

Electrical Engineer

Cryptographer

M.Sc. Thesis (1937)

A Symbolic Analysis of Relay and Switching Circuits

21 years old!

Switching Algebra!

BINARY COMPUTER

Base-2

More Reliability in Engineering

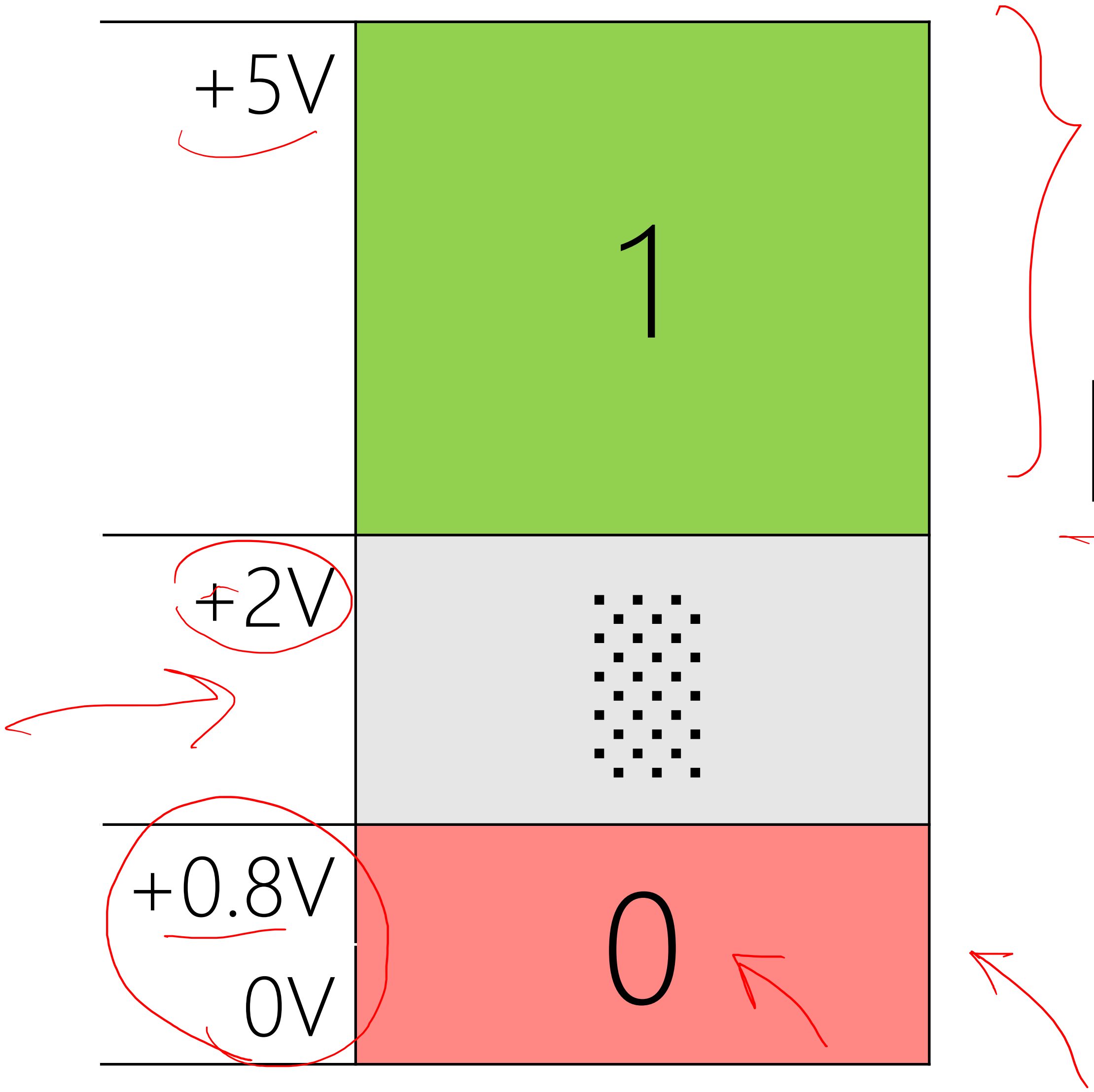
Deep Logic Foundation

2001: A Space Odyssey (1968), Stanley Kubrick, Arthur C. Clarke

Ape Learning Scene: https://www.youtube.com/watch?v=VABNA_an2A0

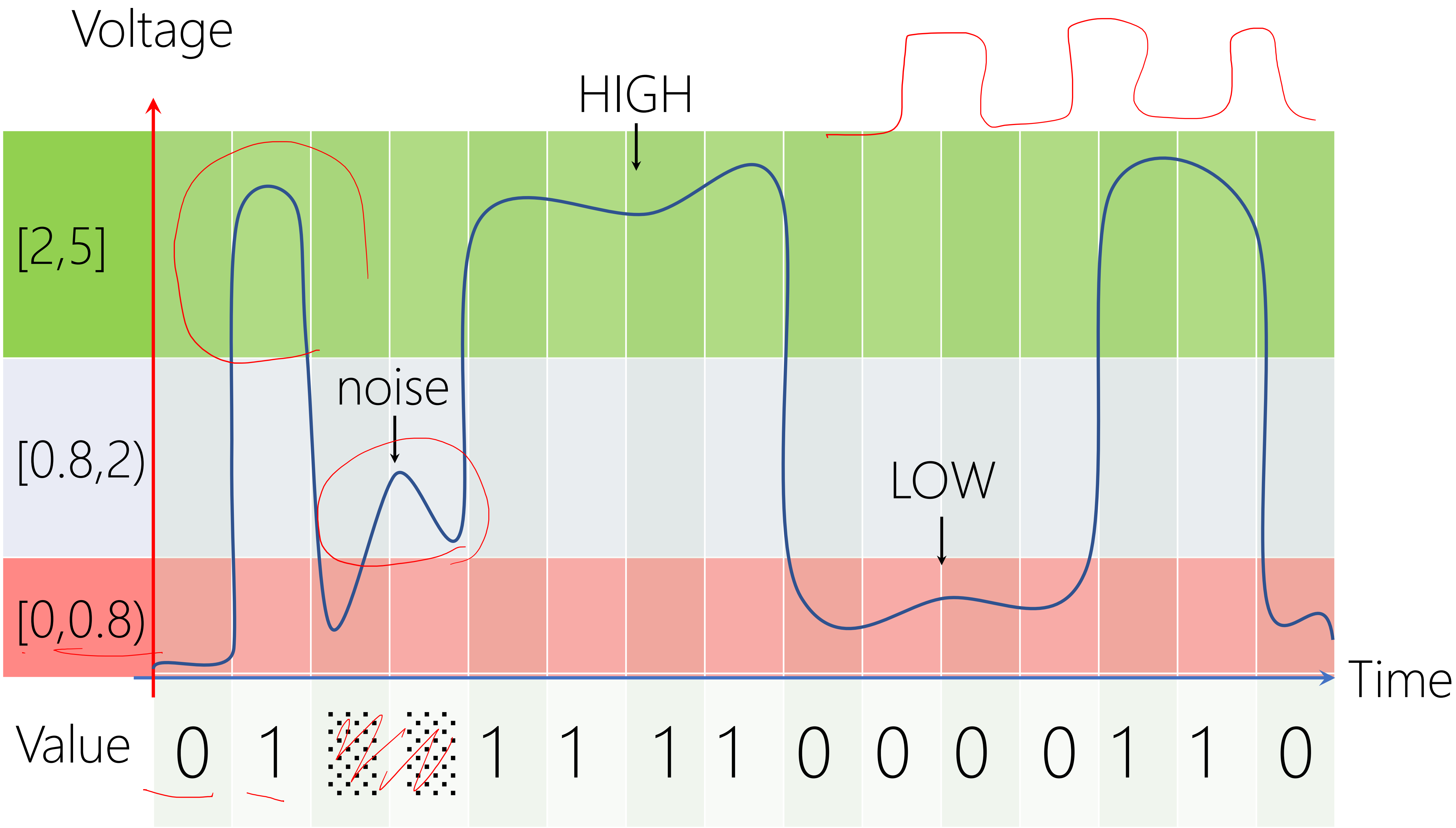
Bone/Satellite Match Cut: https://www.youtube.com/watch?v=avjdKTqiVvQ

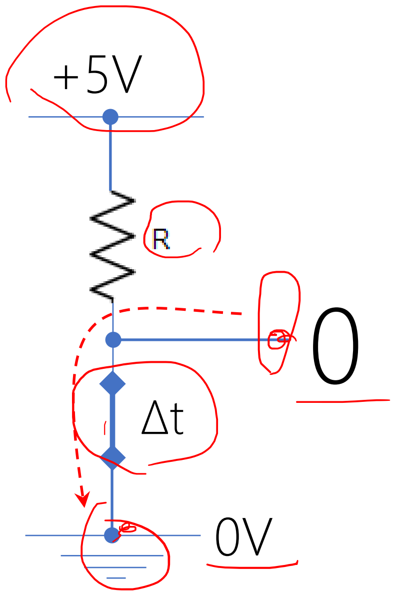
POSITIVE LOGIC



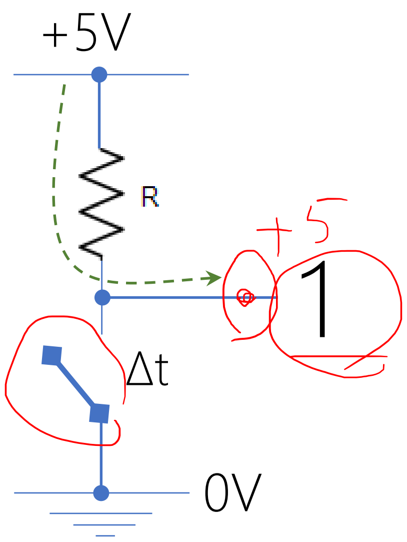
high

https://www.electronics-tutorials.ws/logic/logic_1.html

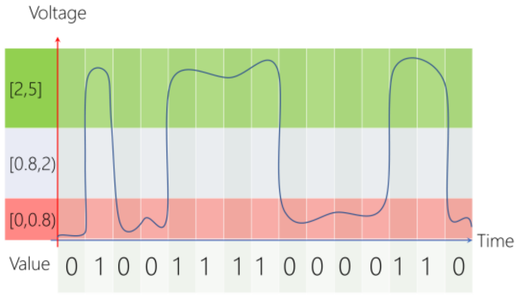

Voltage
HIGH
[2,5]
noise
[0.8,2)
LOW
[0,0.8)
Time
Value
0
1
1
1
1
1
0
0
0
0
1
1
0

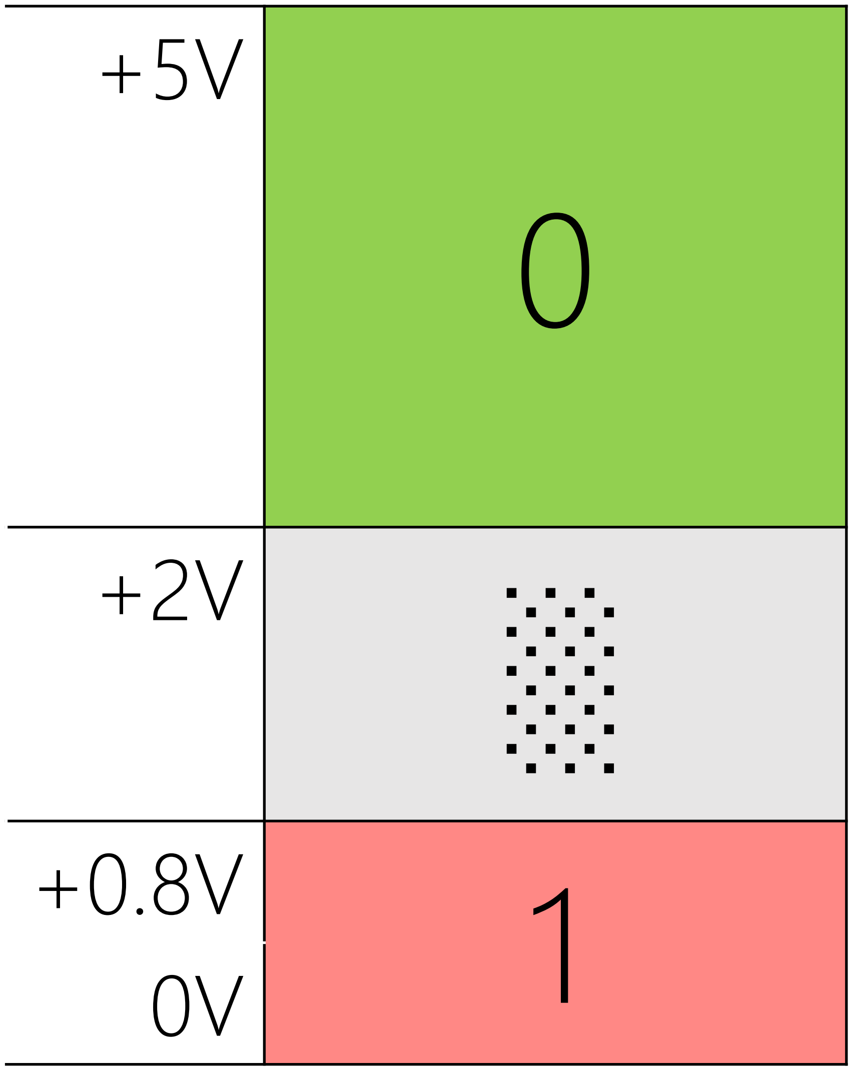


Earth (Ground)



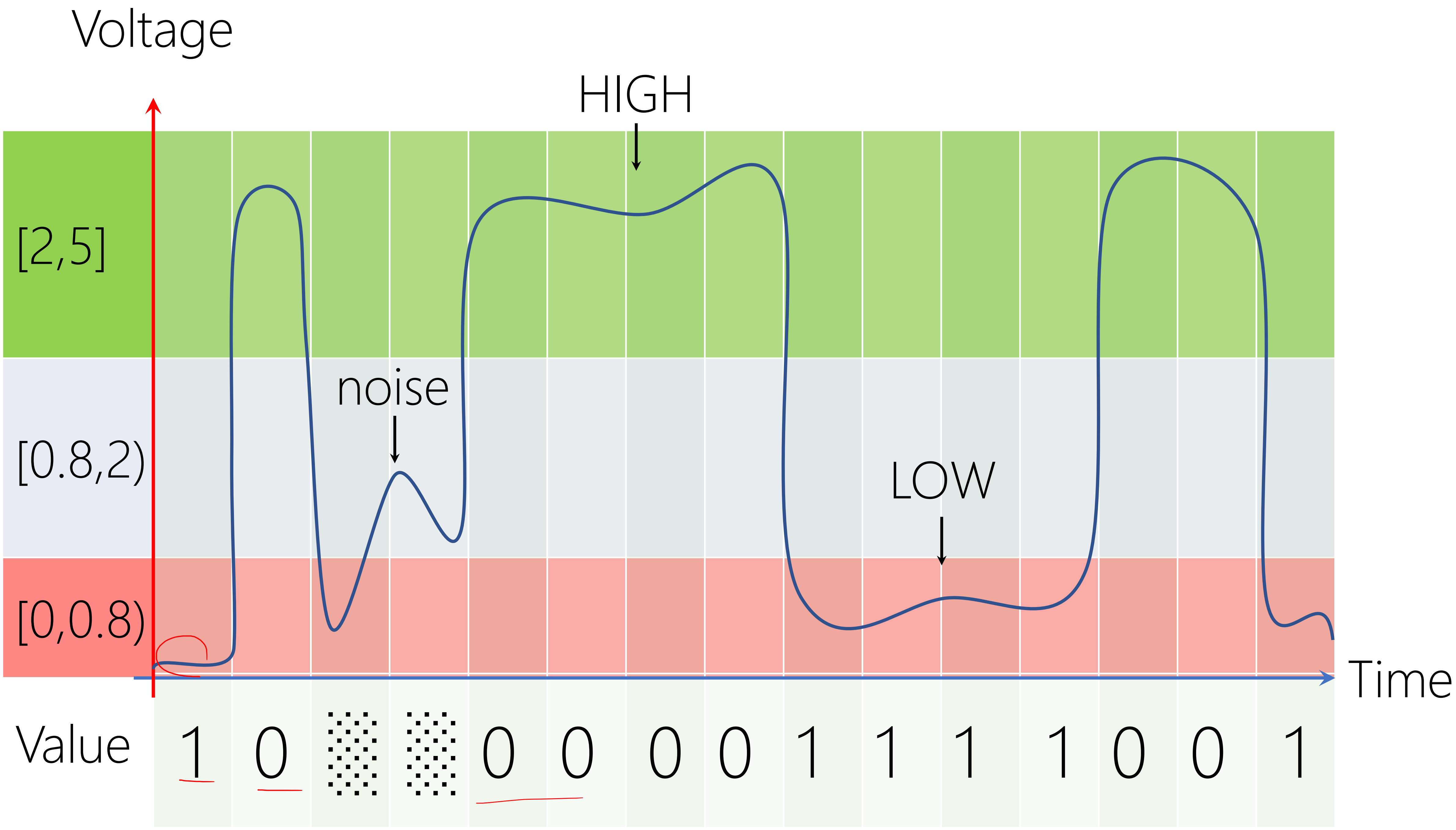
Earth (Ground)

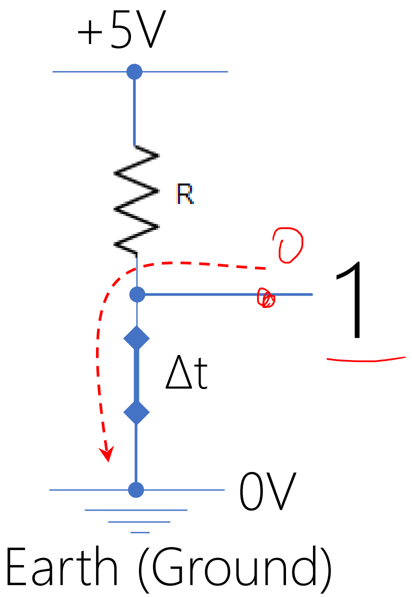




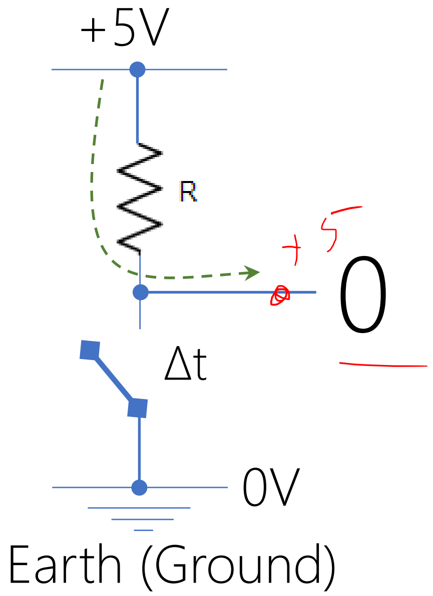
NEGATIVE LOGIC

https://www.electronics-tutorials.ws/logic/logic_1.html

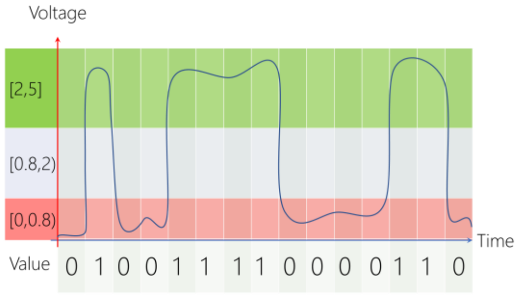

Voltage
HIGH
[2,5]
noise
[0.8,2)
LOW
[0,0.8)
Time
Value
1 0 ▨ ▨ 0 0 0 0 1 1 1 1 0 0 1



+5V
R
0
1
Δt
0V
Earth (Ground)



+5V
R
+5
0
Δt
0V
Earth (Ground)



Voltage
[2,5]
[0.8,2)
[0,0.8)
Time
Value
0 1 0 0 1 1 1 1 0 0 0 0 1 1 0

DESIGN COMPUTER

Positive Logic

Button-Up Approach

DESIGN COMPUTER

Positive Logic
Button-Up Approach

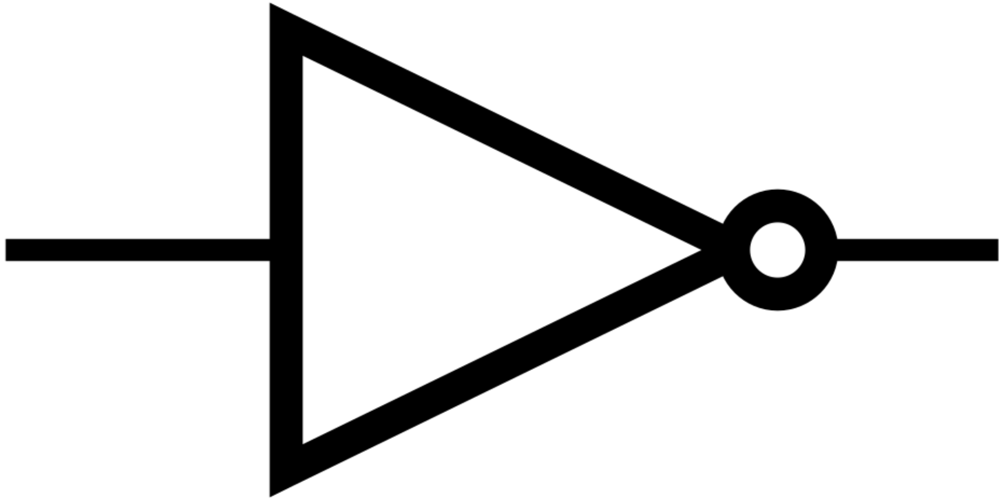
Finding simpler, but equivalent, computers reduces the overall cost!
Rely primarily on mathematical methods in Boolean algebra!

BUILD COMPUTER

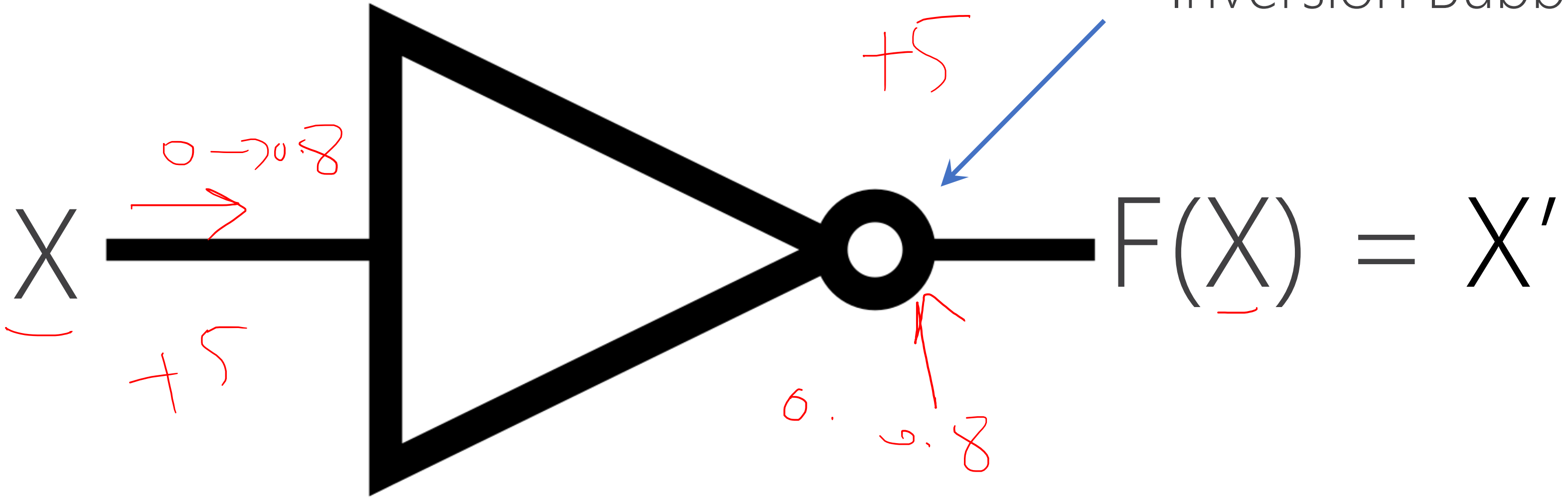
Electrical and Computer Engineering

LOGIC GATES

BOOLEAN GATES



Truth Table

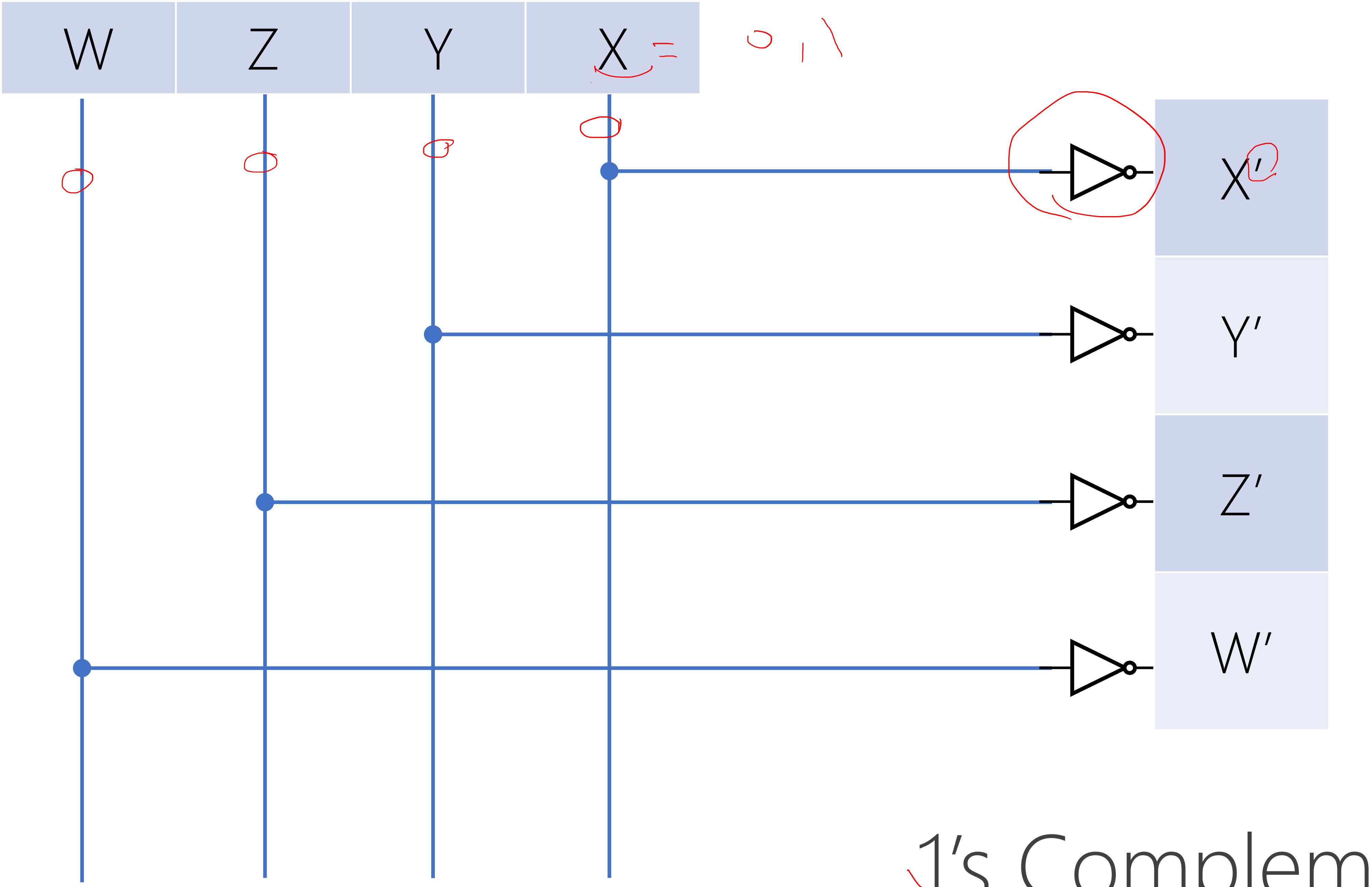


Inversion Bubble

X	NOT X	Invertor X	X'	$\overline{X}$
0	1			
1	0			

Boolean Expression/Function: F(X) = X′

> *inverse of X gives F* <



$F_1 = X'$

$F_2 = Y'$

$F_3 = Z'$

$F_4 = W'$

1′s Complement

BINARY VARIABLE

aka. Boolean variable

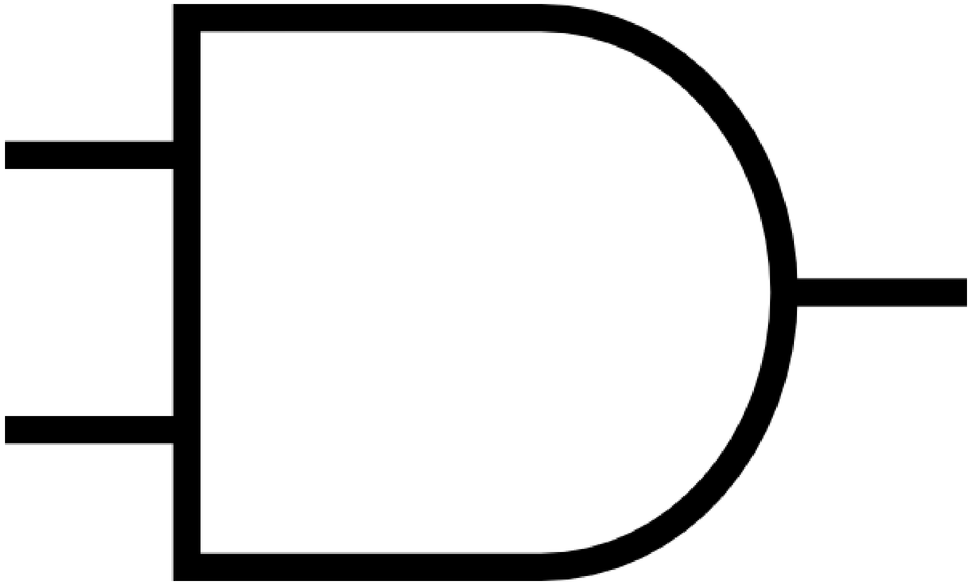
A variable that can have a value in {0,1}

X, Y, Z, W, …

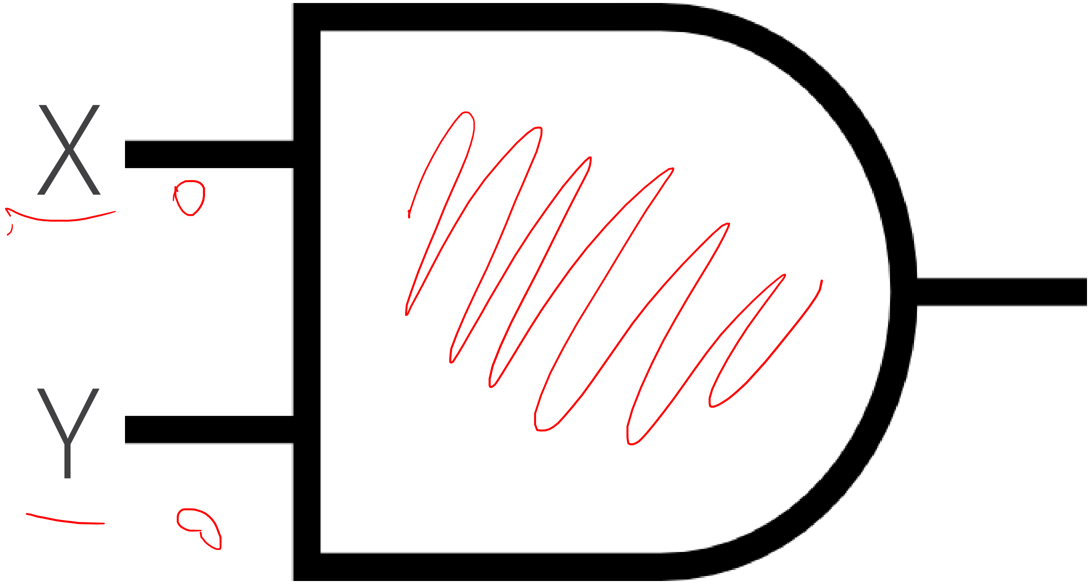
BOOLEAN FUNCTION

aka. Boolean Expression

A function that accept Boolean variable(s) and output a value in $\{0,1\}$, e.g., $F(X) = X'$

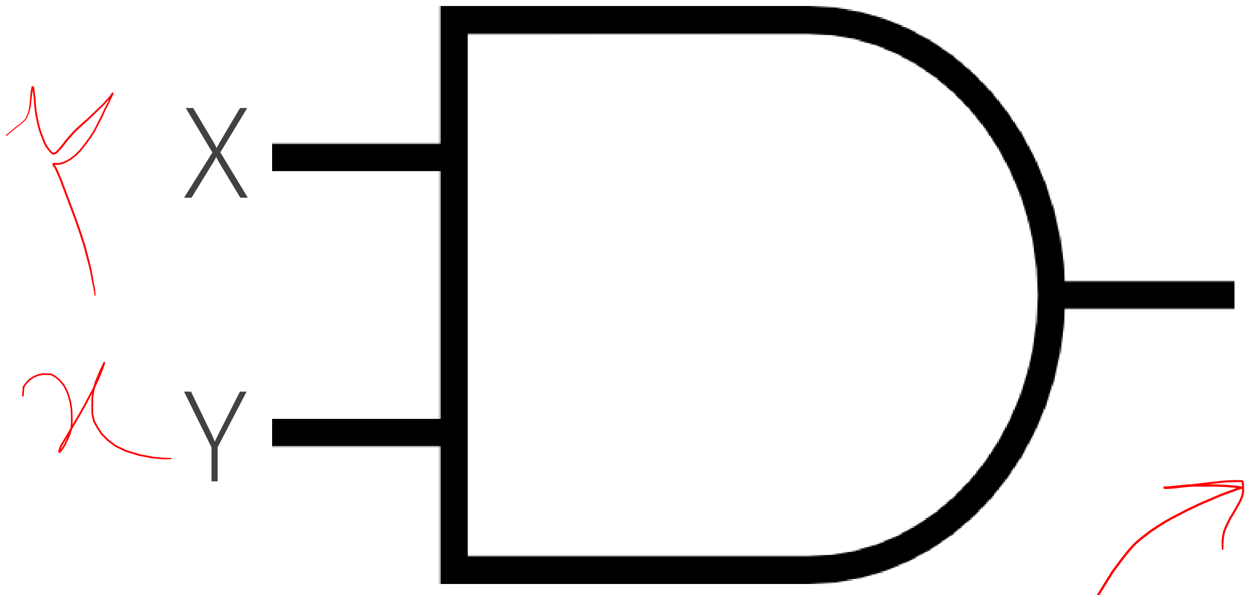


Truth Table



$F(Y, X) = YX$

Y	X	X AND Y	Y · X	Y * X
0	0		0	
0	1		0	
1	0		0	
1	1		1	



$F(Y,X) = YX = XY$

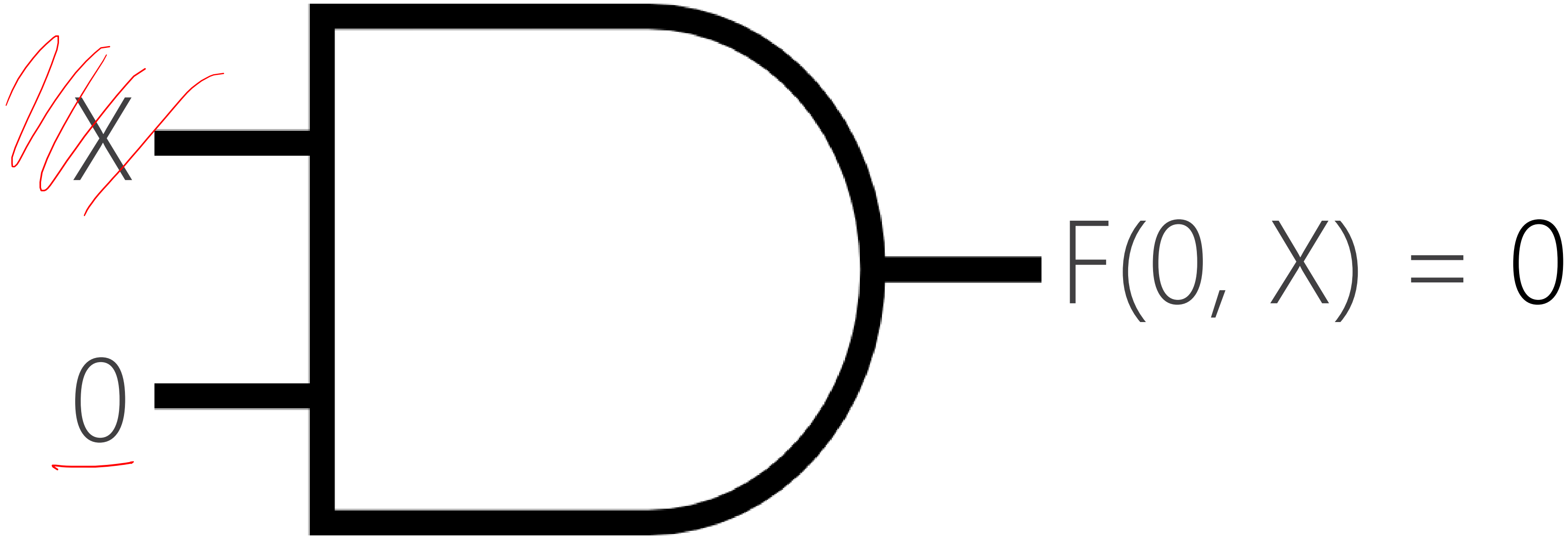
Commutative

X	Y	Y AND X	X · Y	X * Y
0	0		0	
0	1		0	
1	0		0	
1	1		1	

BOOLEAN FUNCTION

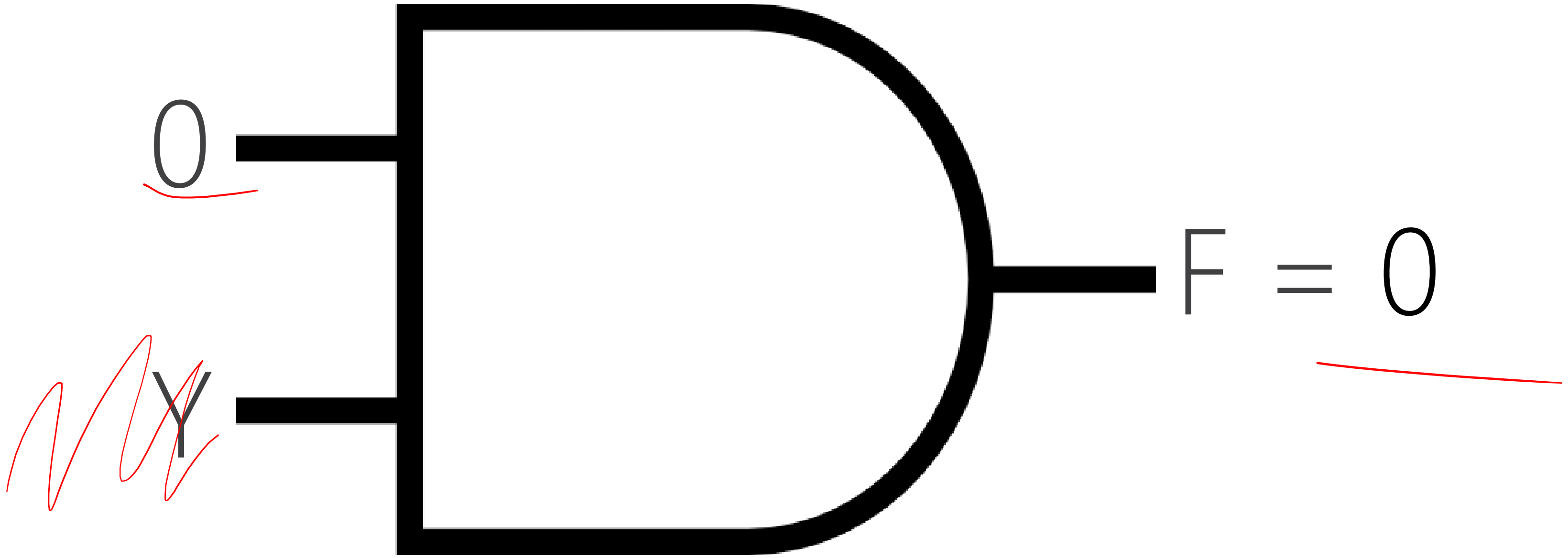
Equality F1 = F2

For the same input(s), F1 and F2 result in same output



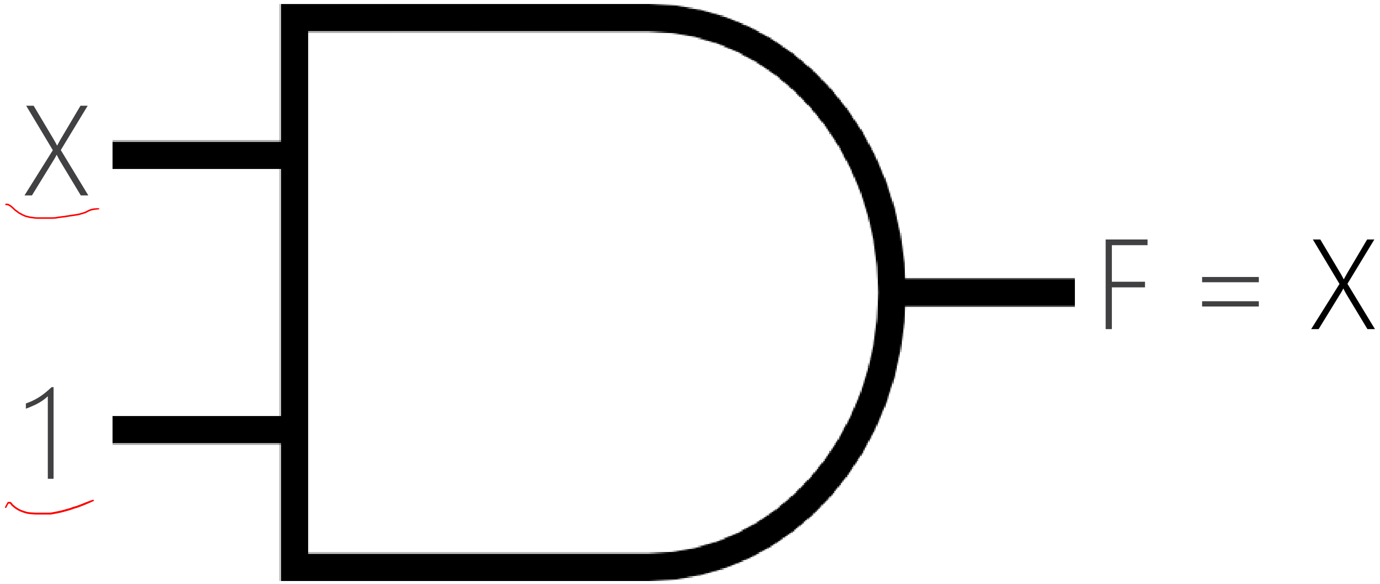
Y	X	YX
0	1	0
0	0	0

$$F(X,0) = X0 = 0$$



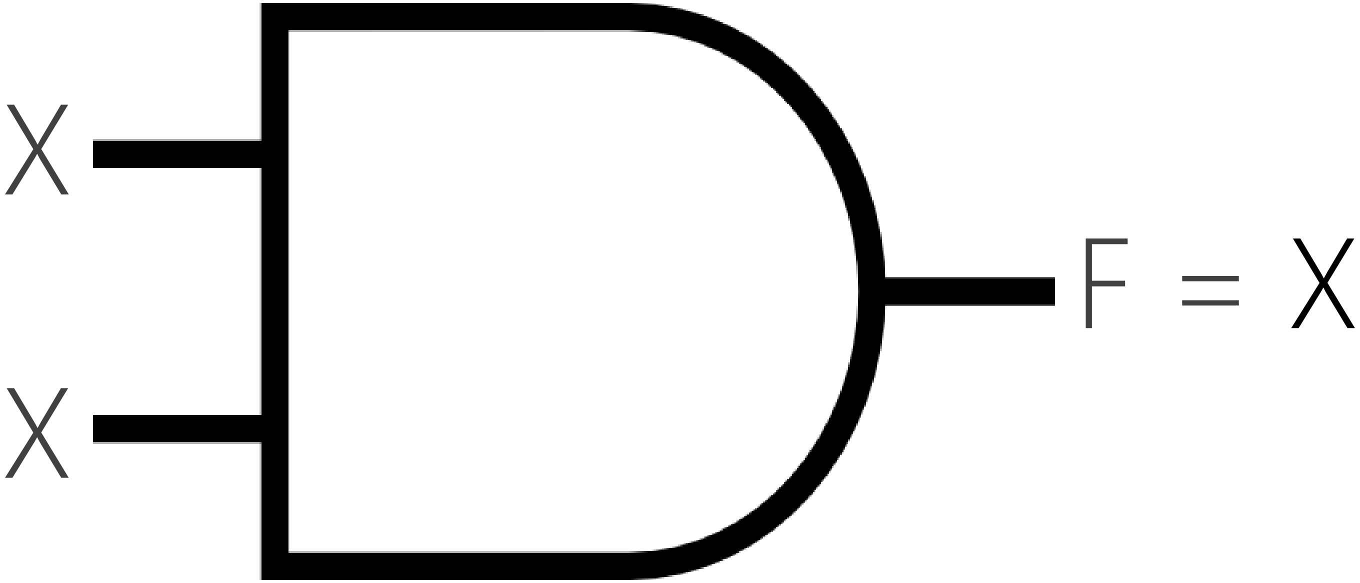
Y	X	YX
0	0	0
1	0	0

$$F = 0Y = 0$$



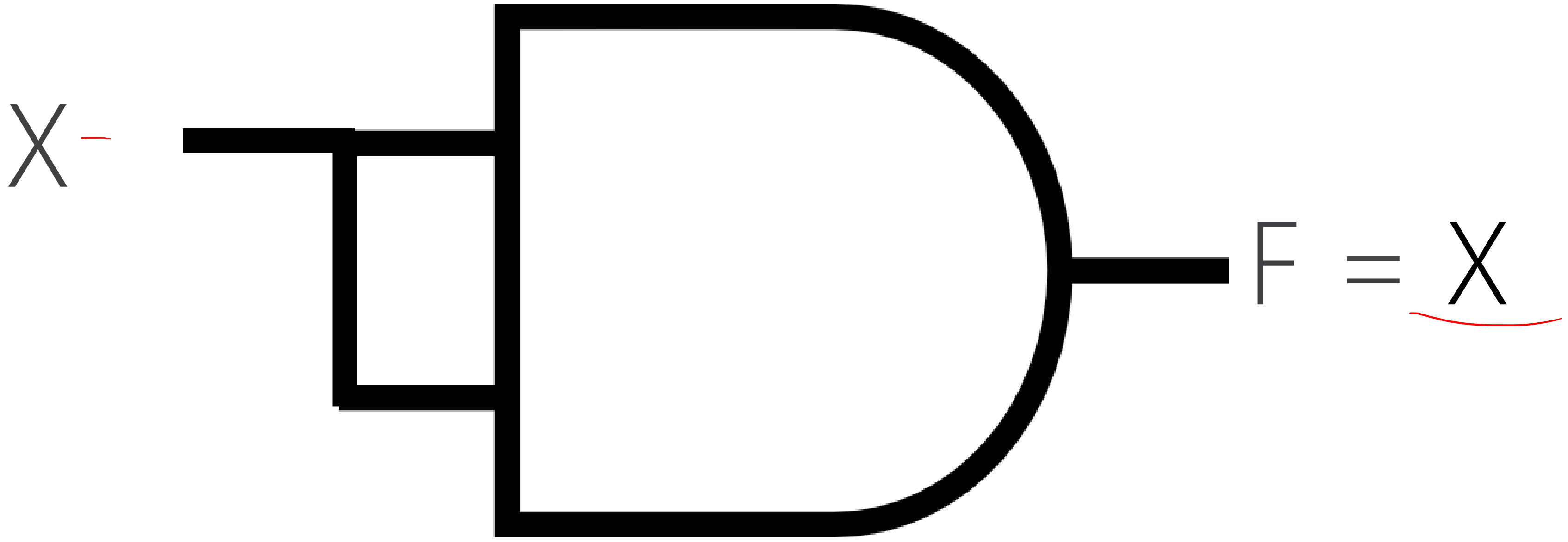
Y	X	YX
1	0	0
1	1	1

$$F = \underline{X1} = 1111X1111 = X$$



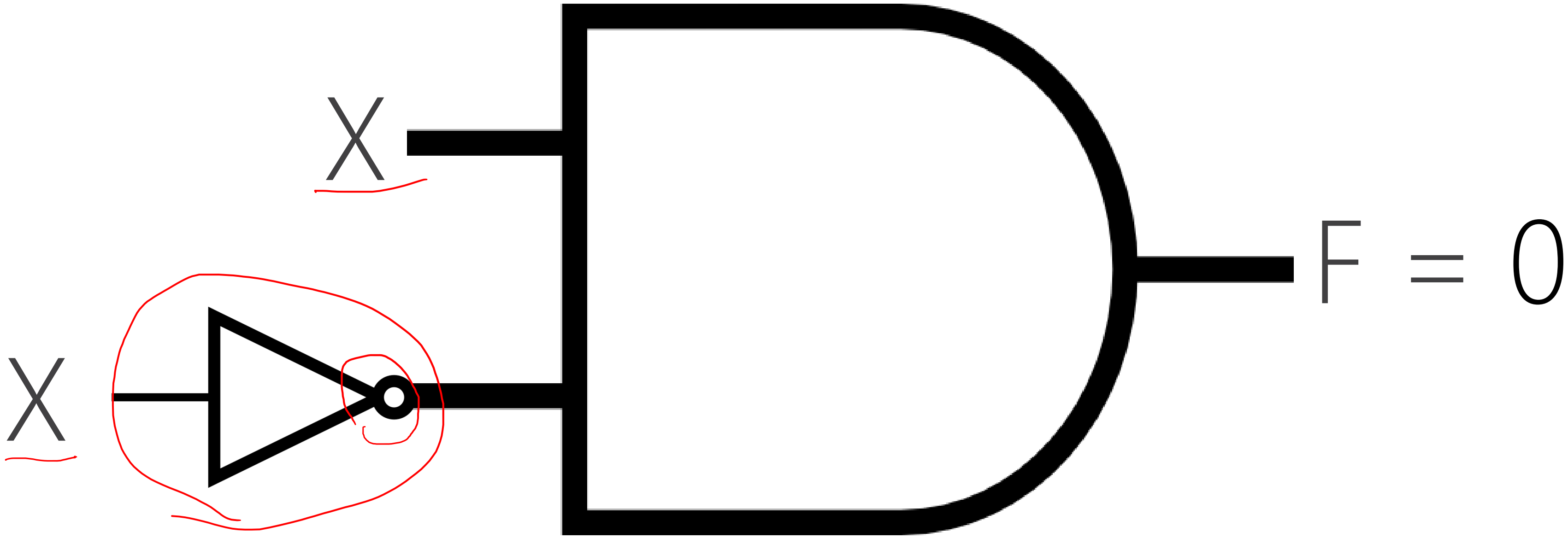
X	X	XX
0	0	0
1	1	1

$$F = \underline{XX} = X$$



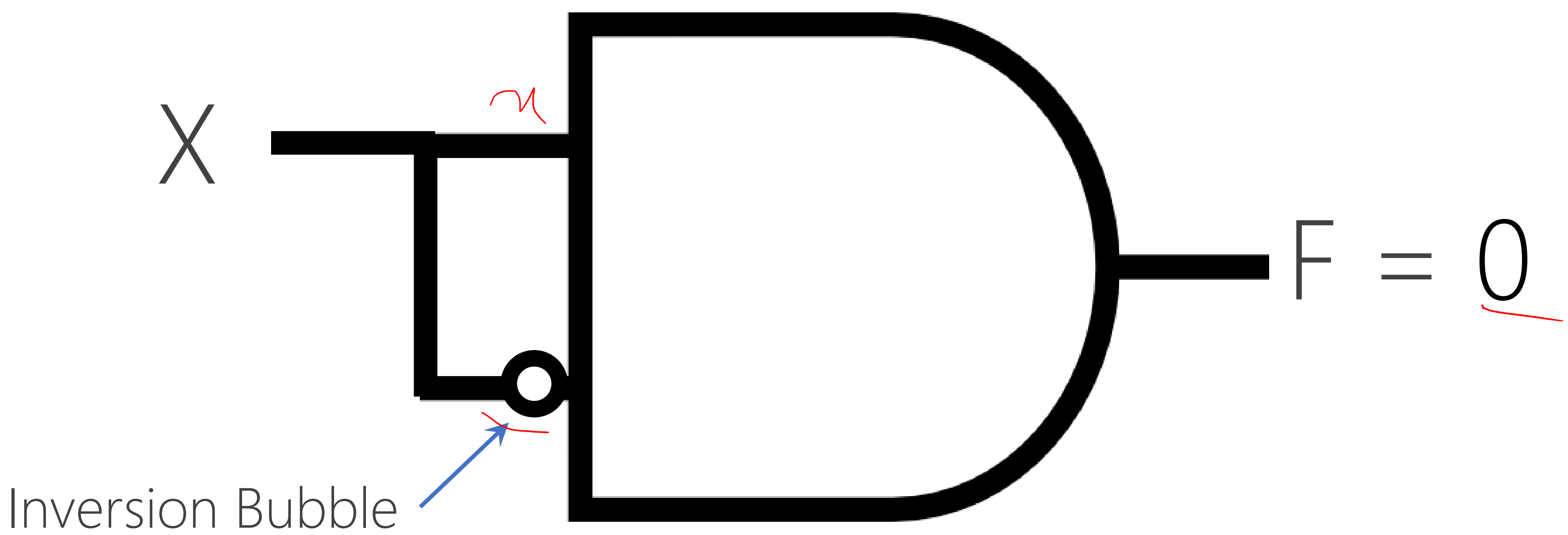
X	X	XX
0	0	0
1	1	1

$$F = XX = X$$



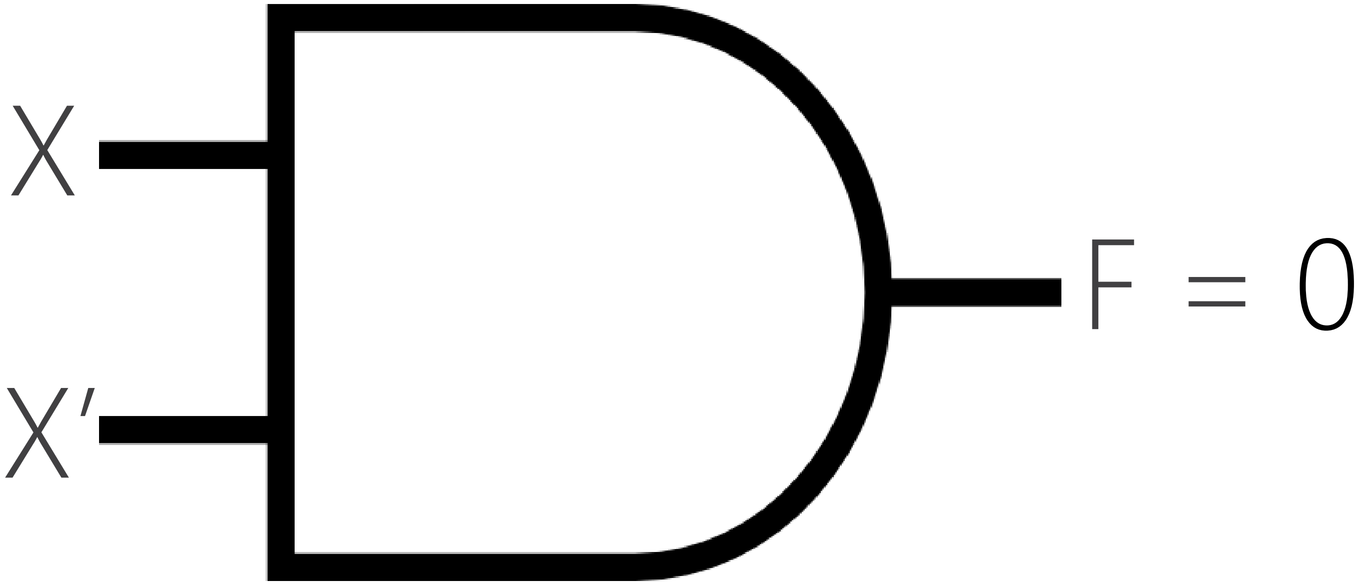
X′	X	X′X
1	0	0
0	1	0

$$F = XX' = 0$$



X′	X	X′X
1	0	0
0	1	0

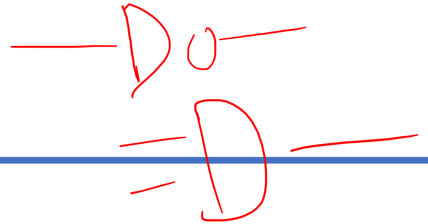
$$F = XX' = 0$$



X′	X	X′X
1	0	0
0	1	0

$$F = XX' = 0$$

DESIGN

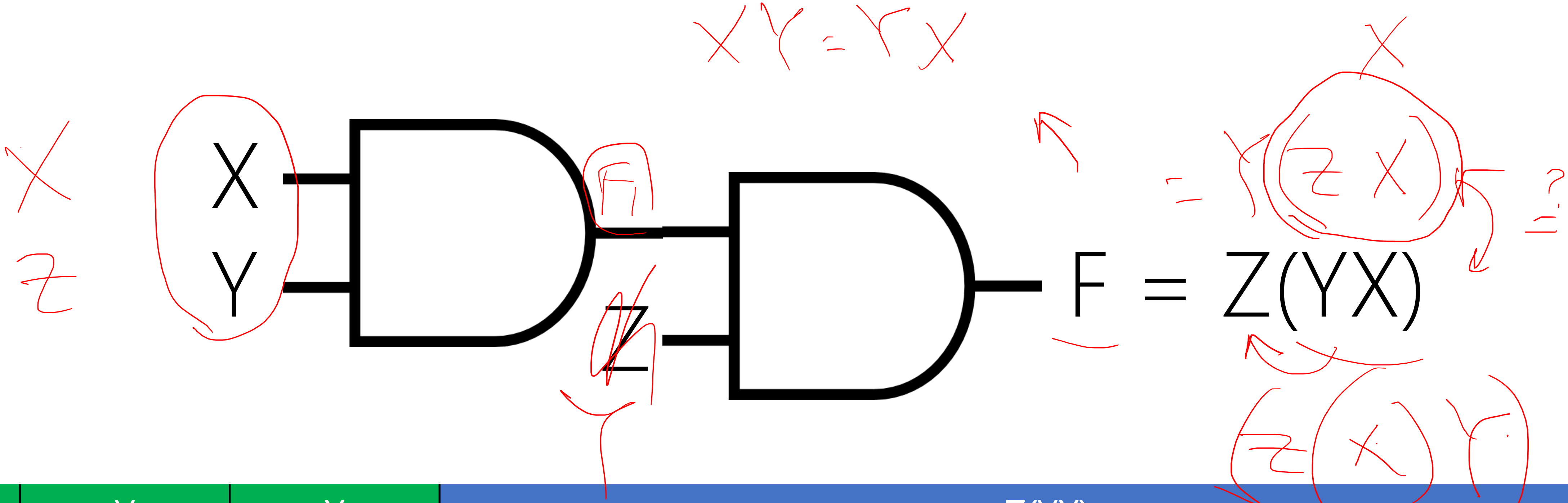


given the functionality, design the structure of a system

3-INPUT AND

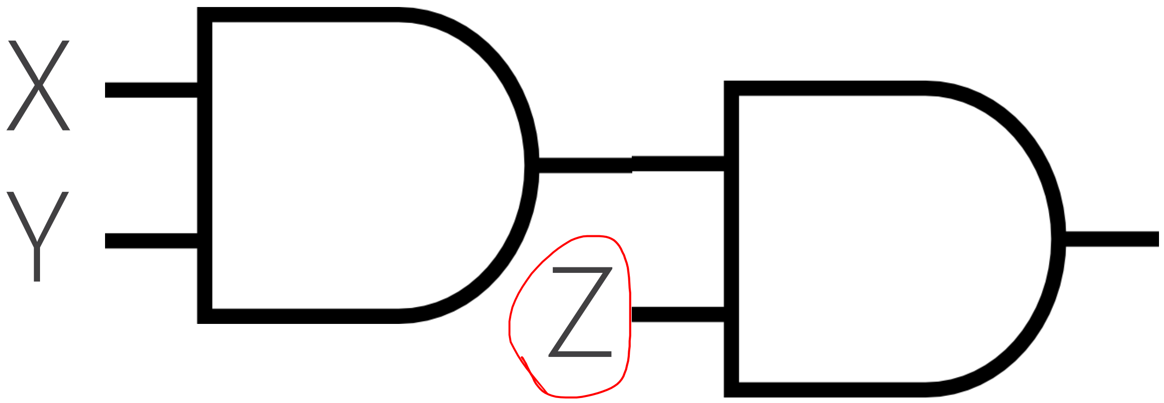
TRUTH TABLE
3-INPUT AND

Z	Y	X	ZYX
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



$F = Z(YX)$

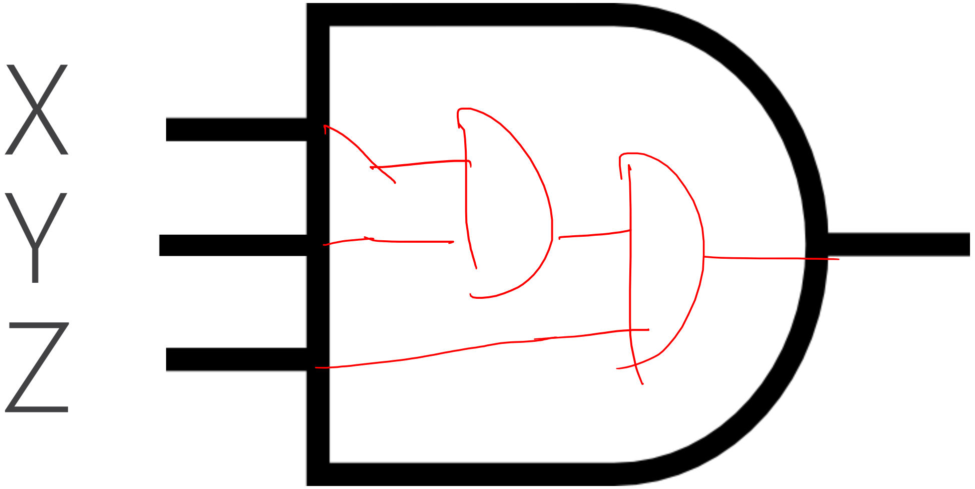
Z	Y	X	Z(YX)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1



F=Z(YX)=Z(XY)=(ZX)Y=(XZ)Y=XZY

Associative

Z	Y	X	Z(YX)	Z(XY)	(ZX)Y	XZY
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	0	0
1	1	1	1	1	1	1



$F = ZYX$

Z	Y	X	ZYX
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

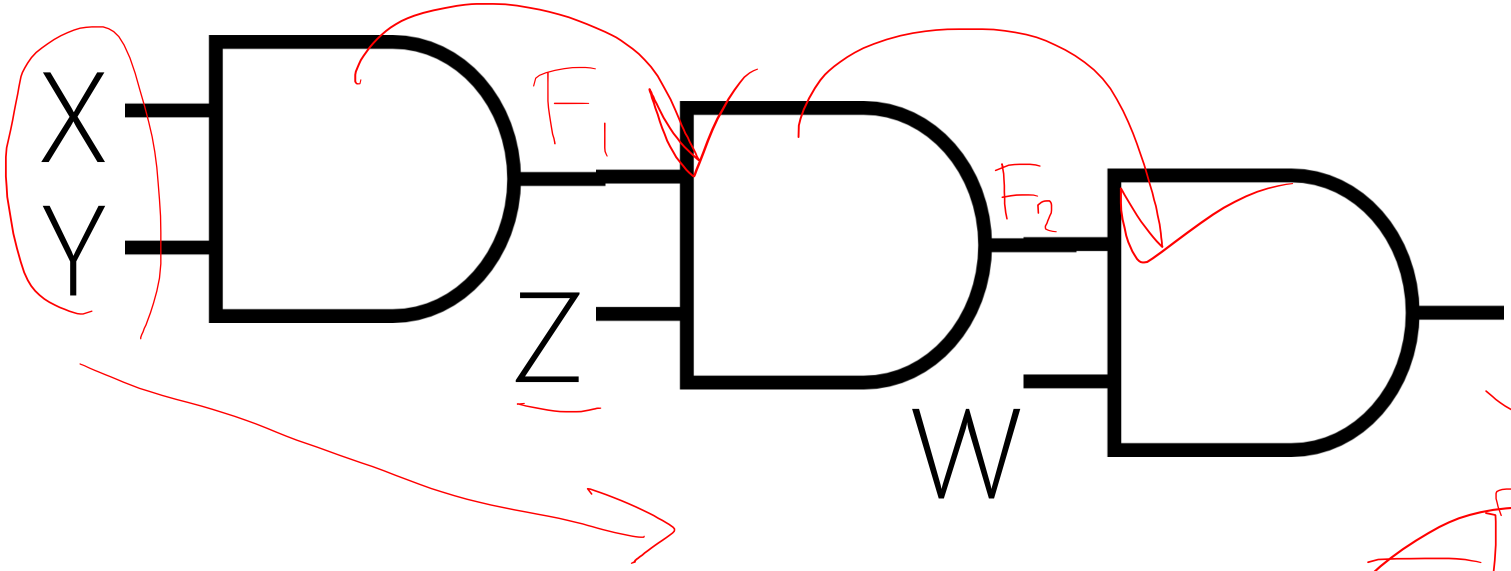
DESIGN

given the functionality, design the structure of a system

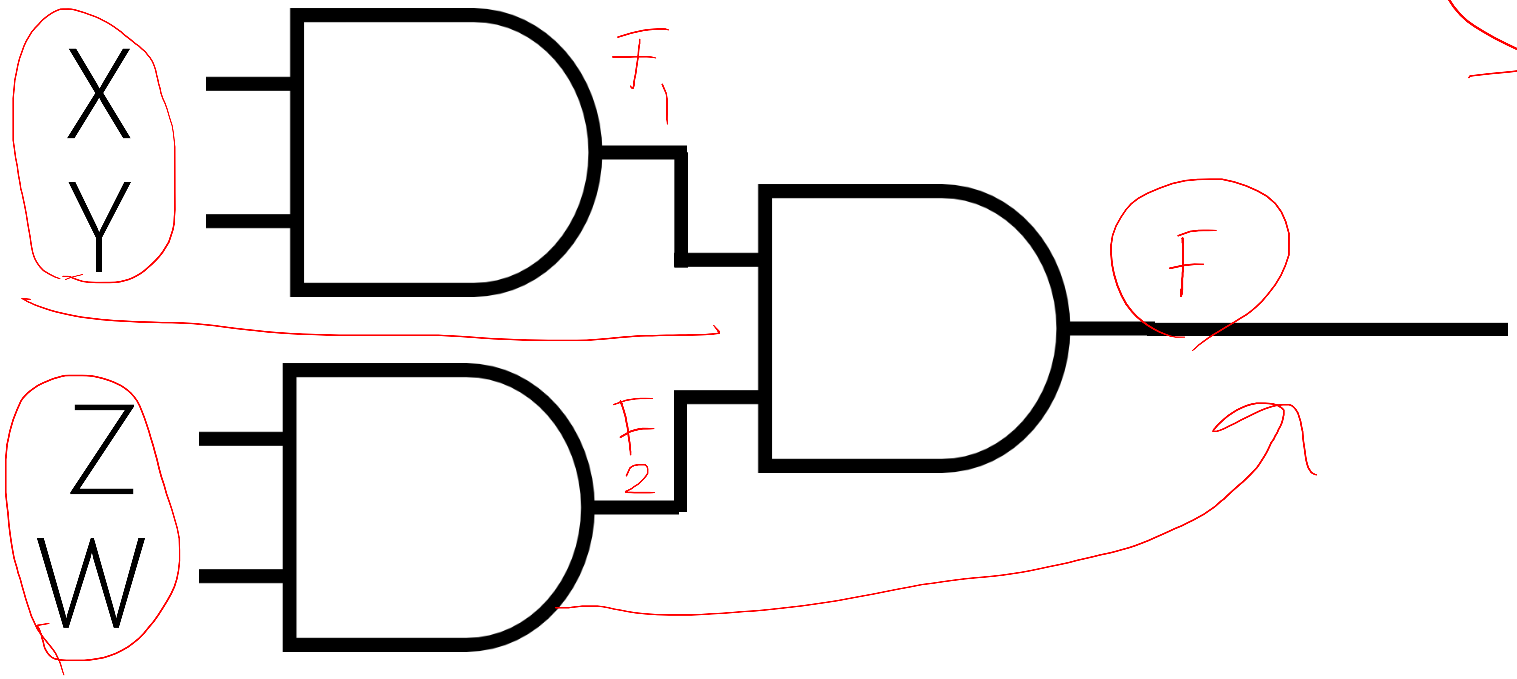
4-INPUT AND

DESIGN PATTERNS

Using Same or Similar Previous Designs for New Designs



$F_1 = W(Z(YX))$



$F_2 = (WZ)(YX)$

ANALYSIS

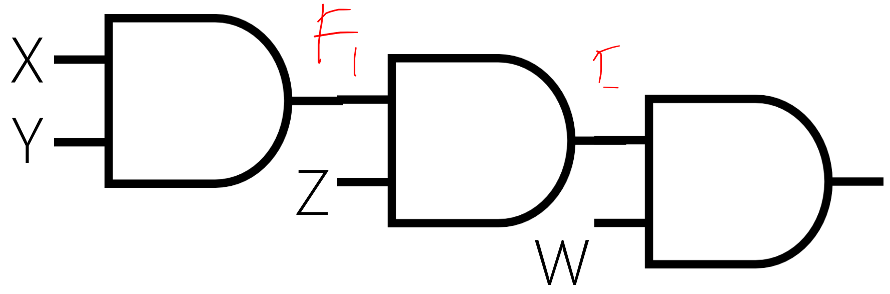
given the structure of a system, find its functionality.

determine the functionality exhibited by a structure.

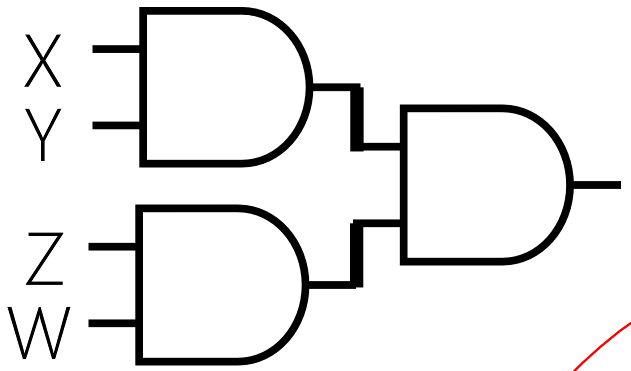
EVALUATION

Correct vs. Wrong

Nice vs. Poor

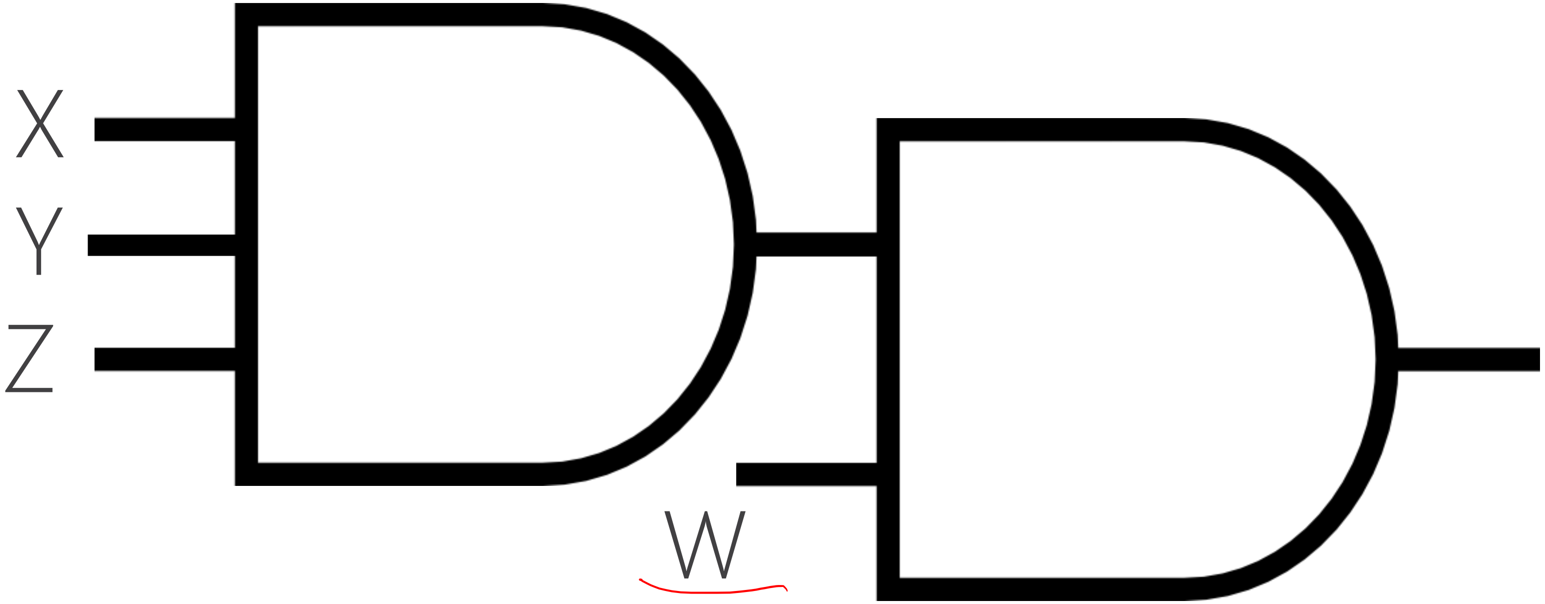


$F_1 = W(Z(YX))$

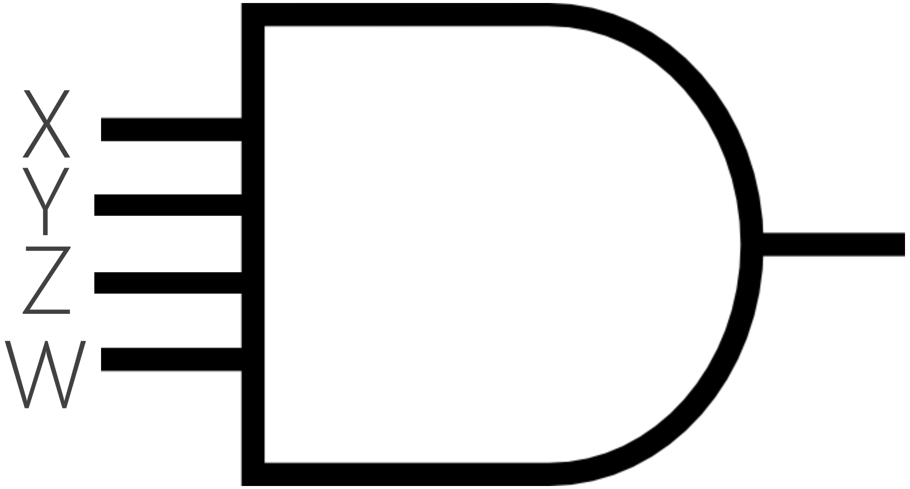


$F_2 = (WZ)(YX)$

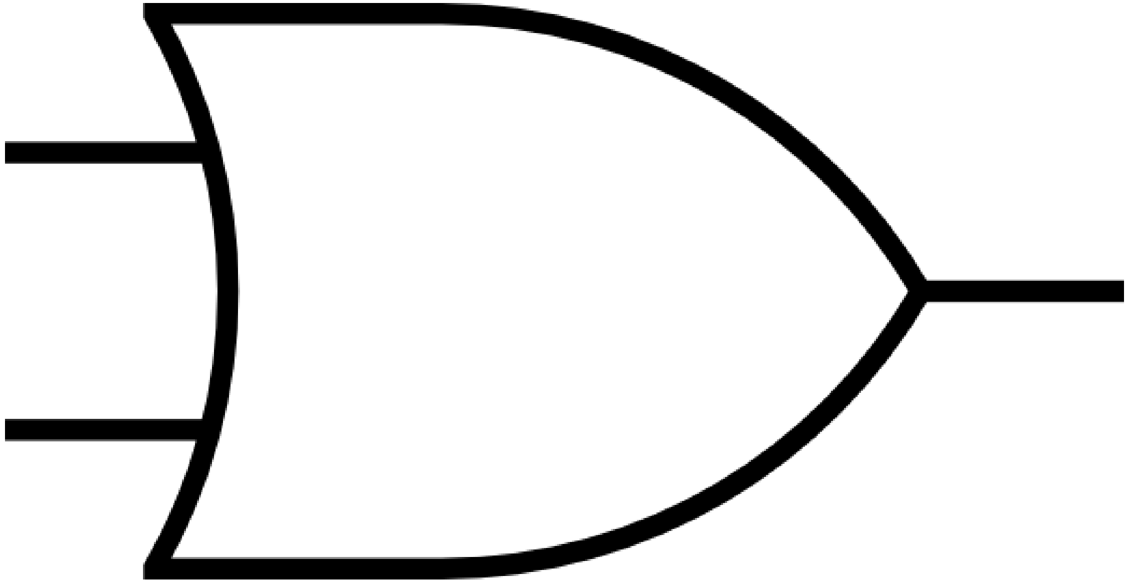
$F = WZYX$	F_1	F_2
Effective (True)	Yes	Yes
Efficient (Fast)	Hmm, 3 levels, No!	Yes! 2 levels
Min. Cost	3 gates, Yes	3 gates, Yes



F = WZYX



F = WZYX



OR