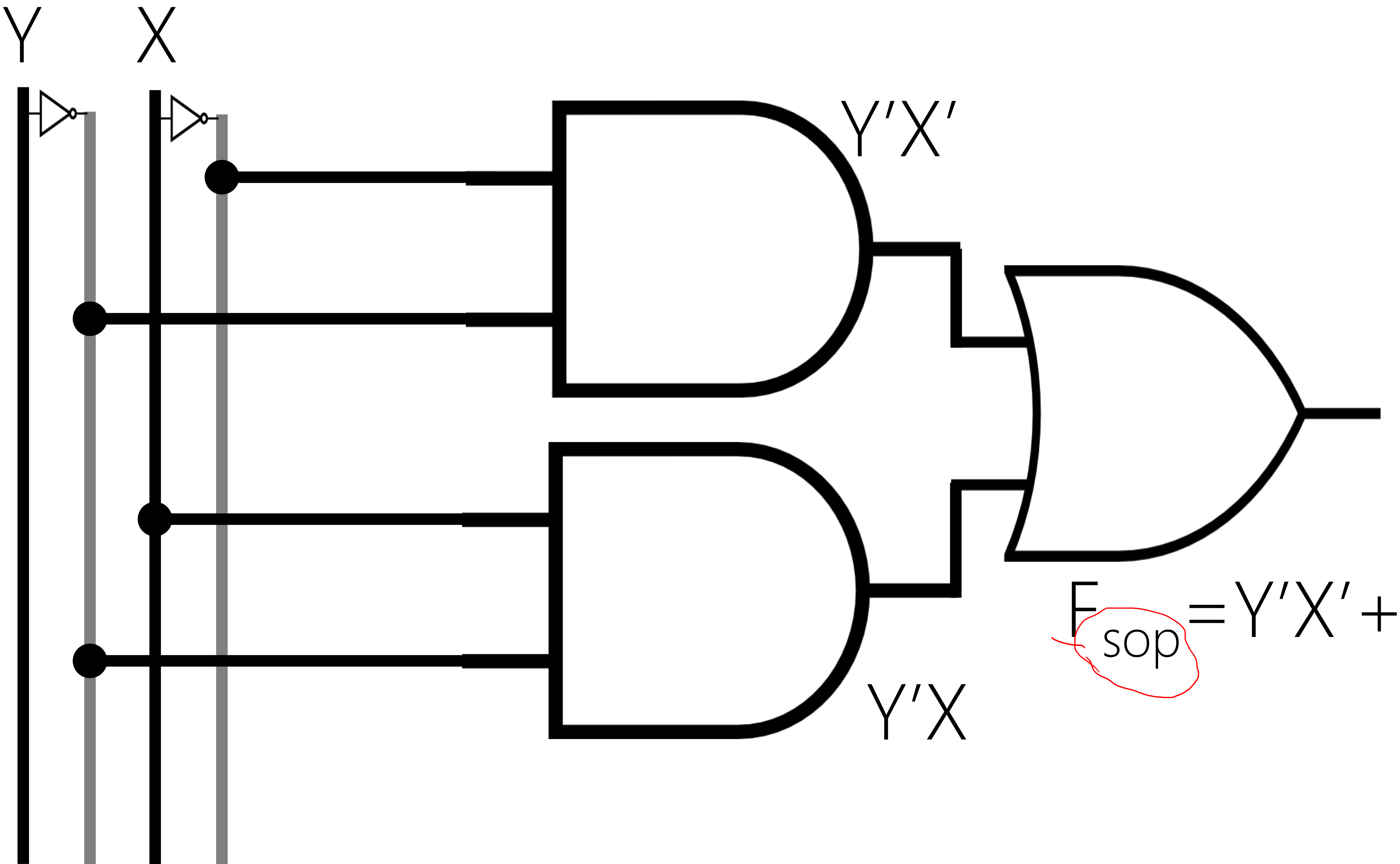
DESIGN II

a new algorithm for designing any logic circuits, given truth table

Y	X	$F=m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0



$$F_{sop} = Y'X' + Y'X$$

Y	X	$F=m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = m_0 + m_1$	$F' = m_2 + m_3$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	1

Y	X	$F=m_0+m_1$	$F'=m_2+m_3$	$(F')'=(m_2+m_3)'$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	0	1	0

1

Augustus De Morgan
(1806–1871)
Mathematician
Logician

2

DE MORGAN'S LAWS

- 1 $(YX)' = Y' + X'$
- 2 $(Y+X)' = Y'X'$

MAXTERM

aka. Standard Sum

$+$

X' vs. X

1 binary variable appear either:
- in its normal form X, or
- in its complement form X'

$M_0 = m'_0$	$(X') = (X')' = X$
$M_1 = m'_1$	X'

$$Y+X \text{ vs. } Y+X' \text{ vs. } Y'+X \text{ vs. } Y'+X'$$

2 binary variables appear either in one of these forms:

Augustus De Morgan (1806–1871) Mathematician Logician

DE MORGAN'S LAWS

- $(YX)' = Y' + X'$
- $(Y+X)' = Y'X'$

$(y')' + (x')'$

$M_0 = m'_0$	$(\overbrace{Y'X'}^{m_0})' = Y+X$
$M_1 = m'_1$	$(Y'X)' = Y+X'$
$M_2 = m'_2$	$(YX')' = Y'+X$
$M_3 = m'_3$	$(\underbrace{YX}_{m_3})' = Y'+X'$

$Z+Y+X$ vs. $Z+Y+X'$ vs. …

3 binary variables appear either in one of these forms: how many?

$Z+Y+X$ vs. $Z+Y+X'$ vs. …

3 binary variables appear either in one of these forms: how many?
Each variable can take 2 forms (normal and complement)
We have 3 variables, $2 \times 2 \times 2 = 2^3 = 8$

$M_0=m'_0$	$(Z'Y'X')'=Z+Y+X$
$M_1=m'_1$	$(Z'Y'X)'=Z+Y+X'$
$M_2=m'_2$	$(Z'YX')'=Z+Y'+X$
$M_3=m'_3$	$(Z'YX)'=Z+Y'+X'$
$M_4=m'_4$	$(ZY'X')'=Z'+Y+X$
$M_5=m'_5$	$(ZY'X)'=Z'+Y+X'$
$M_6=m'_6$	$(ZYX')'=Z'+Y'+X$
$M_7=m'_7$	$(ZYX)'=Z'+Y'+X'$

$$A_n + \cdots A_2 + A_1 + A_0 \text{ vs. } A_n + \cdots A_2 + A_1 + A'_0 \ \ldots$$

n binary variables appear either in one of these forms: how many?

Each variable can take 2 forms (normal and complement)

We have n variables, $2\times2\times2\times \cdots \times2 = 2^n$

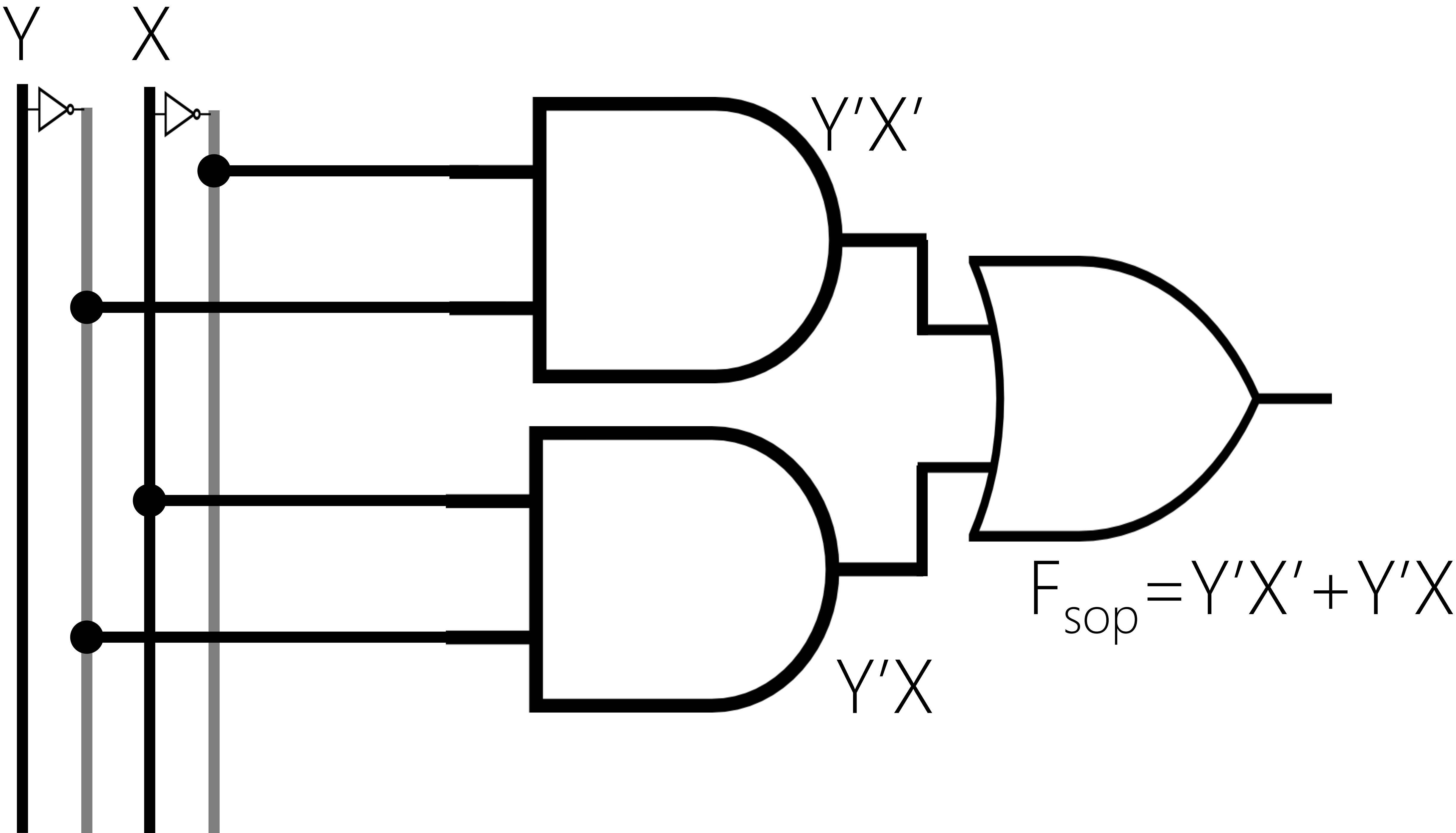
$M_0=m'_0$	$A_n+A_2+A_1+A_0$
$M_1=m'_1$	$A_n+\cdots A_2+A_1+A'_0$
$M_2=m'_2$	$A_n+\cdots A_2+A'_1+A_0$
$\vdots$	$\vdots$
$\vdots$	$\vdots$
$M_{2^n-3}=m'_{2^n-3}$	$A'_n+\cdots A'_2+A_1+A'_0$
$M_{2^n-2}=m'_{2^n-2}$	$A'_n+\cdots A'_2+A'_1+A_0$
$M_{2^n-1}=m'_{2^n-1}$	$A'_n+\cdots A'_2+A'_1+A'_0$

TRUTH TABLE

en.wikipedia.org/wiki/Truth_table

TRUTH TABLE $\leftrightarrow$ MAXTERM

Y	X	$F=m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0


Y
X
Y'X'
Y'X
F_sop = Y'X' + Y'X

Y	X	$F = m_0 + m_1$	$F' = m_2 + m_3$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	0	1

Y	X	$F=m_0+m_1$	$F'=m_2+m_3$	$(F')'=(m_2+m_3)'$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	0	1	0

Y	X	$F=(F')'=(m_2+m_3)'=m'_2m'_3$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F=(F')'=m'_2m'_3=M_2M_3$
0	0	1
0	1	1
1	0	0
1	1	0

$((Y'X')')$

$(Y + X) = 0$

$1 \to 0 \quad 0$

Y	X	$F=(F')'=m'_2m'_3=M_2M_3$
0	0	1
0	1	1
1	0	0
1	1	0

$(Y+X)' = Y' + X'$

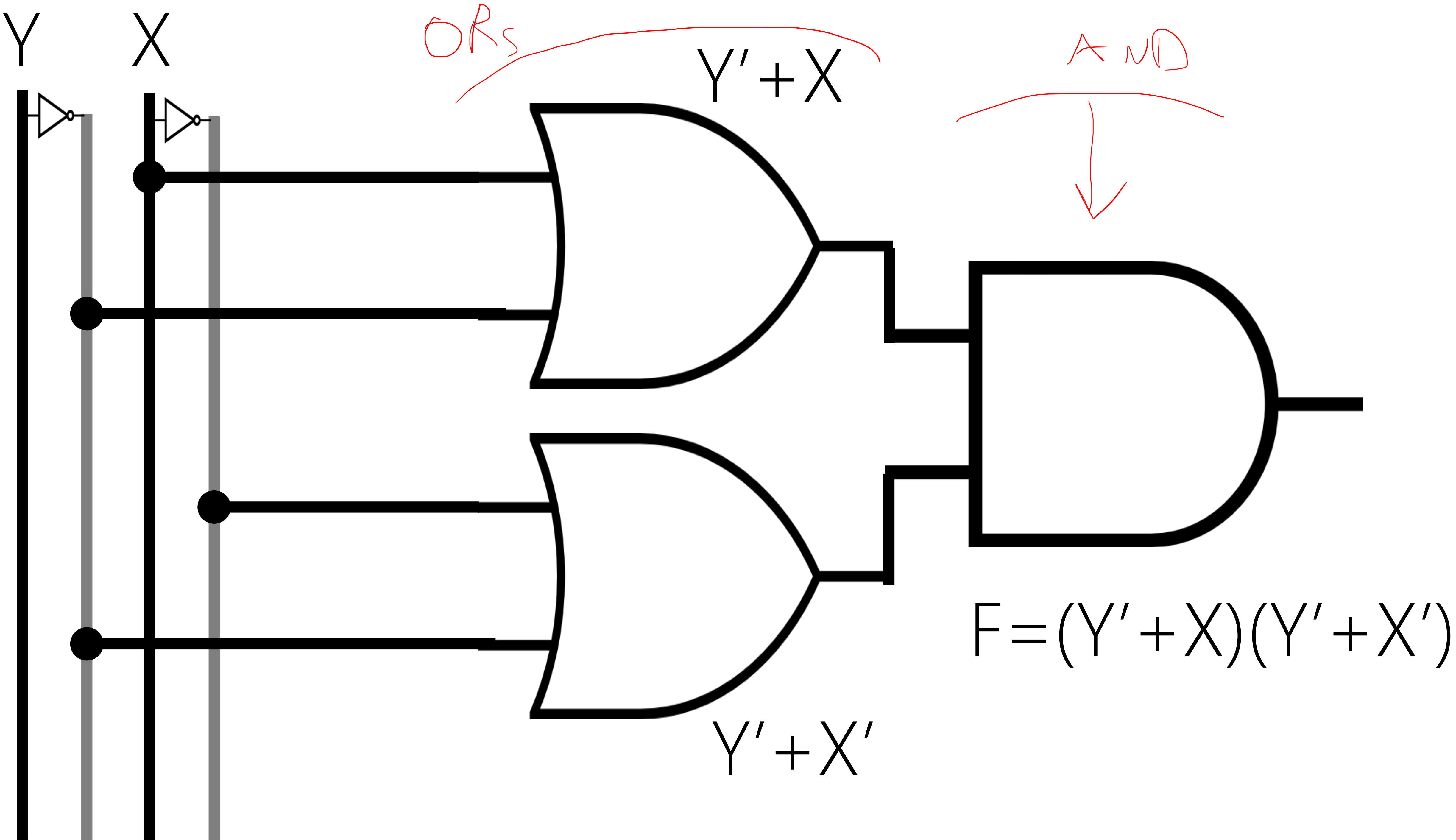
$1' + 1'$

$= 0 + 0 = 0$

Y	X	$F=(F')'=M_2M_3=\prod \mathbf{M}(2,3)$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F=\prod M(2,3)=(Y'+X)(Y'+X')$
0	0	1
0	1	1
1	0	0
1	1	0

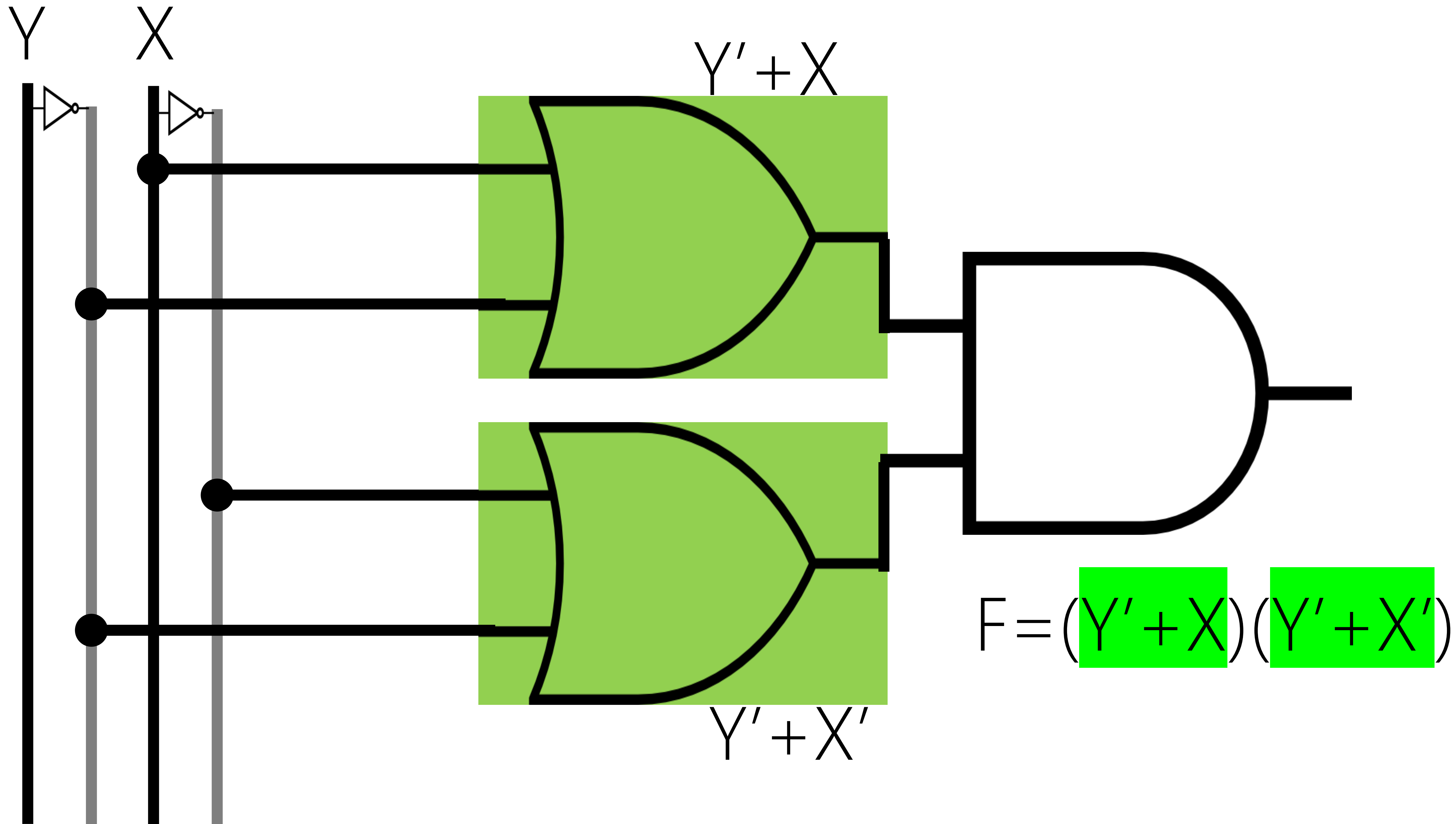
(OR) AND (OR)

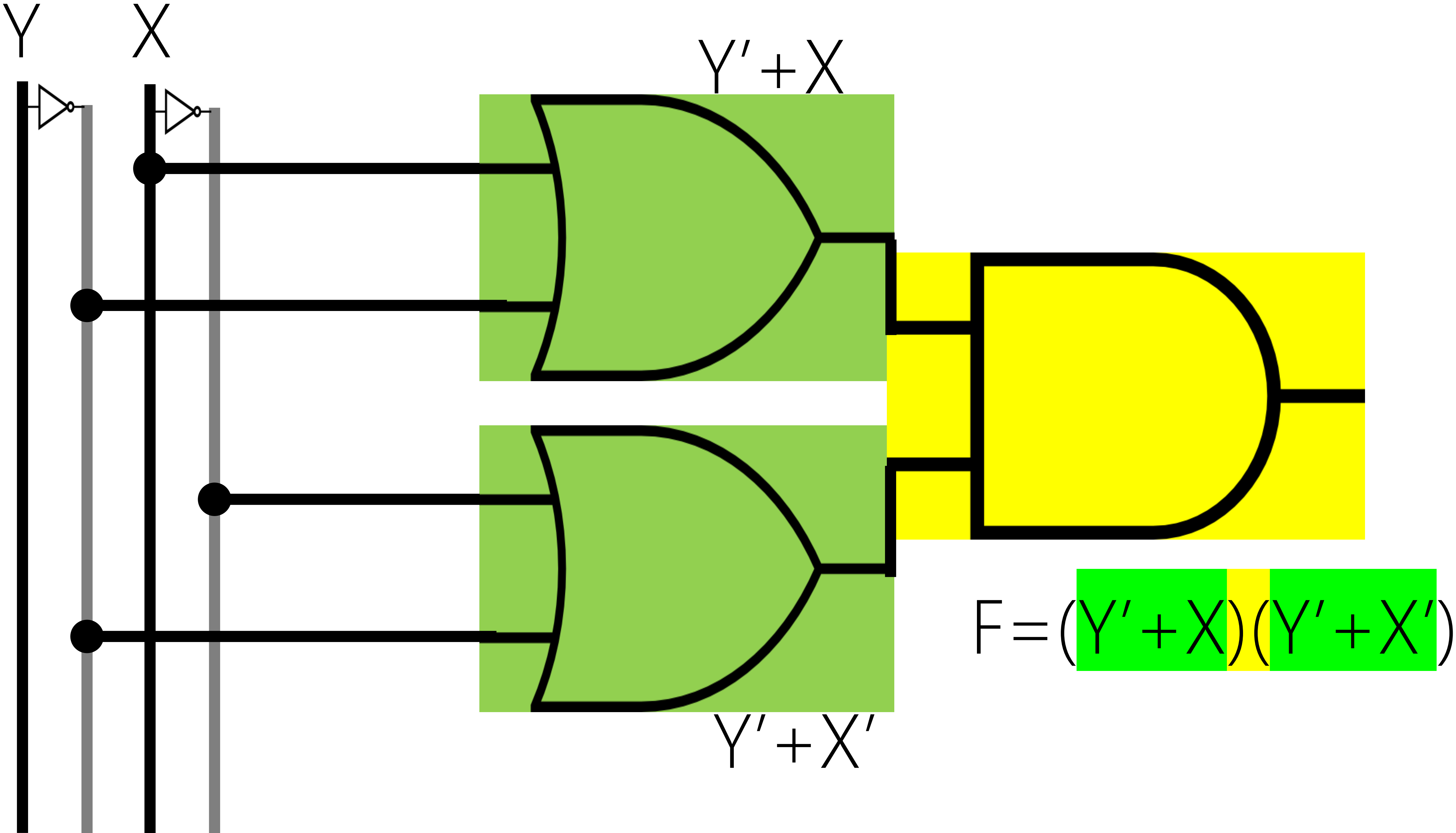

Y
X
ORs
AND
Y'+X
Y'+X'
F=(Y'+X)(Y'+X')

PRODUCT OF SUMS (POS)

AND

OR

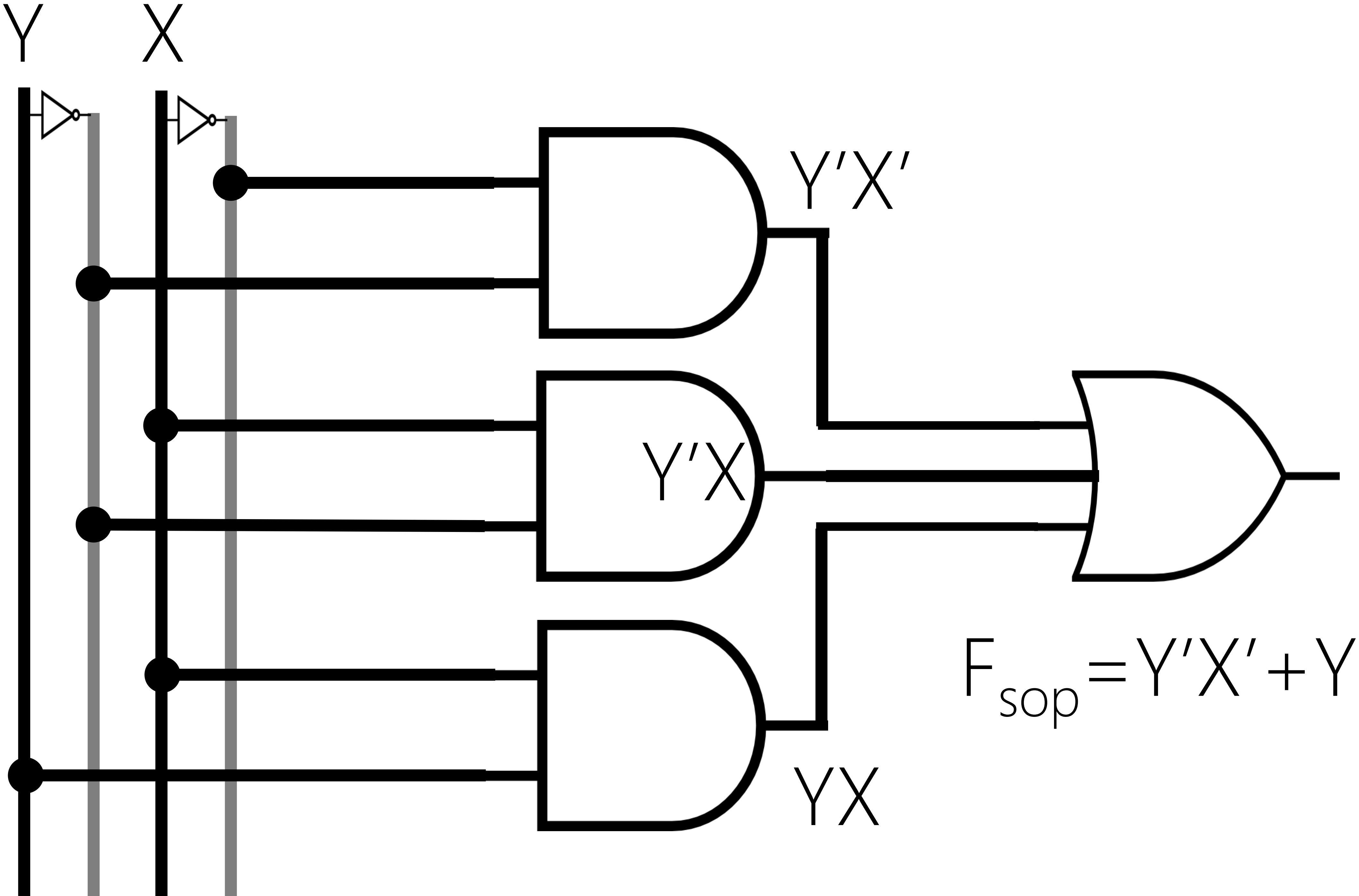

Y
X
Y′+X
Y′+X′
F=(Y′+X)(Y′+X′)


Y
X
Y′+X
Y′+X′
F=(Y′+X)(Y′+X′)

2 LEVELS
OR → AND

Y	X	$F=F(Y,X)=m_0+m_1+m_3=\sum \boldsymbol{m}(0,1,3)$
0	0	1
0	1	1
1	0	0
1	1	1

F =

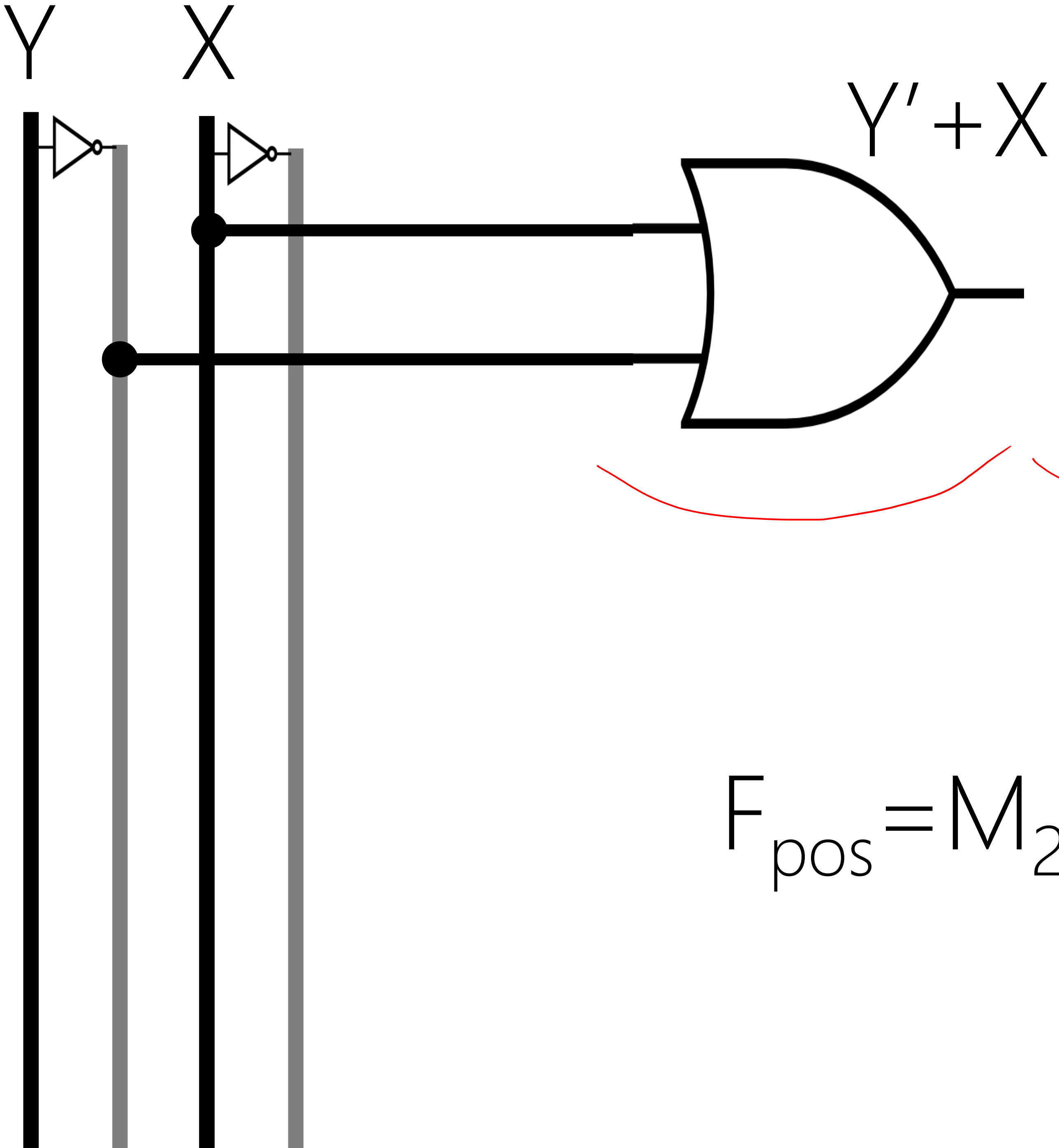


$$F_{sop} = Y'X' + Y'X + YX$$

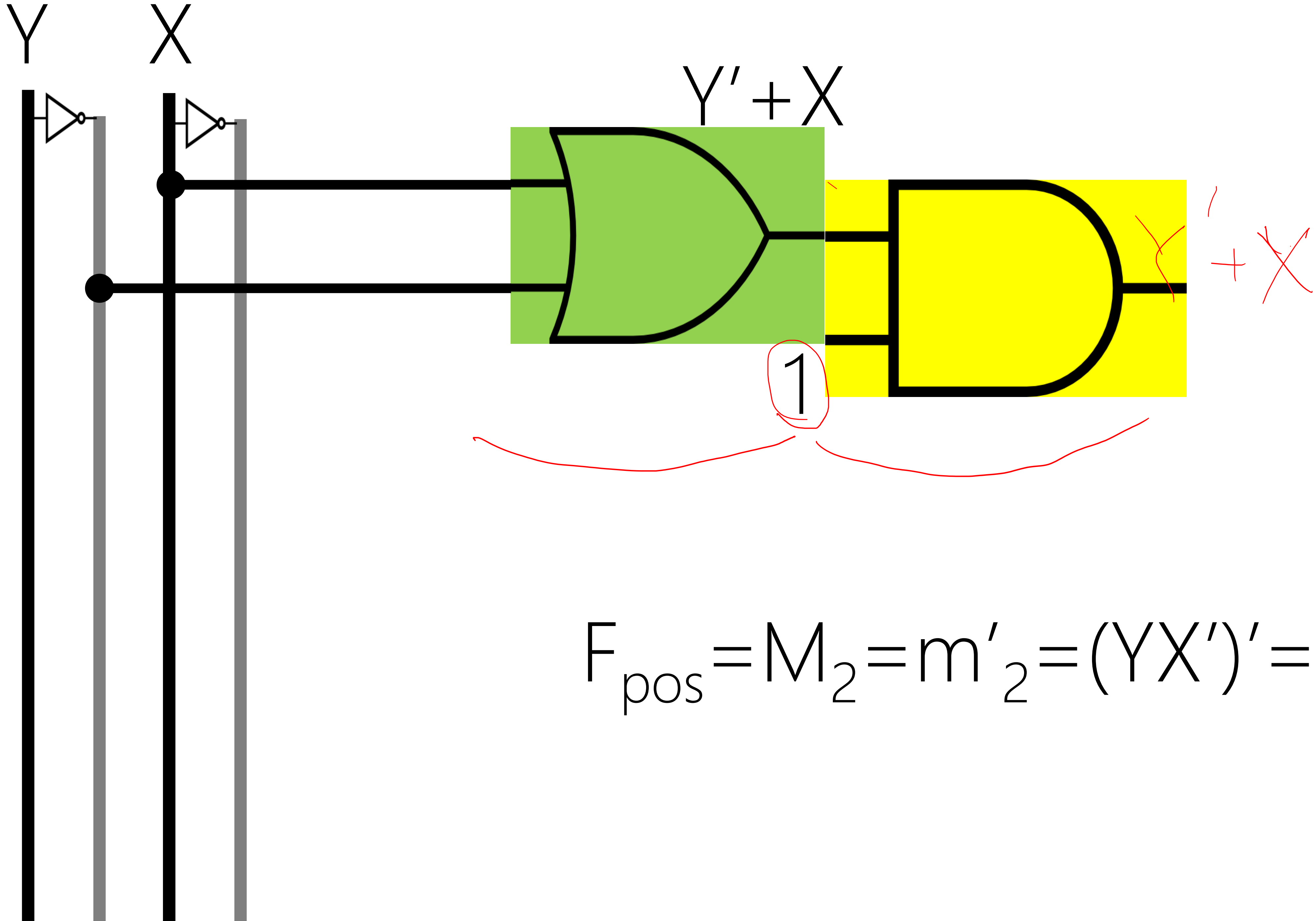
Y	X	$F=\sum m(0,1,3)$	$F'=m_2$
0	0	1	0
0	1	1	0
1	0	0	1
1	1	1	0

Y	X	$F=\sum m(0,1,3)$	$F'=m_2$	$(F')'=m'_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1

Y	X	$F=\sum m(0,1,3)$	$F'=m_2$	$(F')'=M_2$
0	0	1	0	1
0	1	1	0	1
1	0	0	1	0
1	1	1	0	1



$$F_{pos} = M_2 = m'_2 = (YX')' = (Y'+X)$$



$$F_{pos}=M_2=m'_2=(YX')'=(Y'+X)(1)$$

DESIGN I vs. II
SoP vs. PoS

Lecture Assignment

hfani@uwindsor.ca

Given 3 inputs, design a circuit to determine if there is even number of 1

POS $\rightarrow \prod M_i$

100 → 0

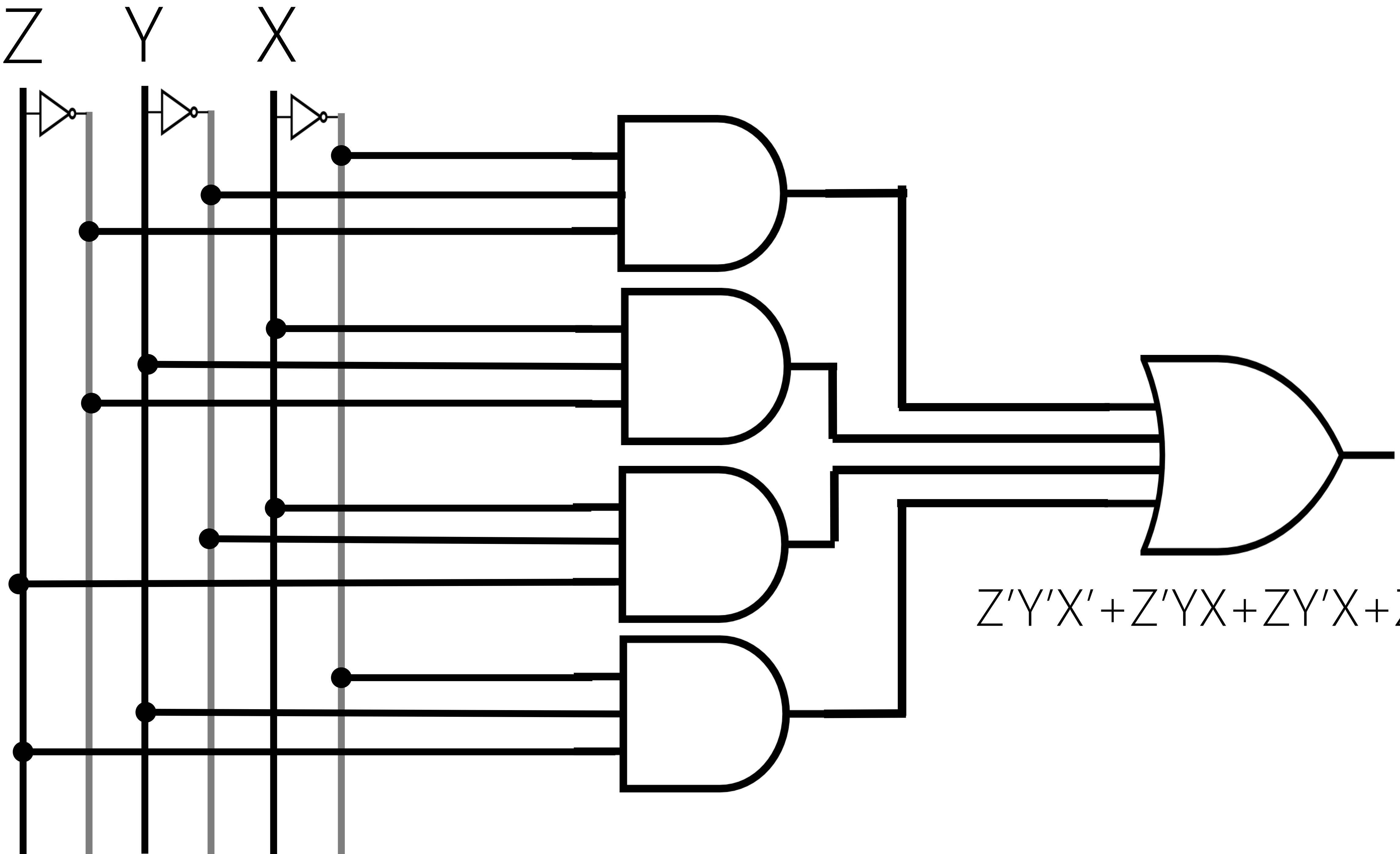
101 → 1

011 → 1

Z	Y	X	F(Z,Y,X)=?
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Z	Y	X	F(Z,Y,X)=?
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=\Sigma m(0,3,5,6)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



$Z'Y'X' + Z'YX + ZY'X + ZYX'$

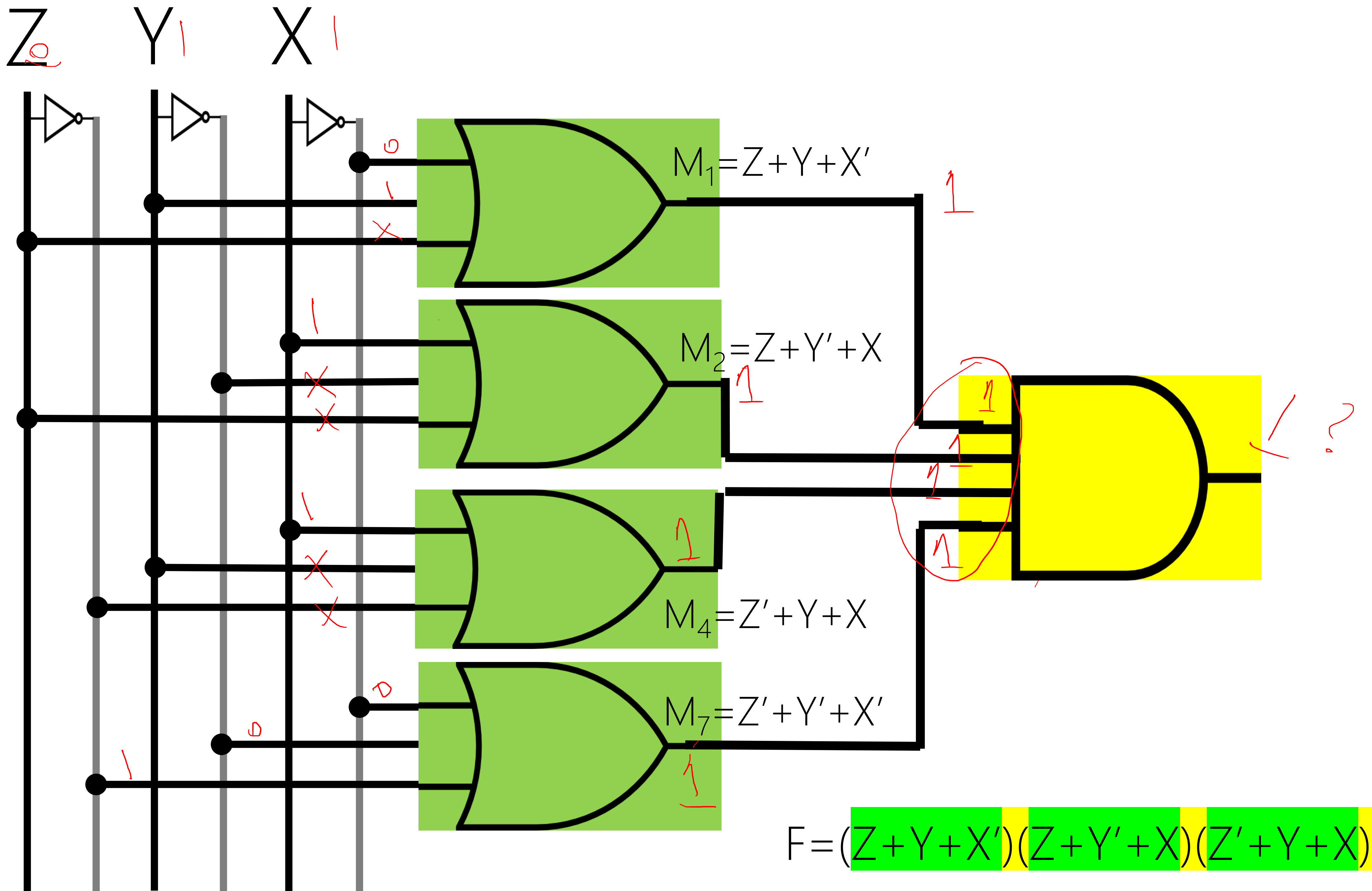
Z	Y	X	$F(Z,Y,X)=M_1$
0	0	0	1
0	0	1	(m_1) 0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2$
0	0	0	1
0	0	1	0
0	1	0	$(m_2)'$ 0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2M_4$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2M_4M_7$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=M_1M_2M_4M_7=\prod \mathbf{M}(1,2,4,7)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



$$F = (Z+Y+X')(Z+Y'+X)(Z'+Y+X)(Z'+Y'+X')$$

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