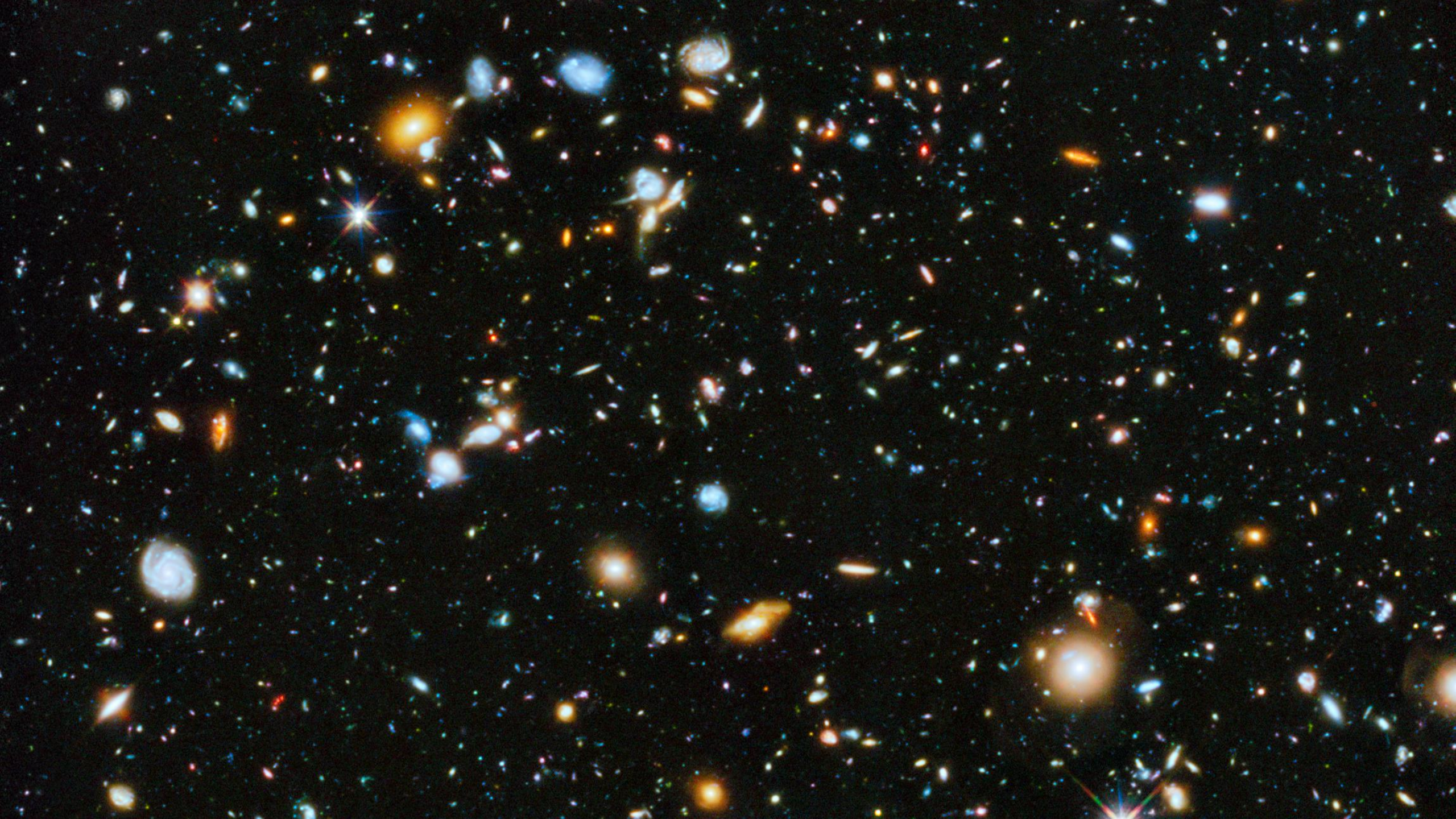


Blackboard will return to service at 6 am EST.

Blackboard is unavailable every weekday (Monday to Friday) from 5 am to 6 am EST for regular maintenance.

At 6 am you can [reload this page](#) to access Blackboard.



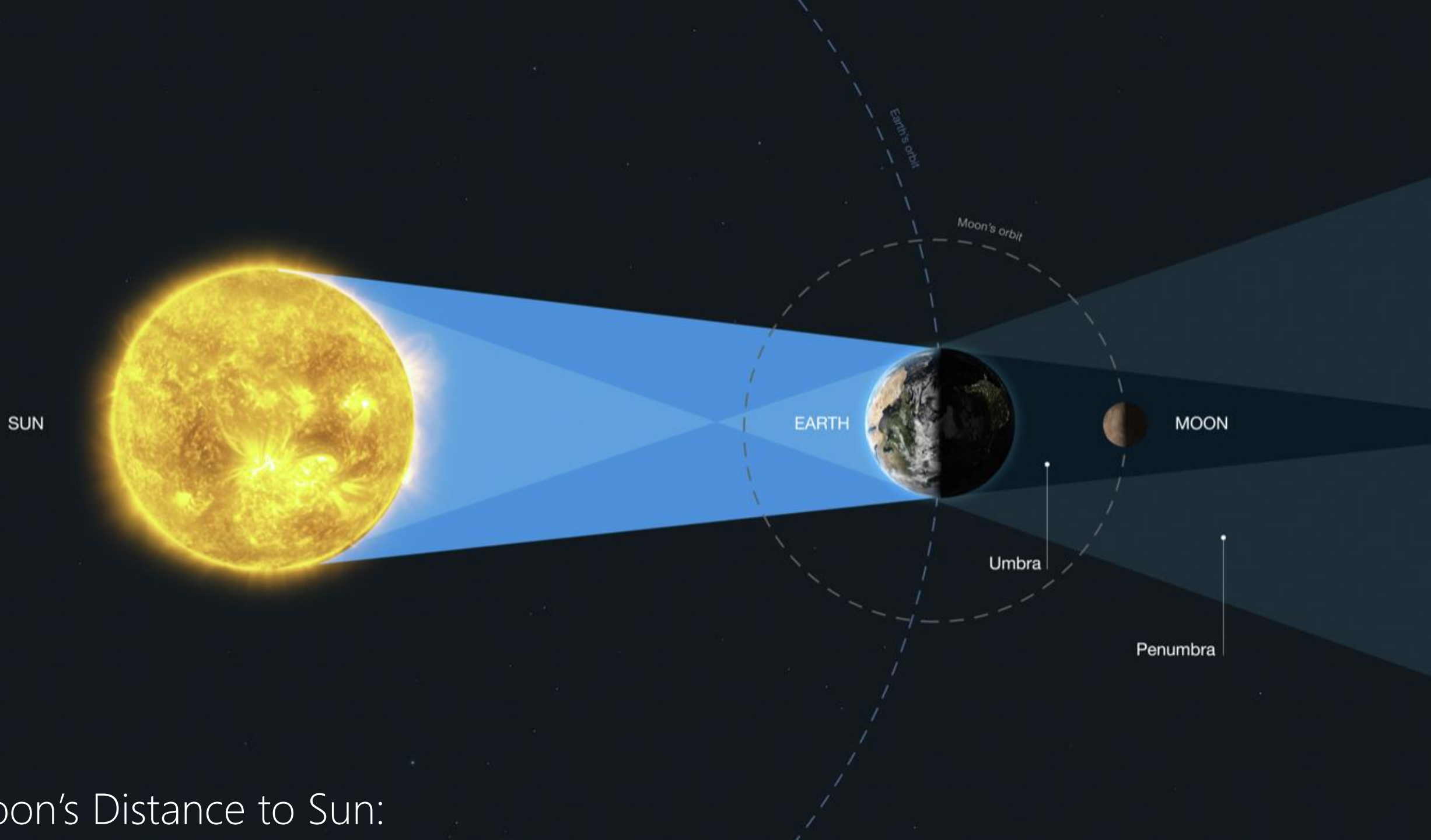
A deep-field astronomical image showing a vast number of galaxies in various colors (blue, orange, white) against a black background. The galaxies are of different shapes and sizes, some appearing as bright, distinct objects while others are fainter. A large, solid black circle is centered in the image, containing the word "Problems" in white text.

Problems

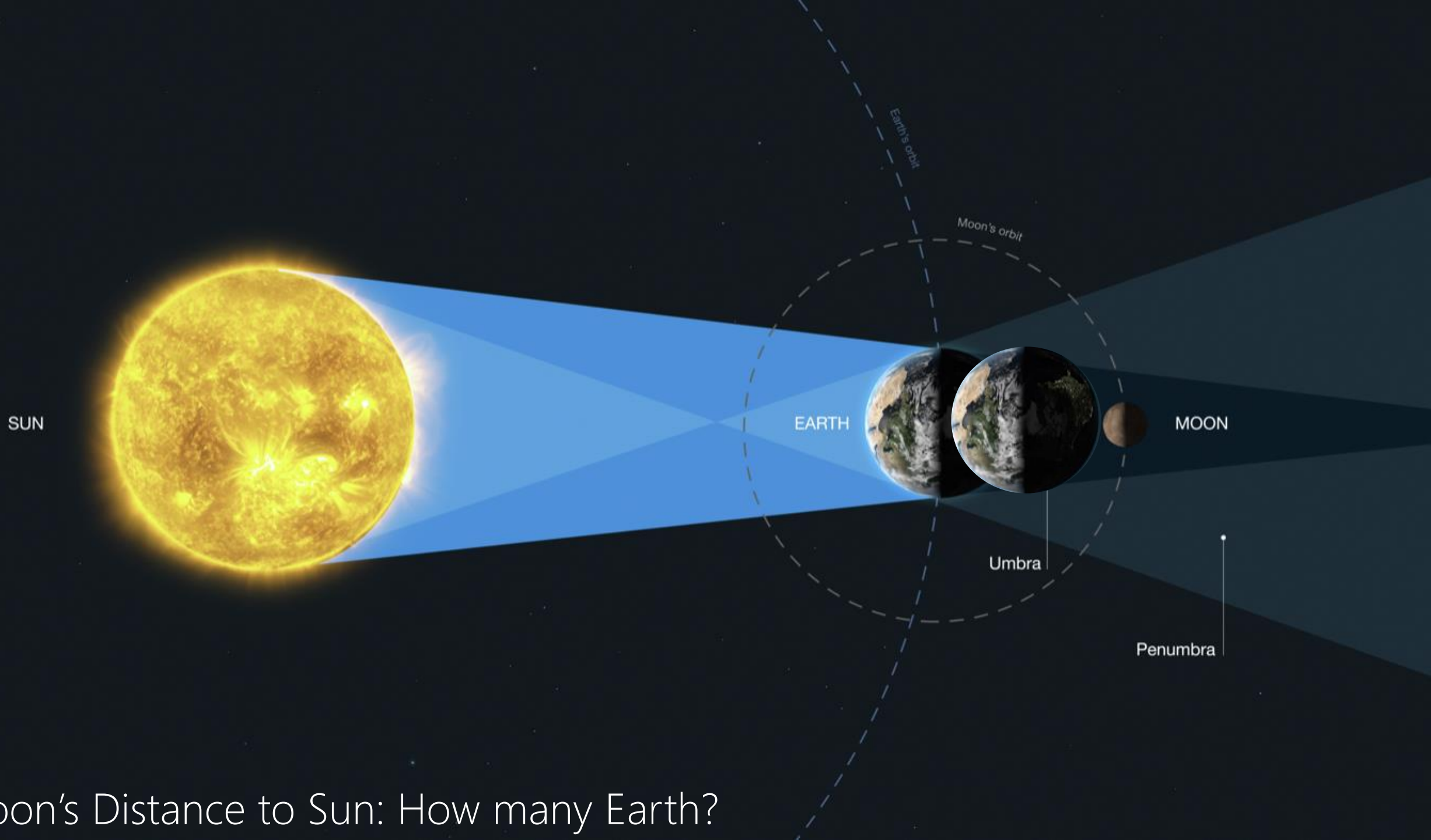
The background is a deep space photograph showing a dense field of galaxies in various colors (blue, orange, white) against a black sky. Overlaid on the center is a diagram consisting of a large black circle with a smaller light blue circle inside it. A line connects the light blue circle to a text box on the right.

Problems

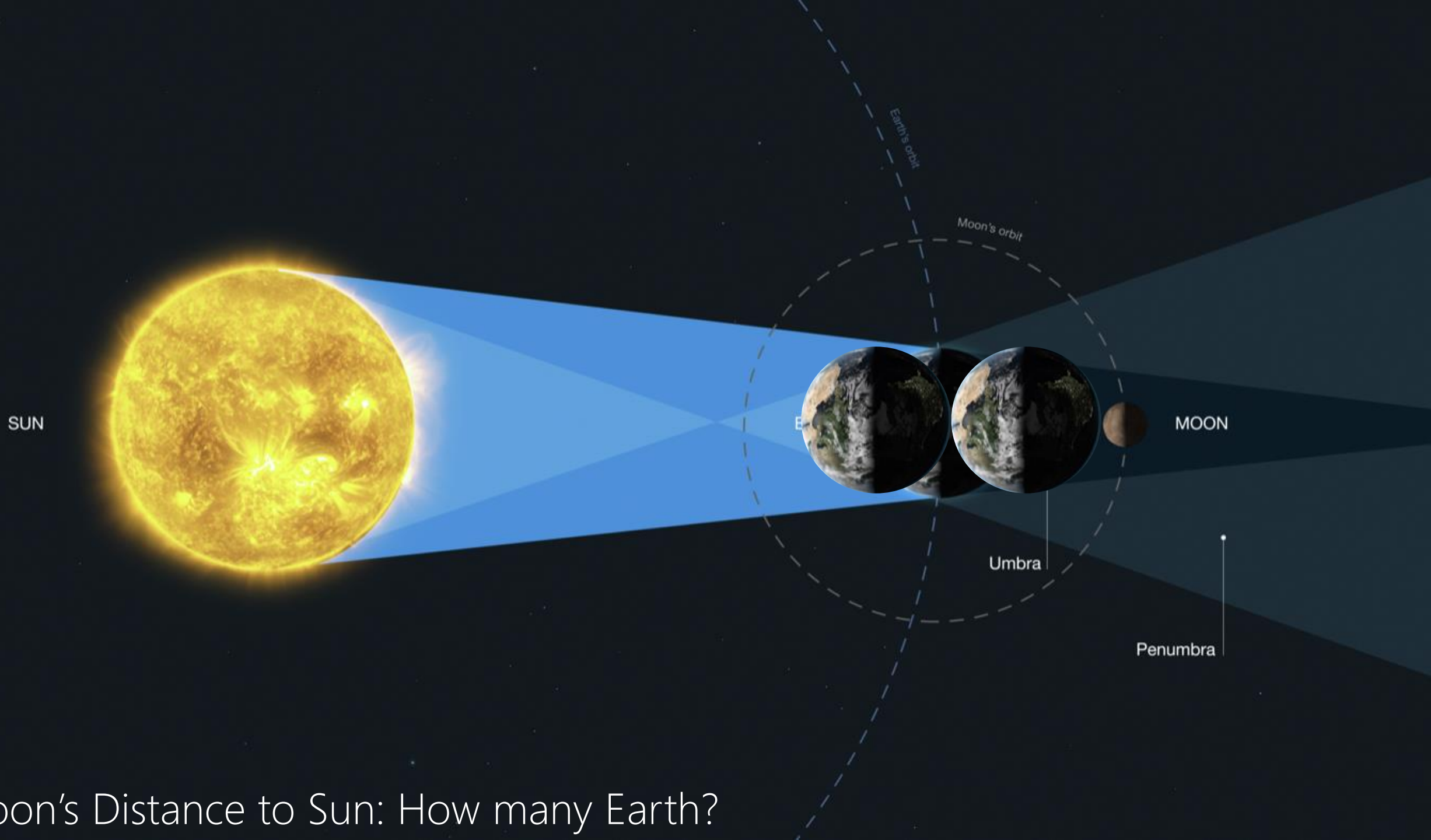
Distance
Height
Length



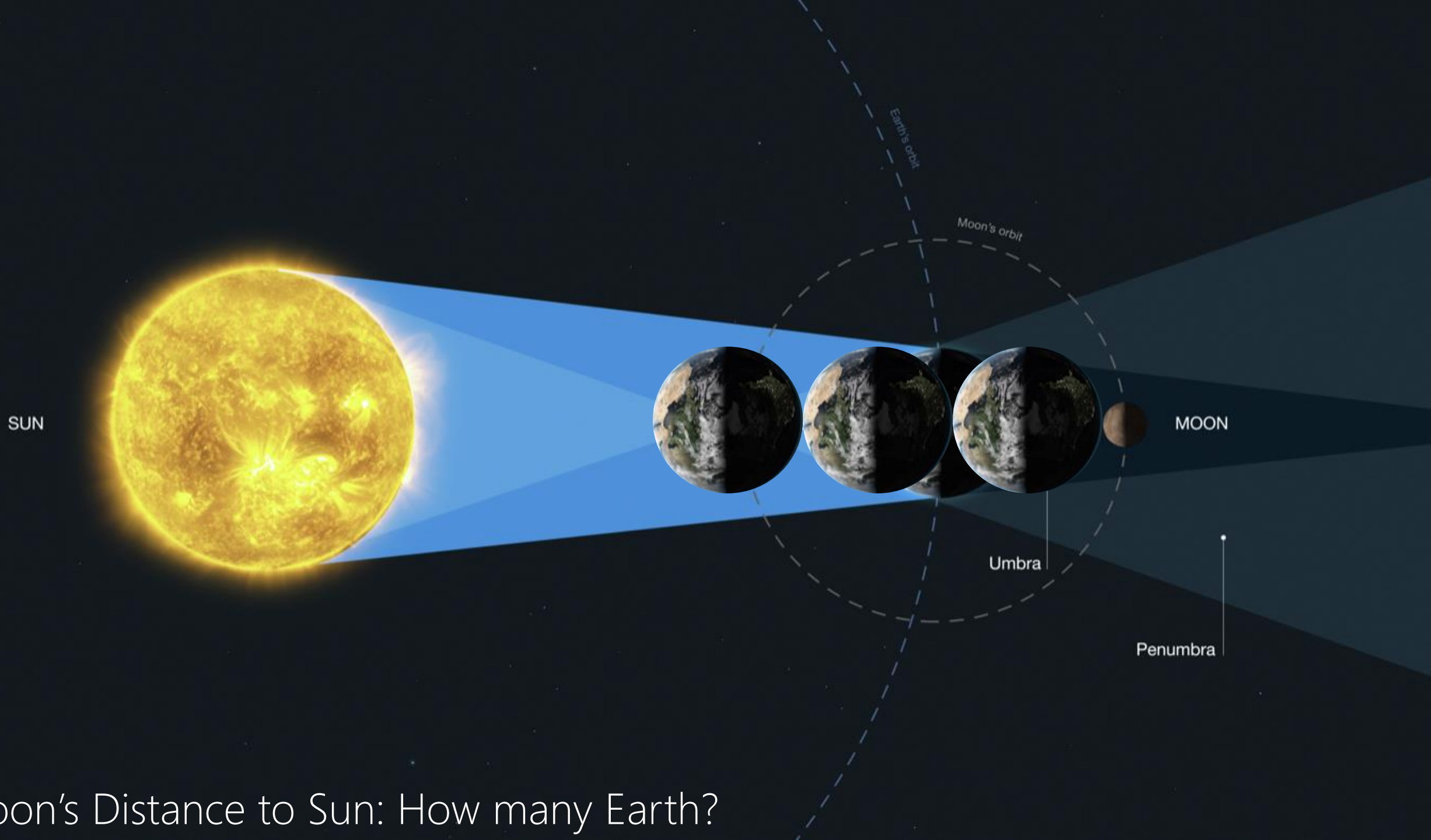
Moon's Distance to Sun:



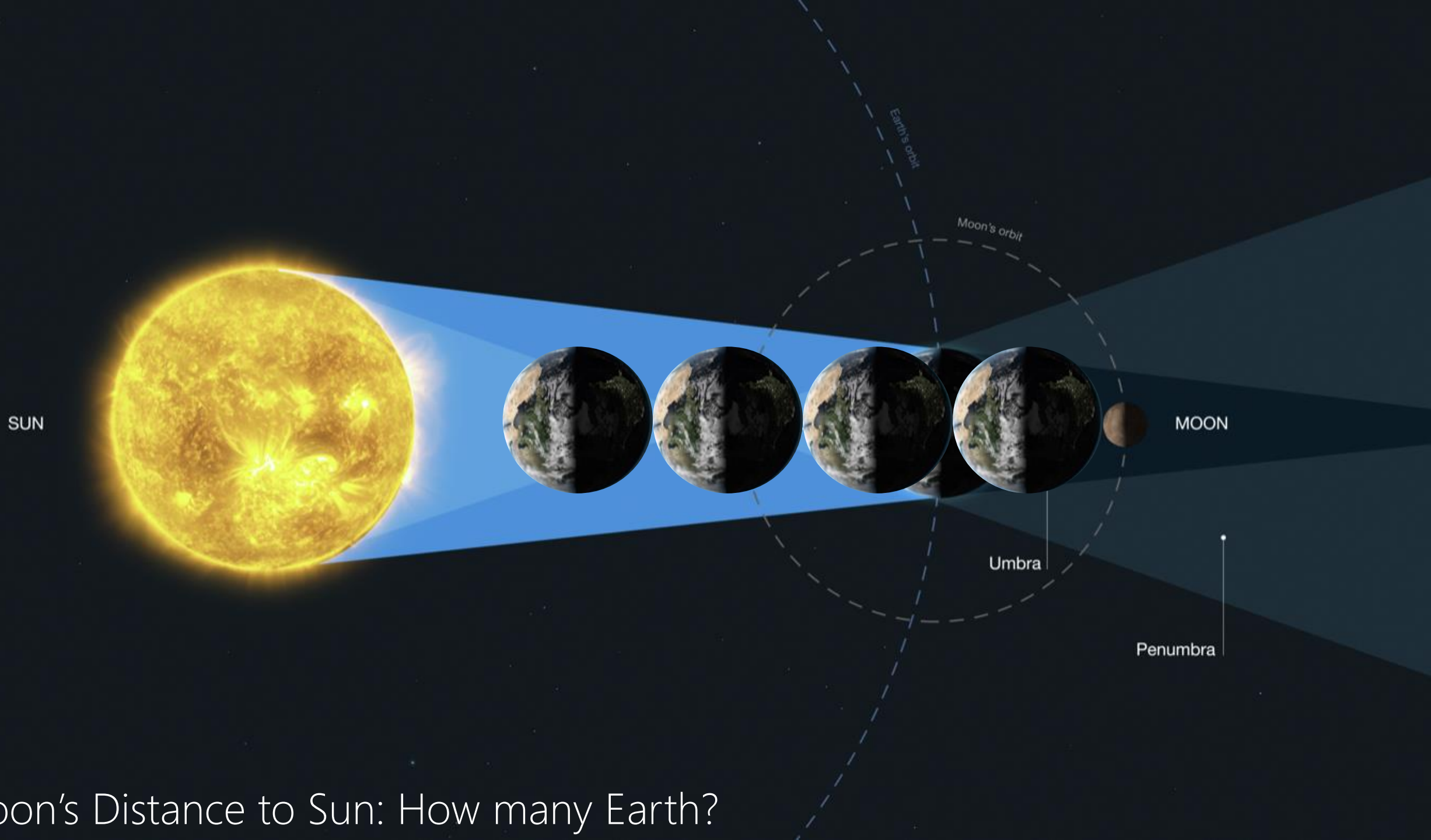
Moon's Distance to Sun: How many Earth?



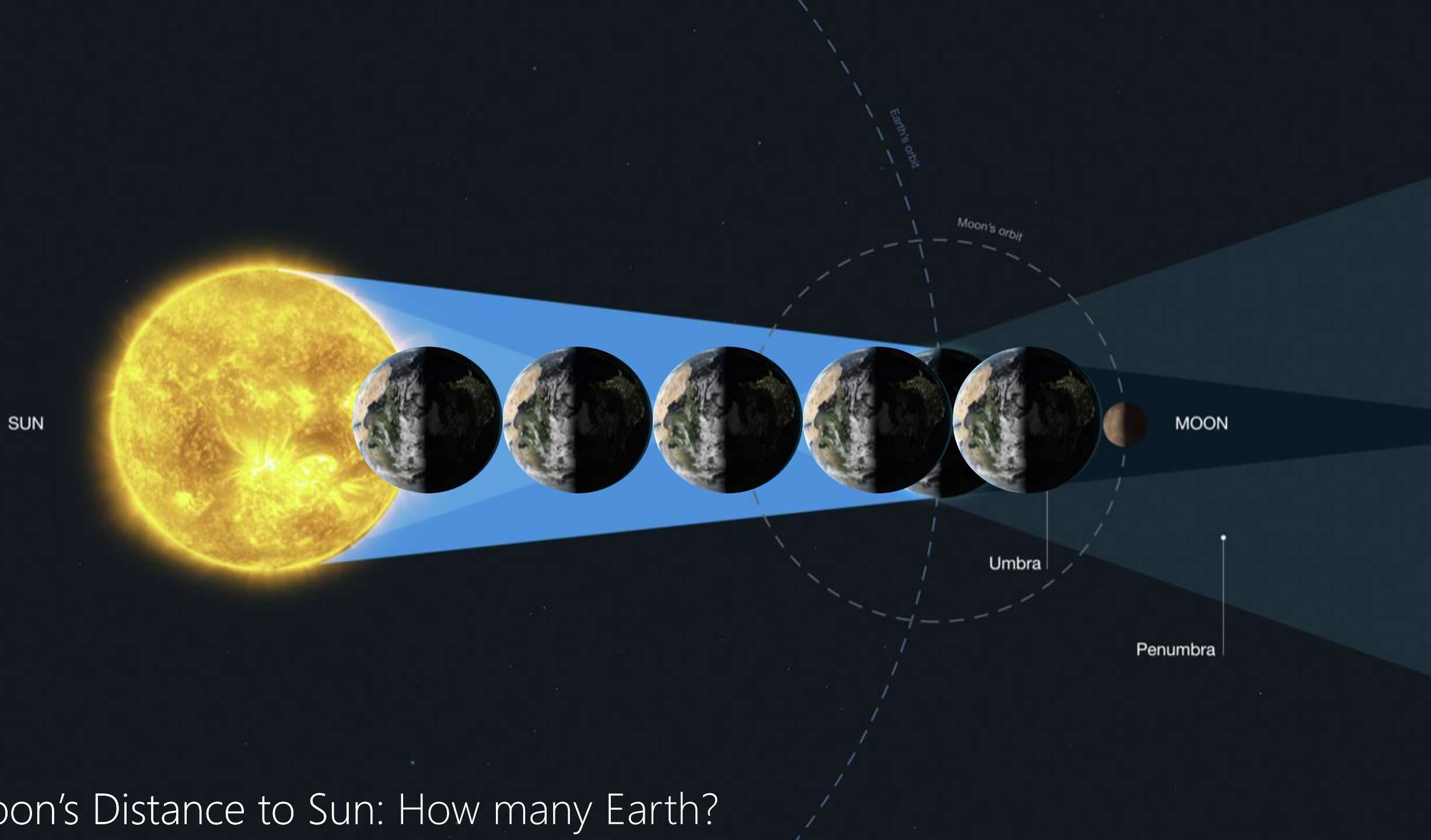
Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?



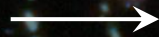
Moon's Distance to Sun: How many Earth?



Moon's Distance to Sun: How many Earth?



Continuous



Discrete

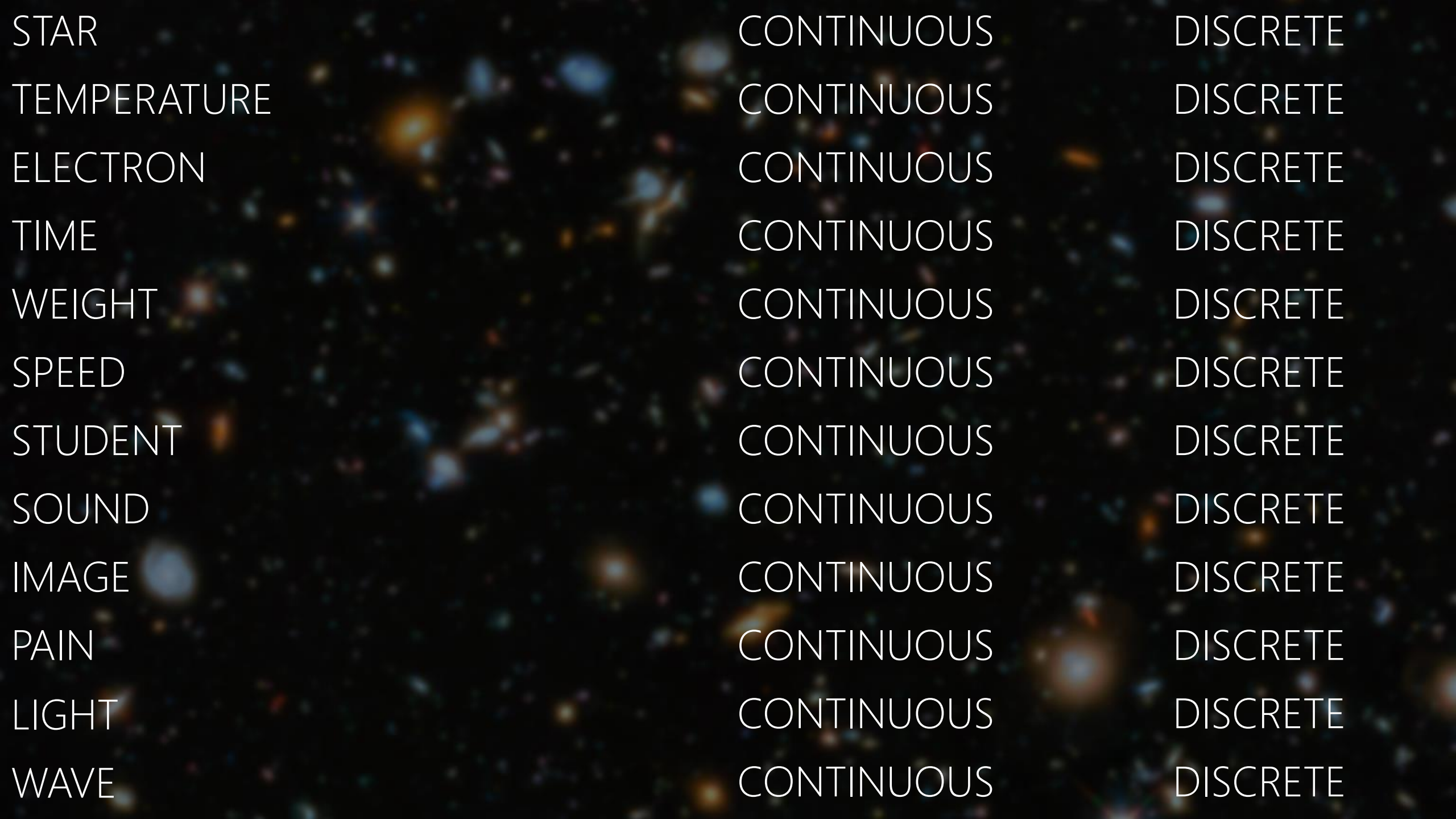
A deep-field astronomical image showing a vast field of galaxies and stars against a black background. The galaxies are in various stages of evolution, with some appearing as bright, diffuse clouds and others as more compact, structured objects. The colors range from bright yellow and orange to deep blue and purple, indicating different temperatures and compositions. Two thin, horizontal blue lines are positioned above and below the central text.

DISCRETE SYSTEMS

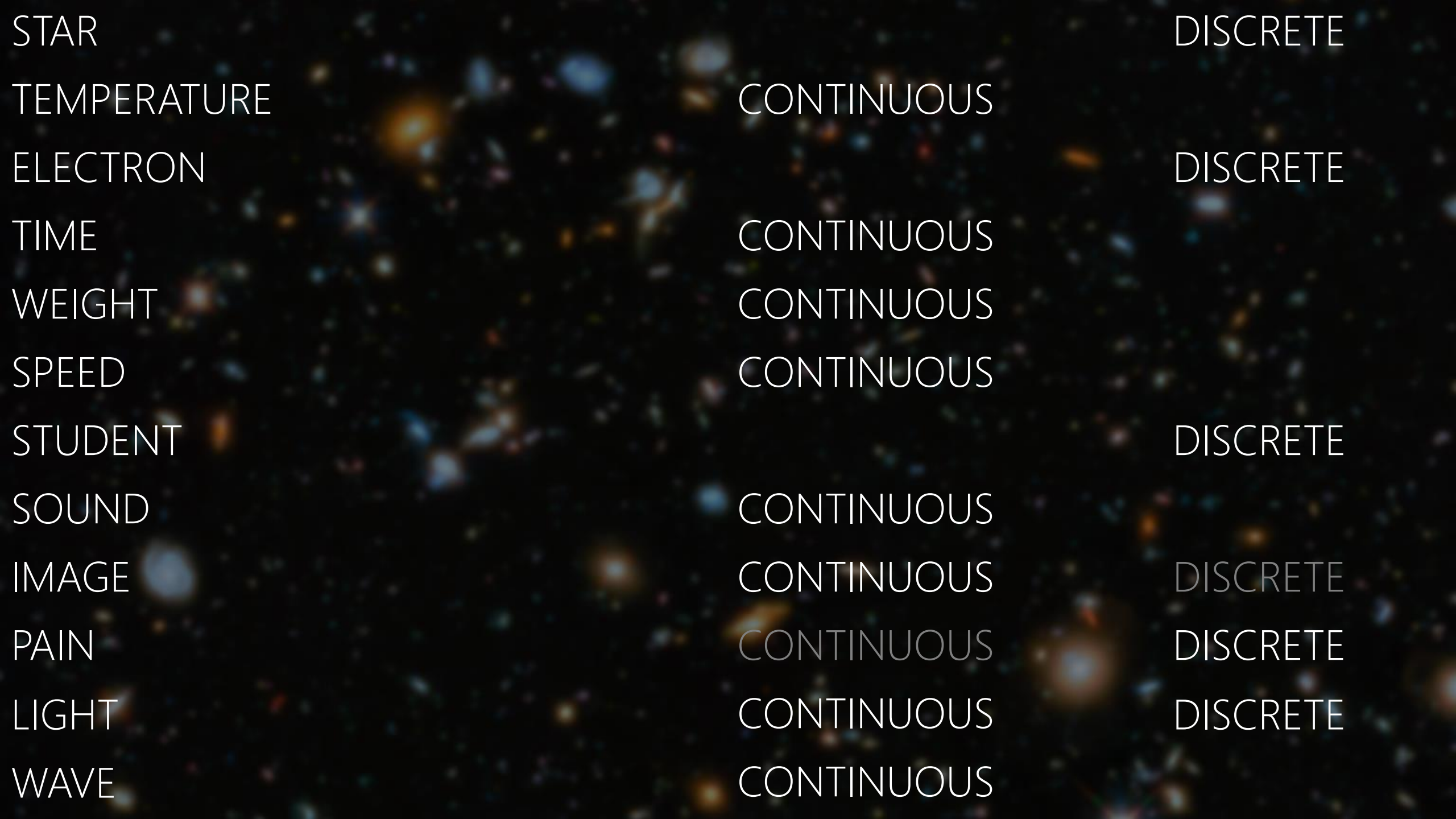


POLL

[Click to Open the Poll](#)



STAR	CONTINUOUS	DISCRETE
TEMPERATURE	CONTINUOUS	DISCRETE
ELECTRON	CONTINUOUS	DISCRETE
TIME	CONTINUOUS	DISCRETE
WEIGHT	CONTINUOUS	DISCRETE
SPEED	CONTINUOUS	DISCRETE
STUDENT	CONTINUOUS	DISCRETE
SOUND	CONTINUOUS	DISCRETE
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	DISCRETE



STAR		DISCRETE
TEMPERATURE	CONTINUOUS	
ELECTRON		DISCRETE
TIME	CONTINUOUS	
WEIGHT	CONTINUOUS	
SPEED	CONTINUOUS	
STUDENT		DISCRETE
SOUND	CONTINUOUS	
IMAGE	CONTINUOUS	DISCRETE
PAIN	CONTINUOUS	DISCRETE
LIGHT	CONTINUOUS	DISCRETE
WAVE	CONTINUOUS	

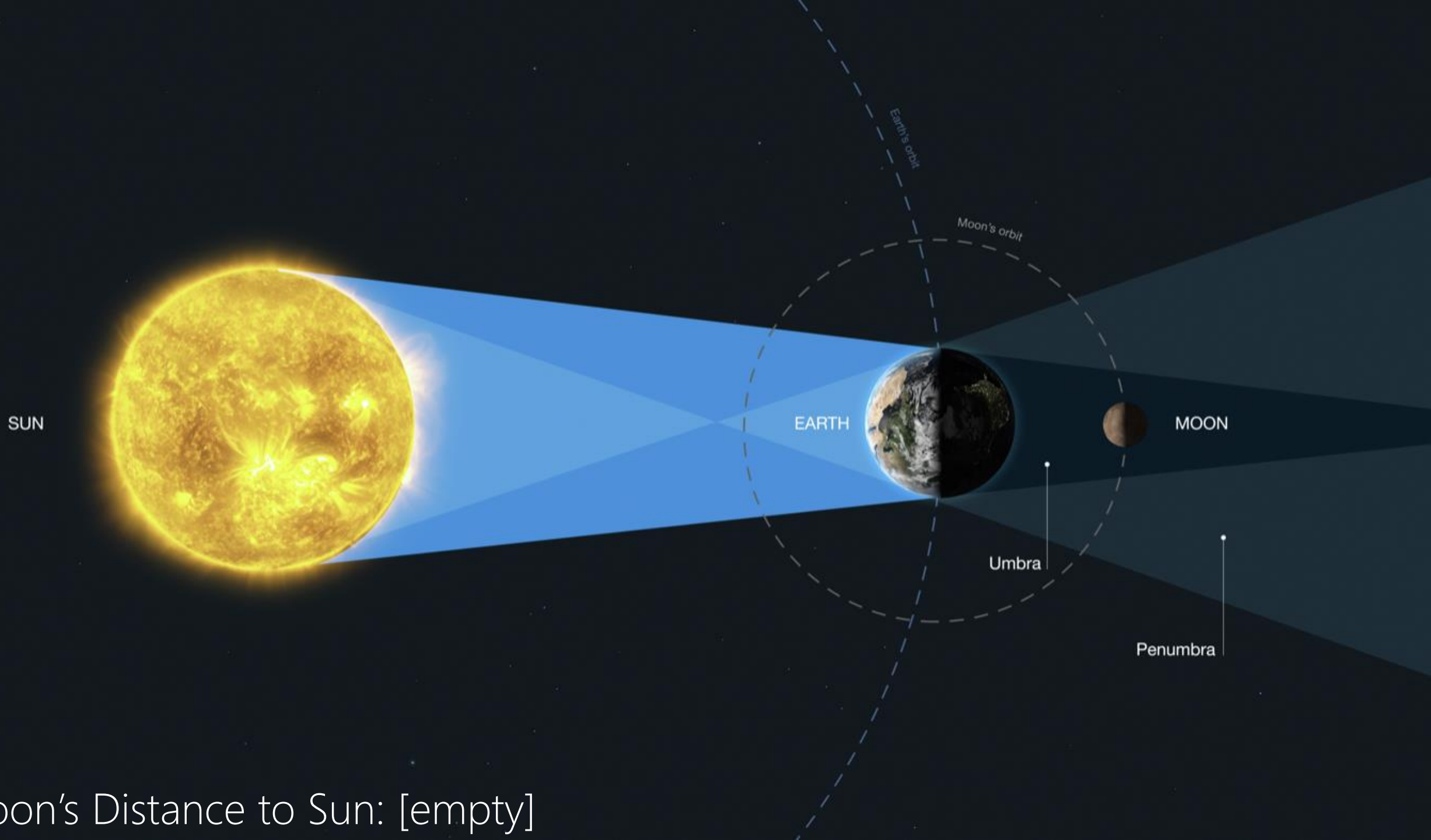


"The world's first photograph—or at least the oldest surviving photo—was taken by Joseph Nicéphore Niépce in 1826 or 1827. Captured using a technique known as heliography, the shot was taken from an upstairs window at Niépce's estate in Burgundy. As heliography produces one-of-a-kind images, there are no duplicates of the piece, which is now part of the permanent collection at the University of Texas-Austin." [18 Famous First Photographs in History: From the Oldest Photo Ever to the World's First Instagram](#)

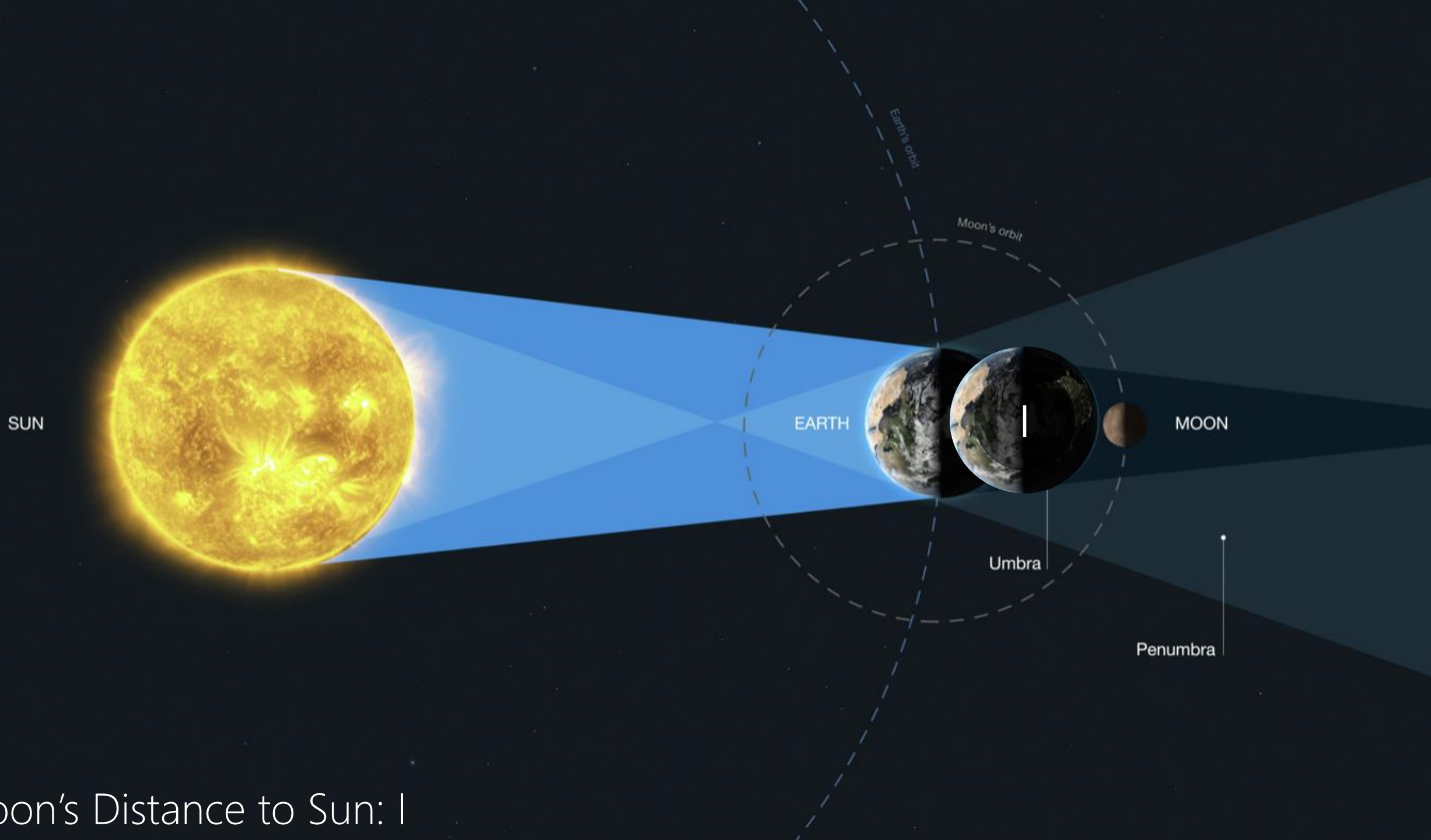




"We may be used to selfies now, but it's Robert Cornelius's 1839 image that lays claim to the first self-portrait. Taken in Philadelphia, Cornelius sat for a little over one minute before covering the lens." [18 Famous First Photographs in History: From the Oldest Photo Ever to the World's First Instagram](#)



Moon's Distance to Sun: [empty]



SUN

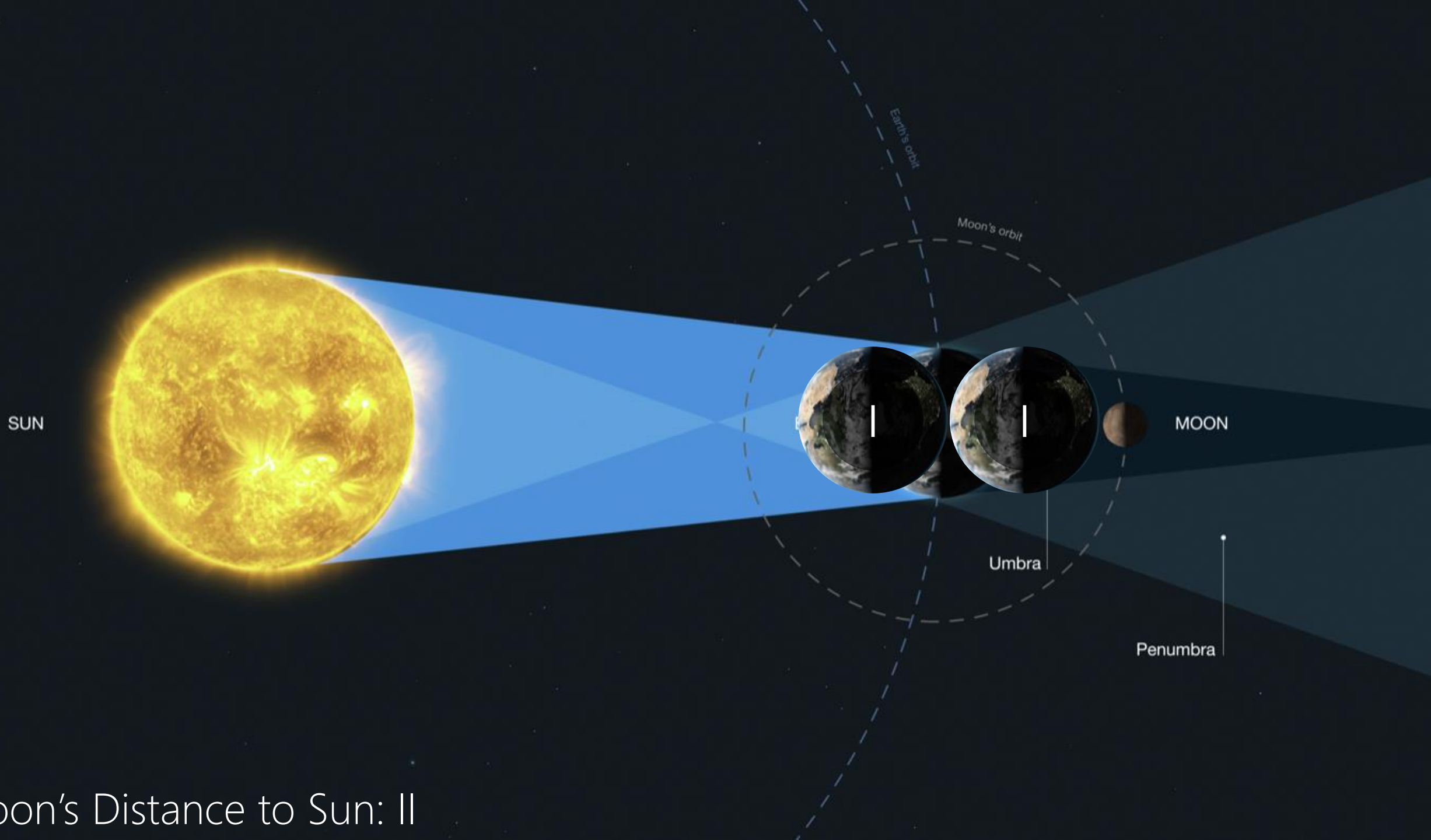
EARTH

MOON

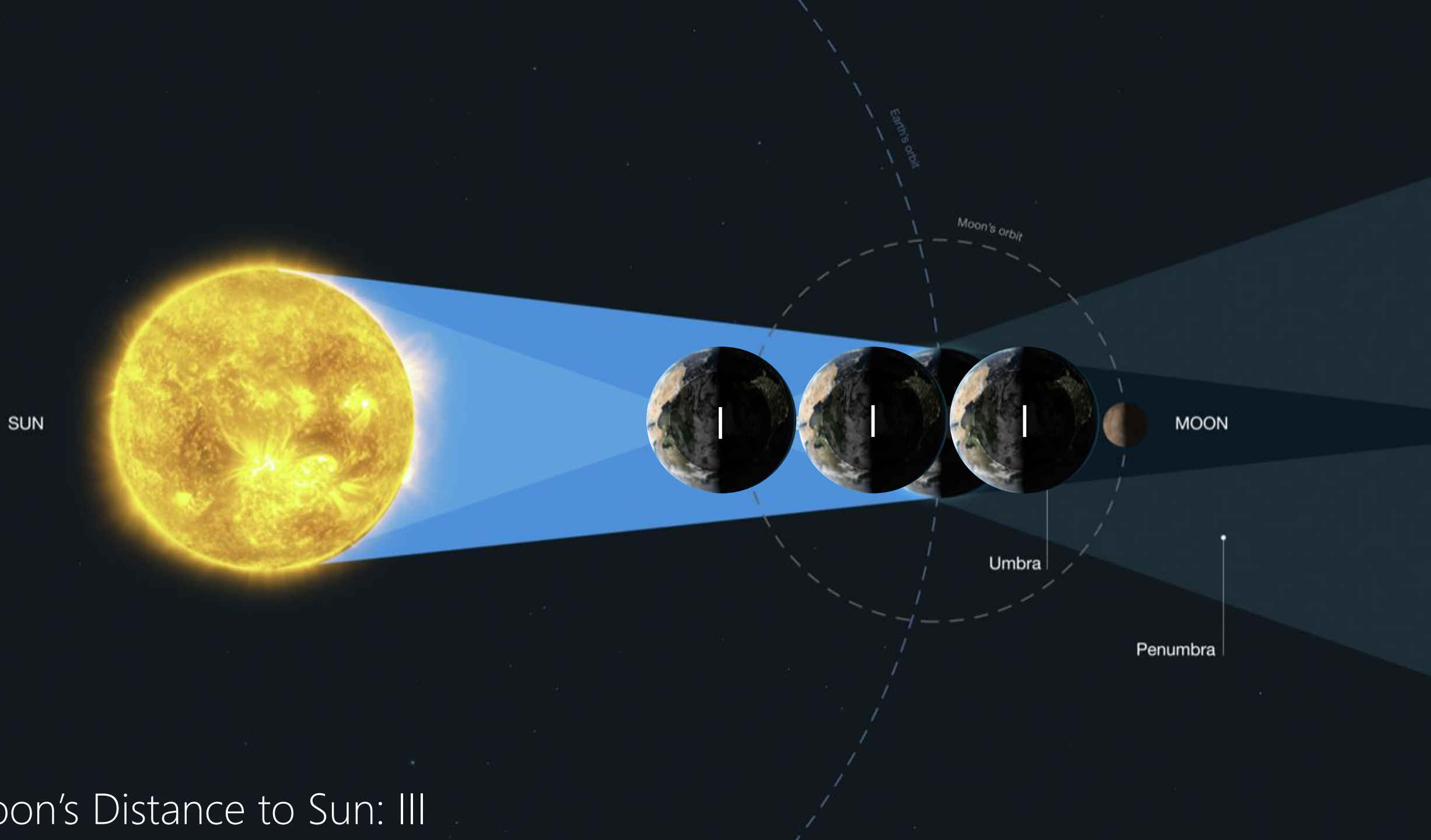
Umbra

Penumbra

Moon's Distance to Sun: 1



Moon's Distance to Sun: II



Moon's Distance to Sun: III



Quantization



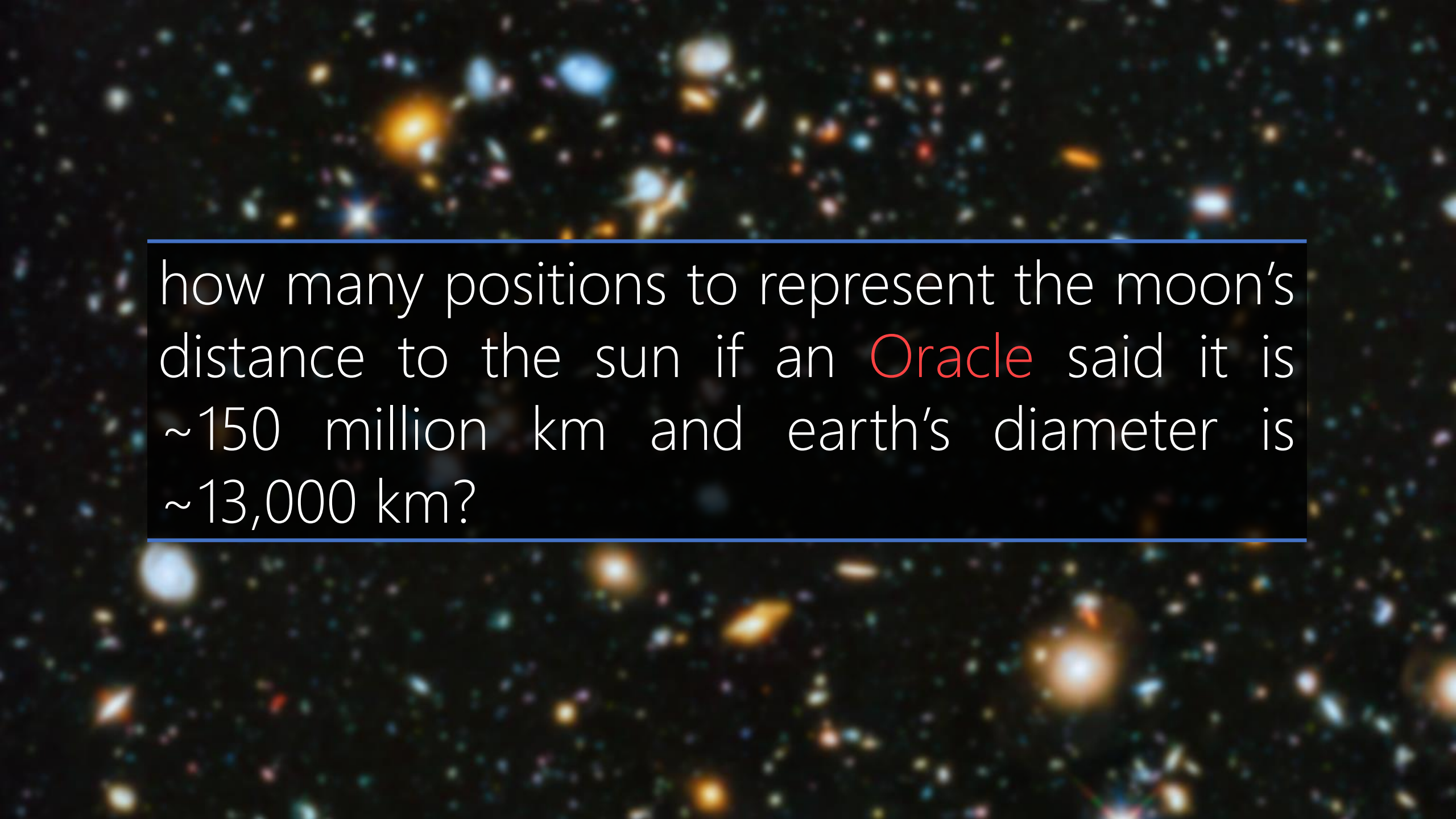
I II III IV V VI VII VIII IX X

Roman Numerals
Originated in Ancient Rome
8th Century BC

The background of the slide is a deep space image filled with numerous galaxies of various shapes and colors, including yellow, orange, blue, and red, set against a black void. Two thin, horizontal blue lines are positioned above and below the central text.

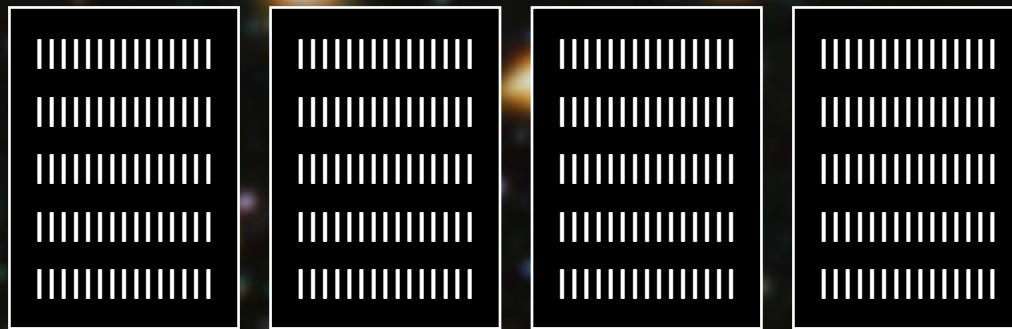
UNARY SYSTEM

aka. Base-1



how many positions to represent the moon's distance to the sun if an **Oracle** said it is ~150 million km and earth's diameter is ~13,000 km?

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$
paper = $\sim 3,000 \text{ positions}$
 $12,000 \div 3,000 = 4 \text{ pages!}$



A deep-field astronomical image showing a vast field of galaxies in various colors (blue, orange, white) against a black background. Two horizontal blue lines frame the central text.

NUMBER SYSTEMS

— = ≡ ≠ ୪ ୮ | ୬ ୭ ୫ ୨

Brahmi

3rd and 7th century AD



୨ ୩ ୪ ୫ ୬ | ୭ ୮ ୯ ୦

Hindu (Gwalior)



୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯ ୦

Sanskrit-Devanagari



୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯

Western Arabic (Gobar)



୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯ ୦

Eastern Arabic

୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯ ୦

11th Century (Apices)

୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯ ୦

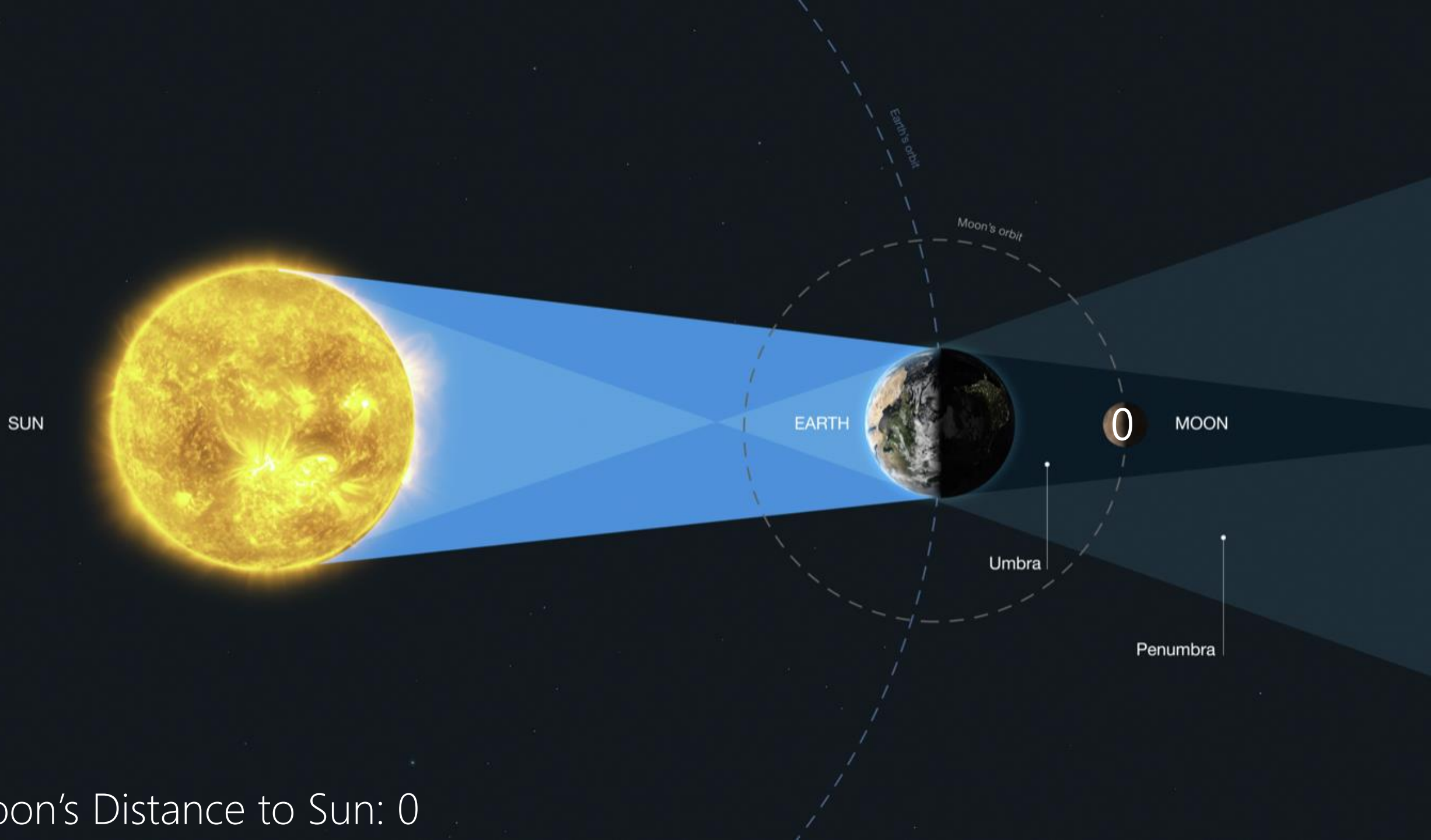
15th Century

୧ ୨ ୩ ୪ ୫ | ୬ ୭ ୮ ୯ ୦

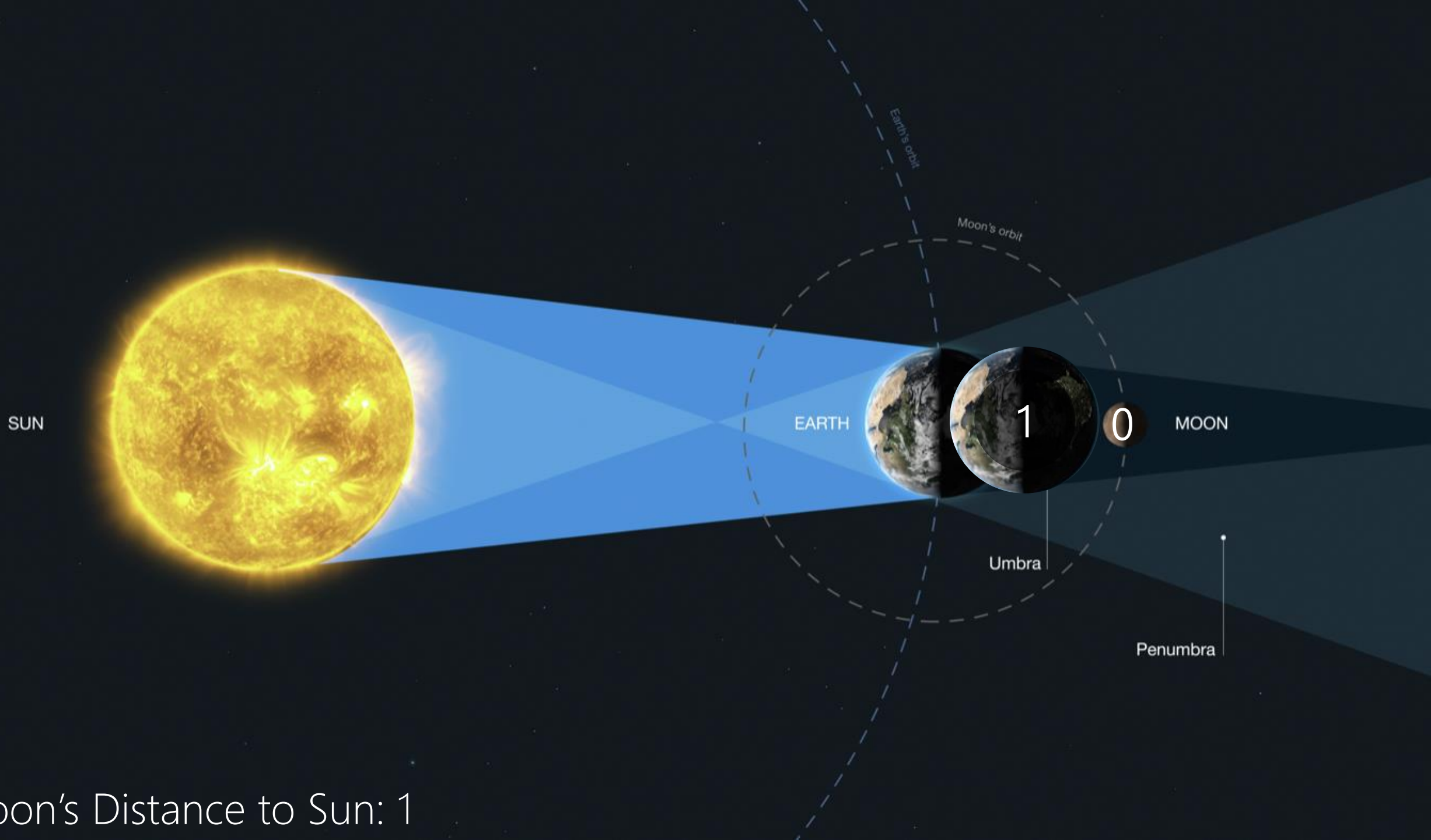
16th Century (Dürer)

0123456789

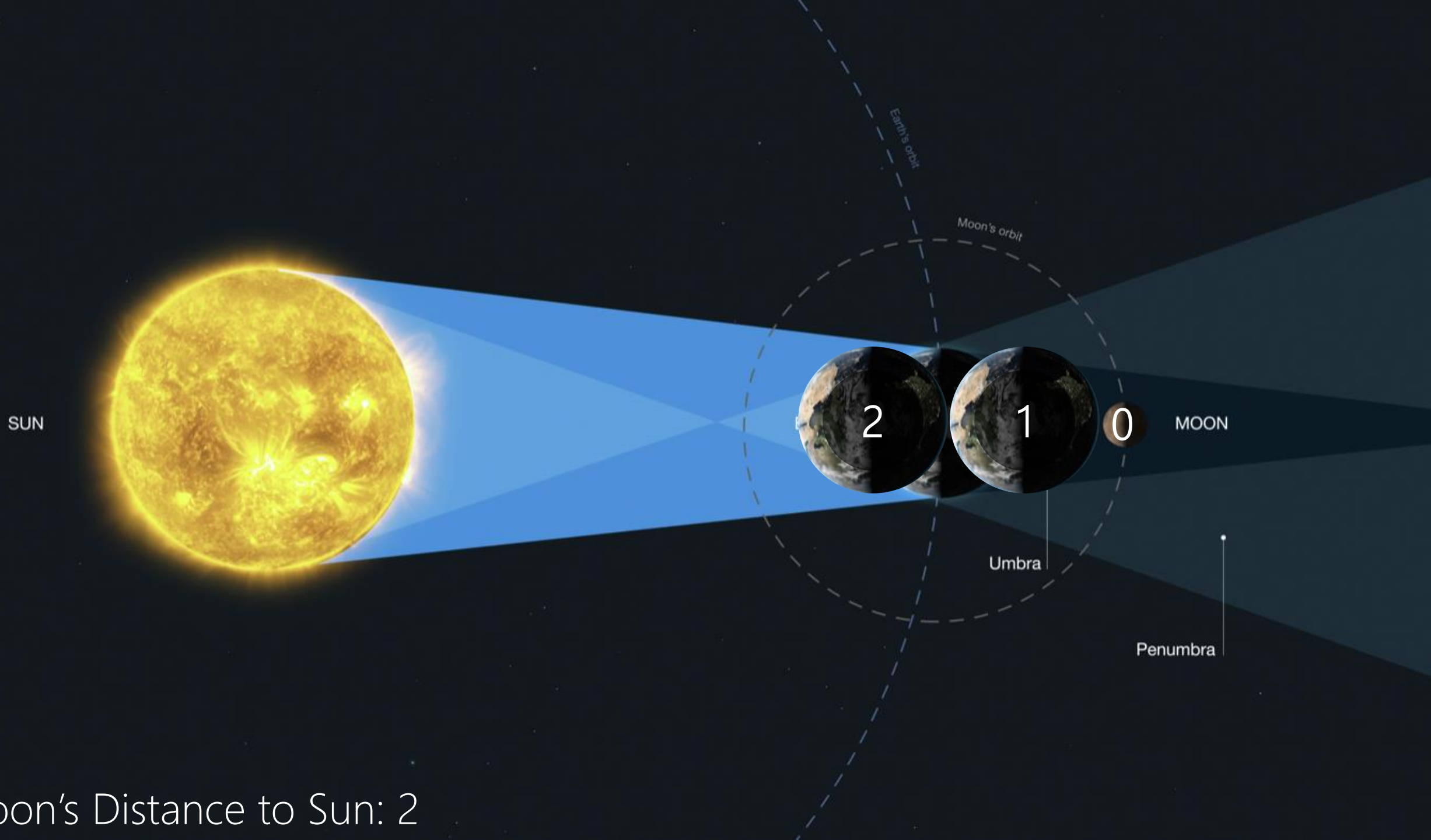
Hossein's Number System



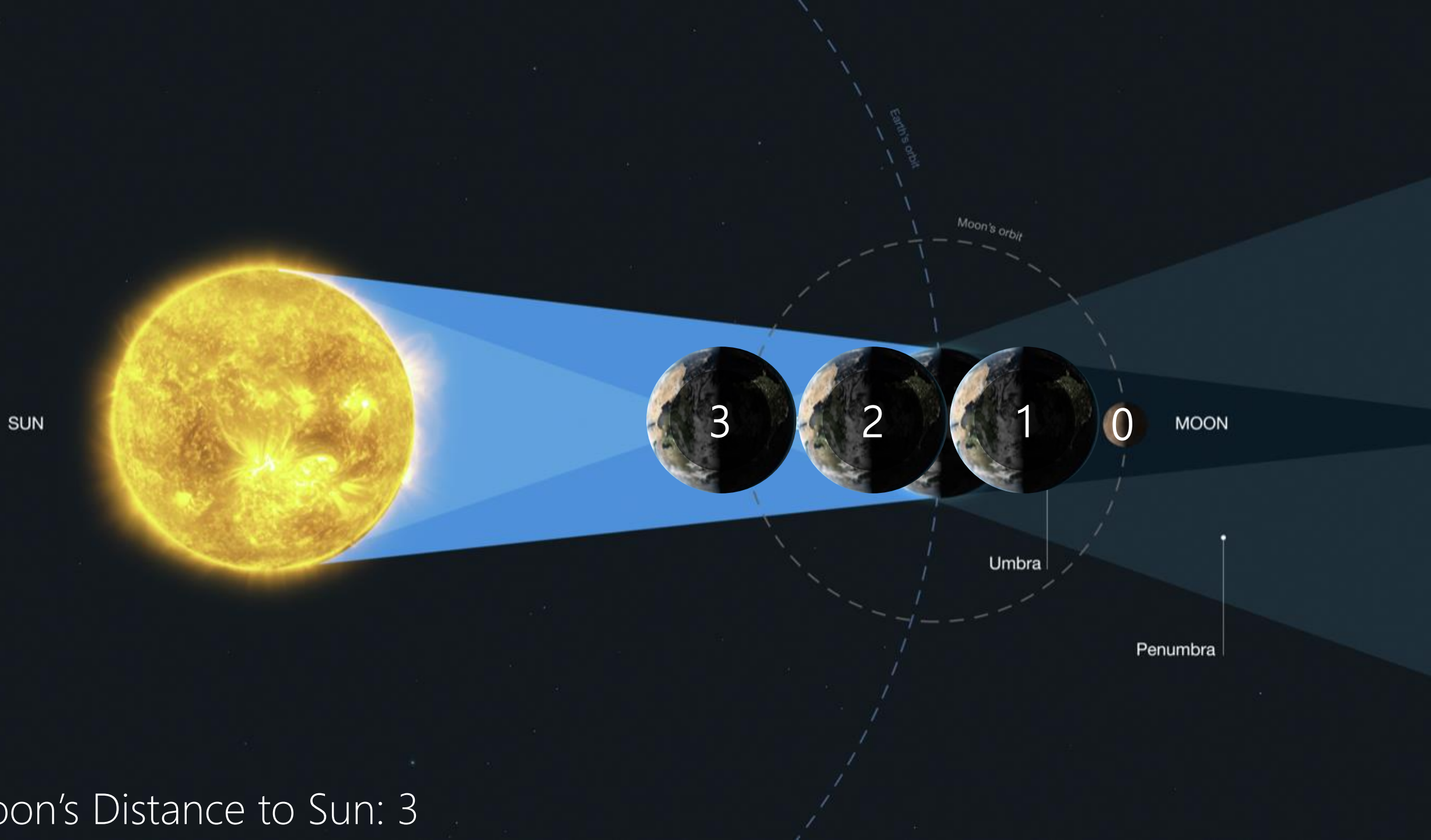
Moon's Distance to Sun: 0



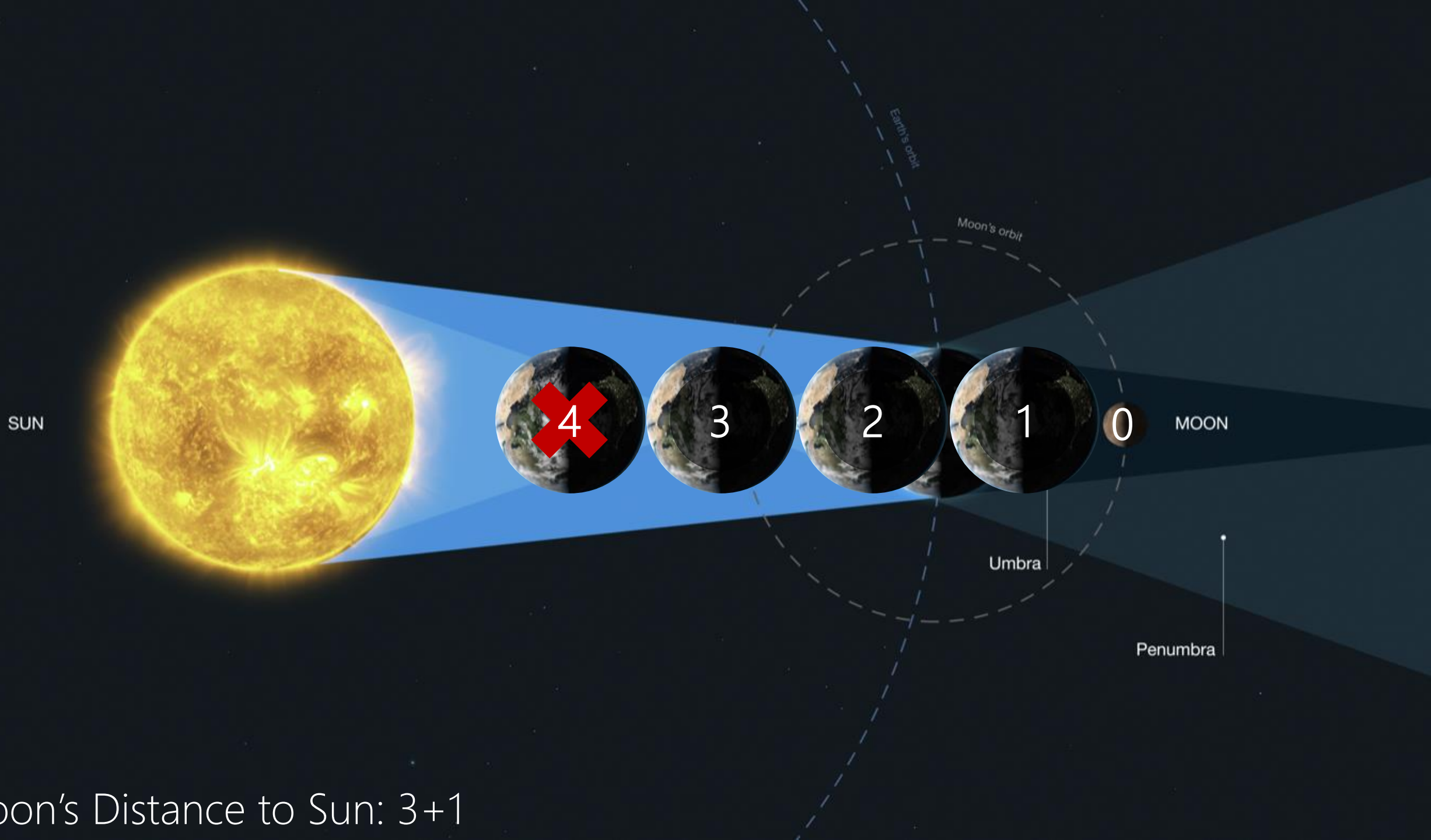
Moon's Distance to Sun: 1



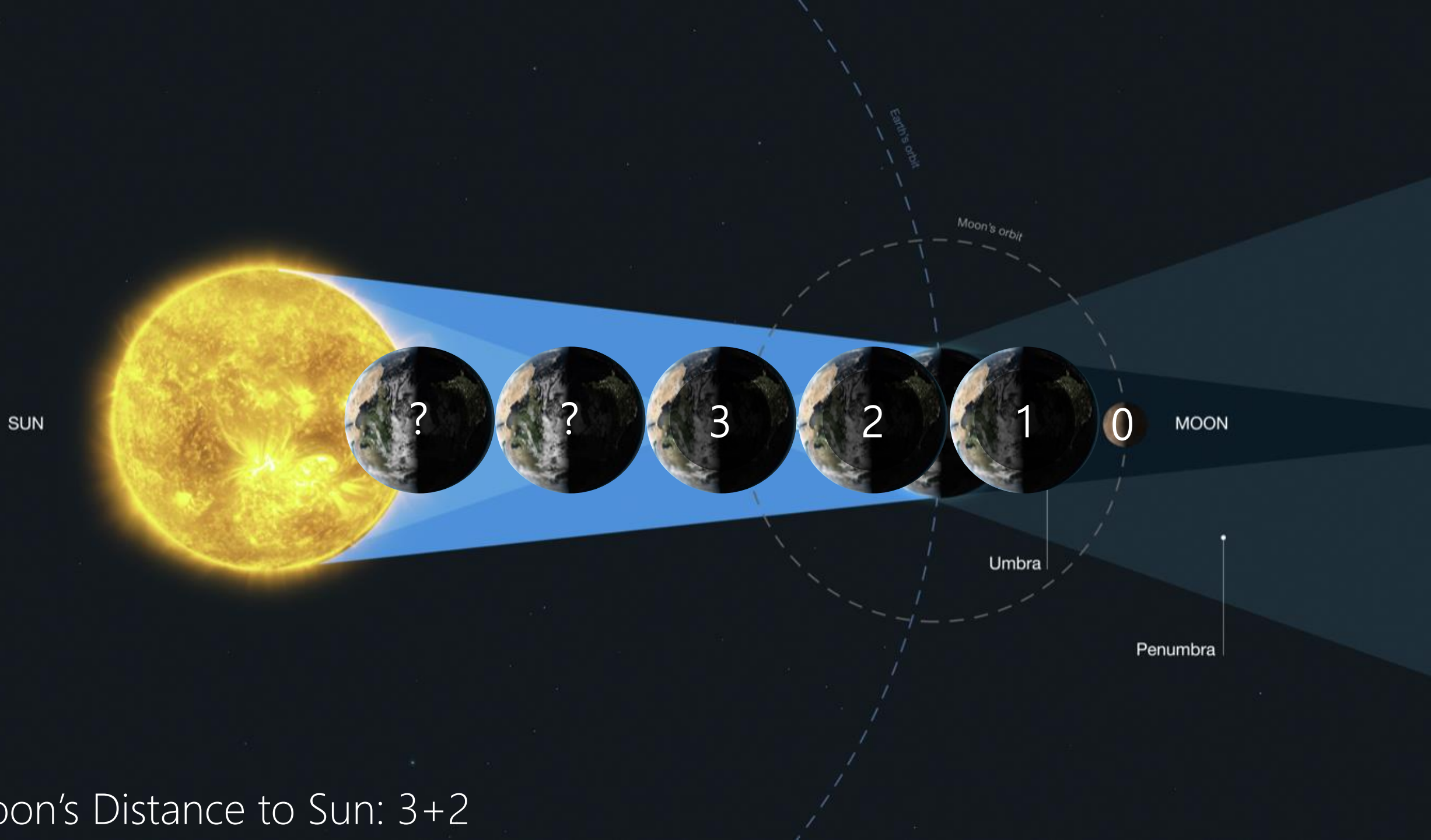
Moon's Distance to Sun: 2



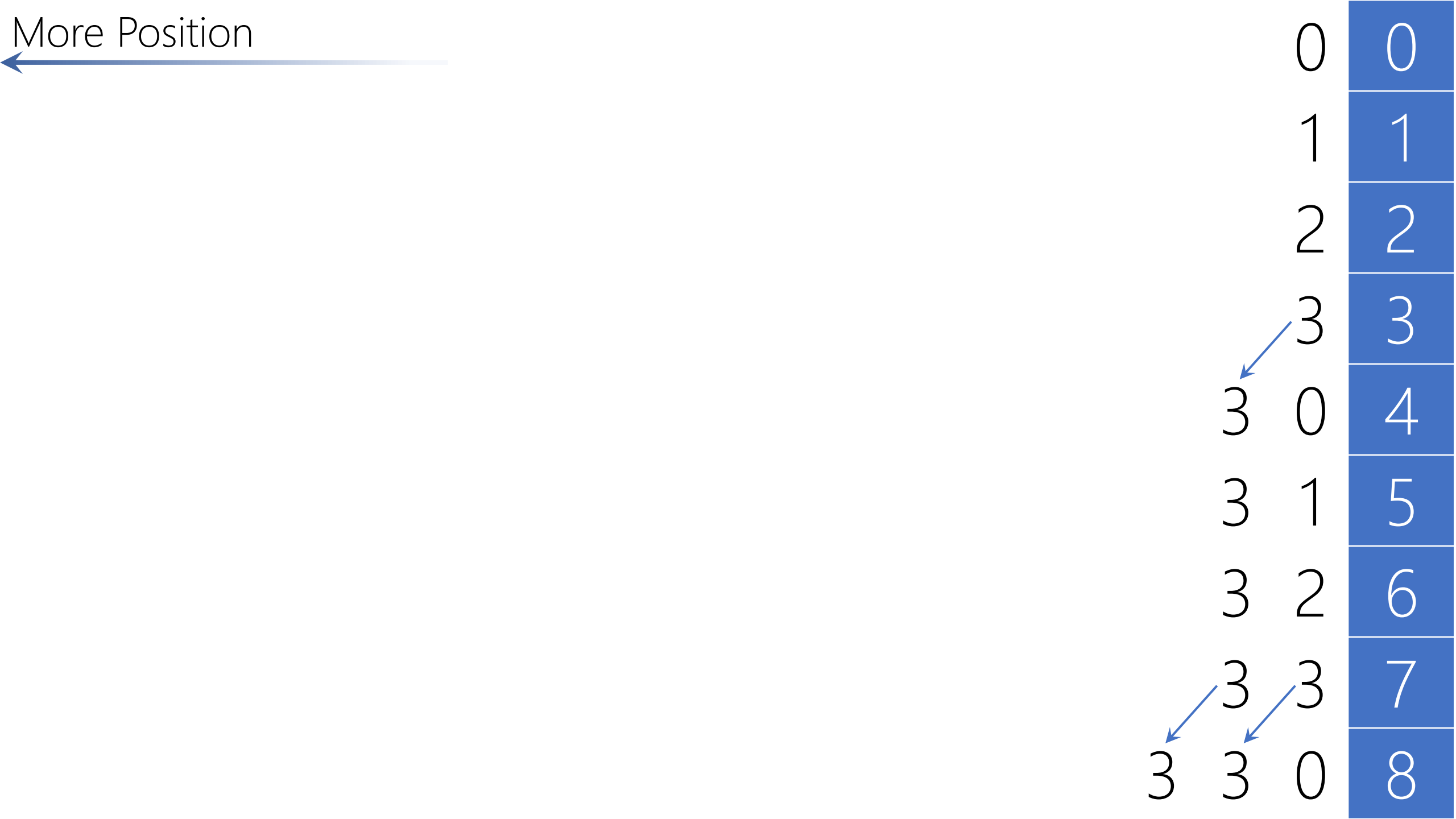
Moon's Distance to Sun: 3

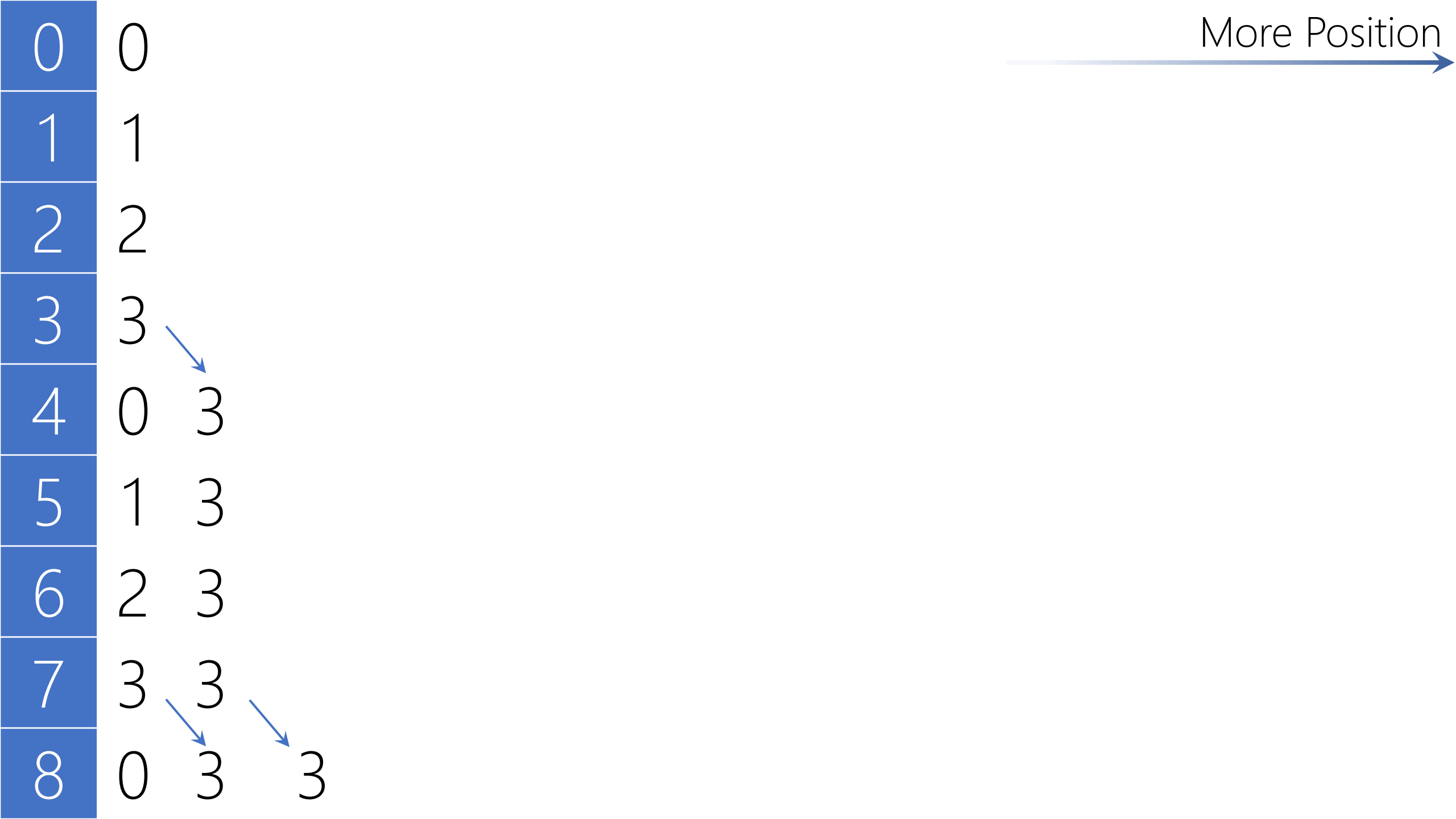


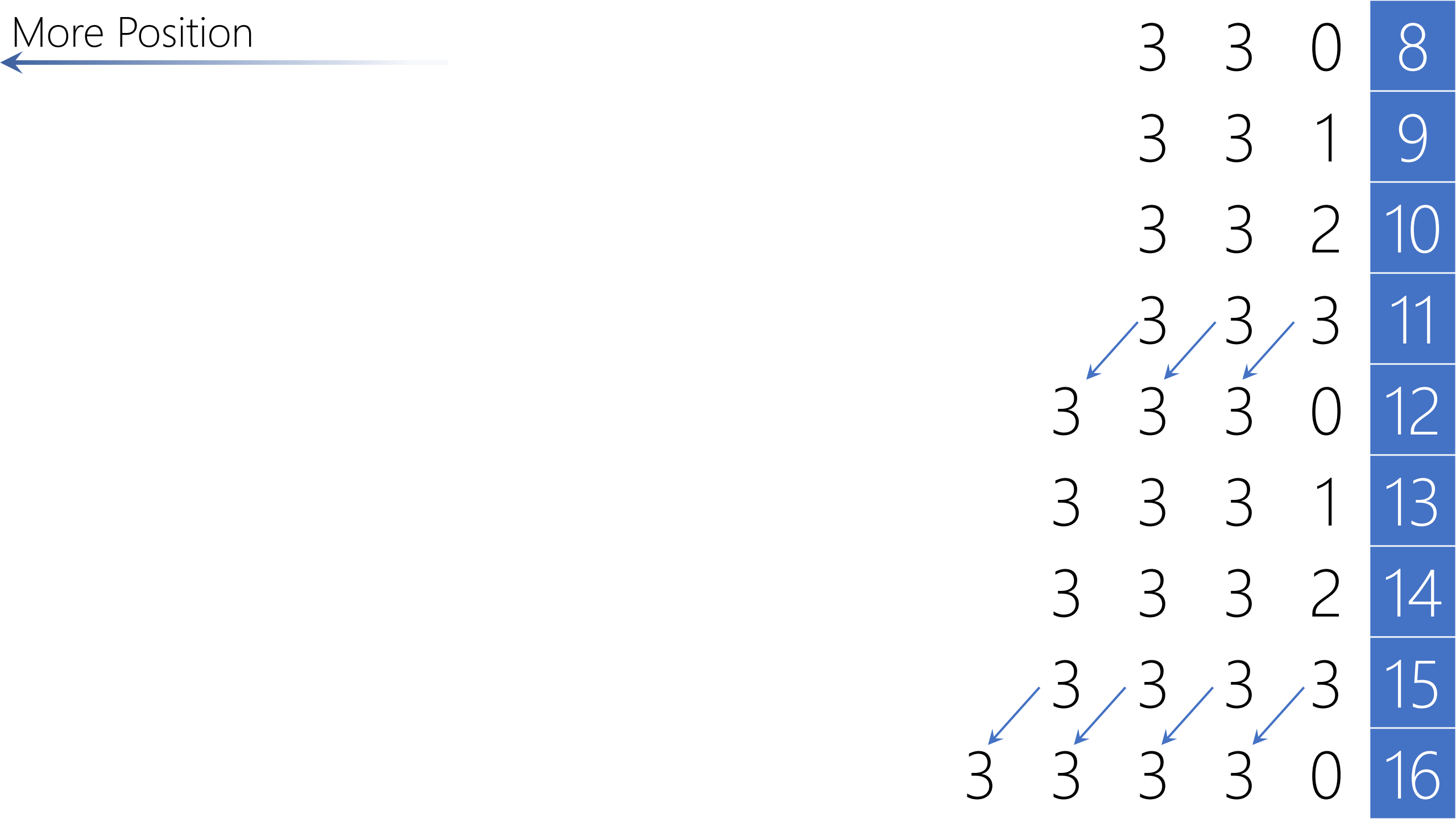
Moon's Distance to Sun: $3+1$



Moon's Distance to Sun: $3+2$







More Position



0	$(1-1)=0$
---	-----------

1	$1+(1-1)=1$
---	-------------

2	$2+(1-1)=2$
---	-------------

3	$3+(1-1)=3$
---	-------------

3	0	$3+0+(2-1)=4$
---	---	---------------

3	1	$3+1+(2-1)=5$
---	---	---------------

3	2	$3+2+(2-1)=6$
---	---	---------------

3	3	$3+3+(2-1)=7$
---	---	---------------

3	3	0	$3+3+(3-1)=8$
---	---	---	---------------

More Position



3 3 0

$$3+3+0+(\textcolor{yellow}{3}-1)=8$$

3 3 1

$$3+3+1+(3-1)=9$$

3 3 2

$$3+3+2+(3-1)=10$$

3 3 3

$$3+3+3+(3-1)=11$$

3 3 3 0

$$3+3+3+0+(\textcolor{yellow}{4}-1)=12$$

3 3 3 1

$$3+3+3+1+(4-1)=13$$

3 3 3 2

$$3+3+3+2+(4-1)=14$$

3 3 3 3

$$3+3+3+3+(4-1)=15$$

3 3 3 3 0

$$3+3+3+3+(\textcolor{yellow}{5}-1)=16$$

[illegible]

[illegible]

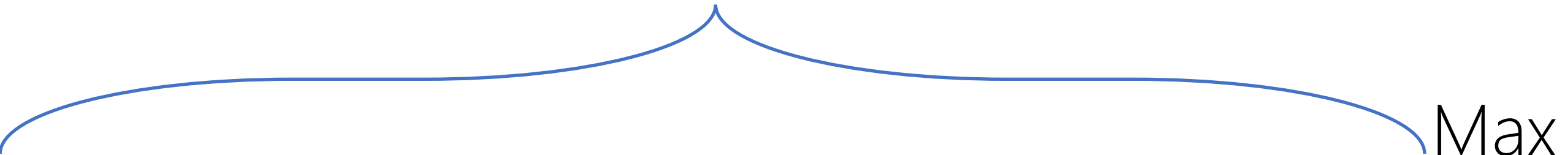
[illegible]

[illegible]

0	0	...			0	0	0
---	---	-----	--	--	---	---	---



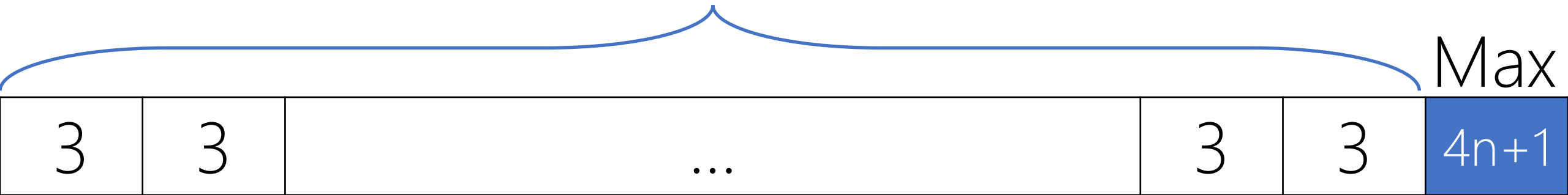
n positions



3	3	...			3	3	?
---	---	-----	--	--	---	---	---

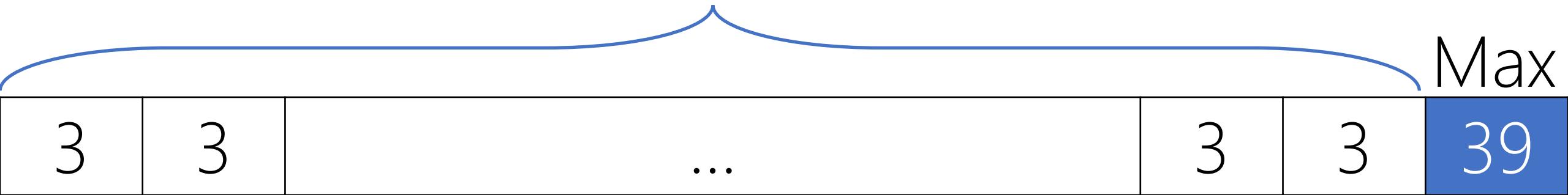
$$3+3+3+\cdots+3+(n-1) = 3 \times n + (n-1) = 4n-1$$

n positions



$$\text{Max} = 4n - 1$$
$$n = 10 \Rightarrow (4 \times 10) - 1 = 39$$

10 positions

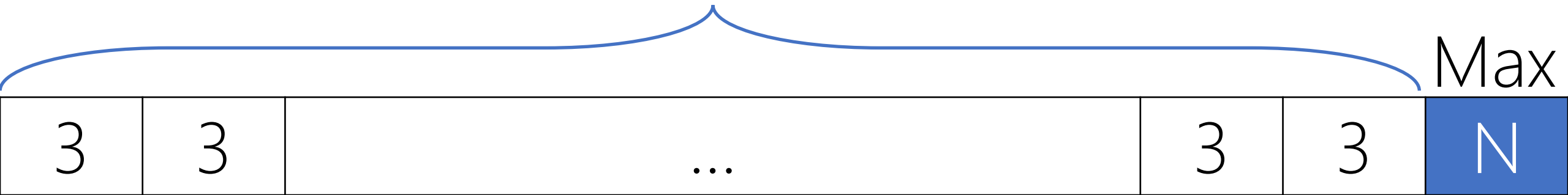


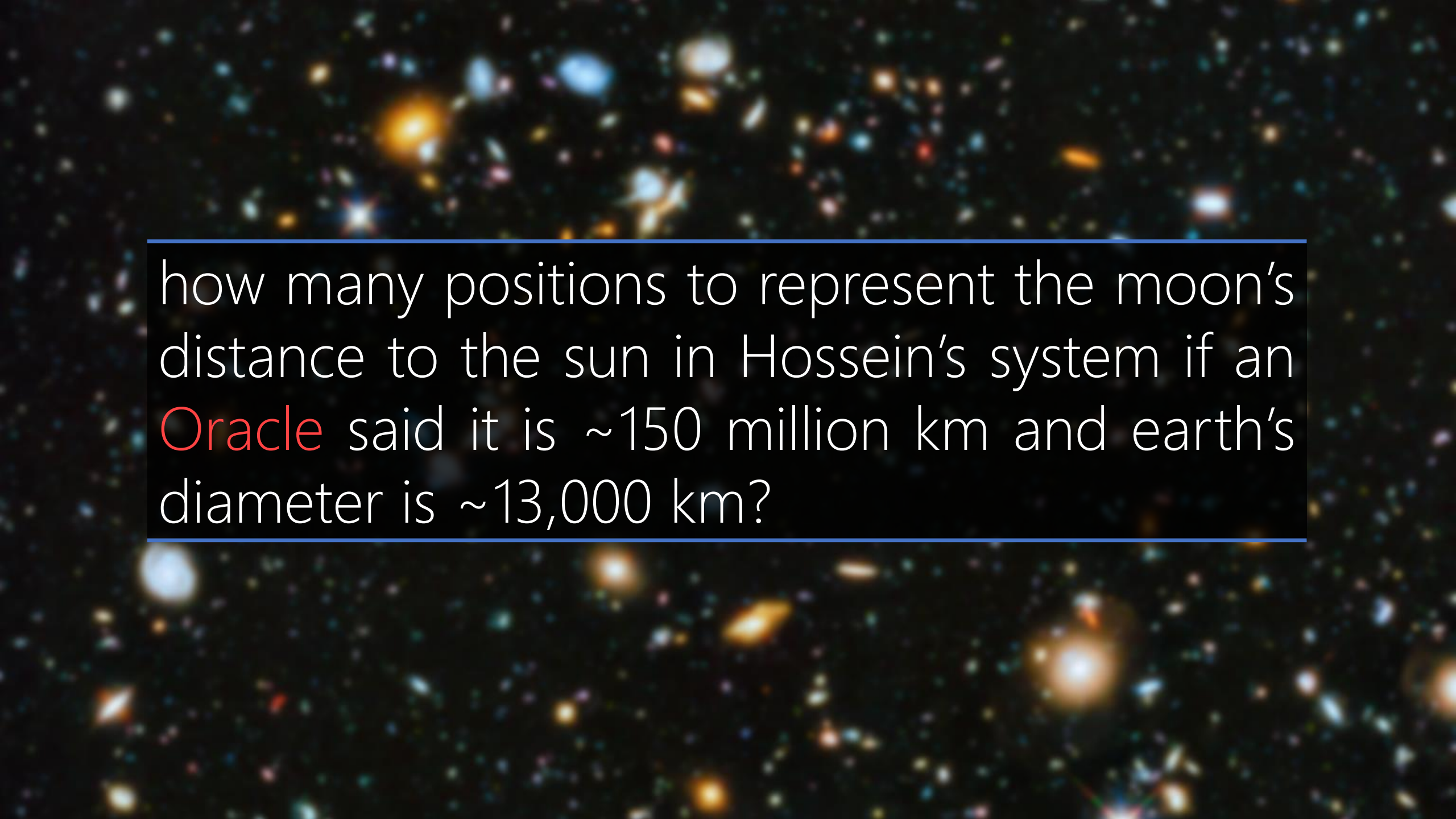
$$\text{Max} = N = 4n - 1$$

$$N + 1 = 4n$$

$$(N + 1) \div 4 = n$$

? positions





how many positions to represent the moon's distance to the sun in Hossein's system if an **Oracle** said it is ~150 million km and earth's diameter is ~13,000 km?

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$N = 12,000$

$n = (N+1) \div 4 = (12,000+1) \div 4 = \sim 3,000 \text{ positions}$

paper = $\sim 3,000 \text{ positions}$

$3,000 \div 3,000 = 1 \text{ pages}$



<div> <div>More Position</div> <div></div> </div>		0	0
		1	1
		2	2
		3	3
1 rounds of all (4) symbols: 1 × 4			
		3	0
		3	1
		3	2
		3	3
2 rounds of all (4) symbols: 2 × 4		3	0
		3	8

2 rounds of all (4) symbols: 2×4

3	3	0	8
3	3	1	9
3	3	2	10
3	3	3	11

3 rounds of all (4) symbols: 3×4

3	3	3	0	12
3	3	3	1	13
3	3	3	2	14
3	3	3	3	15

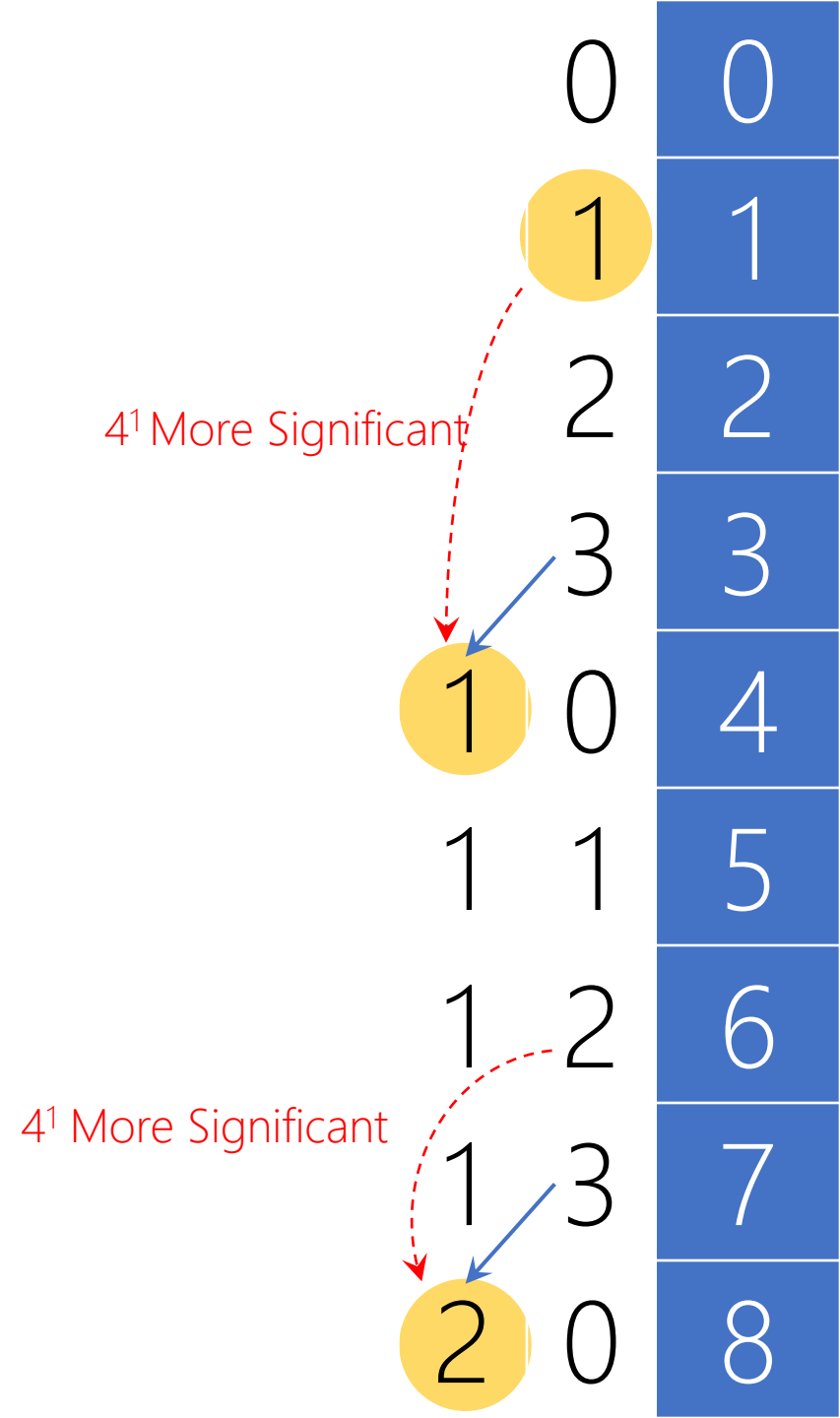
4 rounds of all (4) symbols: 4×4

3	3	3	3	0	16
---	---	---	---	---	----

More Significant Position
←

1 round of all (4) symbols = 1×4^1

2 rounds of all (4) symbols = 2×4^1



2 rounds of all (4) symbols = 2×4^1

2 0 8

2 1 9

2 2 10

4^1 More Significant

2 3 11

3 rounds of all (4) symbols = 3×4^1

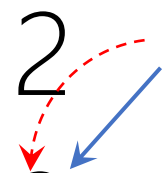
3 0 12

3 1 13

3 2 14

3 3 15

? ? 16



1 rounds of all (4) symbols in the 2nd significant position
 $= 4 \times 4^1 = 1 \times 4^2$

2	0	8
2	1	9
2	2	10
2	3	11
3	0	12
3	1	13
3	2	14
3	3	15
1	0	16

2	0	8
2	1	9
2	2	10
2	3	11
3	0	12
3	1	13
3	2	14
3	3	15
1	0	16

4^2 More Significant

1 rounds of all (4) symbols in the more significant position
 $= 4 \times 4^1 = 1 \times 4^2$

QUATERNARY SYSTEM

/kwaa·tur·neh·ree/

aka. Base-4, Radix-4

$(0,1,2,3)_4$

Hindu-Arabic Numerals
Originated in India
7th Century AD

More Significant Position
←

#Symbols=4
Radix-4
Base-4

4^0

0

$$0 \times 4^0 = 0$$

1

$$1 \times 4^0 = 1$$

2

$$2 \times 4^0 = 2$$

3

$$3 \times 4^0 = 3$$

More Significant Position
←

#Symbols=4
Radix-4
Base-4

4^1	4^0	
	0	$0 \times 4^0 = 0$
	1	$1 \times 4^0 = 1$
	2	$2 \times 4^0 = 2$
	3	$3 \times 4^0 = 3$
1	0	$1 \times 4^1 + 0 \times 4^0 = 4$
1	1	$1 \times 4^1 + 1 \times 4^0 = 5$
1	2	$1 \times 4^1 + 2 \times 4^0 = 6$
1	3	$1 \times 4^1 + 3 \times 4^0 = 7$
2	0	$2 \times 4^1 + 0 \times 4^0 = 8$

More Significant Position
←

#Symbols=4
Radix-4
Base-4

4^2	4^1	4^0	
	2	0	$2 \times 4^1 + 0 \times 4^0 = 8$
	2	1	$2 \times 4^1 + 1 \times 4^0 = 9$
	2	2	$2 \times 4^1 + 2 \times 4^0 = 10$
	2	3	$2 \times 4^1 + 3 \times 4^0 = 11$
	3	0	$3 \times 4^1 + 0 \times 4^0 = 12$
	3	1	$3 \times 4^1 + 1 \times 4^0 = 13$
	3	2	$3 \times 4^1 + 2 \times 4^0 = 14$
	3	3	$3 \times 4^1 + 3 \times 4^0 = 15$
1	0	0	$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$

More Significant Position
←

#Symbols=4
Radix-4
Base-4

4^3	4^2	4^1	4^0	
		2	0	$2 \times 4^1 + 0 \times 4^0 = 8$
		2	1	$2 \times 4^1 + 1 \times 4^0 = 9$
		2	2	$2 \times 4^1 + 2 \times 4^0 = 10$
		2	3	$2 \times 4^1 + 3 \times 4^0 = 11$
		3	0	$3 \times 4^1 + 0 \times 4^0 = 12$
		3	1	$3 \times 4^1 + 1 \times 4^0 = 13$
		3	2	$3 \times 4^1 + 2 \times 4^0 = 14$
		3	3	$3 \times 4^1 + 3 \times 4^0 = 15$
	1	0	0	$1 \times 4^2 + 0 \times 4^1 + 0 \times 4^0 = 16$

[illegible]

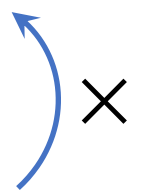
[illegible]

More Significant Position




4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	
3	0	3	0	2	1	3	1	

More Significant Position
←

4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	
3	0	3	0	2	1	3	1	
3×4^7	0×4^6	3×4^5	0×4^4	2×4^3	1×4^2	3×4^1	1×4^0	



4^7	4^6	4^5	4^4	4^3	4^2	4^1	4^0	
3	0	3	0	2	1	3	1	
3×4^7	0×4^6	3×4^5	0×4^4	2×4^3	1×4^2	3×4^1	1×4^0	Σ
								65,437

[illegible]

[illegible]

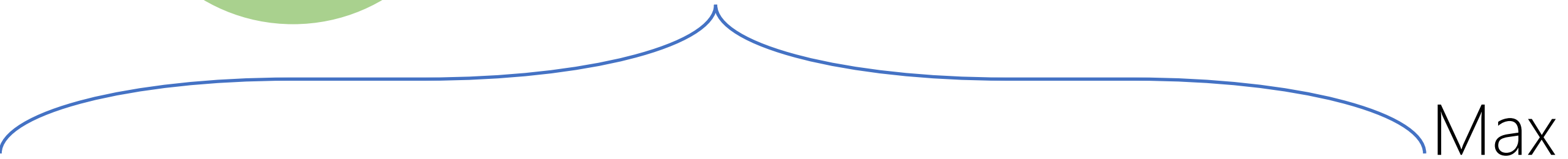
[illegible]

										Base-4	Hossein's Number System
3 0 3 0 2 1 3 1								65,437	-		
				3 3 3 3 1	1,021	17					
			3 3 3 3 3 2	4,094	22						
3 0 0 3 3 3 3 0		50,172	-								
3 3 3 3 3 3 3 3 3 3	1,048,575	39									

0	0	...				0	0	0
---	---	-----	--	--	--	---	---	---



n positions



3	3	...				3	3	3	?
---	---	-----	--	--	--	---	---	---	---

$$N = 3 \times 4^{n-1} + 3 \times 4^{n-2} + \dots + 3 \times 4^2 + 3 \times 4^1 + 3 \times 4^0$$

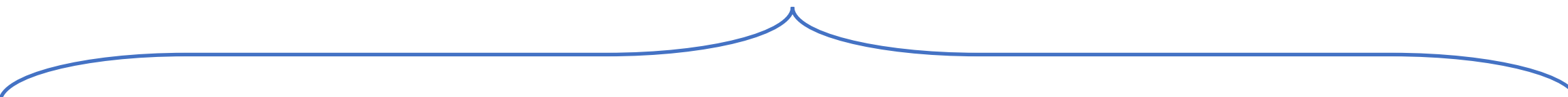
$$N = 3 \times (4^{n-1} + 4^{n-2} + \dots + 4^2 + 4^1 + 4^0)$$

$$N = 3 \times \left(\frac{4^n - 1}{4 - 1} \right)$$

$$N = 4^n - 1$$

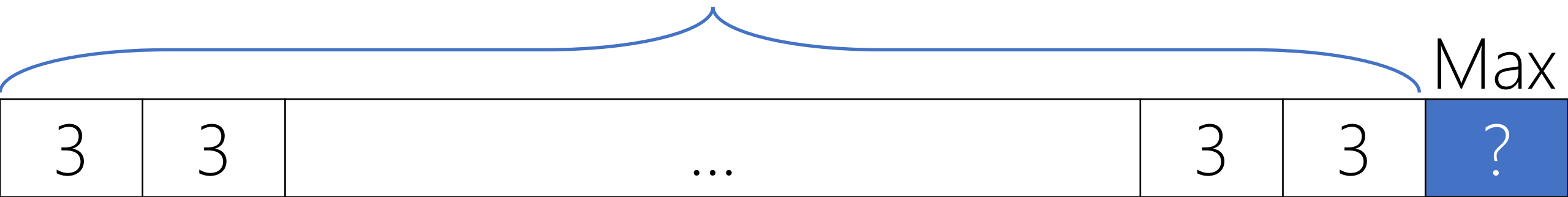
$4n - 1$
Hossein's
System

n positions

							Max		
4^{n-1}	4^{n-2}					4^2	4^1	4^0	
3	3	...				3	3	3	N

$$n = 10 \Rightarrow 4^{10} - 1 = 1,048,575$$

10 positions



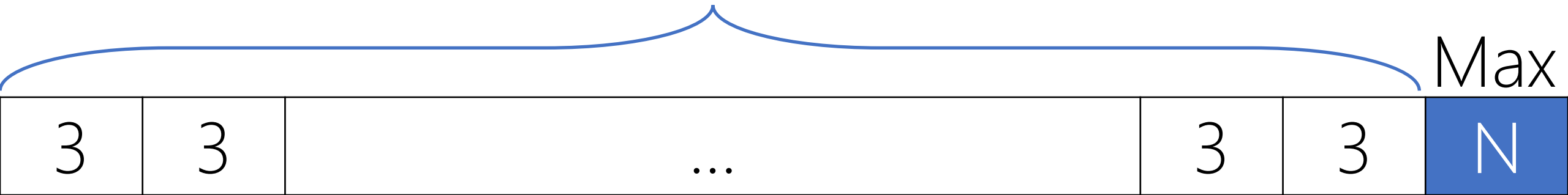
$$4^n - 1 = N$$

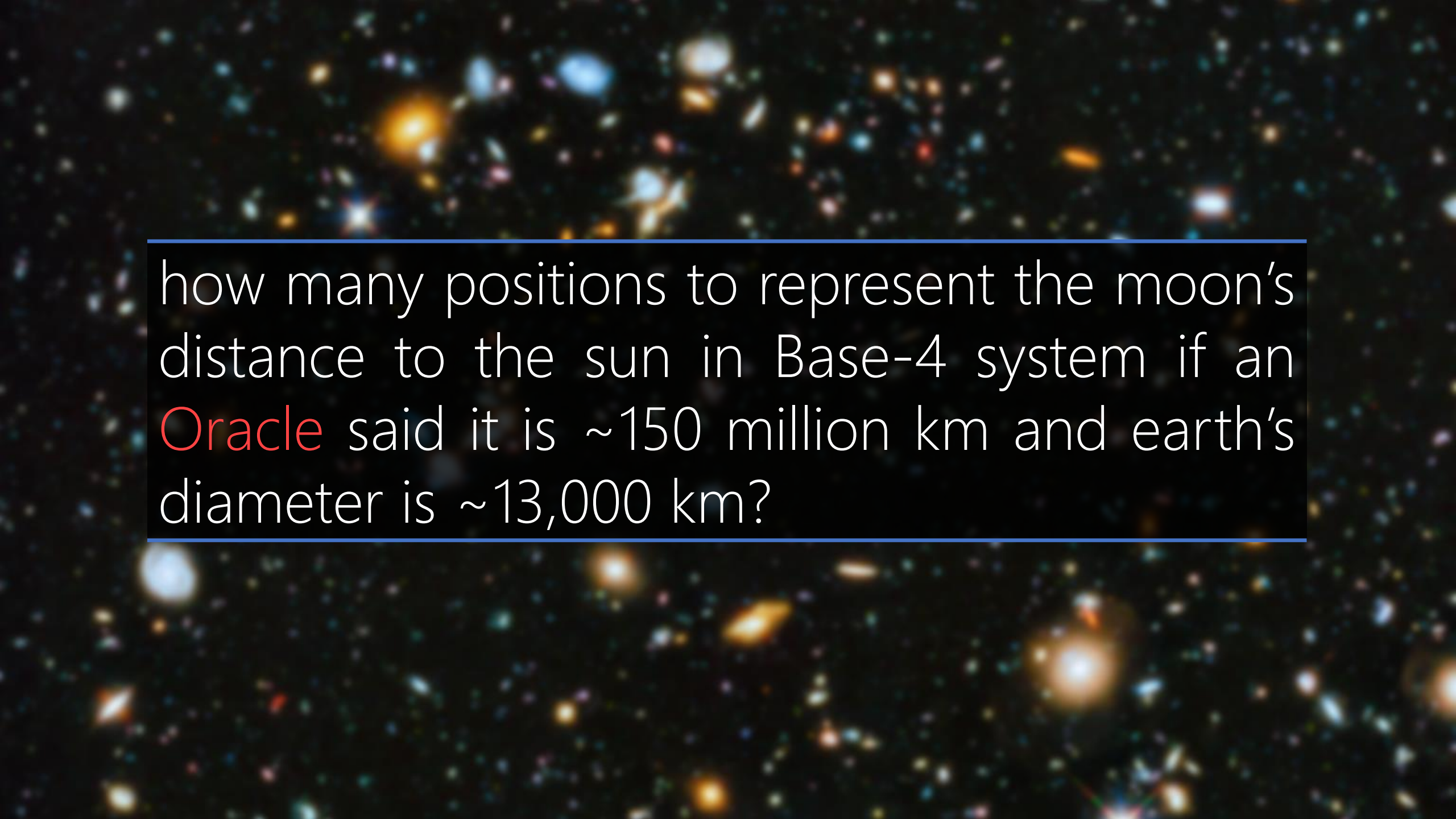
$$4^n = N + 1$$

$$\log_4 4^n = \log(N + 1)$$

$$n = \log_4(N + 1)$$

? positions





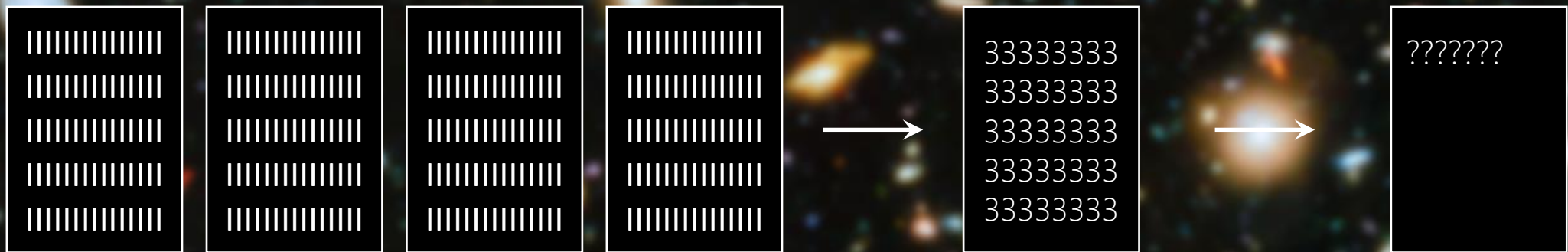
how many positions to represent the moon's distance to the sun in Base-4 system if an **Oracle** said it is ~150 million km and earth's diameter is ~13,000 km?

$\sim 150 \text{ million km} \div \sim 13,000 \text{ km} = \sim 12,000 \text{ Earth}$

$$N = 12,000$$

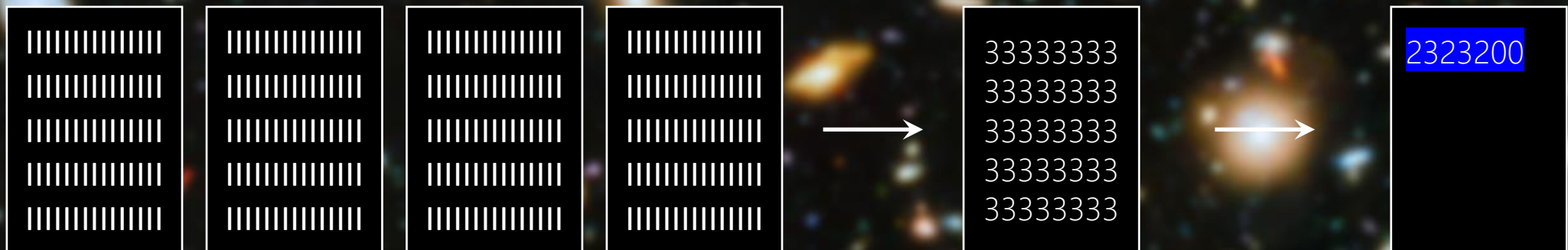
$$n = \text{Log}_4 (12,000+1) = \text{Log}_{10} 12,001 \div \text{Log}_{10} 4 = 4 \div 0.6 = 6.79$$

$\sim 7 \text{ positions}$



$$N = 12,000 \rightarrow (2323200)_4$$

We'll see how to convert from decimal to base-4 or any other number systems later. Stay tuned!





QUESTION