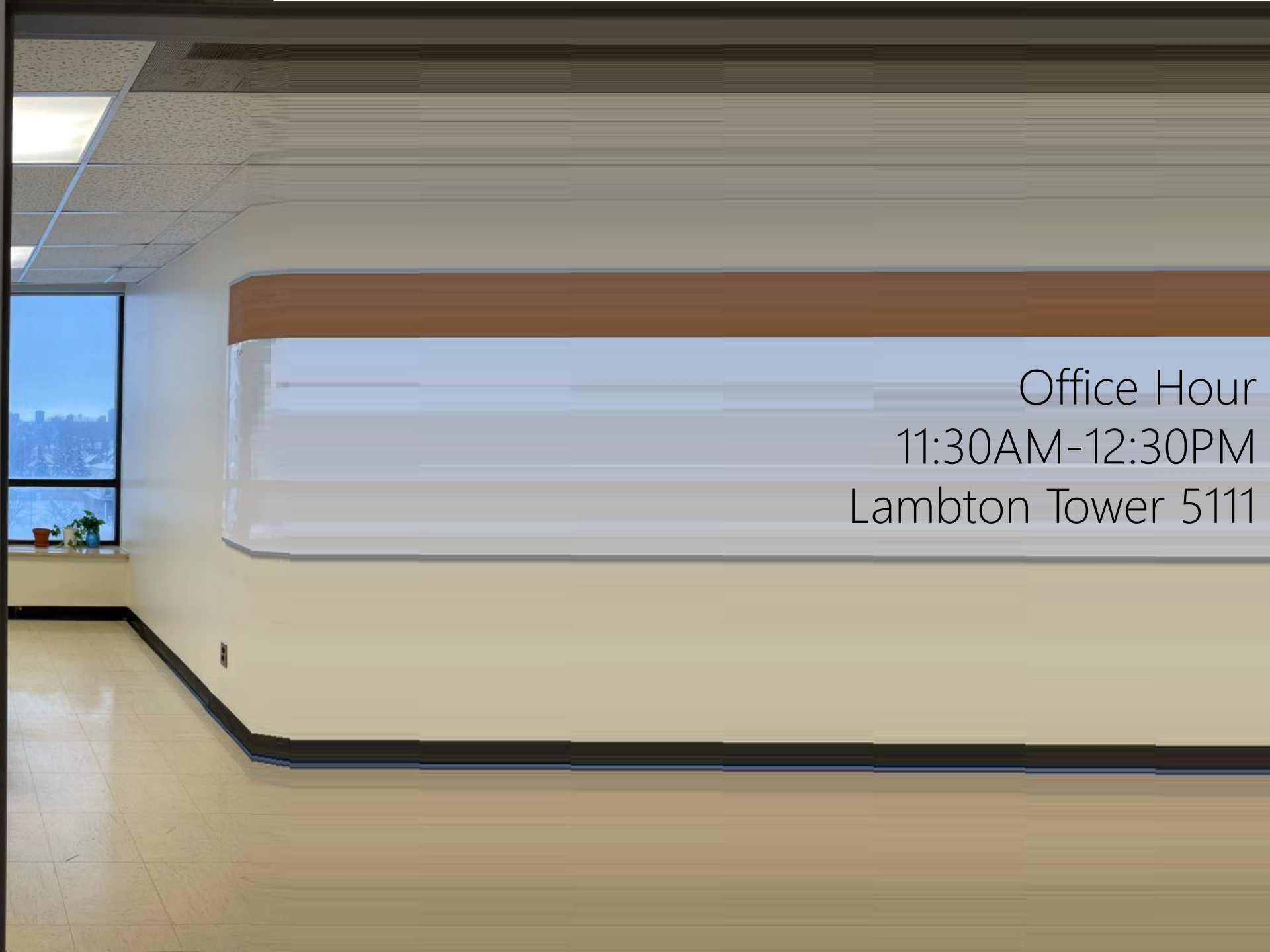




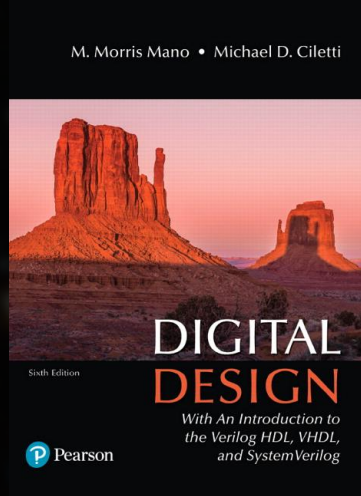
Yao Yao

Rest in Peace

we <https://www.youtube.com/watch?v=etlBZlnTE-I>



Office Hour
11:30AM-12:30PM
Lambton Tower 5111



Chapter 1

Digital Systems and Binary Numbers

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE
SIGNED COMPLEMENT

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

Signed

Magnitude

The background of the slide is a deep space image showing a vast field of galaxies. These galaxies appear as bright, colorful spots and streaks of light against a dark, black background. The colors range from bright yellow and orange to deep blues and purples, representing different wavelengths of light. The galaxies are scattered across the entire frame, creating a sense of depth and cosmic scale.

SIGNED MAGNITUDE ARITHMETIC

0	X
0	X
1	X

+

+

+

0	Y
<u>1</u>	Y
1	Y

=

=

=

0	$X+Y$
$X < Y$ (if borrow)	$X-Y$ (if borrow, apply it)
1	$-(X+Y) = X+Y$

0	X
0	X
1	X

-

-

-

0	Y
1	Y
<u>1</u>	Y

=

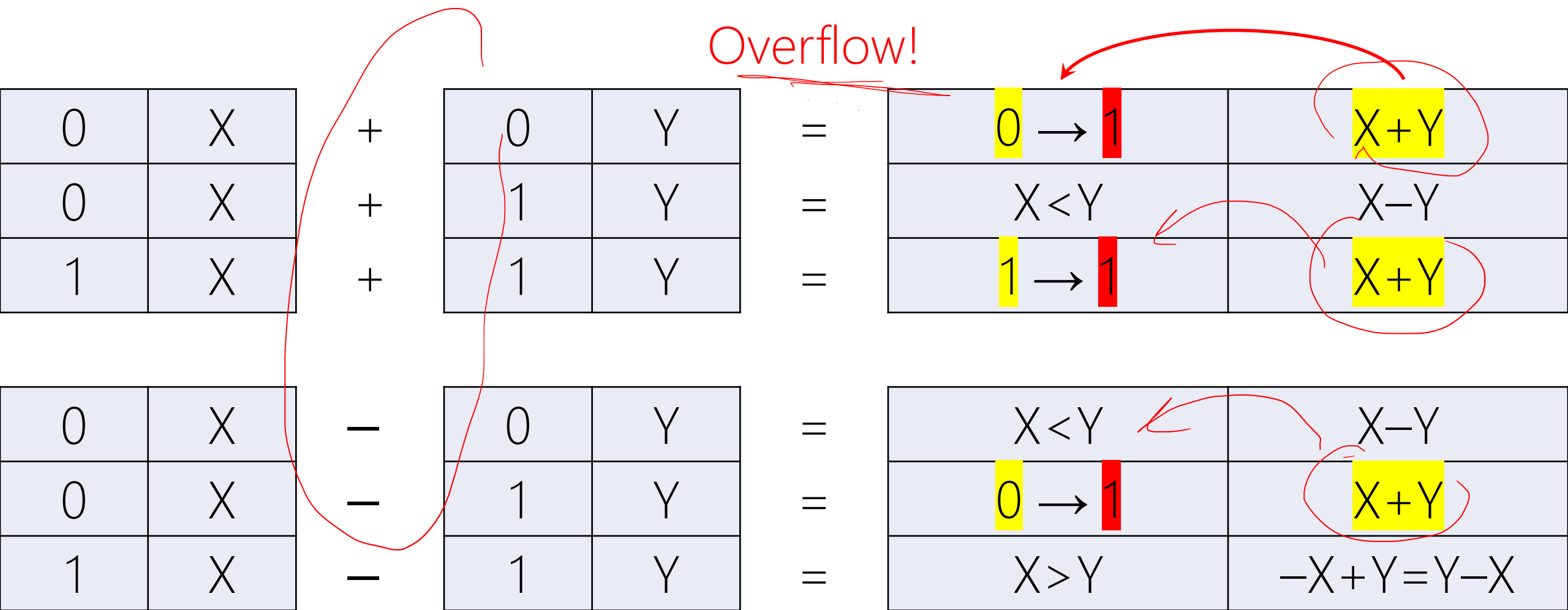
=

=

$X < Y$ (if borrow)	$X-Y$ (if borrow, apply it)
0	$X+Y$
$X > Y$ (if borrow)	$-X+Y = Y-X$ (if borrow, apply it)

The background of the slide is a deep space image showing a vast field of galaxies. These galaxies appear as bright, colorful spots and streaks of light against a dark, black background. The colors range from bright yellow and orange to deep blue and purple. The galaxies are scattered across the entire frame, with some appearing larger and more prominent than others. Two thin, horizontal blue lines are positioned above and below the central text, framing it.

SIGNED MAGNITUDE OVERFLOW





POLL

<http://etc.ch/6pac>

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

WHY NOT SIGNED MAGNITUDE

Give up left most position for sign! What are the wastes?

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

$$+0 \rightarrow \text{Max} = \underline{r^{n-1}} - 1 = \cancel{r^n - 1}$$

$$\text{Min} = -(\underline{r^{n-1} - 1}) \leftarrow \underline{-0}$$

A deep space image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE
SIGNED COMPLEMENT

DIMINISHED RADIX COMPLEMENT

Given $(N)_r$ with n digits, the $(r - 1)$'s complement of N , i.e., its *diminished radix complement*, is defined as $(r^n - 1) - N$.

Base-2	2^5	2^4	2^3	2^2	2^1	2^0
$2^5-1=$		1	1	1	1	1
$N=$		1	0	1	0	1
$(2^5-1)-N=$		0	1	0	1	0

1's complement of $(10101)_2 = (01010)_2 =$ NOT on each digit

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

$(r-1)$'s COMP. BASE- r

Base-r		r^{n-1}	...	r^2	r^1	r^0
$r^n-1=$		$r-1$...	$r-1$	$r-1$	$r-1$
$N=$		d_{n-1}	...	d_2	d_1	d_0
$(r^n-1)-N=$		$r-1-d_{n-1}$...	$r-1-d_{n-1}$	$r-1-d_{n-1}$	$r-1-d_{n-1}$

$(r-1)$'s complement of $(N)_r = (r-1) - \text{Each digit}$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

RADIX COMPLEMENT

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED COMPLEMENT

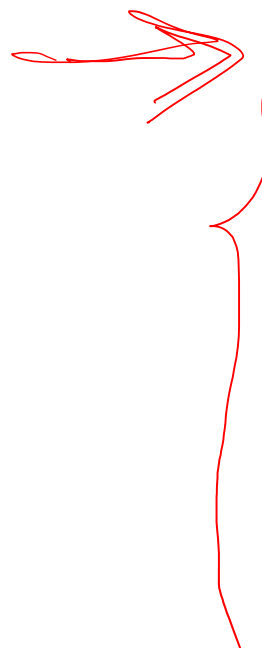
RADIX COMPLEMENT

Given $(N)_r$ with n digits, the r 's complement of N , i.e., its *radix complement*, is defined as $r^n - N$.

Diminished Complement + 1

$$(r-1)\text{'s Complement} + 1 = [(r^n - 1) - N] + 1 = r^n - N$$

10^0		10^0
0	Base-10 Signed 10's comp.	see 0 interpret 0
1		see 1 interpret 1
2		see 2 interpret 2
3		see 3 interpret 3
4		see 4 interpret 4
5 → -5		see 5 interpret -5
6 → -4		see 6 interpret -4
7 → -3		see 7 interpret -3
8 → -2		see 8 interpret -2
9 → -1		see 9 interpret -1



A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

SIGNED 10's COMP. Base-10

n

Signed Radix Complement Base-10

10^{n-1}

10^{n-2}

10^{n-3}

...

10^2

10^1

10^0

0 \leq

Positive Numbers

$\leq (10^n - 1) \div 2$

Nothing to do!

Surface == Meaning

Signed Radix Complement Base-10

 10^{n-1} 10^{n-2} 10^{n-3}

...

 10^2 10^1 10^0

$(10^n - 1) \div 2 + 1 \leq$ Negative Numbers $\leq (10^n - 1)$

Although we see positive numbers, we must interpret them negative!

How?

Surface \neq Meaning

Signed Radix Complement Base-10

 10^{n-1} 10^{n-2} 10^{n-3}

...

 10^2 10^1 10^0

$(10^n - 1) \div 2 + 1 \leq$ Negative Numbers $\leq (10^n - 1)$

— (10's comp. of the number we see)

9 9 9 9 9 9 9

Signed Radix Complement Base-10

10^6	10^5	10^4	10^3	10^2	10^1	10^0
<u>5</u>	<u>8</u>	<u>0</u>	<u>5</u>	<u>0</u>	7	4

$$n = 7$$

$$0 \leq \text{Positive Numbers} \leq (10^7 - 1) \div 2$$

$$(10^7 - 1) \div 2 + 1 \leq \text{Negative Numbers} \leq (10^7 - 1)$$



Signed Radix Complement Base-10

10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$n = 7$$


$$(10^7 - 1) \div 2 = 9,999,999 \div 2 = 4,999,999$$

$$5,805,074 > 4,999,999$$

This number must be interpreted negative!

Signed Radix Complement Base-10

10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4


$$\begin{aligned} &= -(\text{10's comp. } (5,805,074)) \\ &= -(\text{9's comp. } (5,805,074) + 1) \\ &= -(9,999,999 - 5,805,074 + 1) \\ &= -(4,194,925 + 1) \\ &= -(4,194,926) \end{aligned}$$

Signed Radix Complement Base-10

10^6

10^5

10^4

10^3

10^2

10^1

10^0

X

X

X

X

X

X

X

– 3,450,256

Signed Radix Complement Base-10

10^6	10^5	10^4	10^3	10^2	10^1	10^0
X	X	X	X	X	X	X

— 3,450,256

We must find its positive complement to represent it!

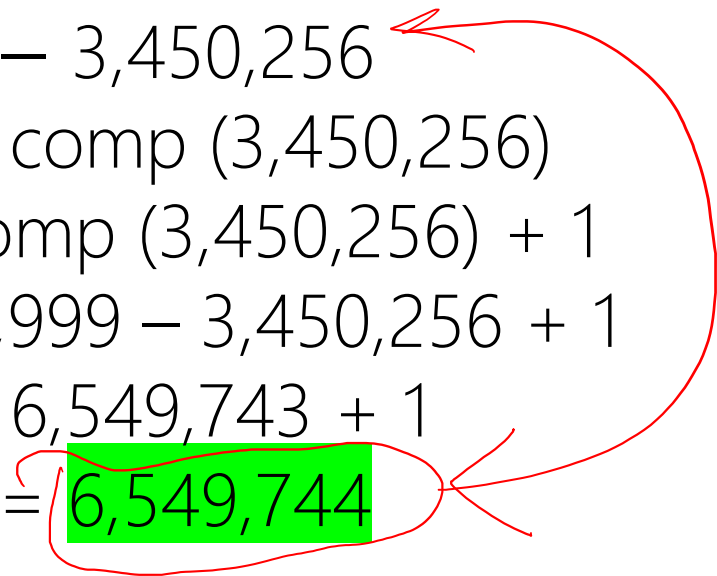
How?

Signed Radix Complement Base-10

10^6	10^5	10^4	10^3	10^2	10^1	10^0
<u>6</u>	<u>5</u>	<u>4</u>	9	7	4	4



$$\begin{aligned} & - 3,450,256 \\ &= 10\text{'s comp } (3,450,256) \\ &= 9\text{'s comp } (3,450,256) + 1 \\ &= 9,999,999 - 3,450,256 + 1 \\ &= 6,549,743 + 1 \\ &= \mathbf{6,549,744} \end{aligned}$$



A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black sky. A horizontal blue line is positioned above the text, and another is below it.

SIGNED 16's COMP. Base-16

Base-16	1	B	F	2	B	4
	Is this a negative number or positive?					

Base-16

1

B

F

2

B

4

How many positions?

$n = 6$

Base-16

F

F

F

F

F

F 52

1

B

F

2

B

4

1F

What is the maximum positive number?

$$(16^6 - 1) \div 2 = (8,388,607)_{10} = (7FF,FFF)_{16}$$

0 0

7

Base-16

1

B

F

2

B

4

This number is less than $(7FF,FFF)_{16}$
 $+ \cancel{(+1BF,2B4)}_{16}$

Base-16	E	B	F	2	B	4
	Is this a negative number or positive?					

Base-16	E	B	F	2	B	4
	How many positions? $n = 6$					

Base-16

E

B

F

2

B

4

What is the maximum positive number?

$$(16^6 - 1) \div 2 = (8,388,608)_{10} = (7FF,FFF)_{16}$$

Base-16

E

B

F

2

B

4

This number is greater than $(7FF,FFF)_{16}$
The number is negative, but what is the number?

Base-16

E

B

F

2

B

4

$$\begin{aligned} &= - (16\text{'s comp. (EBF,2B4)}) \\ &= - (\text{15's comp. (EBF,2B4)} + 1) \\ &= - (FFF,FFF - EBF2B4 + 1) \\ &= - (140,D4B + 1) \\ &= - (140,D4C) \end{aligned}$$

Base-16	X	X	X	X	X	X
	– (140,D4C)					

Base-16

X

X

X

X

X

X

E

B

F

2

B

4

– 140,D4C

= 16's comp(140,D4C)

= 15's comp(140,D4C) + 1

= FFF,FFF – 140,D4C + 1

= EBF,2B3 + 1

= EBF,2B4

Base-16

E

B

F

2

B

4

– 140,D4C

= 16's comp(140,D4C)

= 15's comp(140,D4C) + 1

= FFF,FFF – 140,D4C + 1

= EBF,2B3 + 1

= EBF,2B4

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

SIGNED r 's COMP. Base- r

Base- r in Radix Complement

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
-----------	-----------	-----------	-----	-------	-------	-------

$0 \leq$ Positive Numbers $\leq (r^n - 1) \div 2$

Nothing to do!

Base-2: 0,111,...,111

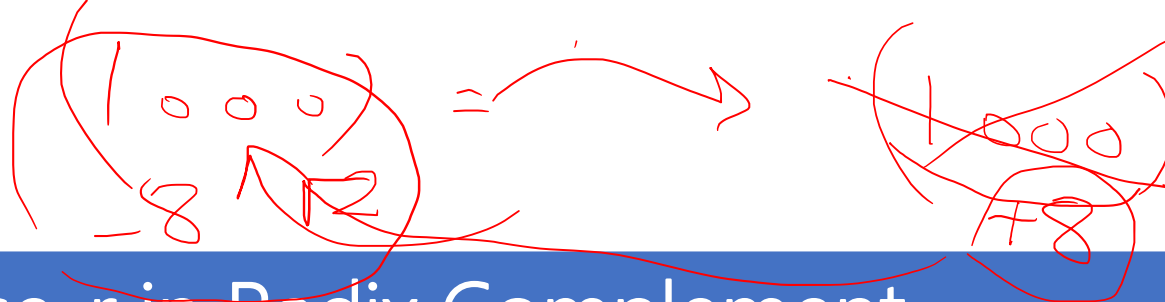
Base-4: 1,333,...,333

Base-8: 3,777,...,777

Base-10: 4,999,...,999

Base-16: 7,FFF,...,FFF

3



Base-r in Radix Complement

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
-----------	-----------	-----------	-----	-------	-------	-------

$(r^n - 1) \div 2 + 1 \leq$ Negative Numbers $\leq (r^n - 1)$

Base-2: ~~1,000, ..., 000~~ \times $\rightarrow 2$'s $\sim (\times) = (\text{1000})$

Base-4: 2,000, ..., 000

Base-8: 4,000, ..., 000

Base-10: 5,000, ..., 000 $\rightarrow (5000) \rightarrow$

Base-16: 8,000, ..., 000

Handwritten calculations for Base-10:

$$\begin{array}{r}
 9999 \\
 - 5000 \\
 \hline
 4999 \\
 + \quad \quad \quad \\
 \hline
 \end{array}$$

The result 4999 is circled in red, and an arrow points from it to the circled 5000 in the Base-10 row. Another arrow points from the circled 5000 to the circled 1000 in the Base-2 row.

We see positive number, but we interpret negative!

$= - (r\text{'s comp. (\#)}) = - ((r-1)\text{'s comp. (\#)} + 1)$

SIGNED 2'S COMPLEMENT BASE-2

6. Show that in 2's complement binary system, the highest significant position acts like a sign (not the same) as in signed-magnitude binary system. Is this true for any radix-r number system? Justify your answer.



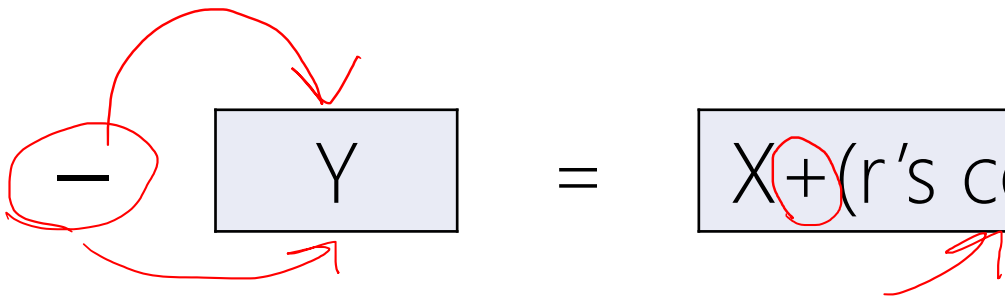
POLL

<http://etc.ch/6pac>

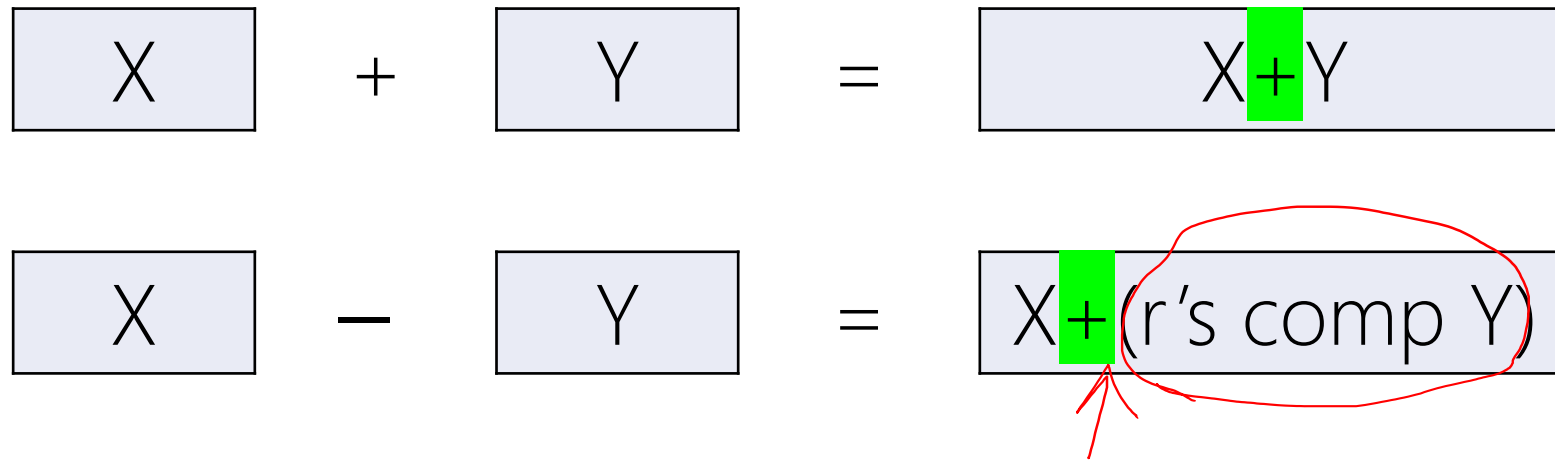
The background of the slide is a deep space image showing a vast field of galaxies. These galaxies appear as bright, colorful spots in various shapes and sizes, including spirals, ellipticals, and irregular forms, set against a dark, star-filled sky. Two thin, horizontal blue lines are positioned above and below the central text, framing it.

SIGNED r 'S COMPLEMENT ARITHMETIC

$$\boxed{X} + \boxed{Y} = \boxed{X+Y}$$

$$\boxed{X} - \boxed{Y} = \boxed{X + (r's \text{ comp } Y)}$$


Last Carry \rightarrow Ignore



One adder for both addition and subtraction!

The background of the slide is a deep space image showing a vast field of galaxies. These galaxies appear as bright, colorful spots (yellow, orange, blue, and white) against a dark, black background. They are scattered across the entire frame, with some appearing more prominent than others. Two thin, horizontal blue lines are positioned above and below the central text, spanning most of the width of the slide.

SIGNED r' S COMPLEMENT ADDITION

+ Base-2		1	0	1	0	1	1
		0	0	1	1	1	0

6 P —

+
Base-2

1

0

1

0

1

1

$$> (2^6 - 1) \div 2 = (011, 111)_2$$

$$= -(2's \text{ comp}(\#))$$

$$= -(010101)_2$$

$$= -(21)_{10}$$

+ Base-2							
		0	0	1	1	1	0
		$< (2^6 - 1) \div 2 = (011, 111)_2$ $= + (001110)_2$					

			1	1	1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1

			1	1	1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1
		$ \begin{aligned} &> (2^6 - 1) \div 2 = (011, 111)_{10} \\ &= -(2's \text{ comp}(\#)) \\ &= -(000111)_2 \\ &= -(7)_{10} \end{aligned} $					

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. The galaxies are of different shapes and sizes, some appearing as bright, fuzzy blobs and others as more distinct, elongated structures. Two thin, horizontal blue lines are positioned above and below the central text.

SIGNED r' S COMPLEMENT SUBTRACTION

Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(8)_{10}$	0	0	1	0	0	0

Base-2

$$+ (8)_{10}$$

0

0

1

0

C

C

Eq., addition with $-(8)_{10}$!

Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(8)_{10}$	0	0	1	0	0	0
$= -(2's \text{ comp. } (001000))$ $= -(1's \text{ comp}(001000) + 1)$ $= -(\text{NOT}(001000) + 1)$ $= -(110111 + 1)$ $= -(111000)$							

Base-2	<div><div>+</div></div>						
	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0

Last Carry → Ignore

+ Base-2	1	1	1				
	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1

		1	1				
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1
		Done! This is the Result.					

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1

$> 2^6 - 1 \div 2 = (011, 111)_{10}$

The number is negative:

$= - (2's \text{ comp. } (100011))$

$= -(011101) + 2^9$

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
	$-(29)_{10}$	1	0	0	0	1	1

The background of the slide is a deep space image showing a vast field of galaxies. These galaxies appear as bright, colorful spots in various shapes and sizes, including spirals, ellipticals, and irregular forms, set against a dark, star-filled sky. Two thin, horizontal blue lines are positioned above and below the central text, framing it.

SIGNED r' S COMPLEMENT OVERFLOW

$$\boxed{X} + \boxed{Y} = \boxed{X+Y}$$

The + result of two negative numbers → positive

The + result of two positive numbers → negative



$$\boxed{X} - \boxed{Y} = \boxed{X + (r's \text{ comp } Y)}$$

The + result of two negative numbers → positive

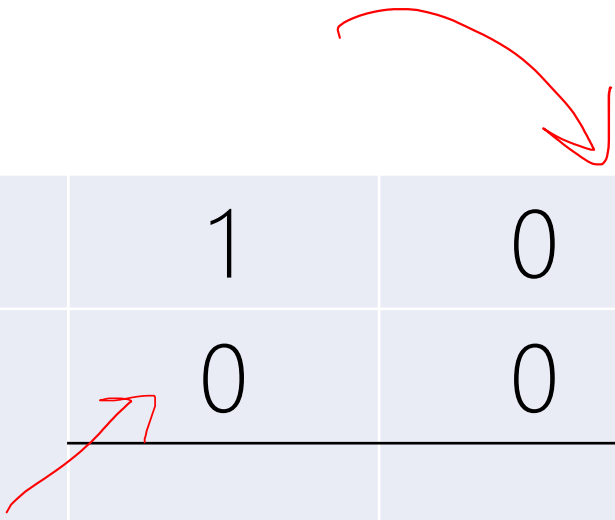
The + result of two positive numbers → negative

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

OVERFLOW

Example

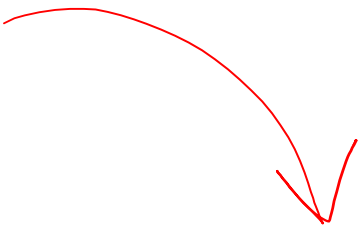

Base-2	<div><div>-</div><div>-(21)₁₀</div></div>	1	0	1	0	1	1
	<div><div>+</div><div>+(14)₁₀</div></div>	0	0	1	1	1	0



Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
We must convert the subtraction to addition. Eq., addition with $-(14)_{10}$!							

<div> <div></div> <div>Base-2</div> </div>	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		$= (2's \text{ comp. } (001110))$ $= (1's \text{ comp}(001110) + 1)$ $= (\text{NOT}(001110) + 1)$ $= (110001 + 1)$ $= (110010)$					

<div>+</div> <div>Base-2</div>	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0


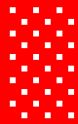

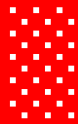
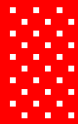
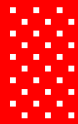



Last Carry → Ignore

		1				1		
Base-2	+	$-(21)_{10}$	1	0	1	0	1	1
		$-(14)_{10}$	1	1	0	0	1	0
			0	1	1	1	0	1

0 < 01111 <

Base-2	+				1		
	—(21) ₁₀	1	0	1	0	1	1
	—(14) ₁₀	1	1	0	0	1	0
		0	1	1	1	0	1
		$< 2^6 - 1 \div 2 = (011, 111)_{10}$ $= (011101)_2$ Positive Number					

Base-2	+				1		
	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0
							
		Overflow! Don't rely on the result!					

A deep space image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the text.

BINARY CODE

Will be covered later. Stay tuned!