University of Windsor
Faculty of Science
School of Computer Science

COMP-2650
Computer Architecture I: Digital Design
Winter 2022

Assignment#	Date	Title	Due Date	Grade Release Date
Lec 07	Week 07	**Algebraic Manipulation**	March 09, 2022, Wednesday 4 AM EDT	March. 14, 2022

The objectives of the lecture (weekly) assignments are to practice on topics covered in the lectures as well as improve the student's critical thinking and problem-solving skills in ad hoc topics that are closely related but not covered in the lectures. Lecture assignments also help students with research skills, including the ability to access, retrieve, and evaluate information (information literacy.)

Deliverables

You should answer 2 of the below questions based on your preference using an editor like MS Word, Notepad, and the likes or pen in papers. In the latter case, you have to write and scan the papers clearly and merge them into a single file. In the end, you have to submit all your answers in one single pdf file `lec07_UWinID.pdf` containing the question ids for the answer. Please note that if your answers cannot be read, you will lose marks. Please follow the naming convention as you lose marks otherwise. Instead of UWinID, use your own UWindsor account name, e.g., mine is hfani@uwindsor.ca, so my submission would be: `lec07_hfani.pdf`

Lecture Assignments

(select only 2 questions based on your preference)

1. Minterm is also called standard product (standard ANDing). Maxterm is also called standard sum (standard ORing). Boolean functions that expressed as a sum of products (SoP) or product of sums (PoS) are said to be in **canonical form** if they are in the forms of minterms or MAXTERMs that include ALL the input binary variables in each term either in normal form or in complement form. For instance, F_1(Y, X)=Y'X+Y'X' is in canonical form because F_1 is in the form of $m_0 + m_1$ and each minterm includes all input variables. However, although F_3(Z,Y,X)=Y'X+ZY is the sum of products, it is not in canonical form since the first term (Y'X) does not have the input variable Z or Z' or the second term (ZY) does not have X or X'. In the design process, we start with the truth table and write the Boolean function in their canonical forms either in the sum of minterms or product of MAXTERMs. <u>Then, through the minimization process, the Boolean function may lose its canonical form in order to have less number of terms or variables instead.</u>

Using truth table, given any two Boolean functions F_1 and F_2, prove that:

a) The Boolean function E = $F_1 + F_2$ contains the sum of the minterms of F1 and F2 to be in the sum of products canonical form.

b) The Boolean function G = F_1F_2 contains only the minterms common to F_1 and F_2.

2. Obtain the truth table of the following functions, and express each function in sum of products (SoP) and product of sums (PoS) in canonical forms:
    a. (c' + d)(b + c')
    b. bd' + acd' + ab'c + a'c'

3. Express the **complement** of the following functions in sum of minterms form:
    a. F(A,B ,C, D) = Σ m(2, 4, 7, 10, 12, 14) → F'(A,B ,C, D) = Σ m(?,...)
    b. F(x, y, z) = Π M(3, 5, 7) → F'(x, y, z) = Σ m(?,...)

4. By default, Σ means the sum of minterms. So, we can drop 'm' and Σ is the same as Σm. Similarly, Π means the product of MAXTERMs and we can drop 'M' and Π is the same as ΠM. Convert each of the following to the other canonical form. Then algebraically simplify them, if possible:

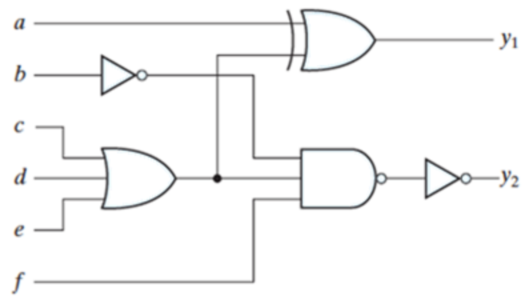
    a. F(x, y, z) = Σ (1, 3, 5)

b. F(A, B, C, D) = Π (3, 5, 8, 11)

5. Determine the number of AND and OR gates as well as the number of their inputs (e.g., 2-input AND, 3-input AND, …) for the design of the following Boolean functions. Try to algebraically simplify them, if possible, and compare the original requirement with the simplified version. The design is <u>not</u> needed.

   a. (x, y, z) = Σ (1, 2, 4, 5)
   b. F(A, B, C, D) = Π (0, 3, 5, 8, 11,13)

6. Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams. Then algebraically simplify the Boolean expression, if possible.



7. **By convention,** when writing a Boolean expression as a function of binary variables, e.g., F(Z, Y, X), we assume the left most binary variable (e.g., Z) has the highest significance in writing the index numbers for minterms and MAXTERMs, e.g., m_2=Z'YX' or M_3=(Z+Y'+X'). However, we know that AND and OR are commutative. Hence, m_2=Z'YX' = YZ'X' = m_4 or M_3= (Z+Y'+X') = (Y'+X'+Z)= M_6. Is this argument correct? Justify your answer.

8. True or False:
   a. If F(X,Y,Z)=X'YZ then F(Z,Y,X)=X'YZ
   b. If F(X,Y,Z)=X'YZ then F(Z,Y,X)=Z'YX
   c. If F(X,Y,Z)=X'YZ then F(Z,Y,X)=YXZ'
   d. If F(X,Y,Z)=m_0+m_1 then F(Z,Y,X)=m_0+m_1
   e. If F(X,Y,Z)=m_0+m_1 then F(Z,Y,X)=m_0+m_4