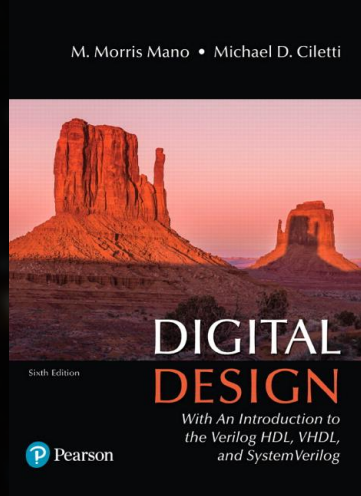




LEC03 & LAB03

Lectures >> Lec03: Signed Numbers Arithmetic
Labs >> Lab03: Number System (Complements)



Chapter 1

Digital Systems and Binary Numbers

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SUBTRACTION

Example I, Base-10

	+10	+10	
-1	-1	-1	+10
—	0	2	1
Base-10	4	3	2
	5	8	9

.

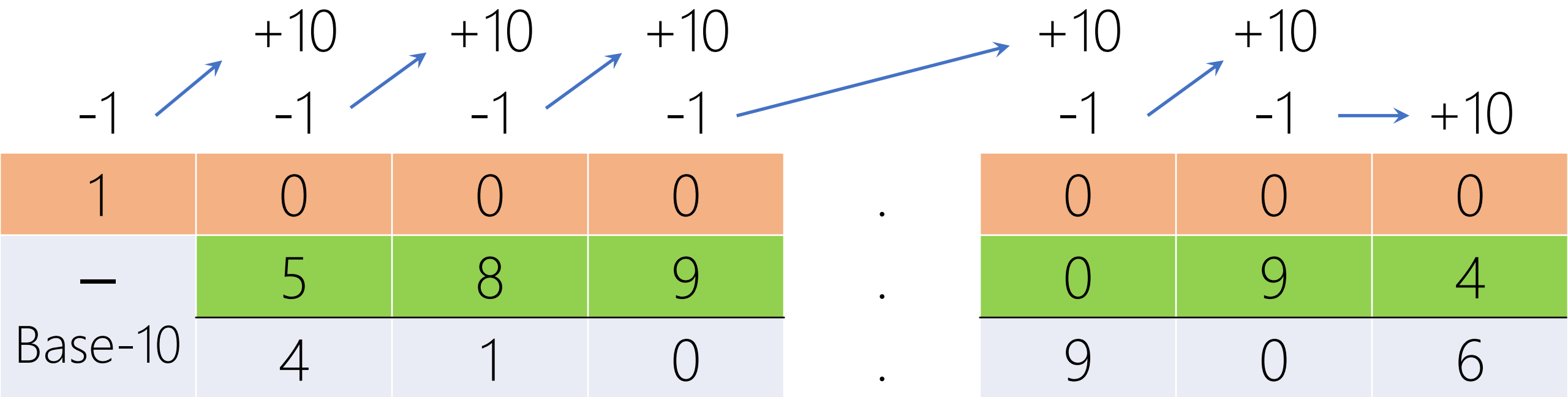
.

.

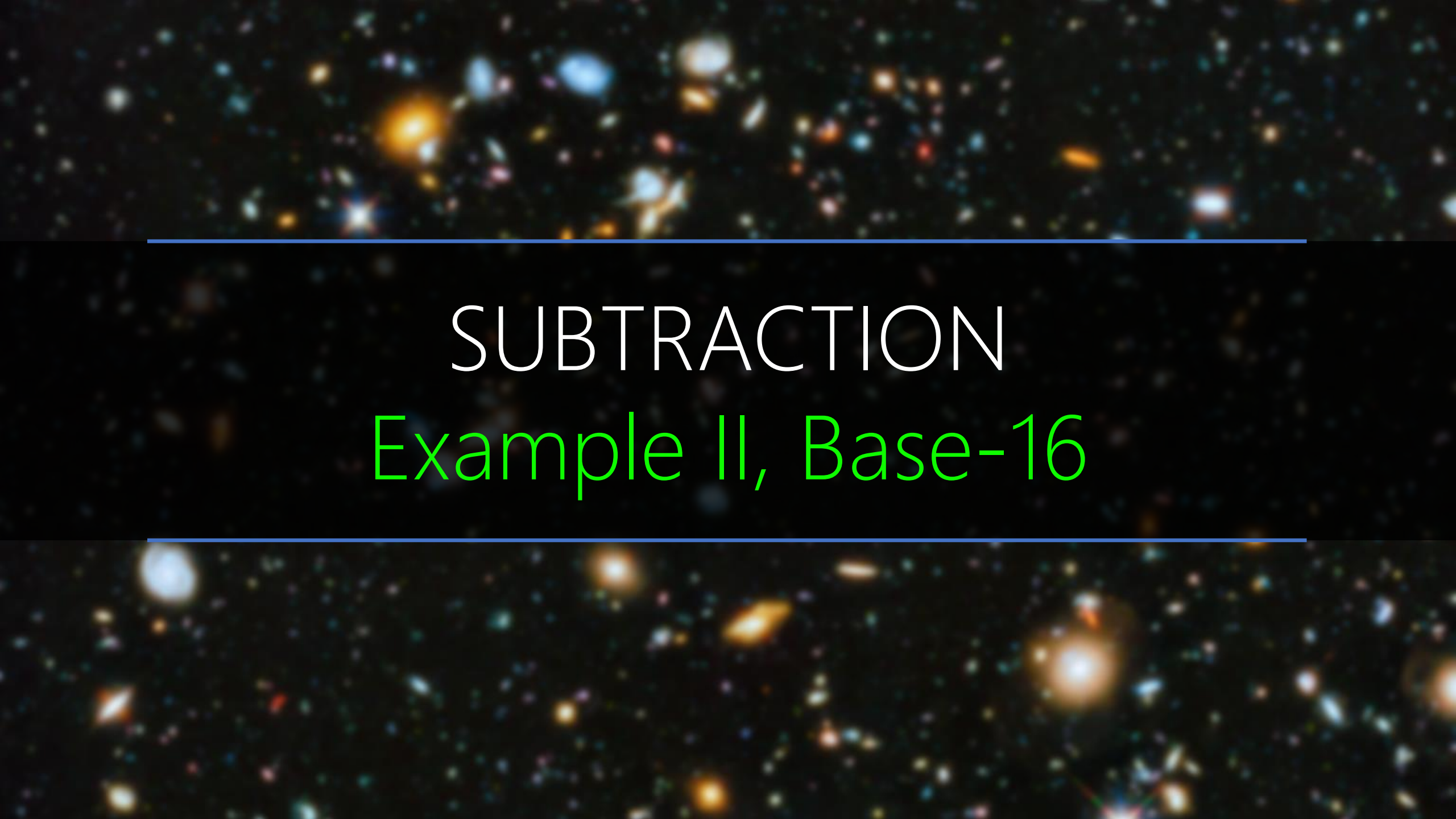
-1	+10	
1	5	4
0	6	0
0	9	4

021.154 < 432.060

Last Borrow → Negative Result



$$\begin{aligned}
 &= (021.154)_{10} - (432.060)_{10} = (021.154)_{10} + (1000.000)_{10} - (1000.000)_{10} - (432.060)_{10} \\
 &= - (1000.000)_{10} + (589.094)_{10} = - [(1000.000)_{10} - (589.094)_{10}] \\
 &= - (410.906)_{10}
 \end{aligned}$$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SUBTRACTION

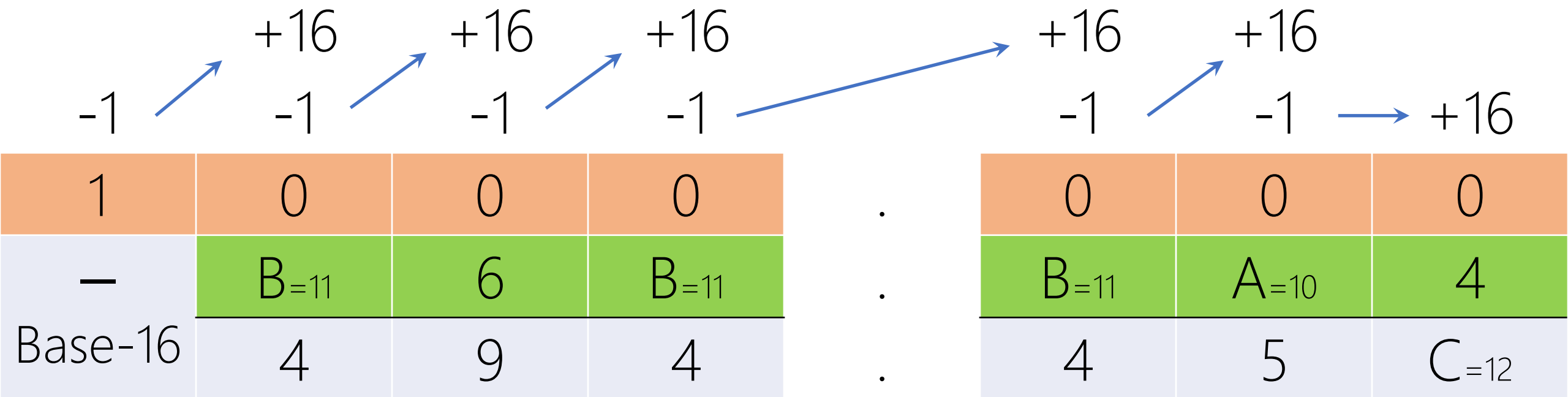
Example II, Base-16

	+16	+16	
-1	-1	-1	+16
—	0	2	A
Base-16	4	B	F
	B	6	B

	-1	+16	
.	E	5	4
.	2	B	0
.	B	A	4

02A.E54 < 4BF.2B0

Last Borrow → Negative Result



$$\begin{aligned}
 &= (2A.E54)_{16} - (4BF.2B0)_{16} = (2A.E54)_{16} + (1000.000)_{16} - (1000.000)_{16} - (4BF.2B0)_{16} \\
 &= -(1000.000)_{16} + (B6B.BA4)_{16} = -[(1000.000)_{16} - (B6B.BA4)_{16}] \\
 &= -(494.45C)_{16}
 \end{aligned}$$

A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black space. Two horizontal blue lines frame the central text.

NEGATIVE NUMBERS

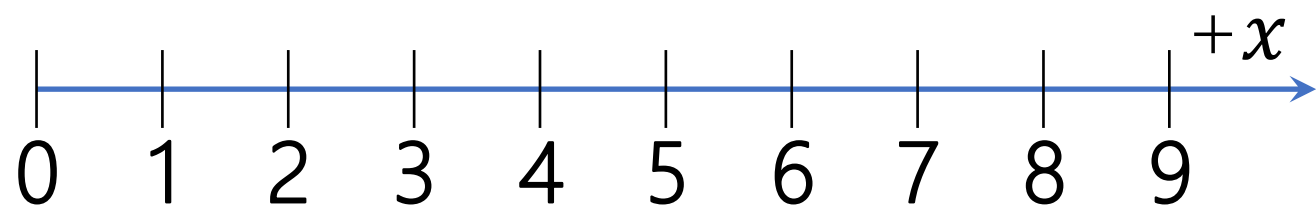
aka Signed Numbers

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

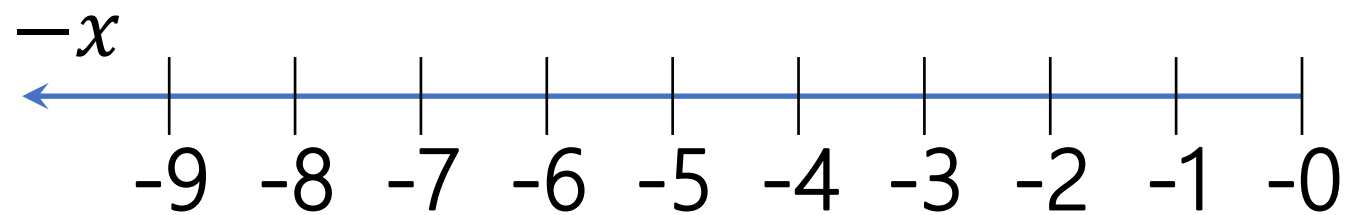
SIGNED MAGNITUDE

SIGNED COMPLEMENT

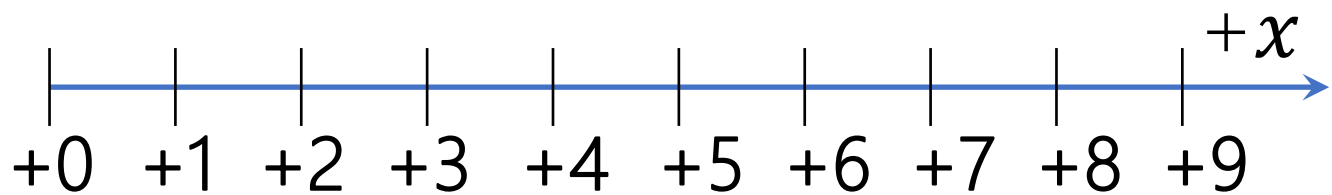
10^0
0
1
2
3
4
5
6
7
8
9



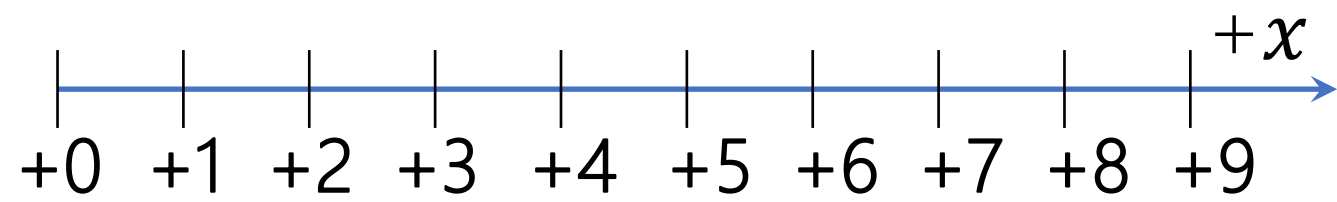
10^1	10^0
—	0
—	1
—	2
—	3
—	4
—	5
—	6
—	7
—	8
—	9



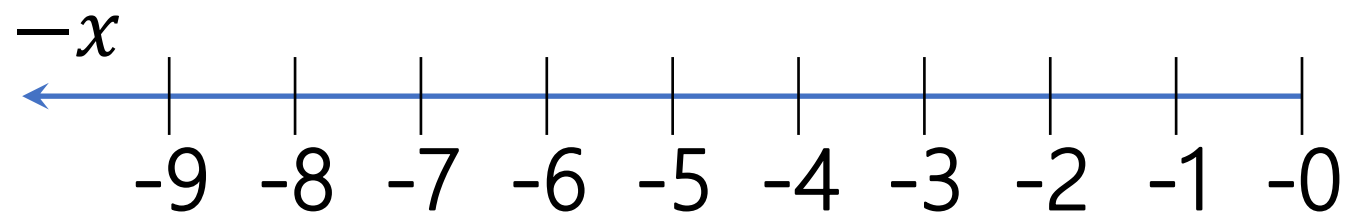
10^1	10^0
+	0
+	1
+	2
+	3
+	4
+	5
+	6
+	7
+	8
+	9



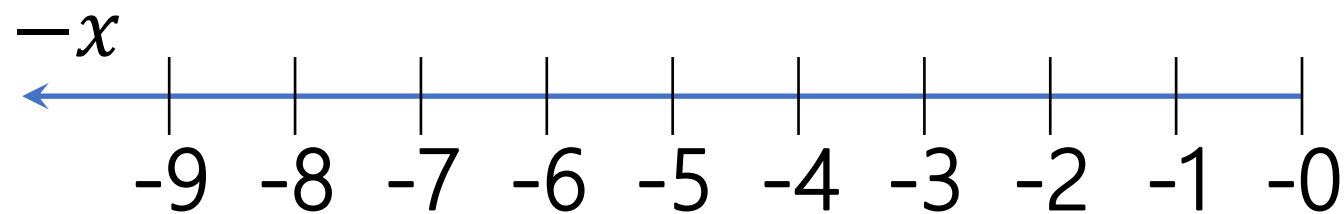
10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9



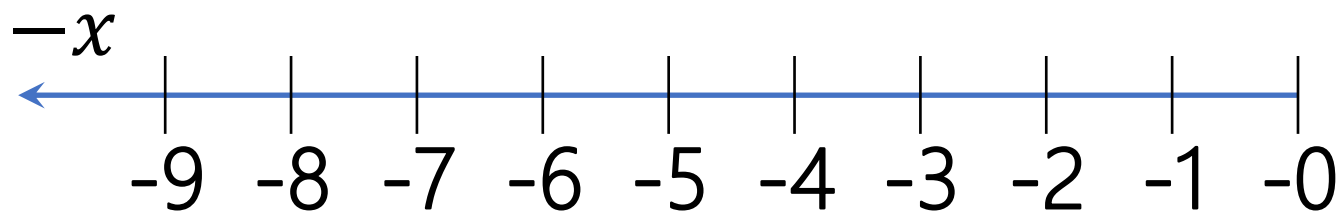
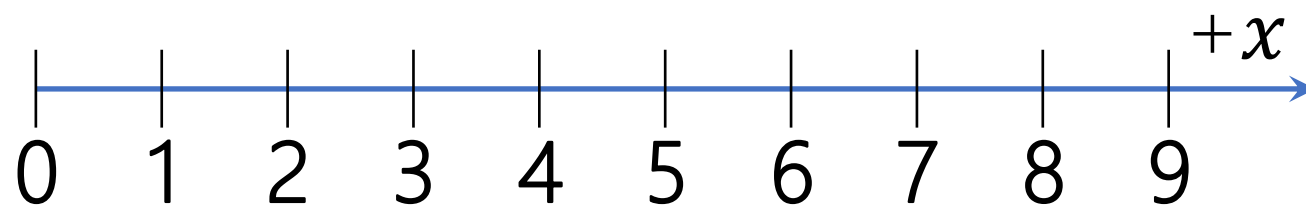
10^1	10^0
1	0
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8
1	9



10^1	10^0
Nonzero	0
Nonzero	1
Nonzero	2
Nonzero	3
Nonzero	4
Nonzero	5
Nonzero	6
Nonzero	7
Nonzero	8
Nonzero	9

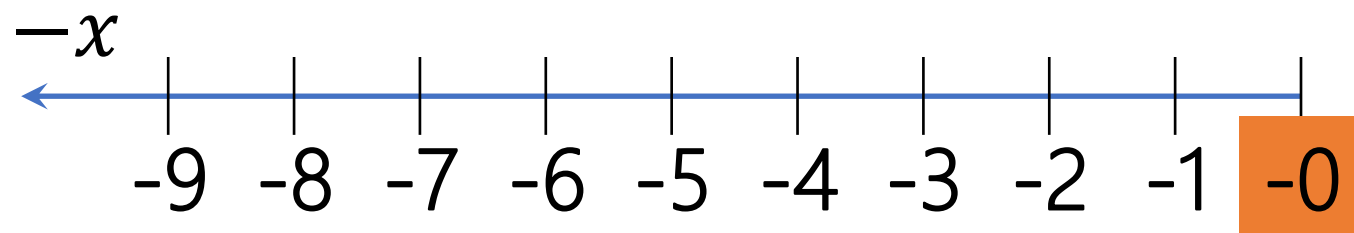
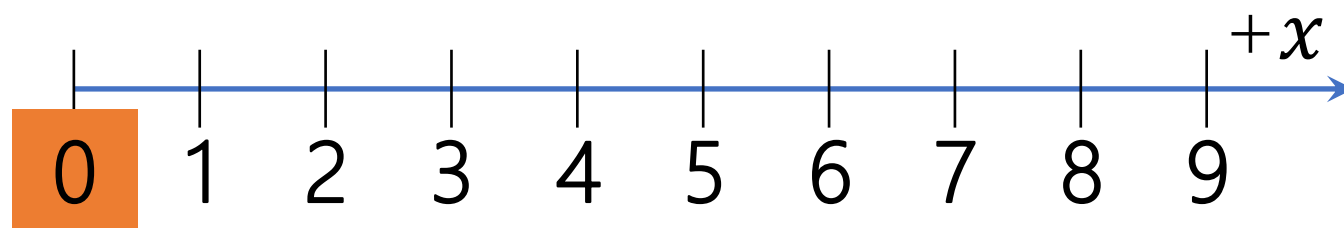


10^1	10^0
1,2,...9	0
1,2,...9	1
1,2,...9	2
1,2,...9	3
1,2,...9	4
1,2,...9	5
1,2,...9	6
1,2,...9	7
1,2,...9	8
1,2,...9	9



10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9

10^1	10^0
1,2,...9	0
1,2,...9	1
1,2,...9	2
1,2,...9	3
1,2,...9	4
1,2,...9	5
1,2,...9	6
1,2,...9	7
1,2,...9	8
1,2,...9	9



10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9

A deep space image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the central text.

SIGNED MAGNITUDE

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

Signed

Magnitude

Give up left most position for sign!

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

$$\text{Min} = -(r^{n-1} - 1) \leftarrow \cancel{0} \rightarrow \text{Max} = r^{n-1} - 1 = \cancel{r^n - 1}$$

2^5	2^4	2^3	2^2	2^1	2^0
0	0	0	0	1	1
1	0	0	1	0	1

Interpretation
+3
-5

7^5	7^4	7^3	7^2	7^1	7^0
0	0	0	0	5	6
1	0	0	2	0	5
4	0	0	2	0	5
6	0	0	2	0	5

Interpretation
+41
-103
-103
-103

A deep space image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE ARITHMETIC

0	X
0	X
1	X

+

0	Y
1	Y
1	Y

=

0	$X+Y$
$X < Y$ (if borrow)	$X-Y$ (if borrow, apply it)
1	$-(X+Y) = X+Y$

0	X
0	X
1	X

-

0	Y
1	Y
1	Y

=

$X < Y$ (if borrow)	$X-Y$ (if borrow, apply it)
0	$X+Y$
$X > Y$ (if borrow)	$-X+Y = Y-X$ (if borrow, apply it)

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE

Example I, Addition, Base-16

+ Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	
				.			

+ Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	
				.			

PADDING

+ Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	0
				.			

SIGNED: $(-X) + (-Y) = -(X + Y)$

+ Base-16	1	2	A	.	E	5	4
	1	B	F	.	2	B	0
	1			.			

		1	1		1		
+ Base-16	1	2	A ₌₁₀	.	E ₌₁₄	5	4
	1	B	F ₌₁₅	.	2	B ₌₁₁	0
	1	E ₌₁₄	A ₌₁₀	.	1	0	4

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE EXAMPLE II

+ Base-16	0	2	A	.	E	5	4
	1	B	F	.	2	B	
				.			

+ Base-16	0	2	A	.	E	5	4
	1	B	F	.	2	B	
				.			

PADDING

+ Base-16	0	2	A	.	E	5	4
	1	B	F	.	2	B	0
				.			

SIGNED: $(+X) + (-Y) = ?(X - Y)$

Base-16	0	2	A	.	E	5	4
	1	B	F	.	2	B	0
	?			.			

		-1	+16					
			-1	+16		-1	+16	
— Base-16	0	2	A ₌₁₀	.	E ₌₁₄	5	4	
	1	B ₌₁₁	F ₌₁₅	.	2	B ₌₁₁	0	
	1	6	B ₌₁₁	.	B ₌₁₁	A ₌₁₀	4	

$$2A.E54 < BF.2B0$$

Last Borrow → Negative Result

		+16	+16		+16	+16	
	-1	-1	-1		-1	-1	+16
— Base-16	1	0	0	.	0	0	0
		6	B=11	.	B=11	A=10	4
		9	4	.	4	5	C=12

$$\begin{aligned}
 &= (2A.E54)_{16} - (BF.2B0)_{16} = (2A.E54)_{16} + (100.000)_{16} - (100.000)_{16} - (BF.2B0)_{16} \\
 &= -(100.000)_{16} + (6B.BA4)_{16} = -[(100.000)_{16} - (6B.BA4)_{16}] \\
 &= -(94.45C)_{16}
 \end{aligned}$$

		-1	+16				
			-1	+16		-1	+16
— Base-16	0	2	A ₌₁₀	.	E ₌₁₄	5	4
	1	B ₌₁₁	F ₌₁₅	.	2	B ₌₁₁	0
	1	6	B ₌₁₁	.	B ₌₁₁	A ₌₁₀	4

2A.E54 < BF.2B0

Last Borrow → Negative Result

1	9	4	.	4	5	C
---	---	---	---	---	---	---

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE OVERFLOW

Overflow!

0	X
0	X
1	X

+

0	Y
1	Y
1	Y

=

0 → 1	X+Y
X < Y	X-Y
1 → 0	X+Y

+

=

+

=

0	X
0	X
1	X

-

0	Y
1	Y
1	Y

=

X < Y	X-Y
0 → 1	X+Y
X > Y	-X+Y=Y-X

-

=

-

=

— Base-2	$+(27)_{10}$	0	1	1	0	1	1
		1	0	1	1	1	0

— Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$-(14)_{10}$	1	0	1	1	1	0

$+X - (-Y)$

— Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$-(14)_{10}$	1	0	1	1	1	0

$+X - (-Y) = +(X+Y)$

Base-2	+(27) ₁₀	0	1	1	0	1	1
	+(14) ₁₀	0	0	1	1	1	0

$+X - (-Y) = + (X + Y)$

Base-2	+	$+(27)_{10}$	0	1	1	0	1	1
		$+(14)_{10}$	0	0	1	1	1	0
			0					

$+X - (-Y) = +(X+Y)$

Base-2	+		1	1	1	1		
			0	1	1	0	1	1
			0	0	1	1	1	0
			0	0	1	0	0	1

$$+X - (-Y) = +(X+Y)$$


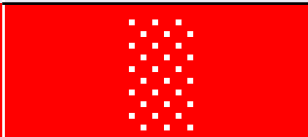
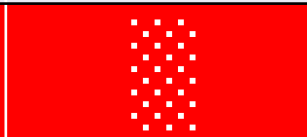

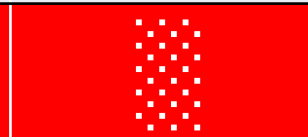
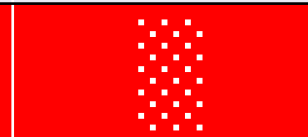
Base-2		1	1	1	1		
	+(27) ₁₀	0	1	1	0	1	1
	+(14) ₁₀	0	0	1	1	1	0
		1	0	1	0	0	1

If you consider the last carry $\rightarrow -(9)_{10} \rightarrow$ Negative!

$$+X - (-Y) = +(X+Y)$$

		1	1	1	1		
+ Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		0	0	1	0	0	1
	If you ignore it $\rightarrow + (9)_{10} \rightarrow$ Result is not correct!						

$$+X - (-Y) = +(X+Y)$$

		1	1	1	1		
+ Base-2	$+(27)_{10}$	0	1	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
							
		Overflow: The result is not reliable!					

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

WHY NOT SIGNED MAGNITUDE

Give up left most position for sign! What are the wastes?

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

$$+0 \rightarrow \text{Max} = r^{n-1} - 1 = r^n - 1$$

$$\text{Min} = -(r^{n-1} - 1) \leftarrow -0$$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE
SIGNED COMPLEMENT

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED COMPLEMENT

DIMINISHED RADIX COMPLEMENT

Given $(N)_r$ with n digits, the $(r - 1)$'s complement of N , i.e., its *diminished radix complement*, is defined as $(r^n - 1) - N$.

Distance to the Largest Number (Largest Digits in Each Position)

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

1's COMP. BASE-2

Base-2		2^4	2^3	2^2	2^1	2^0
$2^5 =$	1	0	0	0	0	0

Base-2		2^4	2^3	2^2	2^1	2^0
$2^5 =$	1	0	0	0	0	0
$1 =$		0	0	0	0	1

Base-2		2^4	2^3	2^2	2^1	2^0
		+2	+2	+2	+2	
	-1	-1	-1	-1	-1	+2
$2^5 =$	1	0	0	0	0	0
$1 =$		0	0	0	0	1
$2^5 - 1 =$		1	1	1	1	1
	5 digits of 1					

Base-2	2^5	2^4	2^3	2^2	2^1	2^0
$2^5-1=$		1	1	1	1	1
N=		1	0	1	0	1
$(2^5-1)-N=$		0	1	0	1	0

1's complement of $(10101)_2 = (01010)_2 =$ NOT on each digit



3's COMP. BASE-4

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5 =$	1	0	0	0	0	0

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5 =$	1	0	0	0	0	0
$1 =$	0	0	0	0	0	1

Base-4		4^4	4^3	4^2	4^1	4^0
		+4	+4	+4	+4	
	-1	-1	-1	-1	-1	+4
$4^5 =$	1	0	0	0	0	0
$1 =$		0	0	0	0	1
$4^5 - 1 =$		3	3	3	3	3
	5 digits of 3					

Base-4		4^4	4^3	4^2	4^1	4^0
$4^5 - 1 =$		3	3	3	3	3
$N =$		1	2	1	3	0
$(4^5 - 1) - N =$		2	1	2	0	3

3's complement of $(12130)_4 = (21203)_4 = 3 - \text{Each digit}$

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

9's COMP. BASE-10

Base-10		10^4	10^3	10^2	10^1	10^0
		+10	+10	+10	+10	
	-1	-1	-1	-1	-1	+10
$10^5 =$	1	0	0	0	0	0
$1 =$		0	0	0	0	1
$10^5 - 1 =$		9	9	9	9	9
	5 digits of 9					

Base-10		10^4	10^3	10^2	10^1	10^0
$10^5 - 1 =$		9	9	9	9	9
N =		1	2	1	3	0
$(10^5 - 1) - N =$		8	7	8	6	9

9's complement of $(12130)_{10} = (87869)_{10} = 9 - \text{Each digit}$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

$(r-1)$'s COMP. BASE- r

Base-r		r^{n-1}	...	r^2	r^1	r^0
$r^n-1=$		$r-1$...	$r-1$	$r-1$	$r-1$
$N=$		d_{n-1}	...	d_2	d_1	d_0
$(r^n-1)-N=$		$r-1-d_{n-1}$...	$r-1-d_{n-1}$	$r-1-d_{n-1}$	$r-1-d_{n-1}$

$(r-1)$'s complement of $(N)_r = (r-1) - \text{Each digit}$

A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black space. Two horizontal blue lines are positioned above and below the text.

RADIX COMPLEMENT

Given $(N)_r$ with n digits, the r 's complement of N , i.e., its *radix complement*, is defined as $r^n - N$.

RADIX COMPLEMENT

Given $(N)_r$ with n digits, the r 's complement of N , i.e., its *radix complement*, is defined as $r^n - N$.

Diminished Complement + 1

$$(r-1)\text{'s Complement} + 1 = [(r^n - 1) - N] + 1 = r^n - N$$



2's COMP. BASE-2

[illegible]

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

4's COMP. BASE-4

[illegible]

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

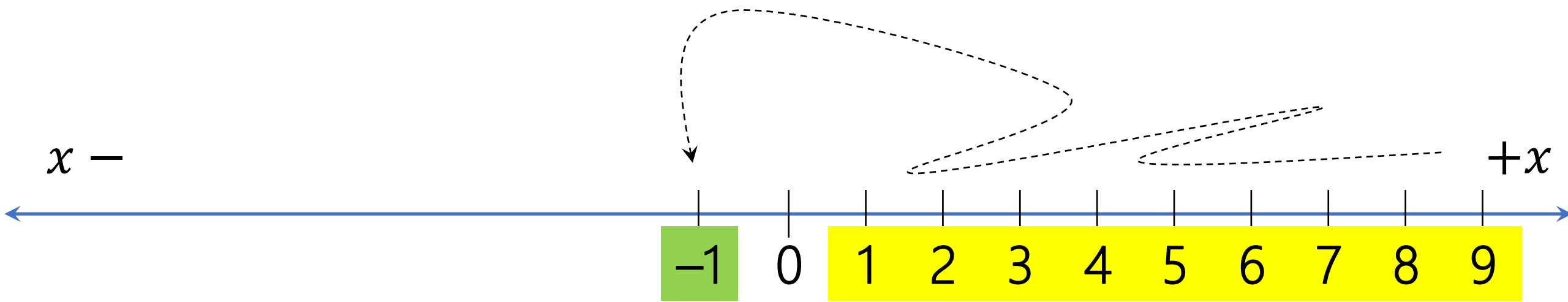
10's COMP. BASE-10

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines are positioned above and below the central text.

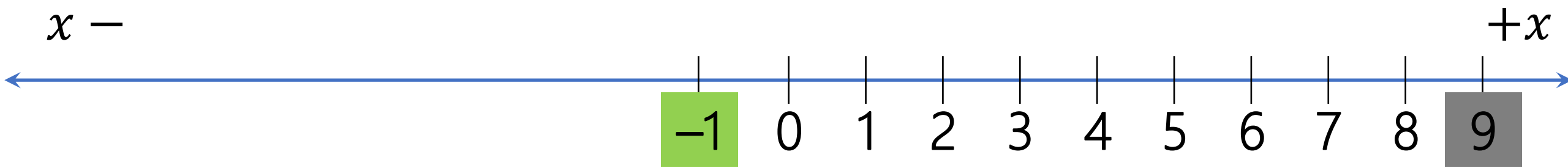
r' 's COMP. BASE- r

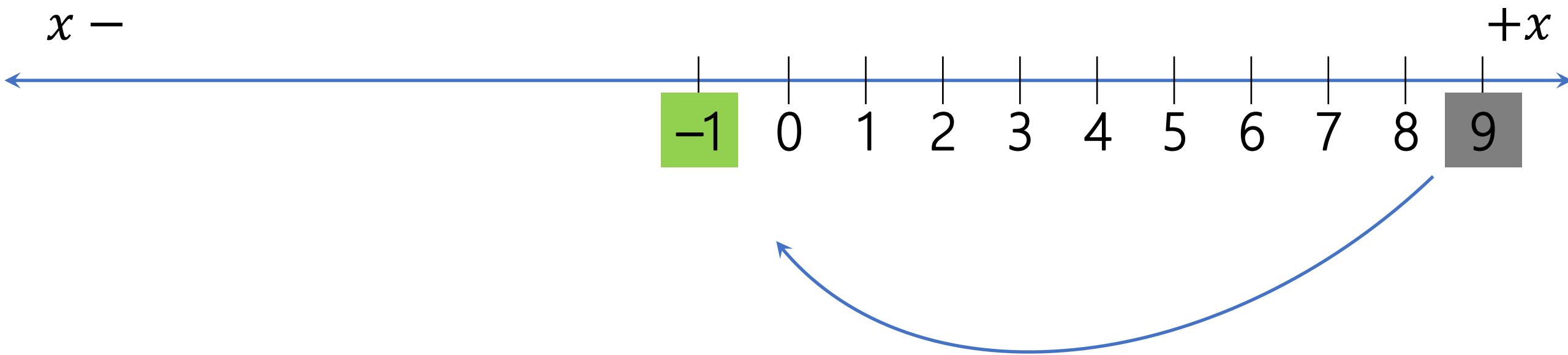
A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED COMPLEMENT

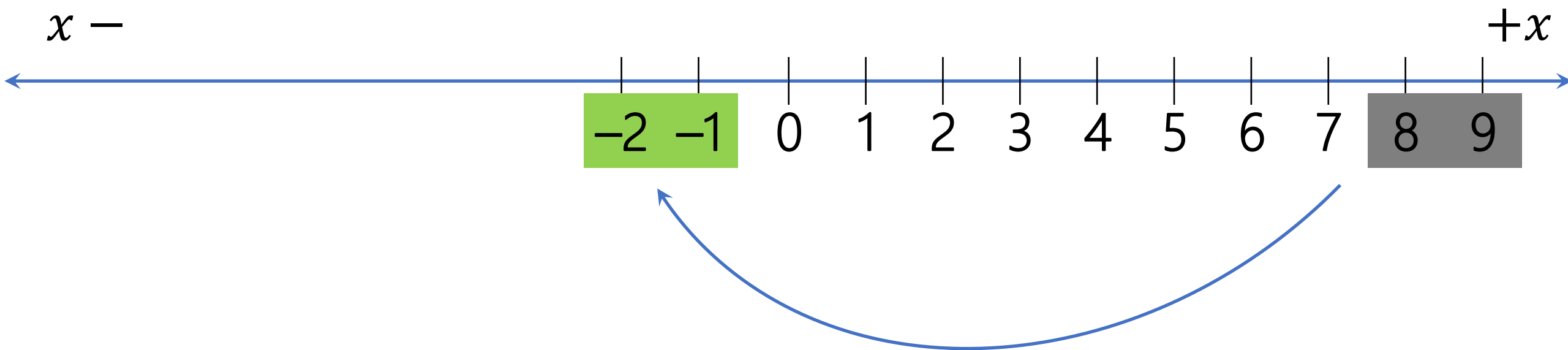


Which one do you scarify?

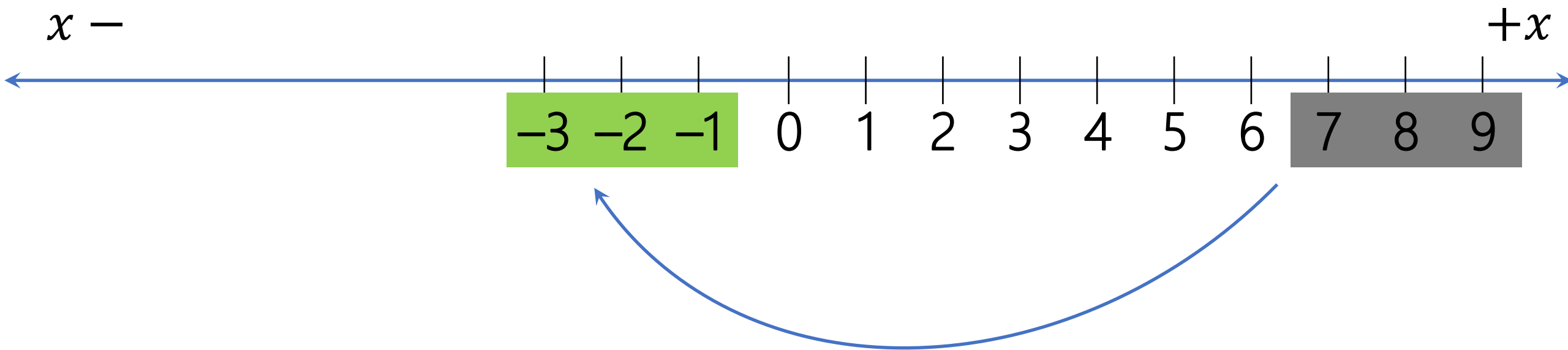




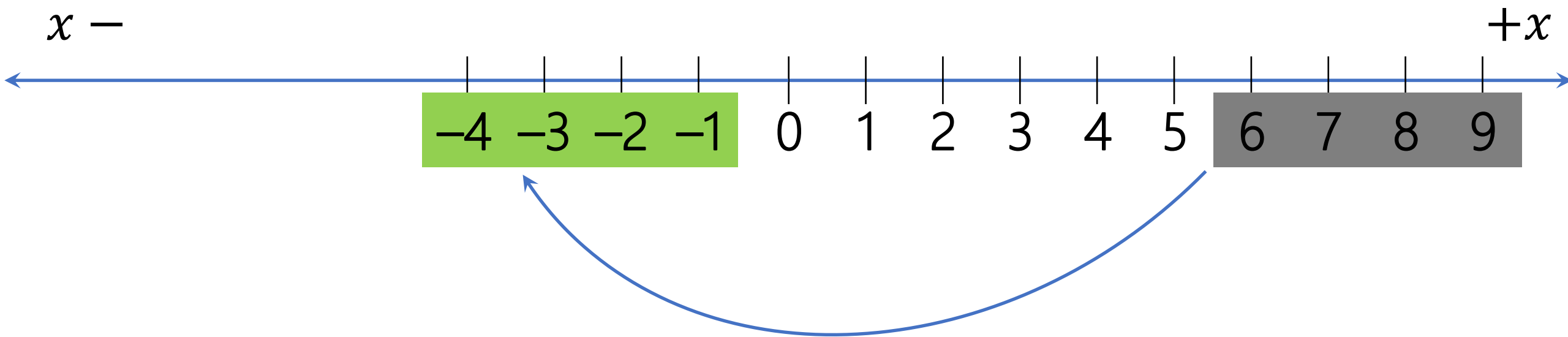
$$\boxed{-1} = -(10^1 - \boxed{9}) = -(10\text{'s comp. } \boxed{9}) = -(9 - \boxed{9} + 1) = - (9\text{'s comp. } \boxed{9} + 1)$$



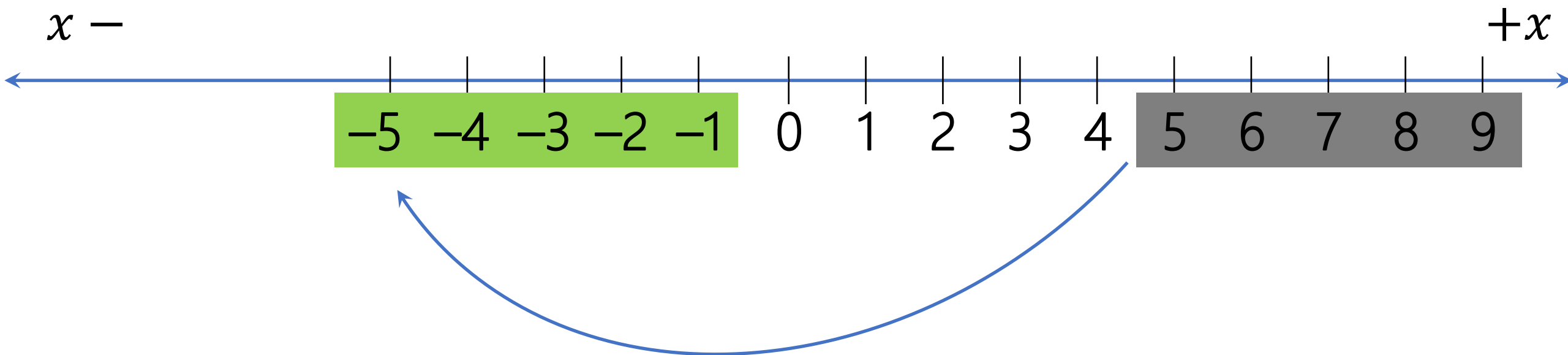
$$\boxed{-2} = -(10^1 - \boxed{8}) = -(\text{10's comp. } \boxed{8}) = -(9 - \boxed{8} + 1) = -(\text{9's comp. } \boxed{8} + 1)$$



$$\boxed{-3} = -(10^1 - \boxed{7}) = -(\text{10's comp. } \boxed{7}) = -(9 - \boxed{7} + 1) = -(\text{9's comp. } \boxed{7} + 1)$$



$$\boxed{-4} = -(10^1 - \boxed{6}) = -(\text{10's comp. } \boxed{6}) = -(9 - \boxed{6} + 1) = -(\text{9's comp. } \boxed{6} + 1)$$



$$\boxed{-5} = -(10^1 - \boxed{5}) = -(10\text{'s comp. } \boxed{5}) = -(9 - \boxed{5} + 1) = -(9\text{'s comp. } \boxed{5} + 1)$$

10^0	$-(10\text{'s comp.})$	10^0
0	→	0
1		−9
2		−8
3		−7
4		−6
5		−5
6		−4
7		−3
8		−2
9		−1

10^0	−(10's comp.)	10^0
0	←	0
1		−9
2		−8
3		−7
4		−6
5		−5
6		−4
7		−3
8		−2
9		−1

10^0		10^0
0	Base-10 Signed 10's comp.	0
1		−9
2		−8
3		−7
4		−6
5		−5
6		−4
7		−3
8		−2
9		−1

10^0		10^0
0	Base-10 Signed 10's comp.	see 0 interpret 0
1		see 1 interpret 1
2		see 2 interpret 2
3		see 3 interpret 3
4		see 4 interpret 4
5 \rightarrow -5		see 5 interpret -5
6 \rightarrow -4		see 6 interpret -4
7 \rightarrow -3		see 7 interpret -3
8 \rightarrow -2		see 8 interpret -2
9 \rightarrow -1		see 9 interpret -1

A deep-field astronomical image showing a vast field of galaxies. The galaxies are of various shapes and sizes, including spiral, elliptical, and irregular forms. They are colored in shades of blue, orange, and white, set against a dark, star-filled background. Two horizontal blue lines are positioned above and below the central text.

SIGNED 10's COMP. Base-10