



LEC02 & LAB02

Lectures >> Lec02: Number Systems

Labs >> Lab02: Programming Environment Setup



Problems

The background is a deep space photograph showing a dense field of galaxies in various colors (blue, orange, white) against a black sky. Overlaid on the center is a diagram consisting of a large black circle with a smaller light blue circle inside it. A line connects the light blue circle to a text box on the right.

Problems

Distance
Height
Length



Quantization

DISCRETE SYSTEMS

"Digital System"

A handwritten red annotation consisting of a circle around the word "Digital" and an arrow pointing to it from the left. The word "System" is written to the right of the circle. The entire annotation is in red ink.

UNARY SYSTEM

aka. Base-1

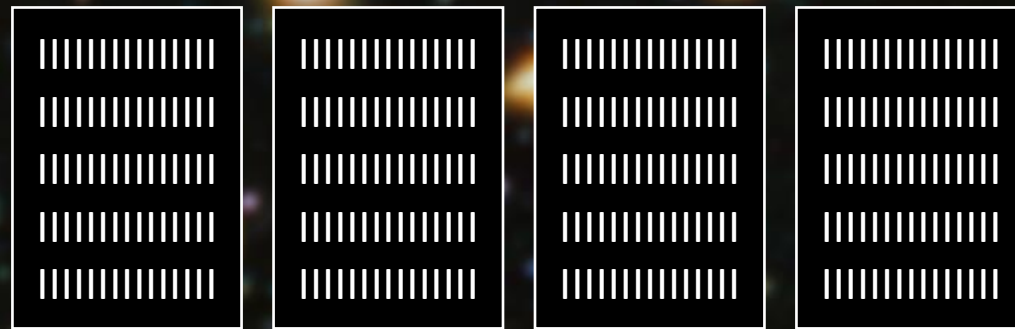


Roman Numerals

Originated in Ancient Rome

8th Century BC

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$
paper = $\sim 3,000 \text{ positions}$
 $12,000$ $\div 3,000 = 4 \text{ pages!}$



The background of the slide is a deep space image showing a dense field of galaxies and stars. The galaxies are in various colors, including blue, orange, and white, and are scattered across the dark background. Two horizontal blue lines are positioned above and below the text.

NUMBER SYSTEMS

— = ≡ ≠ √ √ | 6 7 8 9

Brahmi

3rd and 7th century AD

1 2 3 4 5 | 6 7 8 9 0

Hindu (Gwalior)

1 2 3 4 5 | 6 7 8 9 0

Sanskrit-Devanagari

1 2 3 4 5 | 6 7 8 9

Western Arabic (Gobar)

1 2 3 4 5 | 6 7 8 9 0

Eastern Arabic

1 2 3 4 5 | 6 7 8 9

11th Century (Apices)

1 2 3 4 5 | 6 7 8 9 0

15th Century

1 2 3 4 5 | 6 7 8 9 0

16th Century (Dürer)

0123456789



Hossein's Number System

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$$N = 12,000$$

$$n = (N+1) \div 4 = (\underline{12,000} + 1) \div 4 = \sim \underline{3,000} \text{ positions}$$

paper = $\sim 3,000$ positions

$$3,000 \div 3,000 = \underline{1} \text{ pages}$$



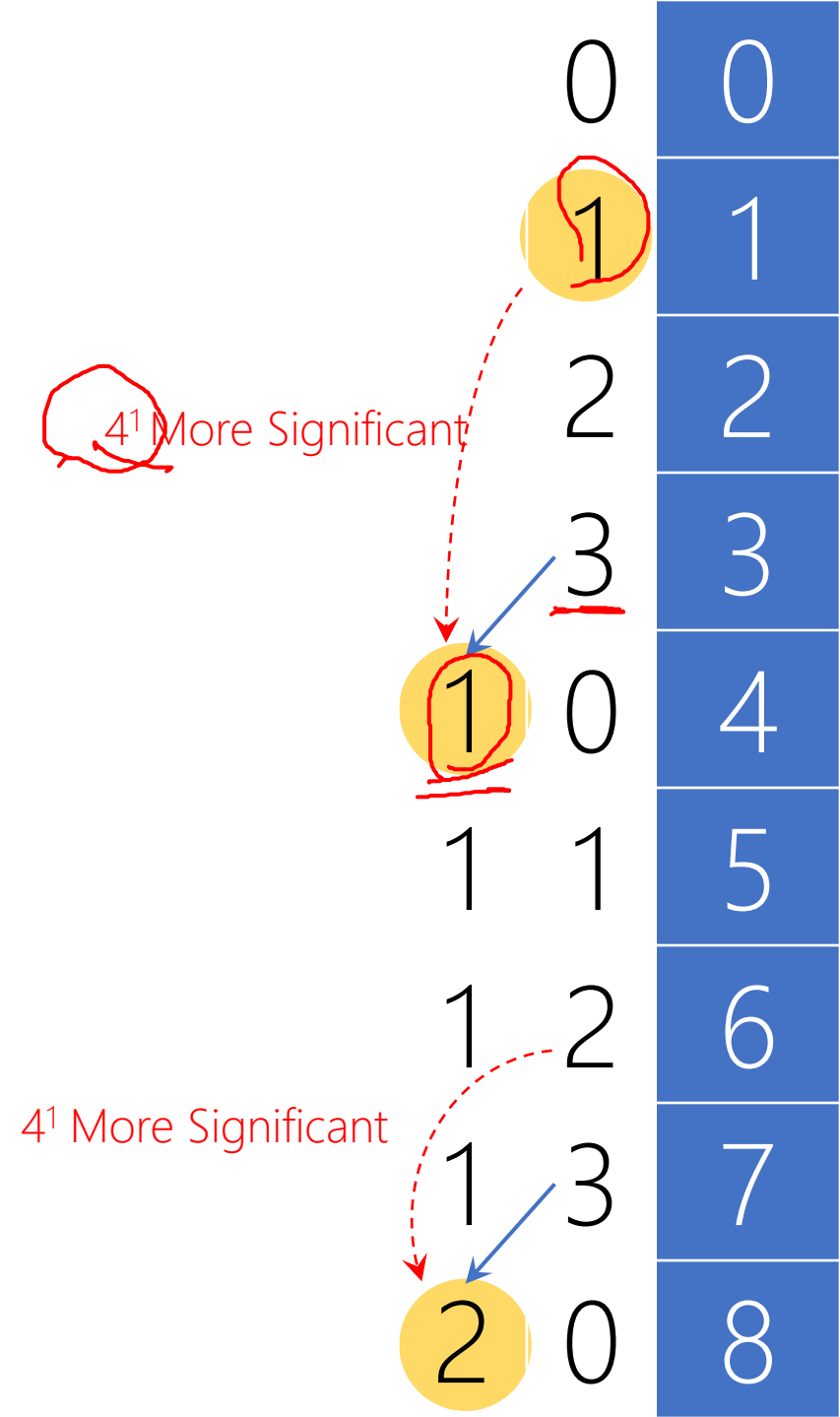
More Significant Position



1 round of all (4) symbols = 1×4^1

2 rounds of all (4) symbols = 2×4^1

0	0
1	1
2	2
3	3
1	0
1	1
1	2
1	3
2	0
	8



QUATERNARY SYSTEM

/kwaa-tur·neh·ree/

aka. Base-4, Radix-4

$(0,1,2,3)_4$

... 4^1 4^0

Hindu-Arabic Numerals
Originated in India
7th Century AD

More Significant Position
←

#Symbols=4
Radix-4
Base-4

4^3	4^2	<u>4^1</u>	<u>4^0</u>
		2	0
		2	1
		2	2
		2	3
		3	0
		3	1
		3	2
		3	3
	<u>1</u>	<u>0</u>	<u>0</u>

$2 \times 4^1 + 0 \times 4^0 = 8$
$2 \times 4^1 + 1 \times 4^0 = 9$
$2 \times 4^1 + 2 \times 4^0 = 10$
$2 \times 4^1 + 3 \times 4^0 = 11$
$3 \times 4^1 + 0 \times 4^0 = 12$
$3 \times 4^1 + 1 \times 4^0 = 13$
$3 \times 4^1 + 2 \times 4^0 = 14$
$3 \times 4^1 + 3 \times 4^0 = 15$
<u>$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$</u>

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	
3	0	3	0	2	1	3	1	\times
3×4^7	0×4^6	3×4^5	0×4^4	2×4^3	1×4^2	3×4^1	1×4^0	Σ
								65,437

More Significant Position
←

										Base-4	Hossein's Number System
3	0	3	0	2	1	3	1			65,437	-
				3	3	3	3	1		1,021	17
					3	3	3	3	2	4,094	22
		3	0	0	3	3	3	3	0	50,172	-
3	3	3	3	3	3	3	3	3	3	1,048,575	39

10

power 4

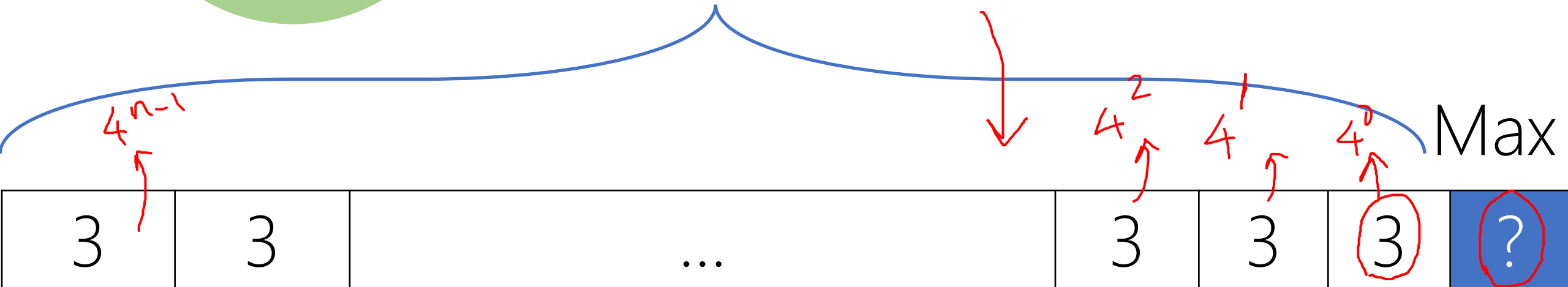
x 4



Min



n positions



$$N = 3 \times 4^{n-1} + 3 \times 4^{n-2} + \dots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$$

$$N = 3 \times (4^{n-1} + 4^{n-2} + \dots + 4^2 + 4^1 + 4^0)$$

$$N = 3 \times \left(\frac{4^n - 1}{4 - 1} \right)$$

$$N = 4^n - 1$$

$4n - 1$
Hossein's
System

n positions

<div> <div></div> <div>Max</div> </div>						
4^{n-1}	4^{n-2}					
3	3	...				N

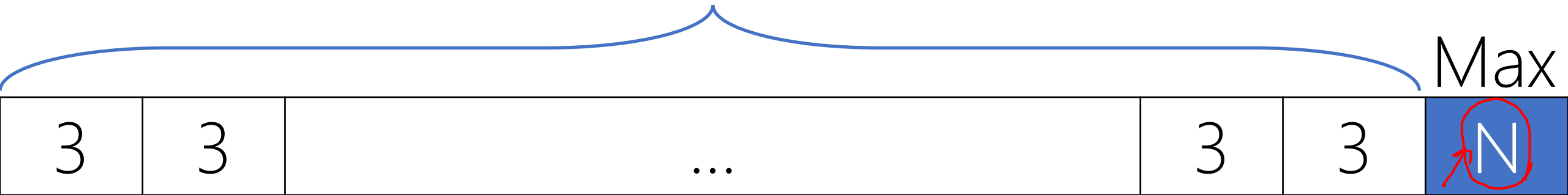
$$\underline{4^n - 1 = N}$$

$$4^n = N + 1$$

$$\log_4 4^n = \log(N + 1)$$

$$\textcircled{n} = \log_{\textcircled{4}}(\underline{N + 1})$$

? positions



$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$$N = 12,000$$

$$n = \text{Log}_4 (12,000 + 1) = \text{Log}_{10} 12,001 \div \text{Log}_{10} 4 = 4 \div 0.6 = \underline{6.79}$$

~ 7 positions



COMMON NUMBER SYSTEMS

Base - 4

- 2

- 3

5

6

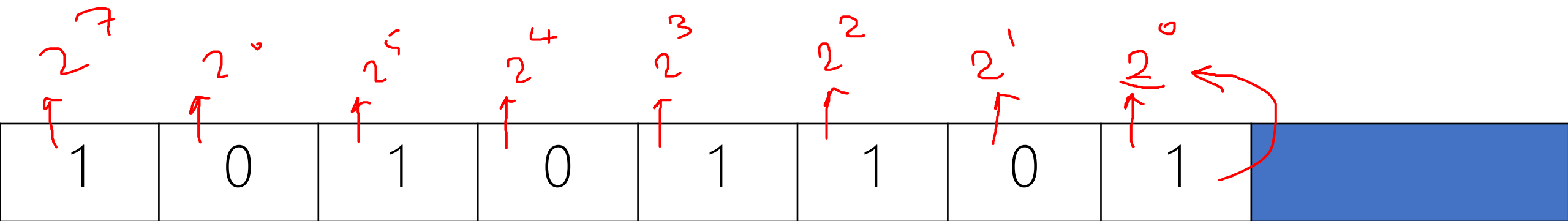
...



A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black space. Two horizontal blue lines frame the central text.

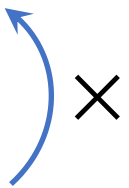
BINARY | BASE-2 | RADIX-2

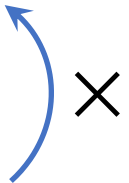
$(0,1)_2$





8

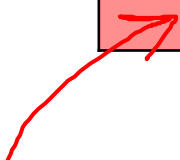
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	


2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	Σ
								173

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
3	0	3	0	2	1	3	1	
								Σ
								?

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	 ×
<u>3</u>	0	<u>3</u>	0	<u>2</u>	1	<u>3</u>	1	
-	0×2^6	-	0×2^4	-	1×2^2	-	1×2^0	Σ
								Not Valid



LET'S COUNT IN BINARY

()₂

$(0000)_2, 0001, 0010, \dots$

$$\begin{array}{r} 0000 \\ 0001 \\ \hline 0000 \\ + 0001 \\ \hline 0001 \end{array}$$

 $\frac{2}{2} = 1 \text{ r } 0$

0010

$(1111)_2$

LET'S COUNT IN BINARY

$(\begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \end{array} \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \end{array} \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \end{array} \begin{array}{c} \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \\ \cdot \cdot \cdot \cdot \end{array})_2$

0000, 0001, 0010, 00011, ..., 1101, 1110, 1111

Increment by 1 → Increment by 0001

$8 + 2 + 1 = 11$

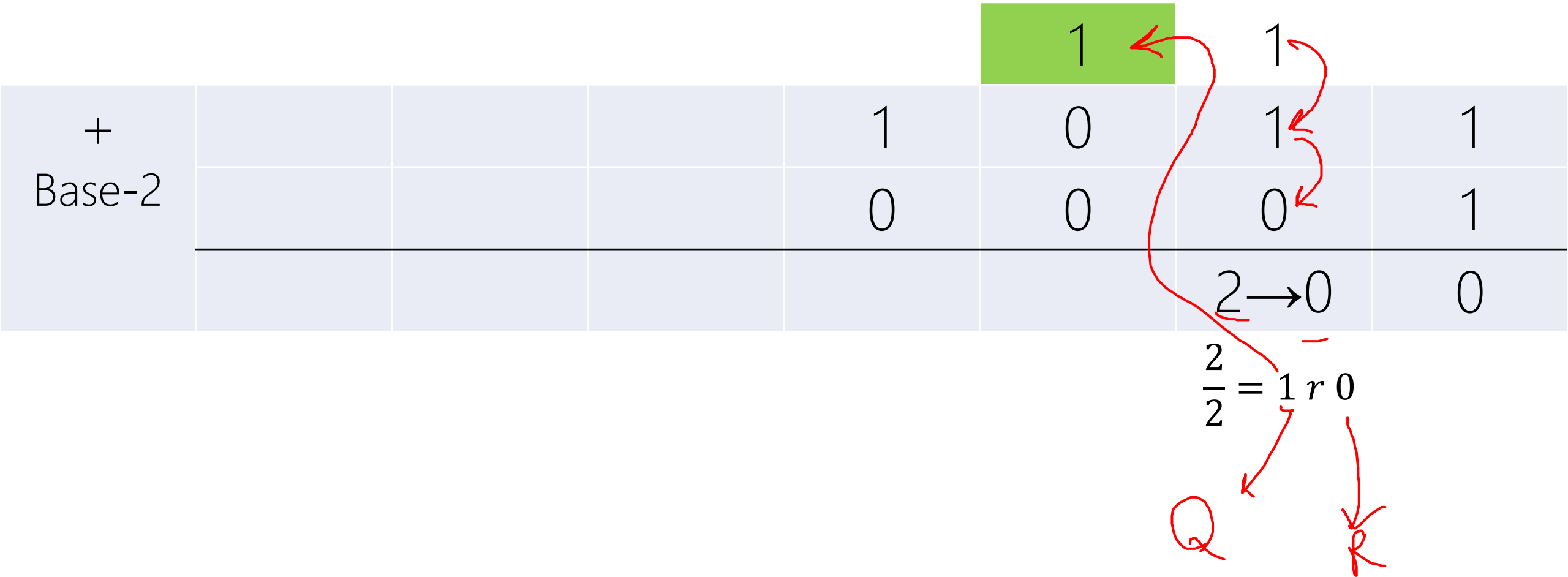
2^3
 2^2 2^2 2^1 2^0

+ Base-2				1	0	1	1
				0	0	0	1
		12					

Increment by 1 → Increment by 0001

						1	
+ Base-2				1	0	1	1
				0	0	0	1
							<u>2</u> → <u>0</u>
							$\frac{2}{2} = \textcircled{1} r \text{ } \underline{0}$

Increment by 1 → Increment by 0001



Increment by 1 → Increment by 0001

+ Base-2							
				1	0	1	1
				0	0	0	1
				1	1	0	0

0000

0001

0010

0011

0100

0101

0110

0111

1000

1001

1010

1011

1100

1101

1110

1111

1011

(7)₁₀

1011

(11)₁₀

2³

8

2²

4

8 + 4 = 12


3, 4, 5, 6, 7


OCTAL | BASE-8 | RADIX-8

(0, 1, 2, 3, 4, 5, 6, 7)₈


3	0	3	0	2	1	3	1	
---	---	---	---	---	---	---	---	--

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	3	0	2	1	3	1	

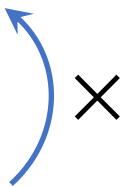
8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	3	0	2	1	3	1	
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0	

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	 \times
3	0	3	0	2	1	3	1	
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0	Σ
								6,390,873

10

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	A	0	8	1	3	1	Σ

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	A	0	8	1	3	1	\times
3×8^7	0×8^6	-	0×8^4	-	1×8^2	3×8^1	1×8^0	Σ
								Not Valid

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	Σ
								<u>2,130,497</u>

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	





2,130,497

Radix-8
vs.
Radix-2

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	

173

LET'S COUNT IN OCTAL

(   )₈

→ (0 0 0 0)₈, 0 0 0 1, 0 0 0 2, 0 0 0 3, 0 0 0 4, 0 0 0 5, (7 7 7 7)₈
0 0 0 6, 0 0 0 7, (0 0 1 0)₈, (0 0 1 1)₈, ...



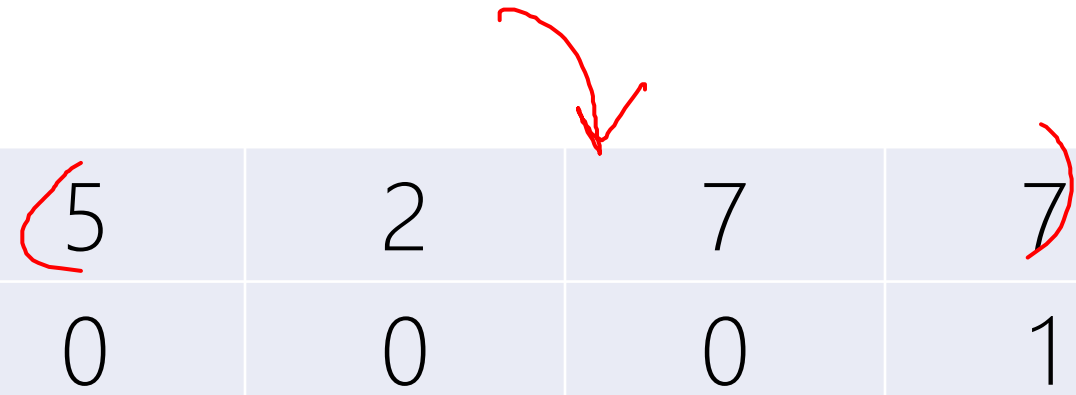
LET'S COUNT IN OCTAL

$(\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix})_8$

0000, 0001, 0002, ..., 0007, 0010, ..., 7775, 7776, 7777

Increment by 1 → Increment by 0001

+ Base-8				5	2	7	7
				0	0	0	1



Increment by 1 \rightarrow Increment by 0001

				5	2	7	1
+				0	0	0	7
Base-8							1
							8 → 0

$\frac{8}{8} = 1 \text{ r } 0$

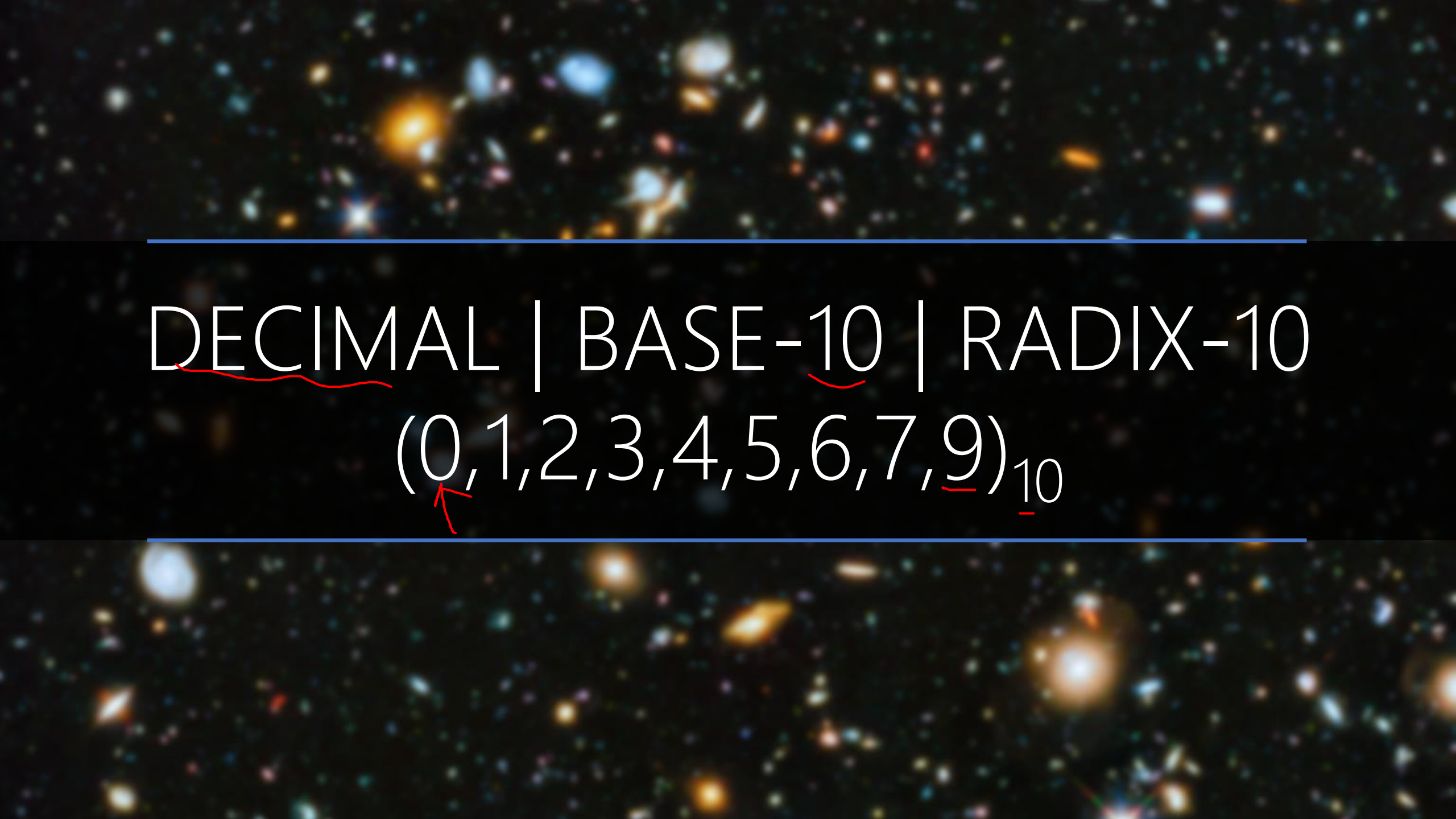
Increment by 1 → Increment by 0001

+ Base-8				5	2	7	7
				0	0	0	1
						<u>8</u> → 0	0

$$\frac{8}{8} = \underline{1} r \underline{0}$$

Increment by 1 → Increment by 0001

+ Base-8						1	1	
				5	2	7	7	
				0	0	0	1	
				5	3	0	0	8



DECIMAL | BASE-10 | RADIX-10

(0,1,2,3,4,5,6,7,9)₁₀

The image features a cosmic background of galaxies. The text is centered on a black horizontal band. The word 'DECIMAL' is underlined with a red wavy line. The '10' in 'BASE-10' is underlined with a red curved line. The '9' in the digit list is underlined with a red horizontal line. A red arrow points to the '0' in the digit list. The subscript '10' in the digit list has a red horizontal line underneath it.

Arabic

→ 0123456789

Eastern Arabic

→ ٠١٢٣٤٥٦٧٨٩

Roman

→ I II III IV V VI VII VIII IX X

Bengali–Assamese

→ ০ ১ ২ ৩ ৪ ৫ ৬ ৭ ৮ ৯

Malayalam

→ ൦ ൧ ൨ ൩ ൪ ൫ ൬ ൭ ൮ ൯

Thai

→ ๐ ๑ ๒ ๓ ๔ ๕ ๖ ๗ ๘ ๙

Chinese

→ 〇 一 二 三 四 五 六 七 八 九

Arabic

→ 0 1 2 3 4 5 6 7 8 9

Eastern Arabic

→ ٠ ١ ٢ ٣ ٤ ٥ ٦ ٧ ٨ ٩

*"the **hand** makes the two complementary aspects of integers entirely intuitive. It serves as an instrument permitting natural movement between cardinal and ordinal numbering. If you need to show that a set contains three, four, seven or ten elements, you raise or bend **simultaneously** three, four, seven or ten fingers, using your hand as cardinal mapping. If you want to count out the same things, then you bend or raise three, four, seven or ten fingers in **succession**, using the hand as an ordinal counting tool."*

- Georges Ifrah The Universal History of Numbers (Wiley, 2000, pp. 21-22)

Thanks Cecelia Nydam and Giavi Tran for the hint!

Thai


→ ๐ ๑ ๒ ๓ ๔ ๕ ๖ ๗ ๘ ๙


Chinese

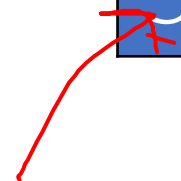
→ 〇 一 二 三 四 五 六 七 八 九


3	0	3	0	2	1	3	1	
---	---	---	---	---	---	---	---	--



10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	 ×
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	Σ
								30,302,131



10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	Σ
								30,302,131



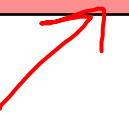
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	 ×
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	Σ
								 10,101,101

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	←
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	A	0	8	1	3	1	\times
								Σ

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	 ×
3	0	A	0	8	1	3	1	
3×10^7	0×10^6	-	0×10^4	8×10^3	1×10^2	3×10^1	1×10^0	Σ
								Not Valid 

YOU KNOW HOW TO COUNT IN DECIMAL!

$$(\begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array} \begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array} \begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array} \begin{array}{c} \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \\ \bullet \bullet \bullet \bullet \end{array})_{10}$$

0 0 0 0 9 9 9 9

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.


Weird NUMBER SYSTEMS


HEXADECIMAL | BASE-16 | RADIX-16

(0,1,2,3,4,5,6,7,9,A,B,C,D,E,F)16

~~10~~

15

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	
								Σ

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A \times 16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	Σ

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	\times
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A \times 16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	Σ

$$A = (9 + 1) = (10)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	\times
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A \times 16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	Σ
								815,829,297

$A = (9 + 1) = (10)_{10}$

$$1, 2, 3, 4, 5, 6, 7, 8, 9, A = 9 + 1 = (10)_{10}$$




$$\underline{B} = A + 1 = (\underline{11})_{10}$$

$$\underline{C} = B + 1 = (\underline{12})_{10}$$

$$\underline{D} = C + 1 = (13)_{10}$$

$$E = D + 1 = (14)_{10}$$

$$F = E + 1 = (15)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	 ×
1	0	1	0	1	1	0	1	
1×16^7	0×16^6	1×16^5	0×16^4	1×16^3	1×16^2	0×16^1	1×16^0	Σ
								 269,488,385

16^7	16^6	16^5	16^4	16^3	16^2	16^1	<u>16^0</u>	
1	0	1	0	1	1	0	1	
1×16^7	0×16^6	1×16^5	0×16^4	1×16^3	1×16^2	0×16^1	1×16^0	
								269,488,385
10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101
8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

LET'S COUNT IN BASE-16

$(\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix} \begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix} \begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix} \begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix})_{16}$

0000,


(FFFF)₁₆

LET'S COUNT IN BASE-16

(   )₁₆

0000, 0001, 0002, ..., 000F, 0010, ..., FFFD, FF FE, FFFF

Increment by 1 → Increment by 0001



+ Base-16				5	C	A	F
				0	0	0	1

Increment by 1 → Increment by 0001

				5	C	A	F ₌₁₅
				0	0	0	1
							16 → 0
							$\frac{16}{16} = \underline{1}r\underline{0}$

Increment by 1 → Increment by 0001

+ Base-16				5	C	A ₌₁₀	F ₌₁₅
				0	0	0	1
						11 → B	0

Increment by 1 \rightarrow Increment by 0001

				5	C	A ₌₁₀	F ₌₁₅
				0	0	0	1
				5	C	B	0



BASE-64 | RADIX-64

(A, B, C, ..., Z, a, b, c, ..., z, 0, 1, 2, ..., 9, +, /) 64

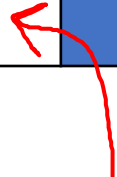
1992: RFC 1341

MIME (MULTIPURPOSE INTERNET MAIL EXTENSIONS)

Mechanisms For Specifying And Describing The Format Of Internet Message Bodies


Digit	Value		Digit	Value		Digit	Value		Digit	Value
A	0		Q	16		g	32		w	48
B	1		R	17		h	33		x	49
C	2		S	18		i	34		y	50
D	3		T	19		j	35		z	51
E	4		U	20		k	36		0	52
F	5		V	21		l	37		1	53
G	6		W	22		m	38		2	54
H	7	→	X	23	→	n	39	→	3	55
I	8		Y	24		o	40		4	56
J	9		Z	25		p	41		5	57
K	10		a	26		q	42		6	58
L	11		b	27		r	43		7	59
M	12		c	28		s	44		8	60
N	13		d	29		t	45		9	61
O	14		e	30		u	46		+	62
P	15		f	31		v	47		/	63



3	a	A	/	d	1	H	+	
---	---	---	---	---	---	---	---	--



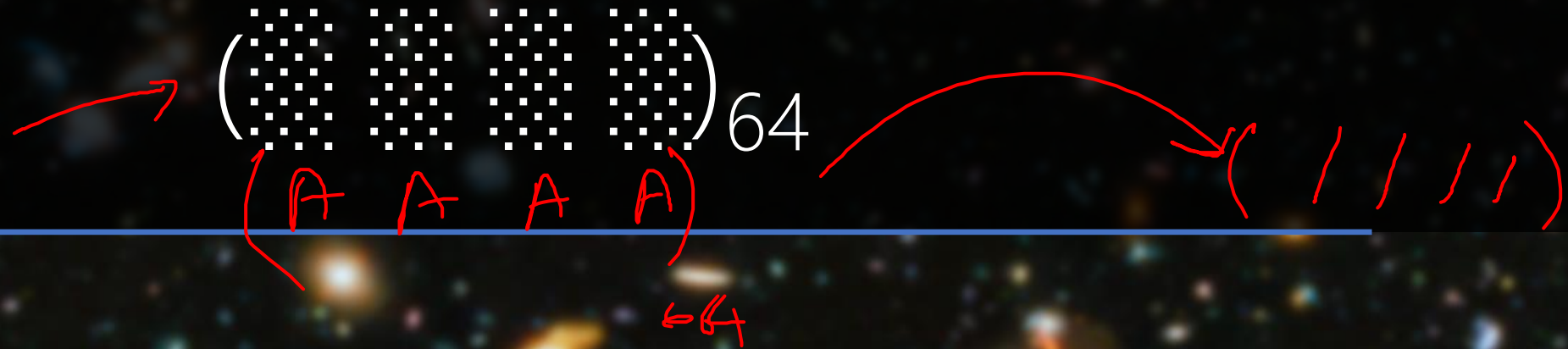
64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	
3	a	A	/	d	1	H	+	
								Σ

Digit	Value		Digit	Value		Digit	Value		Digit	Value
A	0		Q	16		g	32		w	48
B	1		R	17		h	33		x	49
C	2		S	18		i	34		y	50
D	3		T	19		j	35		z	51
E	4		U	20		k	36		0	52
F	5		V	21		l	37		1	53
G	6		W	22		m	38		2	54
H	7	→	X	23	→	n	39	→	3	55
I	8		Y	24		o	40		4	56
J	9		Z	25		p	41		5	57
K	10		a	26		q	42		6	58
L	11		b	27		r	43		7	59
M	12		c	28		s	44		8	60
N	13		d	29		t	45		9	61
O	14		e	30		u	46		+	62
P	15		f	31		v	47		/	63

64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	
3	a	A	/	d	1	H	+	
55 $\times 64^7$	26 $\times 64^6$	0 $\times 64^5$	63 $\times 64^4$	29 $\times 64^3$	53 $\times 64^2$	7 $\times 64^1$	62 $\times \underline{64^0}$	Σ

64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	 ×
3	a	A	/	d	1	H	+	
55 × 64^7	26 × 64^6	0 × 64^5	63 × 64^4	29 × 64^3	53 × 64^2	7 × 64^1	62 × 64^0	Σ
								243,680,329,  290,238

LET'S COUNT IN BASE-64





LET'S COUNT IN BASE-64

(   )₆₄

AAAA, AAAB, AAAC, ..., AAA/, AABA, ..., ///9, ///+, ////

Increment by 1 → Increment by AAAB



+ Base-64				a	+	Z	/
				A=0	A=0	A=0	B=1

Increment by 1 → Increment by **AAAB**

+ Base-64				a ₌₂₆	+ ₌₆₂	Z ₌₂₅	<u>/</u> ₌₆₃
				A ₌₀	A ₌₀	A ₌₀	B ₌₁

+ Base-64				a ₌₂₆	+ ₌₆₂	B ₌₁	/ ₌₆₃
				A ₌₀	A ₌₀	A ₌₀	B ₌₁
							A ₌₀

$$\frac{64}{64} = \underline{1} r 0$$

+ Base-64				a ₌₂₆	<u>+</u> ₌₆₂	Z ₌₂₅	/ ₌₆₃
				A ₌₀	A ₌₀	A ₌₀	B ₌₁
				<u>a</u>	+	<u>a</u> ₌₂₆	A ₌₀

Christina Reynolds-Badder

THE VERTICES NUMBER SYSTEM

0, 1, 2, 3, 4, 5

0 → 0	12 → 10
1 → 1	13 → 11
2 → 2	14 → 12
3 → 3	15 → 13
4 → 4	16 → 14
5 → 5	17 → 15
6 → 10	18 → 20
7 → 11	19 → 21
8 → 12	20 → 22
9 → 13	21 → 23
10 → 14	22 → 30
11 → 15	23 → 31

VERTICES → DECIMAL

n = number of positions

V = number of vertices at position

$$\text{decimal} = V \times 6^n + \dots + V \times 6^2 + V \times 6^1 + V \times 6^0$$

ex 1: convert 15 to decimal.

$$3 \times 6^1 + 5 \times 6^0$$

$$= 18 + 5$$

$$= 23 \quad \checkmark$$

ex 2: convert 3142 to decimal.

$$5 \times 6^3 + 5 \times 6^2 + 4 \times 6^1 + 2 \times 6^0$$

$$= 1080 + 180 + 24 + 2$$

$$= 1286 \quad \checkmark$$

DECIMAL → VERTICES

keep dividing by 6, until you can't anymore. The remainder will be the number of vertices, stop when the solution < 1 .

ex

convert 23 to vertices.

$$23 / 6 = 3 \text{ with remainder } 5$$

$$3 / 6 = 3 \text{ with remainder } 3$$

3, 5

$$= 15 \quad \checkmark$$



0	1	2	3	4
	•	••	•••	••••
5	6	7	8	9
—	•	••	•••	••••
10	11	12	13	14
—	•	••	•••	••••
15	16	17	18	19
—	•	••	•••	••••

Maya cities developed ~750 BC, and by 500 BC these cities possessed monumental architecture. A vigesimal (/vi'dʒesiməl/) or base-20 (base-score) numeral system

400s			
20s	•	•	••
1s	•••	••••	—
	33	429	5125

$1 \times (20^1) + 13$
 $20 + 13 = 33$



RADIX-R NUMBER SYSTEM

aka. Base-r Number System

Hossein's number system is not a Radix-r number system!

Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1}r^{n-1} + d_{n-2}r^{n-2} + \dots + d_i r^i + \dots + d_2 r^2 + d_1 r^1 + d_0 r^0)_{10}$$

where:

r = radix (base)

d_i = digit at position i , $0 \leq d_i \leq r - 1$

r^i = weight (significance) of position i

n = number of digits in N

Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + d_{n-2} r^{n-2} + \dots + d_i r^i + \dots + d_2 r^2 + d_1 r^1 + d_0 r^0)$$

where:

r = radix (base)

d_i = digit at position i , $0 \leq d_i \leq r - 1$

r^i = weight (significance) of position i

n = number of digits in N

Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

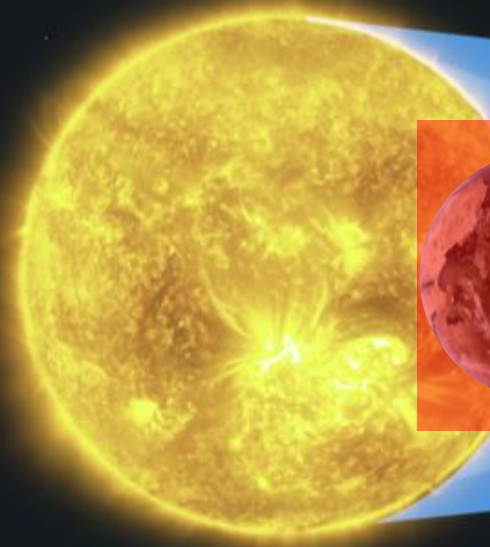
Min	$= (0_{n-1}0_{n-2} \cdots 0_10_0)_r$
Max	$= ((r-1)_{n-1}(r-1)_{n-2} \cdots (r-1)_1(r-1)_0)_r$
Unit	$= (0_{n-1}0_{n-2} \cdots 0_11_0)_r$

$= (0)_{10}$
$= (\cancel{r^n - 1})_{10}$
$= (1)_{10}$

where:

- r = radix (base)
- r^i = weight of position i
- n = number of digits in N

SUN



MOON

Earth's orbit

Moon's orbit

Umbra

Penumbra

Base-2

Base-6

Base-3

$(101)_2$

100

11

10

1

0

5

4

3

2

1

0

$(12)_3$

$(11)_3$

$(10)_3$

2

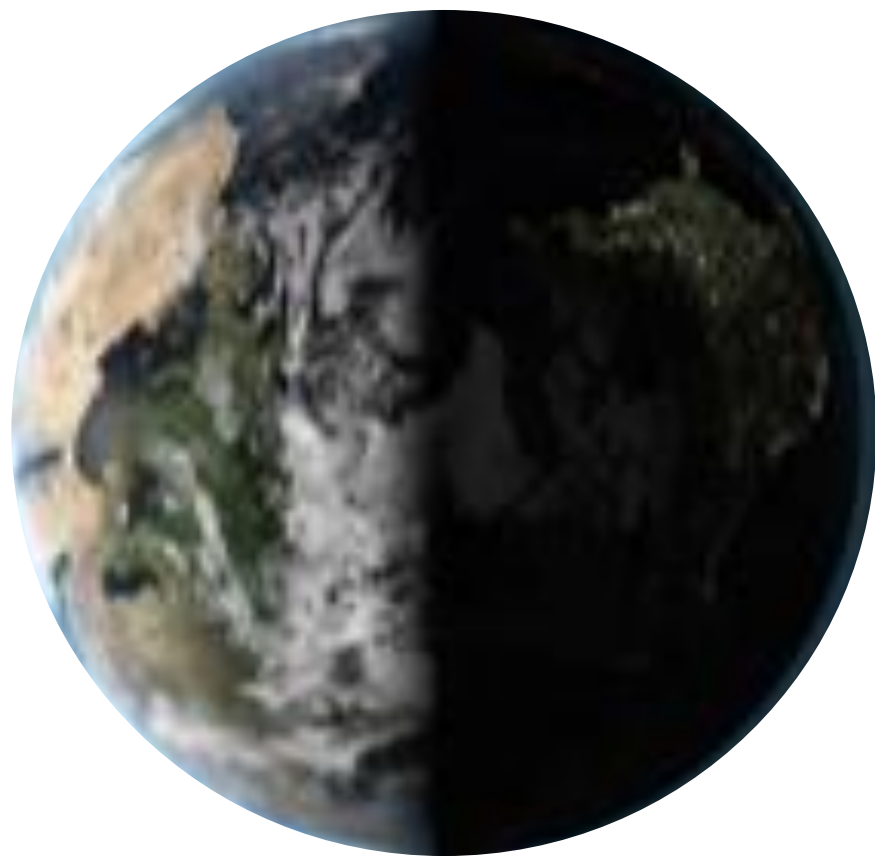
1

0

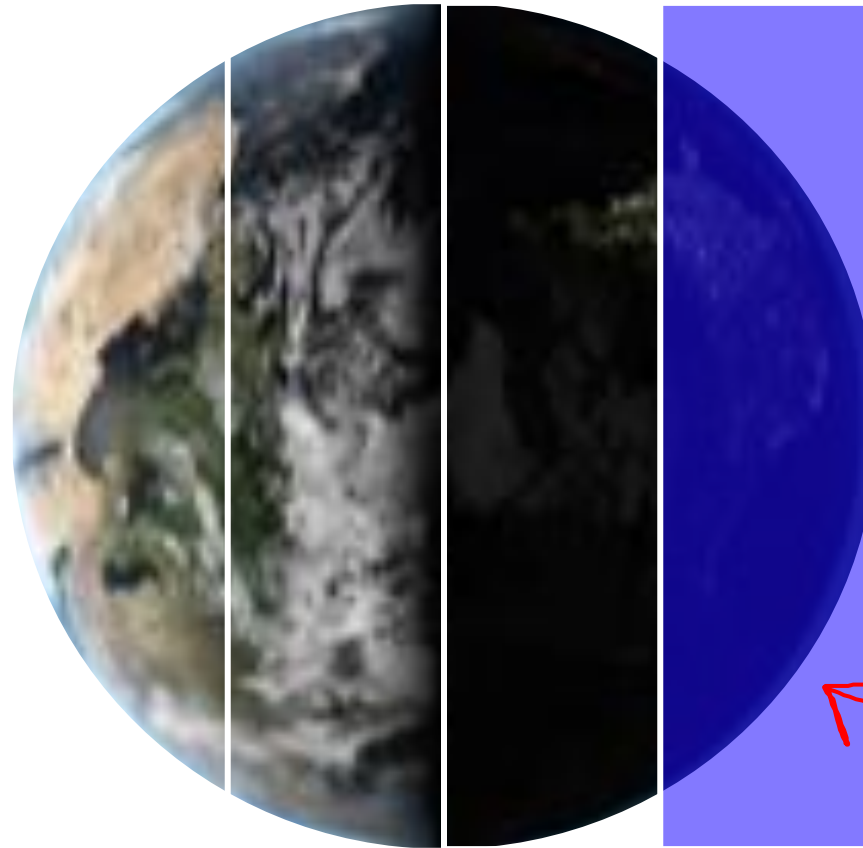
Moon's Distance to Sun: $(10.?)_4 = (4.5)_{10}$



FRACTION

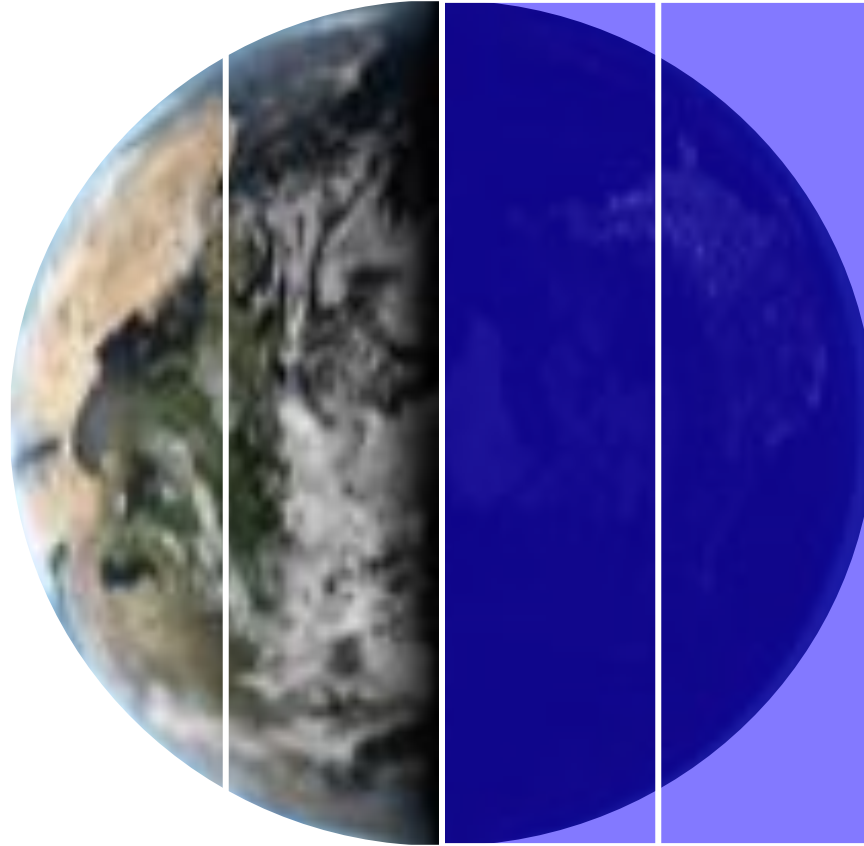


1 Earth

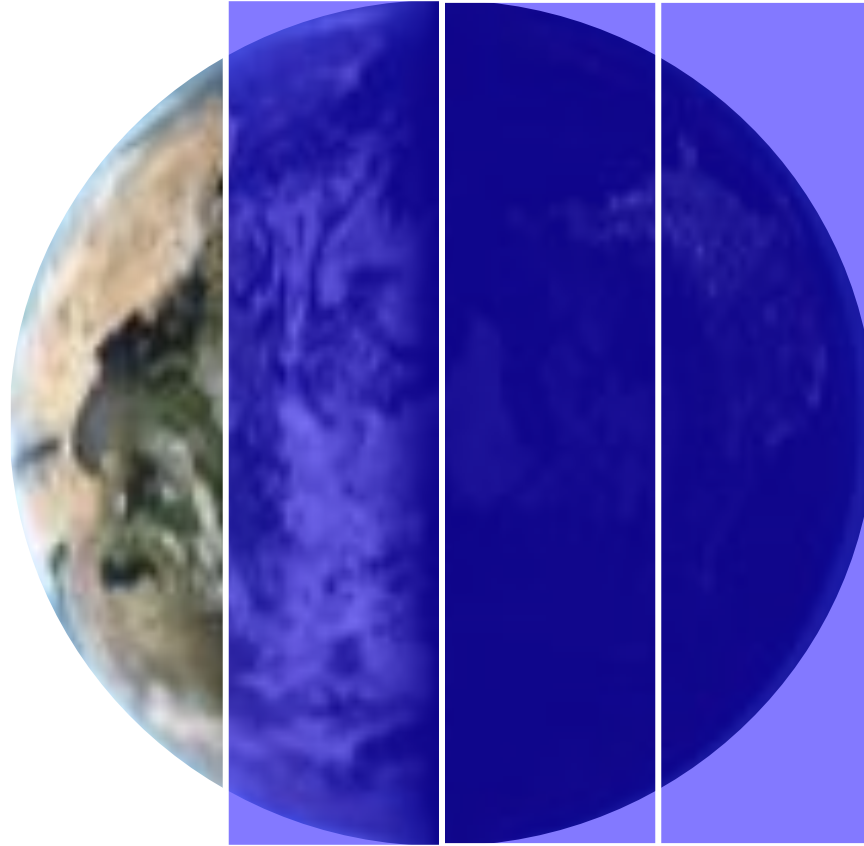


Fraction Point

$$\text{Radix-}\underline{4} \text{ (Base-}\underline{4}\text{)} = 1/\underline{4} \text{ Earth} = \underline{4}^{-1} \text{ Earth} = (\underline{0}\underline{1})_4$$



$$\text{Radix-4 (Base-4)} = \underline{2} \times \underline{1/4} \text{ Earth} = 2 \times \underline{4^{-1}} \text{ Earth} = (. \underline{2})_4$$



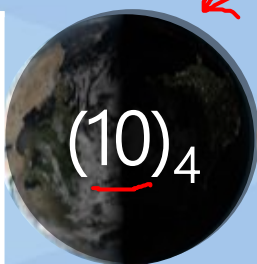
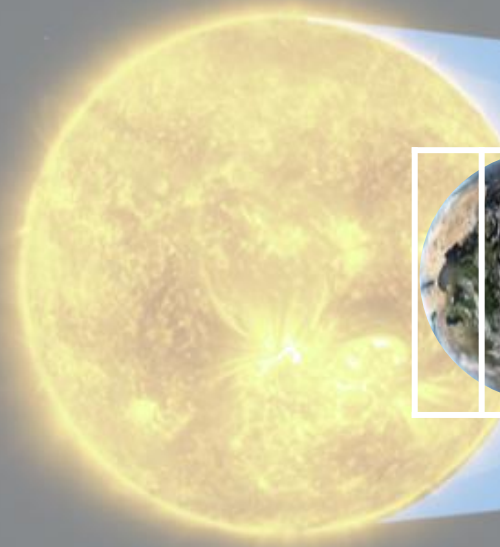
$$\text{Radix-4 (Base-4)} = \underline{3} \times \frac{1}{4} \text{ Earth} = 3 \times \textcircled{4^{-1}} \text{ Earth} = (\underline{.3})_4$$

$\frac{1}{4} = 4^{-1}$
↓



$$\text{Radix-4 (Base-4)} = \underline{4} \times \underline{1/4} \text{ Earth} = 4 \times 4^{-1} \text{ Earth} = \underline{1}_4$$

SUN



MOON

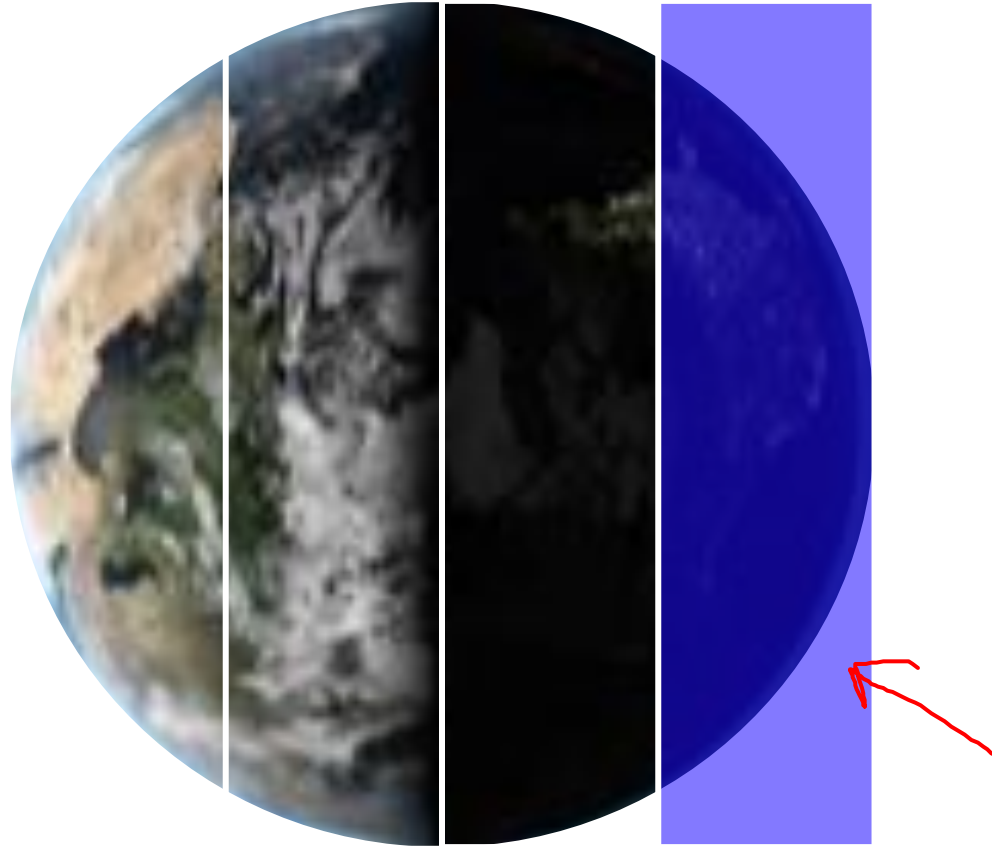
$2 \times \frac{1}{4}$

Umbra

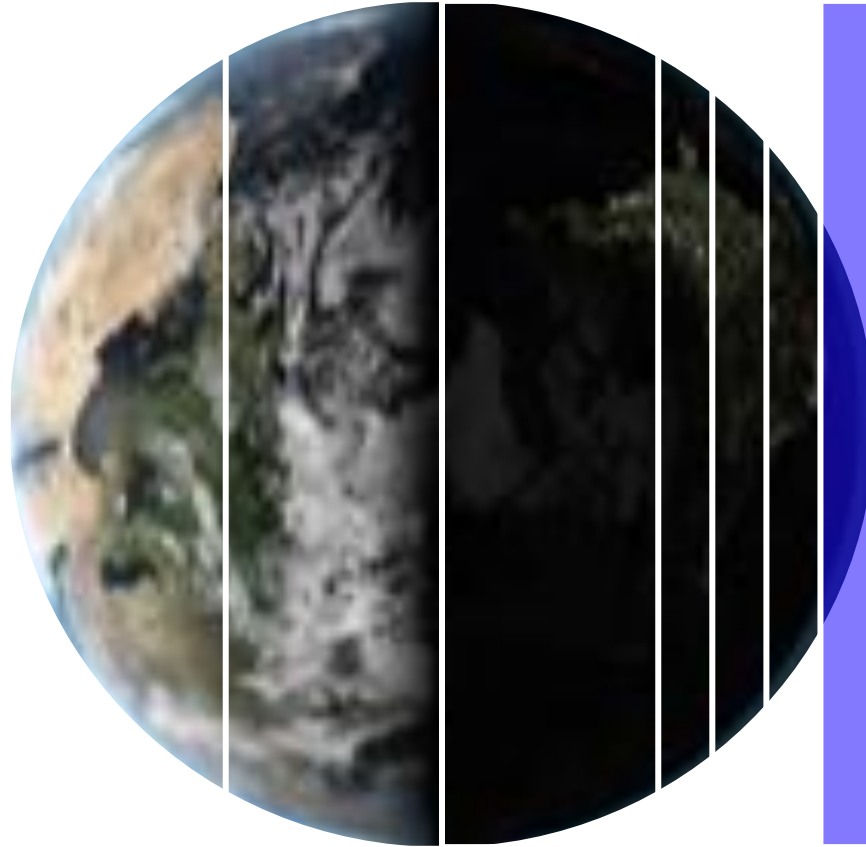
Penumbra

Moon's Distance to Sun in Radix-4: $(10 \cdot 2)_4$

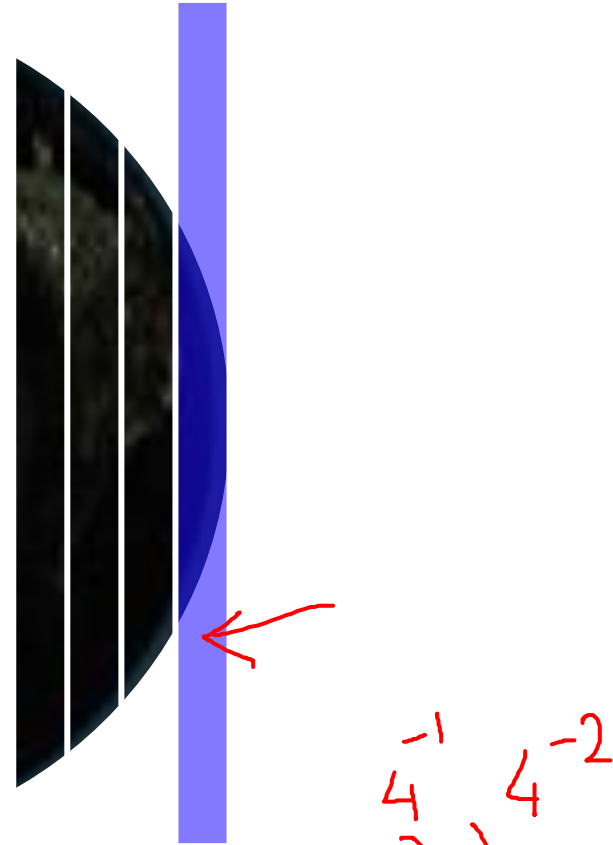
MORE PRECISION



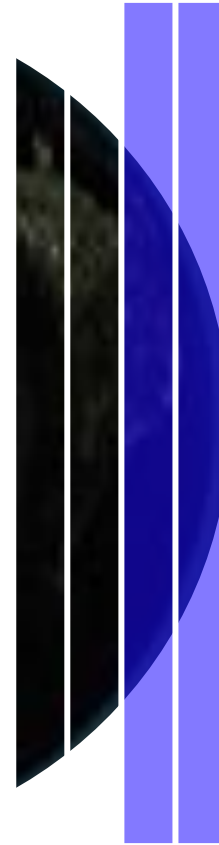
Radix-4 (Base-4) = $\frac{1}{4}$ Earth = 4^{-1} Earth = $(.1)_4$



Radix-4 (Base-4) = $(1/4)/4$ Earth = $1/16$ Earth = 4^{-2} Earth



$$\text{Radix-4 (Base-4)} = \textcircled{1}/16 = 4^{-2} \text{ Earth} = \textcircled{.01}_4$$



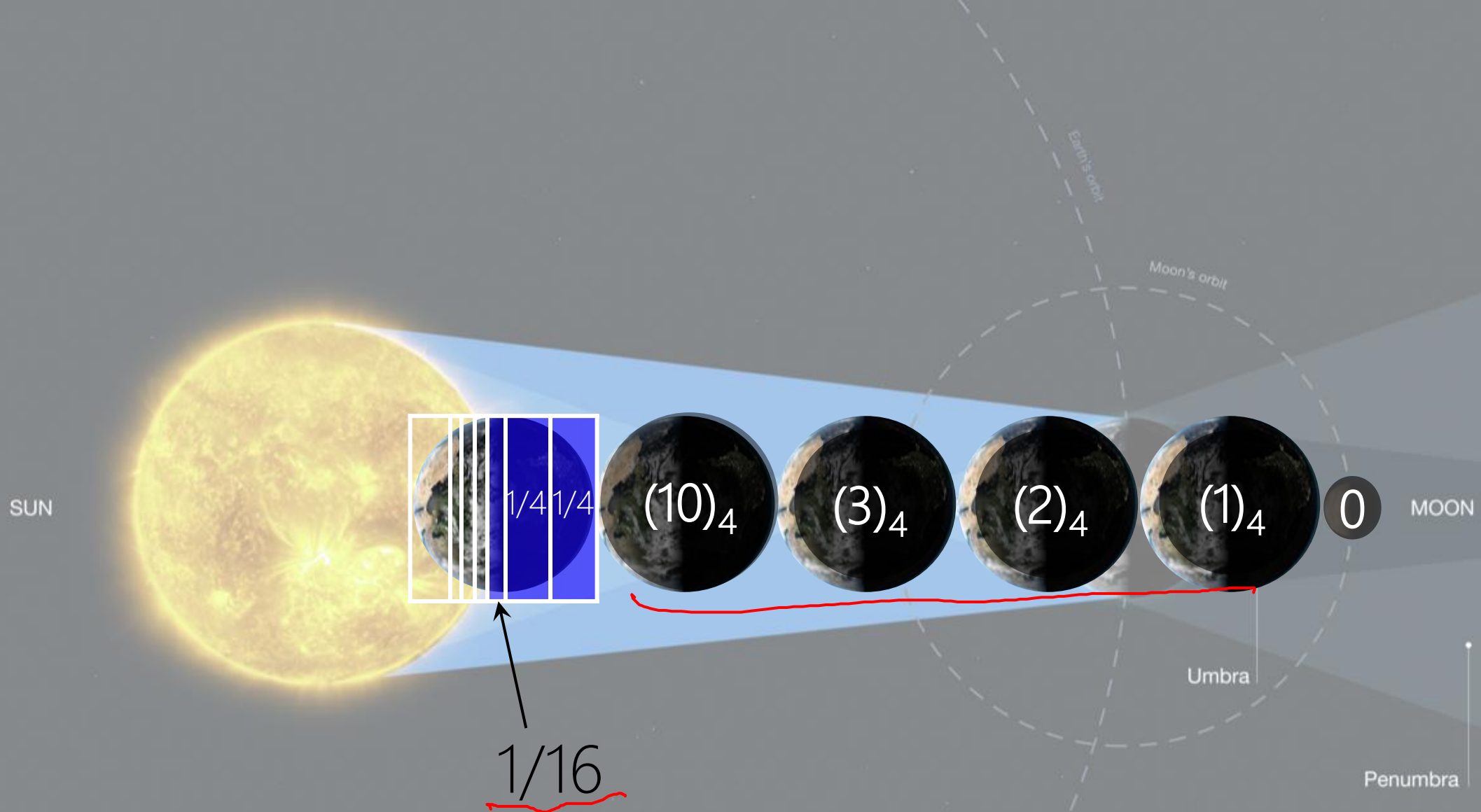
$$\text{Radix-4 (Base-4)} = 2 \times 1/16 = 2 \times 4^{-2} \text{ Earth} = (.02)_4$$



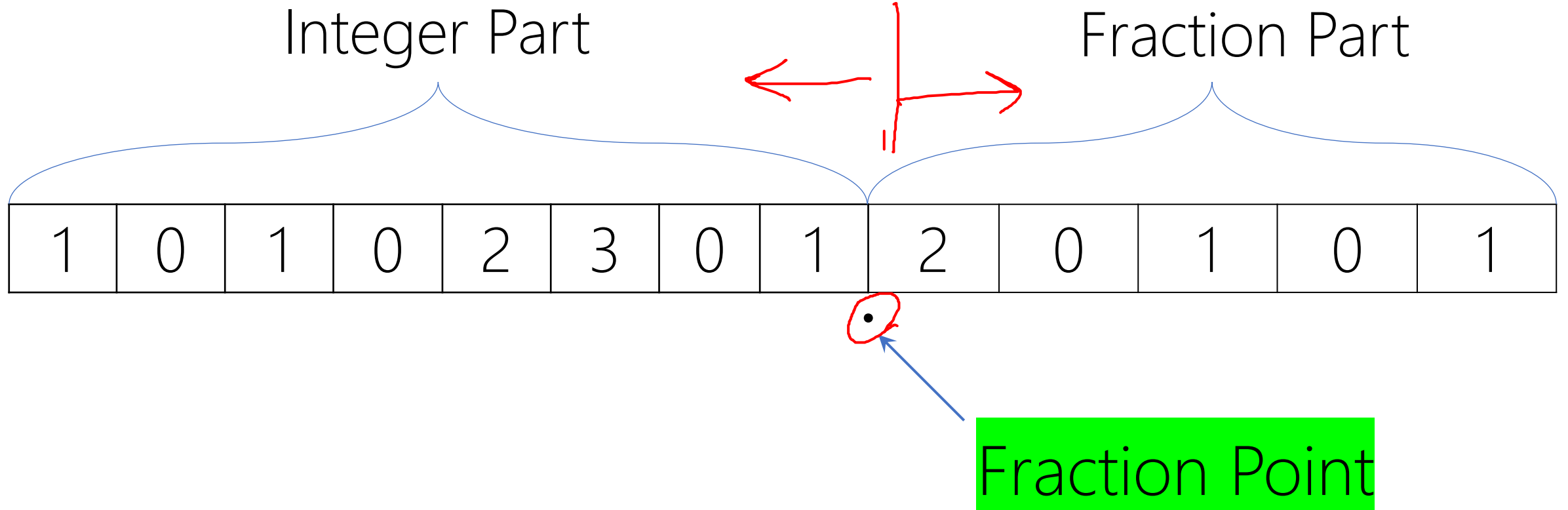
$$\text{Radix-4 (Base-4)} = 3 \times 1/16 = 3 \times 4^{-2} \text{ Earth} = (.03)_4$$



$$\text{Radix-4 (Base-4)} = 4 \times 1/16 = 4 \times 4^{-2} \text{ Earth} = (.1)_4$$

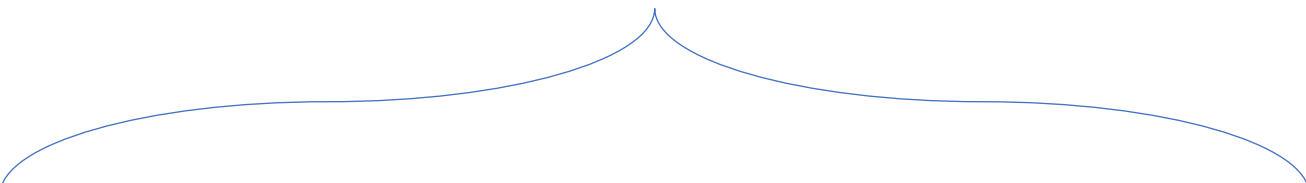


Moon's Distance to Sun in Radix-4: $(10 \cdot 21)_4$



We don't waste a position for the fraction point.
It's location is defined already.

Integer Part

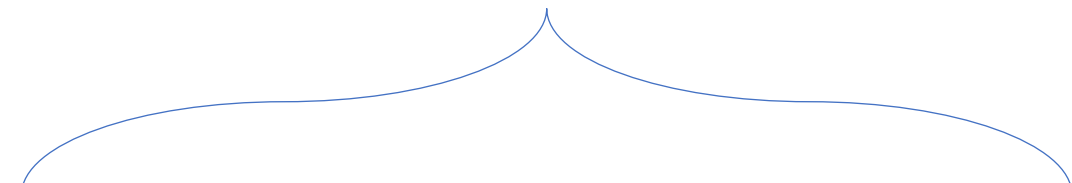


4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1



.

Fraction Part



4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1



Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 $\times 16,384$	0	1 $\times 1,024$	0	2 $\times 64$	3 $\times 16$	0	1

\times

.

Σ

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1,024}$

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 ×16,384	0	1 ×1,024	0	2 ×64	3 ×16	0	1
				17,584			



Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1,024}$

×
.
Σ
.

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 $\times 16,384$	0	1 $\times 1,024$	0	2 $\times 64$	3 $\times 16$	0	1
				17,584			

\times
 \cdot
 Σ
 \cdot

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1,024}$
5166015625				



Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1}r^{n-1} + \dots + d_0r^0 \cdot d_{-1}r^{-1} + d_{-2}r^{-2} + \dots + d_{-m}r^{-m})_{10}$$

where:

r = radix (base)

d_i = digit at position i , $0 \leq d_i \leq r - 1$

r^i = weight of position i

n = number of digits in integer part of N

m = number of digits in fraction part of N

Fraction Point

Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

$$\begin{aligned}
 \text{Min} &= (0_{n-1} \cdots 0_1 0_0 . 0_{-1} 0_{-2} \cdots 0_{-m-1} 0_{-m})_r \\
 \text{Max} &= ((r-1)_{n-1} \cdots (r-1)_0 . (r-1)_{-1} (r-1)_{-2} \cdots (r-1)_{-m-1} (r-1)_{-m})_r = (r^n - 1 . ?)_{10} \\
 \text{Unit} &= (0_{n-1} \cdots 0_1 0_0 . 0_{-1} 0_{-2} \cdots 0_{-m-1} 1_{-m})_r = (r^{-m})_{10}
 \end{aligned}$$

where:

r = radix (base)

r^i = weight of position i

n = number of digits in integer part of N

m = number of digits in fraction part of N

Lecture Assignment

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

PRACTICE RADIX-2

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1 \cdot 2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 = 0.5$
0	0	0	0	.	1	0	1	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1 \cdot 2^0 + 0 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Handwritten notes in red ink:

Top row: 0 0 0 0 . 0 0 0 1

Second row: 0 0 0 0 . 0 0 1

Third row: 0 0 0 0 . 0 1 0

Fourth row: 0 0 0 0 . 0 1 1

Fifth row: 0 0 0 0 . 1 0 0

Sixth row: 0 0 0 0 . 1 0 1

Seventh row: 0 0 0 0 . 1 1 0

Eighth row: 0 0 0 0 . 1 1 1

Ninth row: 0 0 0 1 . 0 0 0

Tenth row: 0 0 0 1 . 0 0 1

Eleventh row: 0 0 0 1 . 0 1 0

Twelfth row: 0 0 0 1 . 0 1 1

Thirteenth row: 0 0 0 1 . 1 0 0

Fourteenth row: 0 0 0 1 . 1 0 1

Fifteenth row: 0 0 0 1 . 1 1 0

Sixteenth row: 0 0 0 1 . 1 1 1

Seventeenth row: 0 0 1 0 . 0 0 0

Eighteenth row: 0 0 1 0 . 0 0 1

Nineteenth row: 0 0 1 0 . 0 1 0

Twentieth row: 0 0 1 0 . 0 1 1

Twenty-first row: 0 0 1 0 . 1 0 0

Twenty-second row: 0 0 1 0 . 1 0 1

Twenty-third row: 0 0 1 0 . 1 1 0

Twenty-fourth row: 0 0 1 0 . 1 1 1

Twenty-fifth row: 0 0 1 1 . 0 0 0

Twenty-sixth row: 0 0 1 1 . 0 0 1

Twenty-seventh row: 0 0 1 1 . 0 1 0

Twenty-eighth row: 0 0 1 1 . 0 1 1

Twenty-ninth row: 0 0 1 1 . 1 0 0

Thirtieth row: 0 0 1 1 . 1 0 1

Thirty-first row: 0 0 1 1 . 1 1 0

Thirty-second row: 0 0 1 1 . 1 1 1

Thirty-third row: 0 0 1 1 . 1 1 1

Thirty-fourth row: 0 0 1 1 . 1 1 1

Thirty-fifth row: 0 0 1 1 . 1 1 1

Thirty-sixth row: 0 0 1 1 . 1 1 1

Thirty-seventh row: 0 0 1 1 . 1 1 1

Thirty-eighth row: 0 0 1 1 . 1 1 1

Thirty-ninth row: 0 0 1 1 . 1 1 1

Fortieth row: 0 0 1 1 . 1 1 1

Forty-first row: 0 0 1 1 . 1 1 1

Forty-second row: 0 0 1 1 . 1 1 1

Forty-third row: 0 0 1 1 . 1 1 1

Forty-fourth row: 0 0 1 1 . 1 1 1

Forty-fifth row: 0 0 1 1 . 1 1 1

Forty-sixth row: 0 0 1 1 . 1 1 1

Forty-seventh row: 0 0 1 1 . 1 1 1

Forty-eighth row: 0 0 1 1 . 1 1 1

Forty-ninth row: 0 0 1 1 . 1 1 1

Fiftieth row: 0 0 1 1 . 1 1 1

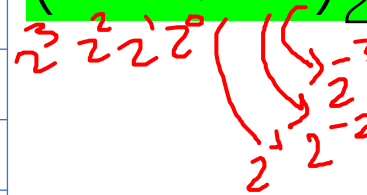
Radix-2									
Integer (n=4)					Fraction (m=3)			Radix-10	
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	$1 \cdot 2^{-3} = 1/8 = 0.125$	
0	0	0	0	.	0	1	0	$1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/4 = 0.25$	
0	0	0	0	.	0	1	1	$1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$	
0	0	0	0	.	1	0	0	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 = 0.5$	
0	0	0	0	.	1	0	1	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$	
0	0	0	0	.	1	1	0	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$	
0	0	0	0	.	1	1	1	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$	
0	0	0	1	.	0	0	0	$1 \cdot 2^0 + 0 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1$	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

What is the max in this system with these spaces?

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

What is the **max** in this system with these spaces?

$(1111.111)_2 = (15.875)_{10}$



Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$	
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$	
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$	
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1/2 = 0.5$	
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$	
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$	
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$	
0	0	0	1	.	0	0	0	$1*2^0 + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

$$\begin{array}{r}
 (0001.000)_2 \\
 + (0000.001)_2 \leftarrow \\
 \hline
 0001.001
 \end{array}$$

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

Radix-2				Radix-10			
Integer (n=4)				Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}
0	0	0	0	.	0	0	0
0	0	0	0	.	0	0	1
0	0	0	0	.	0	1	0
0	0	0	0	.	0	1	1
0	0	0	0	.	1	0	0
0	0	0	0	.	1	0	1
0	0	0	0	.	1	1	0
0	0	0	0	.	1	1	1
0	0	0	1	.	0	0	0
0	0	0	1	.	0	0	1
0	0	0	1	.	0	1	0
0	0	0	1	.	0	1	1
0	0	0	1	.	1	0	0
0	0	0	1	.	1	0	1
0	0	0	1	.	1	1	0
0	0	0	1	.	1	1	1
0	0	1	0	.	0	0	0

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?
A. More precision.

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

- Solution?
- A. More precision.
 - A. More fraction positions.
 - A. More in m!

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	$1 \cdot 2^{-3} = 1/8 = 0.125$	
0	0	0	0	.	0	1	0	$1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/4 = 0.25$	
0	0	0	0	.	0	1	1	$1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$	
0	0	0	0	.	1	0	0	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 = 0.5$	
0	0	0	0	.	1	0	1	$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$	
0	0	0	0	.	1	1	0	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$	
0	0	0	0	.	1	1	1	$1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$	
0	0	0	1	.	0	0	0	$1 \cdot 2^0 + 0 \cdot 2^{-1} + 0 \cdot 2^{-2} + 0 \cdot 2^{-3} = 1$	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

$0.02 \rightarrow 0.01$

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

A. More in m! **How much?**

Radix-2									Radix-10
Integer (n=4)					Fraction (m=3)				
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?
B. Find the closest number

Radix-2									
Integer (n=4)						Fraction (m=3)			Radix-10
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125	
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25	
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375	
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1/2 = 0.5	
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625	
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75	
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875	
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1	
0	0	0	1	.	0	0	1	1.125	
0	0	0	1	.	0	1	0	1.25	
0	0	0	1	.	0	1	1	1.375	
0	0	0	1	.	1	0	0	1.5	
0	0	0	1	.	1	0	1	1.625	
0	0	0	1	.	1	1	0	1.75	
0	0	0	1	.	1	1	1	1.875	
0	0	1	0	.	0	0	0	2	

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number
 $(1.000)_2 = (1)_{10} \Rightarrow \text{Error} = (0.02)_{10}$
 $(1.001)_2 = (1.125)_{10} \Rightarrow \text{Error} = (0.105)_{10}$

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2 ³	2 ²	2 ¹	2 ⁰	.	2 ⁻¹	2 ⁻²	2 ⁻³	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	1*2 ⁻³ = 1/8 = 0.125
0	0	0	0	.	0	1	0	1*2 ⁻² + 0*2 ⁻³ = 1/4 = 0.25
0	0	0	0	.	0	1	1	1*2 ⁻² + 1*2 ⁻³ = 1/4 + 1/8 = 3/8 = 0.375
0	0	0	0	.	1	0	0	1*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = ½ = 0.5
0	0	0	0	.	1	0	1	1*2 ⁻¹ + 0*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/8 = 5/8 = 0.625
0	0	0	0	.	1	1	0	1*2 ⁻¹ + 1*2 ⁻² + 0*2 ⁻³ = 1/2 + 1/4 = 3/4 = 0.75
0	0	0	0	.	1	1	1	1*2 ⁻¹ + 1*2 ⁻² + 1*2 ⁻³ = 1/2 + 1/4 + 1/8 = 0.875
0	0	0	1	.	0	0	0	1*2 ⁰ + 0*2 ⁻¹ + 0*2 ⁻² + 0*2 ⁻³ = 1
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number ~~$(1.02)_{10}$~~ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?
B. Find the closest number
 $(1.000)_2 = (1)_{10} \Rightarrow \text{Error} = 0.02$
 $(1.001)_2 = (1.125)_{10} \Rightarrow \text{Error} = 0.105$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

PRACTICE RADIX-4

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	<u>0</u>
0	0	0	0	.	0	0	<u>1</u>	$1 \cdot 4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2 \cdot 4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3 \cdot 4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1 \cdot 4^{-2} + 0 \cdot 4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1 \cdot 4^{-2} + 1 \cdot 4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1 \cdot 4^{-2} + 2 \cdot 4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1 \cdot 4^{-2} + 3 \cdot 4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2 \cdot 4^{-2} + 0 \cdot 4^{-2} = 2/16 = 0.125$
...								
0	0	0	0	.	3	3	3	$3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 3 \cdot 4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
...								
3	3	3	3	.	3	3	0	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 0 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	1	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 1 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	2	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 2 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	3	<u>255.984375</u>

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1 \cdot 4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2 \cdot 4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3 \cdot 4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1 \cdot 4^{-2} + 0 \cdot 4^{-3} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1 \cdot 4^{-2} + 1 \cdot 4^{-3} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1 \cdot 4^{-2} + 2 \cdot 4^{-3} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1 \cdot 4^{-2} + 3 \cdot 4^{-3} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2 \cdot 4^{-2} + 0 \cdot 4^{-3} = 2/16 = 0.125$
...								
0	0	0	0	.	3	3	3	$3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 3 \cdot 4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
...								
3	3	3	3	.	3	3	0	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 0 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	1	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 1 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	2	$3 \cdot 4^3 + 3 \cdot 4^2 + 3 \cdot 4^1 + 3 \cdot 4^0 + 3 \cdot 4^{-1} + 3 \cdot 4^{-2} + 2 \cdot 4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

Radix-4								Radix-10
Integer (n=4)				Fraction (m=3)				
4 ³	4 ²	4 ¹	4 ⁰	.	4 ⁻¹	4 ⁻²	4 ⁻³	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	1*4 ⁻³ = 1/64 = 0.015625
0	0	0	0	.	0	0	2	2*4 ⁻³ = 2/64 = 0.03125
0	0	0	0	.	0	0	3	3*4 ⁻³ = 3/64 = 0.046875
0	0	0	0	.	0	1	0	1*4 ⁻² + 0*4 ⁻² = 1/16 = 0.0625
0	0	0	0	.	0	1	1	1*4 ⁻² + 1*4 ⁻² = 1/16 + 1/64 = 0.078125
0	0	0	0	.	0	1	2	1*4 ⁻² + 2*4 ⁻² = 1/16 + 2/64 = 0.09375
0	0	0	0	.	0	1	3	1*4 ⁻² + 3*4 ⁻² = 1/16 + 3/64 = 0.109375
0	0	0	0	.	0	2	0	2*4 ⁻² + 0*4 ⁻² = 2/16 = 0.125
...								
0	0	0	0	.	3	3	3	3*4 ⁻¹ + 3*4 ⁻² + 3*4 ⁻³ = 0.984375
0	0	0	1	.	0	0	0	1
...								
3	3	3	3	.	3	3	0	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 0*4 ⁻³ = ?
3	3	3	3	.	3	3	1	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 1*4 ⁻³ = ?
3	3	3	3	.	3	3	2	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 2*4 ⁻³ = ?
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4 ³	4 ²	4 ¹	4 ⁰	.	4 ⁻¹	4 ⁻²	4 ⁻³	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	1*4 ⁻³ = 1/64 = 0.015625
0	0	0	0	.	0	0	2	2*4 ⁻³ = 2/64 = 0.03125
0	0	0	0	.	0	0	3	3*4 ⁻³ = 3/64 = 0.046875
0	0	0	0	.	0	1	0	1*4 ⁻² + 0*4 ⁻² = 1/16 = 0.0625
0	0	0	0	.	0	1	1	1*4 ⁻² + 1*4 ⁻² = 1/16 + 1/64 = 0.078125
0	0	0	0	.	0	1	2	1*4 ⁻² + 2*4 ⁻² = 1/16 + 2/64 = 0.09375
0	0	0	0	.	0	1	3	1*4 ⁻² + 3*4 ⁻² = 1/16 + 3/64 = 0.109375
0	0	0	0	.	0	2	0	2*4 ⁻² + 0*4 ⁻² = 2/16 = 0.125
								...
0	0	0	0	.	3	3	3	3*4 ⁻¹ + 3*4 ⁻² + 3*4 ⁻³ = 0.984375
0	0	0	1	.	0	0	0	1
								...
3	3	3	3	.	3	3	0	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 0*4 ⁻³ = ?
3	3	3	3	.	3	3	1	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 1*4 ⁻³ = ?
3	3	3	3	.	3	3	2	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 2*4 ⁻³ = ?
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

Radix-4									
Integer (n=4)					Fraction (m=3)			Radix-10	
4 ³	4 ²	4 ¹	4 ⁰	.	4 ⁻¹	4 ⁻²	4 ⁻³		
0	0	0	0	.	0	0	0	0	
0	0	0	0	.	0	0	1	1*4 ⁻³ = 1/64 = 0.015625	
0	0	0	0	.	0	0	2	2*4 ⁻³ = 2/64 = 0.03125	
0	0	0	0	.	0	0	3	3*4 ⁻³ = 3/64 = 0.046875	
0	0	0	0	.	0	1	0	1*4 ⁻² + 0*4 ⁻² = 1/16 = 0.0625	
0	0	0	0	.	0	1	1	1*4 ⁻² + 1*4 ⁻² = 1/16 + 1/64 = 0.078125	
0	0	0	0	.	0	1	2	1*4 ⁻² + 2*4 ⁻² = 1/16 + 2/64 = 0.09375	
0	0	0	0	.	0	1	3	1*4 ⁻² + 3*4 ⁻² = 1/16 + 3/64 = 0.109375	
0	0	0	0	.	0	2	0	2*4 ⁻² + 0*4 ⁻² = 2/16 = 0.125	
...									
0	0	0	0	.	3	3	3	3*4 ⁻¹ + 3*4 ⁻² + 3*4 ⁻³ = 0.984375	
0	0	0	1	.	0	0	0	1	
0	0	0	1	.	0	0	1	1.015625 ←	
0	0	0	1	.	0	0	2	1.03125 ←	
...									
3	3	3	3	.	3	3	2	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 2*4 ⁻³ = ?	
3	3	3	3	.	3	3	3	255.984375	

Is it possible to show the number (1.02)₁₀ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

$(1.001)_4 = (1.015625)_{10} \Rightarrow \text{Error} = 0.004375$

$(1.002)_4 = (1.03125)_{10} \Rightarrow \text{Error} = 0.01125$

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4 ³	4 ²	4 ¹	4 ⁰	.	4 ⁻¹	4 ⁻²	4 ⁻³	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	1*4 ⁻³ = 1/64 = 0.015625
0	0	0	0	.	0	0	2	2*4 ⁻³ = 2/64 = 0.03125
0	0	0	0	.	0	0	3	3*4 ⁻³ = 3/64 = 0.046875
0	0	0	0	.	0	1	0	1*4 ⁻² + 0*4 ⁻² = 1/16 = 0.0625
0	0	0	0	.	0	1	1	1*4 ⁻² + 1*4 ⁻² = 1/16 + 1/64 = 0.078125
0	0	0	0	.	0	1	2	1*4 ⁻² + 2*4 ⁻² = 1/16 + 2/64 = 0.09375
0	0	0	0	.	0	1	3	1*4 ⁻² + 3*4 ⁻² = 1/16 + 3/64 = 0.109375
0	0	0	0	.	0	2	0	2*4 ⁻² + 0*4 ⁻² = 2/16 = 0.125
...								
0	0	0	0	.	3	3	3	3*4 ⁻¹ + 3*4 ⁻² + 3*4 ⁻³ = 0.984375
0	0	0	1	.	0	0	0	1
0	0	0	1	.	0	0	1	1.015625 ←
0	0	0	1	.	0	0	2	1.03125
...								
3	3	3	3	.	3	3	2	3*4 ³ + 3*4 ² + 3*4 ¹ +3*4 ⁰ +3*4 ⁻¹ + 3*4 ⁻² + 2*4 ⁻³ = ?
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

$(1.001)_4 = (1.015625)_{10} \Rightarrow \text{Error} = 0.004375$

$(1.002)_4 = (1.03125)_{10} \Rightarrow \text{Error} = 0.01125$

A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black space. Two horizontal blue lines are positioned above and below the text.

PRACTICE RADIX-[8,10,16]

At Home

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines are positioned above and below the central text.

CONVERSION

From Base-r to Base-r'