

Chapter 1

Digital Systems and Binary Numbers

Arabic

→ 0 1 2 3 4 5 6 7 8 9

Eastern Arabic

→ ٠ ١ ٢ ٣ ٤ ٥ ٦ ٧ ٨ ٩

"the hand makes the two complementary aspects of integers entirely intuitive. It serves as an instrument permitting natural movement between cardinal and ordinal numbering. If you need to show that a set contains three, four, seven or ten elements, you raise or bend simultaneously three, four, seven or ten fingers, using your hand as cardinal mapping. If you want to count out the same things, then you bend or raise three, four, seven or ten fingers in succession, using the hand as an ordinal counting tool."

- Georges Ifrah The Universal History of Numbers (Wiley, 2000, pp. 21-22)

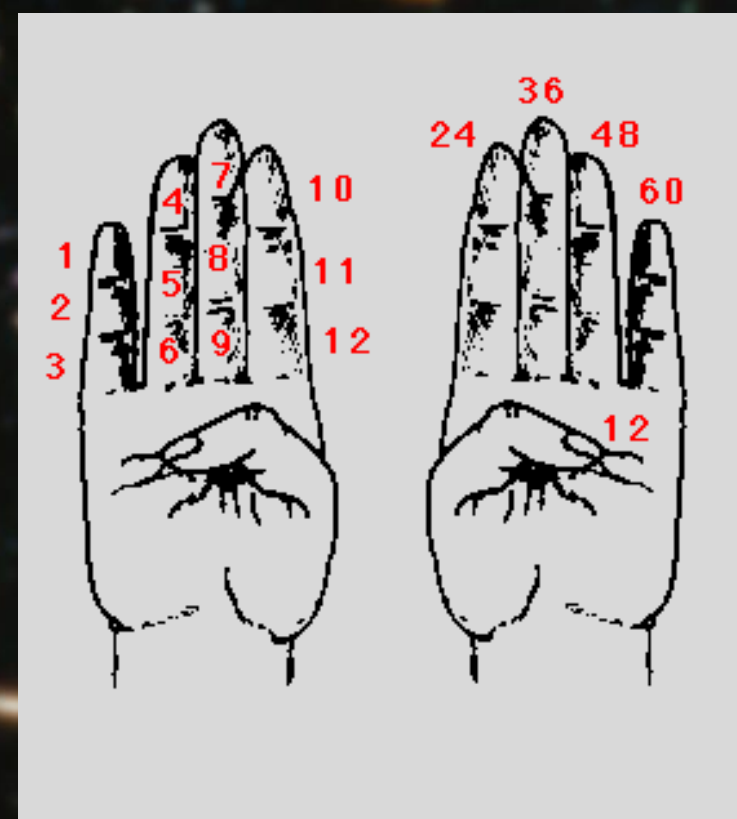
Thanks Cecelia Nydam and Giavi Tran for the hint!

Thai

→ ๐ ๑ ๒ ๓ ๔ ๕ ๖ ๗ ๘ ๙

Chinese

→ 〇 一 二 三 四 五 六 七 八 九



A cosmic background image featuring a dense field of galaxies in various colors (yellow, orange, blue, red) against a black space. Two horizontal blue lines are positioned above and below the central text.

CONVERSION

From Base-r to Base-r'

A diagram illustrating a transformation. On the left, a point cloud is enclosed in large parentheses with a subscript r . The point cloud consists of approximately 25 white square points arranged in a roughly circular pattern. A large white arrow points to the right. On the right, another point cloud is enclosed in large parentheses with a subscript r' . This point cloud consists of approximately 25 blue square points arranged in a similar circular pattern. The background is black.

$$\left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_r \longrightarrow \left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_{10}$$

we already knew that: sum of the powers of r

Let $(N)_r$ be a radix- r (base- r) number in a positional weighting number system, then

$$(N)_r = (d_{n-1} r^{n-1} + \dots + d_0 r^0 . d_{-1} r^{-1} + d_{-2} r^{-2} + \dots + d_{-m} r^{-m})_{10}$$

where:

r = radix (base)

d_i = digit at position i , $0 \leq d_i \leq r - 1$

r^i = weight of position i

n = number of digits in integer part of N

m = number of digits in fraction part of N



Fraction Point

$$\left(\begin{array}{c} \text{5x5 grid of white squares} \end{array} \right)_r \leftarrow \left(\begin{array}{c} \text{10x10 grid of blue squares} \end{array} \right)_{10}$$

Diagram illustrating a mapping from a 10x10 grid of blue squares to a 10x10 grid of white squares. The mapping is indicated by a large white arrow pointing from the blue grid to the white grid. Both grids are enclosed in large parentheses with a subscript 10.

To Come up with an Algorithm!

What is the digit in the position with significance of 10^0 ?

(3 0 3 0 2 1 3 1)₁₀

What is the digit in the position with significance of 10^1 ?

(3 0 3 0 2 1 3 1)₁₀

What is the digit in the position with significance of 10^2 ?

(3 0 3 0 2 1 3 1)₁₀

What is the digit in the position with significance of 10^i ?

$$(30302131)_{10} \rightarrow 10^0 \rightarrow 1$$

$$(30302131)_{10} \rightarrow 10^1 \rightarrow 3$$

$$(30302131)_{10} \rightarrow 10^2 \rightarrow 1$$

$$(30302131)_{10} \rightarrow 10^3 \rightarrow 2$$

$$(30302131)_{10} \rightarrow 10^4 \rightarrow 0$$

$$(30302131)_{10} \rightarrow 10^5 \rightarrow 3$$

$$(30302131)_{10} \rightarrow 10^6 \rightarrow 0$$

$$(30302131)_{10} \rightarrow 10^7 \rightarrow 3$$

What is the digit in the position with significance of 10^i ?

$$(30302131)_{10} \rightarrow 10^0 \rightarrow 1$$

$$(3030213)_{10} \rightarrow 10^0 \rightarrow 3$$

$$(303021)_{10} \rightarrow 10^0 \rightarrow 1$$

$$(30302)_{10} \rightarrow 10^0 \rightarrow 2$$

$$(3030)_{10} \rightarrow 10^0 \rightarrow 0$$

$$(303)_{10} \rightarrow 10^0 \rightarrow 3$$

$$(30)_{10} \rightarrow 10^0 \rightarrow 0$$

$$(3)_{10} \rightarrow 10^0 \rightarrow 3$$

Divide by 10?

(3 0 3 0 2 1 3 1)₁₀

$$30,302,131 \div 10 = 3,030,213$$

$$30,302,131 \% 10 = 1$$

$$\frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1$$

Divide by 10?

(3 0 3 0 2 1 3 1)₁₀

$$3,030,213 \div 10 = 303,021$$

$$3,030,213 \% 10 = 3$$

$$\frac{3,030,213}{10} = 303,021 \text{ } r \text{ } 3$$

Divide by 10?

(3 0 3 0 2 1 3 1)₁₀

$$303,021 \div 10 = 30,302$$

$$303,021 \% 10 = 1$$

$$\frac{303,021}{10} = 30,302 \text{ } r \text{ } 1$$

What is the digit in the position with significance of 10^0 ?

$$\frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1$$

Remainder of the 0+1 division by 10!

What is the digit in the position with significance of 10^1 ?

$$\frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1$$
$$\frac{3,030,213}{10} = 303,021 \text{ } r \text{ } 3$$

Remainder of the $1+1$ division by 10 !

What is the digit in the position with significance of 10^2 ?

$$\begin{array}{r} 30,302,131 \\ \hline 10 \end{array} = 3,030,213 \text{ } r \text{ } 1$$

$$\begin{array}{r} 3,030,213 \\ \hline 10 \end{array} = 303,021 \text{ } r \text{ } 3$$

$$\begin{array}{r} 303,021 \\ \hline 10 \end{array} = 30,302 \text{ } r \text{ } 1$$

Remainder of the $2+1$ division by 10!

What is the digit in the position with significance of 10^3 ?

$$\begin{array}{rcl} \frac{30,302,131}{10} & = & 3,030,213 \text{ } r \text{ } 1 \\ & \frac{3,030,213}{10} & = 303,021 \text{ } r \text{ } 3 \\ & & \frac{303,021}{10} = 30,302 \text{ } r \text{ } 1 \\ & & & \frac{30,302}{10} = 3,030 \text{ } r \text{ } 2 \end{array}$$

Remainder of the $3+1$ division by 10!

What is the digit in the position with significance of 10^4 ?

$$\begin{array}{l} \frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1 \\ \quad \frac{\quad}{10} = 303,021 \text{ } r \text{ } 3 \\ \qquad \frac{\quad}{10} = 30,302 \text{ } r \text{ } 1 \\ \qquad \qquad \frac{\quad}{10} = 3,030 \text{ } r \text{ } 2 \\ \qquad \qquad \qquad \frac{\quad}{10} = 303 \text{ } r \text{ } 0 \end{array}$$

Remainder of the $4+1$ division by $10!$

What is the digit in the position with significance of 10^5 ?

$$\begin{array}{l} \frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1 \\ \quad \frac{\quad}{10} = 303,021 \text{ } r \text{ } 3 \\ \qquad \frac{\quad}{10} = 30,302 \text{ } r \text{ } 1 \\ \qquad \qquad \frac{\quad}{10} = 3,030 \text{ } r \text{ } 2 \\ \qquad \qquad \qquad \frac{\quad}{10} = 303 \text{ } r \text{ } 0 \\ \qquad \qquad \qquad \qquad \frac{\quad}{10} = 30 \text{ } r \text{ } 3 \end{array}$$

Remainder of the $5+1$ division by 10!

What is the digit in the position with significance of 10^6 ?

$$\begin{array}{l} \frac{30,302,131}{10} = 3,030,213 \text{ } r \text{ } 1 \\ \frac{\quad \quad \quad}{10} = 303,021 \text{ } r \text{ } 3 \\ \frac{\quad \quad \quad}{10} = 30,302 \text{ } r \text{ } 1 \\ \frac{\quad \quad \quad}{10} = 3,030 \text{ } r \text{ } 2 \\ \frac{\quad \quad \quad}{10} = 303 \text{ } r \text{ } 0 \\ \frac{\quad \quad \quad}{10} = 30 \text{ } r \text{ } 3 \\ \frac{\quad \quad \quad}{10} = 3 \text{ } r \text{ } 0 \end{array}$$

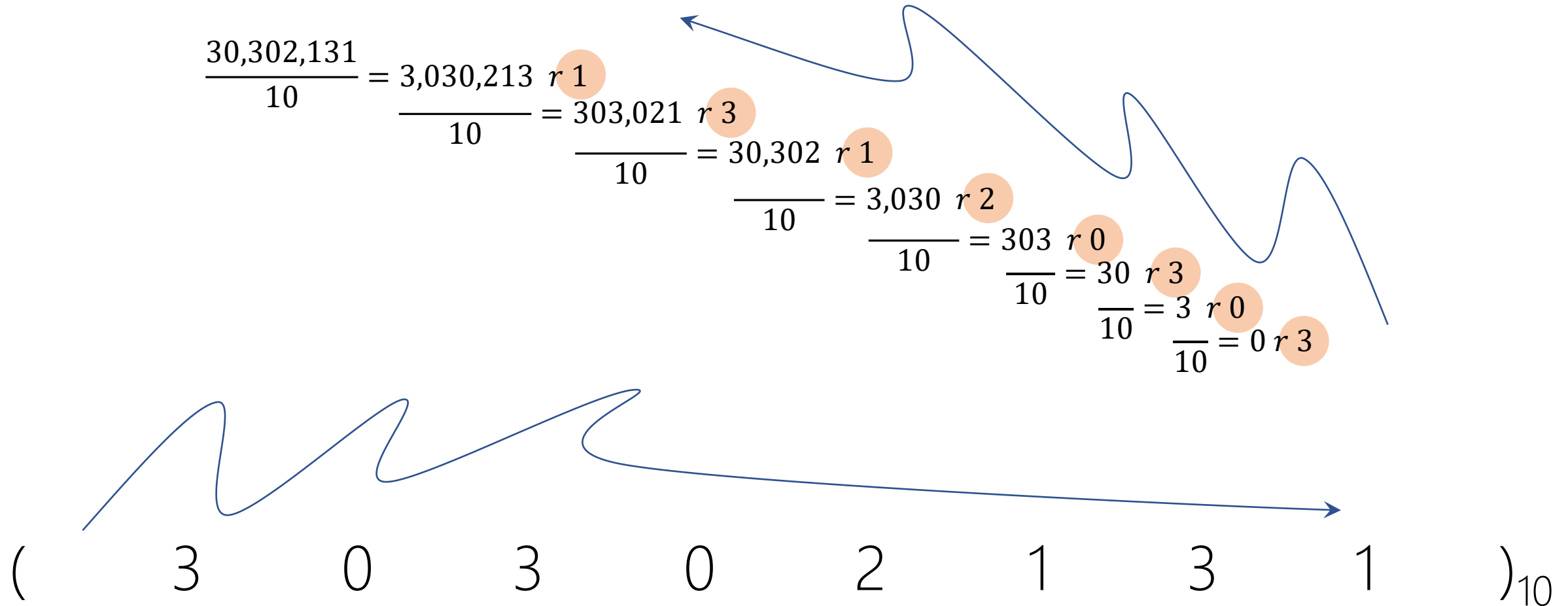
Remainder of the $6+1$ division by 10!

What is the digit in the position with significance of 10^7 ?

$$\begin{aligned}\frac{30,302,131}{10} &= 3,030,213 \text{ } r \text{ } 1 \\ \frac{3,030,213}{10} &= 303,021 \text{ } r \text{ } 3 \\ \frac{303,021}{10} &= 30,302 \text{ } r \text{ } 1 \\ \frac{30,302}{10} &= 3,030 \text{ } r \text{ } 2 \\ \frac{3,030}{10} &= 303 \text{ } r \text{ } 0 \\ \frac{303}{10} &= 30 \text{ } r \text{ } 3 \\ \frac{30}{10} &= 3 \text{ } r \text{ } 0 \\ \frac{3}{10} &= 0 \text{ } r \text{ } 3\end{aligned}$$

Remainder of the $7+1$ division by 10 !

What is the digit in the position with significance of 10^i ?



What is the digit in the position with significance of 16^0 ?

$$\frac{30,302,131}{16} = 1,893,883 \text{ } r \text{ } 3$$

Remainder of the 0+1 division by 16!

What is the digit in the position with significance of 16^1 ?

$$\frac{30,302,131}{16} = 1,893,883 \text{ r } 3$$
$$\frac{\quad}{16} = 118,367 \text{ r } 11$$

Remainder of the $1+1$ division by 16!

What is the digit in the position with significance of 16^2 ?

$$\begin{array}{rcl} \frac{30,302,131}{16} & = & 1,893,883 \text{ } r \text{ } 3 \\ & & \frac{\quad}{16} = 118,367 \text{ } r \text{ } 11 \\ & & \frac{\quad}{16} = 7,397 \text{ } r \text{ } 15 \end{array}$$

Remainder of the $2+1$ division by 16!

What is the digit in the position with significance of 16^3 ?

$$\begin{array}{l} \frac{30,302,131}{16} = 1,893,883 \text{ r } 3 \\ \quad \frac{\quad}{16} = 118,367 \text{ r } 11 \\ \qquad \frac{\quad}{16} = 7,397 \text{ r } 15 \\ \qquad \qquad \frac{\quad}{16} = 462 \text{ r } 5 \end{array}$$

Remainder of the 3+1 division by 16!

What is the digit in the position with significance of 16^4 ?

$$\begin{array}{l} \frac{30,302,131}{16} = 1,893,883 \text{ r } 3 \\ \quad \frac{1,893,883}{16} = 118,367 \text{ r } 11 \\ \qquad \frac{118,367}{16} = 7,397 \text{ r } 15 \\ \qquad \qquad \frac{7,397}{16} = 462 \text{ r } 5 \\ \qquad \qquad \qquad \frac{462}{16} = 28 \text{ r } 14 \end{array}$$

Remainder of the $4+1$ division by 16!

What is the digit in the position with significance of 16^5 ?

$$\begin{aligned}\frac{30,302,131}{16} &= 1,893,883 \text{ r } 3 \\ \frac{1,893,883}{16} &= 118,367 \text{ r } 11 \\ \frac{118,367}{16} &= 7,397 \text{ r } 15 \\ \frac{7,397}{16} &= 462 \text{ r } 5 \\ \frac{462}{16} &= 28 \text{ r } 14 \\ \frac{28}{16} &= 1 \text{ r } 12\end{aligned}$$

Remainder of the $5+1$ division by 16!

What is the digit in the position with significance of 16^6 ?

$$\begin{array}{l} \frac{30,302,131}{16} = 1,893,883 \text{ r } 3 \\ \quad \frac{1,893,883}{16} = 118,367 \text{ r } 11 \\ \qquad \frac{118,367}{16} = 7,397 \text{ r } 15 \\ \qquad \qquad \frac{7,397}{16} = 462 \text{ r } 5 \\ \qquad \qquad \qquad \frac{462}{16} = 28 \text{ r } 14 \\ \qquad \qquad \qquad \qquad \frac{28}{16} = 1 \text{ r } 12 \\ \qquad \qquad \qquad \qquad \qquad \frac{1}{16} = 0 \text{ r } 1 \end{array}$$

Remainder of the $6+1$ division by 16!

What is the digit in the position with significance of 16^6 ?

$$\begin{aligned} \frac{30,302,131}{16} &= 1,893,883 \text{ } r \text{ } 3 \\ \frac{1,893,883}{16} &= 118,367 \text{ } r \text{ } 11 \\ \frac{118,367}{16} &= 7,397 \text{ } r \text{ } 15 \\ \frac{7,397}{16} &= 462 \text{ } r \text{ } 5 \\ \frac{462}{16} &= 28 \text{ } r \text{ } 14 \\ \frac{28}{16} &= 1 \text{ } r \text{ } 12 \\ \frac{1}{16} &= 0 \text{ } r \text{ } 1 \end{aligned}$$

(1 12 14 5 15 11 3)₁₆

What is the digit in the position with significance of 16^6 ?

$$\begin{aligned}\frac{30,302,131}{16} &= 1,893,883 \text{ r } 1 \\ \frac{1,893,883}{16} &= 118,367 \text{ r } 11 \\ \frac{118,367}{16} &= 7,397 \text{ r } 0 \\ \frac{7,397}{16} &= 462 \text{ r } 5 \\ \frac{462}{16} &= 28 \text{ r } 14 \\ \frac{28}{16} &= 1 \text{ r } 12 \\ \frac{1}{16} &= 0 \text{ r } 1\end{aligned}$$

(1 12 14 5 15 11 3)₁₆

$$1, 2, 3, 4, 5, 6, 7, 8, 9, A = 9 + 1 = (10)_{10}$$

$$B = A + 1 = (11)_{10}$$

$$C = B + 1 = (12)_{10}$$

$$D = C + 1 = (13)_{10}$$

$$E = D + 1 = (14)_{10}$$

$$F = E + 1 = (15)_{10}$$

What is the digit in the position with significance of 16^6 ?

$$\begin{aligned}\frac{30,302,131}{16} &= 1,893,883 \text{ r } 1 \\ \frac{1,893,883}{16} &= 118,367 \text{ r } 11 \\ \frac{118,367}{16} &= 7,397 \text{ r } 0 \\ \frac{7,397}{16} &= 462 \text{ r } 5 \\ \frac{462}{16} &= 28 \text{ r } 14 \\ \frac{28}{16} &= 1 \text{ r } 12 \\ \frac{1}{16} &= 0 \text{ r } 1\end{aligned}$$

(1 C E 5 F B 3)₁₆

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. A horizontal blue line is drawn across the middle of the image.
$$\left(\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}\right)_8 \leftarrow \left(\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}\right)_{10}$$

What is the digit in the position with significance of 8^i ?

Quotient	Remainder
$30,302,131 \div 8$	3
$3787766 \div 8$	6
$473470 \div 8$	6
$59183 \div 8$	7
$7397 \div 8$	5
$924 \div 8$	4
$115 \div 8$	3
$14 \div 8$	6
$1 \div 8$	1
0	



$(1CE5FB3)_{16}$
 $(30,302,131)_{10}$
 $(163457663)_8$

The diagram illustrates a mapping from a 10x10 grid of blue squares to a 4x4 grid of white squares. An arrow points from the 10x10 grid to the 4x4 grid.

What is the digit in the position with significance of 4^i ?

Quotient	Remainder
$30,302,131 \div 4$	3
$7575532 \div 4$	0
$1893883 \div 4$	3
$473470 \div 4$	2
$118367 \div 4$	3
$29591 \div 4$	3
$7397 \div 4$	1
$1849 \div 4$	1
$462 \div 4$	2
$115 \div 4$	3
$28 \div 4$	0
$7 \div 4$	3
$1 \div 4$	1
0	



$(1CE5FB3)_{16}$

$(30,302,131)_{10}$

$(163457663)_8$

$(1303211332303)_4$

What is the digit in the position with significance of 2^i ?

Quotient	Remainder
30,302,131÷2	1
15151065÷2	1
7575532÷2	0
3787766÷2	0
1893883÷2	1
946941÷2	1
473470÷2	0
236735÷2	1
118367÷2	1
59183÷2	1
29591÷2	1
14795÷2	1
7397÷2	1
3698÷2	0
1849÷2	1
924÷2	0
462÷2	0
231÷2	1
115÷2	1
57÷2	1
28÷2	0
14÷2	0
7÷2	1
3÷2	1
1÷2	1
0	



$(1CE5FB3)_{16}$

$(30,302,131)_{10}$

$(163457663)_8$

$(1303211332303)_4$

$(111001110010111110110011)_2$

A diagram illustrating a mapping between two sets of points. On the right, there is a \$10 \times 10\$ grid of blue squares enclosed in large parentheses, followed by a subscript 10. An arrow points from this grid to another grid on the left. The second grid is an \$r \times r\$ grid of white squares enclosed in large parentheses, followed by a subscript \$r\$. This visualizes the concept of a point set being mapped or embedded into a higher-dimensional space.

1. Divide quotients by r
2. Keep the remainders as the digits in r
3. Put the remainders in reverse order

For $r > 10$, substitute the digits with valid digits, e.g., in Base-16: $15 \rightarrow F$



$$\left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_r \longrightarrow \left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_{r'}$$

Any Base- r to Any Base- r'



Mediate through Base-10
Base- $r \rightarrow$ Base-10 \rightarrow Base- r'

$$\begin{aligned}
 (1CE5FB3)_{16} &\rightarrow 1 \times 16^6 + C \times 16^5 + E \times 16^4 + 5 \times 16^3 + F \times 16^2 + B \times 16^1 + 3 \times 16^0 \\
 &\rightarrow 1 \times 16^6 + 12 \times 16^5 + 14 \times 16^4 + 5 \times 16^3 + 15 \times 16^2 + 11 \times 16^1 + 3 \times 16^0 \\
 &\rightarrow (30,302,131)_{10}
 \end{aligned}$$

Quotient	Remainder
$30,302,131 \div 8$	3
$3787766 \div 8$	6
$473470 \div 8$	6
$59183 \div 8$	7
$7397 \div 8$	5
$924 \div 8$	4
$115 \div 8$	3
$14 \div 8$	6
$1 \div 8$	1
0	



$$\rightarrow (163457663)_8$$

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

FRACTION CONVERSION



$$(0.\overset{\cdot}{\underset{\cdot}{\text{⦿}}})_{10} \leftarrow (0.\overset{\cdot}{\underset{\cdot}{\text{⦿}}})_{10}$$

To Come up with an Algorithm!

What is the digit in the position with significance of 10^{-i} ?

$$(0.26501)_{10} \rightarrow 10^{-1} \rightarrow 2$$

$$(0.26501)_{10} \rightarrow 10^{-2} \rightarrow 6$$

$$(0.26501)_{10} \rightarrow 10^{-3} \rightarrow 5$$

$$(0.26501)_{10} \rightarrow 10^{-4} \rightarrow 0$$

$$(0.26501)_{10} \rightarrow 10^{-5} \rightarrow 1$$

What is the digit in the position with significance of 10^{-i} ?

$$(0.26501)_{10} \rightarrow 10^{-1} \rightarrow 2$$

$$(0.6501)_{10} \rightarrow 10^{-1} \rightarrow 6$$

$$(0.501)_{10} \rightarrow 10^{-1} \rightarrow 5$$

$$(0.01)_{10} \rightarrow 10^{-1} \rightarrow 0$$

$$(0.1)_{10} \rightarrow 10^{-1} \rightarrow 1$$

Multiply by 10?

(0 . 2 6 5 0 1)₁₀

$$0.26501 \times 10 = 2.6501$$

Multiply by 10?

(0 . 2 6 5 0 1)₁₀

$$0.6501 \times 10 = 6.501$$

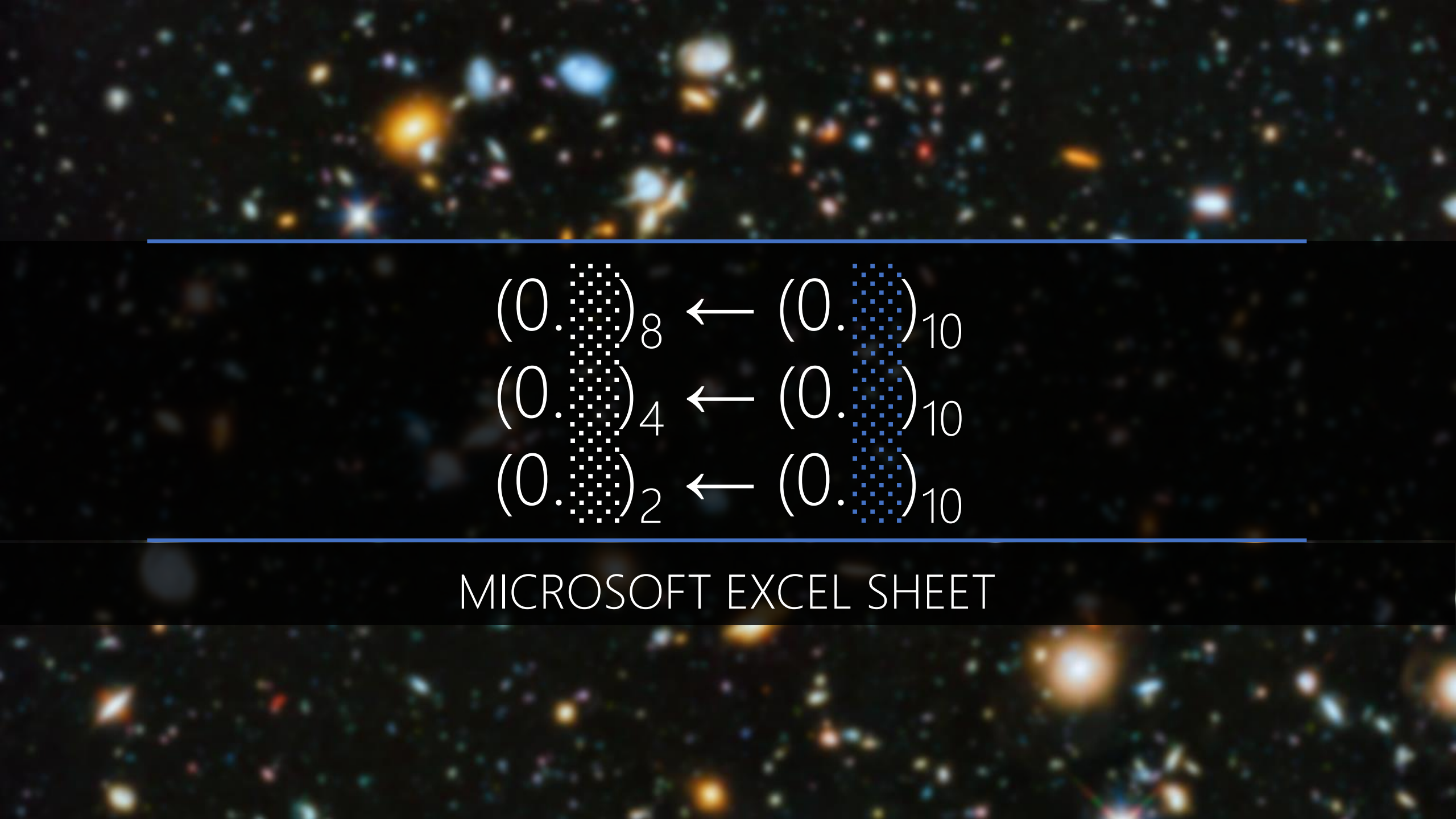
Fraction	Result	Integer Part
0.26501×10	2.6501	2
0.6501×10	6.501	6
0.501×10	5.01	5
0.01×10	0.1	0
0.1×10	1.0	1
0		



Fraction	Result	Integer Part
0.26501×16	4.24016	4
0.24016×16	3.84256	3
0.84256×16	13.48096	13 = D
0.48096×16	7.69536	7
0.69536×16	11.12576	11 = B
0.12576×16	2.01216	2
0.01216×16	0.19456	0
0.194560006×16	3.11296	3
0.1129601×16	1.807362	1
0.807361603×16	12.91779	12 = C
0.917785645×16	14.68457	14 = E
0.684570313×16	10.95313	10 = A
0.953125×16	15.25	15 = F
0.25×16	4	4
0	0	0



$$(0.26501)_{10} \rightarrow (0.43D7B2031CEAF40)_{16}$$


$$\begin{array}{l} (0.\text{[white dots]})_8 \leftarrow (0.\text{[blue dots]})_{10} \\ (0.\text{[white dots]})_4 \leftarrow (0.\text{[blue dots]})_{10} \\ (0.\text{[white dots]})_2 \leftarrow (0.\text{[blue dots]})_{10} \end{array}$$

MICROSOFT EXCEL SHEET



Integer Part independent of Fraction Part!

$$\left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_r \rightarrow \left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_{10} \rightarrow \left(\begin{array}{c} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{array} \right)_{r'}$$

Integer Part

$$\left(\cdot \begin{array}{c} \text{white dotted square} \end{array} \right)_r \rightarrow \left(\cdot \begin{array}{c} \text{green dotted square} \end{array} \right)_{10} \rightarrow \left(\cdot \begin{array}{c} \text{blue dotted square} \end{array} \right)_{r'}$$

Fraction Part

$(30,302,131.26501)_{10}$

$(163457663.20753....)_8$

Quotient	Remainder
$30,302,131 \div 8$	3
$3787766 \div 8$	6
$473470 \div 8$	6
$59183 \div 8$	7
$7397 \div 8$	5
$924 \div 8$	4
$115 \div 8$	3
$14 \div 8$	6
$1 \div 8$	1
0	

Fraction	Result	Integer Part
0.26501×8	2.12008	2
0.12008×8	0.96064	0
0.96064×8	7.68512	7
0.68512×8	5.48096	5
0.48096×8	3.84768	3
0.84768×8	6.78144	6
0.78144×8	6.25152	6
0.25152×8	2.01216	2
0.01216×8	0.09728	0
...

A cosmic background image featuring a dense field of galaxies in various colors (yellow, orange, blue, red) against a dark space. Two horizontal blue lines are positioned above and below the main text.

BASE-2 \rightarrow BASE-[$2^2, 2^3, 2^4, 2^n$]

LECTURE ASSIGNMENT

A cosmic background image featuring a dense field of galaxies in various colors (yellow, orange, blue, red) against a dark space. Two horizontal blue lines are positioned above and below the main text.

BASE-2 \leftarrow BASE-[$2^2, 2^3, 2^4, 2^n$]

LECTURE ASSIGNMENT



<https://planetcalc.com/862/>

The background of the slide is a deep space image showing a dense field of galaxies in various colors (yellow, orange, blue, and red) against a black background. A solid blue horizontal line runs across the middle of the image, positioned behind the title text.

ARITHMETIC

The image features a deep space background filled with numerous galaxies of various shapes and colors, including yellow, orange, blue, and red, set against a black void. Two thin, horizontal blue lines are positioned above and below the central text. The word "ADDITION" is centered in a white, sans-serif font.

ADDITION

+ Base-16		2	A
	4	B	F

.

.

.

E	5	4
2	B	

PADDING

+ Base-16	0	2	A
	4	B	F

.
.
.

E	5	4
2	B	0

+ Base-16	0	2	A
	4	B	F

.

.

.

E	5	4
2	B	0
		4

+ Base-16	0	2	A
	4	B	F

.

.

.

E	5	4
2	B=11	0
	16	4

$$\frac{16}{16} = 1 \text{ } r \text{ } 0$$

+ Base-16	0	2	A
	4	B	F

.	1		
	E	5	4
	2	B	0
		0	4

$$\frac{16}{16} = 1 \text{ } r \text{ } 0$$

+ Base-16	0	2	A
	4	B	F

.	1		
	E=14	5	4
	2	B	0
.	17	0	4

$$\frac{17}{16} = 1 \text{ r } 1$$

			1
+	0	2	A
Base-16	4	B	F

.	1		
.	E	5	4
.	2	B	0
.	1	0	4

$$\frac{17}{16} = 1 \text{ r } 1$$

+ Base-16			1	.	1		
	0	2	A=10		E	5	4
	4	B	F=15		2	B	0
			26	.	1	0	4

$$\frac{26}{16} = 1 \text{ } r \text{ } 10$$

		1	1
+ Base-16	0	2	A
	4	B	F
			10

.
.
.

1		
E	5	4
2	B	0
1	0	4

$$\frac{26}{16} = 1 \text{ } r \text{ } 10$$

		1	1
+ Base-16	0	2	A
	4	B	F
			A

.

.

.

1		
E	5	4
2	B	0
1	0	4

		1	1
+ Base-16	0	2	A
	4	B=11	F
		14	A

.

.

.

1		
E	5	4
2	B	0
1	0	4

		1	1
+ Base-16	0	2	A
	4	B	F
		E	A

.

.

.

1		
E	5	4
2	B	0
1	0	4

		1	1
+ Base-16	0	2	A
	4	B	F
	4	E	A

.

.

.

1		
E	5	4
2	B	0
1	0	4

A deep space image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines are positioned above and below the central text.

MULTIPLY

At Home

A deep-field astronomical image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the central text.

SUBTRACTION

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SUBTRACTION

Example I, Base-10

— Base-10		2	1
	4	3	2

.

.

.

1	5	4
0	6	

PADDING

— Base-10	0	2	1
	4	3	2

.
.
.

1	5	4
0	6	0

— Base-10	0	2	1
	4	3	2

.

.

.

1	5	4
0	6	0
		4

— Base-10	0	2	1
	4	3	2

.

.

.

1	5	4
0	6	0
		4

				BORROW	
—	0	2	1	.	
Base-10	4	3	2	.	
				.	

-1	→ +10	
1	5	4
0	6	0
	= 10+5-6 = 9	4

— Base-10	0	2	1
	4	3	2

.	-1	+10	
.	1	5	4
.	0	6	0
.		9	4

— Base-10	0	2	1
	4	3	2

.	-1	+10	
.	1	5	4
.	0	6	0
.	0	9	4

— Base-10	0	2	1
	4	3	2

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

		-1	+10
— Base-10	0	2	1
	4	3	2
			9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

		-1	+10
— Base-10	0	2	1
	4	3	2
			9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

		+10	
	-1	-1	+10
— Base-10	0	2	1
	4	3	2
		8	9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

		+10	
	-1	-1	+10
— Base-10	0	2	1
	4	3	2
		8	9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

	+10	+10	
-1	-1	-1	+10
— Base-10	0	2	1
	4	3	2
	5	8	9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

	+10	+10	
-1	-1	-1	+10
— Base-10	0	2	1
	4	3	2
	5	8	9

.

.

.

-1	+10	
1	5	4
0	6	0
0	9	4

$$021.154 < 432.060$$

	+10	+10	
-1	-1	-1	+10
—	0	2	1
Base-10	4	3	2
	5	8	9

.

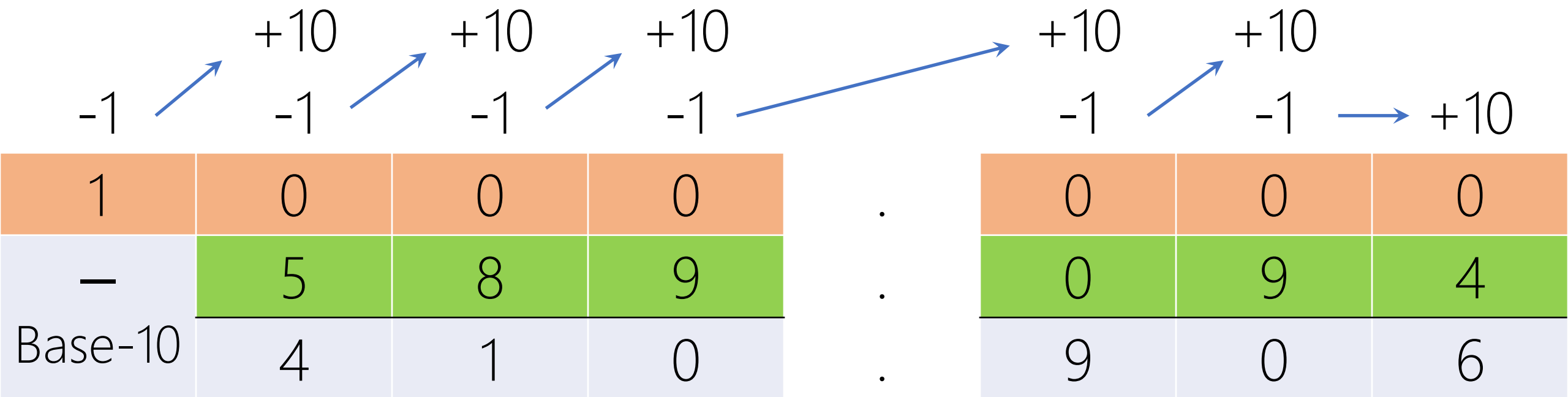
.

.

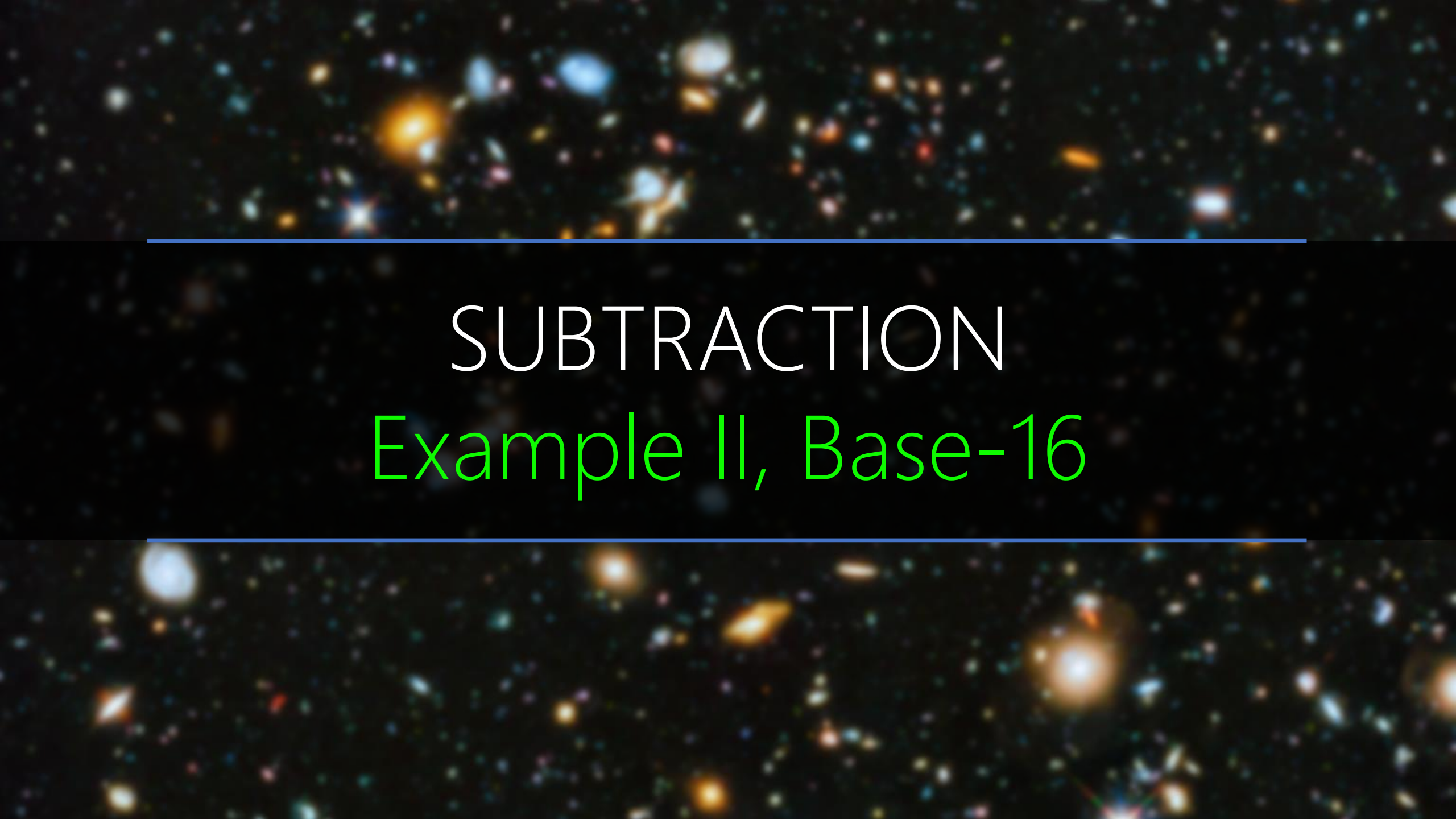
-1	+10	
1	5	4
0	6	0
0	9	4

021.154 < 432.060

Last Borrow → Negative Result



$$\begin{aligned}
 &= (021.154)_{10} - (432.060)_{10} = (021.154)_{10} + (1000.000)_{10} - (1000.000)_{10} - (432.060)_{10} \\
 &= - (1000.000)_{10} + (589.094)_{10} = - [(1000.000)_{10} - (589.094)_{10}] \\
 &= - (410.906)_{10}
 \end{aligned}$$

The background of the slide is a deep space image showing a dense field of galaxies. These galaxies appear in various colors, including bright yellow, orange, and blue, against a dark, star-filled sky. Two thin, horizontal blue lines are positioned above and below the central text.

SUBTRACTION

Example II, Base-16

<div>—</div> <div>Base-16</div>		2	A
	4	B	F

.

.

.

E	5	4
2	B	

PADDING

— Base-16	0	2	A
	4	B	F

.
.
.

E	5	4
2	B	0

— Base-16	0	2	A
	4	B	F

.

.

.

E	5	4
2	B	0
		4

— Base-16	0	2	A
	4	B	F

.

.

.

E	5	4
2	B=11	0
		4

				BORROW	
—	0	2	A	.	
Base-16	4	B	F	.	
				.	

-1	→ +16	
E	5	4
2	B=11	0
	= 5+16-11 = 10 = A	4

— Base-16	0	2	A
	4	B	F

.	-1	+16	
.	E	5	4
.	2	B	0
.		A	4

— Base-16	0	2	A
	4	B	F

.	-1	+16	
.	E=14	5	4
.	2	B	0
.	B=11	A	4

— Base-16	0	2	A=10
	4	B	F=15

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

		-1	+16
— Base-16	0	2	A=10
	4	B	F=15
			B=11

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

		-1	+16
— Base-16	0	2	A
	4	B=11	F
			B

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

		+16	
	-1	-1	+16
— Base-16	0	2	A
	4	B=11	F
		6	B

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

		+16	
	-1	-1	+16
— Base-16	0	2	A
	4	B	F
		6	B

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

	+16	+16	
-1	-1	-1	+16
— Base-16	0	2	A
	4	B	F
	B=11	6	B

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

	+16	+16	
-1	-1	-1	+16
— Base-16	0	2	A
	4	B	F
	B	6	B

.

.

.

-1	+16	
E	5	4
2	B	0
B	A	4

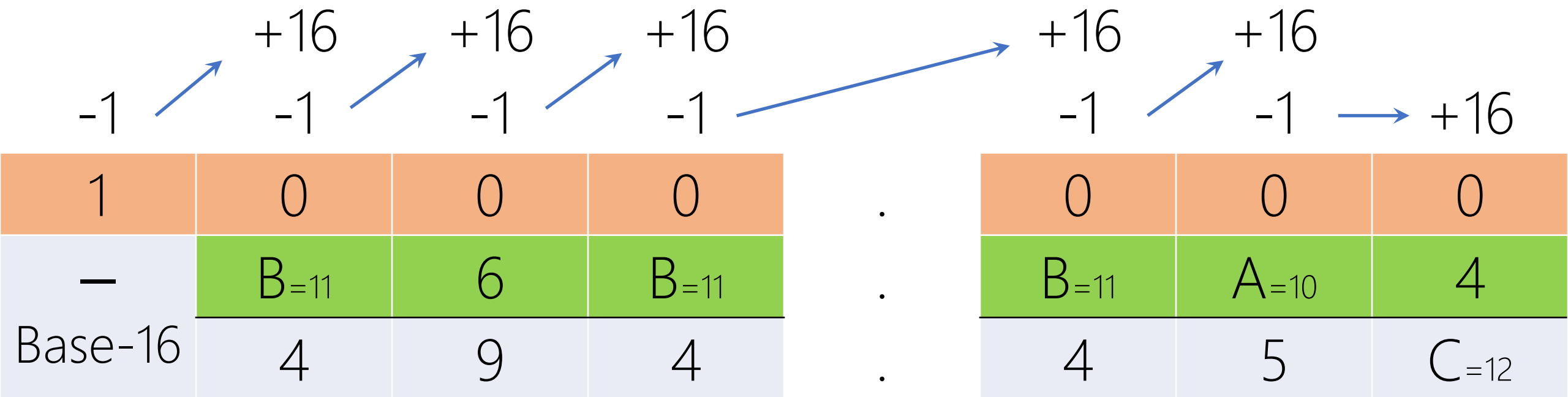
02A.E54 < 4BF.2B0

	+16	+16	
-1	-1	-1	+16
—	0	2	A
Base-16	4	B	F
	B	6	B

	-1	+16	
.	E	5	4
.	2	B	0
.	B	A	4

02A.E54 < 4BF.2B0

Last Borrow → Negative Result



$$\begin{aligned}
 &= (2A.E54)_{16} - (4BF.2B0)_{16} = (2A.E54)_{16} + (1000.000)_{16} - (1000.000)_{16} - (4BF.2B0)_{16} \\
 &= -(1000.000)_{16} + (B6B.BA4)_{16} = -[(1000.000)_{16} - (B6B.BA4)_{16}] \\
 &= -(494.45C)_{16}
 \end{aligned}$$

A deep space image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the central text.

DIVISION

At Home

A cosmic background image featuring a dense field of galaxies in various colors (blue, orange, white) against a black space. Two horizontal blue lines are positioned above and below the central text.

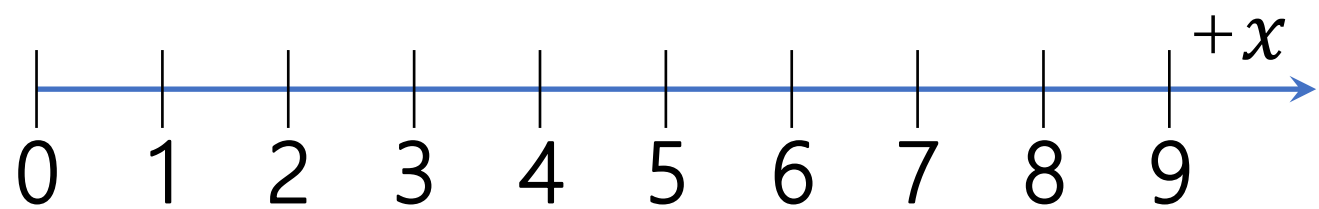
NEGATIVE NUMBERS

aka Signed Numbers

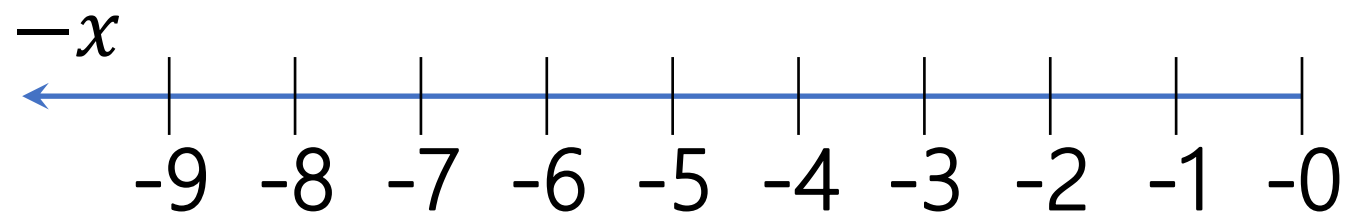
A deep space image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

SIGNED MAGNITUDE
SIGNED COMPLEMENT

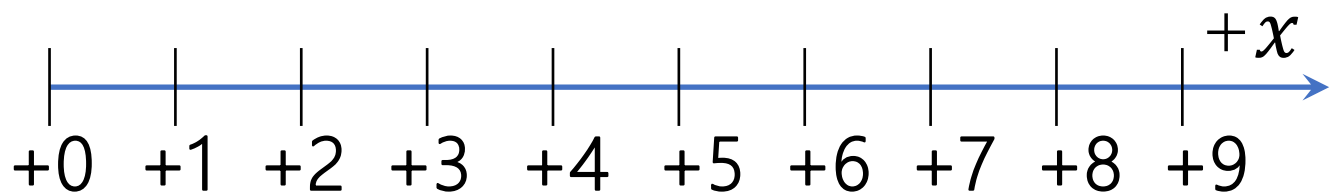
10^0
0
1
2
3
4
5
6
7
8
9



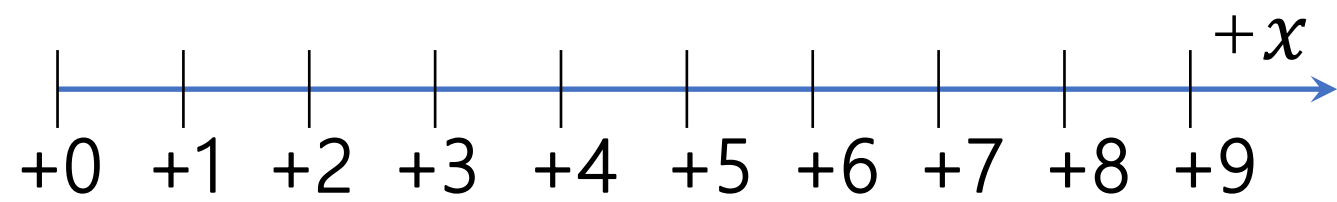
10^1	10^0
—	0
—	1
—	2
—	3
—	4
—	5
—	6
—	7
—	8
—	9



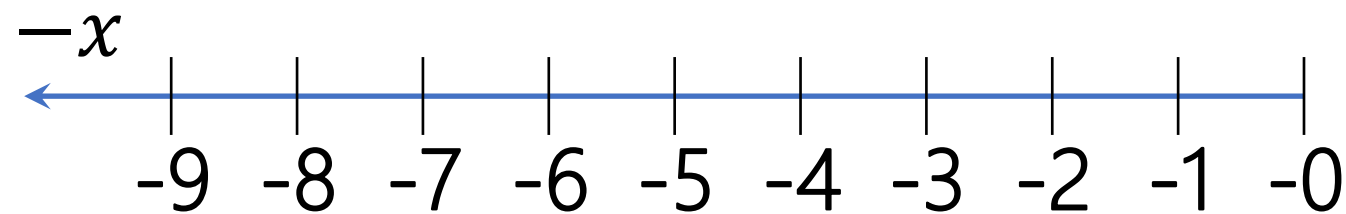
10^1	10^0
+	0
+	1
+	2
+	3
+	4
+	5
+	6
+	7
+	8
+	9



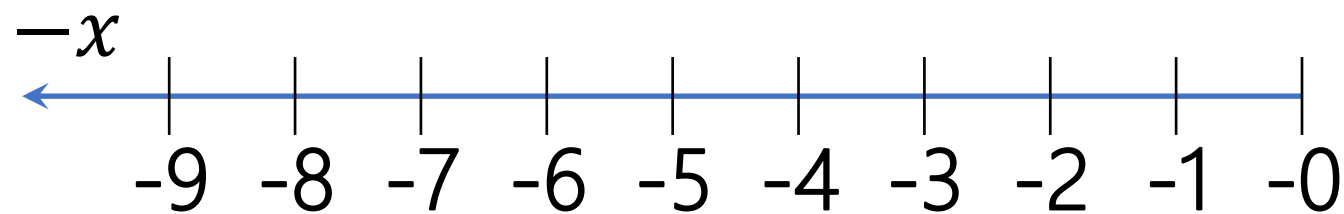
10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9



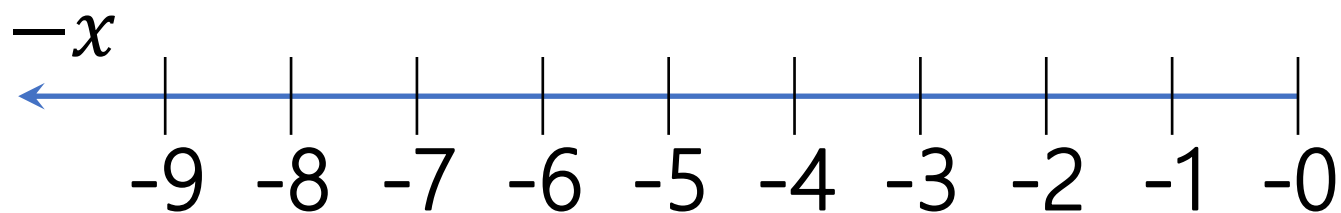
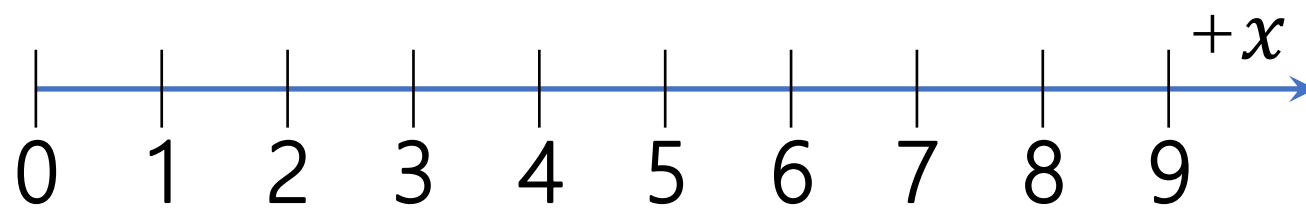
10^1	10^0
1	0
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8
1	9



10^1	10^0
Nonzero	0
Nonzero	1
Nonzero	2
Nonzero	3
Nonzero	4
Nonzero	5
Nonzero	6
Nonzero	7
Nonzero	8
Nonzero	9

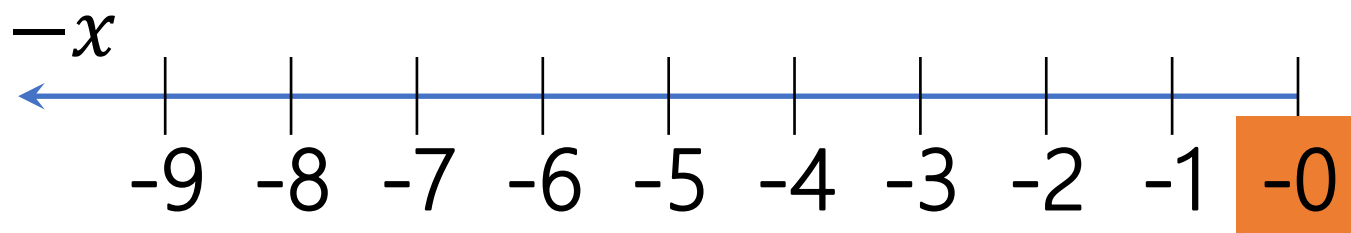
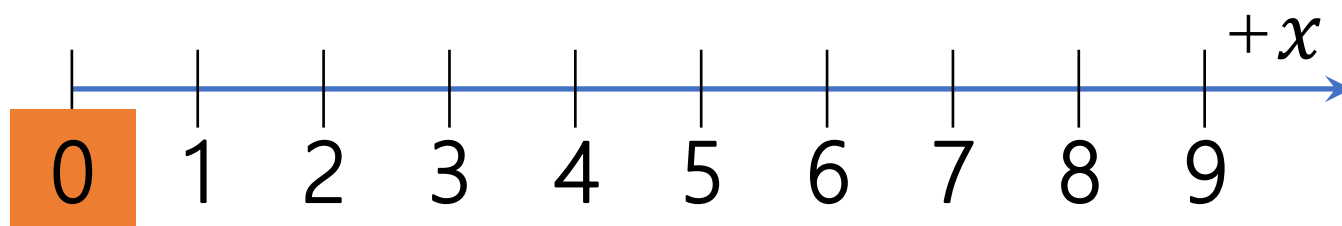


10^1	10^0
1,2,...9	0
1,2,...9	1
1,2,...9	2
1,2,...9	3
1,2,...9	4
1,2,...9	5
1,2,...9	6
1,2,...9	7
1,2,...9	8
1,2,...9	9



10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9

10^1	10^0
1,2,...9	0
1,2,...9	1
1,2,...9	2
1,2,...9	3
1,2,...9	4
1,2,...9	5
1,2,...9	6
1,2,...9	7
1,2,...9	8
1,2,...9	9



10^1	10^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
0	8
0	9

A deep space image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the central text.

SIGNED MAGNITUDE