Summary: Number System

1. Information should be discrete in order to be analyzed or processed by machines
2. Continuous → Discrete: Continues entities or quantities should be broken into discrete units like distance to meters, time to hours, image to pixels.
3. Computer systems are members of the Discrete Systems category
4. Quantization: Continuous → Discrete → Digits/Numbers/Symbols
5. Base-r number system has r symbols from 0 to r-1
6. Base-r number system has positions with significance based on the powers of r
7. Base-r = Radix-r
8. Base-2 → Binary System
9. Base-4 → Quaternary System
10. Base-8 → Octal
11. Base-10 → Decimal
12. Base-16 → Hexadecimal
13. Base-64 number system has 64 symbols but starts from 'A' and ends at '/'
14. Any base-r number → base-10: multiply each digit to the significant of each position
    a. Integer part: increasing powers of r from 0 to n-1
    b. Fraction part: decreasing powers of r from -1 to -m.
15. The min in base-r is 00…000. 00…000
16. The min in base-64 is A…AAA.AA…AAA as A has the value of 0
17. The max in base-r with n integer positions and m fraction positions is r^n-1.1-r^{-m}
18. Hossein's number system is not a base-r (radix-r) system for the positions do not have significance!
19. Given an integer number N in base-10, we need $\log_r(N+1)$ integer positions to show it in base-r
20. The min unit of precision without fraction part is 1 in any base
21. The min unit of precision with m fraction positions in base-r is r^{-m}, e.g., in base-2 with 3 positions is 1/8 = 0.125
22. When converting numbers with fraction parts, there will be more fraction parts, sometimes infinite. Given same or smaller number of fractions, errors happen. We like to minimize the error
23. Base-r → Base-r': Base-r → Base-10 → Base-r'
24. Base-10 → Base-r':
    a. Integer part: repeating division by r on new quotients, put the remainders in reverse
    b. Fraction part: repeating multiplications by r on new fraction parts, put the integer parts in order
25. Addition in base-r
    a. Without negative numbers → normal add X+Y:
        i. simply add each digit as we do in base-10. Create carry if the result is equal or greater than r and put the remainder
    b. With negative number:
        i. Signed-magnitude:
            1. +X+(+Y): first the sign is +, then normal add. *Check for overflow: if there is last carry*
            2. +X+(-Y): this is equal to X-Y.
            3. -X+(+Y): this is equal to Y-X
            4. -X+(-Y): this is equal to –(X+Y). So, the sign is -, then normal add
        ii. Signed-Radix-complement
            1. X+Y: normal add, if carry ignore it. *Check for overflow:*
            2. *Check for overflow:*
                a. *if X and Y were positive but the result is negative*
                b. *if X and Y were negative but the result is positive*
26. Subtraction in base-r
    a. Without negative numbers → normal subtraction X-Y:
        i. simply subtract each digit as we do in base-10. Borrow if the subtraction is not possible (the first digit is smaller than the second). If there is a last borrow, X < Y. Another subtraction with the last borrow is needed to obtain the correct negative number. Eg, 2-9=10+2-9=3 → 10-3=7 → -7
    b. With negative number:
        i. Signed-magnitude:
            1. +X-(+Y): this is equal to X-Y. Normal subtraction. If last borrow, sign position nonzero (-)
            2. +X-(-Y): this is equal to X+Y. Normal addition. Check for overflow

3. -X-(+Y): this is equal to –(X+Y). Sign is nonzero. Normal addition. Check for overflow
4. -X-(-Y): this is equal to Y-X.

ii. Signed-Radix-complement
1. X-Y: X+(r's comp. (Y)): normal addition, if carry ignore it. *Check for overflow:*
a. *if X and* (r's comp. (Y)) *were positive but the result of addition is negative*
b. *if X and* (r's comp. (Y)) *were negative but the result of addition is positive*

27. Diminished-radix-complement in base-r:
a. $(r^n-1)-N$
b. Subtract each digit from r
c. In base-2: NOT each digit
28. Radix-complement in base-r:
a. $(r^n)-N$
b. (Subtract each digit from r) and then + 1
c. Diminished-radix-comp. + 1
d. In base-2: NOT each digit + 1
e. In base-2: move from first position to the last till you see the first one, thereafter NOT the remaining digits
29. Given n positions in base-r:
a. Signed-magnitude:
i. Max: $+r^{(n-1)}-1$
ii. Min: $- r^{(n-1)}-1$
iii. +0, -0
iv. Positive: last position== 0
v. Negative: last position !=0
b. Signed-Radix-Complement:
i. Max: $+ r^{(n-1)}-1$
ii. Min: $- r^{(n-1)}-1 + 1 = - r^{(n-1)}$
iii. 0
iv. Positive: if the number is less or equal Max/2
1. Base-2: less or equal to 01111…111
2. Base-3: less or equal to 11111…111
3. Base-4: less or equal to 13333…333
4. Base-5: less or equal to 22222…222
5. If r is odd: all digits of (r-1)/2
6. If r is even: the significant digit (r-1)/2, all other digits (r-1)
v. Negative: if the number is greater than Max/2
1. In base-2: greater or equal to 10000…000 (looks like signed-magnitude not the same though)
2. Look above
30.