DESIGN

a design algorithm for any digital units (logic circuits), given truth table

minterm

aka. Standard Product

X′ vs. X

1 binary variable appear either:

- in its normal form X, or
- in its complement form X′

m_0	X'
m_1	X

YX vs. YX' vs. $Y'X$ vs. $Y'X'$

2 binary variables appear either in one of these forms:

m_0	$Y'X'$
m_1	$Y'X$
m_2	YX'
m_3	YX

ZYX vs. ZYX’ vs. …

3 binary variables appear either in one of these forms: how many?

ZYX vs. ZYX' vs. …

3 binary variables appear either in one of these forms: how many?
Each variable can take 2 forms (normal and complement)
We have 3 variables, $2 \times 2 \times 2 = 2^3 = 8$

m_0	$Z'Y'X'$
m_1	$Z'Y'X$
m_2	$Z'YX'$
m_3	$Z'YX$
m_4	$ZY'X'$
m_5	$ZY'X$
m_6	ZYX'
m_7	ZYX

$$A_{n-1} \cdots A_2 A_1 A_0 \text{ vs. } A_{n-1} \cdots A_2 A_1 A'_0 \ \cdots$$

n binary variables appear either in one of these forms: how many?

Each variable can take 2 forms (normal and complement)

We have n variables, $2 \times 2 \times 2 \times \cdots \times 2 = 2^n$

m_0	$A'_{n-1} \cdots A'_2 A'_1 A'_0$
m_1	$A'_{n-1} \cdots A'_2 A'_1 A_0$
m_2	$A'_{n-1} \cdots A'_2 A_1 A'_0$
$\vdots$	$\vdots$
$\vdots$	$\vdots$
m_{2^n-3}	$A_{n-1} \cdots A_2 A'_1 A_0$
m_{2^n-2}	$A_{n-1} \cdots A_2 A_1 A'_0$
m_{2^n-1}	$A_{n-1} \cdots A_2 A_1 A_0$

TRUTH TABLE

en.wikipedia.org/wiki/Truth_table

X	F = F(X) = ?
0	?
1	?

X	F = F(X) = 0
0	0
1	0

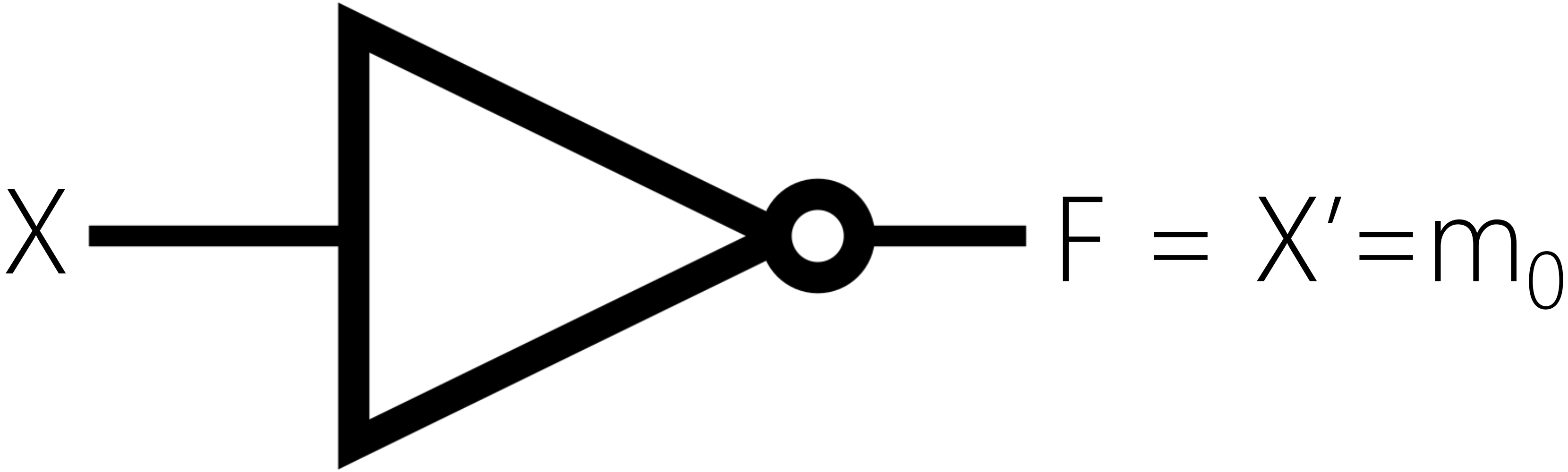
X	F = F(X) =1
0	1
1	1

X	F = F(X) = X′
0	1
1	0

m_0	X'
m_1	X

X	$F = F(X) = X' = m_0$
0	1
1	0

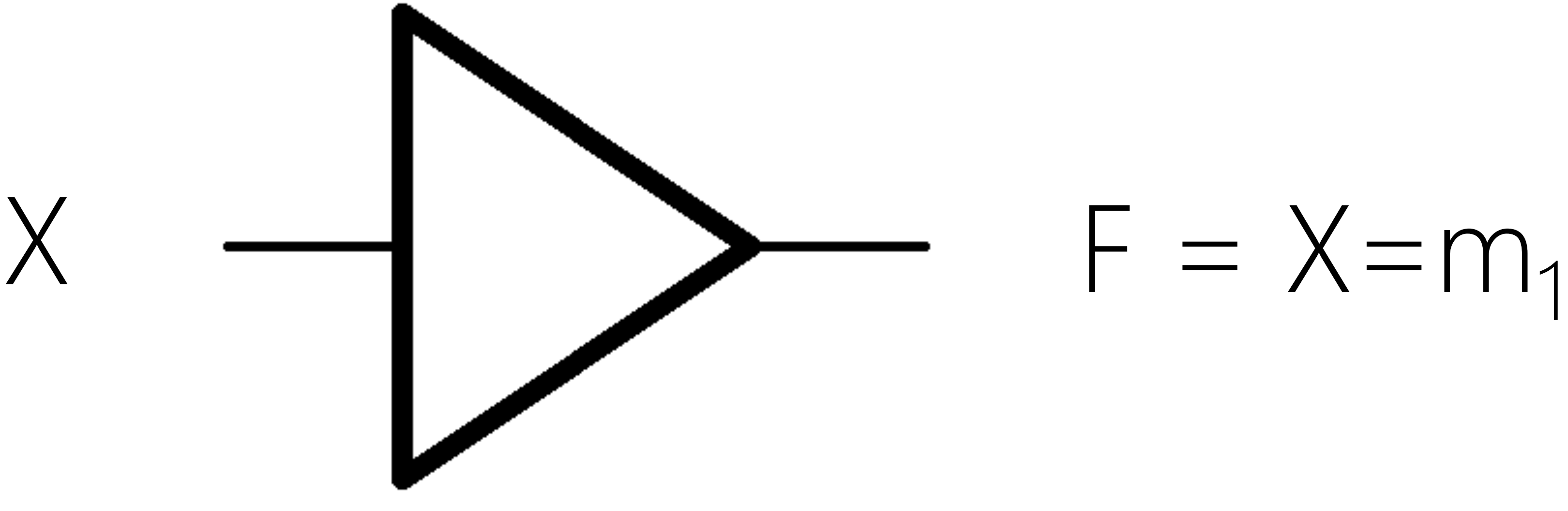
X	$F = F(X) = X' = m_0$
0	1
1	0


X
$F = X' = m_0$

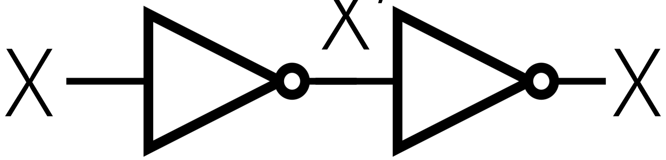
X	F = F(X) = X
0	0
1	1

X	$F = F(X) = X = m_1$
0	0
1	1

X	$F = F(X) = X = m_1$
0	0
1	1



Digital Buffer

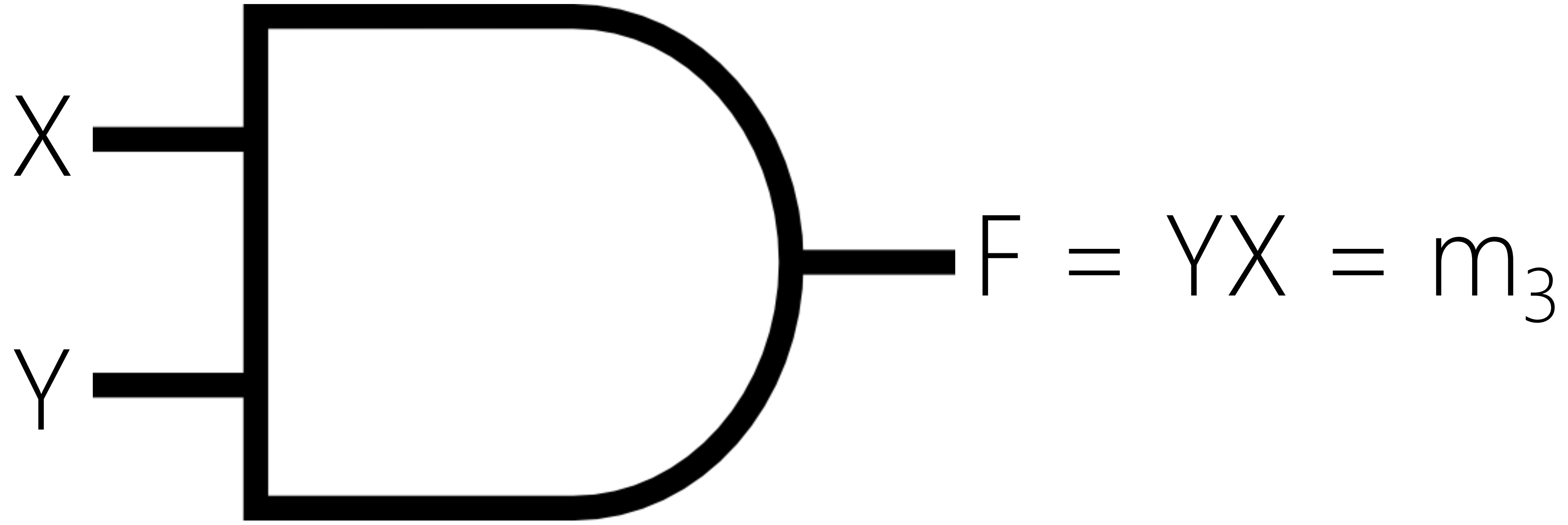


TRUTH TABLE $\leftrightarrow$ minterm

Y	X	F = F(Y,X) = ?
0	0	?
0	1	?
1	0	?
1	1	?

Y	X	$F = F(Y,X) = 0$
0	0	0
0	1	0
1	0	0
1	1	0

Y	X	F = F(Y,X) = YX
0	0	0
0	1	0
1	0	0
1	1	1


X
Y
F = YX = m₃

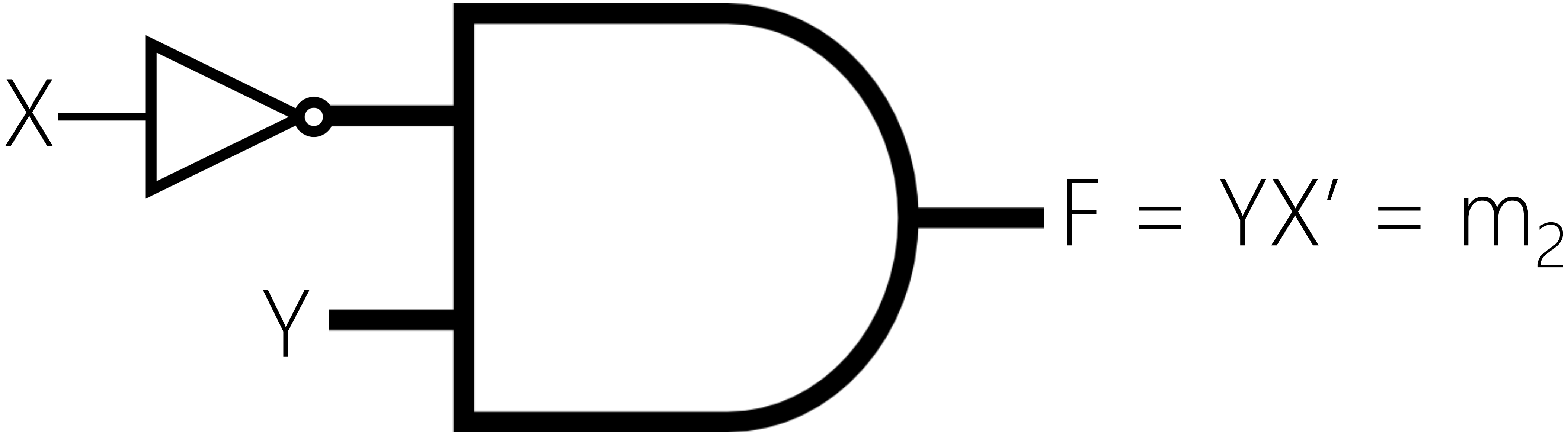
m_0	$Y'X'$
m_1	$Y'X$
m_2	YX'
m_3	YX

Y	X	$F = F(Y,X) = YX = m_3$
0	0	0
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = YX = m_3$
0	0	0
0	1	0
1	0	0
1	1	1

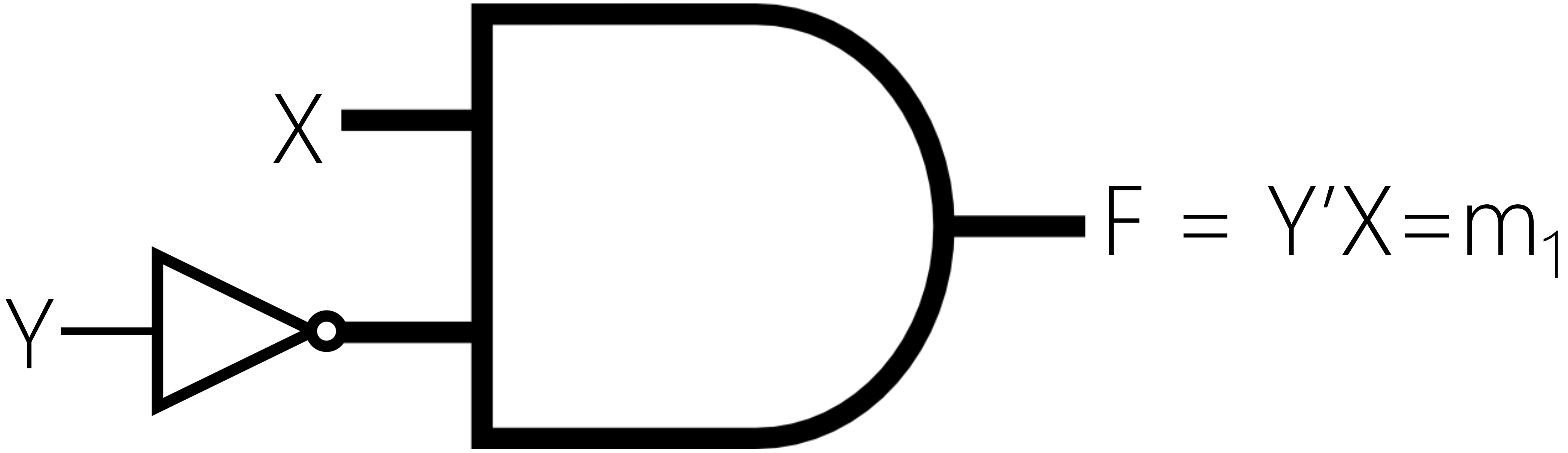
Y	X	$F = F(Y,X) = YX' = m_2$
0	0	0
0	1	0
1	0	1
1	1	0

Y	X	$F = F(Y,X) = YX' = m_2$
0	0	0
0	1	0
1	0	1
1	1	0


X
Y
F = YX′ = m_2

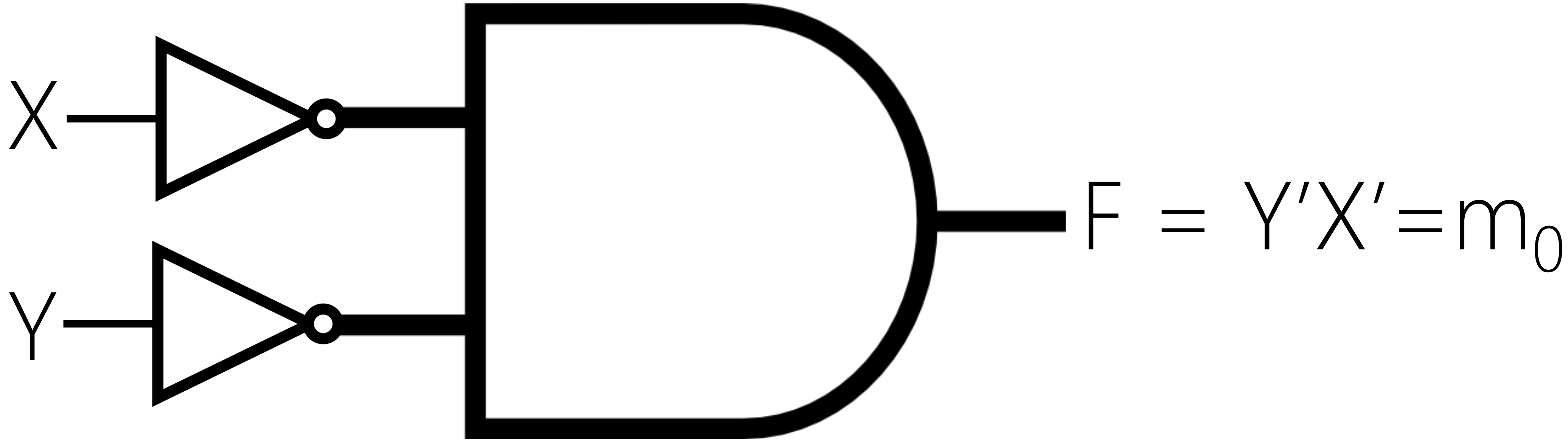
Y	X	$F = F(Y,X) = Y'X = m_1$
0	0	0
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = Y'X = m_1$
0	0	0
0	1	1
1	0	0
1	1	0


X
Y
F = Y'X=m1

Y	X	$F = F(Y,X) = Y'X' = m_0$
0	0	1
0	1	0
1	0	0
1	1	0

Y	X	$F = F(Y,X) = Y'X' = m_0$
0	0	1
0	1	0
1	0	0
1	1	0


X
Y
F = Y'X'=m_0

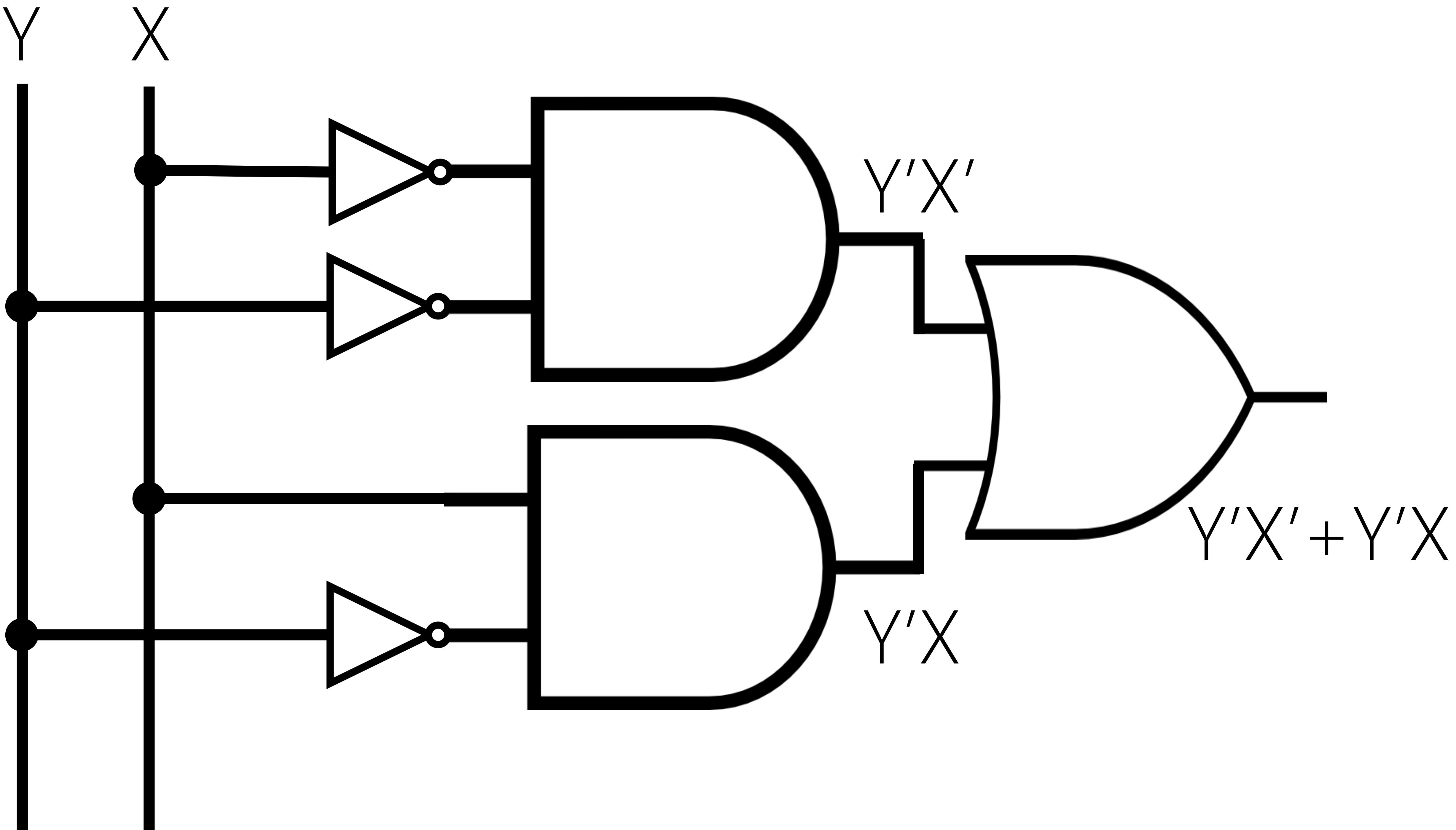
Y	X	F = F(Y,X) = ?
0	0	1
0	1	1
1	0	0
1	1	0

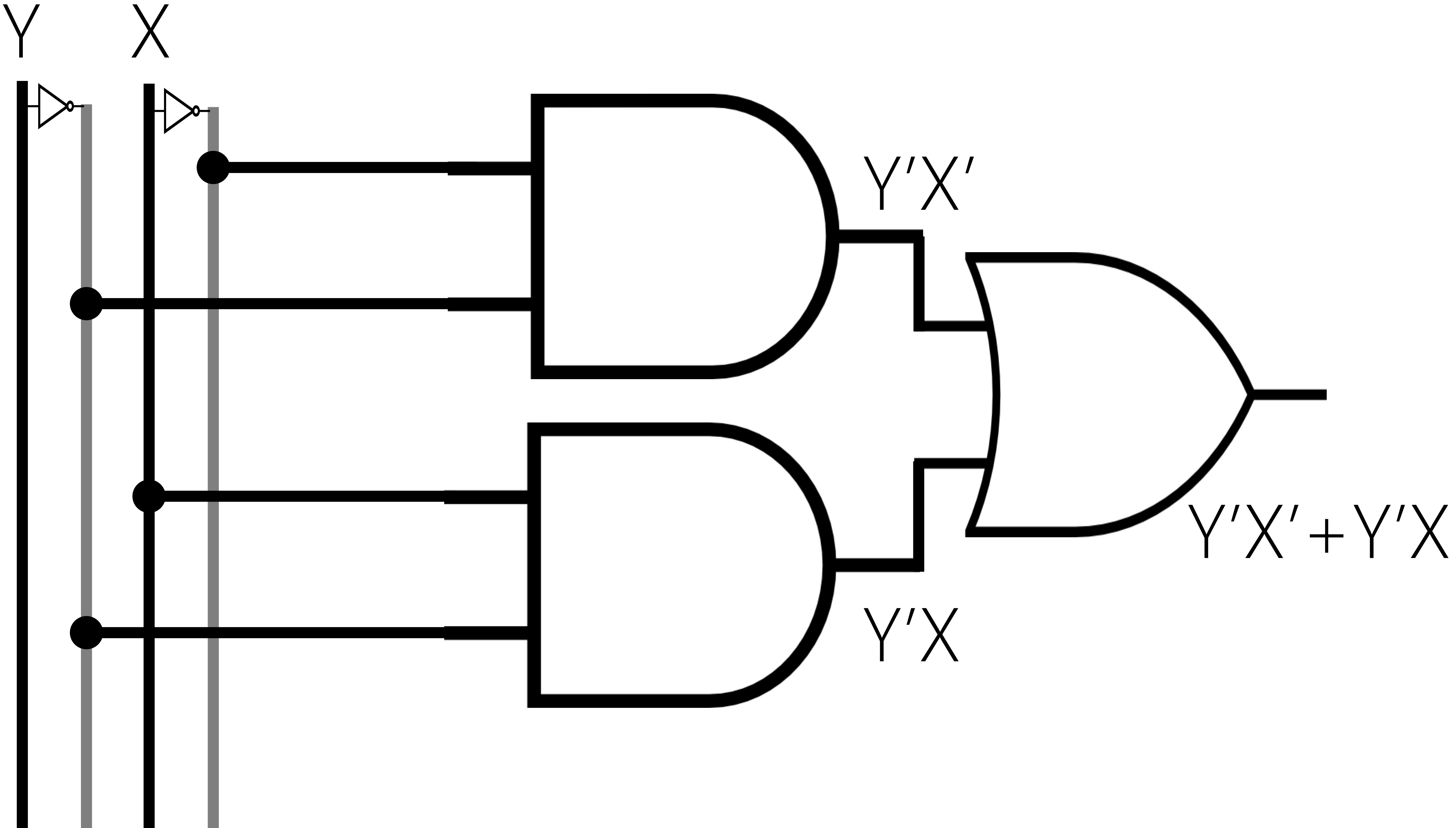
Y	X	F = F(Y,X) = Y'X'
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = Y'X' + Y'X$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = \sum m(0,1)$
0	0	1
0	1	1
1	0	0
1	1	0


Y
X
Y'X'
Y'X
Y'X'+Y'X

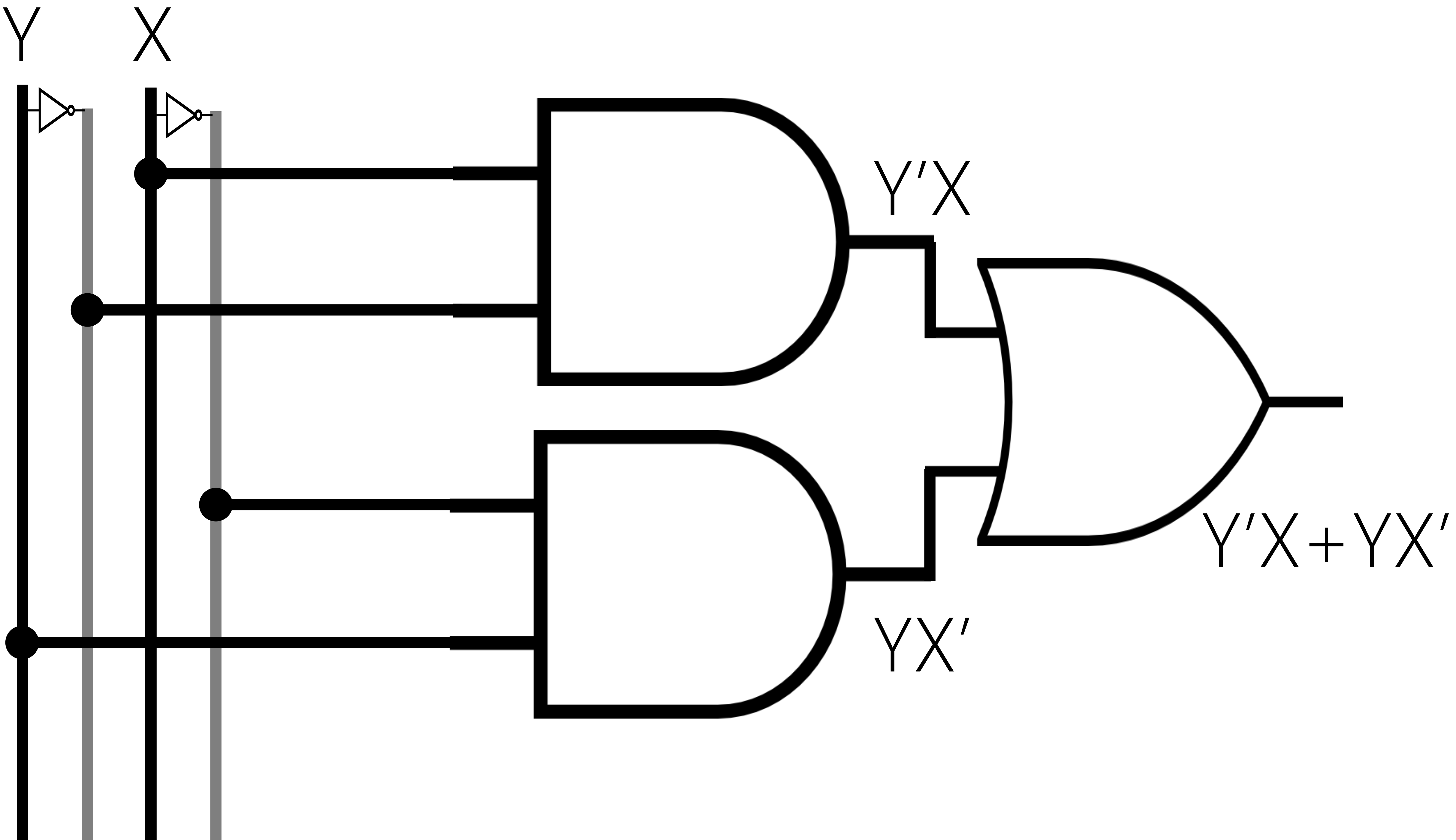

Y
X
Y′X′
Y′X
Y′X′+Y′X

Y	X	F = F(Y,X) = ?
0	0	0
0	1	1
1	0	1
1	1	0

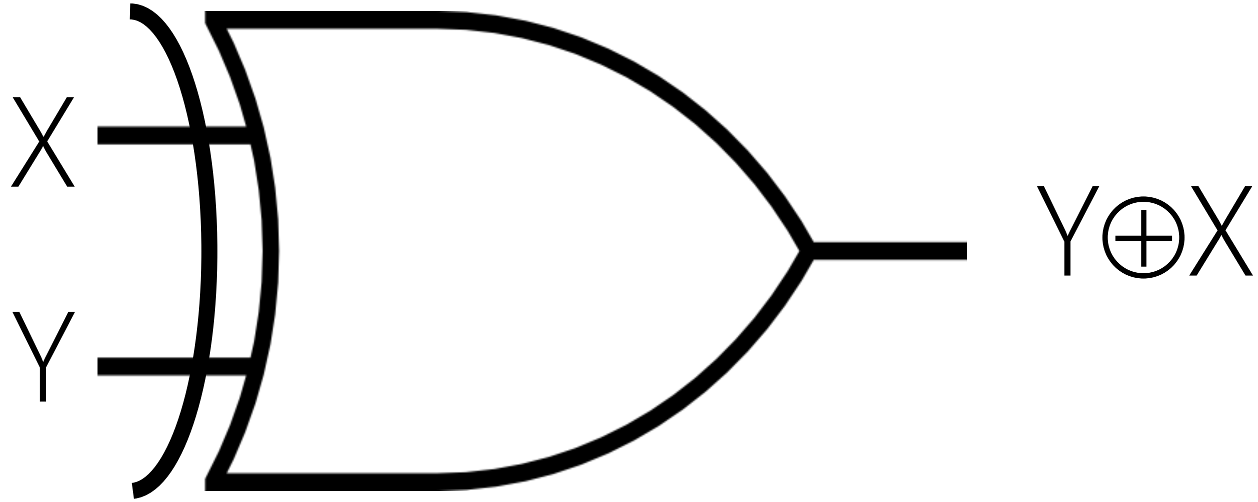
Y	X	F = F(Y,X) = Y’X
0	0	0
0	1	1
1	0	1
1	1	0

Y	X	$F = F(Y,X) = m_1+m_2$
0	0	0
0	1	1
1	0	1
1	1	0

Y	X	$F = F(Y,X) = \sum m(1,2)$
0	0	0
0	1	1
1	0	1
1	1	0


Y
X
Y’X
YX’
Y’X+YX’

Exclusive-OR (XOR)



Y	X	$F = F(Y,X) = Y'X + YX' = m_1 + m_2$
0	0	0
0	1	1
1	0	1
1	1	0

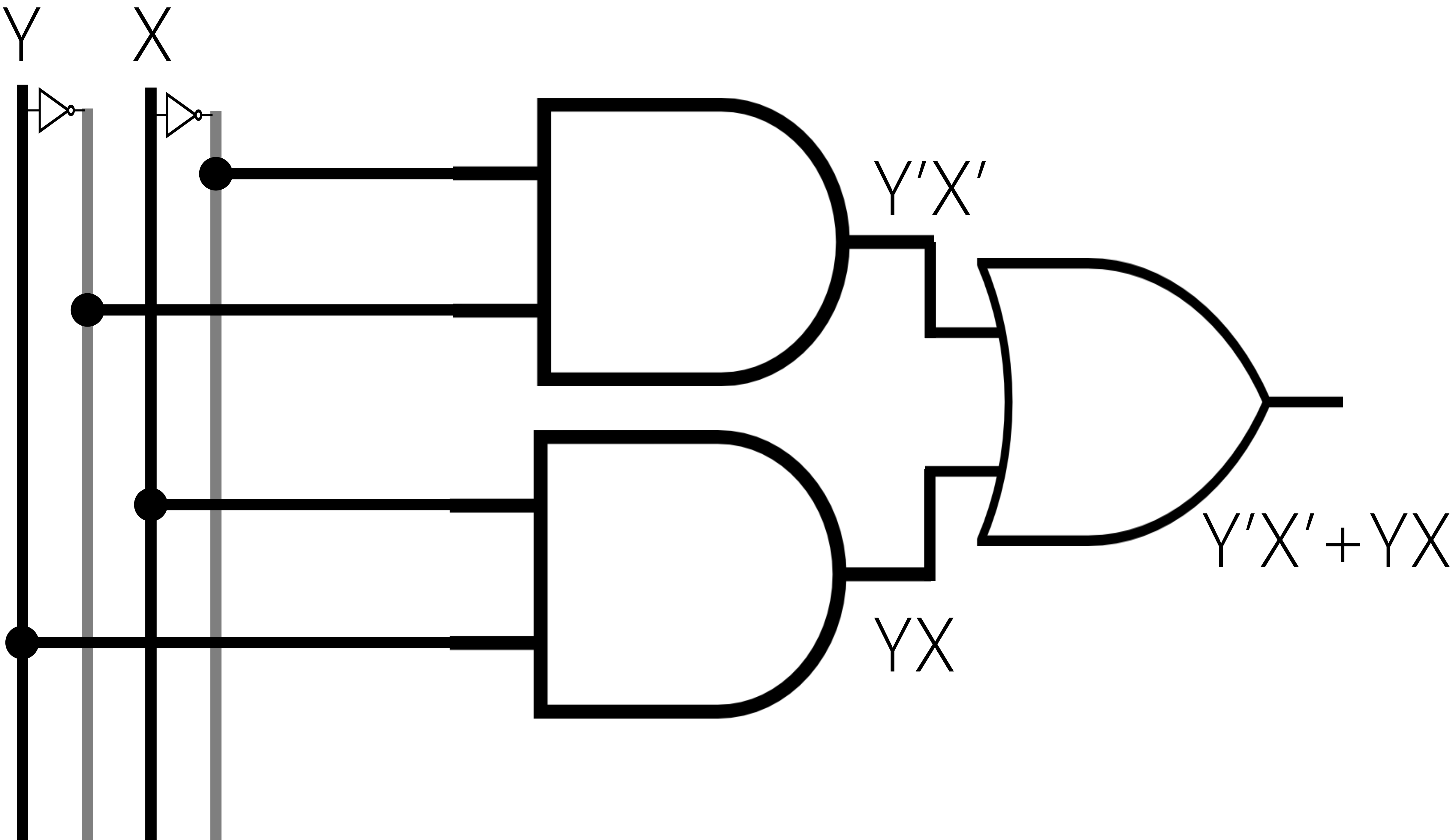
Y	X	F = F(Y,X) = ?
0	0	1
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = Y'X'$
0	0	1
0	1	0
1	0	0
1	1	1

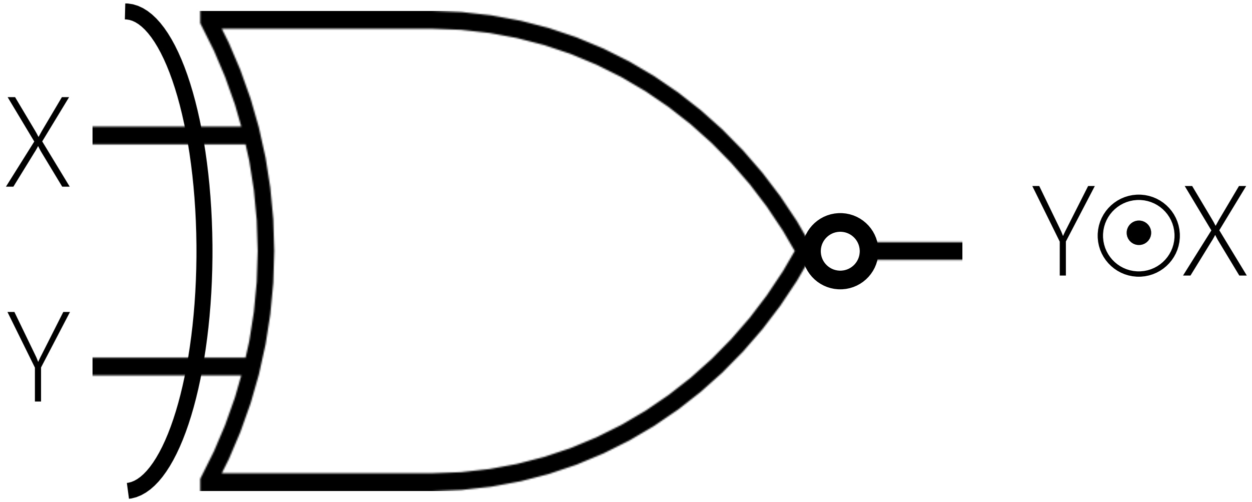
Y	X	F = F(Y,X) = Y’X’+YX
0	0	1
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = m_0 + m_3$
0	0	1
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = \sum m(0,3)$
0	0	1
0	1	0
1	0	0
1	1	1


Y
X
Y′X′
YX
Y′X′+YX

NOT Exclusive-OR (XNOR)



Y	X	F = F(Y,X) = Y'X'+YX = m_0+m_3
0	0	1
0	1	0
1	0	0
1	1	1

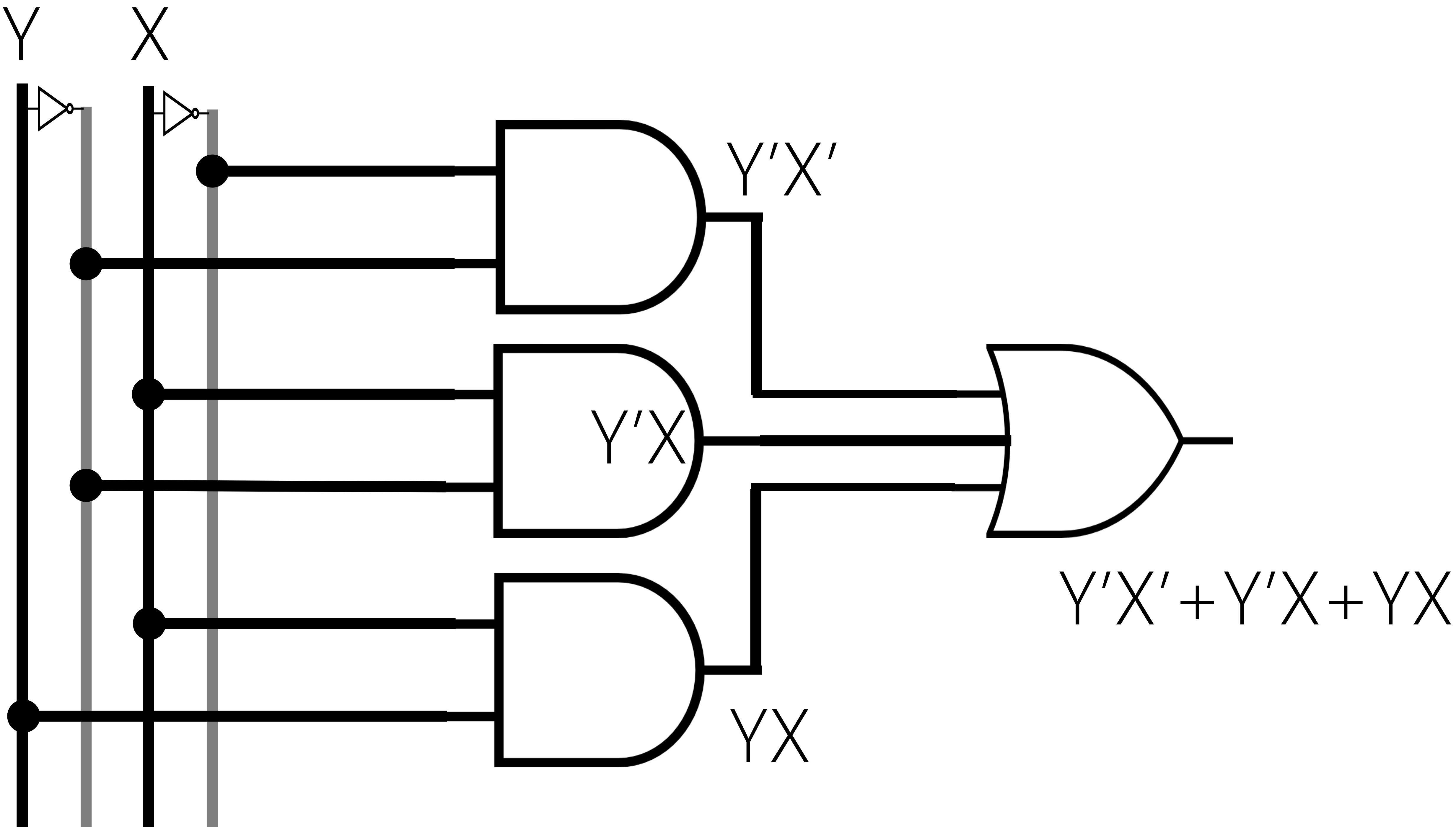
Y	X	F = F(Y,X) = ?
0	0	1
0	1	1
1	0	0
1	1	1

Y	X	F = F(Y,X) = Y'X'
0	0	1
0	1	1
1	0	0
1	1	1

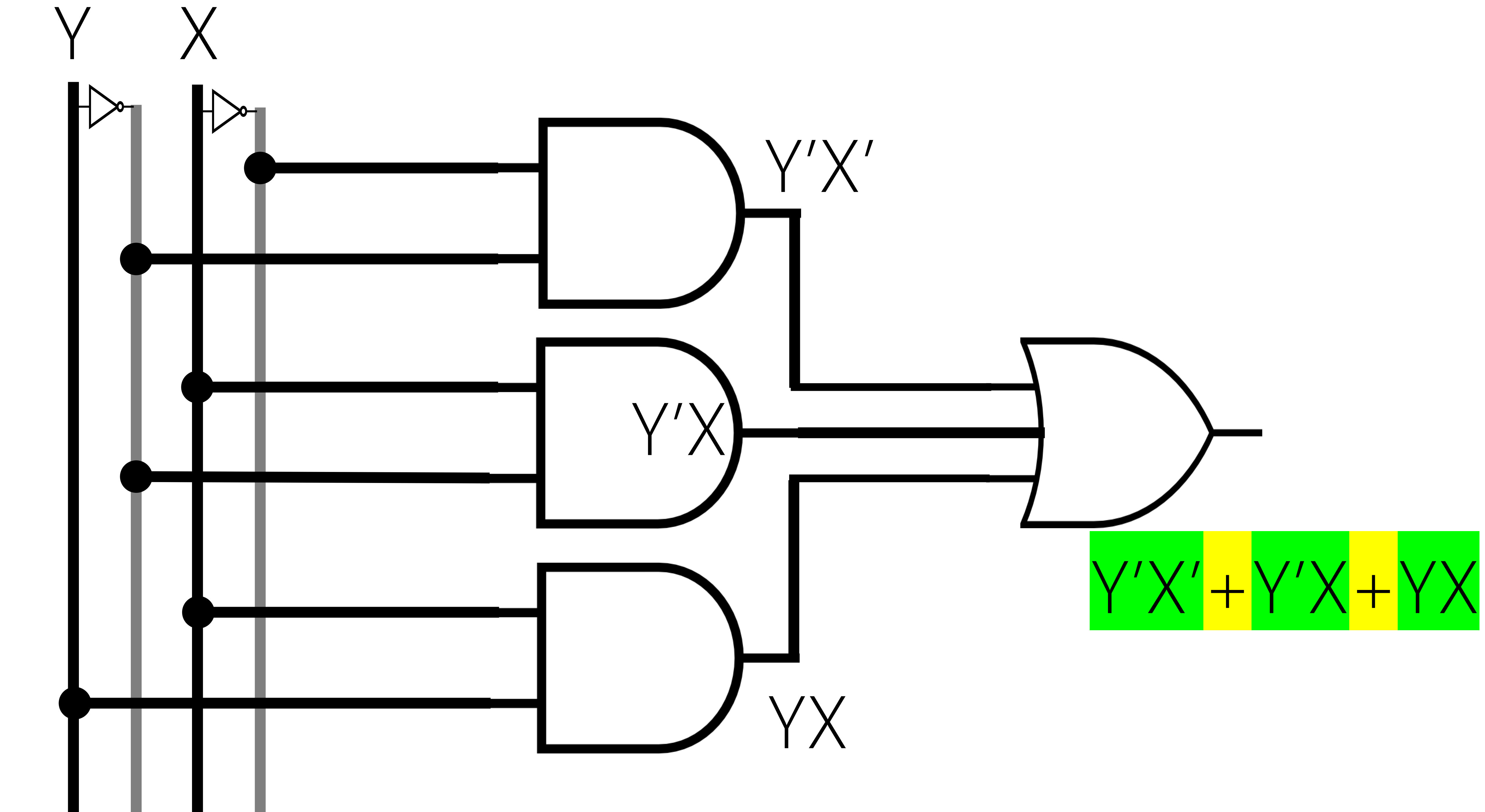
Y	X	F = F(Y,X) = Y′X′+Y′X
0	0	1
0	1	1
1	0	0
1	1	1

Y	X	$F = F(Y,X) = Y'X' + Y'X + YX$
0	0	1
0	1	1
1	0	0
1	1	1

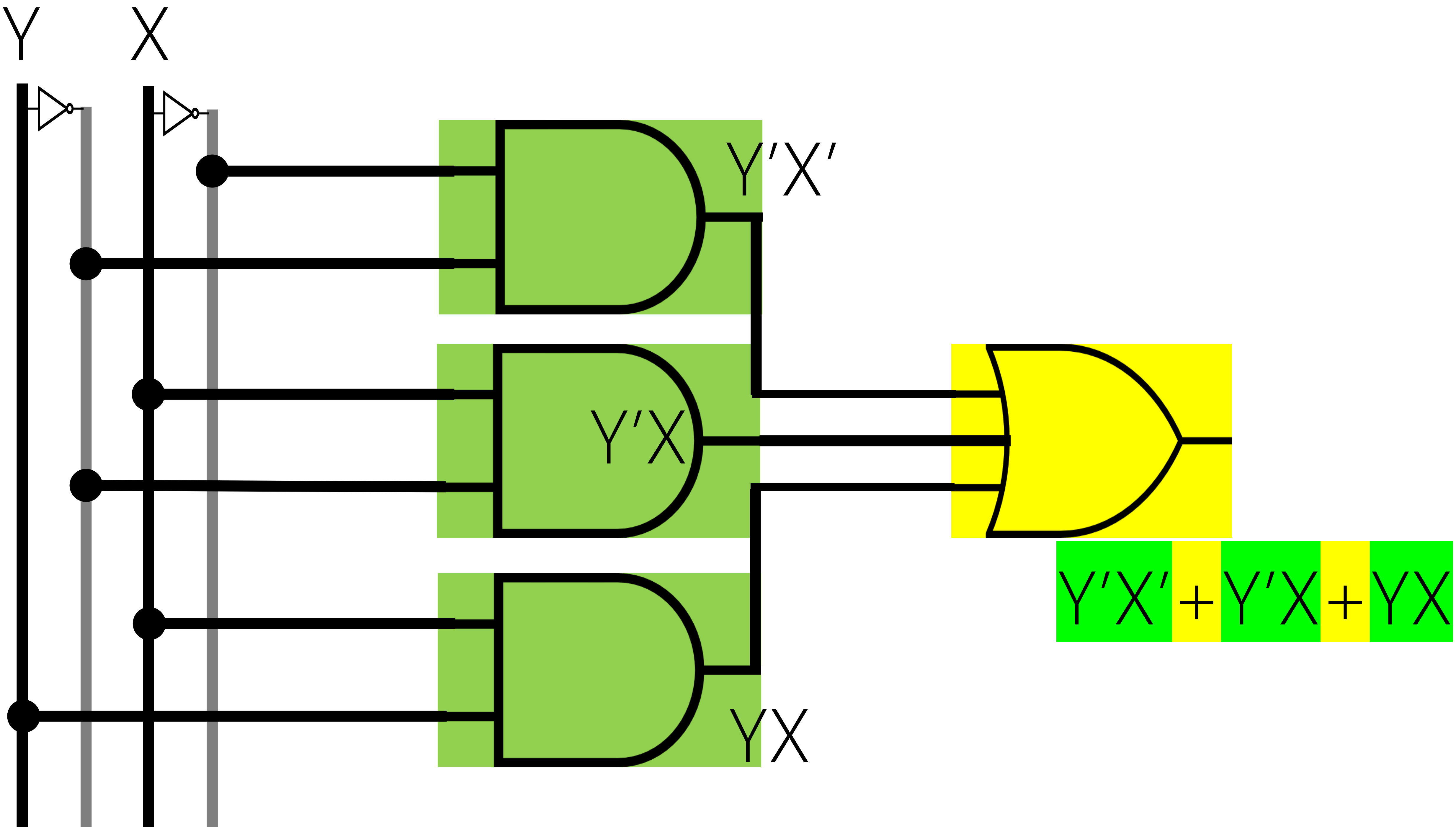
Y	X	$F=F(Y,X)=m_0+m_1+m_3$ $=\sum m(0,1,3)$
0	0	1
0	1	1
1	0	0
1	1	1


Y
X
Y'X'
Y'X
YX
Y'X'+Y'X+YX

SUM OF PRODUCTS (SOP)


Y
X
Y'X'
Y'X
YX
Y'X' + Y'X + YX

2 LEVELS
AND → OR



Y
X
Y'X'
Y'X
YX
Y'X' + Y'X + YX

Given 3 inputs, design a circuit to determine if there is even number of 1

TRUTH TABLE $\leftrightarrow$ minterm

INPUTS/BINARY VARIABLES	OUTPUTS/BOOLEAN FUNCTIONS

Z	Y	X	F(Z,Y,X)=?
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Z	Y	X	F(Z,Y,X)=?
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=Z'Y'X'$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

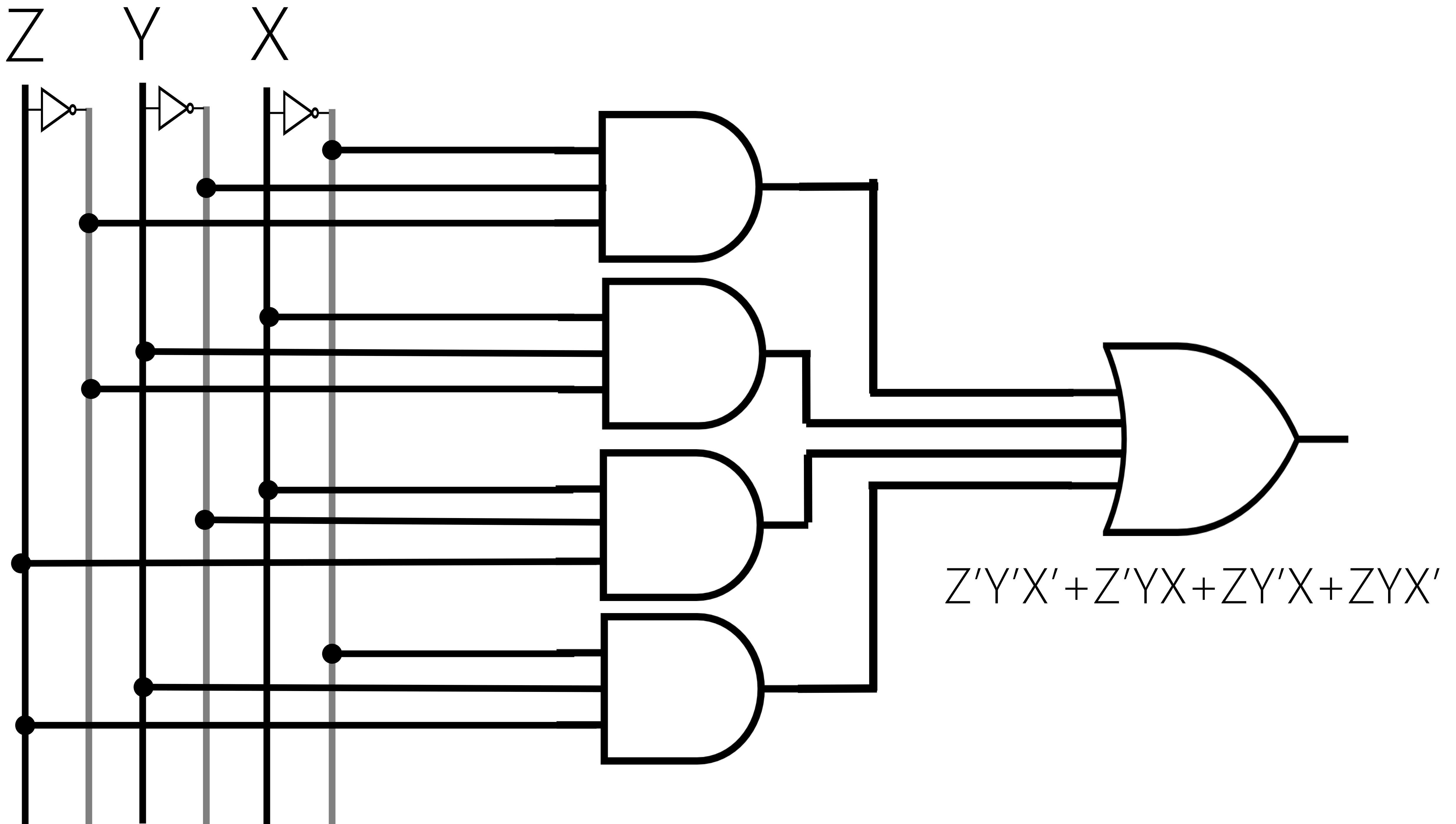
Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX+ZY'X$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

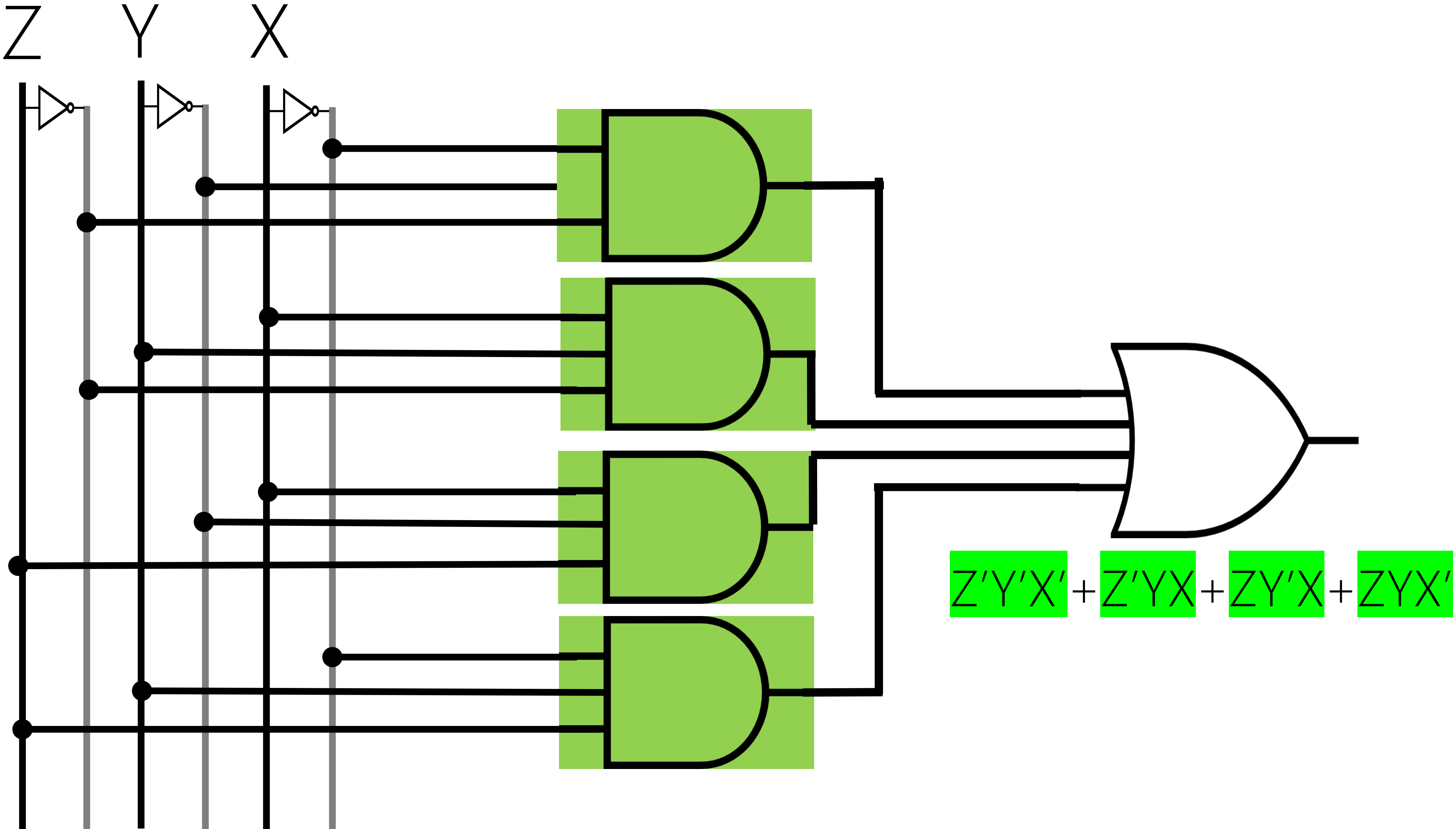
Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX+ZY'X+ZYX'$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

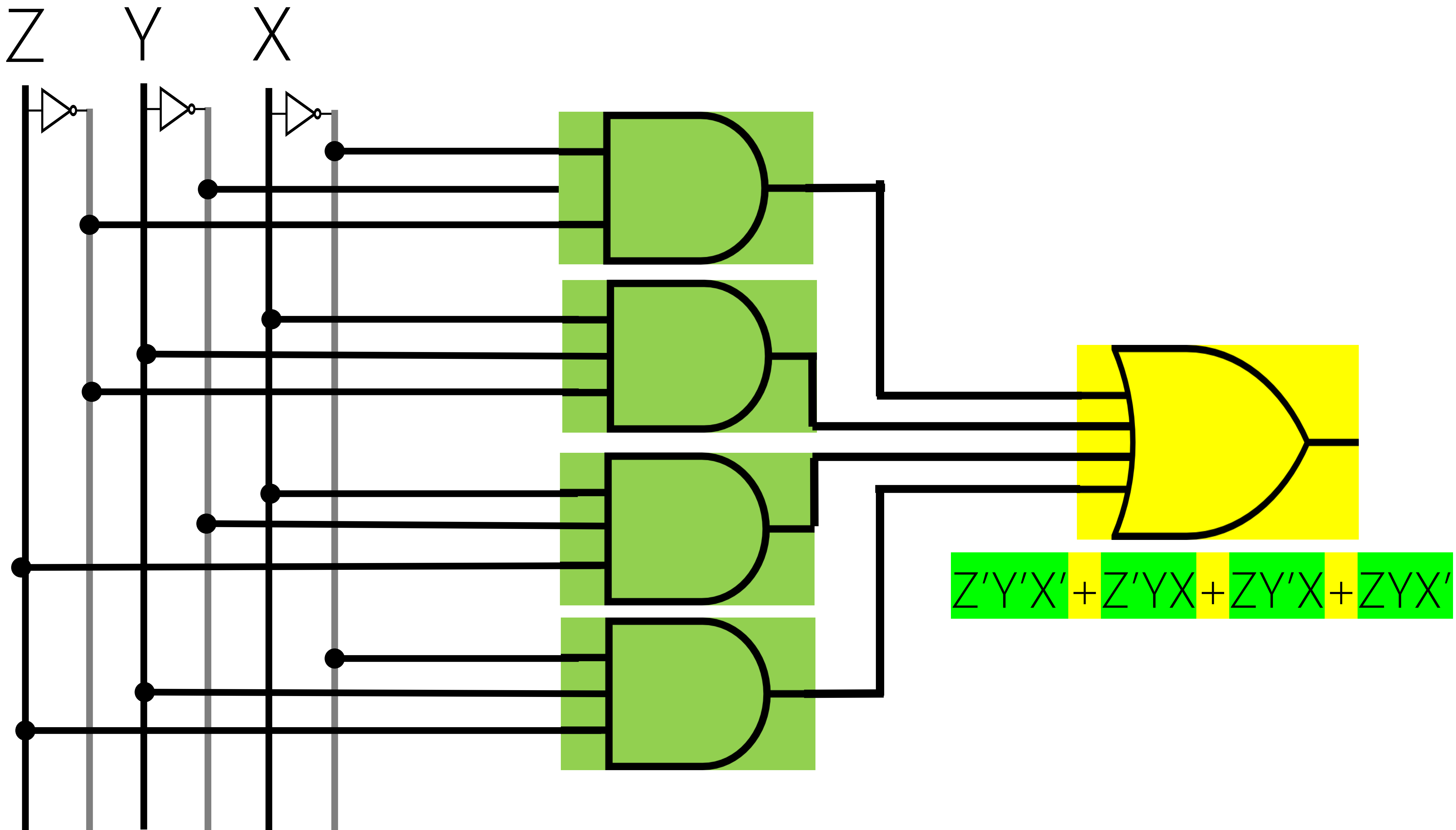
Z	Y	X	$F(Z,Y,X)=m_0+m_3+m_5+m_6$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=m_0+m_3+m_5+m_6=\sum m(0,3,5,6)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0


Z
Y
X
Z′Y′X′+Z′YX+ZY′X+ZYX′

SUM OF PRODUCTS (SOP)
2 LEVELS AND-OR


Z
Y
X
Z'Y'X' + Z'YX + ZY'X + ZYX'


Z
Y
X
Z'Y'X'+Z'YX+ZY'X+ZYX'

SHOW THE REMAINDER (MOD)
NUMBER % 3 = ?

TRUTH TABLE $\leftrightarrow$ minterm

INPUTS/BINARY VARIABLES	OUTPUTS/BOOLEAN FUNCTIONS

WHAT IS THE RANGE OF NUMBERS?

WHAT IS THE RANGE OF NUMBERS?
$[0, 15]_{10}$

HOW MANY INPUT BINARY VARIABLES?

$$[0, 15]_{10} = [0,1111]_2 = [0000,1111]_2$$

W	Z	Y	X
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

WHAT IS THE RANGE OF OUTPUT?

WHAT IS THE RANGE OF OUTPUT?

The remainder of any number divided by 3 is 0, 1, 2

WHAT IS THE RANGE OF OUTPUT?

$[0, 2]_{10}$

HOW MANY BOOLEAN FUNCTION?

$$[0, 2]_{10} = [0, 10]_2 = [00, 10]_2$$

W	Z	Y	X	F_1	F_2
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

minterms

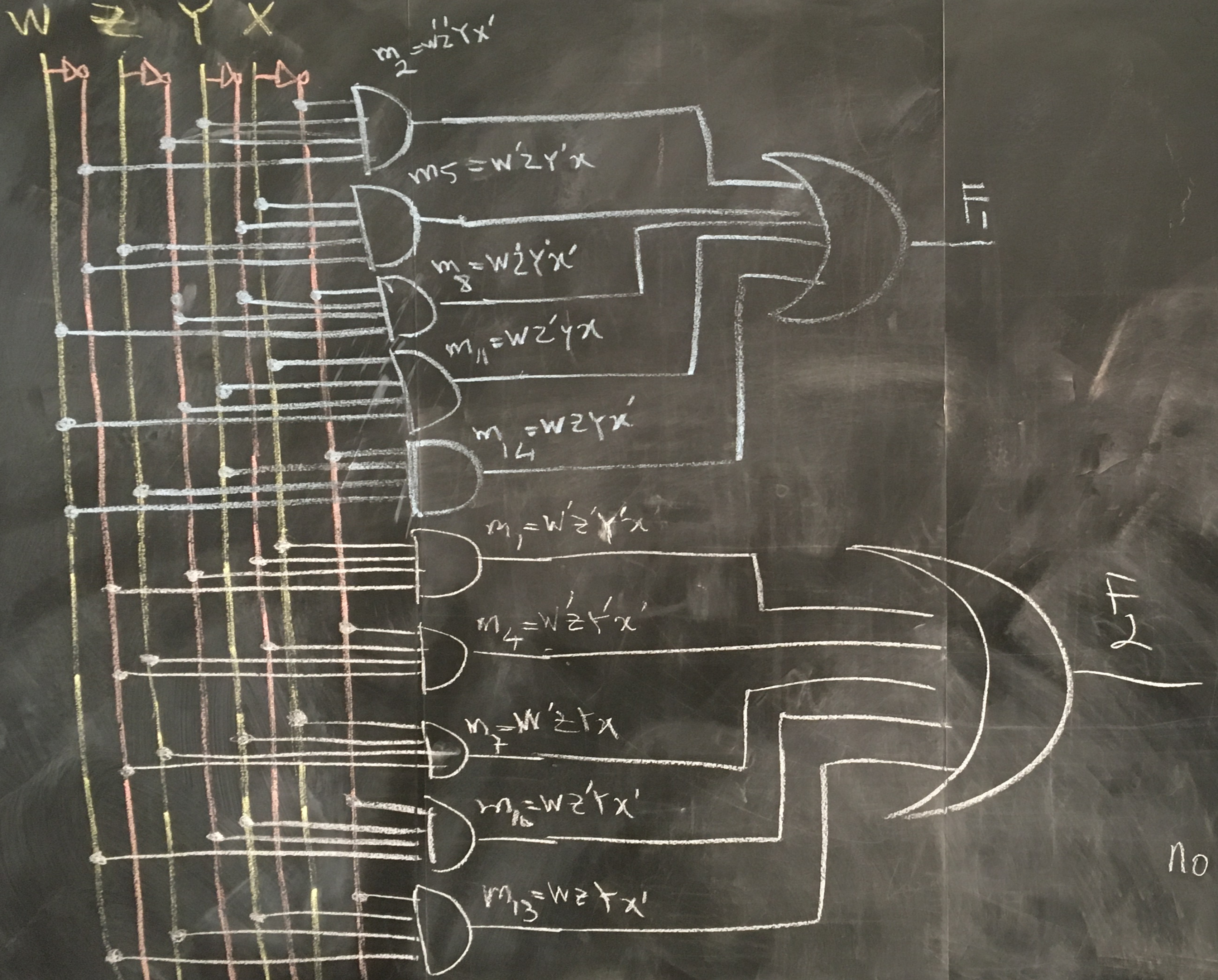
W	Z	Y	X	$F_1=m_2+m_5+m_8+m_{11}+m_{14}$	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1=\sum m(2,5,8,11,14)$	$F_2=m_1+m_4+m_7+m_{10}+m_{13}$
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1=\sum m(2,5,8,11,14)$	$F_2=\sum m(1,4,7,10,13)$
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

$F_1 = \sum m(2,5,8,11,14)$

$F_2 = \sum m(1,4,7,10,13)$



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