1

School of Computer Science
Faculty of Science

COMP-2650: Computer Architecture I: Digital Design
Winter 2021

Assignment#	Date	Title	Due Date	Grade Release Date
Lec 06	Week 06	**W06: Boolean Algebra & Minimization**	Feb. 23, 2021 Tuesday Midnight AoE Wednesday 7 AM EDT	March. 01, 2021

The objectives of the lecture (weekly) assignments are to practice on topics covered in the lectures as well as improving the student's critical thinking and problem-solving skills in ad hoc topics that are closely related but not covered in the lectures. Lecture assignments also help students with research skills, including the ability to access, retrieve, and evaluate information (information literacy).

Lecture Assignments Deliverables

You should answer 4 of the below questions based on your preference using an editor like MS Word, Notepad, and the likes or pen in papers. You have to write and scan the papers clearly and merge them into a single file in the latter case. In the end, you have to submit all your answers in one single pdf file `COMP2650_Lec06_{UWinID}.pdf` containing the following items:

1. Your name, UWinID, and student number
2. The question Id for each answer. Preferably, the questions should be answered in order of increasing Ids. *Please note that if your answers cannot be read, you will lose marks.*
3. Including the questions in your submission pdf file is optional.

Please follow the naming convention as you lose marks otherwise. Instead of {UWinID}, use your own UWindsor account name, e.g., mine is hfani@uwindsor.ca, so, my submission would be: `COMP2650_Lec06_hfani.pdf`

Lecture Assignments
(select only 4 questions based on your preference)

1. Minterm is also called standard product (standard ANDing). Maxterm is also called standard sum (standard ORing). Boolean functions that expressed as a sum of products (SoP) or product of sums (PoS) are said to be in **canonical form** if they are in the forms of minterms or MAXTERMs that include ALL the input binary variables in each term either in normal form or in complement form. For instance, $F_1(Y, X)=Y'X+Y'X'$ is in canonical form because F_1 is in the form of $m_0 + m_1$ and each minterm includes all input variables. However, although $F_3(Z,Y,X)=Y'X+ZY$ is sum of products, it is not in canonical form since the first term (Y'X) does not have the input variable Z or Z' or the second term (ZY) does not have X or X'. In design process, we start with truth table and write the Boolean function in their canonical forms either in sum of minterms or product of MAXTERMs. <u>Then, through the minimization process, the Boolean function may lose its canonical form in order to have less number of terms or variables instead.</u>

   Using truth table, given any two Boolean functions F_1 and F_2, prove that

   a. The Boolean function $E = F_1 + F_2$ contains the sum of the minterms of F1 and F2 in order to be in sum of products canonical form.

b. The Boolean function $G = F_1F_2$ contains only the minterms that are common to F_1 and F_2.

2. Obtain the truth table of the following functions, and express each function in sum of products (SoP) and product of sums (PoS) in canonical forms:
   a. $(c' + d)(b + c')$
   b. $bd' + acd' + ab'c + a'c'$

3. Express the **complement** of the following functions in sum of minterms form:
   a. $F(A,B,C,D) = \Sigma\, m(2, 4, 7, 10, 12, 14) \rightarrow F'(A,B,C,D) = \Sigma\, m(?,\ldots)$
   b. $F(x, y, z) = \Pi\, M(3, 5, 7) \rightarrow F'(x, y, z) = \Sigma\, m(?,\ldots)$

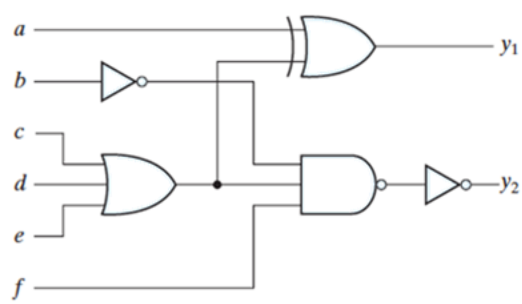
4. By default, Σ means sum of minterms. So, we can drop 'm' and Σ is the same as Σm. Similarly, Π means product of maxterms and we can drop 'M' and Π is the same as ΠM. Convert each of the following to the other canonical form. Then algebraically simplify them, if possible:

   a. $F(x, y, z) = \Sigma\,(1, 3, 5)$
   b. $F(A, B, C, D) = \Pi\,(3, 5, 8, 11)$

5. Determine the number of AND and OR gate as well as the number of their inputs (e.g., 2-input AND, 3-input AND, ...) for the design of the following Boolean functions. Try to algebraically simplify them, if possible, and compare the original requirement with the simplified version. The design is <u>not</u> needed.

   a. $(x, y, z) = \Sigma\,(1, 2, 4, 5)$
   b. $F(A, B, C, D) = \Pi\,(0, 3, 5, 8, 11,13)$

6. Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams. Then algebraically simplify the Boolean expression, if possible.



7. **By convention,** when writing a Boolean expression as a function of binary variables, e.g., F(Z, Y, X), we assume the left most binary variable (e.g., Z) having the highest significance in writing the index numbers for minterms and MAXTERMs, e.g., $m_2=Z'YX'$ or $M_3=(Z+Y'+X')$. However, we know that AND and OR are commutative. Hence, $m_2=Z'YX' = YZ'X' = m_4$ or $M_3= (Z+Y'+X') = (Y'+X'+Z)= M_6$. Is this argument correct? Justify your answer.

8. True or False:
   a. If $F(X,Y,Z)=X'YZ$ then $F(Z,Y,X)=X'YZ$
   b. If $F(X,Y,Z)=X'YZ$ then $F(Z,Y,X)=Z'YX$
   c. If $F(X,Y,Z)=X'YZ$ then $F(Z,Y,X)=YXZ'$
   d. If $F(X,Y,Z)=m_0+m_1$ then $F(Z,Y,X)=m_0+m_1$
   e. If $F(X,Y,Z)=m_0+m_1$ then $F(Z,Y,X)=m_0+m_4$