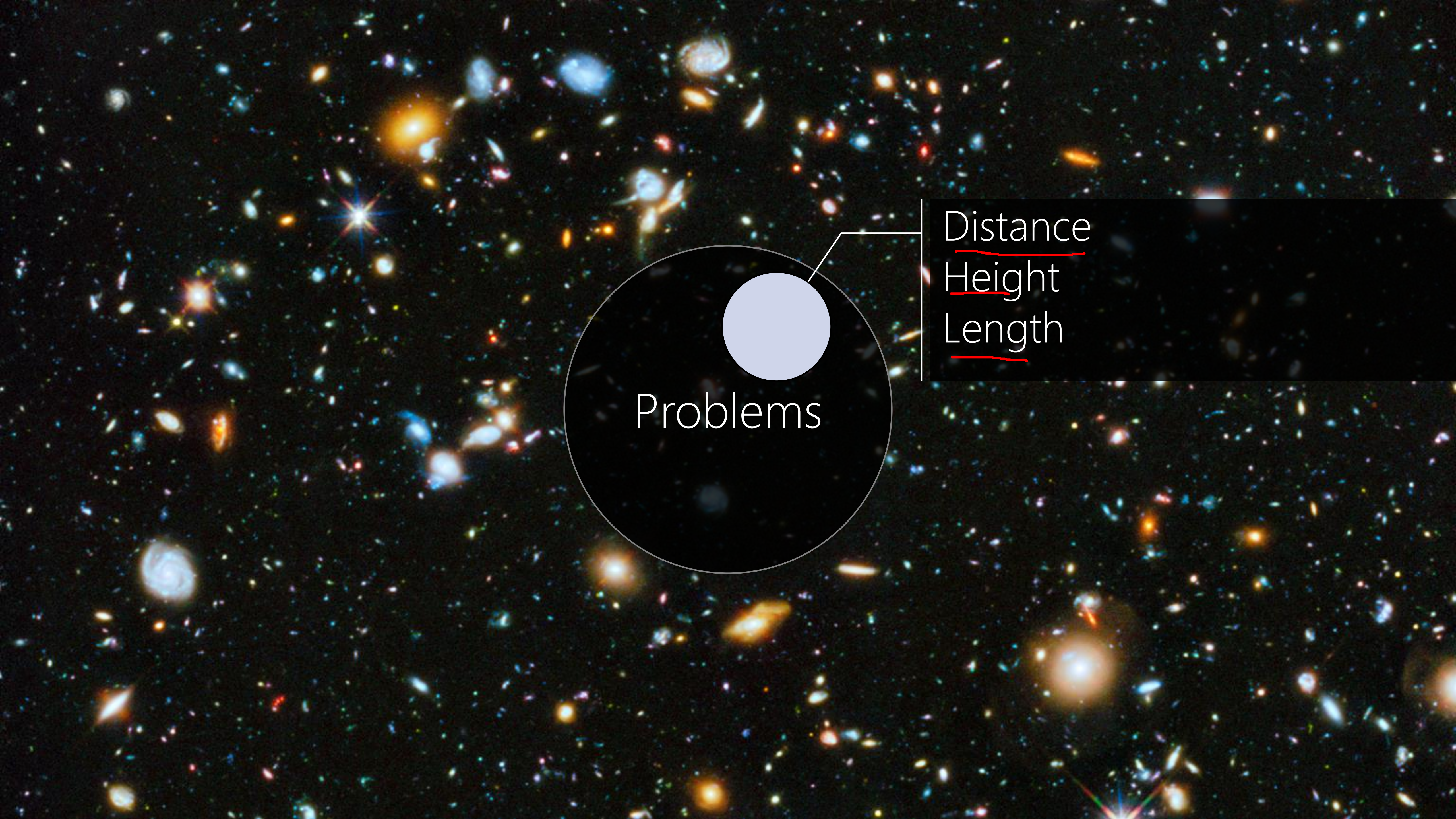
2

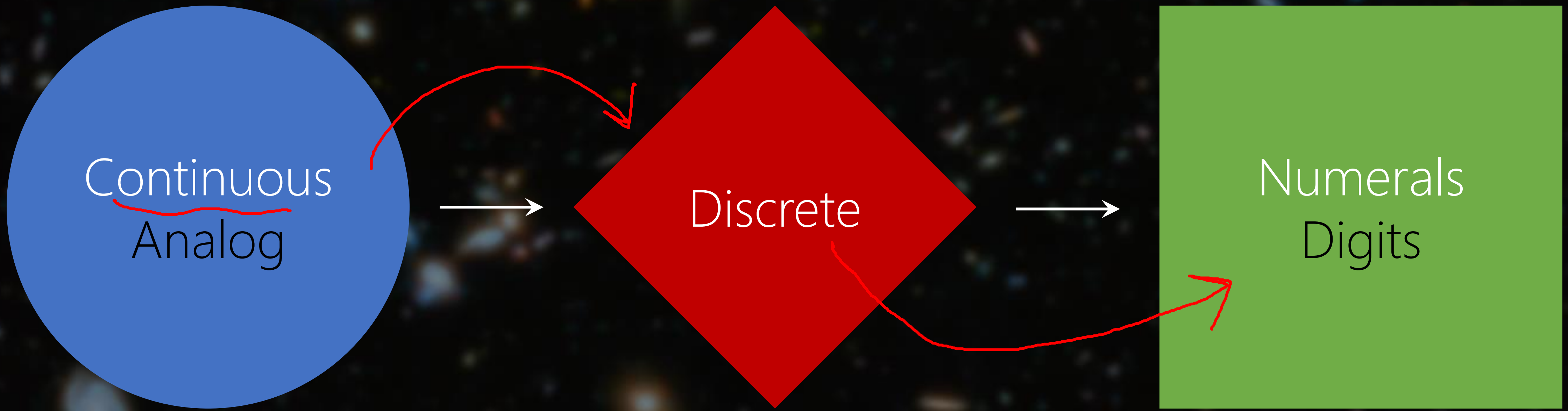
LEC02 & LAB02

Lectures >> Lec02: Number Systems

Labs >> Lab02: Programming Environment Setup

Problems


Problems
Distance
Height
Length



Quantization

DISCRETE SYSTEMS

"Digital System"

UNARY SYSTEM

aka. Base-1

I II III IV V VI VII VIII IX X

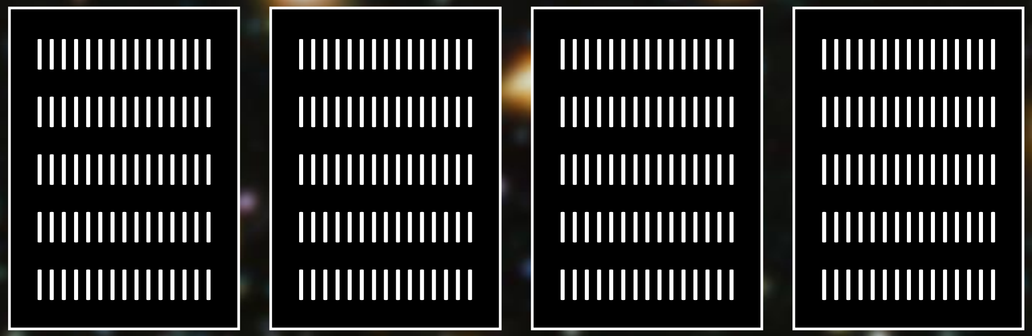
Roman Numerals

Originated in Ancient Rome

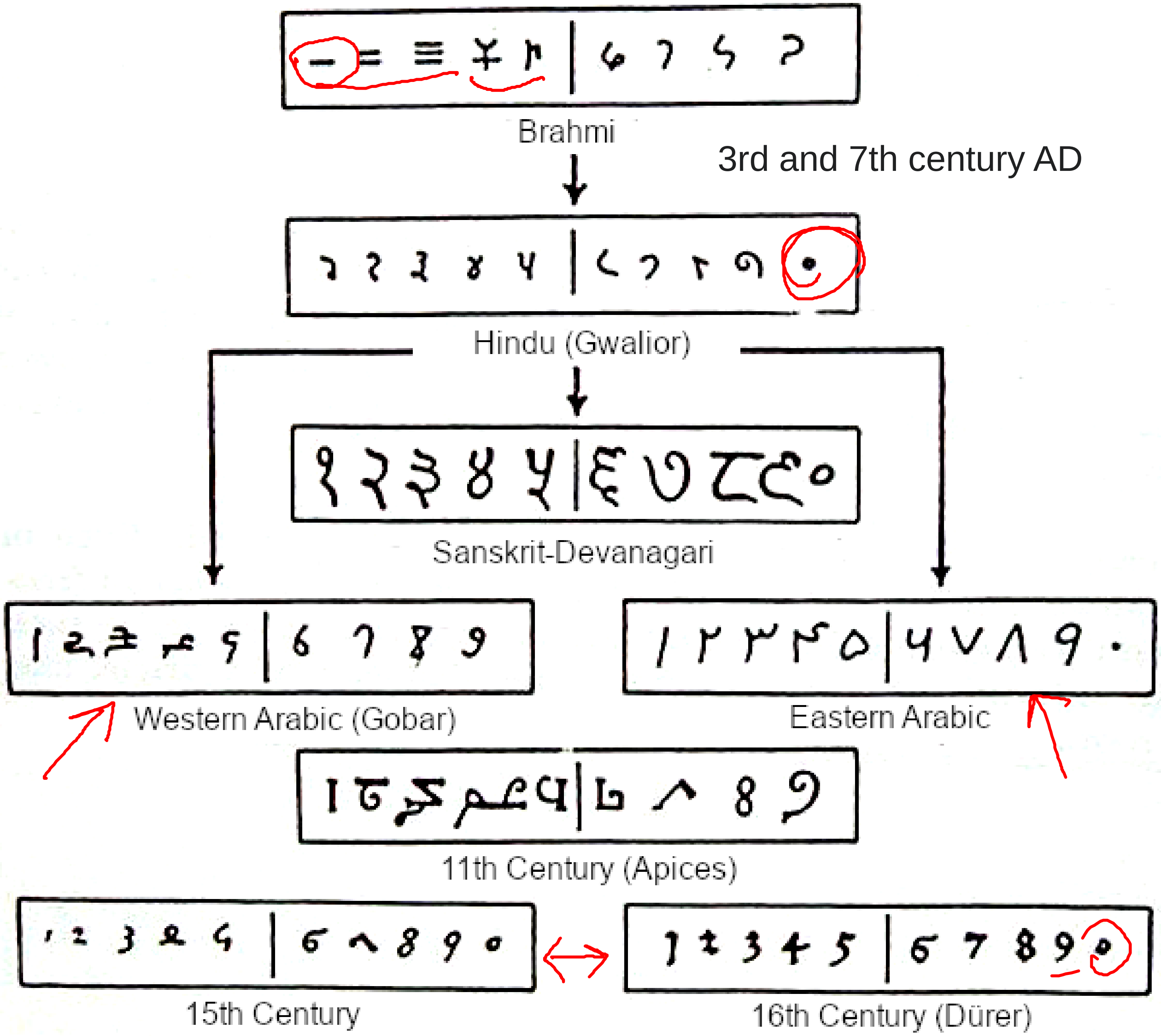
8th Century BC

~150 million km ÷ ~13,000 km = ~12,000 Earth paper = ~3,000 positions

12,000 ÷ 3,000 = 4 pages!



NUMBER SYSTEMS


Brahmi
3rd and 7th century AD
Hindu (Gwalior)
Sanskrit-Devanagari
Western Arabic (Gobar)
Eastern Arabic
11th Century (Apices)
15th Century
16th Century (Dürer)

0123456789

Hossein's Number System

~150 million km ÷ ~13,000 km = ~12,000 Earth

N = 12,000

n = (N+1)÷4 = (12,000+1)÷4 = ~3,000 positions

paper = ~3,000 positions

3,000 ÷ 3,000 = 1 pages

More Significant Position

0	0
1	1
2	2
3	3
10	4
11	5
12	6
13	7
20	8

4^1 More Significant

1 round of all (4) symbols = 1×4^1

4^1 More Significant

2 rounds of all (4) symbols = 2×4^1

QUATERNARY SYSTEM

/kwaa·**tur**·neh·ree/

aka. Base-4, Radix-4

$$(0,1,2,3)_4$$

. . . . 4^1, 4^0

Hindu-Arabic Numerals
Originated in India
7th Century AD

More Significant Position

#Symbols=4
Radix-4
Base-4

4^3	4^2	4^1	4^0	
		2	0	$2\times4^1+0\times4^0=8$
		2	1	$2\times4^1+1\times4^0=9$
		2	2	$2\times4^1+2\times4^0=10$
		2	3	$2\times4^1+3\times4^0=11$
		3	0	$3\times4^1+0\times4^0=12$
		3	1	$3\times4^1+1\times4^0=13$
		3	2	$3\times4^1+2\times4^0=14$
		3	3	$3\times4^1+3\times4^0=15$
	1	0	0	$1\times4^2+0\times4^1+0\times4^0=16$

More Significant Position

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	$\times$
3	0	3	0	2	1	3	1	
3×4^7	0×4^6	3×4^5	0×4^4	2×4^3	1×4^2	3×4^1	1×4^0	$\sum$
								65,437

More Significant Position

										Base-4	Hossein's Number System
		3	0	3	0	2	1	3	1	65,437	-
					3	3	3	3	1	1,021	17
				3	3	3	3	3	2	4,094	22
		3	0	0	3	3	3	3	0	50,172	-
3	3	3	3	3	3	3	3	3	3	1,048,575	39

10

powers 4

x 4

0	0	…	0	0	0

Min

Radix-4
Base-4

n positions

4^{n-1} 4^2 4^1 4^0

Max

3	3	…	3	3	3	?

$$N = 3 \times 4^{n-1} + 3 \times 4^{n-2} + \cdots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$$

$$N = 3 \times (4^{n-1} + 4^{n-2} + \cdots + 4^2 + 4^1 + 4^0)$$

$$N = 3 \times \left(\frac{4^n - 1}{4 - 1}\right)$$

$$N = 4^n - 1$$

$4n - 1$
Hossein's
System

n positions

Max

4^{n-1}	4^{n-2}		4^2	4^1	4^0	
3	3	...	3	3	3	N

$$4^n - 1 = N$$

$$4^n = N + 1$$

$$\log_4 4^n = \log(N + 1)$$

$$n = \log_4(N + 1)$$

? positions

Max

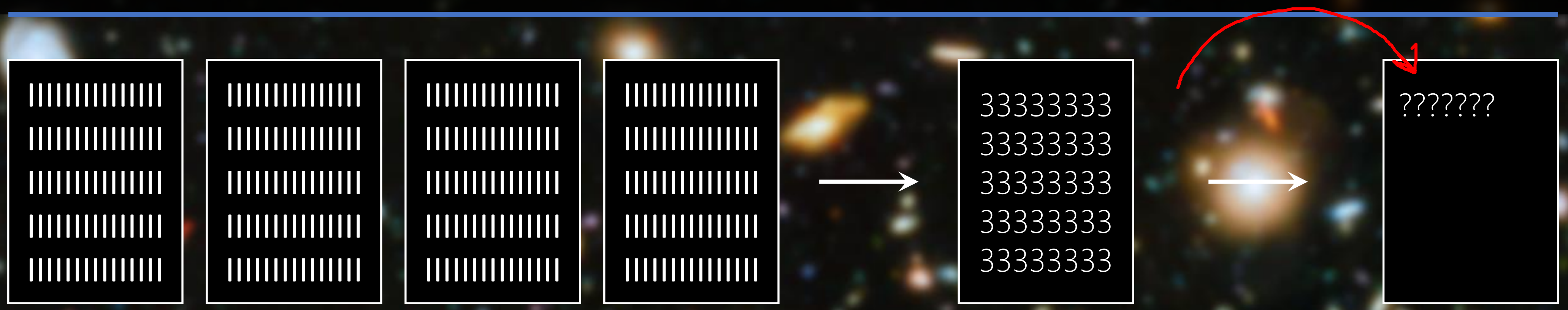
3	3	…	3	3	N

$$\sim 150 \text{ million km} \div \sim 13{,}000 \text{ km} = \sim 12{,}000 \text{ Earth}$$

$$N = 12{,}000$$

$$n = \text{Log}_4\,(12{,}000+1) = \text{Log}_{10}\,12{,}001 \div \text{Log}_{10}\,4 = 4 \div 0.6 = \underline{6.79}$$

$$\sim 7 \text{ positions}$$



COMMON NUMBER SYSTEMS

Base - 4

- 2

- 3

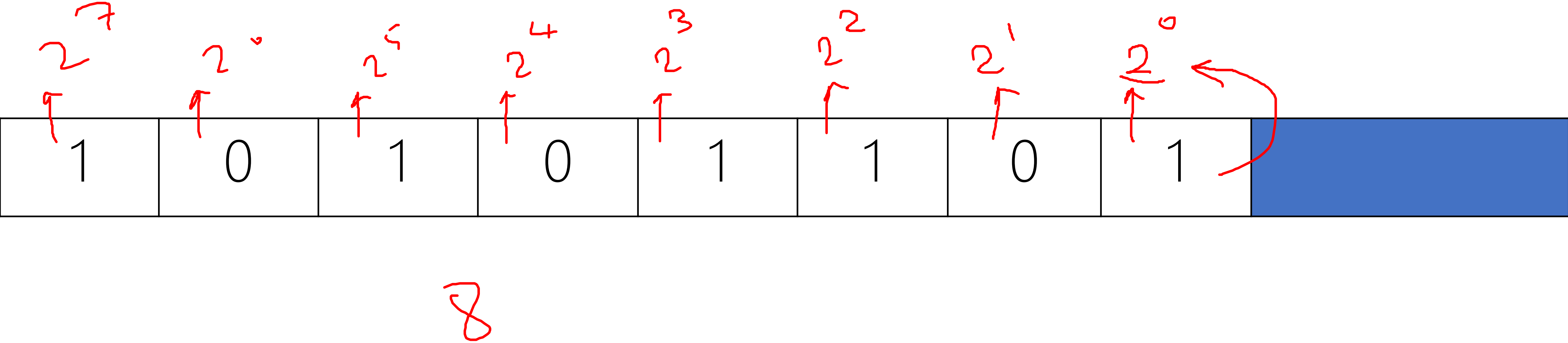
5

6

⋮

BINARY | BASE-2 | RADIX-2

$(0,1)_2$



2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0
1 0 1 0 1 1 0 1
8

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	×
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	$\times$
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	$\sum$
								173

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
3	0	3	0	2	1	3	1	$\times$
								$\sum$
								?

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	$\times$
3	0	3	0	2	1	3	1	
-	0×2^6	-	0×2^4	-	1×2^2	-	1×2^0	$\sum$
								Not Valid

LET'S COUNT IN BINARY

$(\square\square\square\square)_2$

0 0 0 0

1 1 1 1

$(0000)_2$, 0001, 0010, ..., $(1111)_2$

0 0 0 0
0 0 0 1

0 0 1 0
+ 0 0 0 1

$\frac{2}{2} = 1$ r 0

0 0 1 0

LET’S COUNT IN BINARY

$(\square\,\square\,\square\,\square)_2$

0000, 0001, 0010, 00011, …, 1101, 1110, 1111

Increment by 1 → Increment by 0001

$8 + 2 + 1 = 11$

				2^3	2^2	2^1	2^0
+				1	0	1	1
Base-2				0	0	0	1
		12					

Increment by 1 → Increment by 0001

						1	
+				1	0	1	1
Base-2				0	0	0	1
							2→0

$$\frac{2}{2} = 1\ r\ 0$$

Increment by 1 → Increment by 0001

					1	1	
+ Base-2				1	0	1	1
				0	0	0	1
						$2 \rightarrow 0$	0

$$\frac{2}{2} = 1\ r\ 0$$

Q R

Increment by 1 → Increment by 0001

					1	1	
+ Base-2				1	0	1	1
				0	0	0	1
				1	1	0	0

0000
0001
0010
0011
0100
0101
0110
0111 $(7)_{10}$

1000
1001
1010
1011 $(11)_{10}$
1100
1101

1110
1111

2^3 2^2

$8 + 4 = 12$

3, 4, 5, 6, 7

OCTAL | BASE-8 | RADIX-8

$(0,1,2,3,4,5,6,7)_8$

3	0	3	0	2	1	3	1	

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	3	0	2	1	3	1	

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	$\times$
3	0	3	0	2	1	3	1	
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0	

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	$\times$
3	0	3	0	2	1	3	1	
3×8^7	0×8^6	3×8^5	0×8^4	2×8^3	1×8^2	3×8^1	1×8^0	$\sum$
								$\underbrace{6{,}390{,}873}_{10}$

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
3	0	A	0	8	1	3	1	$\times$
								Σ

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	$\curvearrowleft \times$
3	0	A	0	8	1	3	1	
3×8^7	0×8^6	-	0×8^4	-	1×8^2	3×8^1	1×8^0	$\sum$
								Not Valid

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	$\times$
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	$\sum$
								2,130,497

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

Radix-8
vs.
Radix-2

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

LET'S COUNT IN OCTAL

$(\square\,\square\,\square\,\square)_8$

$(0000)_8$, 0001, 0002, 0003, 0004, 0005, 0006, 0007, $(0010)_8$, $(0011)_8$, ..., $(7777)_8$

LET'S COUNT IN OCTAL

$(\ \ \ \ )_8$

0000, 0001, 0002, …, 0007, 0010, …, 7775, 7776, 7777

Increment by 1 → Increment by 0001

+ Base-8							
				5	2	7	7
				0	0	0	1

Increment by 1 $\rightarrow$ Increment by 0001

						1	
+ Base-8				5	2	7	7
				0	0	0	1
							8$\rightarrow$0

$$\frac{8}{8} = 1\ r\ 0$$

Increment by 1 $\longrightarrow$ Increment by 0001

					1	1	
+ Base-8				5	2	7	7
				0	0	0	1
						$8 \longrightarrow 0$	0

$$\frac{8}{8} = 1\ r\ 0$$

Increment by 1 → Increment by 0001

					1	1	
+				5	2	7	7
Base-8				0	0	0	1
				5	3	0	0

8

DECIMAL | BASE-10 | RADIX-10

$(0,1,2,3,4,5,6,7,9)_{10}$

Arabic	0123456789
Eastern Arabic	٠١٢٣٤٥٦٧٨٩
Roman	I II III IV V VI VII VIII IX X
Bengali–Assamese	০১২৩৪৫৬৭৮৯
Malayalam	൦൧൨൩൪൫൬൭൮൯
Thai	๐๑๒๓๔๕๖๗๘๙
Chinese	〇一二三四五六七八九

Arabic → 0123456789

Eastern Arabic → ٠١٢٣٤٥٦٧٨٩

"the hand makes the two complementary aspects of integers entirely intuitive. It serves as an instrument permitting natural movement between cardinal and ordinal numbering. If you need to show that a set contains three, four, seven or ten elements, you raise or bend simultaneously three, four, seven or ten fingers, using your hand as cardinal mapping. If you want to count out the same things, then you bend or raise three, four, seven or ten fingers in succession, using the hand as an ordinal counting tool."

- Georges Ifrah The Universal History of Numbers (Wiley, 2000, pp. 21-22)

Thanks Cecelia Nydam and Giavi Tran for the hint!

Thai → ๐๑๒๓๔๕๖๗๘๙

Chinese → 〇一二三四五六七八九

3	0	3	0	2	1	3	1	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\times$
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
3	0	3	0	2	1	3	1	$\times$
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								30,302,131

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\times$
3	0	3	0	2	1	3	1	
3×10^7	0×10^6	3×10^5	0×10^4	2×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								30,302,131

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\times$
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	$\sum$
								10,101,101

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\times$
3	0	A	0	8	1	3	1	
								Σ

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	$\times$
3	0	A	0	8	1	3	1	
3×10^7	0×10^6	-	0×10^4	8×10^3	1×10^2	3×10^1	1×10^0	$\sum$
								Not Valid

YOU KNOW HOW TO COUNT IN DECIMAL!

$(\quad\quad\quad\quad)_{10}$

0 0 0 0 9 9 9 9

weird NUMBER SYSTEMS

HEXADECIMAL | BASE-16 | RADIX-16

$(0,1,2,3,4,5,6,7,9,A,B,C,D,E,F)_{16}$

~~10~~

15

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	$\times$
								Σ

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
3	0	A	0	9	1	3	1	$\times$
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	$\times$
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$

$$A = (9 + 1) = (10)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	$\times$
3	0	A	0	9	1	3	1	
3×16^7	0×16^6	$A\times16^5$	0×16^4	9×16^3	1×16^2	3×16^1	1×16^0	$\sum$
		10×16^5						815,829,297

$$A = (9 + 1) = (10)_{10}$$

$$1,2,3,4,5,6,7,8,9,A = 9 + 1 = (10)_{10}$$

$$B = A + 1 = (11)_{10}$$

$$C = B + 1 = (12)_{10}$$

$$D = C + 1 = (13)_{10}$$

$$E = D + 1 = (14)_{10}$$

$$F = E + 1 = (15)_{10}$$

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	$\times$
1	0	1	0	1	1	0	1	
1×16^7	0×16^6	1×16^5	0×16^4	1×16^3	1×16^2	0×16^1	1×16^0	$\sum$
								269,488,385

16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0	
1	0	1	0	1	1	0	1	
1×16^7	0×16^6	1×16^5	0×16^4	1×16^3	1×16^2	0×16^1	1×16^0	
								269,488,385

10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0	
1	0	1	0	1	1	0	1	
1×10^7	0×10^6	1×10^5	0×10^4	1×10^3	1×10^2	0×10^1	1×10^0	
								10,101,101

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0	
1	0	1	0	1	1	0	1	
1×8^7	0×8^6	1×8^5	0×8^4	1×8^3	1×8^2	0×8^1	1×8^0	
								2,130,497

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	
1	0	1	0	1	1	0	1	
1×2^7	0×2^6	1×2^5	0×2^4	1×2^3	1×2^2	0×2^1	1×2^0	
								173

LET'S COUNT IN BASE-16

$(\square\,\square\,\square\,\square)_{16}$

0000; , $(FFFF)_{16}$

LET'S COUNT IN BASE-16

$(\quad\quad\quad\quad)_{16}$

0000, 0001, 0002, …, 000F, 0010, …, FFFD, FFFE, FFFF

Increment by 1 → Increment by 0001

+ Base-16				5	C	A	F
				0	0	0	1

Increment by 1 → Increment by 0001

						1	
+				5	C	A	F $_{=15}$
Base-16				0	0	0	1
							16→0

$$\frac{16}{16} = 1\ r\ 0$$

Increment by 1 → Increment by 0001

							1	
+				5	C	$A_{=10}$	$F_{=15}$
Base-16				0	0	0	1
						$\underbrace{11}\rightarrow\underbrace{B}$	0

Increment by 1 → Increment by 0001

						1	
+ Base-16				5	C	$A_{=10}$	$F_{=15}$
				0	0	0	1
				5	C	B	0

0 1 2 63

BASE-64 | RADIX-64

$(A,B,C, \ldots,Z, a,b,c,\ldots,z,0,1,2,\ldots,9,+,/)_{64}$

1992: RFC 1341

MIME (MULTIPURPOSE INTERNET MAIL EXTENSIONS)

Mechanisms For Specifying And Describing The Format Of Internet Message Bodies

Digit	Value
A	0
B	1
C	2
D	3
E	4
F	5
G	6
H	7
I	8
J	9
K	10
L	11
M	12
N	13
O	14
P	15

→

Digit	Value
Q	16
R	17
S	18
T	19
U	20
V	21
W	22
X	23
Y	24
Z	25
a	26
b	27
c	28
d	29
e	30
f	31

→

Digit	Value
g	32
h	33
i	34
j	35
k	36
l	37
m	38
n	39
o	40
p	41
q	42
r	43
s	44
t	45
u	46
v	47

→

Digit	Value
w	48
x	49
y	50
z	51
0	52
1	53
2	54
3	55
4	56
5	57
6	58
7	59
8	60
9	61
+	62
/	63

3	a	A	/	d	1	H	+	

64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	
3	a	A	/	d	1	H	+	$\times$
								Σ

Digit	Value
A	0
B	1
C	2
D	3
E	4
F	5
G	6
H	7
I	8
J	9
K	10
L	11
M	12
N	13
O	14
P	15

→

Digit	Value
Q	16
R	17
S	18
T	19
U	20
V	21
W	22
X	23
Y	24
Z	25
a	26
b	27
c	28
d	29
e	30
f	31

→

Digit	Value
g	32
h	33
i	34
j	35
k	36
l	37
m	38
n	39
o	40
p	41
q	42
r	43
s	44
t	45
u	46
v	47

→

Digit	Value
w	48
x	49
y	50
z	51
0	52
1	53
2	54
3	55
4	56
5	57
6	58
7	59
8	60
9	61
+	62
/	63

64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	
3	a	A	/	d	1	H	+	$\times$
55 $\times 64^7$	26 $\times 64^6$	0 $\times 64^5$	63 $\times 64^4$	29 $\times 64^3$	53 $\times 64^2$	7 $\times 64^1$	62 $\times 64^0$	Σ

64^7	64^6	64^5	64^4	64^3	64^2	64^1	64^0	
3	a	A	/	d	1	H	+	$\times$
55 $\times 64^7$	26 $\times 64^6$	0 $\times 64^5$	63 $\times 64^4$	29 $\times 64^3$	53 $\times 64^2$	7 $\times 64^1$	62 $\times 64^0$	$\sum$
								243,680,329,290,238

LET'S COUNT IN BASE-64

$(\quad\quad\quad\quad)_{64}$

$(A\ A\ A\ A)_{64}$

(/ / / /)

LET'S COUNT IN BASE-64

(░░░░)$_{64}$

AAAA, AAAB, AAAC, …, AAA/, AABA, …, ///9, ///+, ////

Increment by 1 → Increment by AAAB

+				a	+	Z	/
Base-64				$A_{=0}$	$A_{=0}$	$A_{=0}$	$B_{=1}$

Increment by 1 → Increment by AAAB

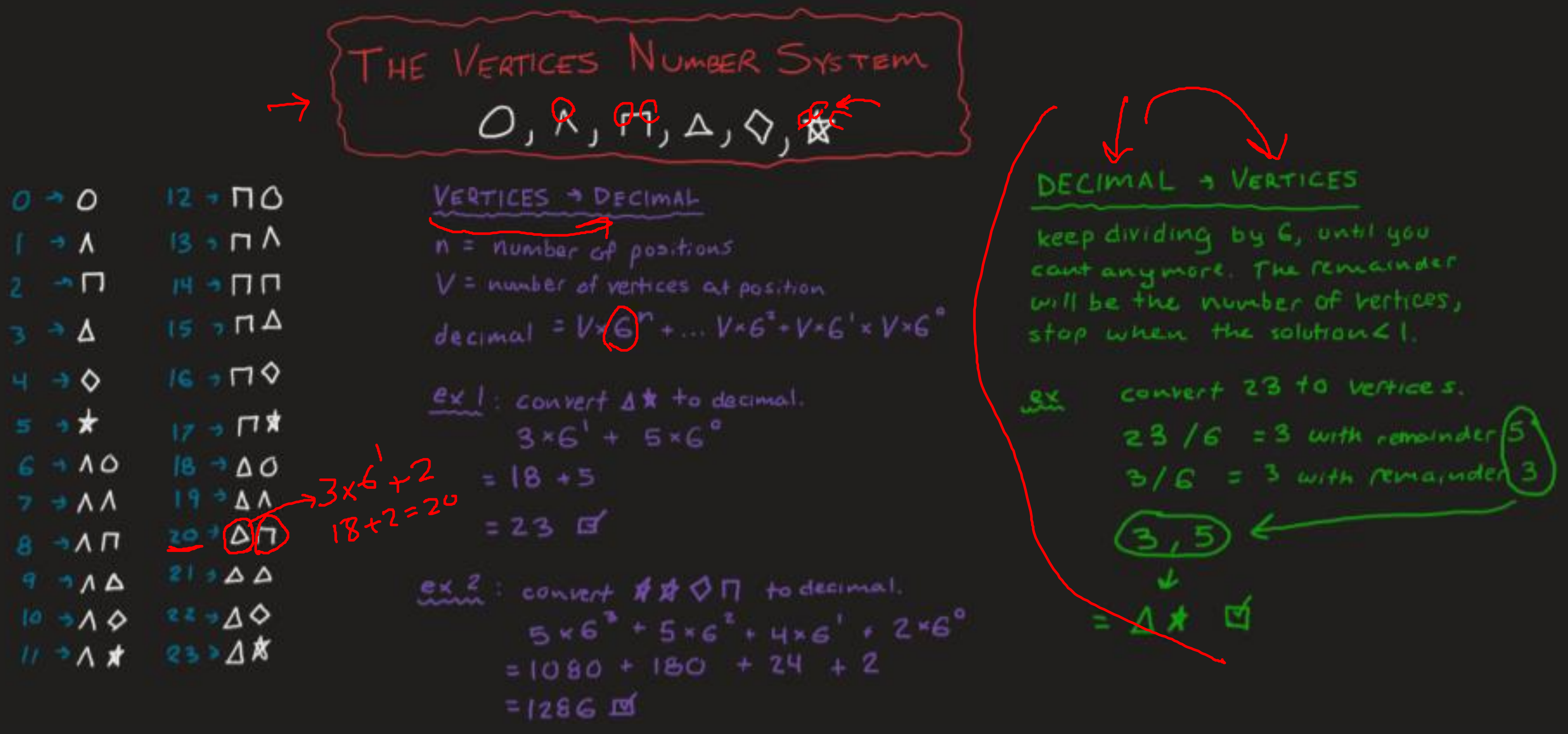
+ Base-64				a=26	+ =62	Z=25	/=63
				A=0	A=0	A=0	B=1

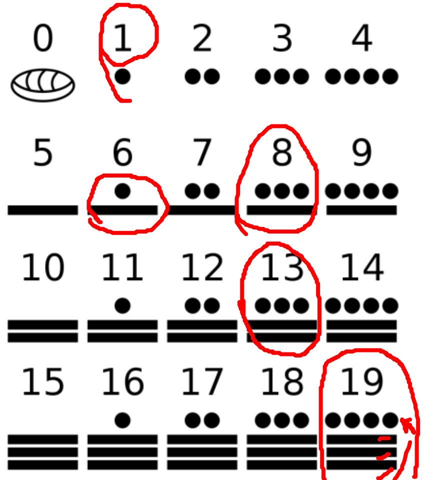
						B=1	
+ Base-64				a=26	+=62	Z=25	/=63
				A=0	A=0	A=0	B=1
							A=0

$$\frac{64}{64} = 1\ r\ 0$$

						B=1	
+				a=26	+=62	Z=25	/=63
Base-64				A=0	A=0	A=0	B=1
				a	+	a=26	A=0

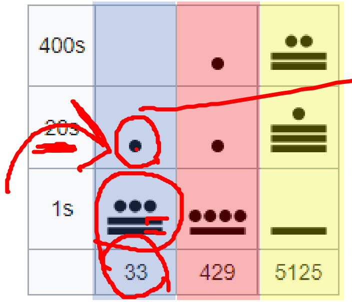
Christina Reynolds-Badder





Maya cities developed ~750 BC, and by 500 BC these cities possessed monumental architecture.

A vigesimal (/vɪˈdʒɛsɪməl/) or base-20 (base-score) numeral system



$1 \times (20)^1 + 13$

$20 + 13 = 33$

r^1

RADIX-R NUMBER SYSTEM

aka. Base-r Number System

Hossein's number system is not a Radix-r number system!

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + d_{n-2} r^{n-2} + \cdots + d_i r^i + \cdots + d_2 r^2 + d_1 r^1 + d_0 r^0)_{10}$$

where:

r	=	radix (base)
d_i	=	digit at position i, $0 \le d_i \le r - 1$
r^i	=	weight (significance) of position i
n	=	number of digits in N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1}\, r^{n-1} + d_{n-2}\, r^{n-2} + \cdots + d_i\, r^i + \cdots + d_2\, r^2 + d_1\, r^1 + d_0 r^0)_{10}$$

where:

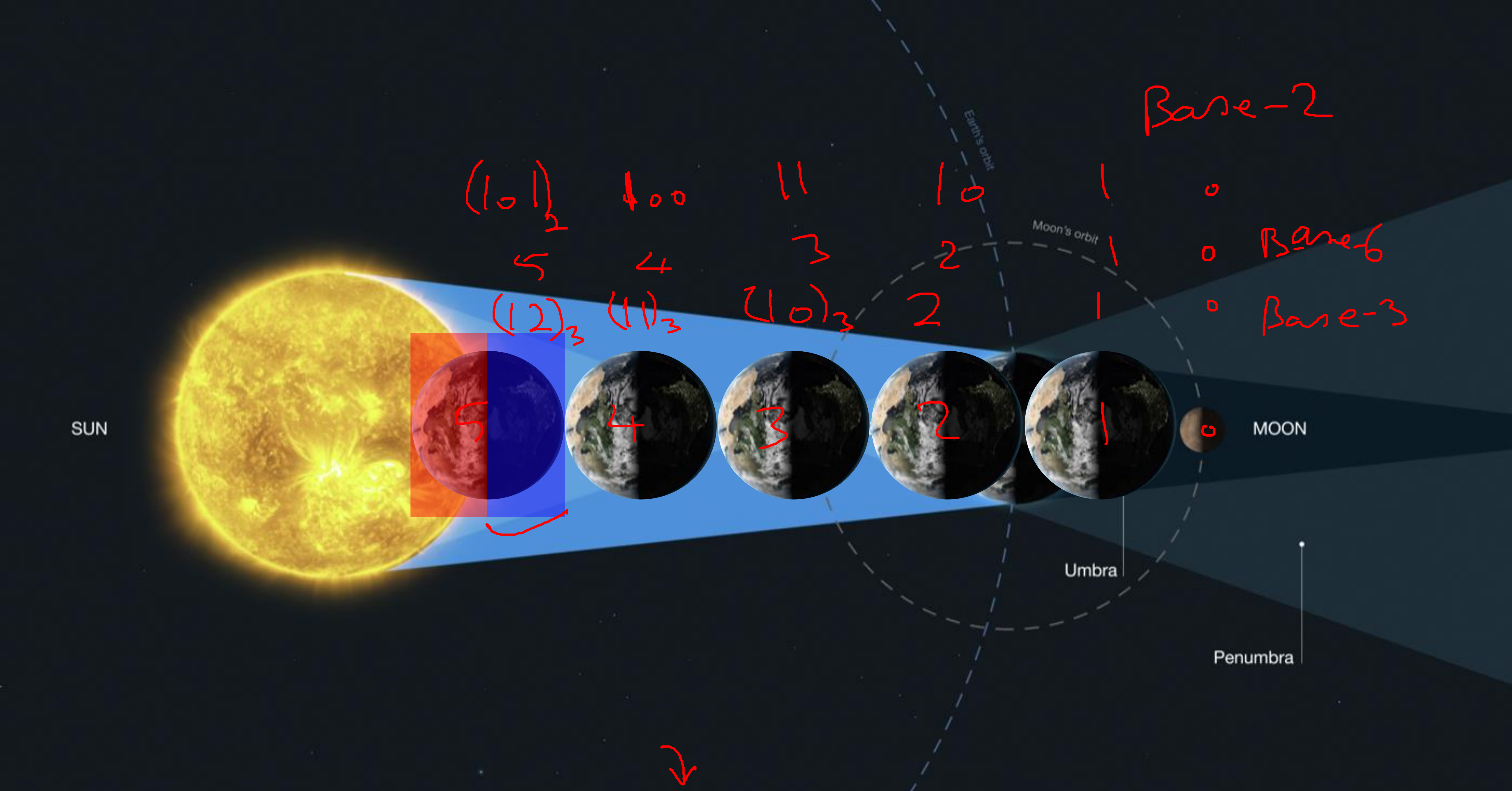
- r = radix (base)
- d_i = digit at position i, $0 \le d_i \le r - 1$
- r^i = weight (significance) of position i
- n = number of digits in N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

Min	$= (0_{n-1}0_{n-2} \cdots 0_1 0_0)_r$	$= (0)_{10}$
Max	$= ((r-1)_{n-1}(r-1)_{n-2} \cdots (r-1)_1(r-1)_0)_r$	$= (r^n-1)_{10}$
Unit	$= (0_{n-1}0_{n-2} \cdots 0_1 1_0)_r$	$= (1)_{10}$

where:

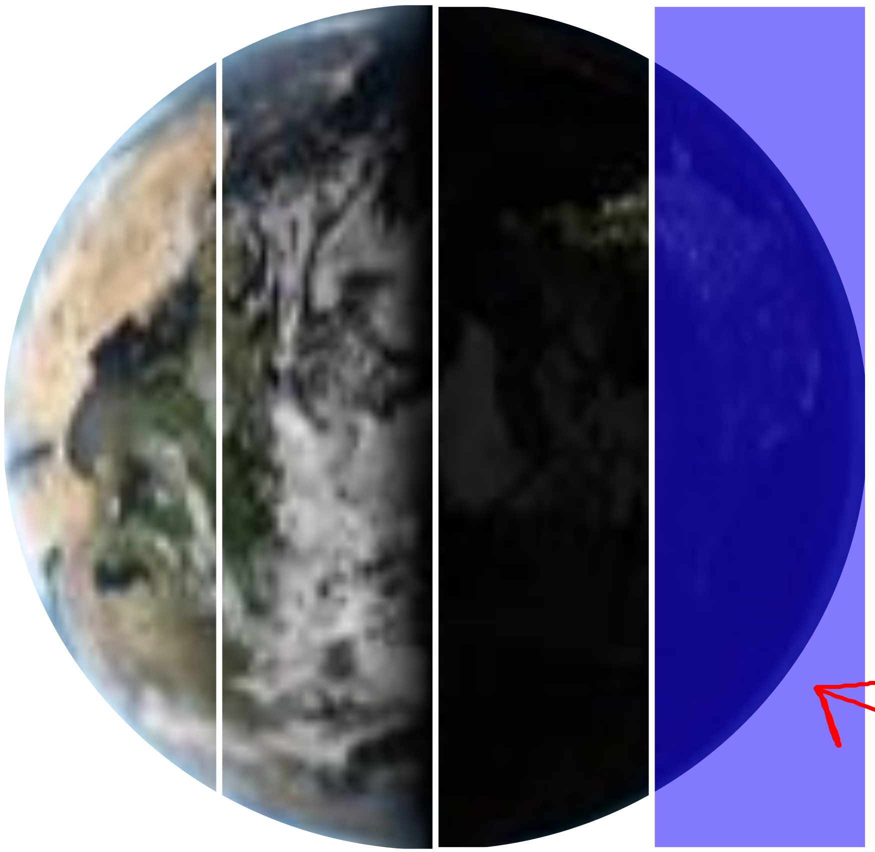
- r = radix (base)
- r^i = weight of position i
- n = number of digits in N



Moon's Distance to Sun: $(10.?)_4 = (4.5)_{10}$

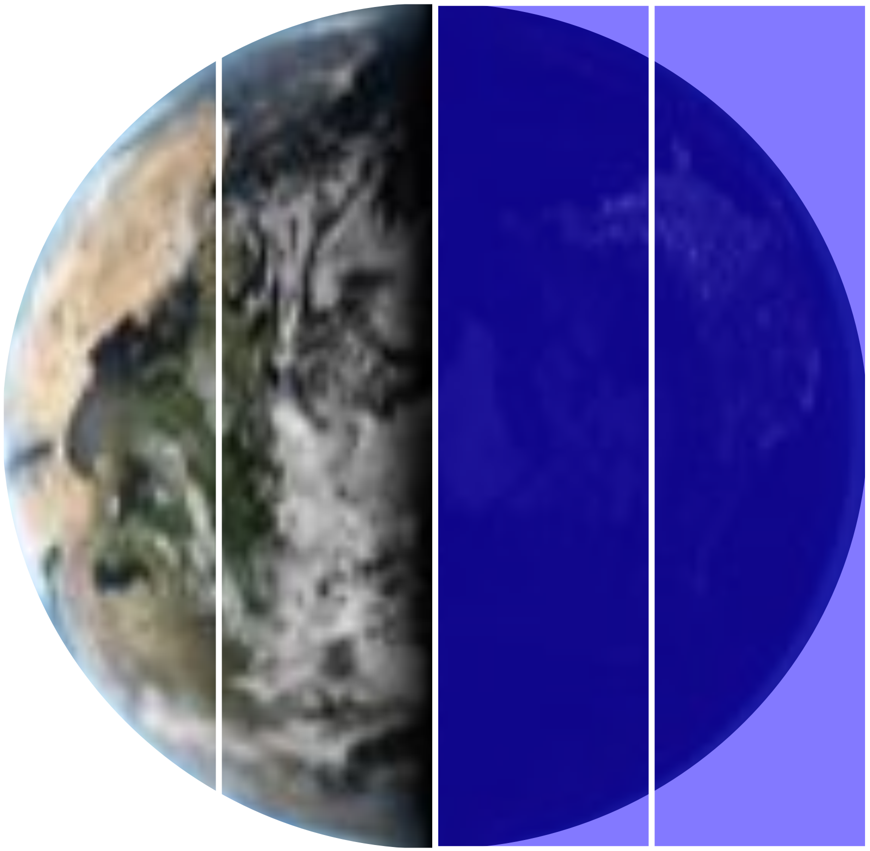
FRACTION

1 Earth

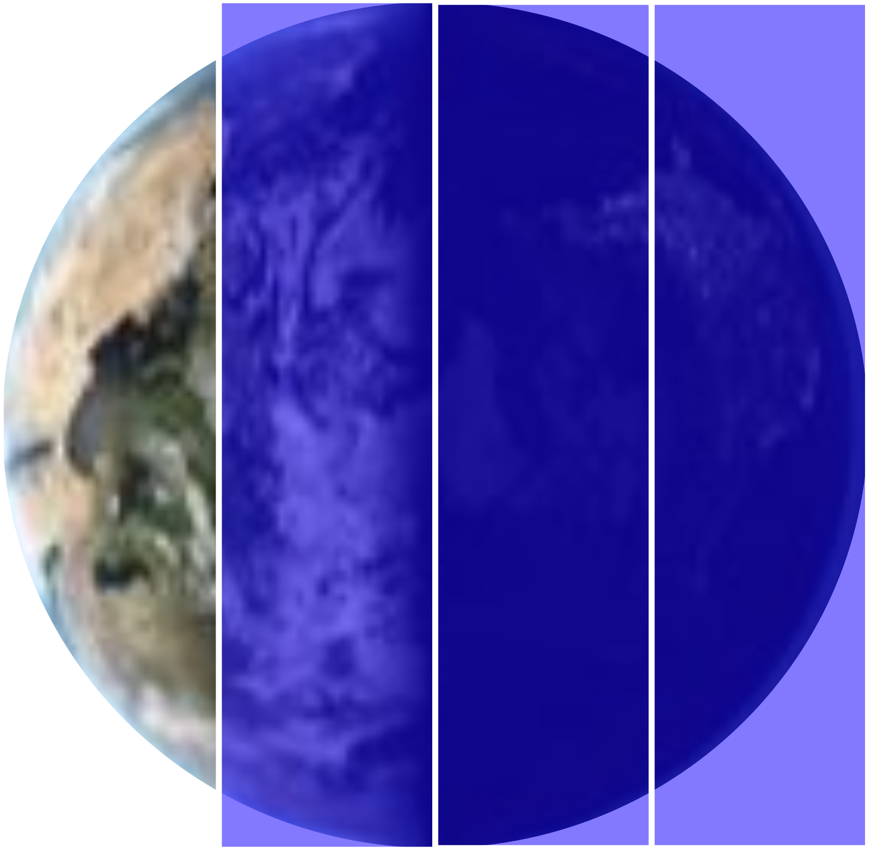


Fraction Point

Radix-4 (Base-4) = 1/4 Earth = 4^{-1} Earth = $(.1)_4$

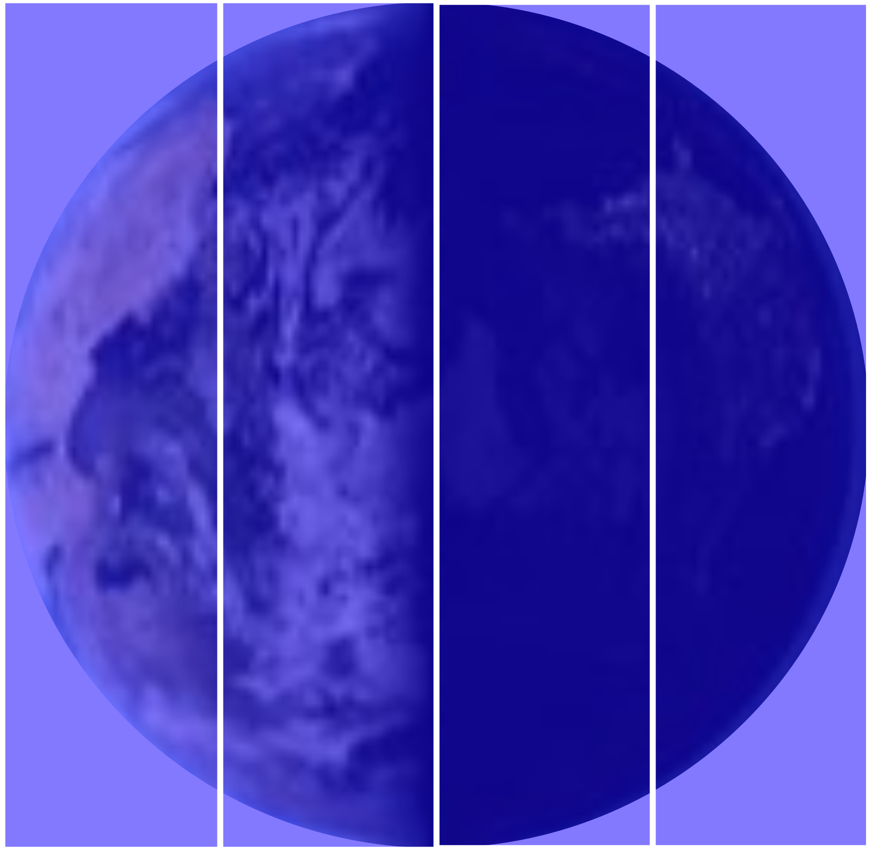


Radix-4 (Base-4) $= 2 \times 1/4$ Earth $= 2 \times 4^{-1}$ Earth$=(.2)_4$

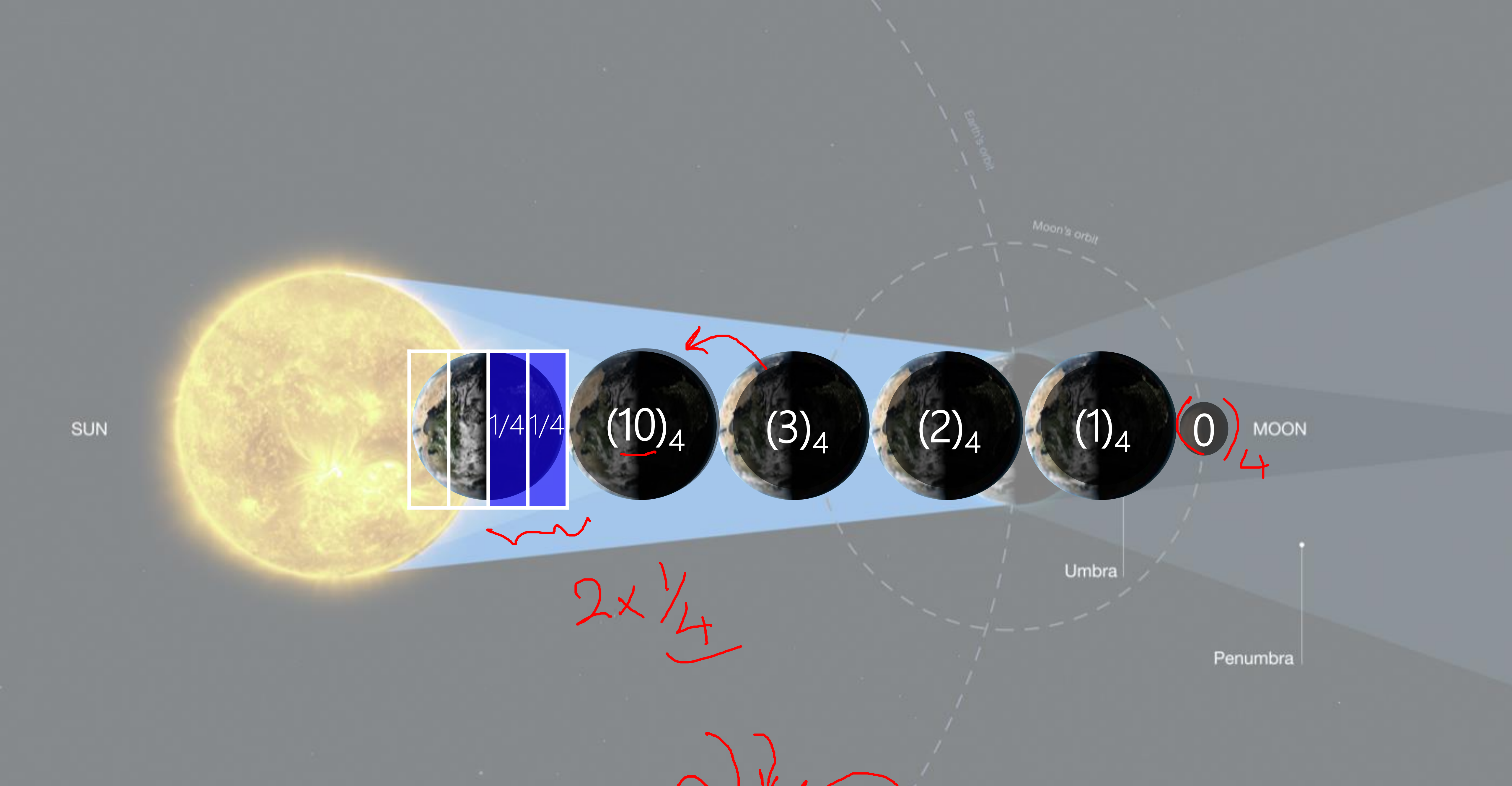


$1/4 = 4^{-1}$

Radix-4 (Base-4) = $3 \times 1/4$ Earth = 3×4^{-1} Earth=$(.3)_4$

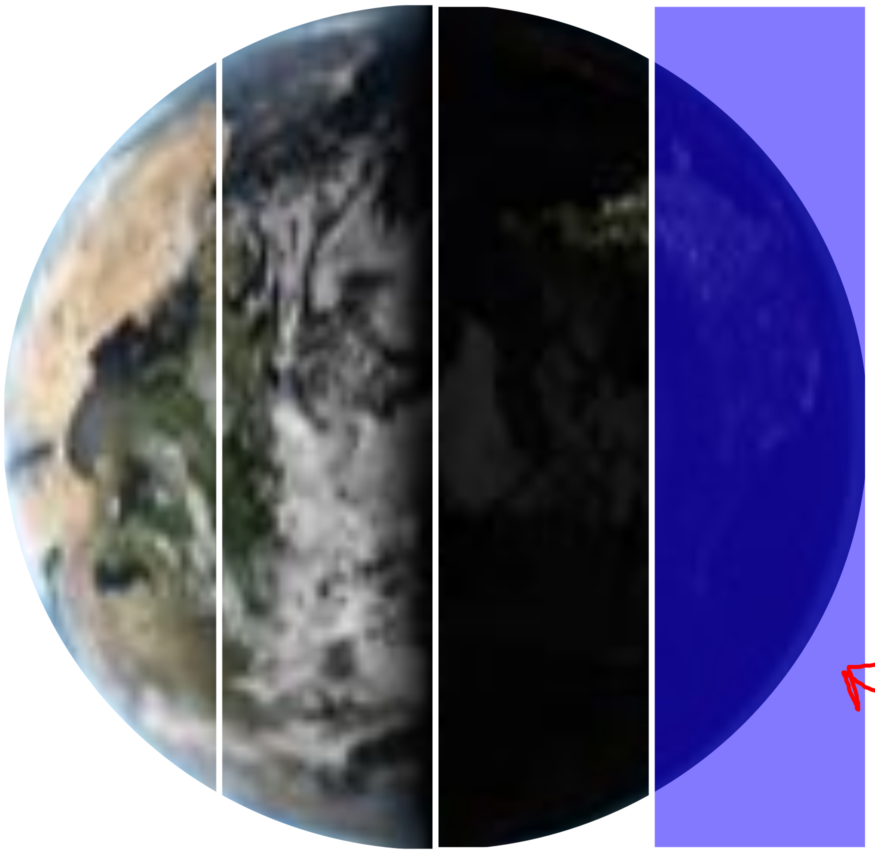


Radix-4 (Base-4) = $\underbrace{4} \times \underbrace{1/4}$ Earth = 4×4^{-1} Earth=$(\underline{1})_4$

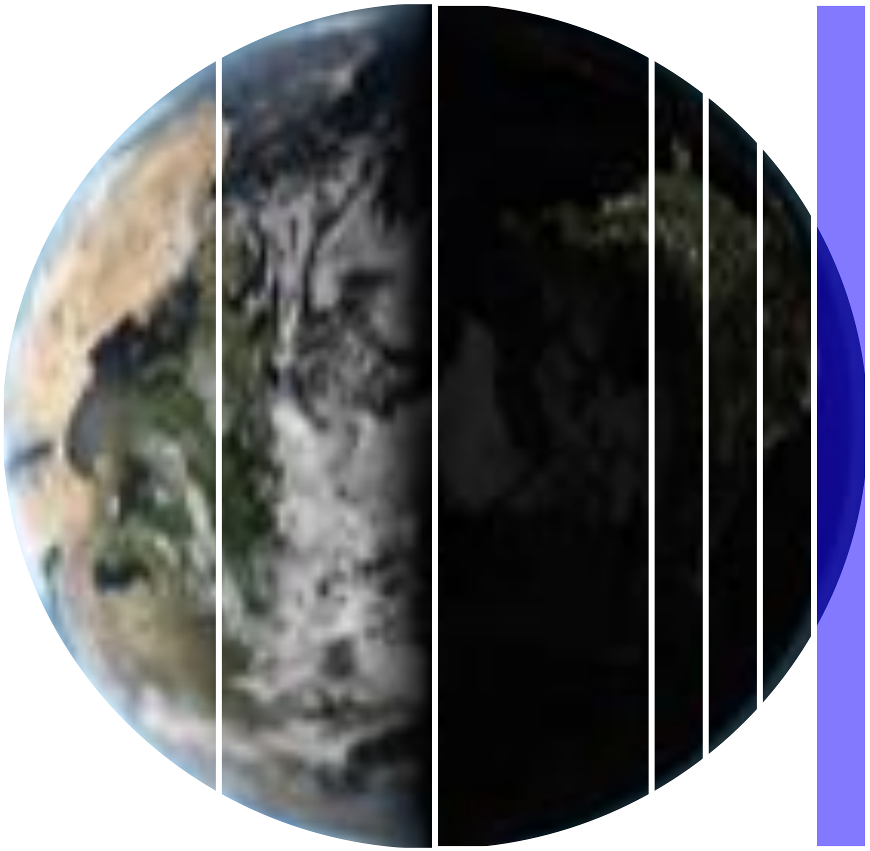


Moon's Distance to Sun in Radix-4: $(10.2)_4$

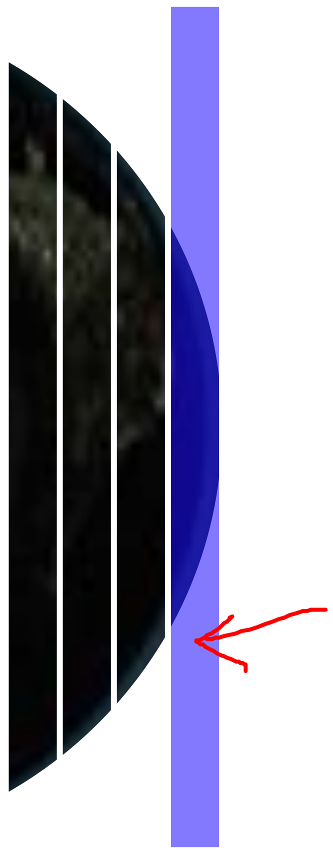
MORE PRECISION



Radix-4 (Base-4) = 1/4 Earth = 4^{-1} Earth = $(.1)_4$

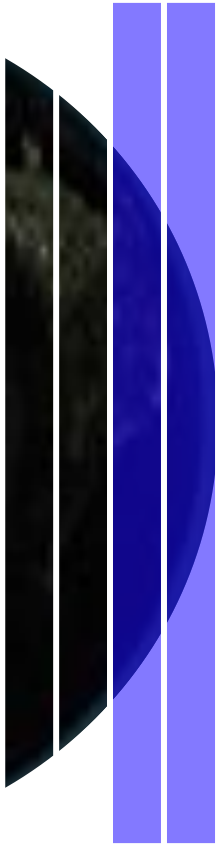


Radix-4 (Base-4) = (1/4)/4 Earth = 1/16 Earth = 4^{-2} Earth

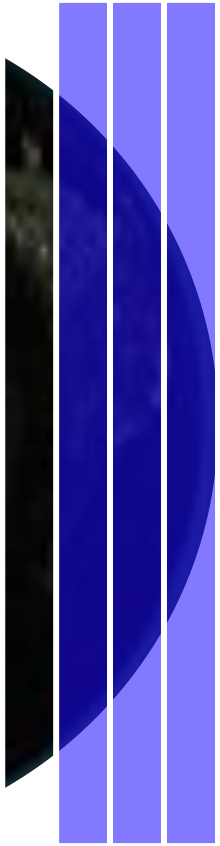


4^{-1} 4^{-2}

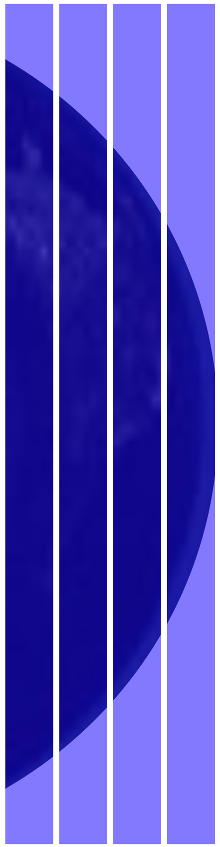
Radix-4 (Base-4) = 1/16 = 4^{-2} Earth = $(.01)_4$



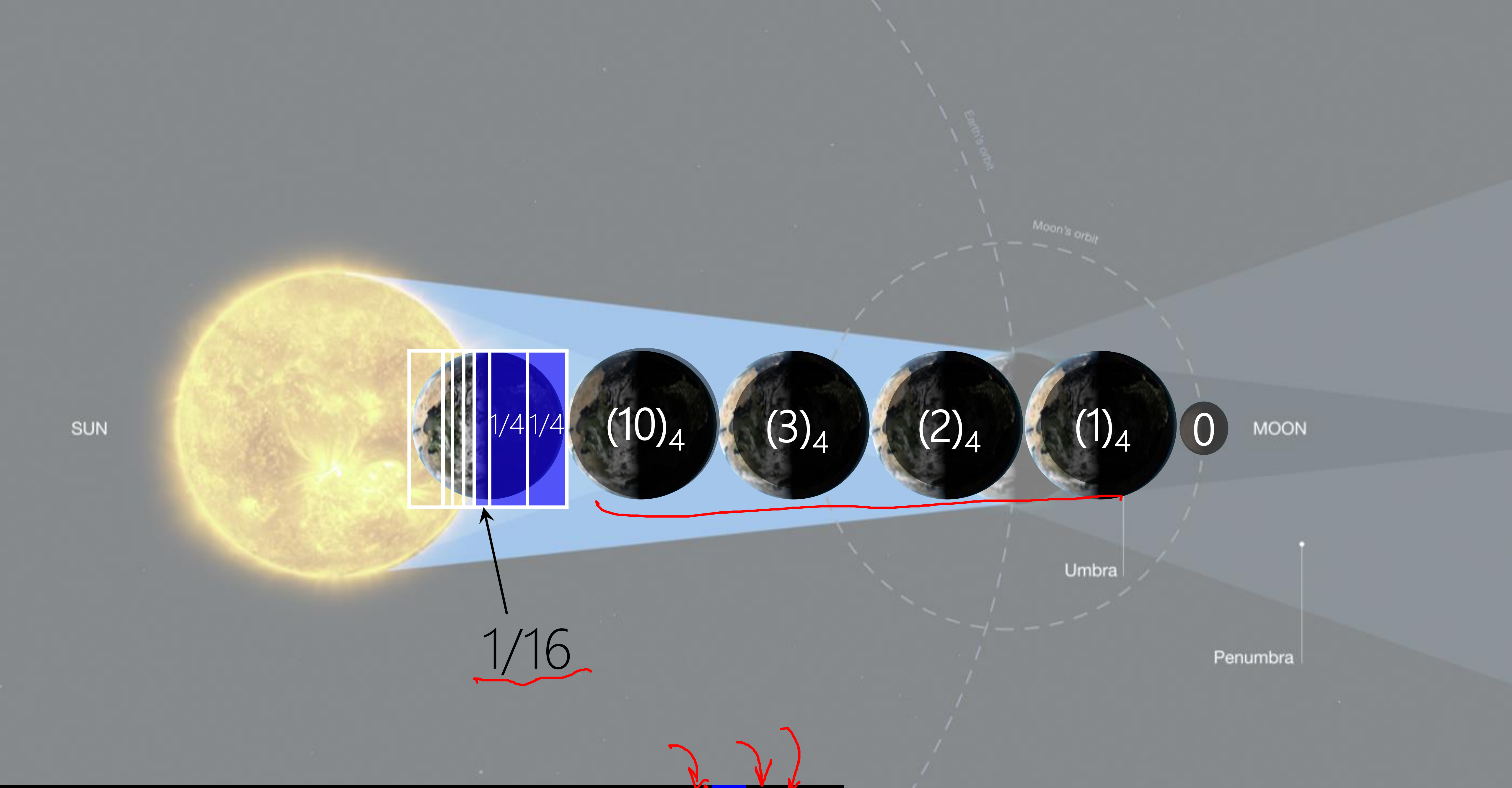
Radix-4 (Base-4) = $2 \times 1/16 = 2 \times 4^{-2}$ Earth = $(.02)_4$



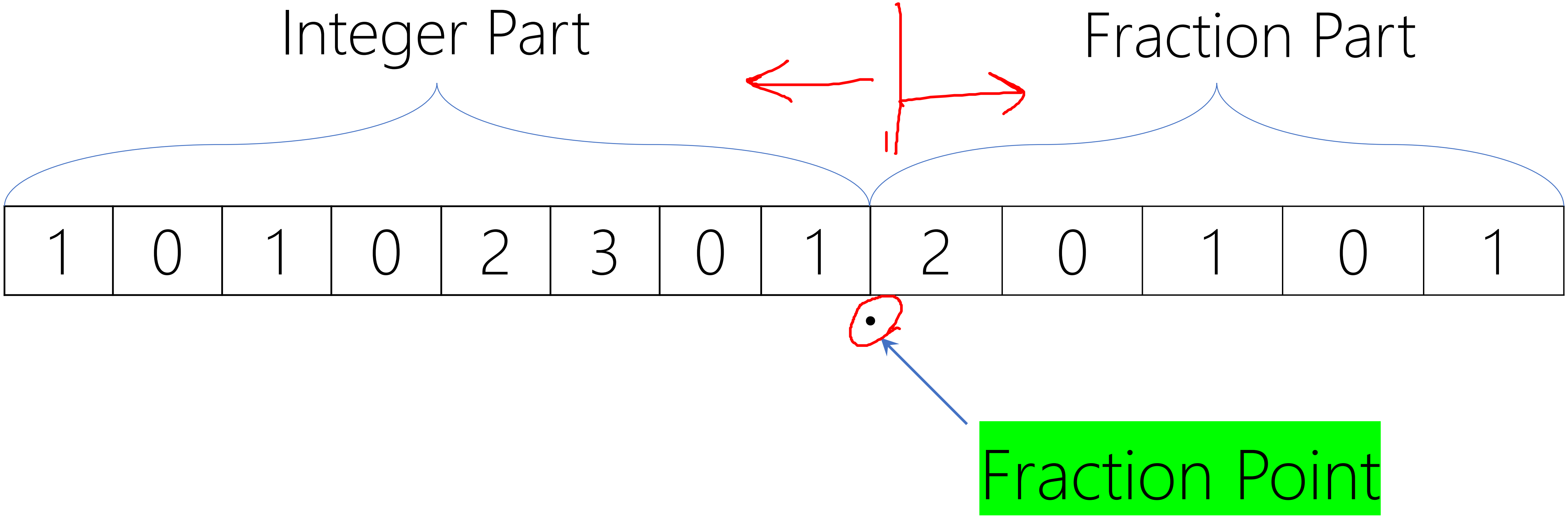
Radix-4 (Base-4) = $3 \times 1/16 = 3 \times 4^{-2}$ Earth = $(.03)_4$



Radix-4 (Base-4) = $4 \times 1/16 = 4 \times 4^{-2}$ Earth = $(.1)_4$



Moon's Distance to Sun in Radix-4: $(10.21)_4$



We don’t waste a position for the fraction point.
It’s location is defined already.

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1

.

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 ×16,384	0	1 ×1,024	0	2 ×64	3 ×16	0	1

$\times$

$\cdot$

$\sum$

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1{,}024}$

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 ×16,384	0	1 ×1,024	0	2 ×64	3 ×16	0	1
				17,584			

$\times$ · $\sum$ ·

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1,024}$

Integer Part

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0
1	0	1	0	2	3	0	1
1 ×16,384	0	1 ×1,024	0	2 ×64	3 ×16	0	1
				17,584			

$\times$

$\sum$

.

Fraction Part

4^{-1}	4^{-2}	4^{-3}	4^{-4}	4^{-5}
2	0	1	0	1
$\frac{2}{4}$	0	$\frac{1}{64}$	0	$\frac{1}{1{,}024}$
5166015625				

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + \cdots + d_0 r^0 . d_{-1} r^{-1} + d_{-2} r^{-2} + \cdots + d_{-m} r^{-m})_{10}$$

n

Fraction Point

where:

r = radix (base)

d_i = digit at position i, $0 \le d_i \le r - 1$

r^i = weight of position i

n = number of digits in integer part of N

m = number of digits in fraction part of N

Let $(N)_r$ be a radix-r (base-r) number in a positional weighting number system, then

$(-m-1)$ $-m$

$\times r$ $+$ r

Min $= (0_{n-1} \cdots 0_1 0_0 \, . \, 0_{-1} 0_{-2} \cdots 0_{-m-1} 0_{-m})_r = (0 \, . \, 0)_{10}$

Max $= ((r\text{-}1)_{n-1} \cdots (r\text{-}1)_0 \, . \, (r\text{-}1)_{-1} (r\text{-}1)_{-2} \cdots (r\text{-}1)_{-m-1} (r\text{-}1)_{-m})_r = (r^n\text{-}1 \, . \, ?\,)_{10}$

Unit $= (0_{n-1} \cdots 0_1 0_0 \, . \, 0_{-1} 0_{-2} \cdots 0_{-m-1} 1_{-m})_r = (r^{-m})_{10}$

$0 \, .$ $1 \times 1/r^m$

Lecture Assignment

where:

- r = radix (base)
- r^i = weight of position i
- n = number of digits in integer part of N
- m = number of digits in fraction part of N

PRACTICE RADIX-2

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

0000.001
0000.001 +

0000.010

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

What is the max in this system with these spaces?

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^0 + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

What is the max in this system with these spaces?

$$(1111.111)_2 = (15.875)_{10}$$

$2^3\ 2^2\ 2^1\ 2^0\ 2^{-1}\ 2^{-2}\ 2^{-3}$

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3}$ = 1/8 = 0.125
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3}$ = 1/4 = 0.25
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3}$ = 1/4 + 1/8 = 3/8 = 0.375
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3}$ = ½ = 0.5
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3}$ = 1/2 + 1/8 = 5/8 = 0.625
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3}$ = 1/2 + 1/4 = 3/4 = 0.75
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3}$ = 1/2 + 1/4 + 1/8 = 0.875
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3}$ = 1
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

$$
\begin{array}{r}
(0001.000)_2 \\
+\,(0000.001) \\
\hline
0001.001
\end{array}
$$

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^0 + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

Radix-2								Radix-10
Integer (n=4)					Fraction (m=3)			
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3}$ = 1/8 = 0.125
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3}$ = 1/4 = 0.25
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3}$ = 1/4 + 1/8 = 3/8 = 0.375
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3}$ = ½ = 0.5
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3}$ = 1/2 + 1/8 = 5/8 = 0.625
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3}$ = 1/2 + 1/4 = 3/4 = 0.75
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3}$ = 1/2 + 1/4 + 1/8 = 0.875
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3}$ = 1
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

1/8/2 2^{-4} 2^{-5} 1/4 = 1/16 /2 1/5 = 1/32 /2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

A. More in m!

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	2^{-4} 2^{-5}
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

0.02 → 0.01

No! The numbers in this system increments by 0.125 unit.

Solution?

A. More precision.

A. More fraction positions.

A. More in m! How much?

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

$(1.000)_2 = (1)_{10}$ => Error = 0.02_{10}

$(1.001)_2 = (1.125)_{10}$ => Error = $(0.105)_{10}$

Radix-2								
Integer (n=4)					Fraction (m=3)			Radix-10
2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*2^{-3} = 1/8 = 0.125$
0	0	0	0	.	0	1	0	$1*2^{-2} + 0*2^{-3} = 1/4 = 0.25$
0	0	0	0	.	0	1	1	$1*2^{-2} + 1*2^{-3} = 1/4 + 1/8 = 3/8 = 0.375$
0	0	0	0	.	1	0	0	$1*2^{-1} + 0*2^{-2} + 0*2^{-3} = ½ = 0.5$
0	0	0	0	.	1	0	1	$1*2^{-1} + 0*2^{-2} + 1*2^{-3} = 1/2 + 1/8 = 5/8 = 0.625$
0	0	0	0	.	1	1	0	$1*2^{-1} + 1*2^{-2} + 0*2^{-3} = 1/2 + 1/4 = 3/4 = 0.75$
0	0	0	0	.	1	1	1	$1*2^{-1} + 1*2^{-2} + 1*2^{-3} = 1/2 + 1/4 + 1/8 = 0.875$
0	0	0	1	.	0	0	0	$1*2^{0} + 0*2^{-1} + 0*2^{-2} + 0*2^{-3} = 1$
0	0	0	1	.	0	0	1	1.125
0	0	0	1	.	0	1	0	1.25
0	0	0	1	.	0	1	1	1.375
0	0	0	1	.	1	0	0	1.5
0	0	0	1	.	1	0	1	1.625
0	0	0	1	.	1	1	0	1.75
0	0	0	1	.	1	1	1	1.875
0	0	1	0	.	0	0	0	2

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! The numbers in this system increments by 0.125 unit.

Solution?

B. Find the closest number

$(1.000)_2 = (1)_{10}$ => Error = 0.02

$(1.001)_2 = (1.125)_{10}$ => Error = 0.105

PRACTICE RADIX-4

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
…								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
…								
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
…								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
…								
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
…								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
…								
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Radix-4								
Integer (n=4)					Fraction (m=3)			Radix-10
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
...								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
...								
3	3	3	3	.	3	3	0	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 0*4^{-3} = ?$
3	3	3	3	.	3	3	1	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 1*4^{-3} = ?$
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
...								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
0	0	0	1	.	0	0	1	1.015625
0	0	0	1	.	0	0	2	1.03125
...								
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

$(1.001)_4 = (1.015625)_{10}$ => Error = 0.004375

$(1.002)_4 = (1.03125)_{10}$ => Error = 0.01125

Radix-4								Radix-10
Integer (n=4)					Fraction (m=3)			
4^3	4^2	4^1	4^0	.	4^{-1}	4^{-2}	4^{-3}	
0	0	0	0	.	0	0	0	0
0	0	0	0	.	0	0	1	$1*4^{-3} = 1/64 = 0.015625$
0	0	0	0	.	0	0	2	$2*4^{-3} = 2/64 = 0.03125$
0	0	0	0	.	0	0	3	$3*4^{-3} = 3/64 = 0.046875$
0	0	0	0	.	0	1	0	$1*4^{-2} + 0*4^{-2} = 1/16 = 0.0625$
0	0	0	0	.	0	1	1	$1*4^{-2} + 1*4^{-2} = 1/16 + 1/64 = 0.078125$
0	0	0	0	.	0	1	2	$1*4^{-2} + 2*4^{-2} = 1/16 + 2/64 = 0.09375$
0	0	0	0	.	0	1	3	$1*4^{-2} + 3*4^{-2} = 1/16 + 3/64 = 0.109375$
0	0	0	0	.	0	2	0	$2*4^{-2} + 0*4^{-2} = 2/16 = 0.125$
...								
0	0	0	0	.	3	3	3	$3*4^{-1} + 3*4^{-2} + 3*4^{-3} = 0.984375$
0	0	0	1	.	0	0	0	1
0	0	0	1	.	0	0	1	1.015625
0	0	0	1	.	0	0	2	1.03125
...								
3	3	3	3	.	3	3	2	$3*4^3 + 3*4^2 + 3*4^1+3*4^0+3*4^{-1} + 3*4^{-2} + 2*4^{-3} = ?$
3	3	3	3	.	3	3	3	255.984375

Is it possible to show the number $(1.02)_{10}$ in this system with these spaces?

No! Why?

Solution:

A. More in m

B. Find the closest number

$(1.001)_4 = (1.015625)_{10}$ => Error = 0.004375

$(1.002)_4 = (1.03125)_{10}$ => Error = 0.01125

PRACTICE RADIX-[8,10,16]

At Home

CONVERSION

From Base-r to Base-r′