

HEADS UP



The Bonnie Situation, Pulp Fiction (1994), Quentin Tarantino

- Blackboard is unavailable every weekdays (Monday to Friday) from 5 AM to 6 AM for regular maintenance!
- Change of classroom and labrooms
- Assignments
  - For marking and any related issue, foremost reach out to your lab instructor, not the marker. Then me!
  - For opening another submissions
  - Pay attention to naming of files!
  - Some topics are not covered in the lecture, but in labs or lecture assignments
  - Lab may have dependencies.
  - No key to the labs!
  - Key to the assignments based on highest selected assignments
  - Questions about the lab or lec in discussion board.
    - Subscribe to the forums to receive emails when somebody put sth
  - Academic Integrity (plagiarism)
- Keys to the exams (midterm and final)

- Blackboard is unavailable every weekdays (Monday to Friday) from 5 AM to 6 AM for regular maintenance!
- Change of classroom and labrooms
- Assignments
  - For marking and any related issue, foremost reach out to your lab instructor, not the marker. Then me!
  - For opening another submissions
  - Pay attention to version of files

SLIDES ARE NOT LIKE THIS

- No key to the labs.
  - Key to the assignments based on highest selected assignments
  - Questions about the lab or lec in discussion board.
    - Subscribe to the forums to receive emails when somebody put sth
- Keys to the exams (midterm and final)



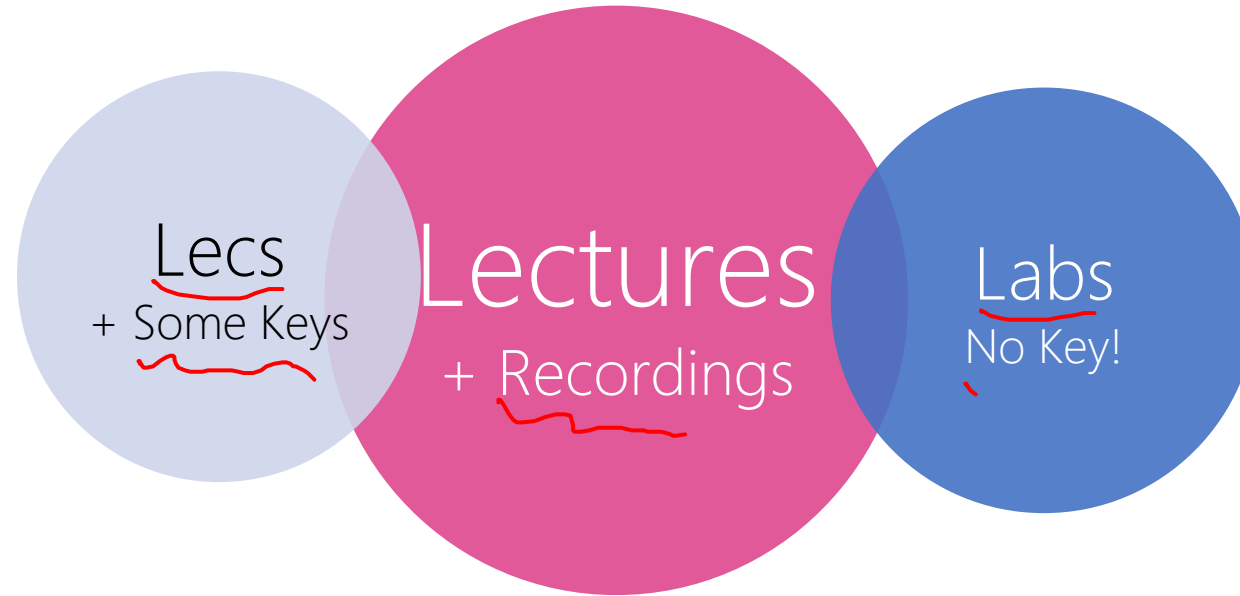
Pink Floyd - Another Brick In The Wall

Are robots becoming more human or humans becoming more robotic?



Slides are not books, summaries, ...  
Slides are not for last night before exam  
Slides are to be presented! Lecture = Slide + Talk

SOME TOPICS ARE NOT COVERED IN THE LECTURE, BUT IN LABS OR LECTURE ASSIGNMENTS



NEXT  
WEEK



Zahra Taherikhonakdar ([taherik@uwindsor.ca](mailto:taherik@uwindsor.ca))  
Morning Labs  
Section 51, Tuesdays 11:30 AM - 12:50 PM  
Section 52, Thursdays 11:30 AM - 12:50 PM



Roonak Moasses ([moasses@uwindsor.ca](mailto:moasses@uwindsor.ca))  
Afternoon Labs  
Section 53, Tuesdays 01:00 PM - 02:20 PM  
Section 54, Thursdays 01:00 PM - 02:20 PM

# Discussion Board

Discussions are a good way to encourage students to think critically about your coursework and interact with each others' ideas. You can create discussions around individual course lessons or for your course in general. [More Help](#)

Create Forum

Search

Discover Content



	Delete					
<input type="checkbox"/>	FORUM	DESCRIPTION	TOTAL POSTS	UNREAD POSTS	UNREAD REPLIES TO ME	TOTAL PARTICIPANTS
<input type="checkbox"/>	Lectures		126	0	0	25
<input type="checkbox"/>	Laboratories		199	0	0	35
<input type="checkbox"/>	Midterm Exam		31	0	0	16
<input type="checkbox"/>	Final Exam		60	0	0	27
<input type="checkbox"/>	Miscellaneous		35	0	0	14
	Delete					















Handwritten notes: "20" above the Unread Posts column, "5" circled in the Unread Posts column for Lectures, "subscrip" with an arrow pointing to Lectures, and "+5%" with an arrow pointing to Laboratories.

Displaying 1 to 5 of 5 items

Show All Edit Paging...



# THERE MIGHT BE A CHANGE TO CLASSROOM AND LABROOM

Personalize   View All      							First	 1-12 of 12  Last
	Class	Class Title	Enrolled	Days & Times	Room	Class Dates		
	COMP 2650-1A (5115)	Comp Architre I:Digital Design (Lecture)	62	TuTh 10:00AM - 11:20AM	Erie Hall 3123	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-1O (6123)	Comp Architre I:Digital Design (Lecture)	72	TuTh 10:00AM - 11:20AM	Livestream (Hyflex)	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-51A (5116)	Comp Architre I:Digital Design (Laboratory)	19	Tu 11:30AM - 12:50PM	Erie Hall 2125	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-51O (6124)	Comp Architre I:Digital Design (Laboratory)	20	Tu 11:30AM - 12:50PM	Livestream (Hyflex)	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-52A (5117)	Comp Architre I:Digital Design (Laboratory)	19	Th 11:30AM - 12:50PM	Erie Hall 2125	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-52O (6125)	Comp Architre I:Digital Design (Laboratory)	14	Th 11:30AM - 12:50PM	Livestream (Hyflex)	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-53A (5118)	Comp Architre I:Digital Design (Laboratory)	17	Tu 1:00PM - 2:20PM	Erie Hall 2125	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-53O (6126)	Comp Architre I:Digital Design (Laboratory)	14	Tu 1:00PM - 2:20PM	Livestream (Hyflex)	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-54A (5119)	Comp Architre I:Digital Design (Laboratory)	18	Th 1:00PM - 2:20PM	Dillon Hall 256	Jan 17, 2022-Apr 18, 2022		
	COMP 2650-54O (6127)	Comp Architre I:Digital Design (Laboratory)	13	Th 1:00PM - 2:20PM	Livestream (Hyflex)	Jan 17, 2022-Apr 18, 2022		

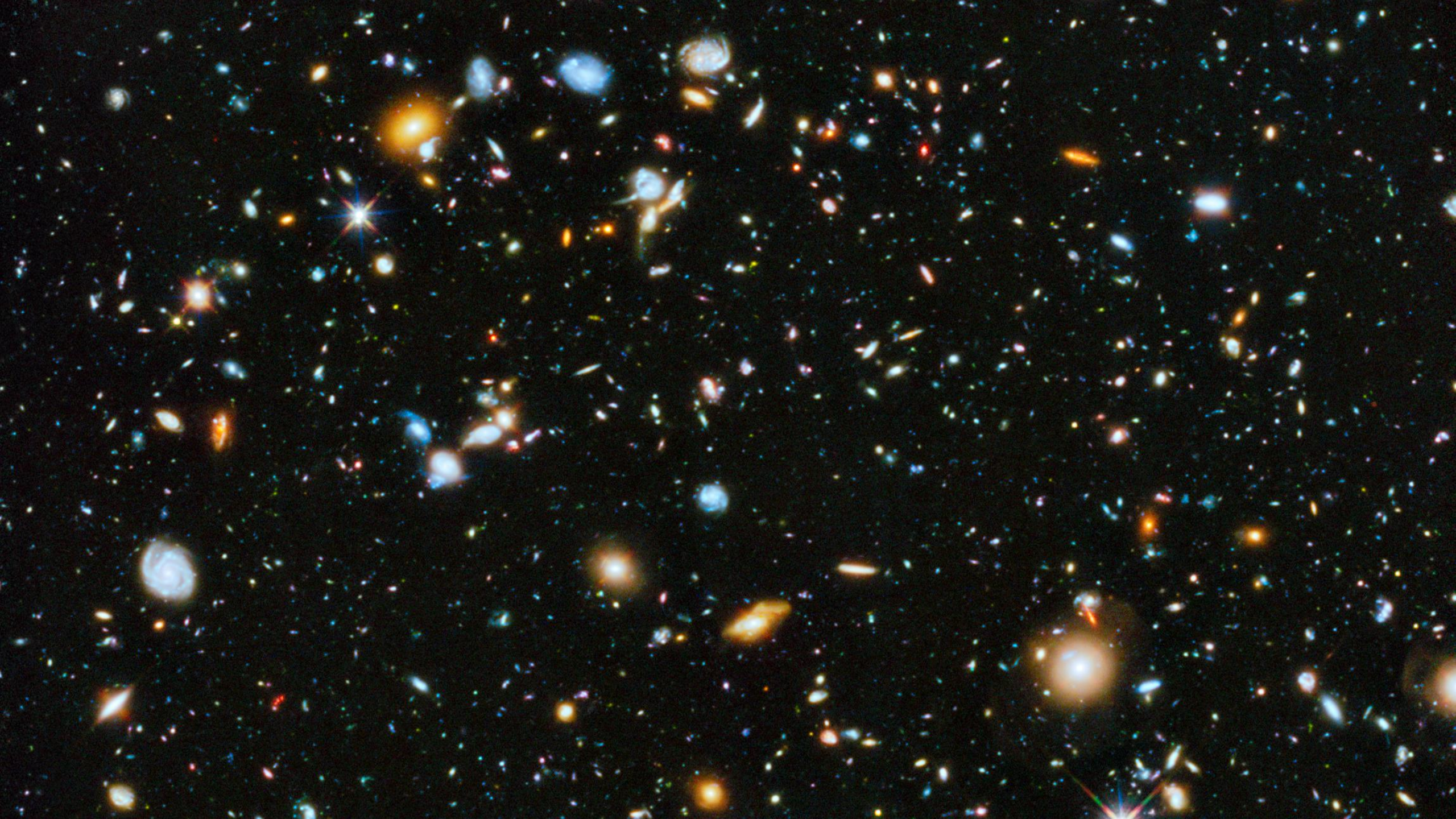
4 AM

**Blackboard will return to service at 6 am EST.**

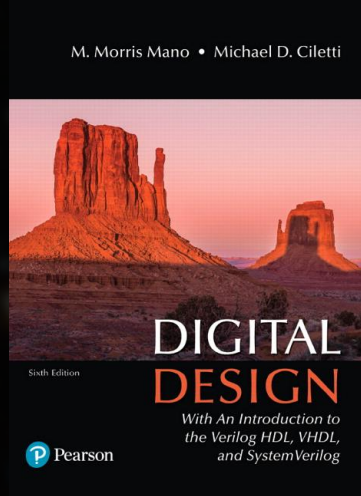
Blackboard is unavailable every weekday (Monday to Friday) from 5 am to 6 am EST for regular maintenance.

At 6 am you can [reload this page](#) to access Blackboard.









# Chapter 1

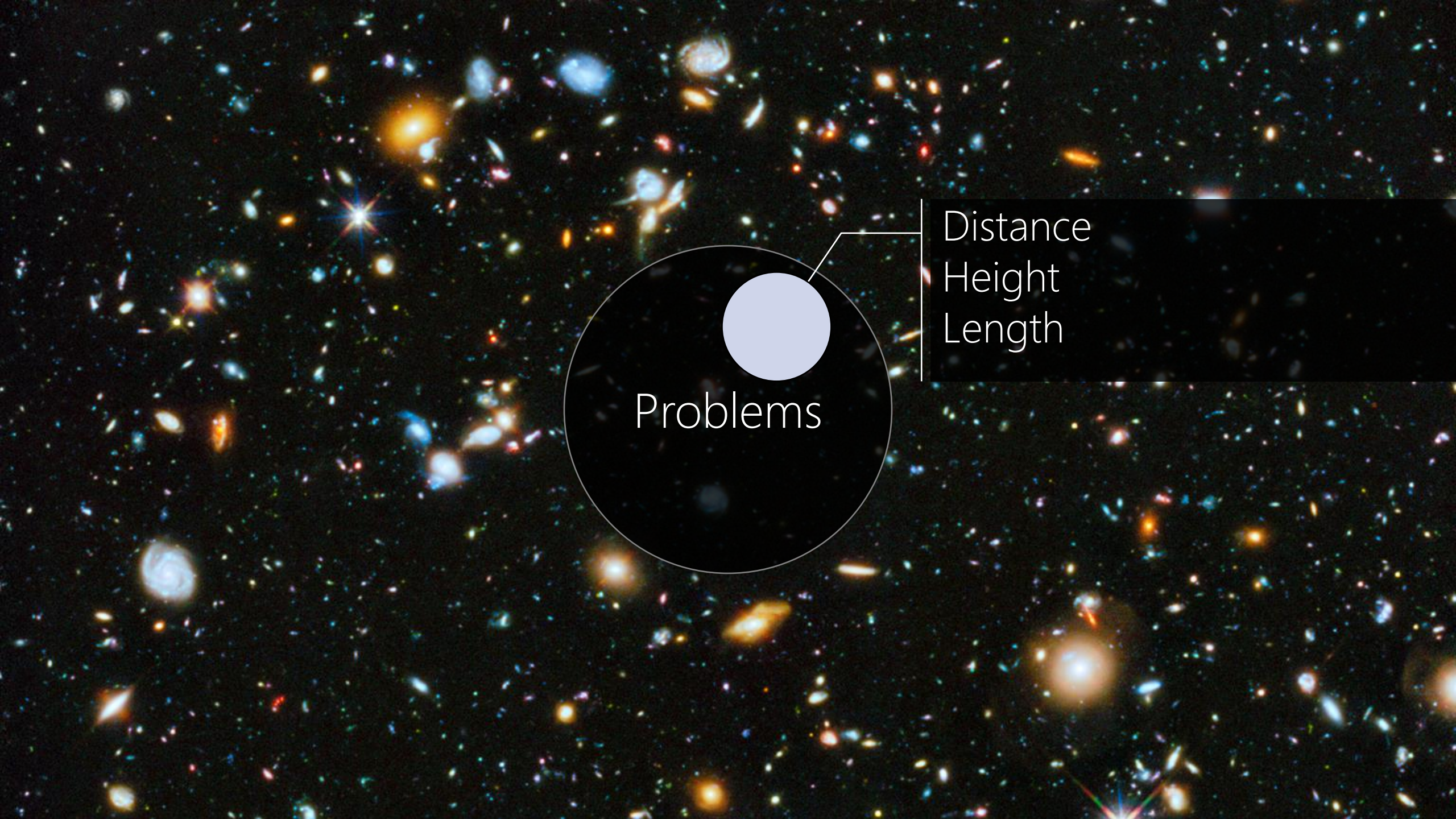
## Digital Systems and Binary Numbers



A deep-field astronomical image showing a vast number of galaxies in various colors (blue, orange, white) against a black background. The galaxies are of different shapes and sizes, some appearing as bright, distinct objects while others are fainter. A large, solid black circle is centered in the image, containing the word "Problems" in white text.

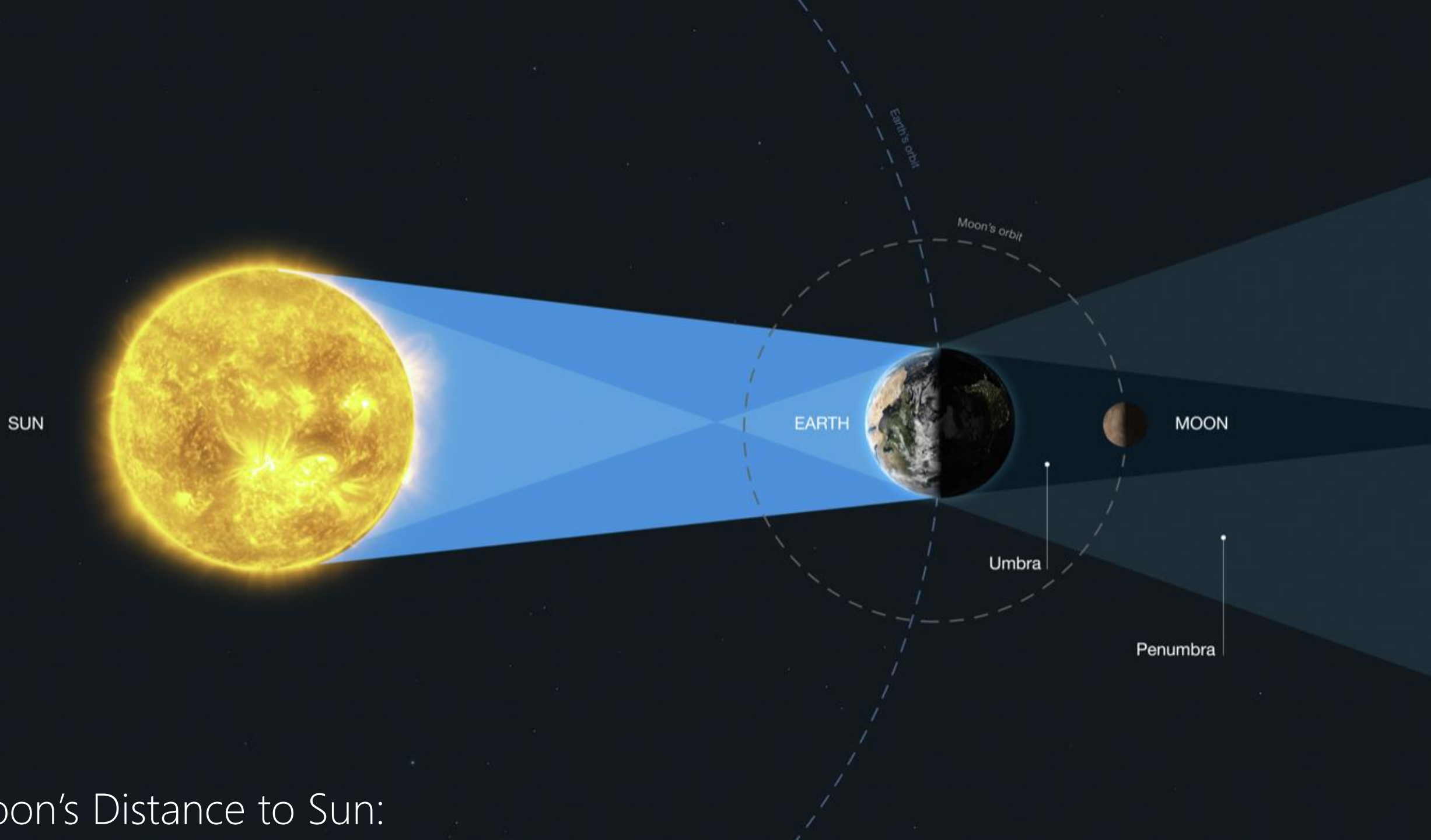
Problems



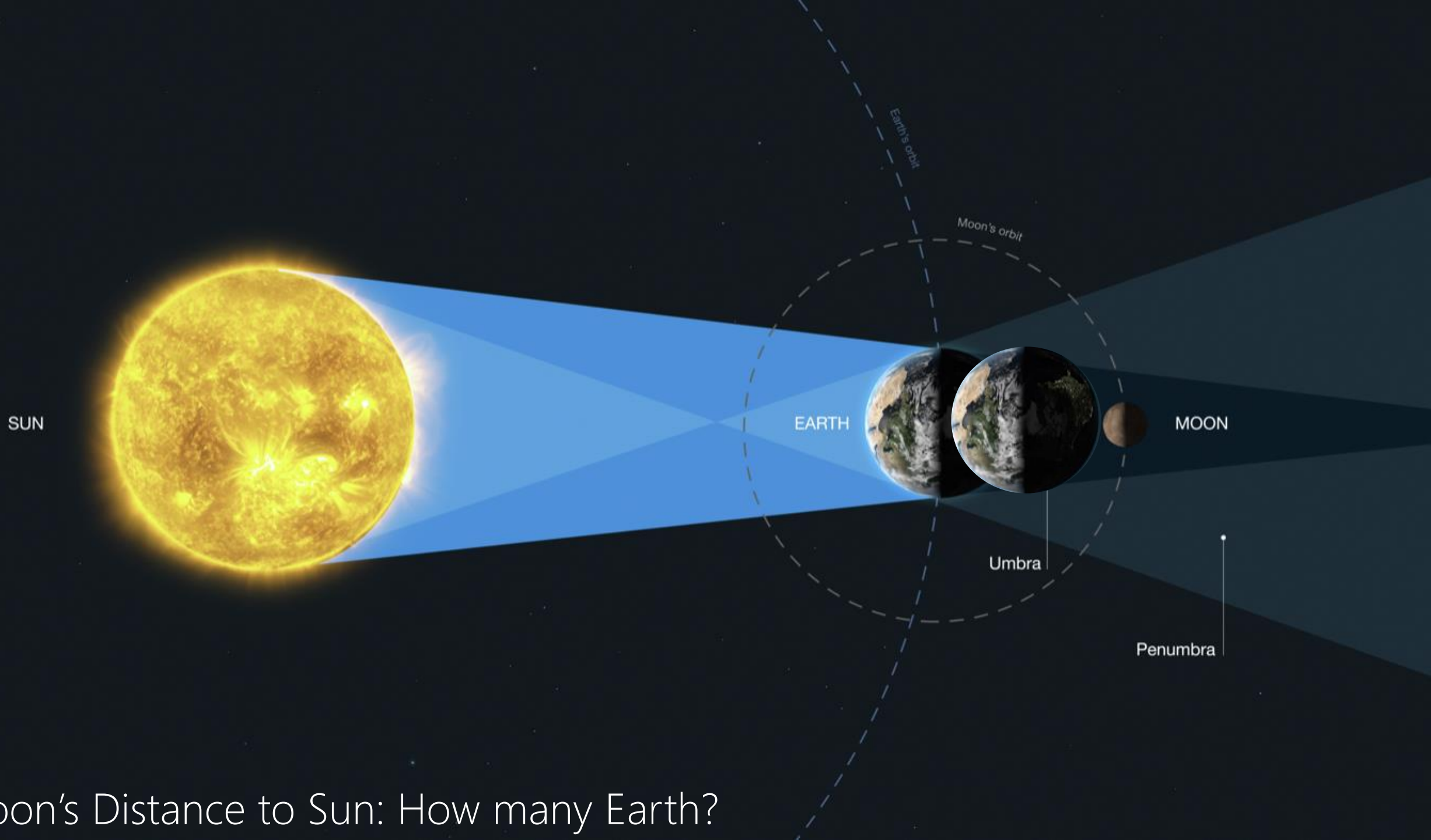


Distance  
Height  
Length



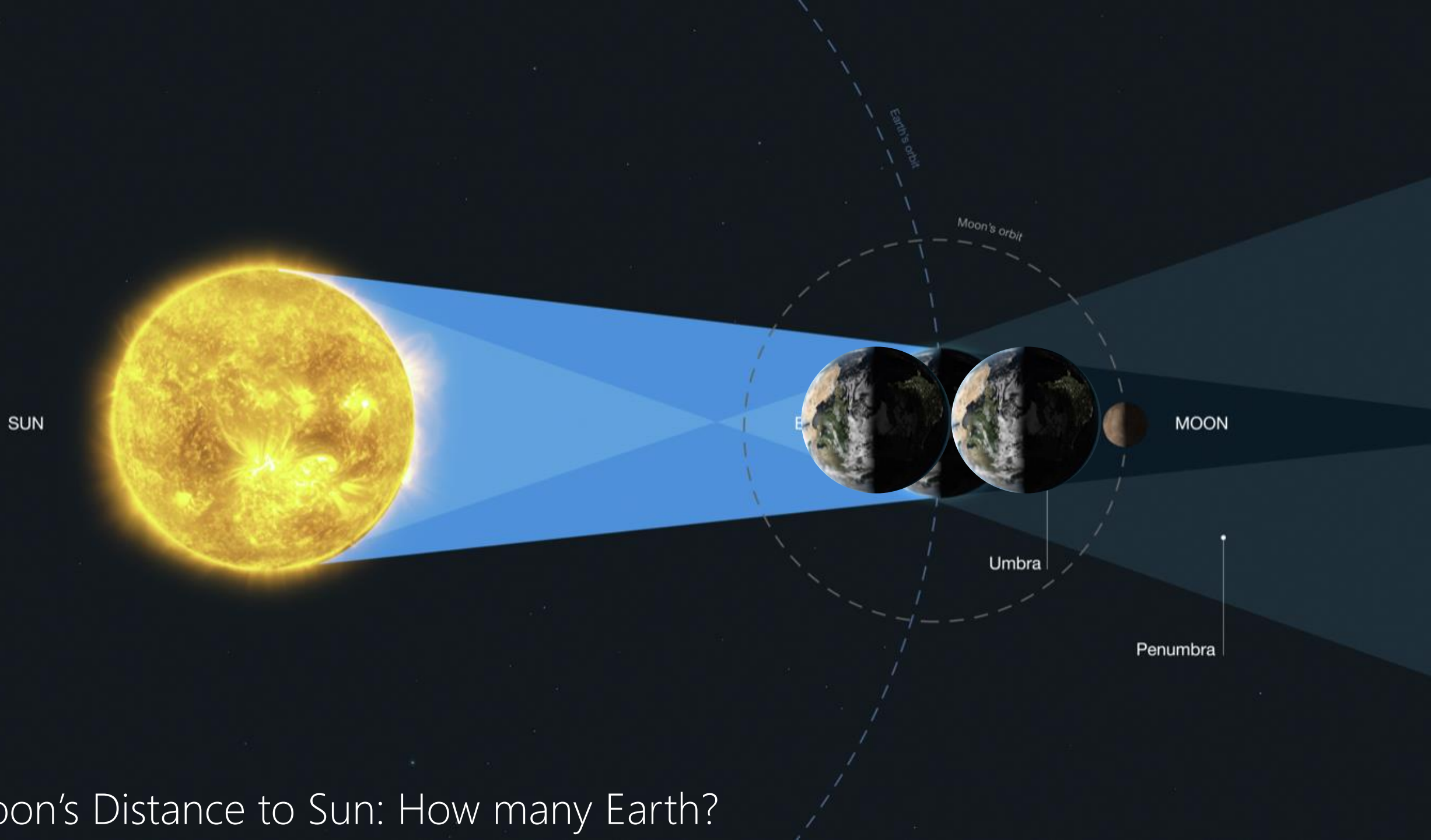


Moon's Distance to Sun:

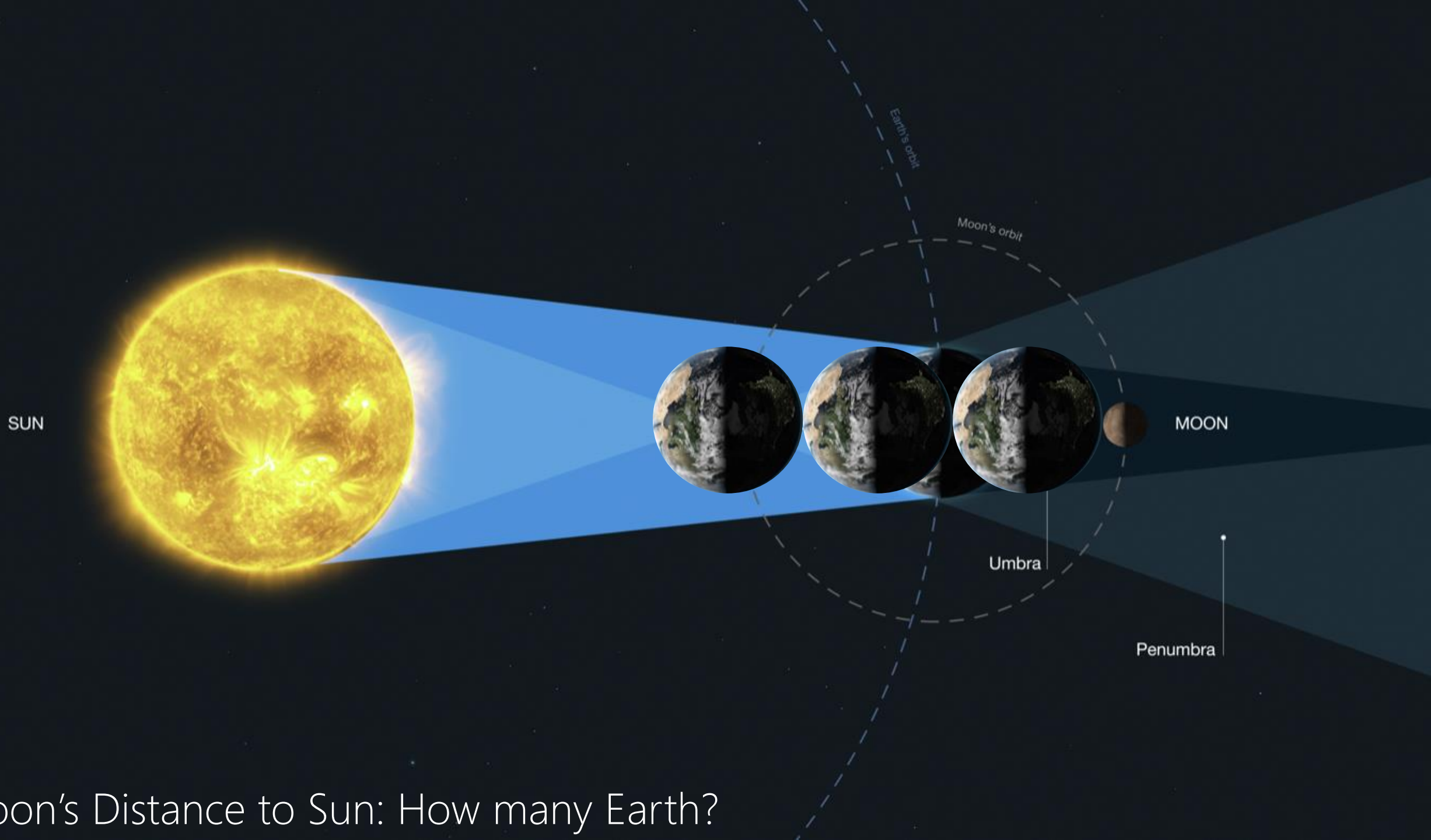


Moon's Distance to Sun: How many Earth?

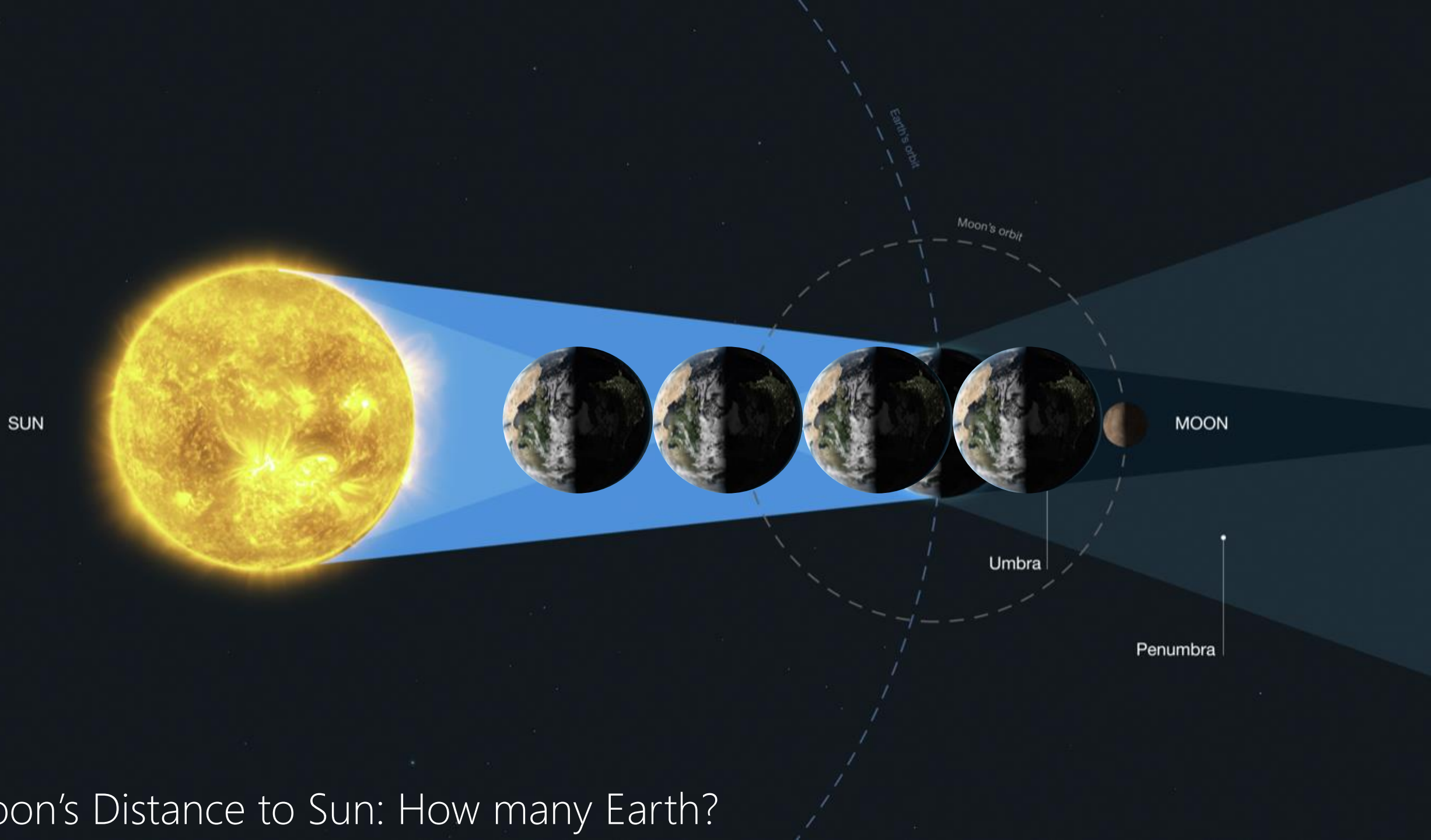




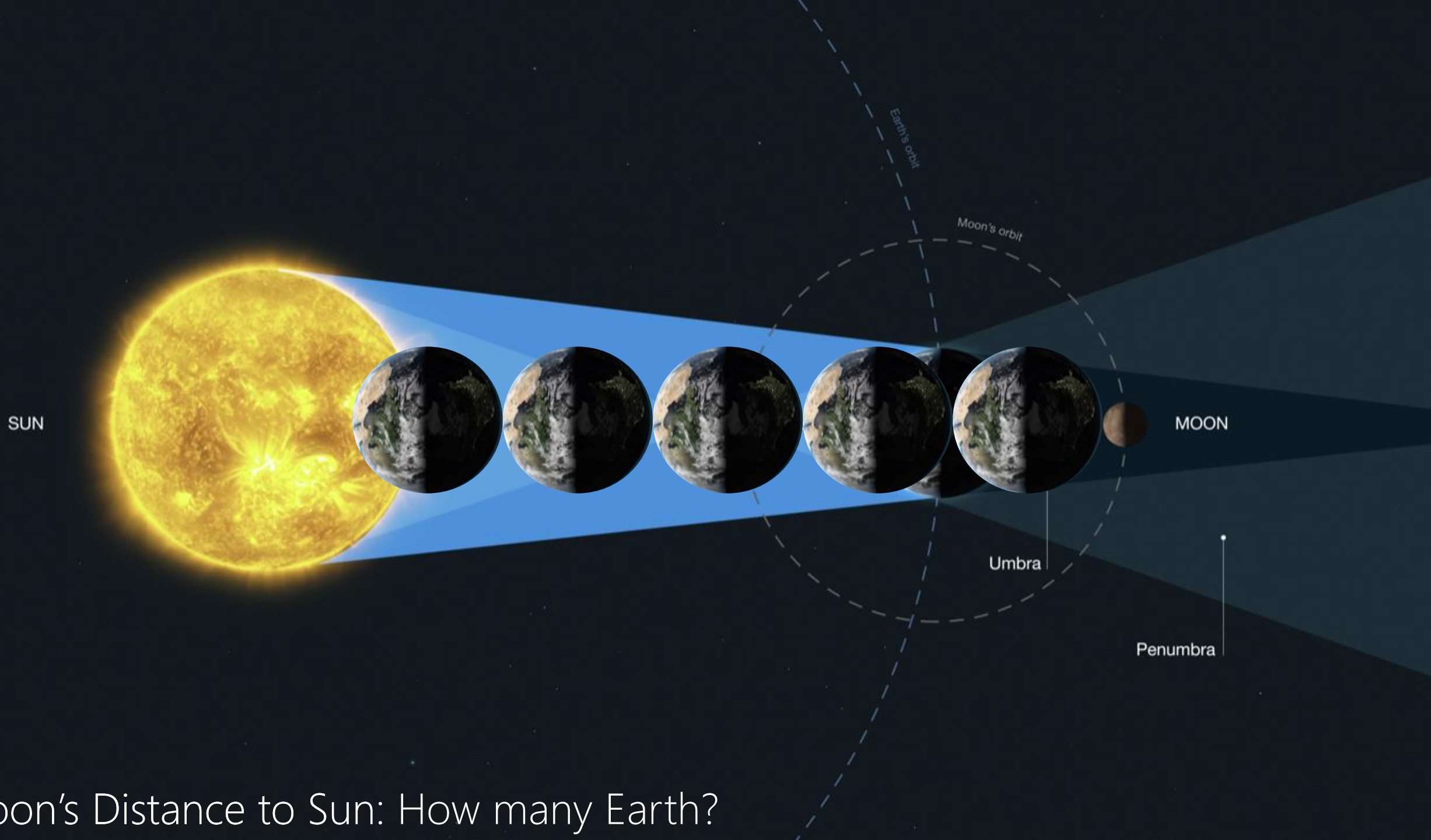
Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?

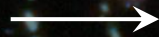


Moon's Distance to Sun: How many Earth?





Continuous



Discrete



A deep-field astronomical image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the central text.

# DISCRETE SYSTEMS

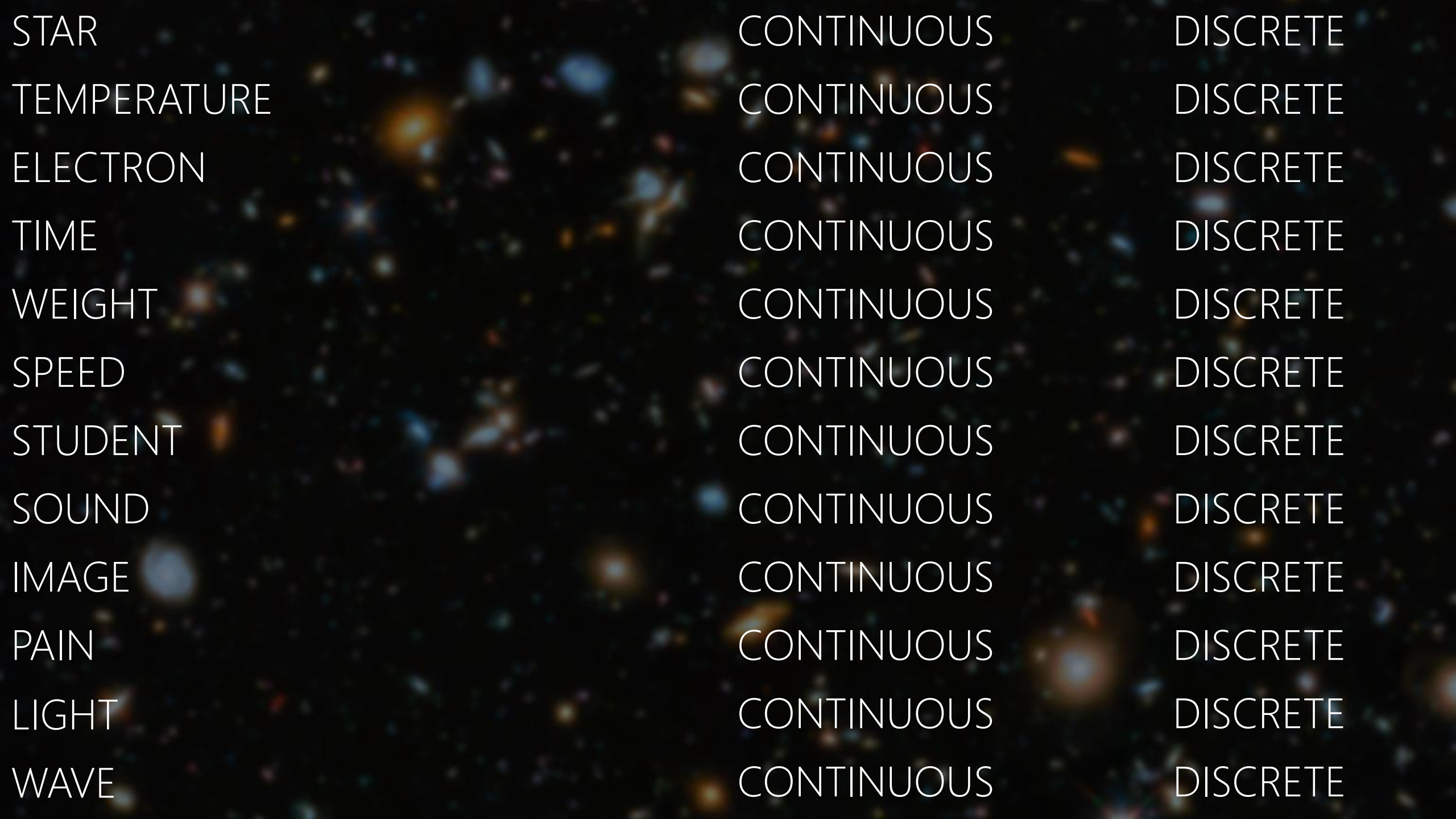


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# POLL

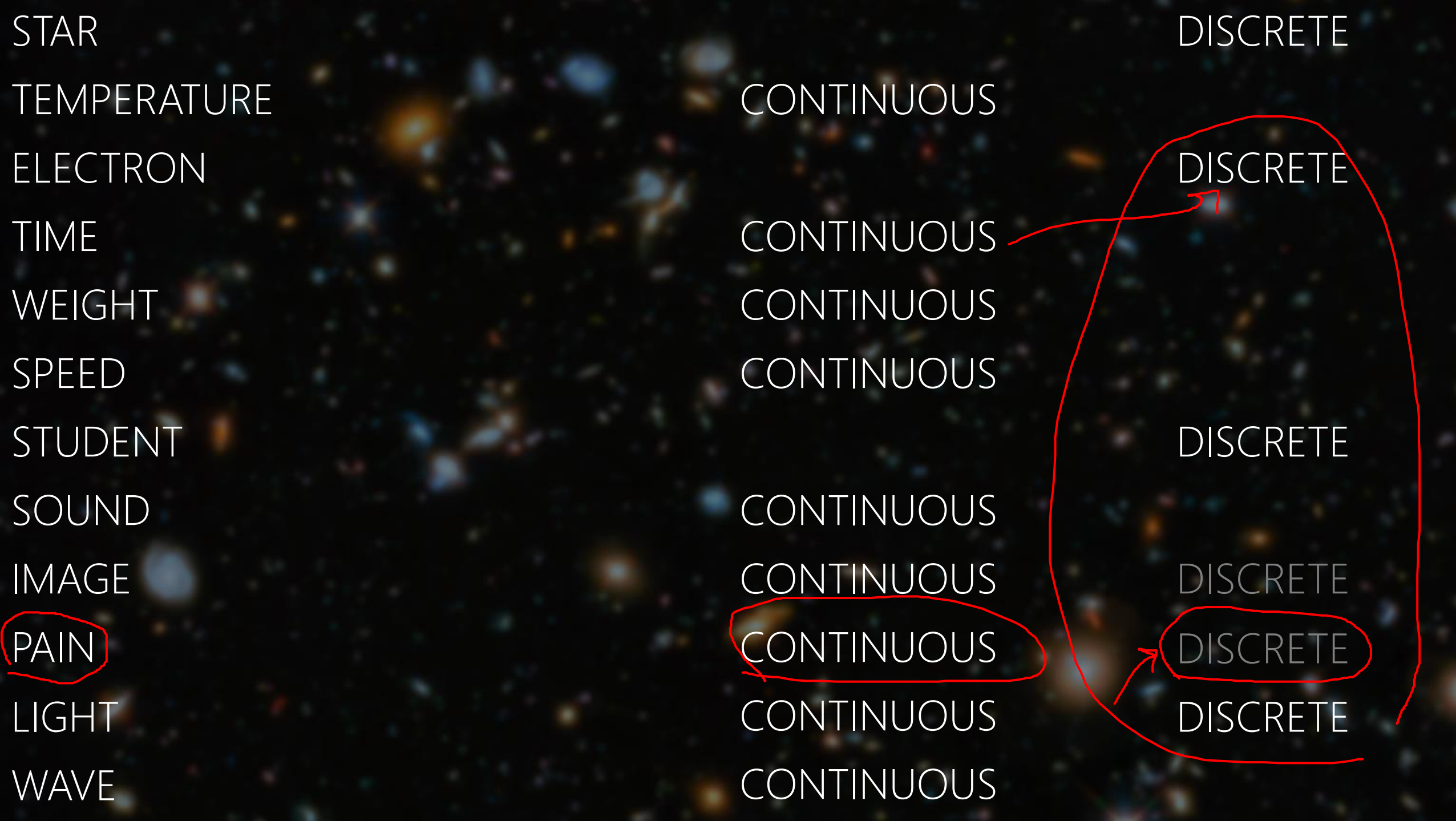
[Click to Open the Poll](#)

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STAR	CONTINUOUS	DISCRETE
TEMPERATURE	CONTINUOUS	DISCRETE
ELECTRON	CONTINUOUS	DISCRETE
TIME	CONTINUOUS	DISCRETE
WEIGHT	CONTINUOUS	DISCRETE
SPEED	CONTINUOUS	DISCRETE
STUDENT	CONTINUOUS	DISCRETE
SOUND	CONTINUOUS	DISCRETE
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	DISCRETE





STAR		DISCRETE
TEMPERATURE	CONTINUOUS	
ELECTRON		DISCRETE
TIME	CONTINUOUS	
WEIGHT	CONTINUOUS	
SPEED	CONTINUOUS	
STUDENT		DISCRETE
SOUND	CONTINUOUS	
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	

A red hand-drawn oval encloses the following cells in the table:

- Row 3: ELECTRON (empty), DISCRETE
- Row 7: STUDENT (empty), DISCRETE
- Row 8: IMAGE (CONTINUOUS), DISCRETE
- Row 9: PAIN (CONTINUOUS), DISCRETE
- Row 10: LIGHT (CONTINUOUS), DISCRETE

Red arrows point from the following cells to the enclosed area:

- Row 4: TIME (CONTINUOUS) points to the top of the oval.
- Row 9: PAIN (CONTINUOUS) points to the bottom-left of the oval.
- Row 10: LIGHT (CONTINUOUS) points to the bottom-right of the oval.



"The world's first photograph—or at least the oldest surviving photo—was taken by Joseph Nicéphore Niépce in 1826 or 1827. Captured using a technique known as heliography, the shot was taken from an upstairs window at Niépce's estate in Burgundy. As heliography produces one-of-a-kind images, there are no duplicates of the piece, which is now part of the permanent collection at the University of Texas-Austin." [18 Famous First Photographs in History: From the Oldest Photo Ever to the World's First Instagram](#)



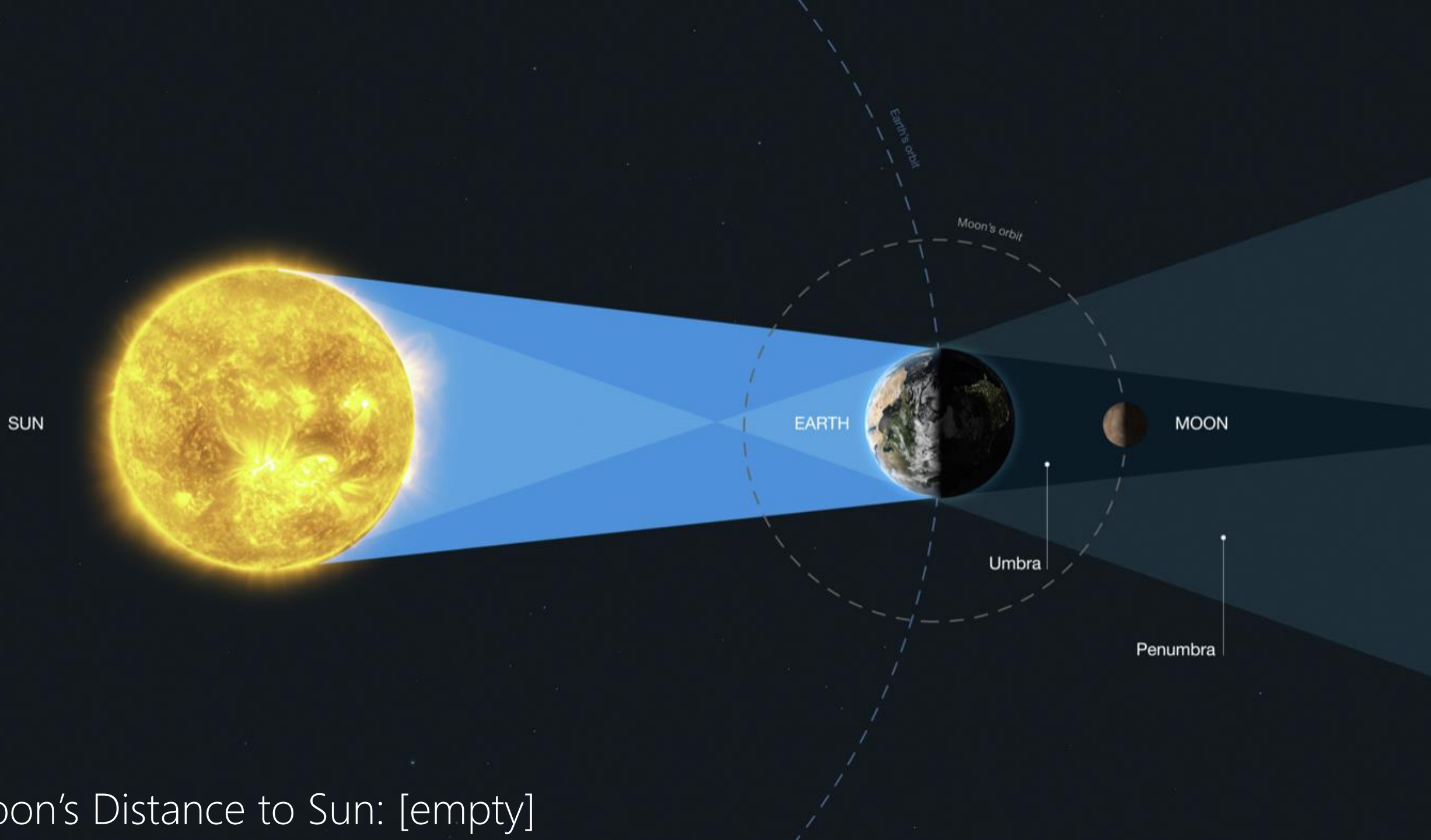




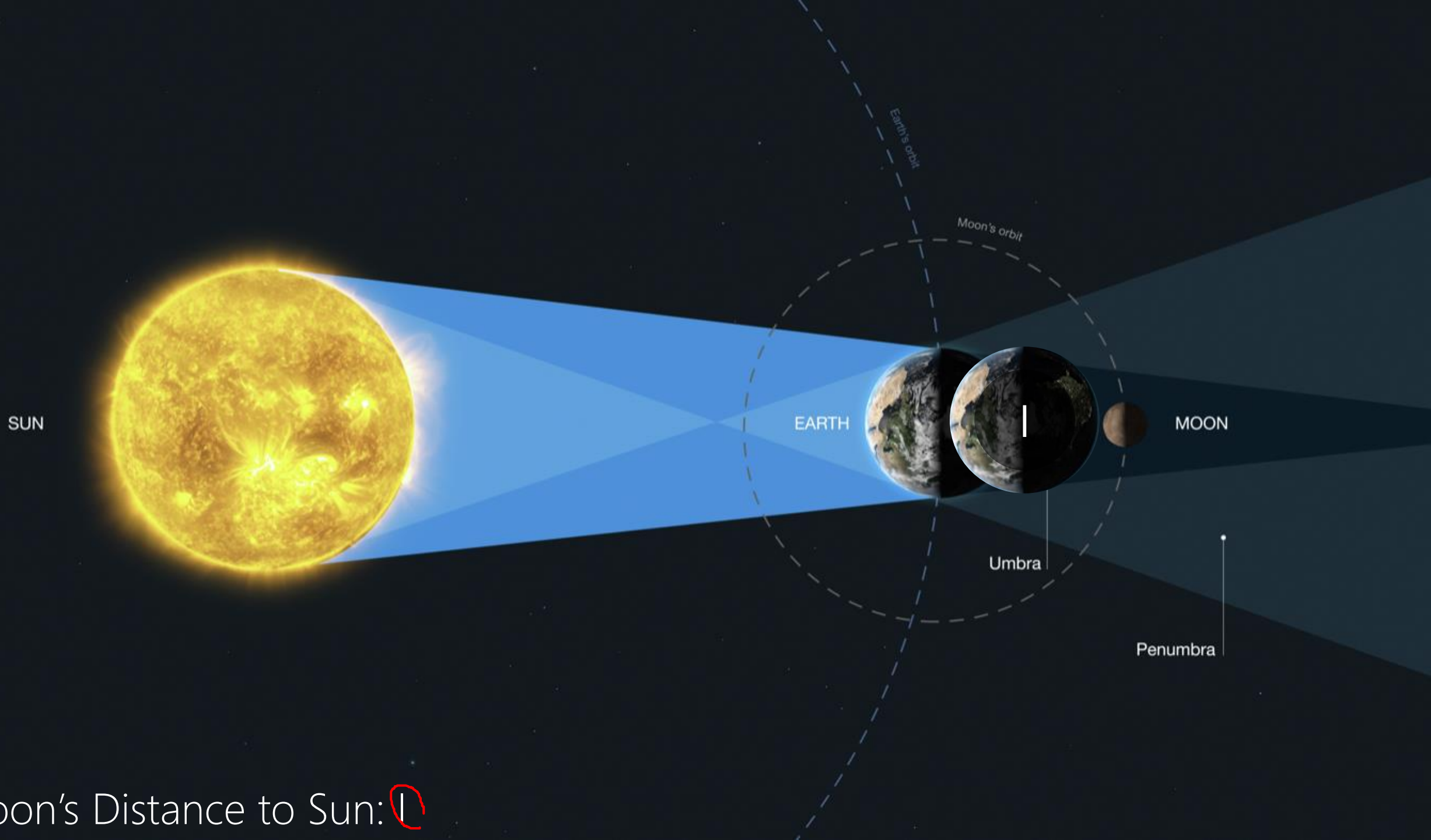


"We may be used to selfies now, but it's Robert Cornelius's 1839 image that lays claim to the first self-portrait. Taken in Philadelphia, Cornelius sat for a little over one minute before covering the lens." [18 Famous First Photographs in History: From the Oldest Photo Ever to the World's First Instagram](#)



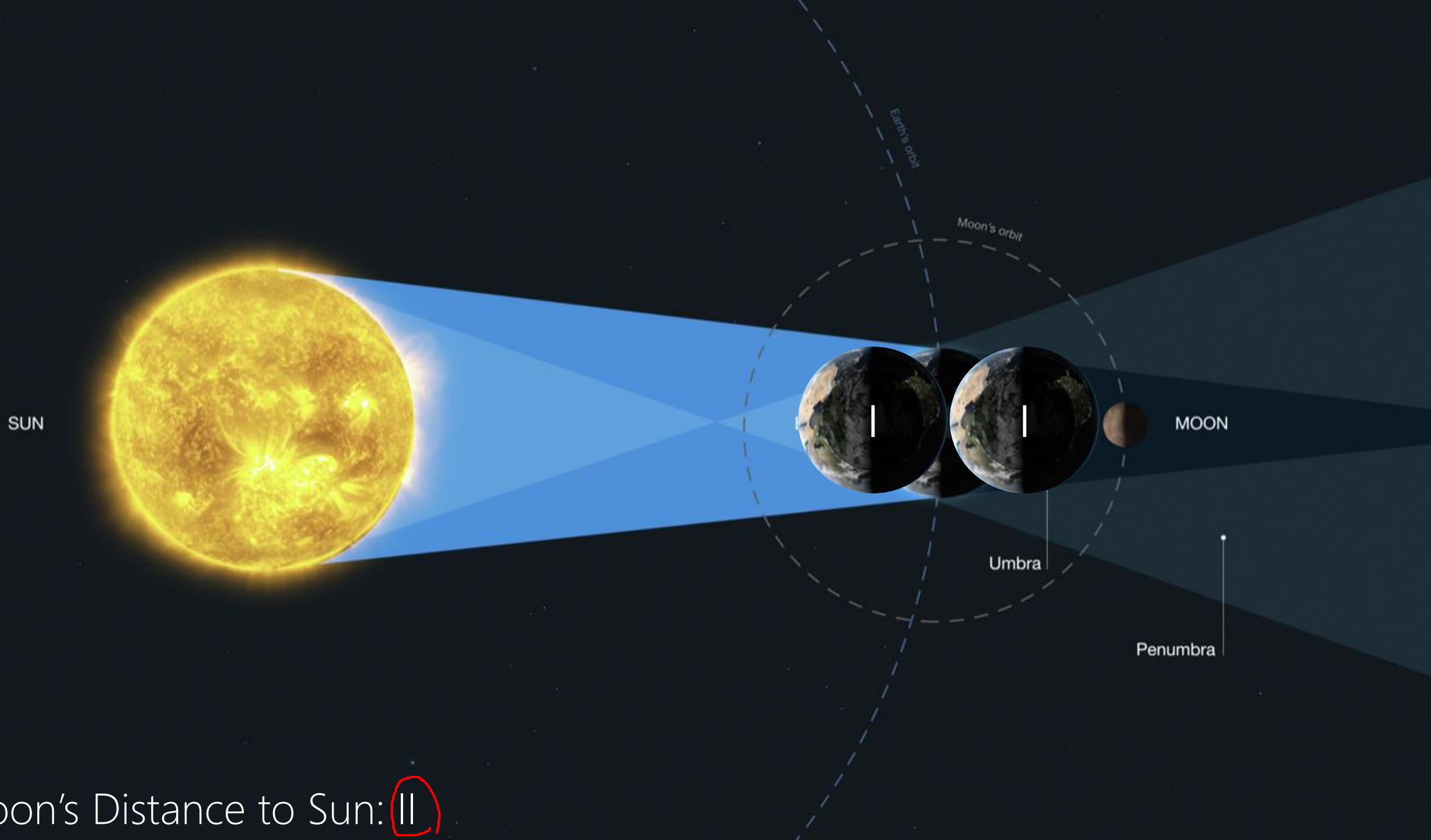


Moon's Distance to Sun: [empty]



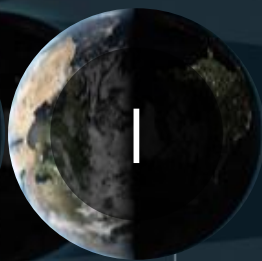
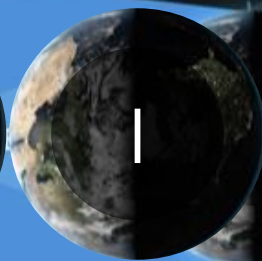
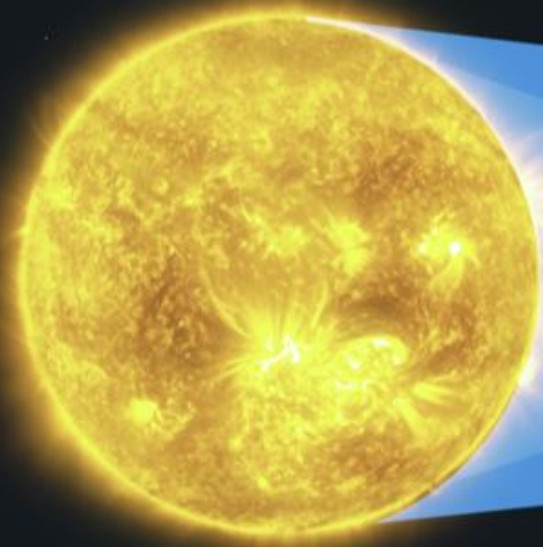
Moon's Distance to Sun: 1





Moon's Distance to Sun: II

SUN



MOON

Earth's orbit

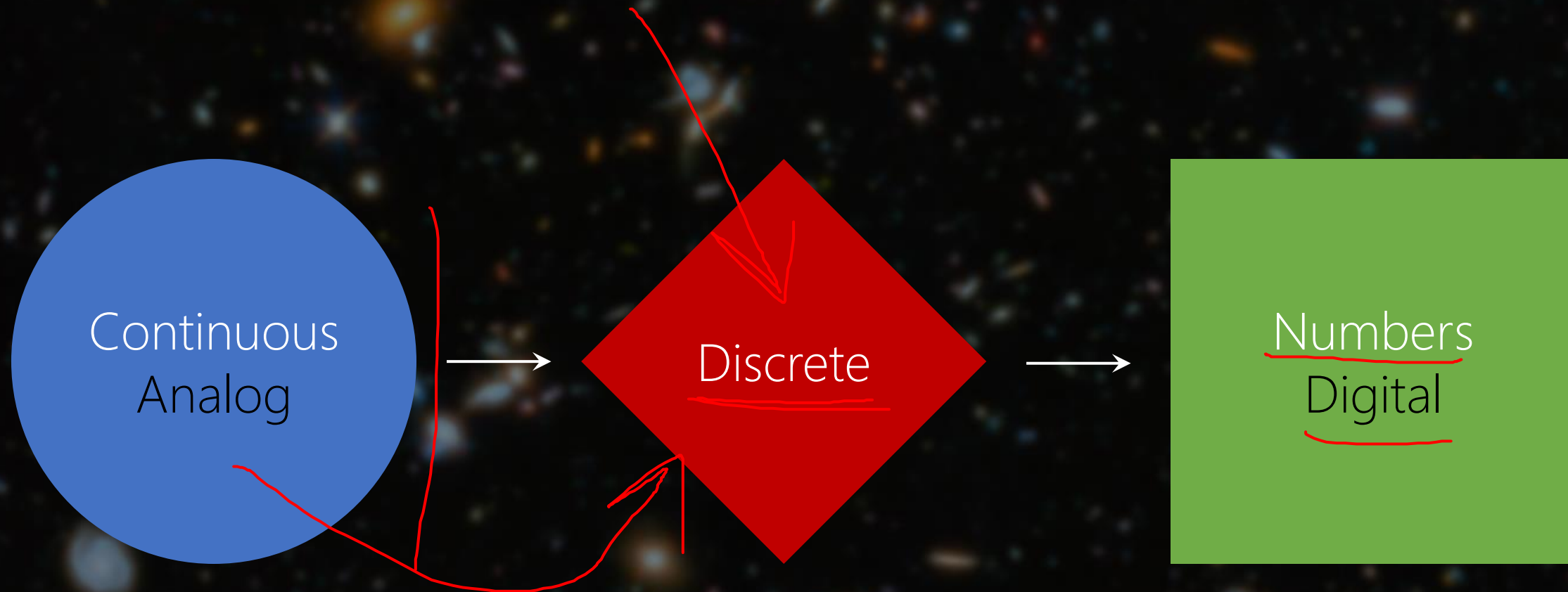
Moon's orbit

Umbra

Penumbra

Moon's Distance to Sun: III





Quantization



Roman Numerals  
Originated in Ancient Rome  
8th Century BC





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# UNARY SYSTEM

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aka. Base-1

how many positions to represent the moon's distance to the sun if an Oracle said it is ~150 million km and earth's diameter is ~13,000 km?





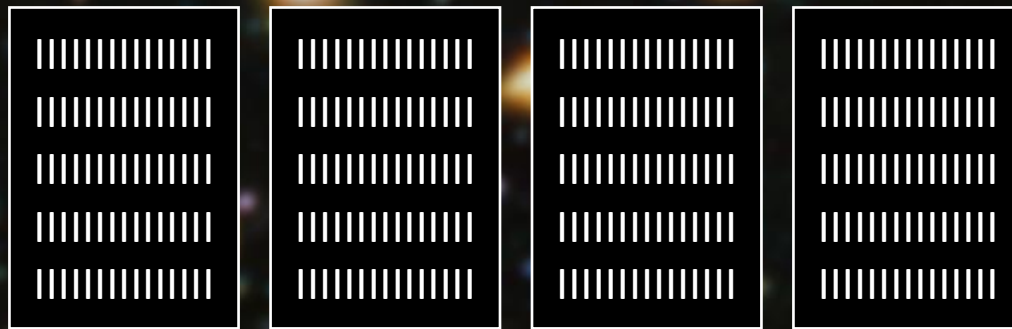


# POLL

[Click to Open the Poll](#)



$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$   
paper =  $\sim 3,000 \text{ positions}$   
 $12,000 \div 3,000 = 4 \text{ pages!}$



A deep space image showing a vast field of galaxies and stars against a black background. The galaxies are in various colors, including blue, orange, and white, and are scattered across the frame. Two horizontal blue lines are positioned above and below the text.

# NUMBER SYSTEMS



— = ≡ ≠ √ √ | 6 7 8 9

Brahmi

3rd and 7th century AD

ॐ ॐ ॐ ॐ ॐ ॐ | ॐ ॐ ॐ ॐ ॐ

XX

Hindu (Gwalior)

ॐ ॐ ॐ ॐ ॐ ॐ | ॐ ॐ ॐ ॐ ॐ

Sanskrit-Devanagari

۱ ۲ ۳ ۴ ۵ | ۶ ۷ ۸ ۹

Western Arabic (Gobar)

۱ ۲ ۳ ۴ ۵ | ۶ ۷ ۸ ۹ ۰

Eastern Arabic

۱ ۲ ۳ ۴ ۵ | ۶ ۷ ۸ ۹ ۰

11th Century (Apices)

۱ ۲ ۳ ۴ ۵ | ۶ ۷ ۸ ۹ ۰

15th Century

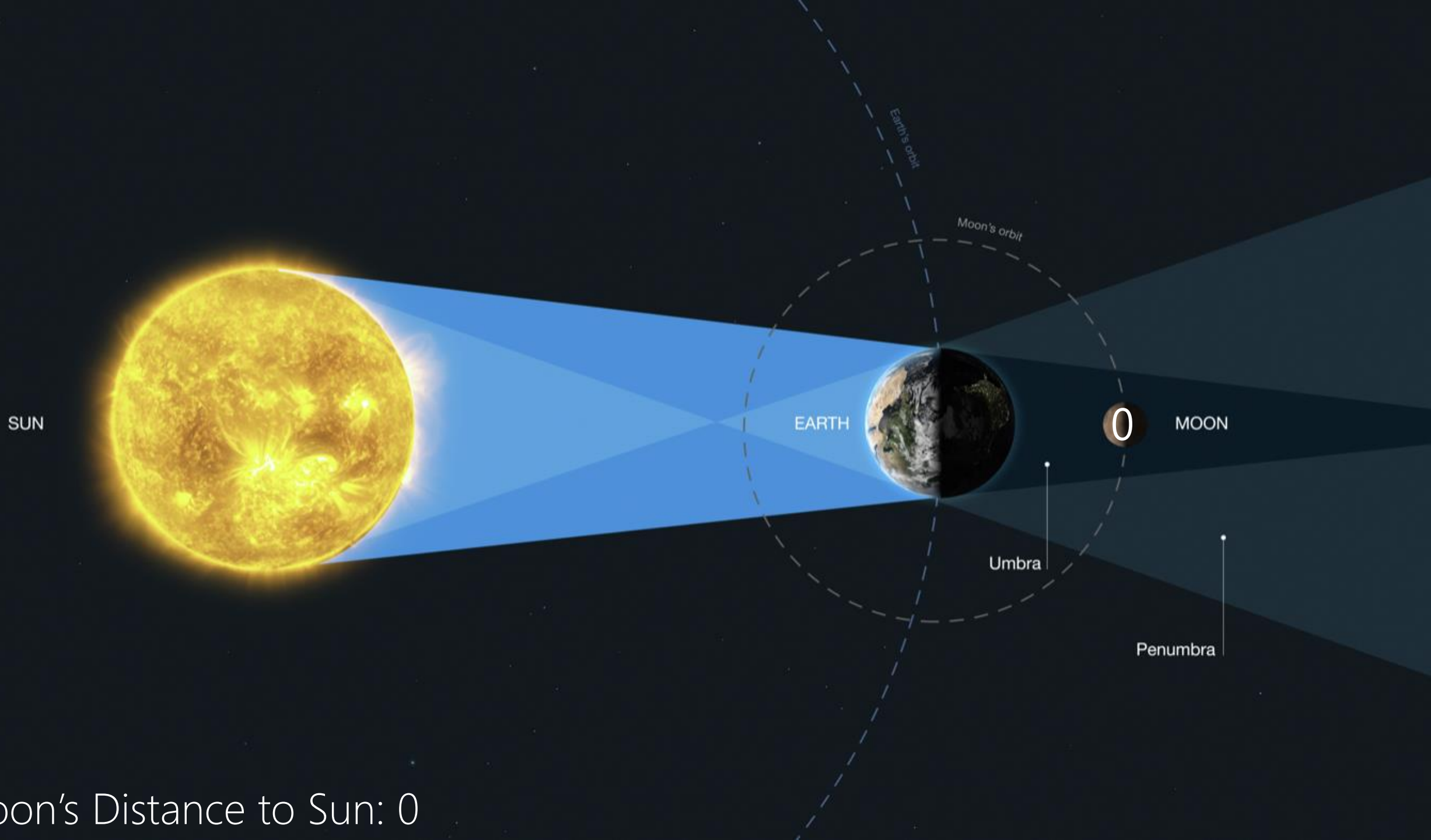
۱ ۲ ۳ ۴ ۵ | ۶ ۷ ۸ ۹ ۰

16th Century (Dürer)

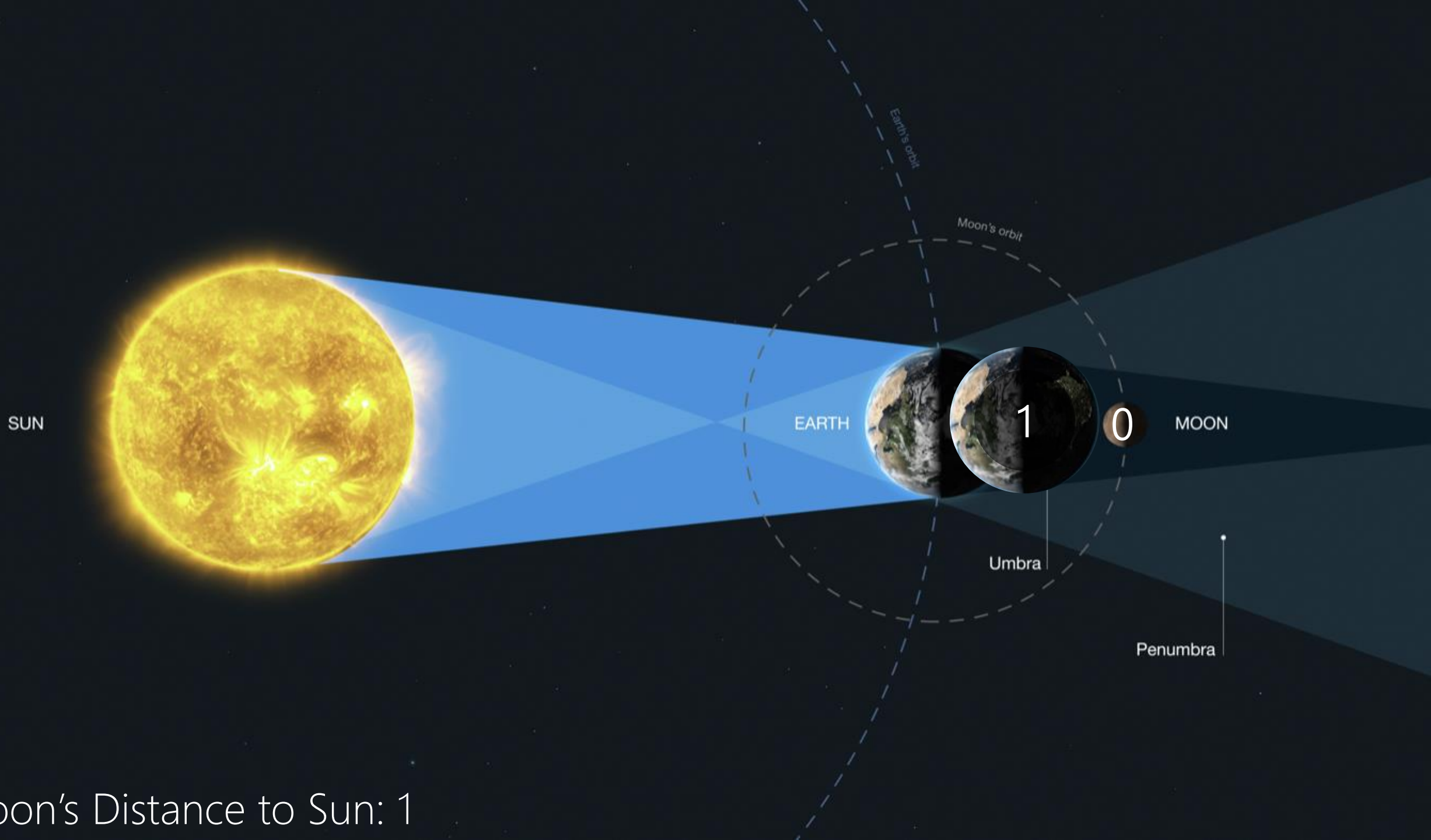
0123456789

Hossein's Number System



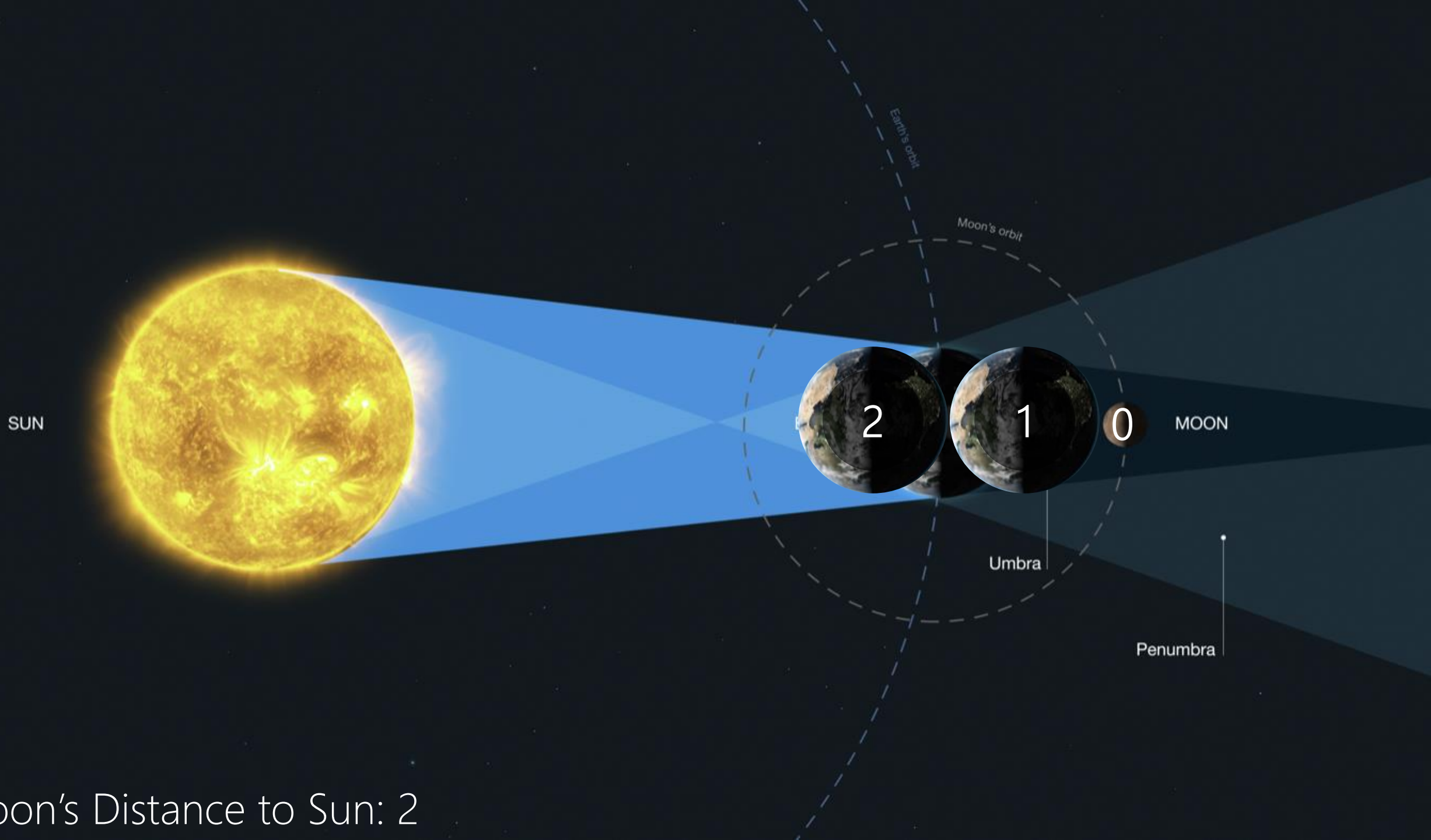


Moon's Distance to Sun: 0

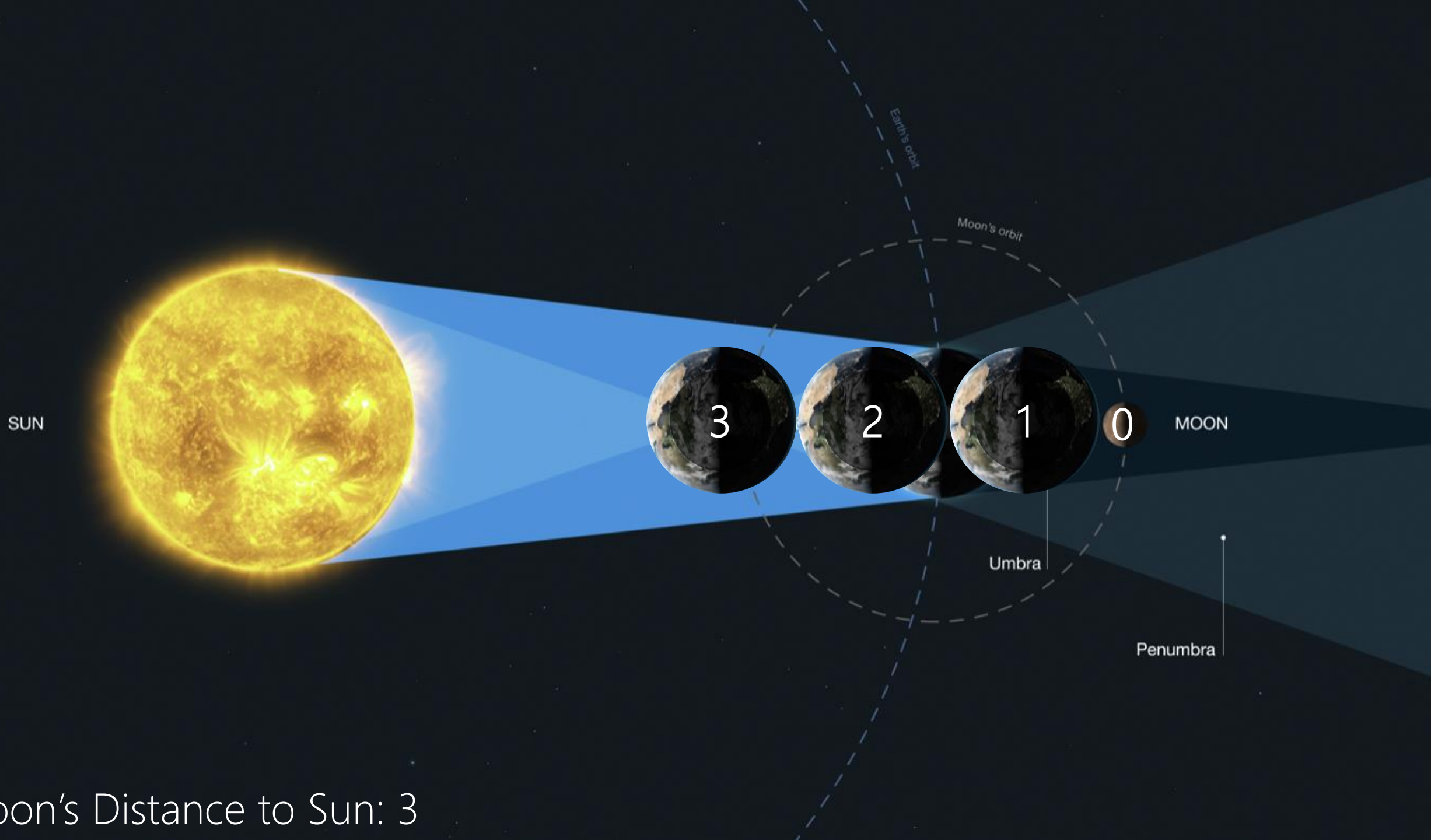


Moon's Distance to Sun: 1



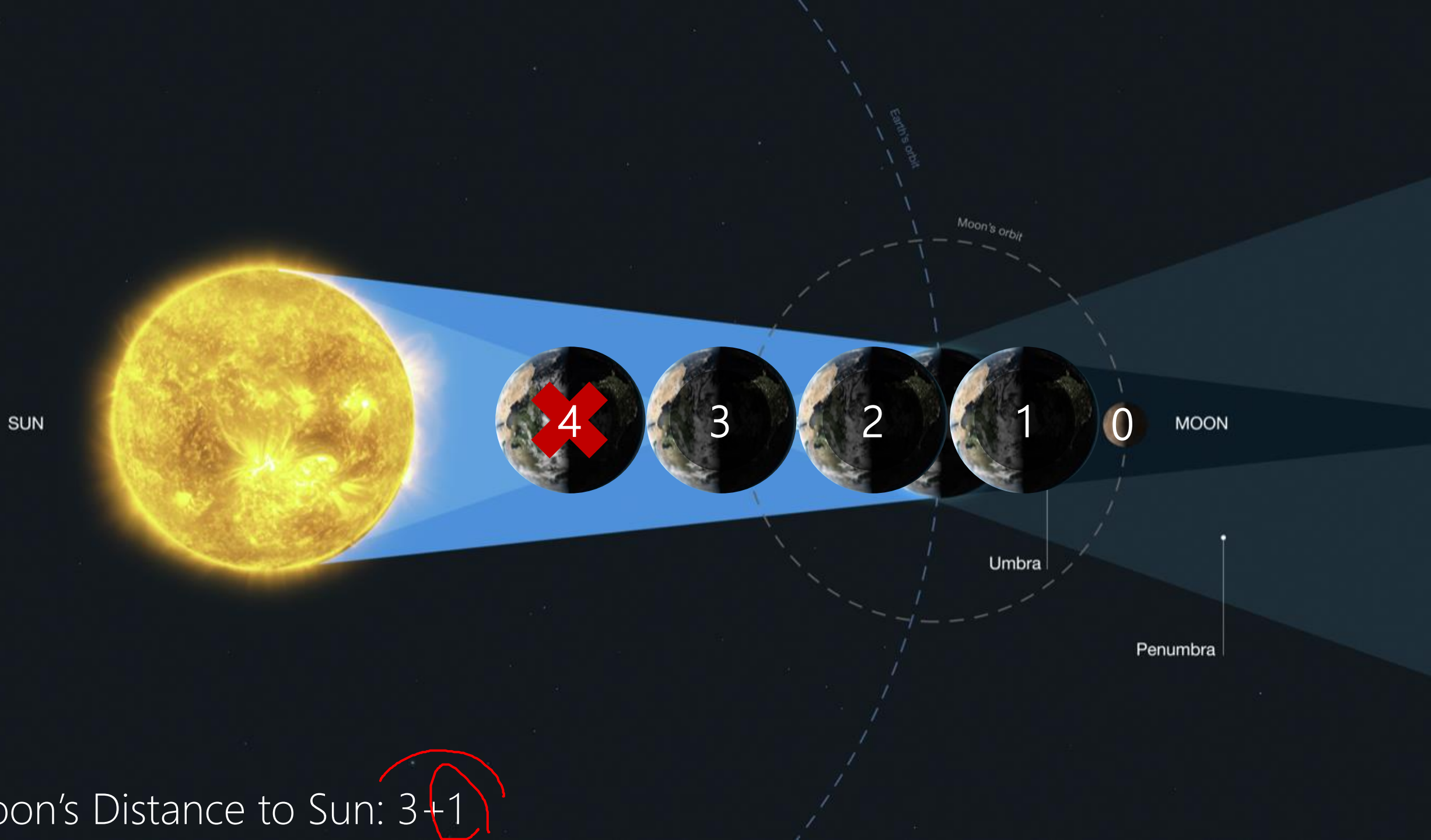


Moon's Distance to Sun: 2

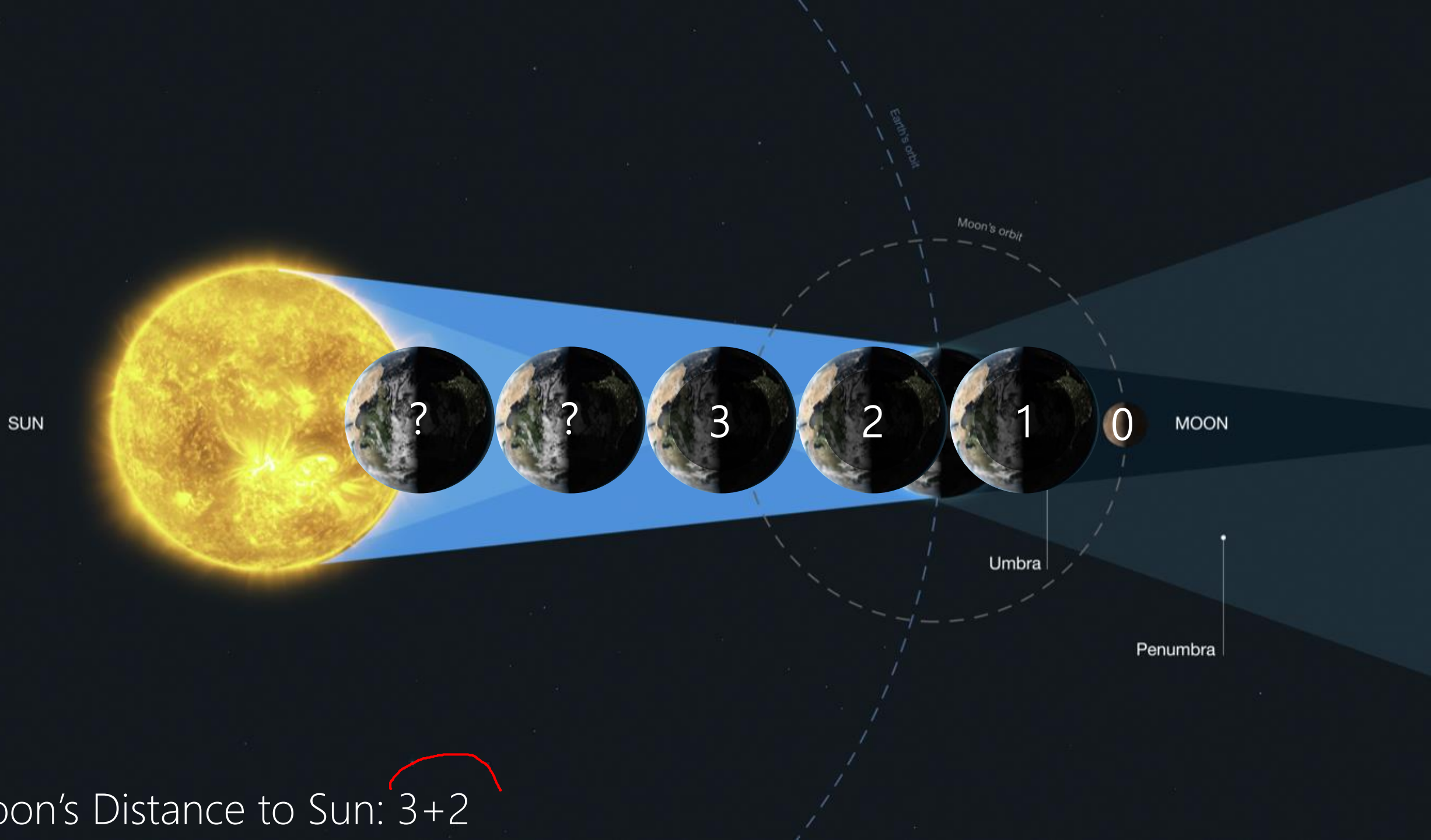


Moon's Distance to Sun: 3





Moon's Distance to Sun:  $3+1$



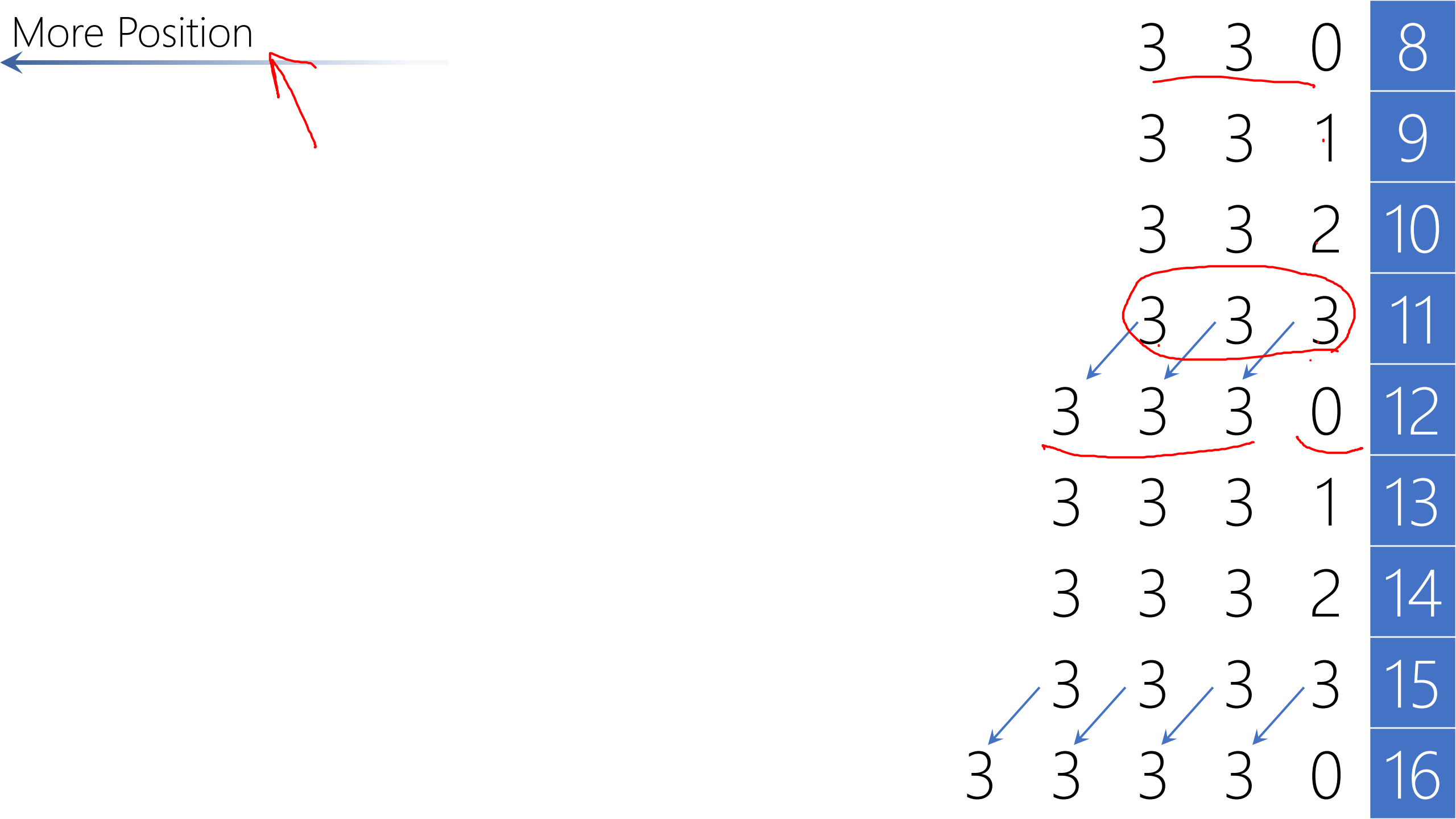
Moon's Distance to Sun:  $3+2$

More Position  
←

<u>0</u>	0
<u>1</u>	1
<u>2</u>	2
<u>3</u>	3
3 <u>0</u>	4
<u>3 1</u>	5
3 2	6
3 <u>3</u>	7
<u>3 3 0</u>	8







More Position



$$0 \quad (1-1)=0$$

$$1 \quad 1+(1-1)=1$$

$$2 \quad 2+(1-1)=2$$

$$3 \quad 3+(1-1)=3$$

$$3 \quad 0 \quad 3+0+(2-1)=4$$

$$3 \quad 1 \quad 3+1+(2-1)=5$$

$$3 \quad 2 \quad 3+2+(2-1)=6$$

$$3 \quad 3 \quad 3+3+(2-1)=7$$

$$3 \quad 3 \quad 0 \quad 3+3+(3-1)=8$$



More Position

3 3 0

$$3+3+0+(3-1)=8$$

3 3 1

$$3+3+1+(3-1)=9$$

3 3 2

$$3+3+2+(3-1)=10$$

3 3 3

$$3+3+3+(3-1)=11$$

3 3 3 0

$$3+3+3+0+(4-1)=12$$

3 3 3 1

$$3+3+3+1+(4-1)=13$$

3 3 3 2

$$3+3+3+2+(4-1)=14$$

3 3 3 3

$$3+3+3+3+(4-1)=15$$

3 3 3 3 0

$$3+3+3+3+(5-1)=16$$

[illegible]

A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. The galaxies are of different shapes and sizes, some appearing as bright, fuzzy blobs and others as more distinct, elongated structures. A thin blue horizontal line is positioned above the word 'POLL'.

# POLL

[Click to Open the Poll](#)



[illegible]

[illegible]





0	0	...			0	0	0
---	---	-----	--	--	---	---	---

$$0 + (n-1) =$$

Min

n positions

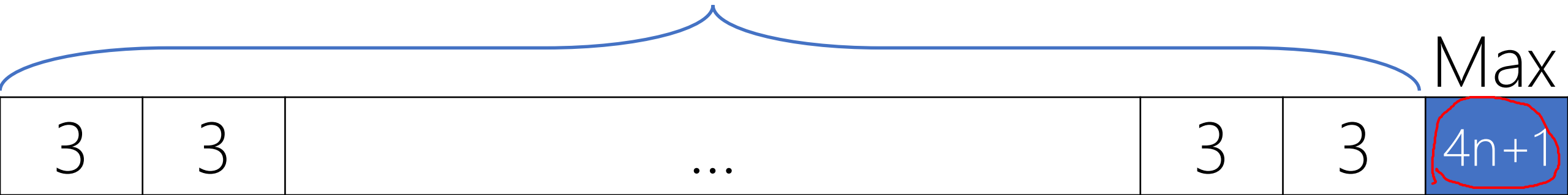
3	3	...			3	3	?
---	---	-----	--	--	---	---	---

$$3 + 3 + 3 + \dots + 3 + (n-1)$$

Max

$$3+3+3+\dots+3+(n-1) = \underline{3} \times \underline{n} + \underline{(n-1)} = \underline{4n-1}$$

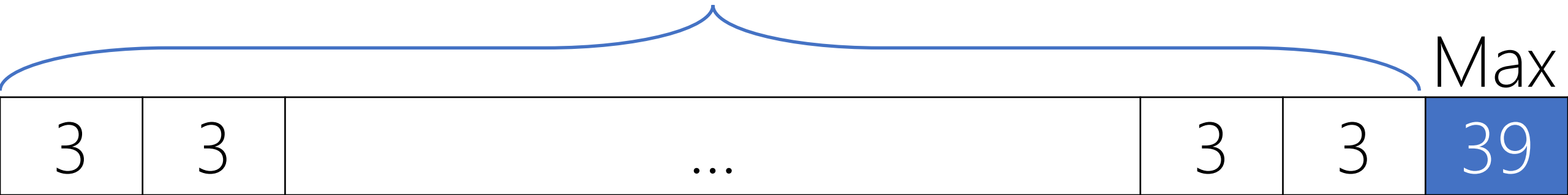
n positions



$$\text{Max} = 4n - 1$$

$$n = \underline{10} \Rightarrow (4 \times 10) - 1 = 39$$

10 positions



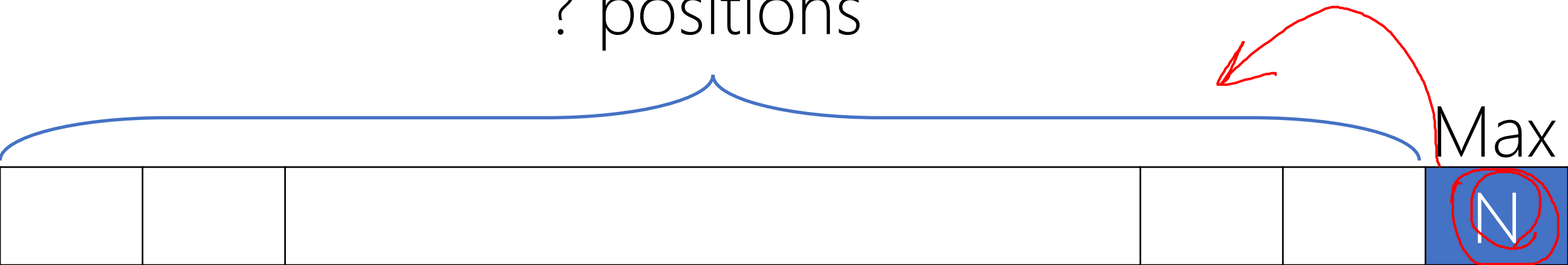


$$\text{Max} = N = 4n - 1$$

$$N + 1 = 4n$$

$$(N + 1) \div 4 = n$$

? positions



how many positions to represent the moon's distance to the sun in Hossein's system if an Oracle said it is ~150 million km and earth's diameter is ~13,000 km? 12,000 earth

4 pages → ? pages

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$N = 12,000$

$n = (N+1) \div 4 = (12,000+1) \div 4 = \sim 3,000 \text{ positions}$

paper =  $\sim 3,000 \text{ positions}$

$3,000 \div 3,000 = 1 \text{ pages}$

0, 2, 3





More Position



1 rounds of all (4) symbols:  $1 \times 4$

2 rounds of all (4) symbols:  $2 \times 4$

0	0
1	1
2	2
3	3
<u>3</u>	<u>0</u>
3	1
3	2
3	3
<u>3</u>	<u>0</u>
3	0

2 rounds of all (4) symbols:  $2 \times 4$

2	3	3	0	8
	3	3	1	9
	3	3	2	10
	3	3	3	11
	<del>3</del>	<del>3</del>	<del>3</del>	12
	3	3	1	13
	3	3	2	14
	3	3	3	15
	<del>3</del>	<del>3</del>	<del>3</del>	16

3 rounds of all (4) symbols:  $3 \times 4$

4 rounds of all (4) symbols:  $4 \times 4$

More Significant Position

←

1 round of all (4) symbols =  $1 \times 4^1$

2 rounds of all (4) symbols =  $2 \times 4^1$

0	0
1	1
2	2
3	3
4	0
5	1
6	2
7	3
8	0

Diagram illustrating the significance of positions in a 4-point DFT. The positions are numbered 0 to 8. The diagram shows the progression of significance over time (rounds).

At  $t=0$ , position 1 is highlighted (yellow circle) and labeled "1".

At  $t=1$ , position 4 is highlighted (yellow circle) and labeled "1". A red dashed arrow points from position 1 at  $t=0$  to position 4 at  $t=1$ , labeled  $4^1$  More Significant.

At  $t=2$ , position 6 is highlighted (yellow circle) and labeled "2". A red dashed arrow points from position 4 at  $t=1$  to position 6 at  $t=2$ , labeled  $4^1$  More Significant.

At  $t=3$ , position 8 is highlighted (yellow circle) and labeled "2". A red dashed arrow points from position 6 at  $t=2$  to position 8 at  $t=3$ , labeled  $4^1$  More Significant.

Blue arrows indicate the shift of the highlighted position from 1 to 4, 4 to 6, and 6 to 8, showing a consistent shift of 2 positions (or  $4^0$  more significant) per round.

1 round of all (4) symbols =  $1 \times 4^1$

2 rounds of all (4) symbols =  $2 \times 4^1$

The diagram shows a 4x8 grid with rows 0-3 and columns 0-7. A path is highlighted with red circles and arrows, starting at (0,1), moving down to (1,1), then down to (2,1), and finally down to (3,2). Red annotations include the text "4<sup>1</sup> More Significant" at (0,1) and (3,2), and "4<sup>0</sup> More Significant" at (1,1). A red box encloses the cells from (1,0) to (3,7). A red line is drawn under the entire row 3.



2 rounds of all (4) symbols =  $2 \times 4^1$

2	0	8
---	---	---

2	1	9
---	---	---

2	2	10
---	---	----

$4^1$  More Significant

2	3	11
---	---	----

3 rounds of all (4) symbols =  $3 \times 4^1$

3	0	12
---	---	----

3	1	13
---	---	----

3	2	14
---	---	----

3	3	15
---	---	----

?	?	16
---	---	----

1 rounds of all (4) symbols in the 2nd significant position  
 $= 4 \times 4^1 = 1 \times 4^2$

2	0	8
2	1	9
2	2	10
2	3	11
3	0	12
3	1	13
3	2	14
3	3	15
0	0	16

4x4  
 16  
 1

2	0	8
2	1	9
2	2	10
2	3	11
3	0	12
3	1	13
3	2	14
3	3	15
1	0	16

$4^2$  More Significant

1 rounds of all (4) symbols in the more significant position  
 $= 4 \times 4^1 = 1 \times 4^2$



# QUATERNARY SYSTEM

/kwaa·tur·neh·ree/

aka. Base-4, Radix-4

(0,1,2,3)<sub>4</sub>

Hindu-Arabic Numerals

Originated in India

7th Century AD



More Significant Position  
←

#Symbols=4

Radix-4

Base-4

<u>4<sup>0</sup></u>	<u>× 1</u>
<u>0</u>	0 × <u>4<sup>0</sup></u> = 0
<u>1</u>	1 × 4 <sup>0</sup> = 1
<u>2</u>	2 × 4 <sup>0</sup> = 2
<u>3</u>	3 × 4 <sup>0</sup> = 3

More Significant Position  
←

#Symbols=4  
Radix-4  
Base-4

<u>4<sup>1</sup></u>	4 <sup>0</sup>	
	0	$0 \times 4^0 = 0$
	1	$1 \times 4^0 = 1$
	2	$2 \times 4^0 = 2$
	3	$3 \times 4^0 = 3$
<u>1</u>	<u>0</u>	$1 \times 4^1 + 0 \times 4^0 = 4$
1	1	$1 \times 4^1 + 1 \times 4^0 = 5$
1	2	$1 \times 4^1 + 2 \times 4^0 = 6$
1	3	$1 \times 4^1 + 3 \times 4^0 = 7$
<u>2</u>	<u>0</u>	$2 \times 4^1 + 0 \times 4^0 = 8$



More Significant Position  
←

#Symbols=4  
Radix-4  
Base-4

<u>4<sup>2</sup></u>	4 <sup>1</sup>	4 <sup>0</sup>	
	2	0	$2 \times 4^1 + 0 \times 4^0 = 8$
	<u>2</u>	1	$2 \times 4^1 + 1 \times 4^0 = 9$
	<u>2</u>	2	$2 \times 4^1 + 2 \times 4^0 = 10$
	2	3	$2 \times 4^1 + 3 \times 4^0 = 11$
	3	0	$3 \times 4^1 + 0 \times 4^0 = 12$
	3	1	$3 \times 4^1 + 1 \times 4^0 = 13$
	3	2	$3 \times 4^1 + 2 \times 4^0 = 14$
	<u>3</u>	<u>3</u>	$3 \times 4^1 + 3 \times 4^0 = 15$
<u>1</u>	<u>0</u>	<u>0</u>	$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$

More Significant Position  
←

#Symbols=4  
Radix-4  
Base-4

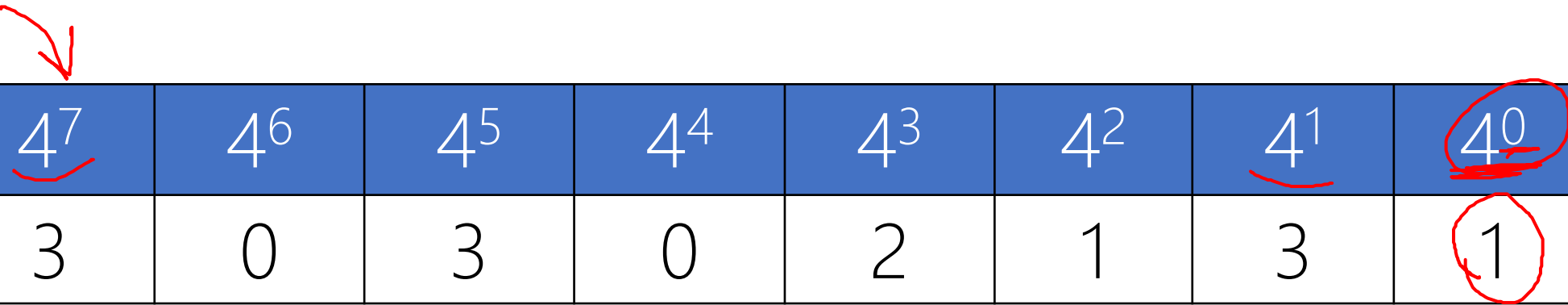
$4^3$	$4^2$	$4^1$	$4^0$	
		2	0	$2 \times 4^1 + 0 \times 4^0 = 8$
		2	1	$2 \times 4^1 + 1 \times 4^0 = 9$
		2	2	$2 \times 4^1 + 2 \times 4^0 = 10$
		2	3	$2 \times 4^1 + 3 \times 4^0 = 11$
		3	0	$3 \times 4^1 + 0 \times 4^0 = 12$
		3	1	$3 \times 4^1 + 1 \times 4^0 = 13$
		3	2	$3 \times 4^1 + 2 \times 4^0 = 14$
		3	3	$3 \times 4^1 + 3 \times 4^0 = 15$
	1	0	0	$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$

[illegible]

[illegible]

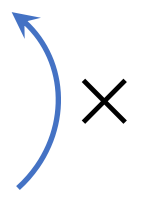


More Significant Position  
←



$4^7$	$4^6$	$4^5$	$4^4$	$4^3$	$4^2$	$4^1$	$4^0$	
3	0	3	0	2	1	3	1	

More Significant Position  
←

$4^7$	$4^6$	$4^5$	$4^4$	$4^3$	$4^2$	$4^1$	$4^0$	
3	0	3	0	2	1	3	1	
$3 \times 4^7$	$0 \times 4^6$	$3 \times 4^5$	$0 \times 4^4$	$2 \times 4^3$	$1 \times 4^2$	$3 \times 4^1$	<u><math>1 \times 4^0</math></u>	

More Significant Position  
←

$4^7$	$4^6$	$4^5$	$4^4$	$4^3$	$4^2$	$4^1$	$4^0$	$\times$
3	0	3	0	2	1	3	1	
$3 \times 4^7$	$0 \times 4^6$	$3 \times 4^5$	$0 \times 4^4$	$2 \times 4^3$	$1 \times 4^2$	$3 \times 4^1$	$1 \times 4^0$	$\Sigma$
								65,437

~~Natural  
H/oss~~



More Significant Position  
←

3	0	3	0	2	1	3	1	65,437	
			3	3	3	3	1	?	
			3	3	3	3	2	?	
		3	0	0	3	3	3	0	?
3	3	3	3	3	3	3	3	3	?

Handwritten red annotations:

- Arrows pointing from the right towards the digits 3, 3, 3, 3, 1 in the second row.
- Labels  $4^4$ ,  $4^3$ ,  $4^2$ ,  $4^1$ ,  $4^0$  above the digits 3, 3, 3, 3, 2 in the third row.
- A red 'x' mark above the question mark in the second row.
- A red '+' sign above the first '0' in the fourth row.
- A red line connecting the first '0' in the fourth row to the '0' in the fifth row.





[illegible]

More Significant Position  
←

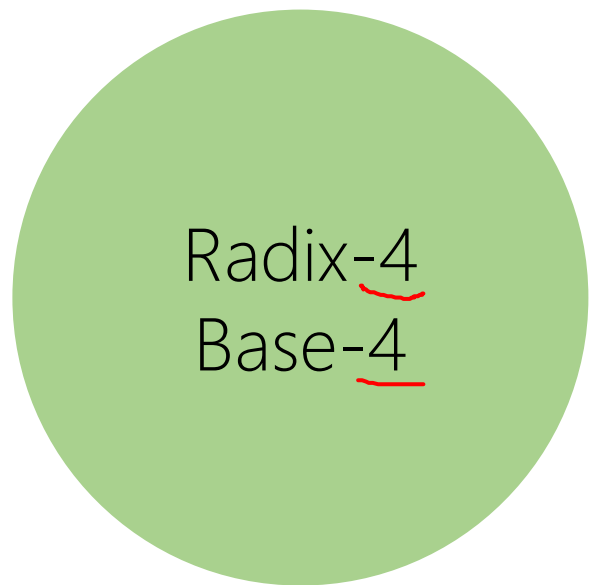
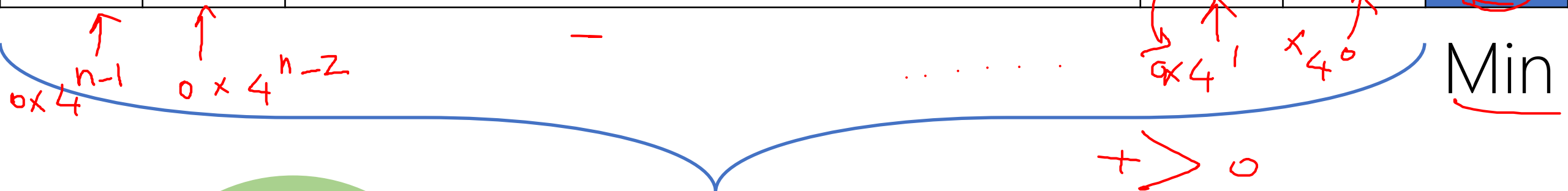
								Base-4	Hossein's Number System		
3	0	3	0	2	1	3	1	65,437 ↗	X -		
			↗ 3	3	3	3	1	<u>1,021</u>	<u>17</u> ↗		
				3	3	3	3	2	4,094	22	
		3	0	0	3	3	3	3	0	50,172 ↗	X -
3	3	3	3	3	3	3	3	3	3	<u>1,048,575</u>	<u>39</u> ↗
<u>(D)</u>											



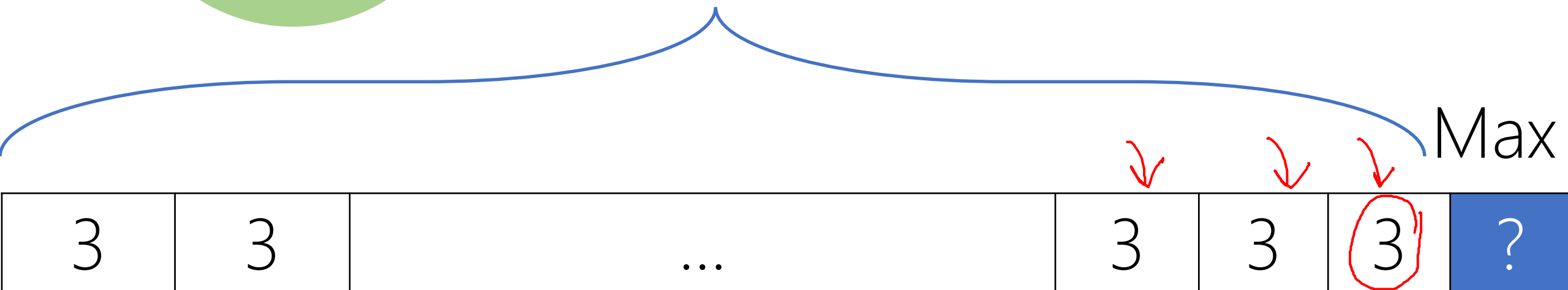
# POLL

[Click to Open the Poll](#)





n positions



Max

$$N = 3 \times 4^{n-1} + 3 \times 4^{n-2} + \dots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$$

$$N = 3 \times (4^{n-1} + 4^{n-2} + \dots + 4^2 + 4^1 + 4^0) \rightarrow \text{geometric series}$$

$$N = 3 \times \left( \frac{4^n - 1}{4 - 1} \right)$$

$$N = 4^n - 1$$

$$\begin{aligned} & 1 + r^1 + r^2 + \dots + r^{n-1} \\ &= \frac{r^n - 1}{r - 1} \end{aligned}$$

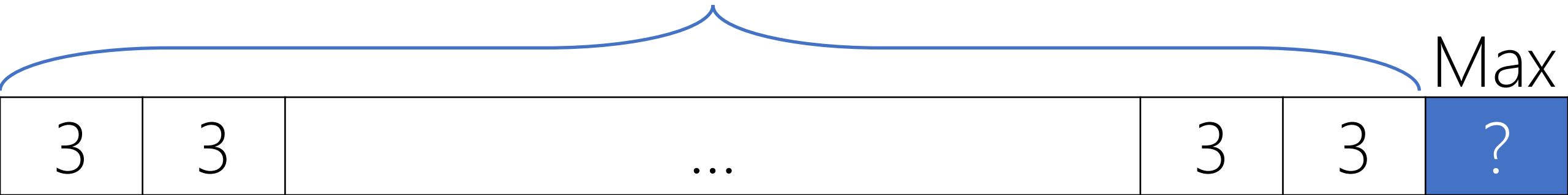
$4^n - 1$   
Hossein's  
System

n positions

						Max
$4^{n-1}$	$4^{n-2}$					
3	3	...				N

$$n = 10 \Rightarrow \underline{4^{10}} - 1 = \underline{1,048,575}$$

10 positions



$$4^n - 1 = N$$

$$4^n = N + 1$$

$$\log_4 4^n = \log(N + 1)$$

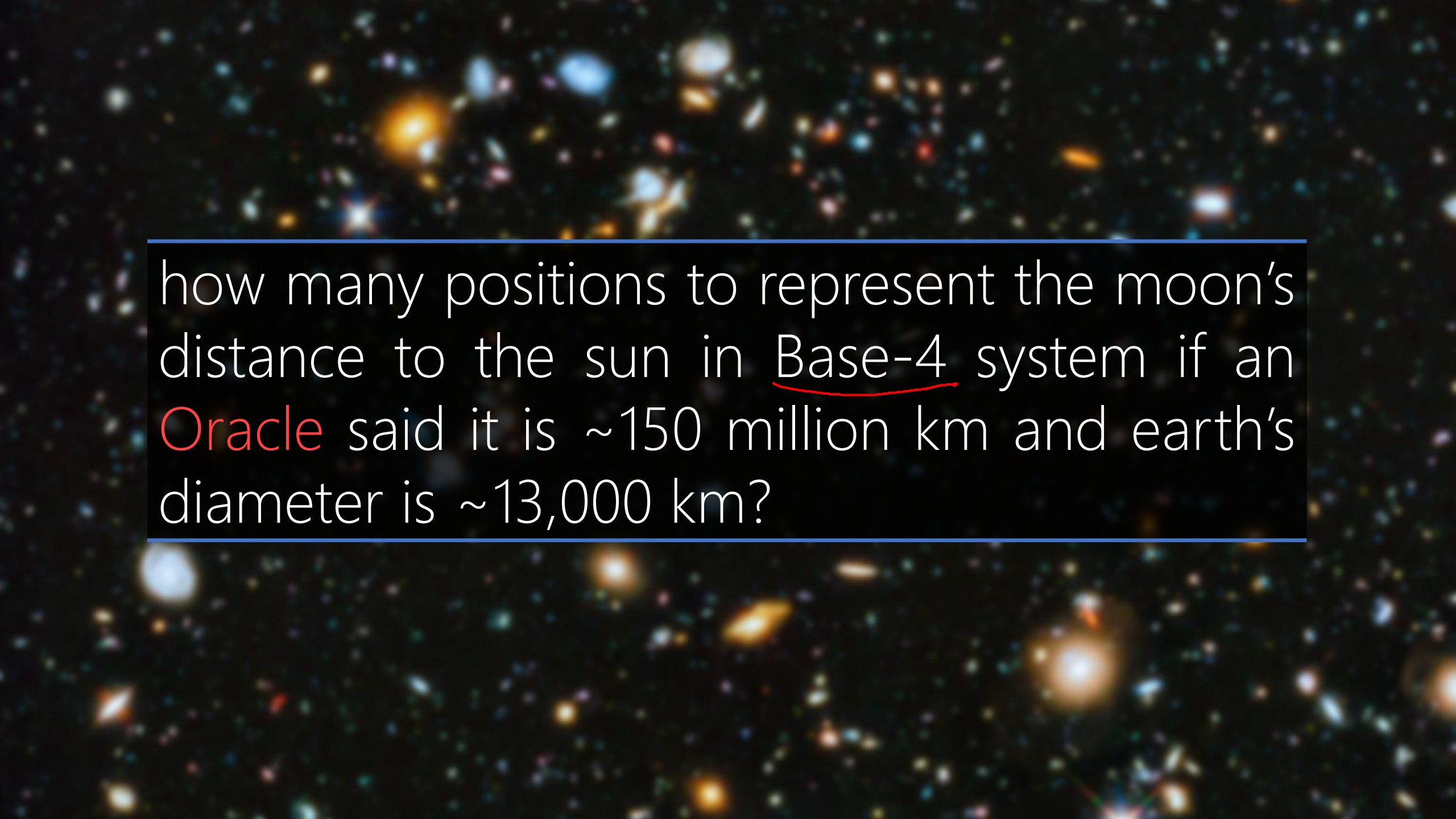
$$n = \log_4(N + 1)$$

? positions

Max

N





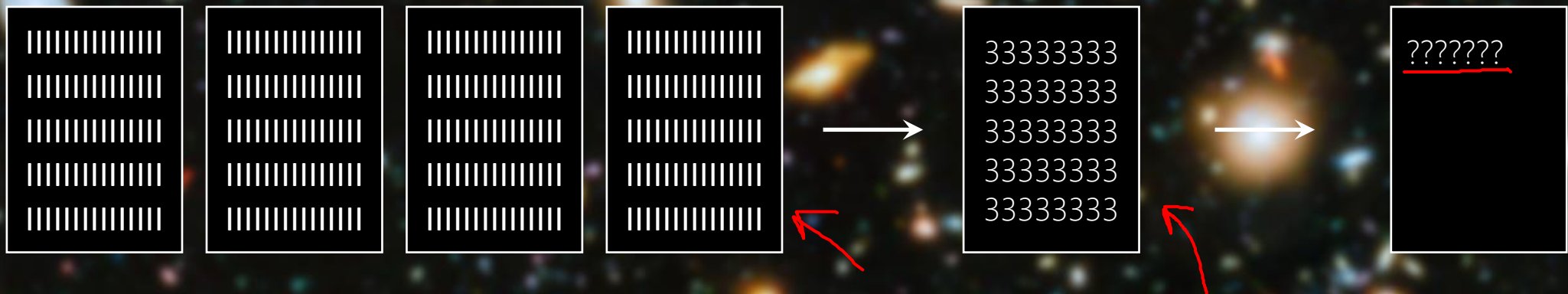
how many positions to represent the moon's distance to the sun in Base-4 system if an **Oracle** said it is ~150 million km and earth's diameter is ~13,000 km?

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$$N = \underline{12,000}$$

$$n = \text{Log}_{\underline{4}} (12,000+1) = \text{Log}_{10} 12,001 \div \text{Log}_{10} 4 = 4 \div 0.6 = \underline{6.79}$$

$\sim \underline{7}$  positions

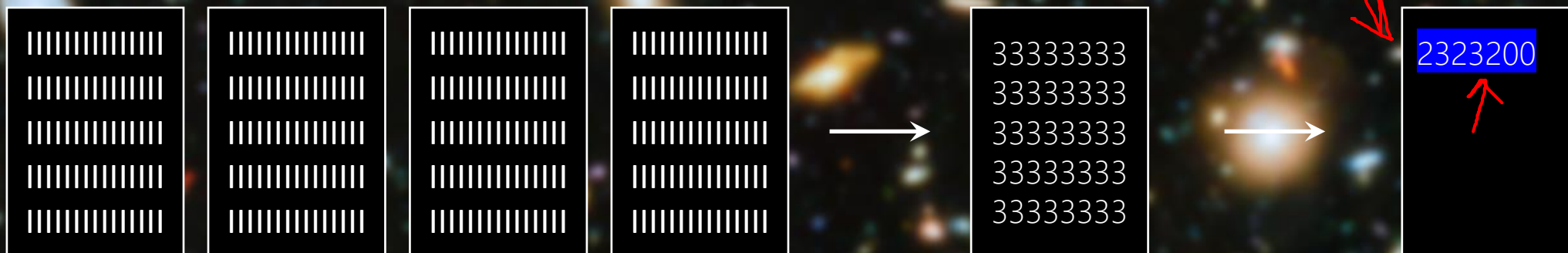




0 1 2 3 4 10 11 12 ...

$$N = 12,000 \rightarrow (2323200)_4$$

We'll see how to convert from decimal to base-4 or any other number systems later. Stay tuned!





QUESTION