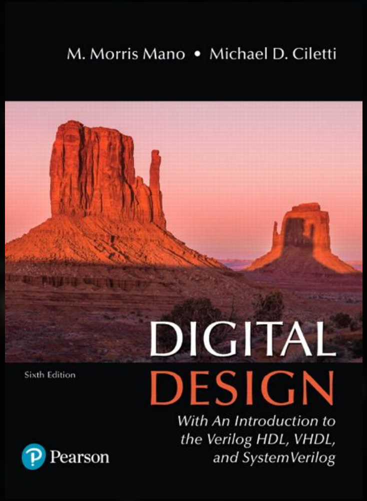
Yao Yao

Rest in Peace

https://www.youtube.com/watch?v=etlBZInTE-I


Office Hour
11:30AM-12:30PM
Lambton Tower 5111



Chapter 1
Digital Systems and Binary Numbers

SIGNED MAGNITUDE

SIGNED COMPLEMENT

r^{n-1}	r^{n-2}	r^{n-3}	...	r^2	r^1	r^0
0	Positive Numbers					
Nonzero	Negative Numbers					

Signed Magnitude

SIGNED MAGNITUDE ARITHMETIC

0	X	+	0	Y	=	0	X+Y
0	X	+	1	Y	=	X<Y (if borrow)	X−Y (if borrow, apply it)
1	X	+	1	Y	=	1	−(X+Y) = X+Y

0	X	−	0	Y	=	X<Y (if borrow)	X−Y (if borrow, apply it)
0	X	−	1	Y	=	0	X+Y
1	X	−	1	Y	=	X>Y (if borrow)	−X+Y=Y−X (if borrow, apply it)

SIGNED MAGNITUDE OVERFLOW

Overflow!

0	X	+	0	Y	=	$0 \to 1$	$X+Y$
0	X	+	1	Y	=	$X<Y$	$X-Y$
1	X	+	1	Y	=	$1 \to 1$	$X+Y$

0	X	−	0	Y	=	$X<Y$	$X-Y$
0	X	−	1	Y	=	$0 \to 1$	$X+Y$
1	X	−	1	Y	=	$X>Y$	$-X+Y=Y-X$

POLL

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WHY NOT
SIGNED MAGNITUDE

Give up left most position for sign! What are the wastes?

<table>
<tr><th>r^{n-1}</th><th>r^{n-2}</th><th>r^{n-3}</th><th>…</th><th>r^2</th><th>r^1</th><th>r^0</th></tr>
<tr><td>0</td><td colspan="6">Positive Numbers</td></tr>
<tr><td>Nonzero</td><td colspan="6">Negative Numbers</td></tr>
</table>

$$+0 \longrightarrow \text{Max} = r^{n-1} - 1 = \cancel{r^{n} - 1}$$

$$\text{Min} = -(r^{n-1} - 1) \longleftarrow -0$$

SIGNED MAGNITUDE

SIGNED COMPLEMENT

DIMINISHED RADIX COMPLEMENT

Given $(N)_r$ with n digits, the $(r - 1)$'s complement of N, i.e., its *diminished radix complement*, is defined as $(r^n - 1) - N$.

Base-2	2^5	2^4	2^3	2^2	2^1	2^0
$2^5-1=$		1	1	1	1	1
$N=$		1	0	1	0	1
$(2^5-1)-N=$		0	1	0	1	0
1's complement of $(10101)_2=(01010)_2=$ NOT on each digit						

(r–1)’s COMP. BASE-r

Base-r		r^{n-1}	…	r^2	r^1	r^0
$r^n-1=$		$r-1$	…	$r-1$	$r-1$	$r-1$
$N=$		d_{n-1}	…	d_2	d_1	d_0
$(r^n-1)-N=$		$r-1-d_{n-1}$	…	$r-1-d_{n-1}$	$r-1-d_{n-1}$	$r-1-d_{n-1}$
$(r-1)$'s complement of $(N)_r = (r-1) -$ Each digit						

RADIX COMPLEMENT

SIGNED COMPLEMENT

RADIX COMPLEMENT

Given $(N)_r$ with n digits, the r's complement of N, i.e., its *radix complement*, is defined as $r^n - N$.

Diminished Complement + 1

(r–1)'s Complement + 1 = $[(r^n - 1) - N] + 1 = r^n - N$

10^0		10^0
0	Base-10 Signed 10's comp.	see 0 interpret 0
1		see 1 interpret 1
2		see 2 interpret 2
3		see 3 interpret 3
4		see 4 interpret 4
5→–5		see 5 interpret –5
6→–4		see 6 interpret –4
7→–3		see 7 interpret –3
8→–2		see 8 interpret –2
9→–1		see 9 interpret –1

SIGNED 10's COMP. Base-10

n

Signed Radix Complement Base-10						
10^{n-1}	10^{n-2}	10^{n-3}	...	10^{2}	10^{1}	10^{0}

$0 <=$	Positive Numbers	$<= (10^{n}-1) \div 2$

Nothing to do!

Surface == Meaning

Signed Radix Complement Base-10						
10^{n-1}	10^{n-2}	10^{n-3}	…	10^2	10^1	10^0
$(10^n - 1) \div 2 + 1 <=$			Negative Numbers		$<= (10^n - 1)$	

Although we see positive numbers, we must interpret them negative!

How?

Surface != Meaning

Signed Radix Complement Base-10						
10^{n-1}	10^{n-2}	10^{n-3}	…	10^2	10^1	10^0

$(10^n - 1) \div 2 + 1 <=$	Negative Numbers	$<= (10^n - 1)$

– (10's comp. of the number we see)

9 9 9 9 9 9 9

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$n = 7$

$$0 <= \text{Positive Numbers} <= (10^7 - 1) \div 2$$

$$(10^7 - 1) \div 2 + 1 <= \text{Negative Numbers} <= (10^7 - 1)$$

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$n = 7$$

$$(10^7 - 1) \div 2 = 9{,}999{,}999 \div 2 = 4{,}999{,}999$$

$$5{,}805{,}074 > 4{,}999{,}999$$

This number must be interpreted negative!

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
5	8	0	5	0	7	4

$$= -\,(10\text{'s comp. }(5{,}805{,}074))$$

$$= -\,(9\text{'s comp. }(5{,}805{,}074) + 1)$$

$$= -\,(9{,}999{,}999 - 5{,}805{,}074 + 1)$$

$$= -\,(4{,}194{,}925 + 1)$$

$$= -\,(4{,}194{,}926)$$

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
X	X	X	X	X	X	X

– 3,450,256

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
X	X	X	X	X	X	X

– 3,450,256

We must find its positive complement to represent it!

How?

Signed Radix Complement Base-10						
10^6	10^5	10^4	10^3	10^2	10^1	10^0
6	5	4	9	7	4	4

$$
\begin{aligned}
&- 3{,}450{,}256 \\
&= 10\text{'s comp } (3{,}450{,}256) \\
&= 9\text{'s comp } (3{,}450{,}256) + 1 \\
&= 9{,}999{,}999 - 3{,}450{,}256 + 1 \\
&= 6{,}549{,}743 + 1 \\
&= 6{,}549{,}744
\end{aligned}
$$

SIGNED 16’s COMP. Base-16

Base-16	1	B	F	2	B	4
	Is this a negative number or positive?					

<table>
<tr><td rowspan="2">Base-16</td><td>1</td><td>B</td><td>F</td><td>2</td><td>B</td><td>4</td></tr>
<tr><td colspan="6">How many positions?
n = 6</td></tr>
</table>

Base-16	1	B	F	2	B	4
	What is the maximum positive number? $(16^6-1)\div 2 = (8{,}388{,}607)_{10} = (7FF{,}FFF)_{16}$					

Base-16	1	B	F	2	B	4
	This number is less than $(7FF,FFF)_{16}$					

$+$ $(\cancel{+}1BF,2B4)_{16}$

Base-16	E	B	F	2	B	4
	Is this a negative number or positive?					

<table>
<tr><td rowspan="2">Base-16</td><td>E</td><td>B</td><td>F</td><td>2</td><td>B</td><td>4</td></tr>
<tr><td colspan="6">How many positions?
$n = 6$</td></tr>
</table>

Base-16	E	B	F	2	B	4
	What is the maximum positive number? $(16^6 - 1) \div 2 = (8{,}388{,}608)_{10} = (7FF{,}FFF)_{16}$					

Base-16	E	B	F	2	B	4
	This number is greater than $(7FF,FFF)_{16}$ The number is negative, but what is the number?					

Base-16	E	B	F	2	B	4
	$= -\,(16\text{'s comp. }(\text{EBF,2B4}))$ $= -\,(15\text{'s comp. }(\text{EBF,2B4}) + 1)$ $= -\,(\text{FFF,FFF} - \text{EBF2B4} + 1)$ $= -\,(\text{140,D4B} + 1)$ $= -\,(\text{140,D4C})$					

Base-16	X	X	X	X	X	X
	– (140,D4C)					

	E	B	F	2	B	4
Base-16	X	X	X	X	X	X
	$- 140,D4C$ $= 16\text{'s comp}(140,D4C)$ $= 15\text{'s comp}(140,D4C) + 1$ $= FFF,FFF - 140,D4C + 1$ $= EBF,2B3 + 1$ $= EBF,2B4$					

<table>
<tr><td rowspan="2">Base-16</td><td>E</td><td>B</td><td>F</td><td>2</td><td>B</td><td>4</td></tr>
<tr><td colspan="6">– 140,D4C
= 16’s comp(140,D4C)
= 15’s comp(140,D4C) + 1
= FFF,FFF – 140,D4C + 1
= EBF,2B3 + 1
= EBF,2B4</td></tr>
</table>

SIGNED r's COMP. Base-r

Base-r in Radix Complement

r^{n-1}	r^{n-2}	r^{n-3}	…	r^2	r^1	r^0

$$0 <= \text{Positive Numbers} <= (r^n - 1) \div 2$$

Nothing to do!

Base-2: 0,111,…,111
Base-4: 1,333,…,333
Base-8: 3,777,…,777
Base-10: 4,999,…,999
Base-16: 7,FFF,…,FFF

3

Base-r in Radix Complement

r^{n-1}	r^{n-2}	r^{n-3}	…	r^2	r^1	r^0

$(r^n - 1) \div 2 + 1 <=$ Negative Numbers $<= (r^n - 1)$

Base-2: 1,000,…,000

Base-4: 2,000,…,000

Base-8: 4,000,…,000

Base-10: 5,000,…,000

Base-16: 8,000,…,000

(5000)

(−5000)

9999
− 5000
4999
+ 1

We see positive number, but we interpret negative!

$= -$ (r's comp. (#)) $= -$ ((r–1)'s comp. (#) + 1)

SIGNED 2'S COMPLEMENT BASE-2

6. Show that in 2's complement binary system, the highest significant position acts like a sign (not the same) as in signed-magnitude binary system. Is this true for any radix-r number system? Justify your answer.

POLL

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SIGNED r'S COMPLEMENT ARITHMETIC

$$X + Y = X+Y$$

$$X - Y = X + (r\text{'s comp } Y)$$

Last Carry $\longrightarrow$ Ignore

X + Y = X+Y

X − Y = X+(r's comp Y)

One adder for both addition and subtraction!

SIGNED r'S COMPLEMENT ADDITION

+ Base-2		1	0	1	0	1	1
		0	0	1	1	1	0

6 p —

+ Base-2		1	0	1	0	1	1
		$> (2^6 - 1) \div 2 = (011{,}111)_2$ $= -(2\text{'s comp}(\#))$ $= -(010101)_2$ $= -(21)_{10}$					

+ Base-2							
		0	0	1	1	1	0
		$< (2^6-1)\div 2 = (011{,}111)_2$ $= +(001110)_2$					

			1	1	1		
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1

			1	1	1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		1	1	1	0	0	1

$> (2^6 - 1) \div 2 = (011,111)_{10}$

$= -(2\text{'s comp}(\#))$

$= -(000111)_2$

$= -(7)_{10}$

SIGNED r'S COMPLEMENT SUBTRACTION

$-$	$-\cancel{(21)_{10}}$	1	0	1	0	1	1
Base-2	$+\cancel{(8)_{10}}$	0	0	1	0	0	0

$-$ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(8)_{10}$	0	0	1	0	0	0
		We must convert the subtraction to addition. Eq., addition with $-(8)_{10}$!					

$-$	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$+(8)_{10}$	0	0	1	0	0	0

$= -\ (2\text{'s comp. } (001000))$

$= -\ (1\text{'s comp}(001000) + 1)$

$= -\ (\text{NOT}(001000) + 1)$

$= -\ (110111 + 1)$

$= -\ (111000)$

+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0

Last Carry → Ignore

	$\cancel{1}$	1	1				
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1
		Done! This is the Result.					

		1	1				
+	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$-(8)_{10}$	1	1	1	0	0	0
		1	0	0	0	1	1

$> 2^6 - 1 \div 2 = (011{,}111)_{10}$

The number is negative:

= − (2's comp. (100011))

= −(011101) +29

		1	1				
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(8)_{10}$	1	1	1	0	0	0
	$-(29)_{10}$	1	0	0	0	1	1

SIGNED r'S COMPLEMENT OVERFLOW

X	+	Y	=	X+Y

The + result of two negative numbers → positive

The + result of two positive numbers → negative

$$X - Y = X + (r\text{'s comp } Y)$$

The + result of two negative numbers → positive

The + result of two positive numbers → negative

OVERFLOW

Example

$-$ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0

$-$ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$+(14)_{10}$	0	0	1	1	1	0
		We must convert the subtraction to addition. Eq., addition with $-(14)_{10}$!					

−	$-(21)_{10}$	1	0	1	0	1	1
Base-2	$+(14)_{10}$	0	0	1	1	1	0
		= (2's comp. (001110)) = (1's comp(001110) + 1) = (NOT(001110) + 1) = (110001 + 1) = (110010)					

0 11111

+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0

0 1111

Last Carry → Ignore

	1				1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0
		0	1	1	1	0	1

					1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0
		0	1	1	1	0	1

$< 2^6 - 1 \div 2 = (011{,}111)_{10}$

$= (011101)_2$ Positive Number

					1		
+ Base-2	$-(21)_{10}$	1	0	1	0	1	1
	$-(14)_{10}$	1	1	0	0	1	0
		Overflow! Don't rely on the result!					

BINARY CODE

Will be covered later. Stay tuned!