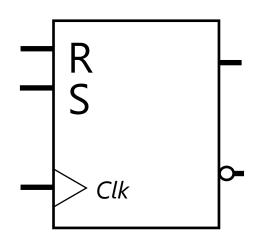
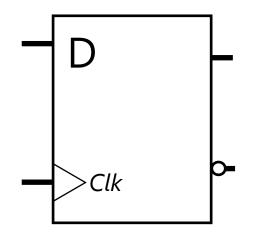
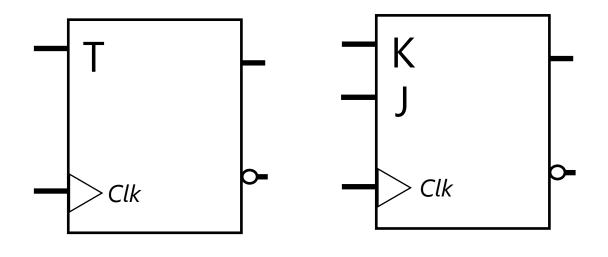


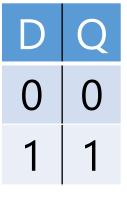
# Flip-Flop A single edge triggered latch







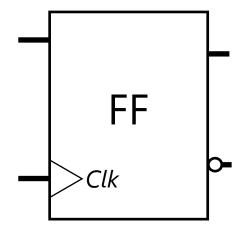
S	R	Q
0	0	$Q_{t}$
0	1	0
1	0	1
1	1	X



Т	Q
0	Q <sub>t</sub>
1	$Q'_t$

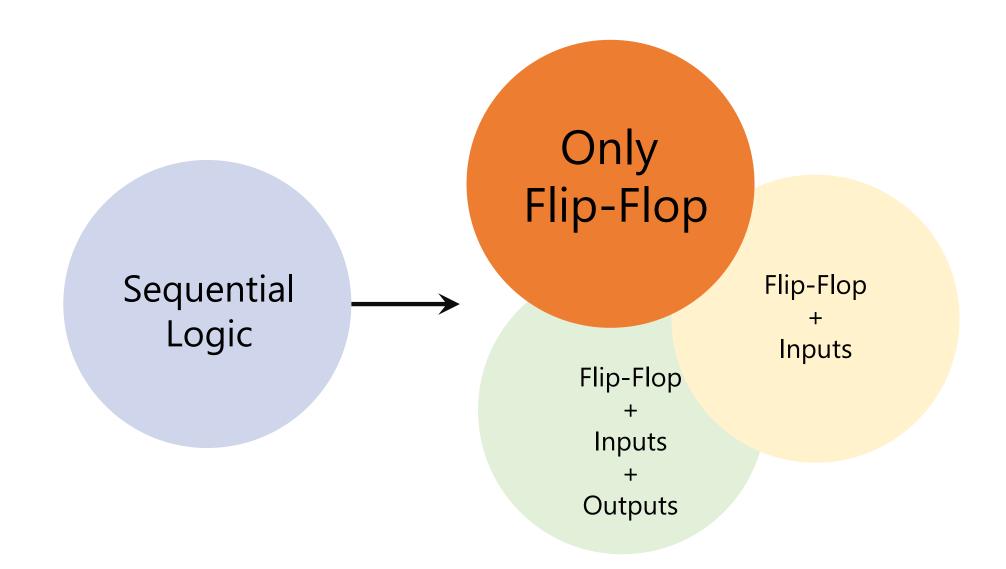
J	K	Q
0	0	$Q_t$
0	1	0
1	0	1
1	1	Q' <sub>t</sub>

### Single Edge Positive



Analysis: Given a sequential circuit, show the behavior vs.

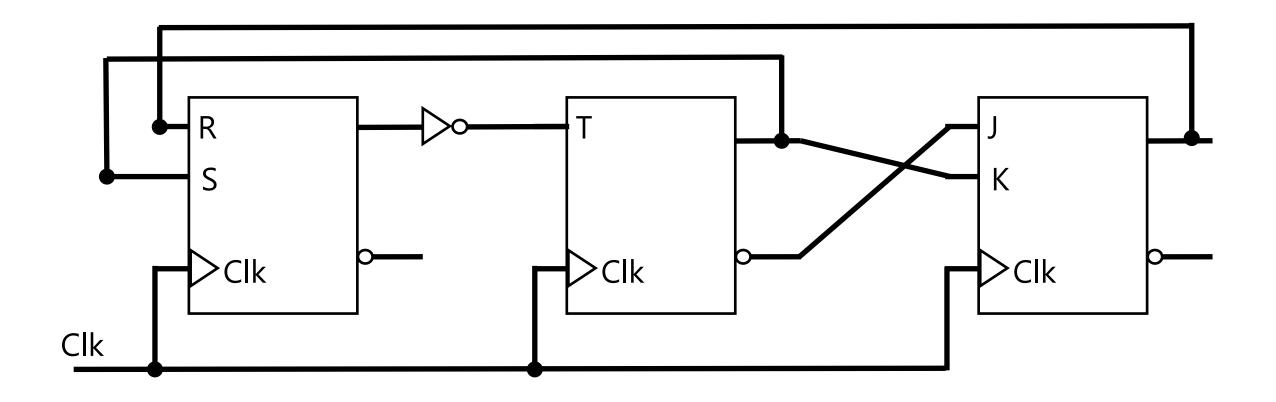
Design: Given a behavior, build the sequential circuit



#### Analysis (Recap)

- 0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
- 1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, ...)
- 2. What are the state combinations? 2#FF
- 3. Form "State" table:
  - a) Columns: for each FF, two columns:
    - o one for current state,
    - o one for next state
  - b) Rows: for each state combination
    - o In total: 2<sup>#FF</sup>
- 4. Fill the state table for next state columns based on:
  - a) the current state
  - b) the inputs to the FFs
- 5. Form State Transition Diagram
- 6. (Optional) Analyze paths and states in state transition diagram

### Analysis by an example

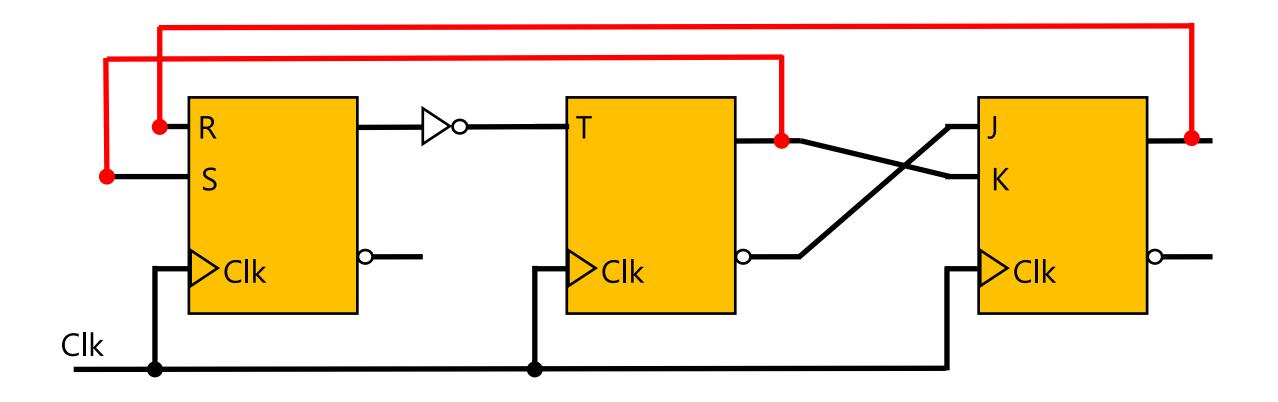


0) Is it sequential circuit?

At least one FF → Yes

At least one feedback → Yes

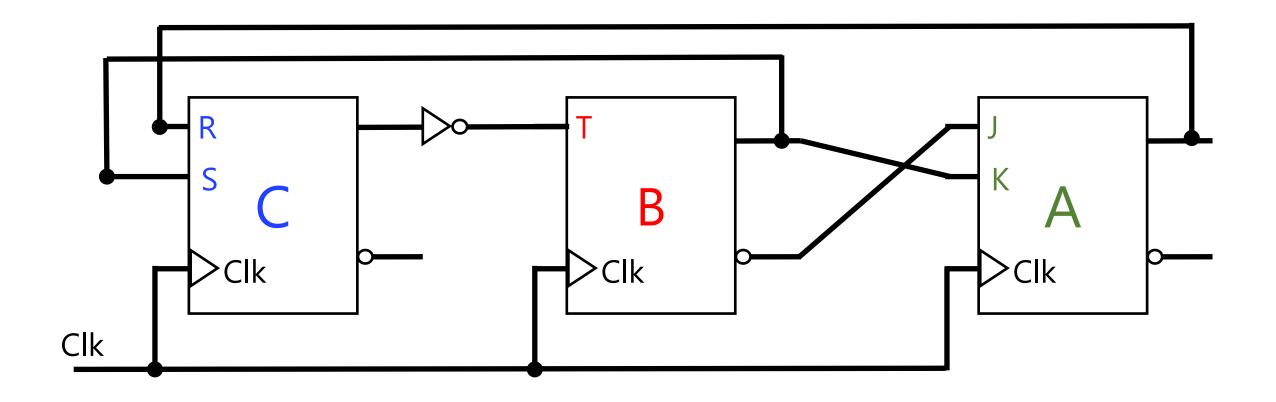
Otherwise → No



1) What are the FFs?

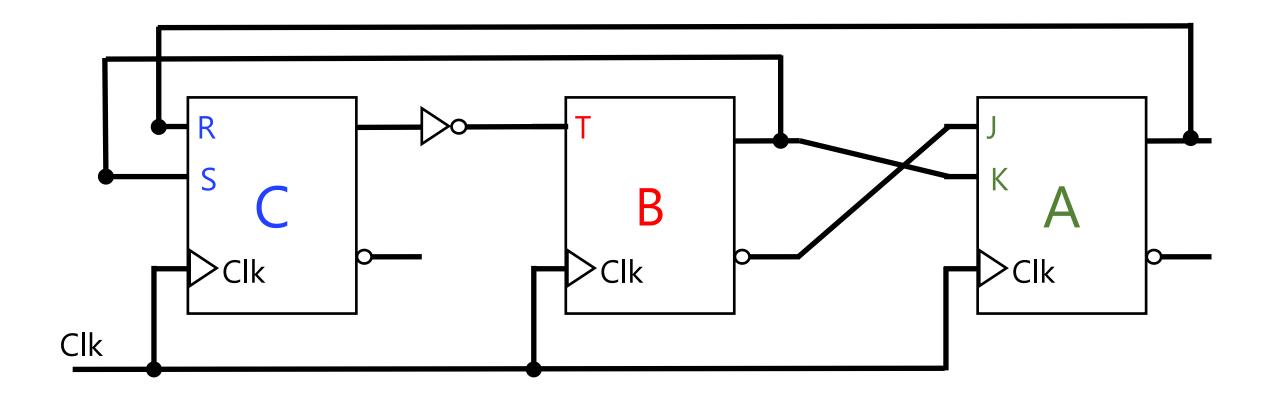
1.1. We pick a name for each FF

1.2. We note the type of FF



2) What are the state combinations (possibilities)?

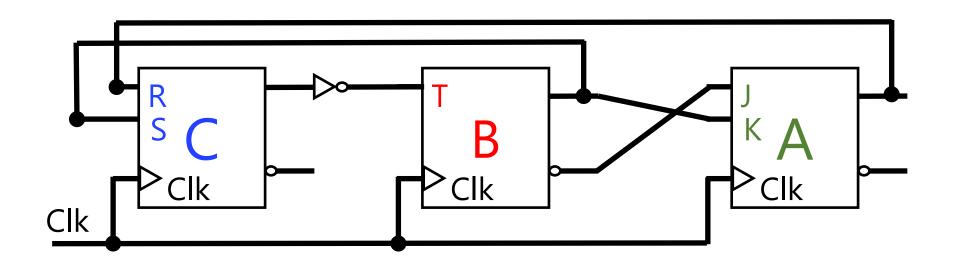
Each FF can have {0,1} states In total, 2#FFs



#FFs =  $3 \rightarrow 2^3 = 8$  combinations

3) Form a 'State' Table

- 3.1. For each FF, one column for current state
- 3.2. For each FF, one column for next state
- 3.3. For each combination of current state one row

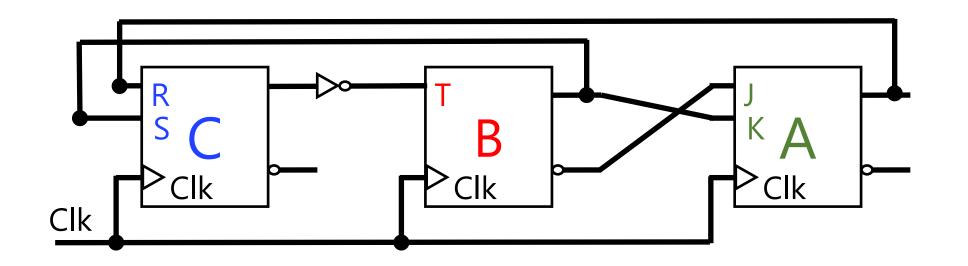


Q(T)			Q(T+1)			
С	В	Α	С	В	Α	

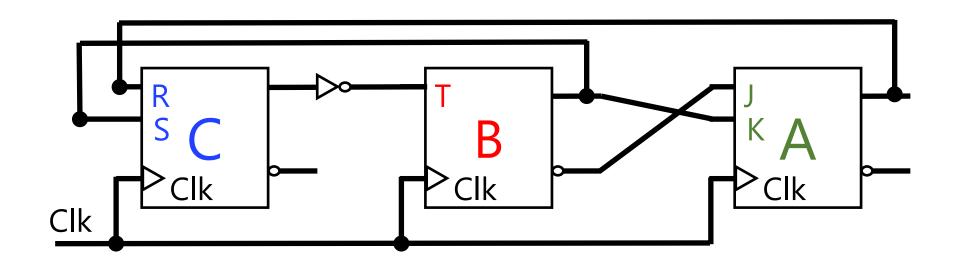
4) Fill the 'State' table

For each FF, we determine the next state based on

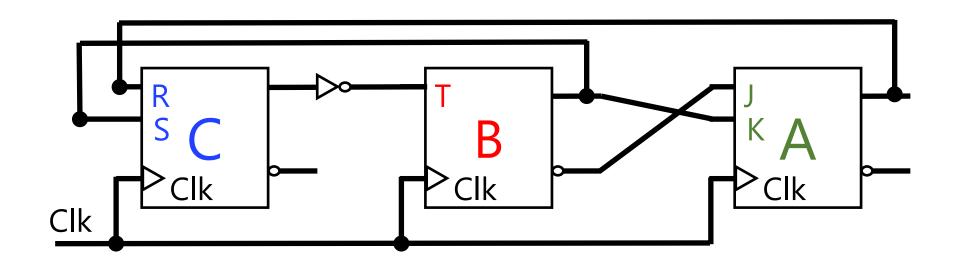
- l) current state
- II) the current value of inputs to the FF



	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0			$Q_{A}(T)=0$ $J_{A}=Q'_{B}(T)=1$ $K_{A}=Q_{B}(T)=0$ Set Action: 1
0	0	1			
0	1	0			
0	1	1			
1	0	0			



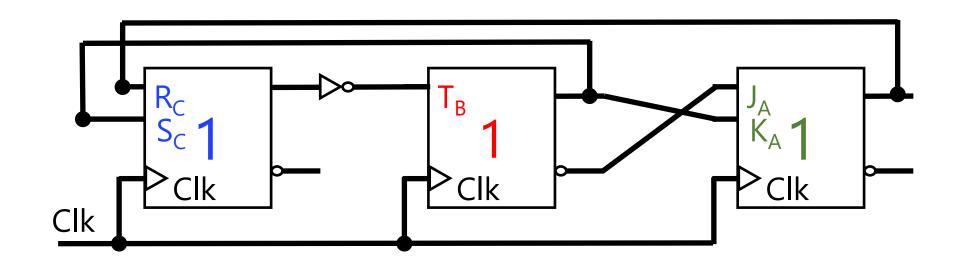
Q(T)			Q(T+1)			
С	В	Α	С	В	Α	
0	0	0		$Q_{B}(T)=0$ $T_{B}=Q'_{C}(T)=1$ Comp. $(Q_{B}(T))=1$	1	
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				



Q(T)			Q(T+1)			
С	В	А	С	В	А	
0	0	0	$Q_{C}(T)=0$ $R_{C}=Q_{A}(T)=0$ $S_{C}=Q_{B}(T)=0$ Store $Q_{C}(T)=0$	1	1	
0	0	1				
0	1	0				
0	1	1				
1	0	0				

Analysis
$$Q_{A}(T) = A, Q'_{A}(T) = A'$$

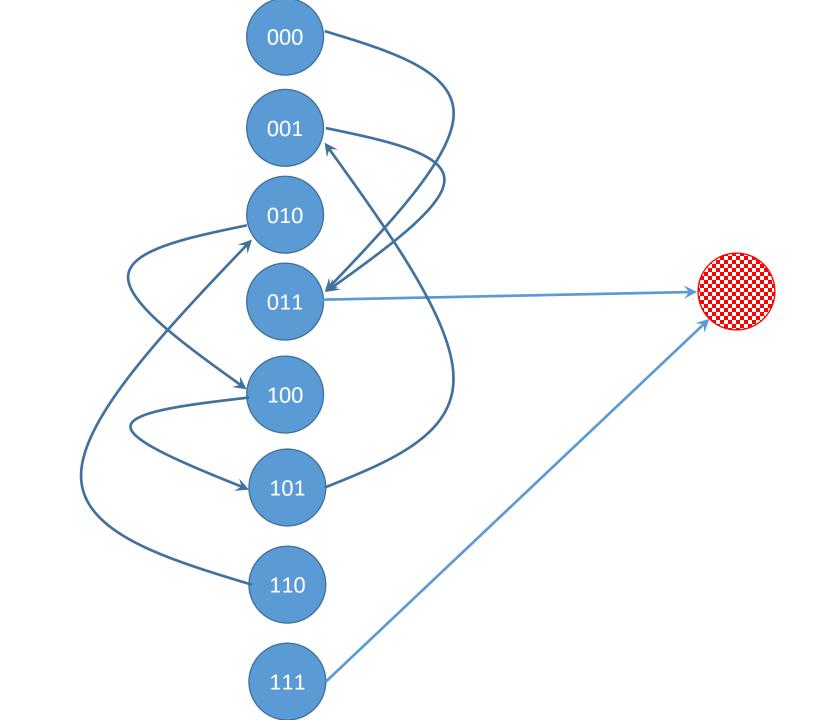
For simplicity, the current status of a FF can be assume to be as a binary variable

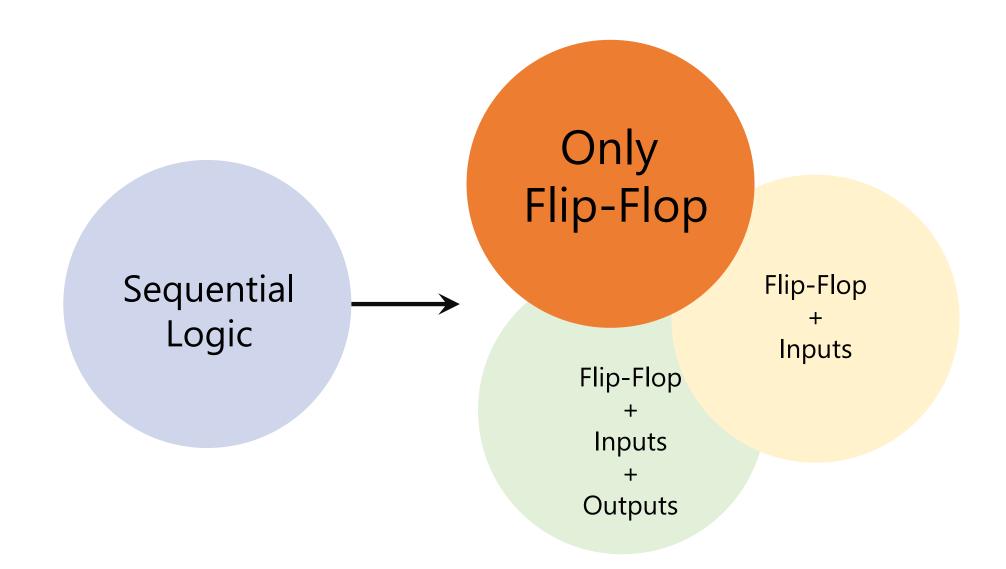


Q(T)		Q(T+1)			
С	В	А	С	В	А
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	X	0	0
1	0	0	1	0	1
1	0	1	0	0	1
1	1	0	0	1	0
1	1	1	X	1	0

5) State Transition Diagram

- 5.1. for each state combination (each row), a node
- 5.2. from one state (node) to another state, a directed edge





## Design by an example

### Counter Count from 0 to N

### Design

0. Do we need combinational logic or sequential logic?

Do we need memory?

# Counter Count from 0 to N $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow ... \rightarrow N-1 \rightarrow N$

# Counter Count from 0 to N $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow ... \rightarrow N-1 \rightarrow N$

At each step, we have to see at number we are and then move to next number:  $i \rightarrow i+1$ 

### Counter

### Count from 0 to N We need a storage to store current number.

We need a sequential circuit!

### Design

1. How many storage (flip-flops)?

Depends on the storage you need to store the current state in binary system!

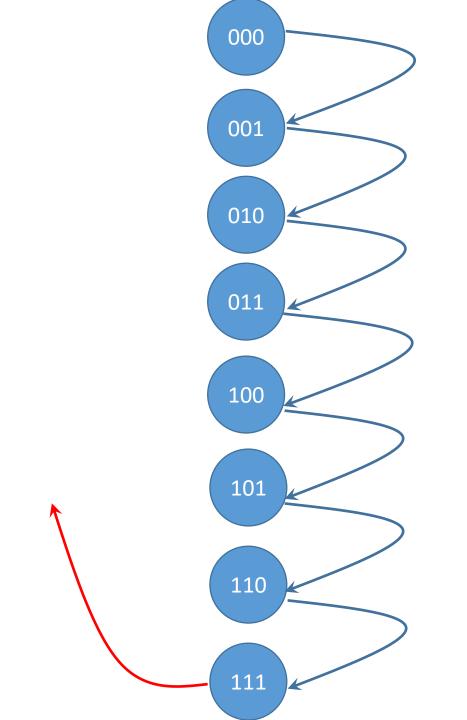
### Counter Count from 0 to N N = 7

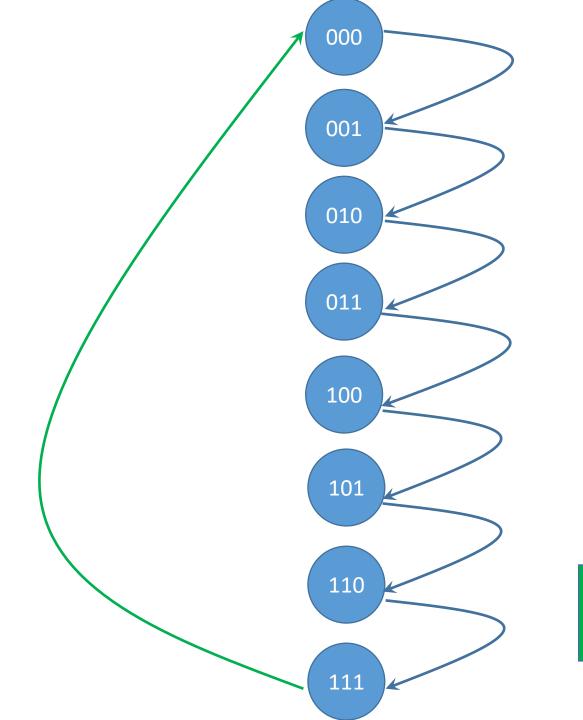
 $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7$   $000 \rightarrow 001 \rightarrow 010 \rightarrow 011 \rightarrow 100 \rightarrow 101 \rightarrow 110 \rightarrow 111$ For each intermediate state, we need 3 bits  $\rightarrow$  3 flip-flops

2. Form the state (transition) diagram

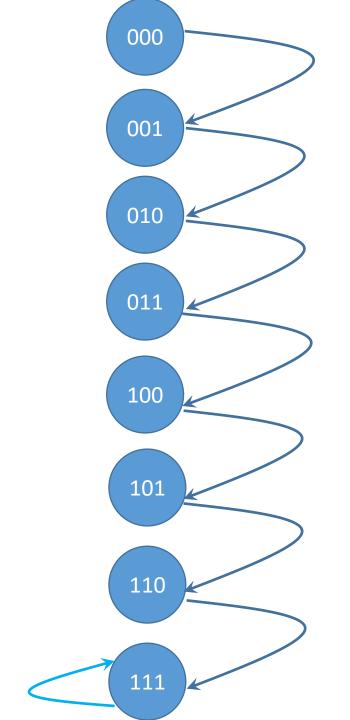
#### Same as analysis,

- For each state → one node
- For each state transition to next state → a directed edge





Loop to the beginning!

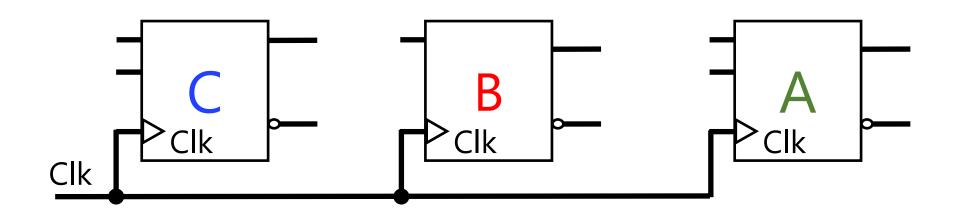


Stuck in 7
Just one time counter!

3. Form the state table

Same as analysis, two columns for each flip-flop (storage unit)

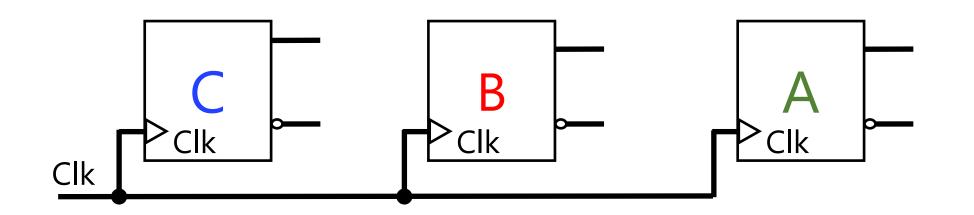
- a) One for current state Q(T)
- o) One for next state Q(T+1)



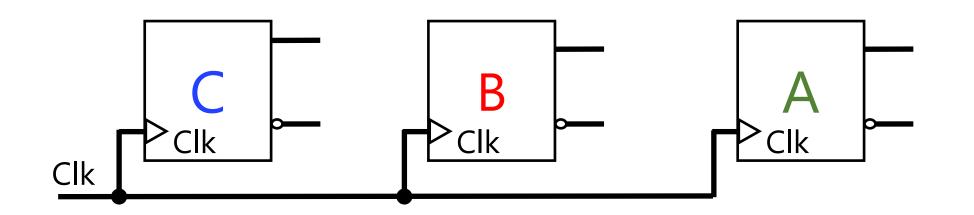
Q(T)			Q(T+1)			
С	В	Α	С	В	Α	

4. Fill the state table

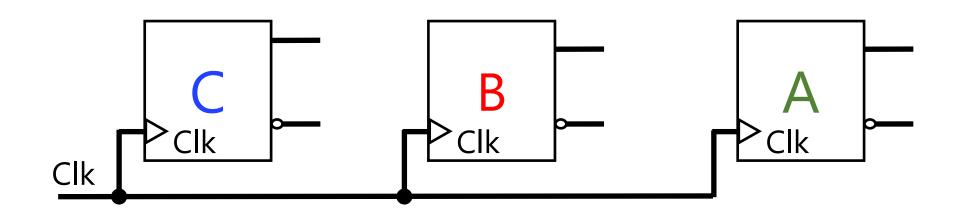
Unlike analysis, here we already know what is going to be the next state Q(T+1) based on current state Q(T)



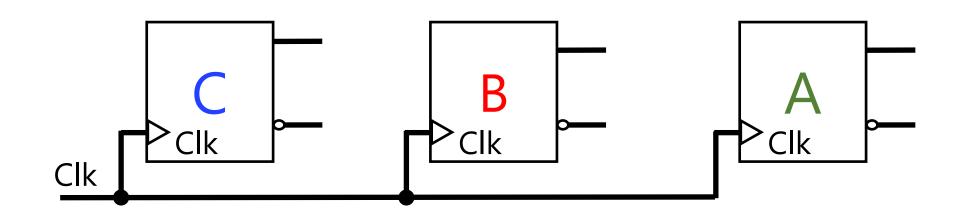
	Q(T)			Q(T+1)	
С	В	Α	С	В	Α
0	0	0	0	0	1
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



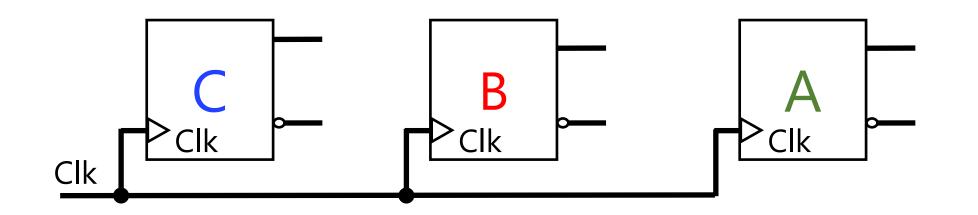
	Q(T)			Q(T+1)	
С	В	Α	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



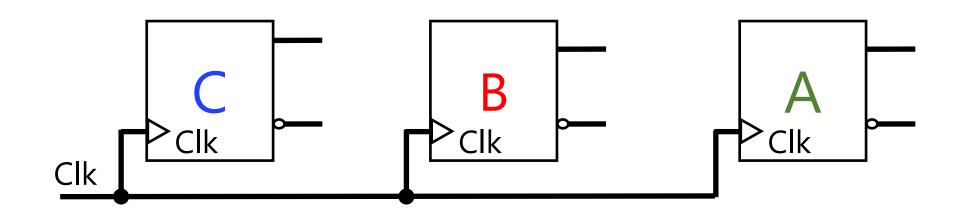
	Q(T)			Q(T+1)	
С	В	Α	С	В	Α
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



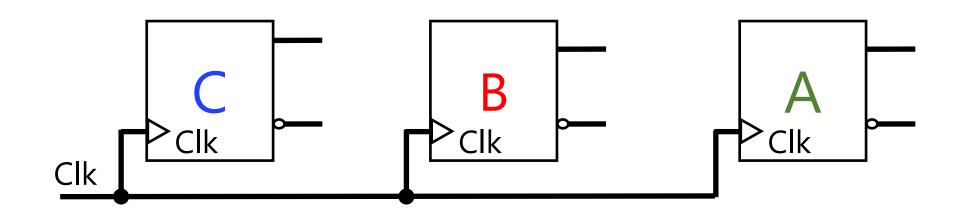
	Q(T)			Q(T+1)	
С	В	Α	С	В	Α
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0			
1	0	1			
1	1	0			
1	1	1			



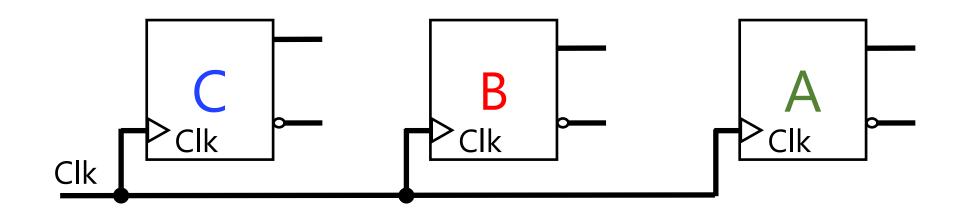
Q(T)				Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1			
1	1	0			
1	1	1			



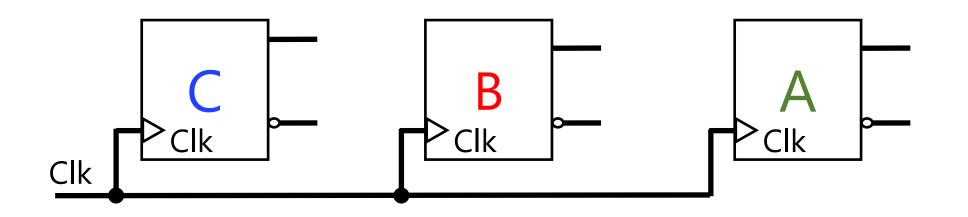
	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0			
1	1	1			



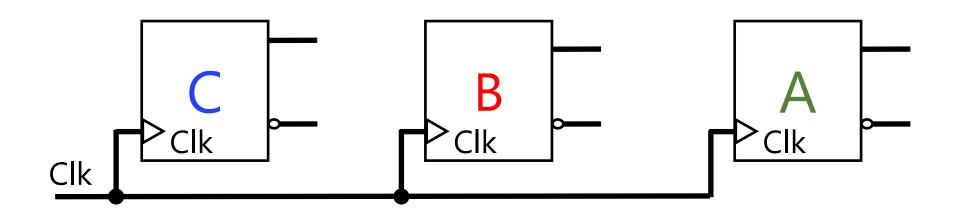
	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1			



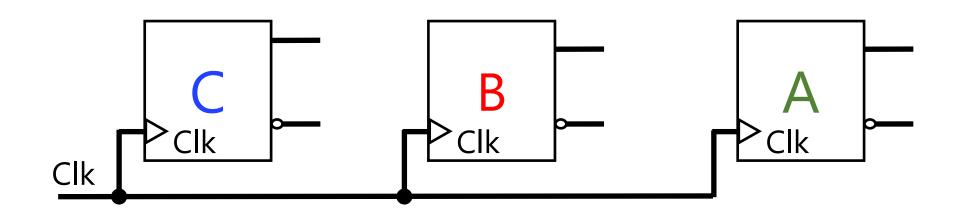
	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	?	?	?



	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	Lanca de Ale	1
1	0	1	1	Loop to the beginning	ne o
1	1	0	1	beginning	1
1	1	1	0	0	0



	Q(T)			Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	Church in 7	1
1	0	1	1	Stuck in 7 Just one time cou	interl 0
1	1	0	1		1
1	1	1	1	1	1



Q(T)				Q(T+1)	
С	В	А	С	В	А
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	Our Design Ch	oicel 1
1	0	1	1	Our Design Ch	0
1	1	0	1	1	1
1	1	1	0	0	0

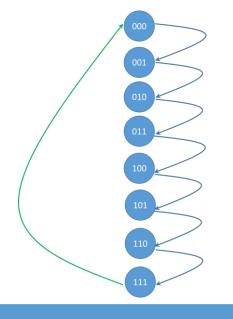
5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed

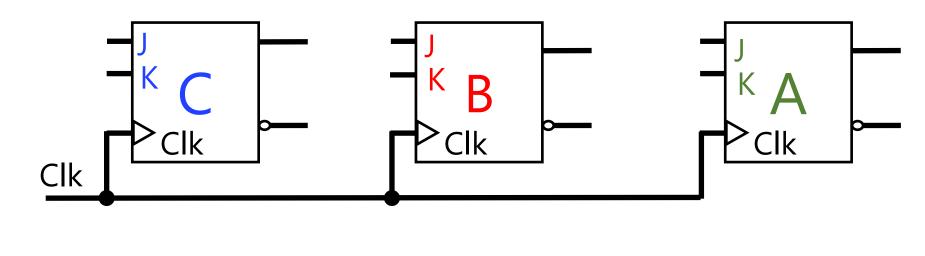
5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed

In terms of design, does <u>not</u> matter. In terms of efficiency, matters!

#### Counter

Count from 0 to N=7Let's select JK, the complete FF.





	Q(T)		Q(T+1)				
С	В	А	С	В	Α		
0	0	0	0	0	1		
0	0	1	0	1	0		
0	1	0	0	1	1		
0	1	1	1	0	0		
1	0	0	1	0	1		
1	0	1	1	1	0		
1	1	0	1	1	1		
1	1	1	0	0	0		

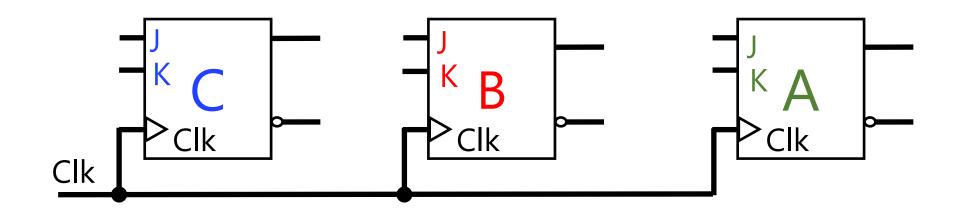
6. Boolean expression for the flip-flops' input? input equations, aka, *excitation* equations

# Counter Count from 0 to N=7

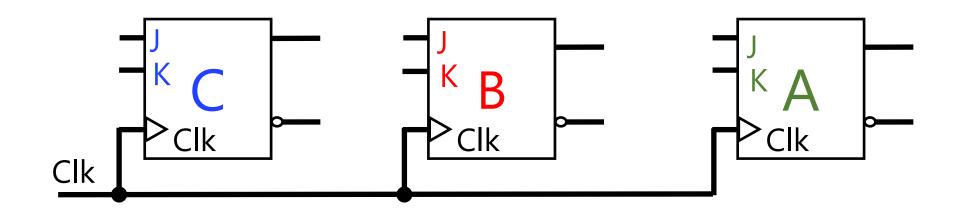
Α	$J_A=$	K <sub>A</sub> =
В	$J_{B}=$	$K_B =$
С	$J_{C}=$	$K_C =$

# Counter Count from 0 to N=7

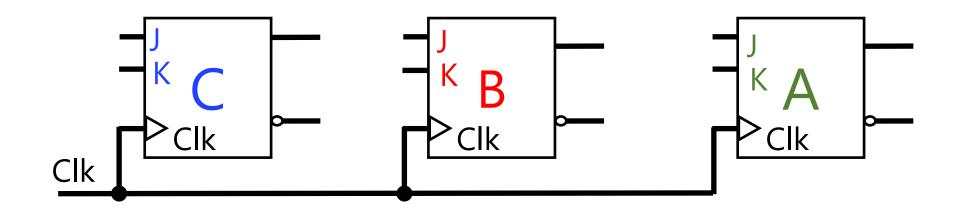
Α	$J_A =$	K <sub>A</sub> =
В	$J_B =$	$K_B =$
С	J <sub>C</sub> =	$K_C =$



	Q(T)		Q(T+1)				
С	В	Α	С	В	Α		
0	0	0	0	0	1		
0	0	1	0	1	0		
0	1	0	0	1	1		
0	1	1	1	0	0		
1	0	0	1	0	1		
1	0	1	1	1	0		
1	1	0	1	1	1		
1	1	1	0	0	0		

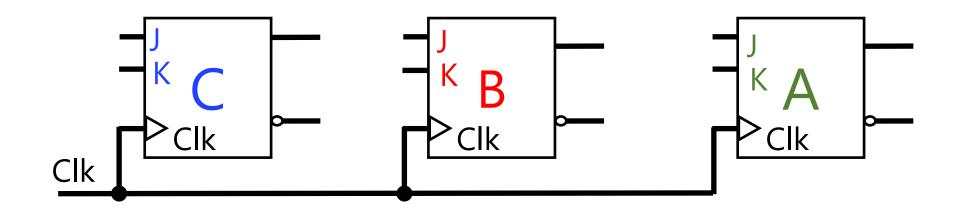


Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	А	Action	$J_A$	$K_A$
0	0	0	0	0	1			
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

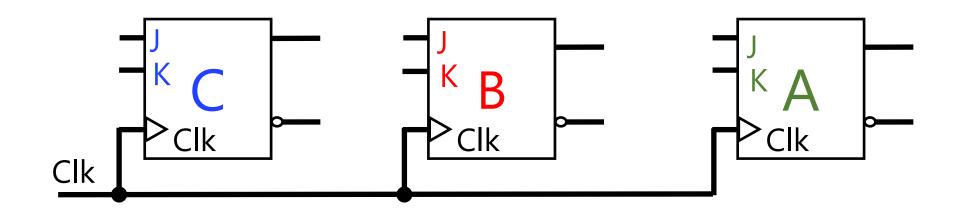


Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	А	Action	$J_A$	$K_A$
0	0	0	0	0	1	Set	1	0
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

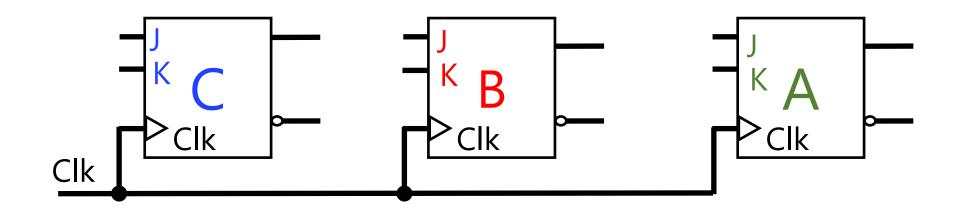
## OR



Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	А	Action	$J_A$	$K_A$
0	0	0	0	0	1	Comp	1	1
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

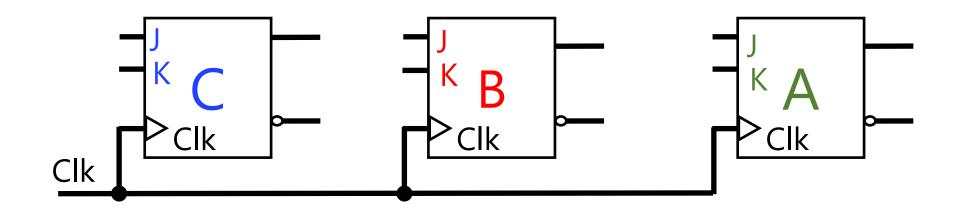


Q(T)			Q(T+1)			Not part of state table!		
С	В	А	C	В	Α	Action	$J_A$	K <sub>A</sub>
0	0	0	0	0	1	Set/Comp	1	0/1 <b>→</b> ×
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

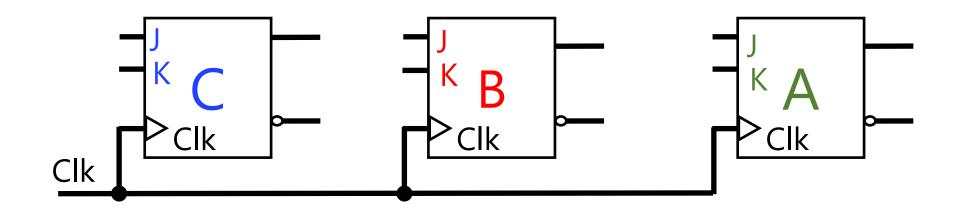


	Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	А	Action	$J_A$	$K_A$	
0	0	0	0	0	1	Set/Comp	1	X	
0	0	1	0	1	0	Reset	0	1	
0	1	0	0	1	1				
0	1	1	1	0	0				
1	0	0	1	0	1				
1	0	1	1	1	0				
1	1	0	1	1	1				
1	1	1	0	0	0				

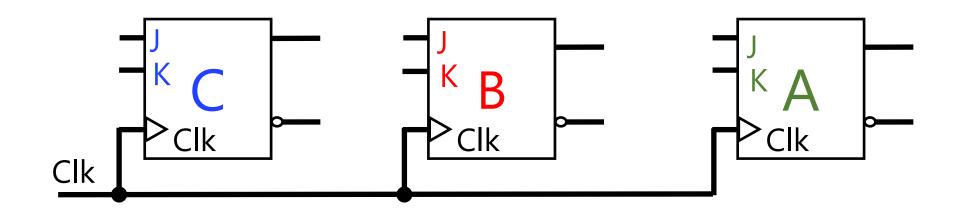
## OR



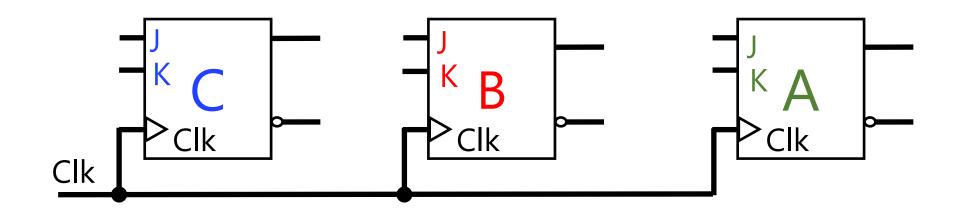
Q(T)			Q(T+1)			Not part of state table!		
С	В	Α	С	В	Α	Action	J <sub>A</sub>	K <sub>A</sub>
0	0	0	0	0	1	Set/Comp	1	X
0	0	1	0	1	0	Comp.	1	1
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



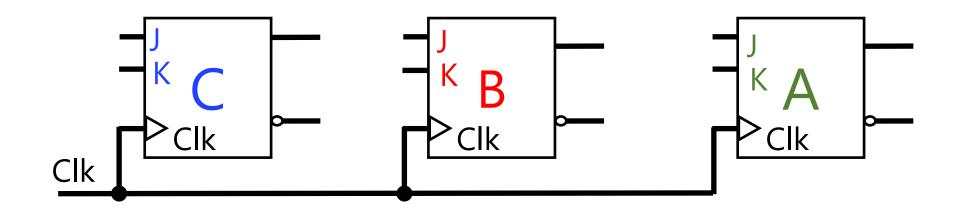
	Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	Α	Action	$J_A$	$K_A$	
0	0	0	0	0	1	Set/Comp	1	X	
0	0	1	0	1	0	Reset/Comp	X	1	
0	1	0	0	1	1				
0	1	1	1	0	0				
1	0	0	1	0	1				
1	0	1	1	1	0				
1	1	0	1	1	1				
1	1	1	0	0	0				



Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	Α	Action	$J_A$	$K_A$
0	0	0	0	0	1	Set/Comp	1	X
0	0	1	0	1	0	Reset/Comp	X	1
0	1	0	0	1	1	Set/Comp	1	X
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	Α	Action	$J_A$	$K_A$
0	0	0	0	0	1	Set/Comp	1	X
0	0	1	0	1	0	Reset/Comp	X	1
0	1	0	0	1	1	Set/Comp	1	X
0	1	1	1	0	0	Reset/Comp	X	1
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

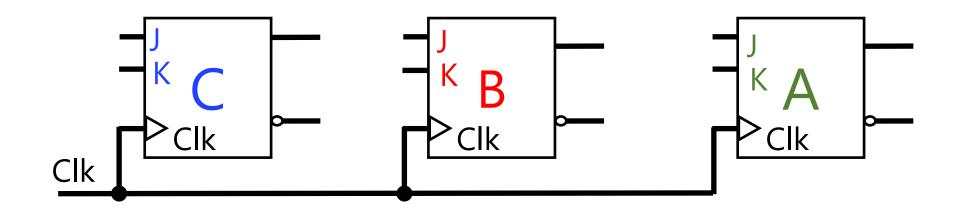


Q(T)			Q(T+1)			Not part of state table!		
С	В	А	С	В	А	Action	$J_A$	$K_A$
0	0	0	0	0	1	Set/Comp	1	X
0	0	1	0	1	0	Reset/Comp	X	1
0	1	0	0	1	1	Set/Comp	1	X
0	1	1	1	0	0	Reset/Comp	X	1
1	0	0	1	0	1	Set/Comp	1	X
1	0	1	1	1	0	Reset/Comp	X	1
1	1	0	1	1	1	Set/Comp	1	X
1	1	1	0	0	0	Reset/Comp	X	1

Flip-Flops

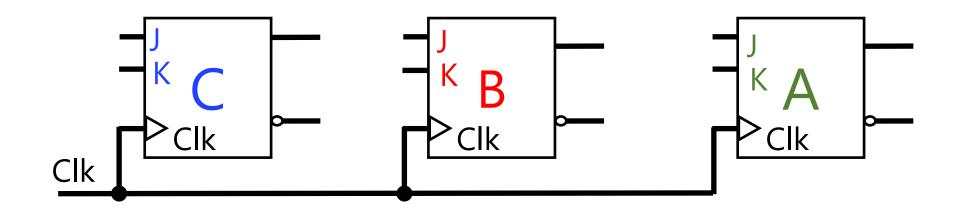
Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_B =$	$K_B =$
C	$J_{C}=$	$K_C =$

Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_{B}=$	$K_B =$
C	$J_{C}=$	$K_C =$

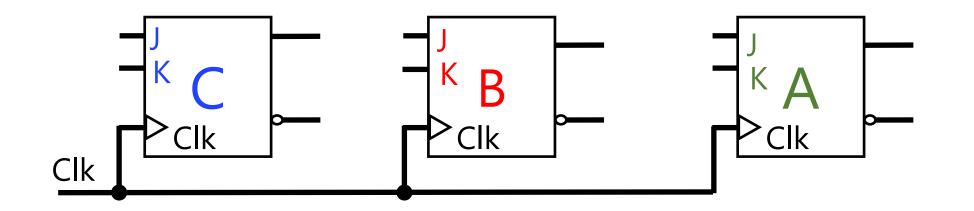


Q(T)			Q(T+1)			Not part of state table!		
С	В	Α	С	В	A	Action	$J_B$	K <sub>B</sub>
0	0	0	0	0	1	Store	0	0
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

### OR



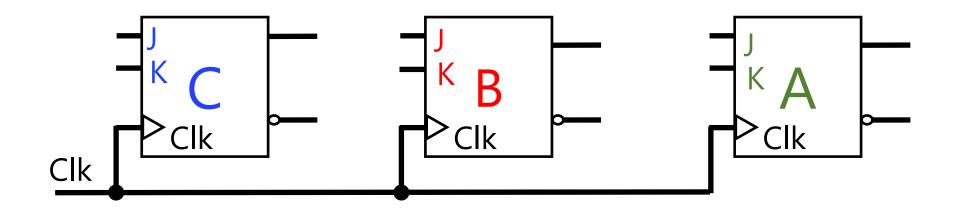
Q(T)			Q(T+1)			Not part of state table!		
С	В	Α	С	В	A	Action	$J_B$	$K_B$
0	0	0	0	0	1	Reset	0	1
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



Q(T)			Q(T+1)		Not part of state table!			
С	В	А	С	В	A	Action	$J_B$	$K_{B}$
0	0	0	0	<b>0</b>	1	Store/Reset	0	X
0	0	1	0	1	0	Set/Comp	1	X
0	1 —	0	0	1	1	Store/Set	X	0
0	1 —	1	1	<b>0</b>	0	Reset/Comp	X	1
1	0	0	1	<b>0</b>	1	Store/Reset	0	X
1	0	1	1	1	0	Set/Comp	1	X
1	1 —	0	1	1	1	Store/Set	X	0
1	1	1	0	<b>0</b>	0	Reset/Comp	X	1

Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_B = F(C,B,A) = \sum (1,5) + d(2,3,6,7)$	$K_B = F(C,B,A) = \sum (3,7) + d(0,1,4,5)$
С	$J_{C}=$	$K_C =$

Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_B = F(C,B,A) = \sum (1,5) + d(2,3,6,7)$	$K_B = F(C,B,A) = \sum (3,7) + d(0,1,4,5)$
С	$J_{C}=$	$K_C =$



Q(T)			Q(T+1)			Not part of state table!		
С	В	Α	С	В	A	Action	$J_{C}$	K <sub>C</sub>
0 /	0	0	<b>0</b>	0	1	Store/Reset	0	X
0	0	1	<b>0</b>	1	0	Store/Reset	0	X
0	1	0	0	1	1	Store/Reset	0	X
0	1	1	1	0	0	Comp/Set	1	X
1 —	0	0	1	0	1	Store/Set	X	0
1	0	1	1	1	0	Store/Set	X	0
1	1	0	1	1	1	Store/Set	X	0
1	1	1	<b>0</b>	0	0	Comp/Reset	X	1

Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_B = F(C,B,A) = \sum (1,5) + d(2,3,6,7)$	$K_B = F(C,B,A) = \sum (3,7) + d(0,1,4,5)$
C	$J_C = F(C,B,A) = \sum (3) + d(4,5,6,7)$	$K_C = F(C,B,A) = \sum (7) + d(0,1,2,3)$

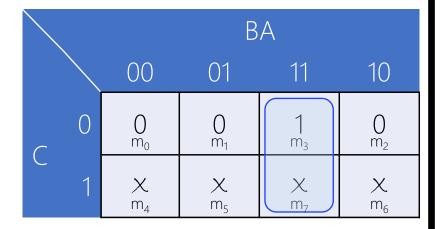
### Design

7. Minimization of input (excitation) equations

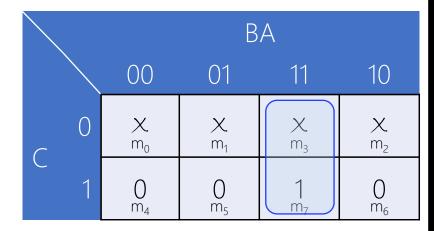
### Counter

# Count from 0 to N=73-Variable K-Map

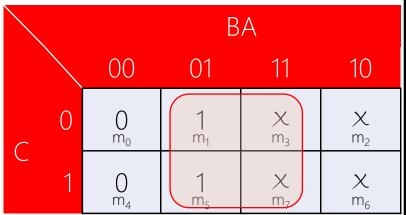
Α	$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$
В	$J_B = F(C,B,A) = \sum (1,5) + d(2,3,6,7)$	$K_B = F(C,B,A) = \sum (3,7) + d(0,1,4,5)$
С	$J_C = F(C,B,A) = \sum(3) + d(4,5,6,7)$	$K_C = F(C,B,A) = \sum (7) + d(0,1,2,3)$



$$J_C = F(C,B,A) = \sum (3) + d(4,5,6,7)$$
  
 $J_C = BA$ 

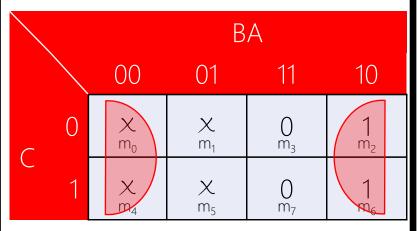


$$K_C = F(C,B,A) = \sum (7) + d(0,1,2,3)$$
  
 $K_C = BA$ 

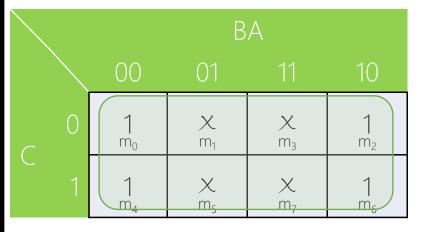


$$J_B = F(C,B,A) = \sum (1,5) + d(2,3,6,7)$$

$$J_B = A$$

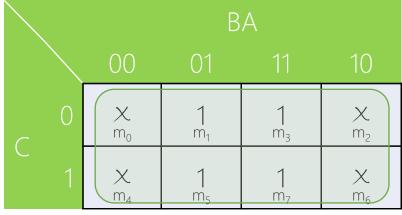


$$K_B = F(C,B,A) = \sum (3,7) + d(0,1,4,5)$$
  
 $K_B = A'$ 



$$J_A = F(C,B,A) = \sum (0,2,4,6) + d(1,3,5,7)$$

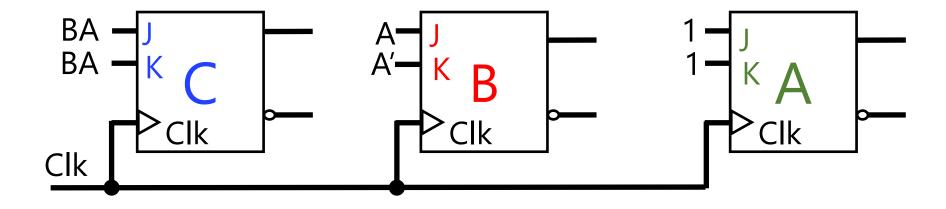
$$J_A = 1$$

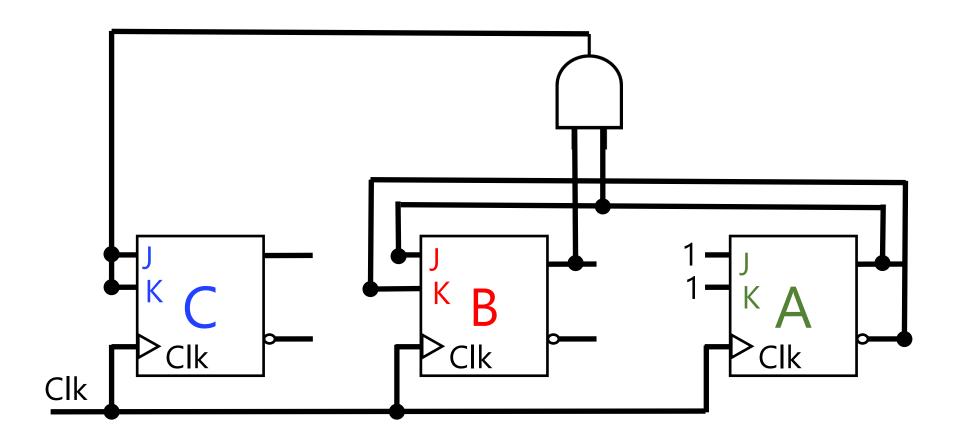


$$K_A = F(C,B,A) = \sum (1,3,5,7) + d(0,2,4,6)$$
  
 $K_A = 1$ 

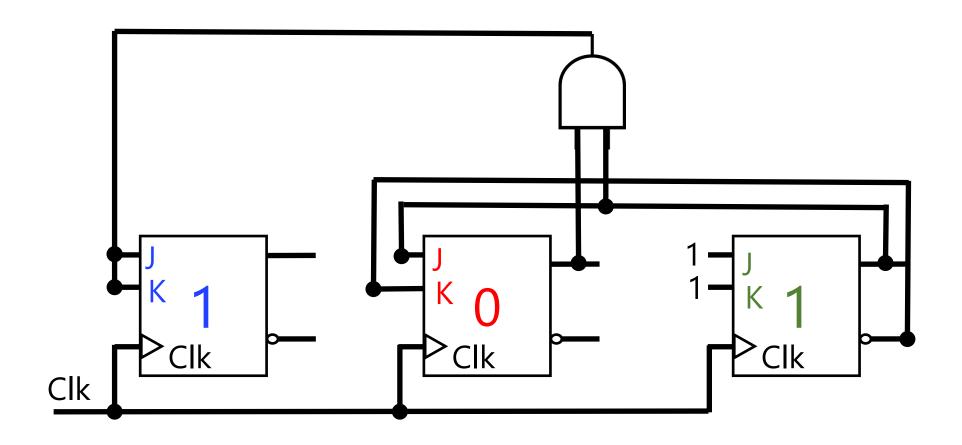
## Design

8. Draw/Sketch Logic Circuit

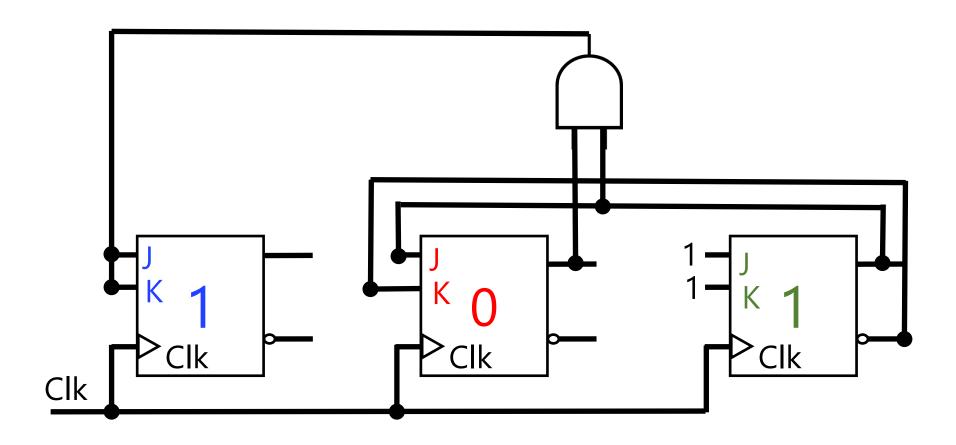




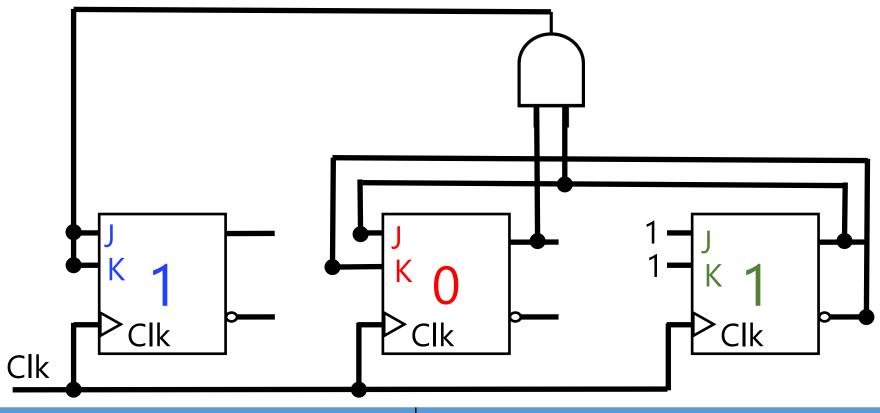
Design
9. (Optional) Test



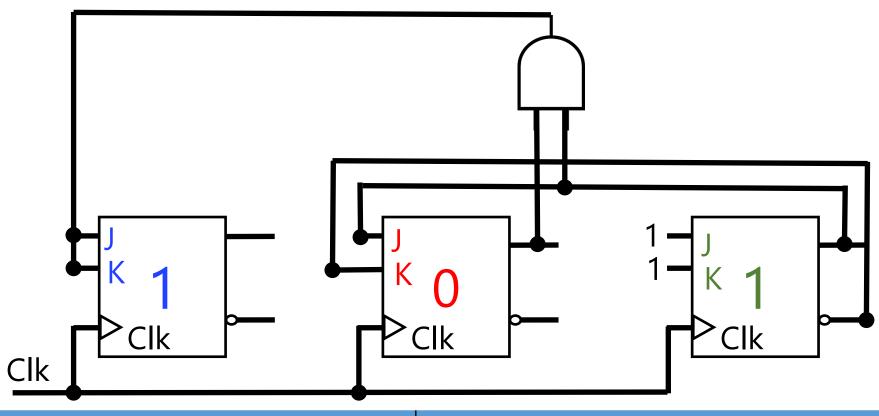
Q(T)			Q(T+1)			
С	В	Α	С	В	Α	
1	0	1	?	?	?	
5 → ?						



Q(T)			Q(T+1)			
С	В	Α	С	В	Α	
1	0	1	?	?	?	
5 → ?						

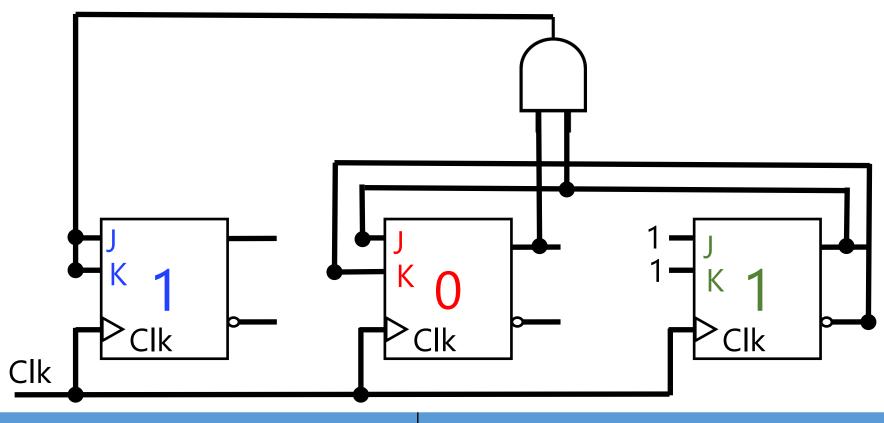


Q(T)			Q(T+1)			
С	В	Α	С	В	Α	
1	0	1	?	?	$A=1, J_A=1, K_A=1$	
					Comp. → 0	

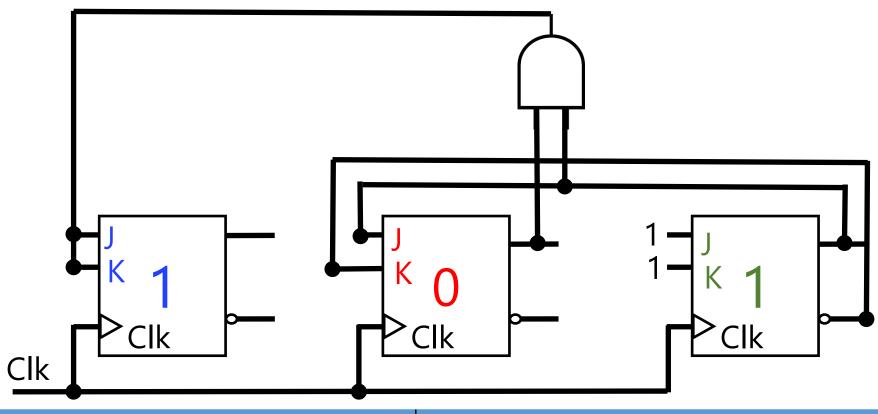


	Q(T)			Q(T+1)		
	С	В	Α	С	В	Α
	1	0	1	?	?	0

 $5 \rightarrow ?$ 

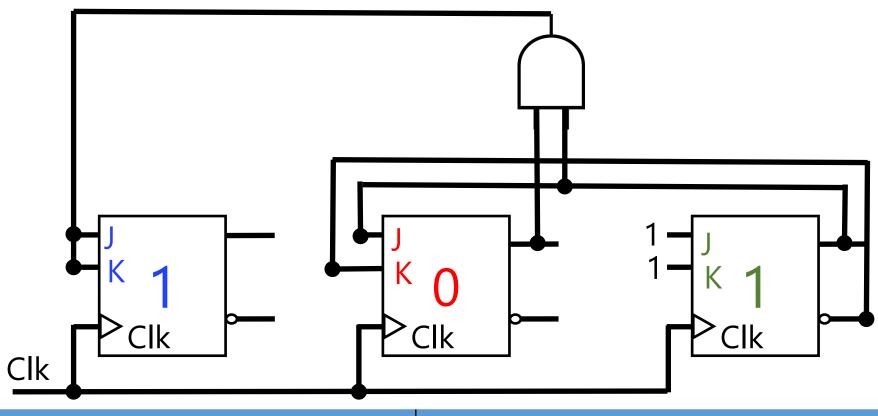


Q(T)			Q(T+1)			
С	В	А	С	В	Α	
1	0	1	?	B=0, $J_B=A=1$ , $K_B=A'=0$  Set $\rightarrow$ 1	0	



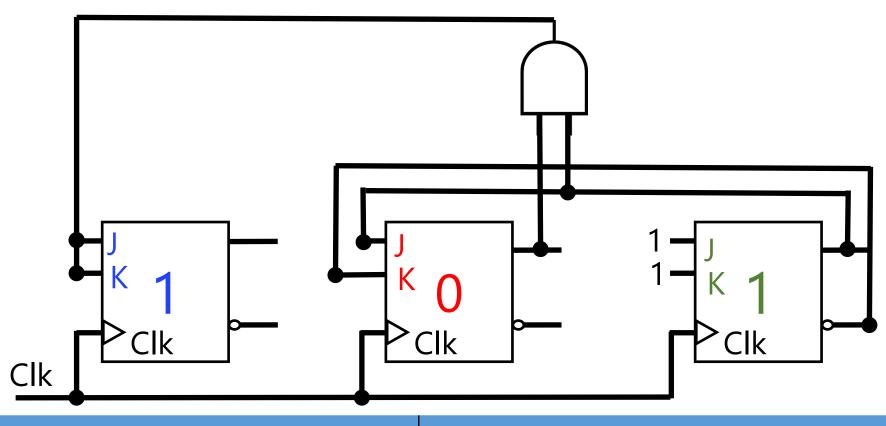
Q(T)			Q(T+1)				
	С	В	Α	С	В	А	
	1	0	1	?	1	0	

 $5 \rightarrow ?$ 

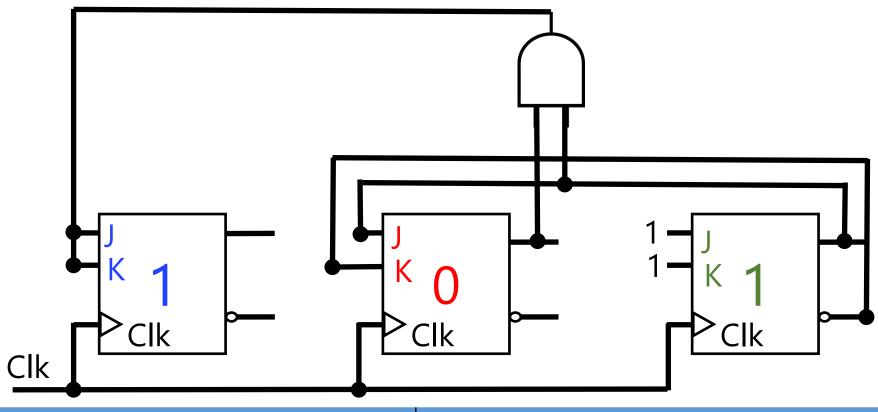


Q(T)			Q(T+1)		
С	В	Α	С	В	А
1	0	1	?	1	0

 $5 \rightarrow ?$ 



Q(T)			Q(T+1)		
С	В	Α	С	В	А
1	0	1	C=1, $J_C = BA = 01 = 0$ $K_C = BA = 01 = 0$  Store $\rightarrow$ 1	1	0



Q(T)			Q(T+1)				
С	В	Α	С	В	Α		
1	0	1	1	1	0		

 $5 \rightarrow 6$ 

#### Design (Recap)

- 0. Do we need combinational logic or sequential logic? Do we need memory?
- 1. How many storage (flip-flops)? #FF
- 2. Form the state (transition) diagram
- 3. Form the state table
- 4. Fill the state table
- 5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
- 6. Input (excitation) equations for each FF
- 7. Minimization of input (excitation) equations
- 8. Draw/Sketch Logic Circuit
- 9. (Optional) Test

#### Design (Recap)

- 0. Do we need combinational logic or sequential logic? Do we need memory?
- 1. How many storage (flip-flops)? #FF
- 2. Form the state (transition) diagram
- 3. Form the state table
- 4. Fill the state table
- 5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
- 6. Input (excitation) equations for each FF
- 7. Minimization of input (excitation) equations
- 8. Draw/Sketch Logic Circuit
- 9. (Optional) Test

#### Design (Advanced)

- 0. Do we need combinational logic or sequential logic? Do we need memory?
- 1. How many storage (flip-flops)? #FF
- 2. Form the state (transition) diagram
- 2.1. State Reduction

Theory of Automata

COMP-2140: Computer Languages, Grammars, and Translators

- 3. Form the state table
- 4. Fill the state table
- 5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
- 6. Input (excitation) equations for each FF
- 7. Minimization of input (excitation) equations
- 8. Draw/Sketch Logic Circuit
- 9. (Optional) Test

