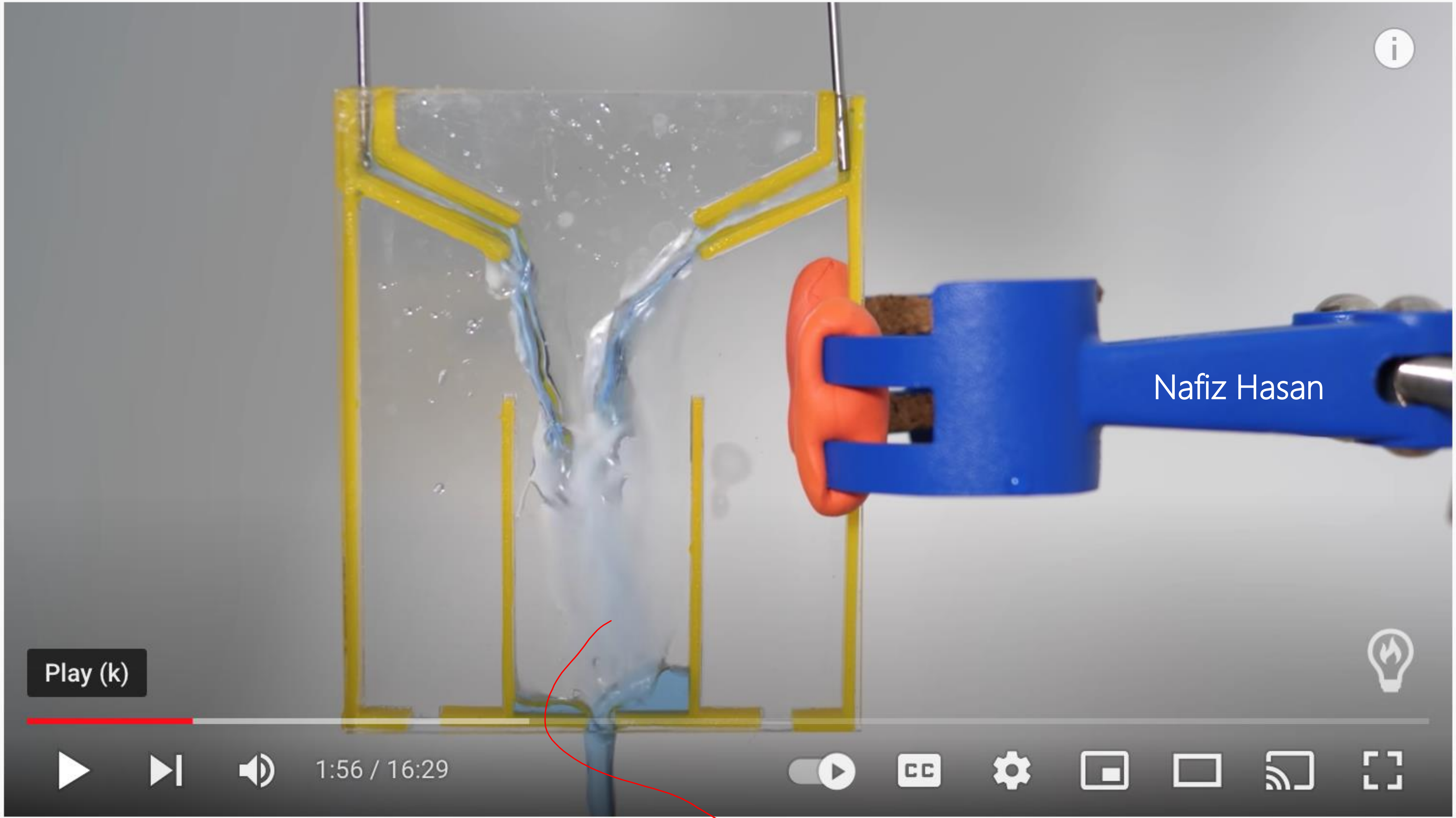

I Made A Water Computer And It Actually Works

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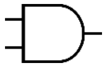
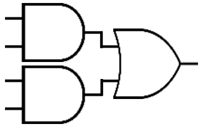
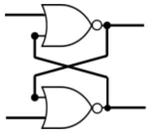
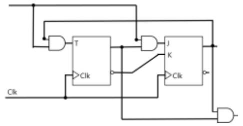
Number Systems | $(12)_{10} \rightarrow (1100)_2$

| Logic Gates | 

| Combinational Logic | 

| Flip-Flops | 

| Sequential Logic | 

| Computer Systems

MINIMIZATION

Number of Gates

Number of Inputs (2-input vs 4-input)

Number of Interconnections

Propagation Time

Cost of Gates

Circuit Area

. . .

A circuit may not satisfy all due to conflicting constraints!

$\{AND, OR\} \leftarrow \{x, y, \dots\}, \{\cdot, +\}, \{0, 1\}$

$x \neq y$

$e_{\cdot} \quad e_{+}$

$0' \Rightarrow 1 \quad 1' \Rightarrow 0$

$0 + 1 = e_{AND}$

$0 \cdot 1 = e_{OR}$

MINIMIZATION

I) Boolean Algebra (algebraically)

- Needs to be smart. It is hard due to guesswork (which rules to apply?)
- If the number of variables (ABCDEF…) and/or number of minterms (MAXTERMS) grows
- No Algorithm
- Is the result minimal?!

BASIC THEOREMS

Prove by postulates

$$X + X = X$$

$$X + X + X + \dots + X = X$$

$$X + X = X$$

$$X + X + X + \ldots + X = X$$

$X + X \;=$

$= (X + X)\, 1$ using identity $e_{\times} = 1$

$$X + X = X$$

$$X + X + X + \ldots + X = X$$

$X + X =$

$= (X + X)\, 1$ using identity $e_\times = 1$

$= (X + X)\, (X + X')$ using complement property

$$X + X = X$$

$$X + X + X + \ldots + X = X$$

$X + X \;=$

$= (X + X)\,1$ using identity $e_\times = 1$

$= (X + X)\,(X + X')$ using complement property

$= X + (XX')$ using distributive property of $+$ over $\times$

$$X + X = X$$

$$X + X + X + \ldots + X = X$$

$X + X =$

$= (X + X)\,1$ using identity $e_\times = 1$

$= (X + X)\,(X + X')$ using complement property

$= X + (XX')$ using distributive property of $+$ over $\times$

$= X + 0$ using complement property

$$X + X = X$$

$$X + X + X + \ldots + X = X$$

$X + X =$

$= (X + X)\,1$ using identity $e_\times = 1$

$= (X + X)\,(X + X')$ using complement property

$= X + (XX')$ using distributive property of $+$ over $\times$

$= X + 0$ using complement property

$= X$ using identity property of $e_+ = 0$

$$X + 1 = 1$$

$$X + Y + Z + \ldots + 1 = 1$$

$$X + 1 = 1$$

$$X + Y + Z + \ldots + 1 = 1$$

$X + 1 \quad =$

$= (X + 1)\, 1$ using identity $e_x = 1$

$$X + 1 = 1$$
$$X + Y + Z + \ldots + 1 = 1$$

$X + 1 =$

$= (X + 1)\ 1$ using identity $e_x = 1$

$= (X + 1)\ (X + X')$ using complement property

$$X + 1 = 1$$

$$X + Y + Z + \ldots + 1 = 1$$

$X + 1 \quad =$

$= (X + 1)\,1$ using identity $e_{\times} = 1$

$= (X + 1)\,(X+X')$ using complement property

$= X + (1X')$ using distributive property of + over ×

$$X + 1 = 1$$

$$X + Y + Z + \ldots + 1 = 1$$

$X + 1 =$

$= (X + 1)\,1$ using identity $e_{\times}=1$

$= (X + 1)\,(X+X')$ using complement property

$= X + (1X')$ using distributive property of + over $\times$

$= X + X'$ using identity $e_{\times}=1$

$$X + 1 = 1$$

$$X + Y + Z + \ldots + 1 = 1$$

$X + 1 =$

$= (X + 1)\ 1$ using identity $e_{\times}=1$

$= (X + 1)\ (X+X')$ using complement property

$= X + (1X')$ using distributive property of $+$ over $\times$

$= X + X'$ using identity $e_{\times}=1$

$= 1$ using complement property

$$X + XY = X$$

$$X + XY + XZW + \ldots + XWAD = X$$

Absorption

$$X + XY = X$$

$$X + XY + XZW + \dots + XWAD = X$$

$X + XY =$

$= X1 + XY$ using identity $e_\times = 1$

$= X(1 + Y)$ using distributive property of $\times$ over $+$

$= X1$ using previous theorem $x+1=1$

$= X$ using identity $e_\times = 1$

Absorption

DUALITY THEORY

DUALITY

Dual(F) = OR ⇄ AND, 1 ⇄ 0

Dual(F) may or may not equal to F!

DUALITY

$$X+1 \rightleftarrows X0$$

$$X+X' \rightleftarrows XX'$$

$$(X+Y)' \rightleftarrows (XY)'$$

DUALITY
FOR COMPLEMENT (NOT)

DUALITY
FOR COMPLEMENT (NOT)

$$F = A + (BC) \rightarrow F' = [A + (BC)]' = [A'(BC)'] = [A'(B' + C')]$$

DUALITY
FOR COMPLEMENT (NOT)

$$F = A + (BC) \rightarrow \mathrm{Dual}(F) \rightarrow A(B+C) \rightarrow F' = A'(B' + C')$$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$F' = [AB(C+(DL'G(B'+A+E)))(H+(J'A'B))]'$$
$$F' = [AB(\ldots)(H+(J'A'B))]'$$
$$F' = [A'+B'+(\ldots)'+(H+(J'A'B))']$$
$$F' = [A'+B'+(\ldots)'+(H'(J'A'B)')]$$
$$F' = [A'+B'+(\ldots)'+(H'(J+A+B'))]$$
$$F' = [A'+B'+(C+(\ldots))'+(H'(J+A+B'))] \text{ OMG!}$$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\text{Dual}(F) = A$$

$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$

What's F'?

$\mathrm{Dual}(F) = A+$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\mathrm{Dual}(F) = A+B$$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\mathrm{Dual}(F) = A+B+($$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\mathrm{Dual}(F) = A+B+(C$$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\mathrm{Dual}(F) = A+B+(C($$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$Dual(F) = A+B+(C(D+L'+G+(B'AE)))(H(J'+A'+B))$$

$$F = AB(C+(DL'G(B'+A+E)))(H+(J'A'B))$$

What's F'?

$$\text{Dual}(F) = A+B+(C(D+L'+G+(B'AE)))(H(J'+A'+B))$$

Complement all variables only!

$$F' = A'+B'+(C'(D'+L+G'+(BA'E')))(H'(J+A+B'))$$

DUALITY THEOREM

A postulate or a proved theorem for F, also a postulate or a proved theorem for Dual(F)

DUALITY

$$\{X+1=1\} \rightleftarrows \{X0 = 0\}$$

$$\{X+X'=1\} \rightleftarrows \{XX'=0\}$$

$$\{(X+Y)'=X'Y'\} \rightleftarrows \{(XY)'=X'+Y'\}$$

$x + xy + xyz + \dots = x$

$$X(X+Y) = X$$

$$X(X+Y)(X+Z) \dots (X+W) = X$$

$\{X(X+Y) = X\}$ $\rightleftarrows$ $\{X+XY=X\}$

$\rightleftarrows$ We proved the dual version

$\rightleftarrows$ Using the duality property, this is also true!

Absorption

MINIMIZATION

I) Boolean Algebra (algebraically)

aka. Algebraic Manipulation

EXAMPLE I

$$F = \underbrace{\overset{1}{Z}\overset{0}{Y'}\overset{0}{X'}}_{m_4} + \underbrace{ZYX}_{m_7} + \underbrace{ZYX'}_{m_6} + \underbrace{ZY'X}_{m_5}$$

4 × 3-input-AND

1 × 4-input-OR

$$F = ZY'X' + ZYX + ZYX' + ZY'X$$

$$F = Z\,(Y'X' + YX + YX' + Y'X)$$

$$F = Z\,(Y'X' + YX + YX' + Y'X)$$

$$F = Z\,(Y'(X'+X) + YX + YX')$$

$$F = Z\,(Y'(X'+X) + YX + YX')$$

$$F = Z\,(Y'1 + YX + YX')$$

$$F = Z\,(Y'\,1 + YX + YX')$$

$$F = Z\,(Y' + YX + YX')$$

$$F = Z(Y' + YX + YX')$$

$$F = Z\,(Y' + Y\,(X+X'))$$

$$F = Z(Y' + Y(X+X'))$$

$$F = Z\,(Y' + Y1)$$

$$F = Z\,(Y' + Y)$$

$$F = Z\,(Y' + Y)$$

$$F = Z1$$

$$F = \boxed{Z}$$

0 gates!

$$F = ZY'X' + ZYX + ZYX' + ZY'X$$

4 × 3-input-AND
1 × 4-input-OR

EXAMPLE I

Another Approach

$$F = ZY'X' + ZYX + ZYX' + ZY'X$$

$$F = ZY'(X'+X) + ZYX + ZYX'$$

$$F = ZY'(X'+X) + ZYX + ZYX'$$

$$F = \boxed{ZY'}(X'+X) + \boxed{ZY}(X+X')$$

$$F = ZY'(X'+X) + ZY(X+X')$$

$$F = ZY'1 + ZY1$$

$$F = ZY' + ZY$$

$$F = \boxed{Z}\,(\boxed{Y' + Y})$$

$$F = Z1$$

$$F = Z$$

0 gates!

$$F = ZY'X' + ZYX + ZYX' + ZY'X$$

4 × 3-input-AND
1 × 4-input-OR

EXAMPLE I

Another Approach

$$F = ZY'X' + ZYX + ZYX' + ZY'X$$

Absorption?

EXAMPLE II

$zx' + zy'x$

$$F = \underbrace{ZY'X'}_{m_4} + \underbrace{ZYX'}_{m_6} + \underbrace{Z'Y'X}_{m_1}$$

$\overset{1\,1\,0}{}$

3 × 3-input-AND

1 × 3-input-OR

$$F = ZY'X' + ZYX' + Z'Y'X$$

$$F = ZX'(Y'+Y) + Z'Y'X$$

$$F = ZX'(Y'+Y) + Z'Y'X$$

$$F = ZX'1 + Z'Y'X$$

$$F = ZX' + Z'Y'X$$

1 × 2-input-AND
1 × 3-input-AND
1 × 2-input-OR

$$F = ZYX' + ZYX' + Z'Y'X$$

3 × 3-input-AND
1 × 3-input-OR

DESIGN RECAP

SoP (ANDs-OR) → NAND
PoS (ORs-AND) → NOR

Comparator

Given two unsigned numbers x and y, design a logic circuit to see

$$x \overset{?}{\geq} y$$

What is the range of x and y?

$$x \overset{?}{\geq} y$$

What is the range of x and y?

$$x, y \in [0, 3]_{10}$$

What is the range of x and y?

$$x, y \in [0, 3]_{10} = [00, 11]_2$$

What is the range of output?

$$F \in \{0, 1\}$$

Y2	Y1	X2	X1	F(Y2,Y1,X2,X1)=?
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Y2	Y1	X2	X1	F(Y2,Y1,X2,X1)=Σ m(0,1,2,3,5,6,7,10,11,15)
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Y2	Y1	X2	X1	$F(Y2,Y1,X2,X1)=\Pi\ M(4,8,9,12,13,14)$
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Y2	Y1	X2	X1	$F(Y2,Y1,X2,X1)=\Sigma\ m(0,1,2,3,5,6,7,10,11,15)$	$F(Y2,Y1,X2,X1)=\Pi\ M(4,8,9,12,13,14)$
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1

Which design?

Which design?

SoP and PoS are both effective

SoP and PoS have same efficiency (2-levels)

Which design?

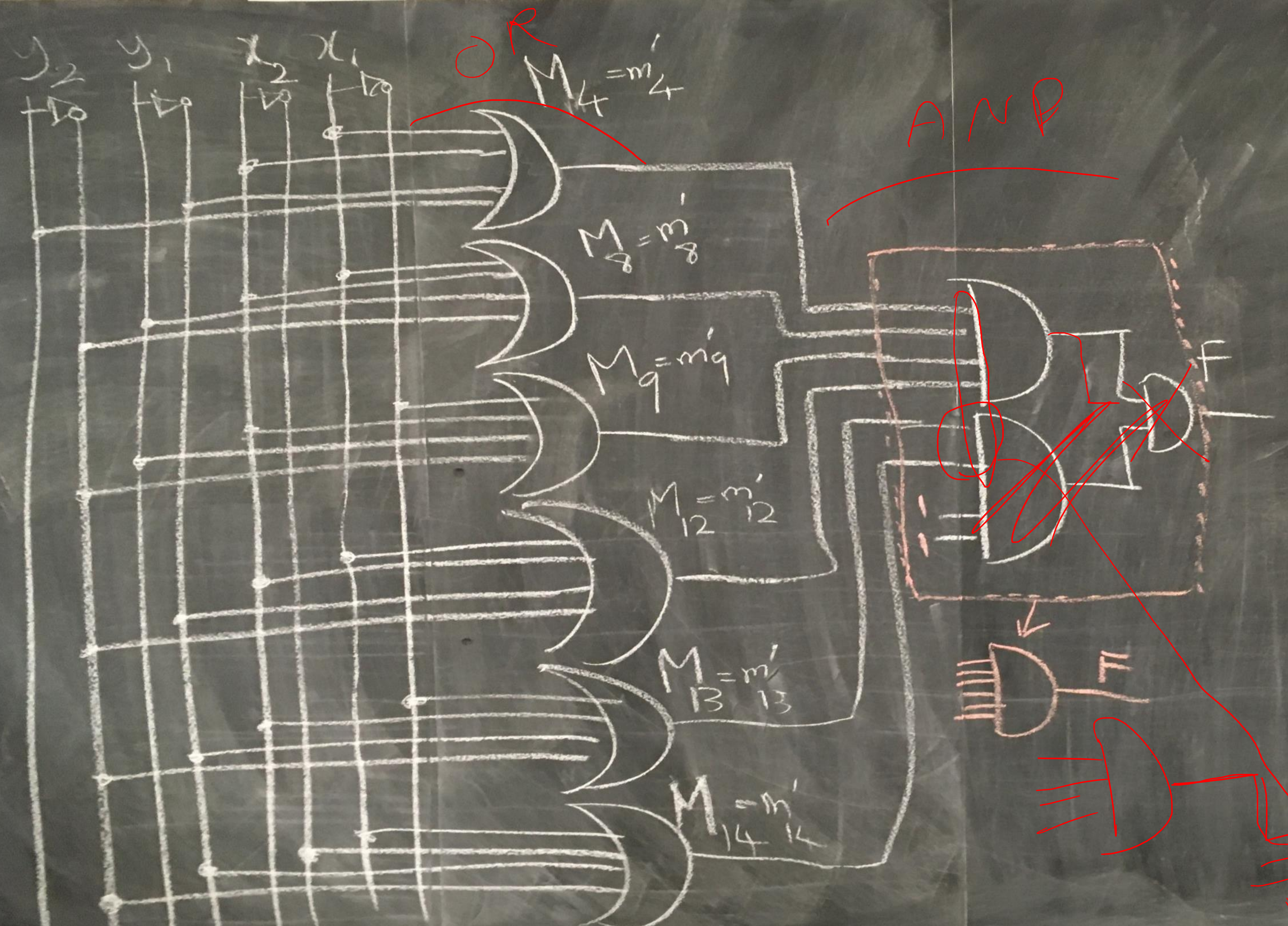
Cost: How many gates in SoP vs. PoS?

	F(Y2,Y1,X2,X1)= Σ m(0,1,2,3,5,6,7,10,11,15)	F(Y2,Y1,X2,X1)= Π M(4,8,9,12,13,14)
AND	?	?
OR	?	?
NOT	?	?
NAND	?	?
NOR	?	?

	$F(Y2,Y1,X2,X1)= \Sigma\, m(0,1,2,3,5,6,7,10,11,15)$	$F(Y2,Y1,X2,X1)= \Pi\, M(4,8,9,12,13,14)$
AND	Each minterm one 4-input-AND	Each MAXTERM one 4-input-OR
OR	One final 10-input-OR	One final 6-input-AND
NOT	Doesn't Matter Much	Doesn't Matter Much

	$F(Y2,Y1,X2,X1)=$ $\Sigma m(0,1,2,3,5,6,7,10,11,15)$	$F(Y2,Y1,X2,X1)=$ $\Pi M(4,8,9,12,13,14)$
AND	10 × 4-input-AND	6 × 4-input-OR
OR	10-input-OR = 3 × 4-input-OR	6-input-AND = 2 × 4-input-AND
NOT	Doesn't Matter Much	Doesn't Matter Much

	$F(Y2,Y1,X2,X1)=$ $\boldsymbol{\Sigma}\, m(0,1,2,3,5,6,7,10,11,15)$	$F(Y2,Y1,X2,X1)=$ $\boldsymbol{\Pi}\, M(4,8,9,12,13,14)$
AND	10 ✕ 4-input-AND	6 ✕ 4-input-OR
OR	10-input-OR = 3 ✕ 4-input-OR	6-input-AND = 2 ✕ 4-input-AND
NOT	Doesn’t Matter Much	Doesn’t Matter Much



$$F = \prod M(4,8,9,12,13,14)$$

$$= M_4 M_8 M_9 M_{12} M_{13} M_{14}$$

$$= m'_4 m'_8 m'_9 m'_{12} m'_{13} m'_{14}$$

$= (y'_2 y_1 x'_2 x'_1)'$ $\quad m'_4 \rightarrow y_2 + y'_1 + x_2 + x_1$

$(y_2 y'_1 x'_2 x'_1)'$ $\quad m'_8 \rightarrow y'_2 + y_1 + x_2 + x_1$

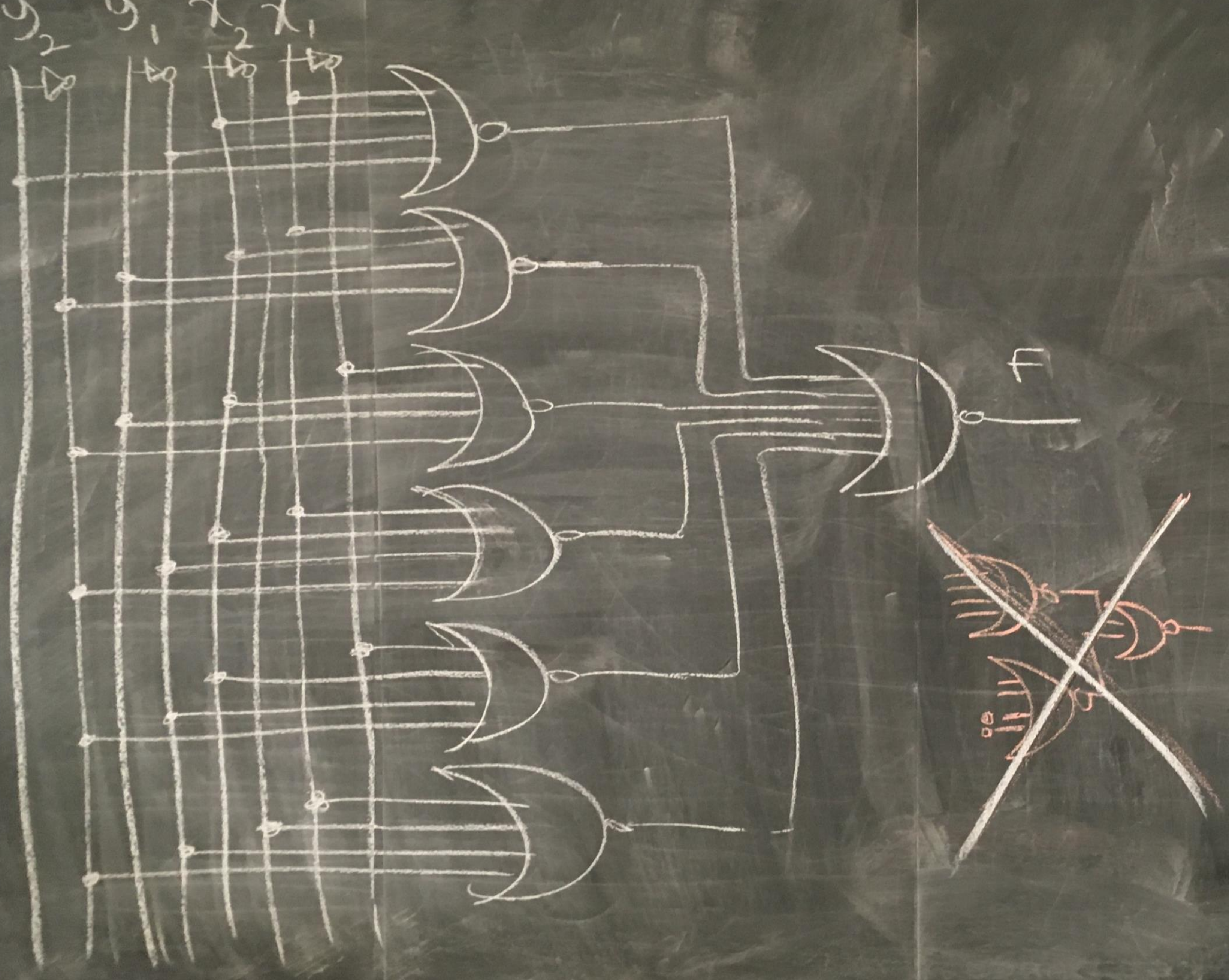
$(y_2 y'_1 x'_2 x_1)'$ $\quad m'_9 \rightarrow y'_2 + y_1 + x_2 + x'_1$

$(y_2 y_1 x'_2 x'_1)'$ $\quad m'_{12} \rightarrow y'_2 + y'_1 + x_2 + x_1$

$(y_2 y_1 x'_2 x_1)'$ $\quad m'_{13} \rightarrow y'_2 + y'_1 + x_2 + x'_1$

$(y_2 y_1 x_2 x'_1)'$ $\quad m'_{14} \rightarrow y'_2 + y'_1 + x'_2 + x_1$

	F(Y2,Y1,X2,X1)= $\Sigma\, m(0,1,2,3,5,6,7,10,11,15)$	F(Y2,Y1,X2,X1)= $\Pi\, M(4,8,9,12,13,14)$
NAND	10 × 4-input-NAND + 1 × 10-input-NAND	
NOR		6 × 4-input-NOR + 1 × 6-input-NOR



$$F = \prod M(4,8,9,12,13,14)$$

$$= M_4 M_8 M_9 M_{12} M_{13} M_{14}$$

$$F = (F')'$$

$$= \left(\left(M_4 M_8 M_9 M_{12} M_{13} M_{14} \right)' \right)'$$

$$= \left(M_4' + M_8' + M_9' + M_{12}' + M_{13}' + M_{14}' \right)'$$

$$= \Big((y_2 + y_1' + x_2 + x_1)' + (y_2' + y_1 + x_2 + x_1)' + (y_2' + y_1 + x_2 + x_1')' + (y_2' + y_1' + x_2 + x_1)' + (y_2' + y_1' + x_2 + x_1')' + (y_2' + y_1' + x_2' + x_1)' \Big)'$$

$F(Y_2,Y_1,X_2,X_1)= \mathbf{\Pi}\, M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

OMG!!

$F(Y_2,Y_1,X_2,X_1) = \Pi\, M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$F(Y_2,Y_1,X_2,X_1)= \Pi\ M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$F(Y_2,Y_1,X_2,X_1)= \Pi M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$F(Y_2,Y_1,X_2,X_1) = \Pi\ M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$F(Y_2,Y_1,X_2,X_1)= \Pi\, M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual → $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

Are we done?!

$F(Y_2,Y_1,X_2,X_1)= \Pi\ M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual → $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)\ +(Y'_2Y_1X_2X_1)$

OMG!!

$F(Y_2,Y_1,X_2,X_1)= \Pi\, M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1) +(Y'_2Y_1X_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1) + (Y'_2Y'_1X_2X_1)$

$F(Y_2,Y_1,X_2,X_1)= \Pi\ M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)\ +(Y'_2Y_1X_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)\ +\ (Y'_2Y'_1X_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2(X'_1+X_1))+(Y'_2Y'_1X'_2X_1)$

$$F(Y_2,Y_1,X_2,X_1)= \prod M(4,8,9,12,13,14)$$
$$= (M4)(M8)(M9)(M12)(M13)(M14)$$
$$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$$

Dual → $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1) +(Y'_2Y_1X_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1) + (Y'_2Y'_1X_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2(X'_1+X_1))+(Y'_2Y'_1X'_2X_1)$$
$$= (Y_2Y'_1X_2X_1)+(X_2X_1Y'_2)+(Y'_2Y'_1X_2(1))+(Y'_2Y'_1X'_2X_1)$$

OMG!!

$F(Y_2,Y_1,X_2,X_1) = \Pi M(4,8,9,12,13,14)$

$= (M4)(M8)(M9)(M12)(M13)(M14)$

$= (Y_2+Y'_1+X_2+X_1)(Y'_2+Y_1+X_2+X_1)(Y'_2+Y'_1+X_2+X_1)(Y'_2+Y'_1+X_2+X'_1)(Y'_2+Y'_1+X'_2+X_1)$

Dual $\rightarrow$ $(Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2Y_1X_2X_1)+(Y'_2Y'_1X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(Y_1+Y'_1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1(1))+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$ $+(Y'_2Y_1X_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2X'_1)+(Y'_2Y'_1X'_2X_1)$ $+$ $(Y'_2Y'_1X_2X_1)$

$= (Y_2Y'_1X_2X_1)+(Y'_2X_2X_1)+(Y'_2Y'_1X_2(X'_1+X_1))+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(X_2X_1Y'_2)+(Y'_2Y'_1X_2(1))+(Y'_2Y'_1X'_2X_1)$

$= (Y_2Y'_1X_2X_1)+(X_2X_1Y'_2)+(Y'_2Y'_1X_2)+(Y'_2Y'_1X'_2X_1)$

Are we done?! Honestly, I don't know ☹ ☺

MINIMIZATION

I) Boolean Algebra (algebraically)

- Needs to be smart. It is hard due to guesswork (which rules to apply?)
- If the number of variables (ABCDEF...) and/or number of minterms (MAXTERMS) grows
- No Algorithm
- Is the result minimal?!

MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

Algorithm; Straightforward, up to six variables

Result is always minimal

MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

Algorithm; Straightforward, up to six variables

Result is always minimal

Maurice Karnaugh

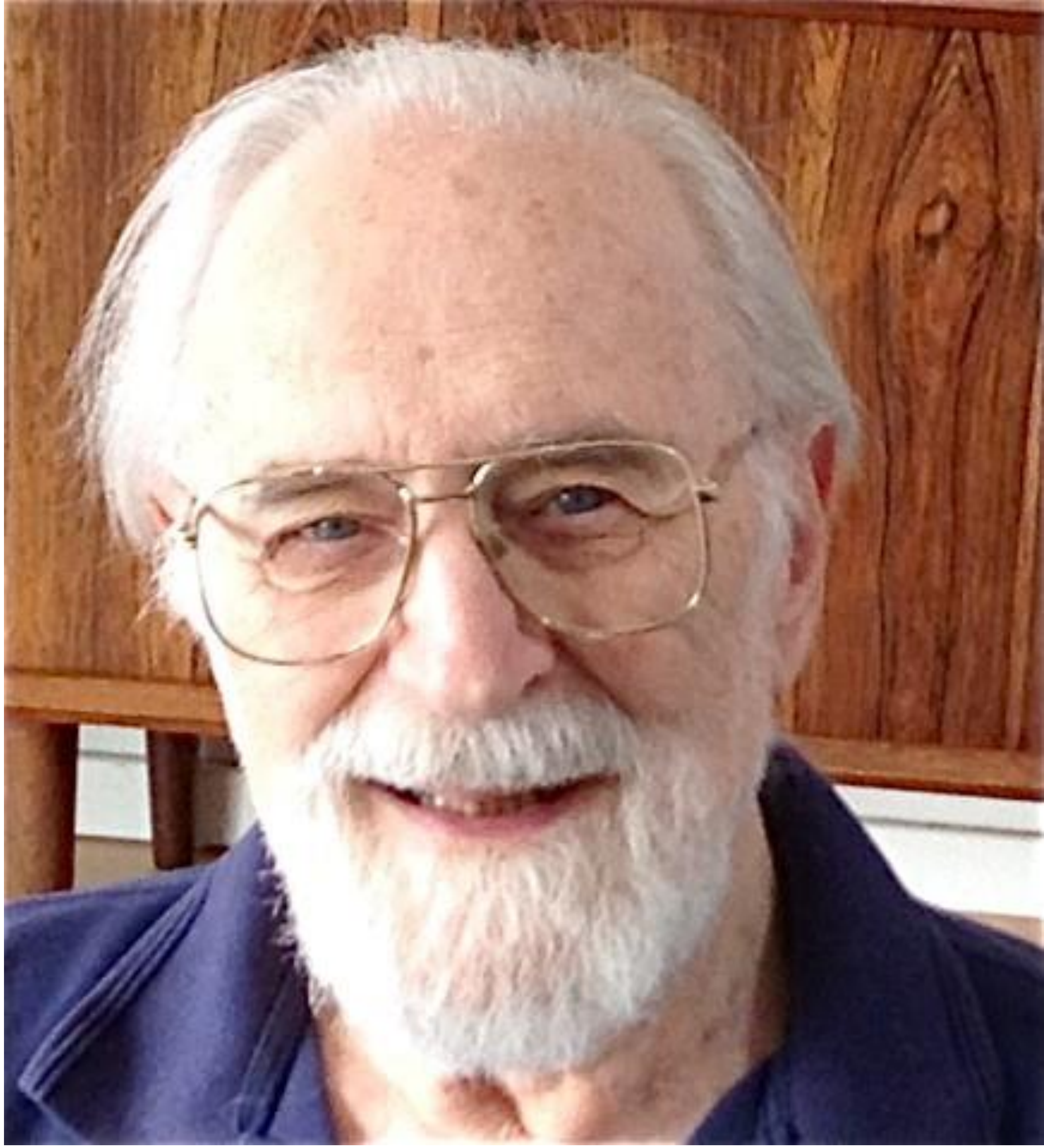
Physicist

Mathematician

Inventor

Bell Labs (1954)

"The Map Method for Synthesis of Combinational Logic Circuits"



MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

KARNAUGH MAP

/ˈkɑːrnɔː/

1-Variable KARNAUGH MAP

X	F
0	m_0
1	m_1

K-map

X = 0	X = 1
m_0	m_1

2-Variable KARNAUGH MAP

Y	X	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y	X	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y	X	F
0	0	1
0	1	0
1	0	0
1	1	0

$$F(Y,X) = m_0 = Y'X'$$

Y \ X	0	1
0	1	0
1	0	0

$$F(Y,X) = Y'X'$$

Y	X	F
0	0	0
0	1	1
1	0	0
1	1	0

$$F(Y,X) = m_1 = Y'X$$

Y \ X	0	1
0	0	1
1	0	0

$$F(Y,X) = Y'X$$

Y	X	F
0	0	0
0	1	0
1	0	1
1	1	0

$F(Y,X) = m_2 = YX'$

Y \ X	0	1
0	0	0
1	1	0

$F(Y,X) = YX'$

Y	X	F
0	0	0
0	1	0
1	0	0
1	1	1

$$F(Y,X) = m_3 = YX$$

Y \ X	0	1
0	0	0
1	0	1

$$F(Y,X) = YX$$

Y	X	F
0	0	1
0	1	1
1	0	0
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 \\
&= Y'X' + Y'X \\
&= Y'(X' + X) \\
&= Y'
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	0	0

$$F(Y,X) = Y'$$

Y	X	F
0	0	0
0	1	0
1	0	1
1	1	1

$$\begin{aligned} F(Y,X) &= m_2 + m_3 \\ &= YX' + YX \\ &= Y(X' + X) \\ &= Y \end{aligned}$$

Y \ X	0	1
0	0	0
1	1	1

$$F(Y,X) = Y$$

Y	X	F
0	0	0
0	1	1
1	0	0
1	1	1

$$\begin{aligned} F(Y,X) &= m_1 + m_3 \\ &= Y'X + YX \\ &= X(Y' + Y) \\ &= X \end{aligned}$$

		X	
		0	1
Y	0	0	1
	1	0	1

$$F(Y,X) = X$$

Y	X	F
0	0	1
0	1	0
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_2 \\
&= Y'X' + YX' \\
&= X'(Y' + Y) \\
&= X'
\end{aligned}
$$

Y \ X	0	1
0	1	0
1	1	0

$$F(Y,X) = X'$$

WHAT IF

Y	X	F	
0	0	1	m_0
0	1	1	m_1
1	0	1	m_2
1	1	0	

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= Y'(X' + X) + YX' \\
&= Y' + YX'
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	0

$$F(Y,X) = Y' + YX'$$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= X'(Y' + Y) + Y'X \\
&= X' + Y'X
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	0

$$F(Y,X) = X' + Y'X$$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= Y'X' + Y'X' + Y'X + YX' \\
&= Y'(X' + X) + Y'X' + YX' \\
&= Y' + Y'X' + YX' \\
&= Y' + X'(Y' + Y) \\
&= Y' + X'
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	0

$$F(Y,X) = Y' + X'$$

WHAT IF

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	1

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 + m_3 \\
&= Y'X' + Y'X + YX' + YX \\
&= Y'(X' + X) + Y(X' + X) \\
&= Y' + Y \\
&= 1
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	1

$$
\begin{aligned}
F(Y,X) &= m_0+m_1+m_2+m_3 \\
&= 1
\end{aligned}
$$

Y	X	F
0	0	0
0	1	1
1	0	1
1	1	0

$$F(Y,X) = m_1 + m_2$$
$$= Y'X + YX'$$

		X	
		0	1
Y	0	0	1
	1	1	0

$$F(Y,X) = m_1 + m_2$$
$$= Y'X + YX'$$

Y	X	F
0	0	1
0	1	0
1	0	0
1	1	1

$$F(Y,X) = m_0 + m_3$$
$$= Y'X' + YX$$
$$=$$

		X	
		0	1
Y	0	1	0
	1	0	1

$$F(Y,X) = m_0 + m_2$$
$$= Y'X' + YX$$

3-Variable KARNAUGH MAP

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

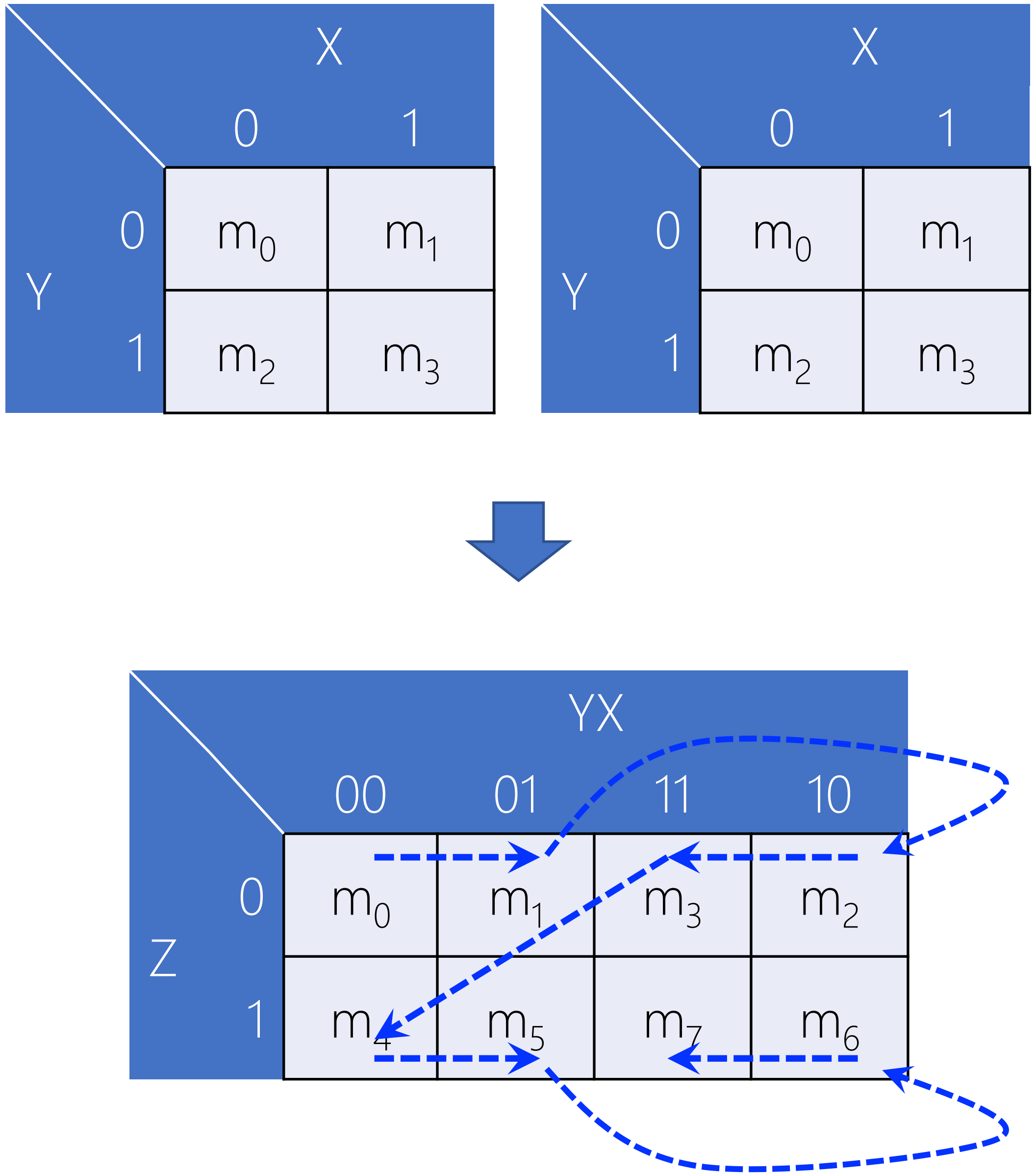
Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

11

101

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$\}\ Z$

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10	
0	m_0	m_1	m_3	m_2	Z'
1	m_4	m_5	m_7	m_6	

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Y

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Y'

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

X

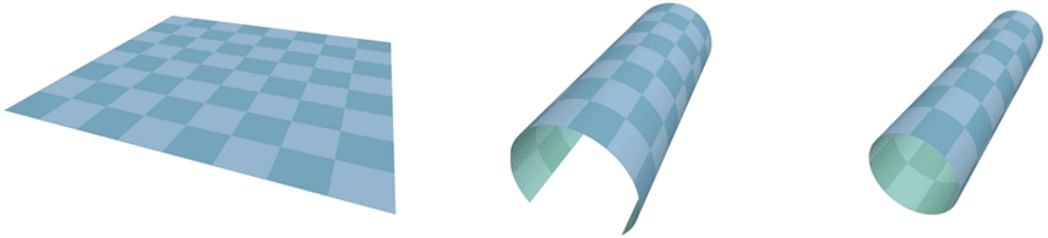
Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

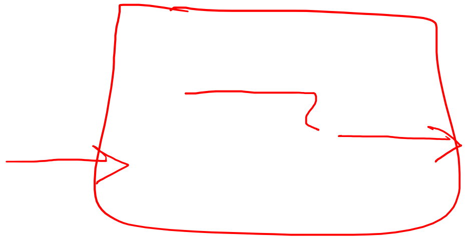
Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

X′ ?

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6





Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10	Z
0	m_0	m_1	m_3	m_2	0
1	m_4	m_5	m_7	m_6	1

X' X'

WHAT IF

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z'Y'X'+Z'Y'X+Z'YX'+Z'YX+ZYX'+ZYX$$
$$= ?$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX$$
$$= ?$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z' +$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z' + ZY$$

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX$$
$$= ?$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX$$
$$= ?$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z' + Y$$