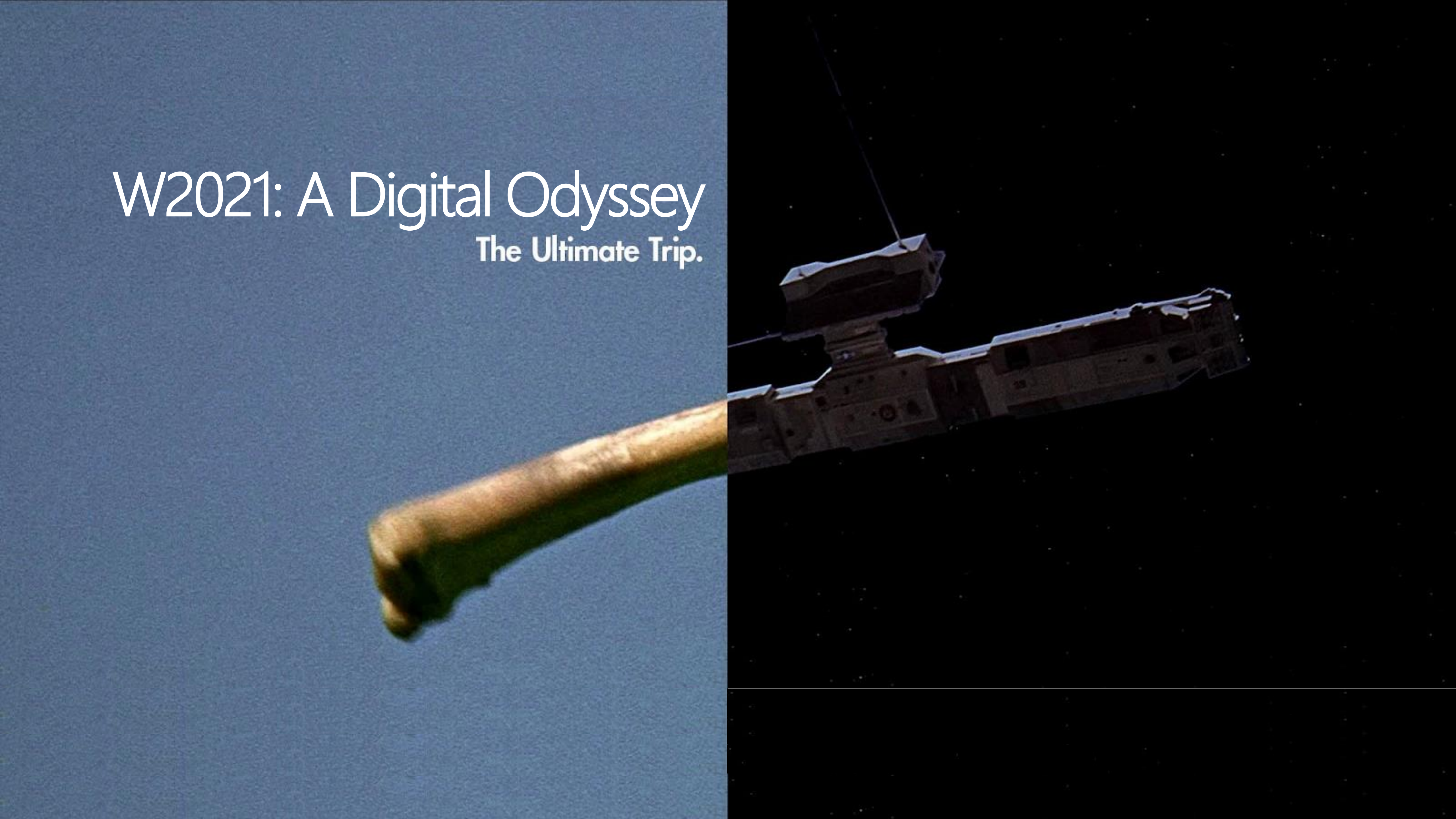
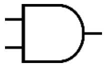
W2021: A Digital Odyssey

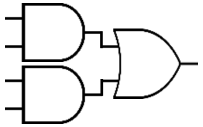


Number Systems | $(12)_{10} \rightarrow (1100)_2$

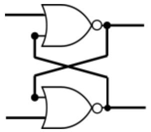
| Logic Gates |



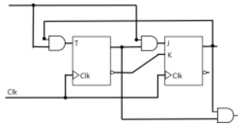
| Combinational Logic |



| Flip-Flops |



| Sequential Logic |



| Computer Systems

MINIMIZATION

aka. Simplification

MINIMIZATION

Number of Gates

Number of Inputs (2-input vs 4-input)

Number of Interconnections

Propagation Time

Cost of Gates

Circuit Area

. . .

A circuit may not satisfy all due to conflicting constraints!

MINIMIZATION

I) Boolean Algebra (algebraically)

- Needs to be smart. It is hard due to guesswork (which rules to apply?)
- If the number of variables (ABCDEF…) and/or number of minterms (MAXTERMS) grows
- No Algorithm
- Is the result minimal?!

MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

MINIMIZATION

II) Map (Karnaugh map, K-map)

aka. Graphical Manipulation

Algorithm; Straightforward, up to six variables

Result is always minimal

Maurice Karnaugh

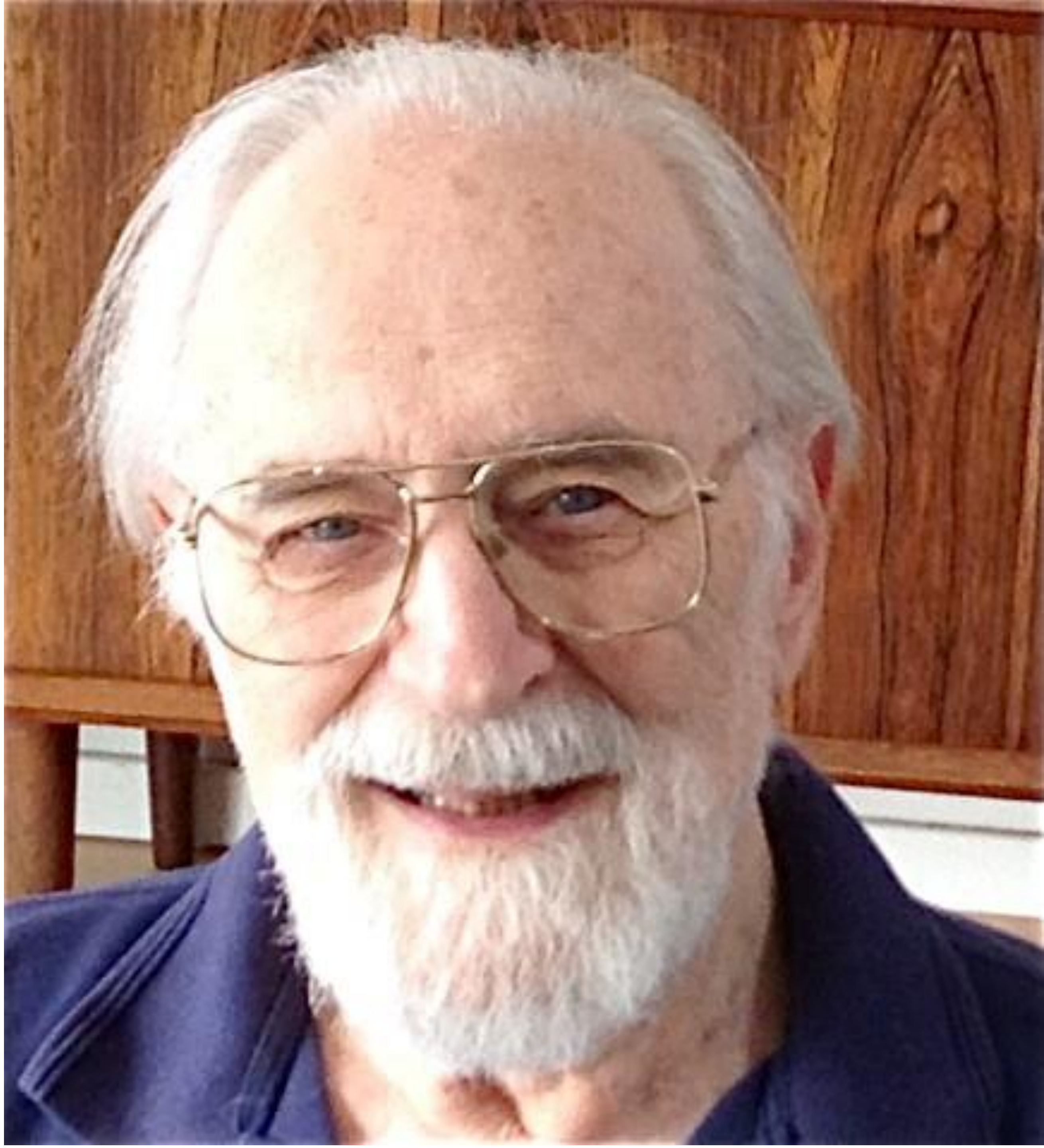
Physicist

Mathematician

Inventor

Bell Labs (1954)

"The Map Method for Synthesis of Combinational Logic Circuits”



KARNAUGH MAP

/ˈkɑːrnɔː/

1-Variable KARNAUGH MAP

X	F
0	m_0
1	m_1

X	
0	1
m_0	m_1

2-Variable KARNAUGH MAP

Y	X	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y	X	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y	X	F
0	0	1
0	1	0
1	0	0
1	1	0

$F(Y,X) = m_0 = Y'X'$

		X	
		0	1
Y	0	1	0
	1	0	0

$F(Y,X) = Y'X'$

Y	X	F
0	0	0
0	1	1
1	0	0
1	1	0

$$F(Y,X) = m_1 = Y'X$$

		X	
		0	1
Y	0	0	1
	1	0	0

$$F(Y,X) = Y'X$$

Y	X	F
0	0	0
0	1	0
1	0	1
1	1	0

$$F(Y,X) = m_2 = YX'$$

Y \ X	0	1
0	0	0
1	1	0

$$F(Y,X) = YX'$$

Y	X	F
0	0	0
0	1	0
1	0	0
1	1	1

$$F(Y,X) = m_3 = YX$$

		X	
		0	1
Y	0	0	0
	1	0	1

$$F(Y,X) = YX$$

Y	X	F
0	0	1
0	1	1
1	0	0
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 \\
&= Y'X' + Y'X \\
&= Y'(X' + X) \\
&= Y'
\end{aligned}
$$

		X	
		0	1
Y	0	1	1
	1	0	0

$$F(Y,X) = Y'$$

Y	X	F
0	0	0
0	1	0
1	0	1
1	1	1

$$\begin{aligned} F(Y,X) &= m_2 + m_3 \\ &= YX' + YX \\ &= Y(X' + X) \\ &= Y \end{aligned}$$

		X	
		0	1
Y	0	0	0
	1	1	1

$$F(Y,X) = Y$$

Y	X	F
0	0	0
0	1	1
1	0	0
1	1	1

$$
\begin{aligned}
F(Y,X) &= m_1 + m_3 \\
&= Y'X + YX \\
&= X(Y' + Y) \\
&= X
\end{aligned}
$$

		X	
		0	1
Y	0	0	1
	1	0	1

$$F(Y,X) = X$$

Y	X	F
0	0	1
0	1	0
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_2 \\
&= Y'X' + YX' \\
&= X'(Y' + Y) \\
&= X'
\end{aligned}
$$

Y \ X	0	1
0	1	0
1	1	0

$$F(Y,X) = X'$$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= Y'(X' + X) + YX' \\
&= Y' + YX'
\end{aligned}
$$

		X	
		0	1
Y	0	1	1
	1	1	0

$$F(Y,X) = Y' + YX'$$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	0

$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= X'(Y' + Y) + Y'X \\
&= X' + Y'X
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	0

$F(Y,X) = X' + Y'X$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	0

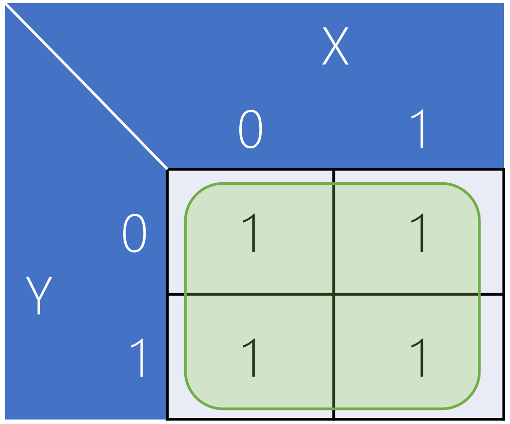
$$
\begin{aligned}
F(Y,X) &= m_0 + m_1 + m_2 \\
&= Y'X' + Y'X + YX' \\
&= Y'X' + Y'X' + Y'X + YX' \\
&= Y'(X' + X) + Y'X' + YX' \\
&= Y' + Y'X' + YX' \\
&= Y' + X'(Y' + Y) \\
&= Y' + X'
\end{aligned}
$$

Y \ X	0	1
0	1	1
1	1	0

$$F(Y,X) = Y' + X'$$

Y	X	F
0	0	1
0	1	1
1	0	1
1	1	1

$$\begin{aligned} F(Y,X) &= m_0 + m_1 + m_2 + m_3 \\ &= Y'X' + Y'X + YX' + YX \\ &= Y'(X' + X) + Y(X' + X) \\ &= Y' + Y \\ &= 1 \end{aligned}$$



$$\begin{aligned} F(Y,X) &= m_0+m_1+m_2+m_3 \\ &= 1 \end{aligned}$$

Y	X	F
0	0	0
0	1	1
1	0	1
1	1	0

$$F(Y,X) = m_1 + m_2$$
$$= Y'X + YX'$$

		X	
		0	1
Y	0	0	1
	1	1	0

$$F(Y,X) = m_1 + m_2$$
$$= Y'X + YX'$$

Y	X	F
0	0	1
0	1	0
1	0	0
1	1	1

$$F(Y,X) = m_0 + m_3$$
$$= Y'X' + YX$$

		X	
		0	1
Y	0	1	0
	1	0	1

$$F(Y,X) = m_0 + m_2$$
$$= Y'X' + YX$$

3-Variable KARNAUGH MAP

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Y \ X	0	1
0	m_0	m_1
1	m_2	m_3

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Z

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10	
0	m_0	m_1	m_3	m_2	Z'
1	m_4	m_5	m_7	m_6	

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Y

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Y'

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

X

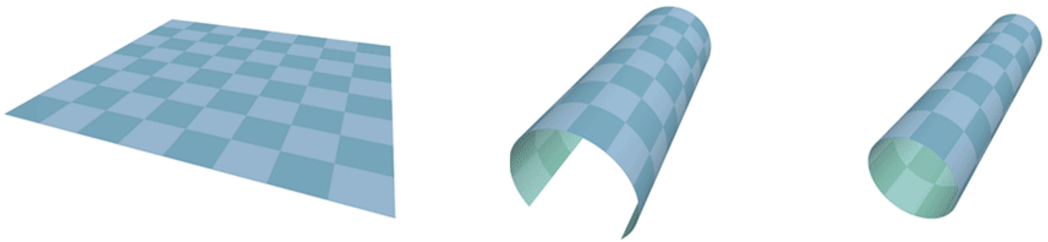
Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

X′ ?

Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

		YX			
		00	01	11	10
Z	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6



Z	Y	X	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Z \ YX	00	01	11	10	Z
0	m_0	m_1	m_3	m_2	0
1	m_4	m_5	m_7	m_6	1

X' X'

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$
\begin{aligned}
F(Z,Y,X) &= \sum m(0,1,2,3,6,7) \\
&= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX \\
&= ?
\end{aligned}
$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$
\begin{aligned}
F(Z,Y,X) &= \sum m(0,1,2,3,6,7) \\
&= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX \\
&= ?
\end{aligned}
$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$
\begin{aligned}
F(Z,Y,X) &= \sum m(0,1,2,3,6,7) \\
&= Z' +
\end{aligned}
$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$\begin{aligned} F(Z,Y,X) &= \sum m(0,1,2,3,6,7) \\ &= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX \\ &= ? \end{aligned}$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$\begin{aligned} F(Z,Y,X) &= \sum m(0,1,2,3,6,7) \\ &= Z' + ZY \end{aligned}$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX$$
$$= ?$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$$
$$= Z' + Y$$

MAXTERMS

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(Z,Y,X) = \prod M(4,5)$$
$$= (Z'+Y+X)\ (Z'+Y+X')$$
$$= ?$$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$F(Z,Y,X) = \prod M(4,5)$$
$$= ?$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$F(Z,Y,X) = \prod M(4,5)$$
$$= (Z'+Y+X)\,(Z'+Y+X')$$
$$= ?$$

Z \ YX	00	01	11	10
0	0	0	0	0
1	1	1	0	0

$$F'(Z,Y,X) = \sum m(4,5)$$
$$= ZY'$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$
\begin{aligned}
F(Z,Y,X) &= \prod M(4,5) \\
&= (Z'+Y+X)\,(Z'+Y+X') \\
&= ?
\end{aligned}
$$

		YX			
		00	01	11	10
Z	0	1	1	1	1
	1	0	0	1	1

$$
\begin{aligned}
F(Z,Y,X) &= \prod M(4,5) \\
&= (F')' \\
&= (ZY')' \\
&= Z'+Y
\end{aligned}
$$

Z	Y	X	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$
$= Z'Y'X' + Z'Y'X + Z'YX' + Z'YX + ZYX' + ZYX$
$= ?$

$F(Z,Y,X) = \prod M(4,5)$
$= (Z'+Y+X)\,(Z'+Y+X')$
$= ?$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$F(Z,Y,X) = \sum m(0,1,2,3,6,7)$
$= Z' + Y$

Z \ YX	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$F(Z,Y,X) = \prod M(4,5)$
$= (F')'$
$= (ZY')'$
$= Z'+Y$

Z	Y	X	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$
\begin{aligned}
F(Z,Y,X) &= \sum m(0,2,4,6) \\
&= Z'Y'X' + Z'YX' + ZY'X' + ZYX' \\
&= ?
\end{aligned}
$$

		YX			
		00	01	11	10
Z	0	1	0	0	1
	1	1	0	0	1

Z	Y	X	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$F(Z,Y,X) = \sum m(0,2,4,6)$$
$$= Z'Y'X' + Z'YX' + ZY'X' + ZYX'$$
$$= ?$$

		YX			
		00	01	11	10
Z	0	1	0	0	1
	1	1	0	0	1

$$F(Z,Y,X) = \sum m(0,2,4,6)$$
$$= X'$$

MAXTERMS

Z	Y	X	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

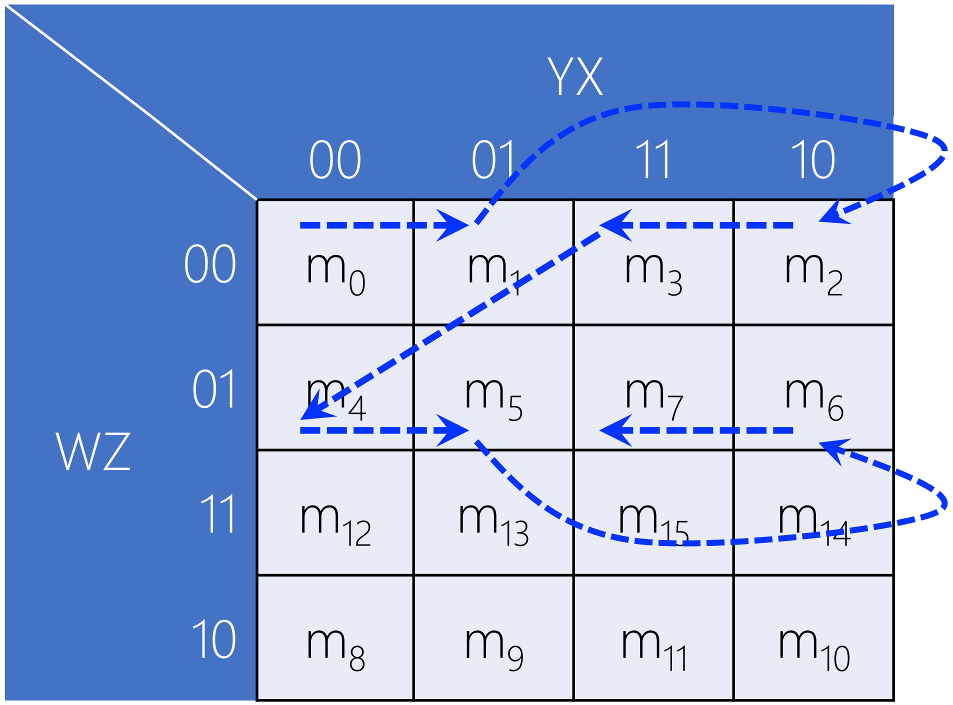
$$
\begin{aligned}
F(Z,Y,X) &= \Pi\, M(1,3,5,7) \\
&= (Z+Y+X')(Z+Y'+X')(Z'+Y+X')(Z'+Y'+X') \\
&= ?
\end{aligned}
$$

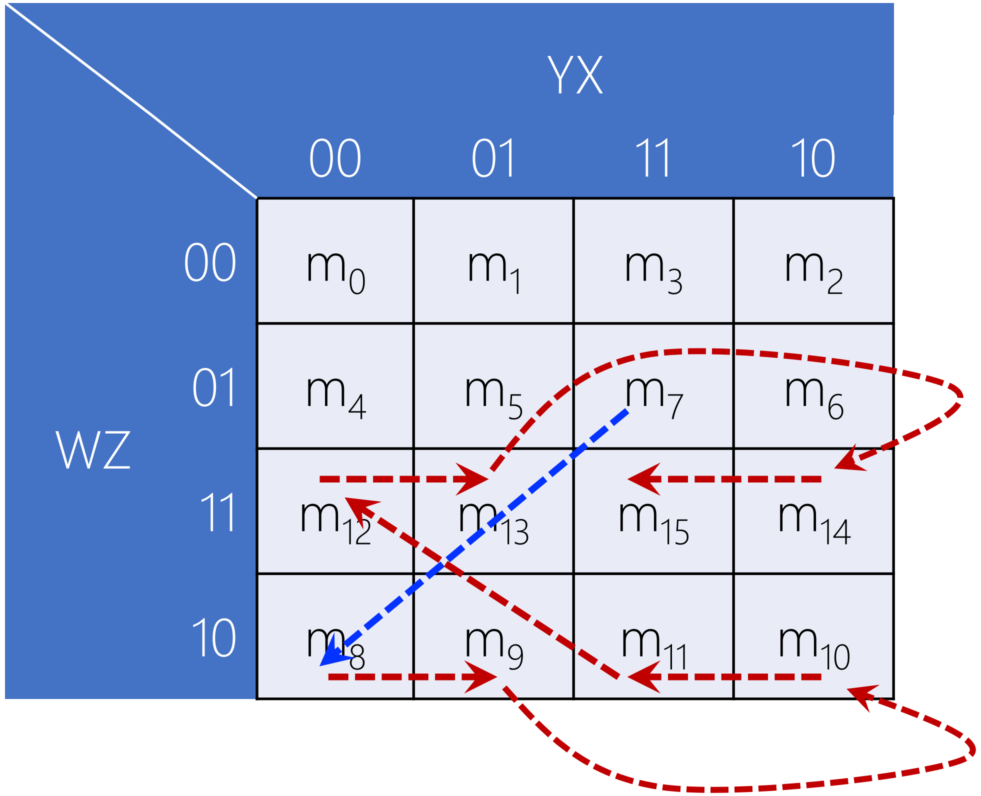
Z \ YX	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$$
\begin{aligned}
F(Z,Y,X) &= \Pi\, M(1,3,5,7) \\
&= (X)' \\
&= X'
\end{aligned}
$$

4-Variable KARNAUGH MAP

WZ \ YX	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}


YX
00
01
11
10
WZ
00
01
11
10
m0 m1 m3 m2
m4 m5 m7 m6
m12 m13 m15 m14
m8 m9 m11 m10



YX
00
01
11
10
WZ
00
01
11
10
m_0 m_1 m_3 m_2
m_4 m_5 m_7 m_6
m_{12} m_{13} m_{15} m_{14}
m_8 m_9 m_{11} m_{10}

WZ \ YX	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F\,(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' +$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

Warning!

$W=\{0,1\}$, the value of W matters in m_{10}

$Z=\{0,1\}$

$Y=\{1\}$

$X=\{0\}$

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' +$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX' + WZYX'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

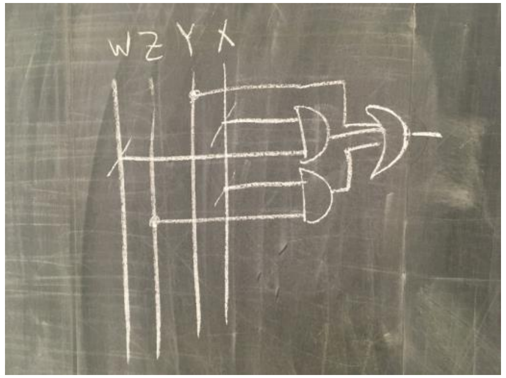
$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'YX' + ZYX'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F\,(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'X' + WYX'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

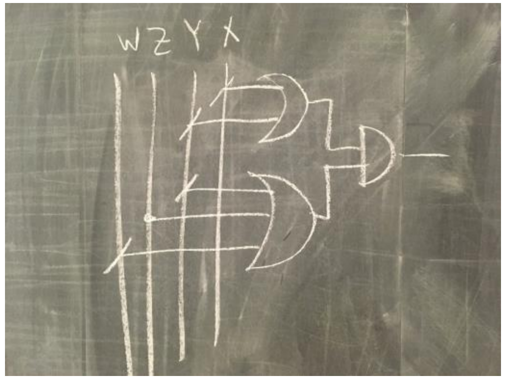
$$F(W,Z,Y,X) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$$
$$= Y' + W'X' + ZX'$$



MAXTERMS

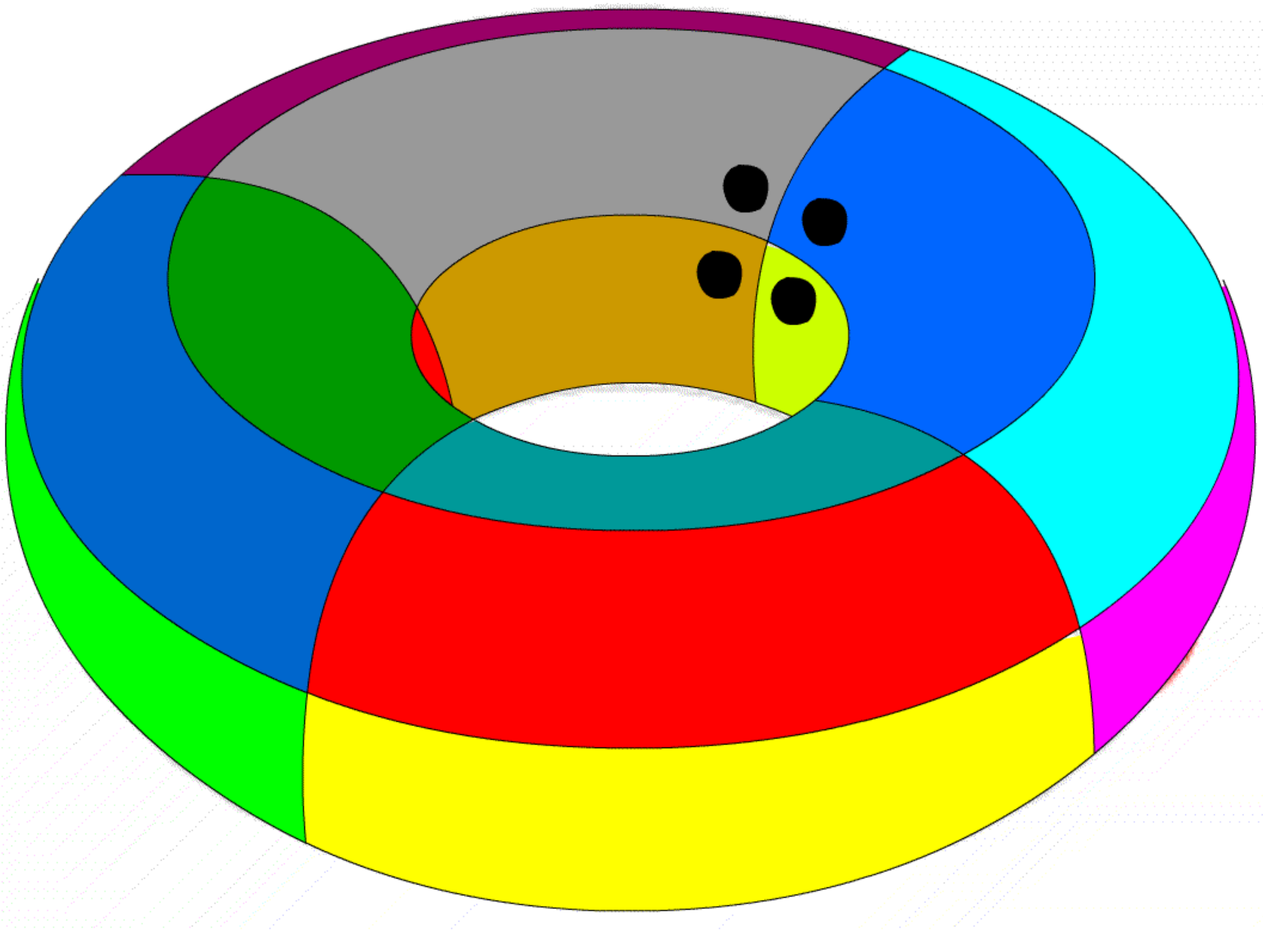
WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	0 m_3	1 m_2
01	1 m_4	1 m_5	0 m_7	1 m_6
11	1 m_{12}	1 m_{13}	0 m_{15}	1 m_{14}
10	1 m_8	1 m_9	0 m_{11}	0 m_{10}

$$F(W,Z,Y,X) = \prod M(3, 7, 10, 11, 15)$$
$$= (YX)'(WZ'Y)'$$
$$= (Y'+X')(W'+Z+Y')$$



Click to Play!

https://en.wikipedia.org/wiki/Karnaugh_map#/media/File:Torus_from_rectangle.gif

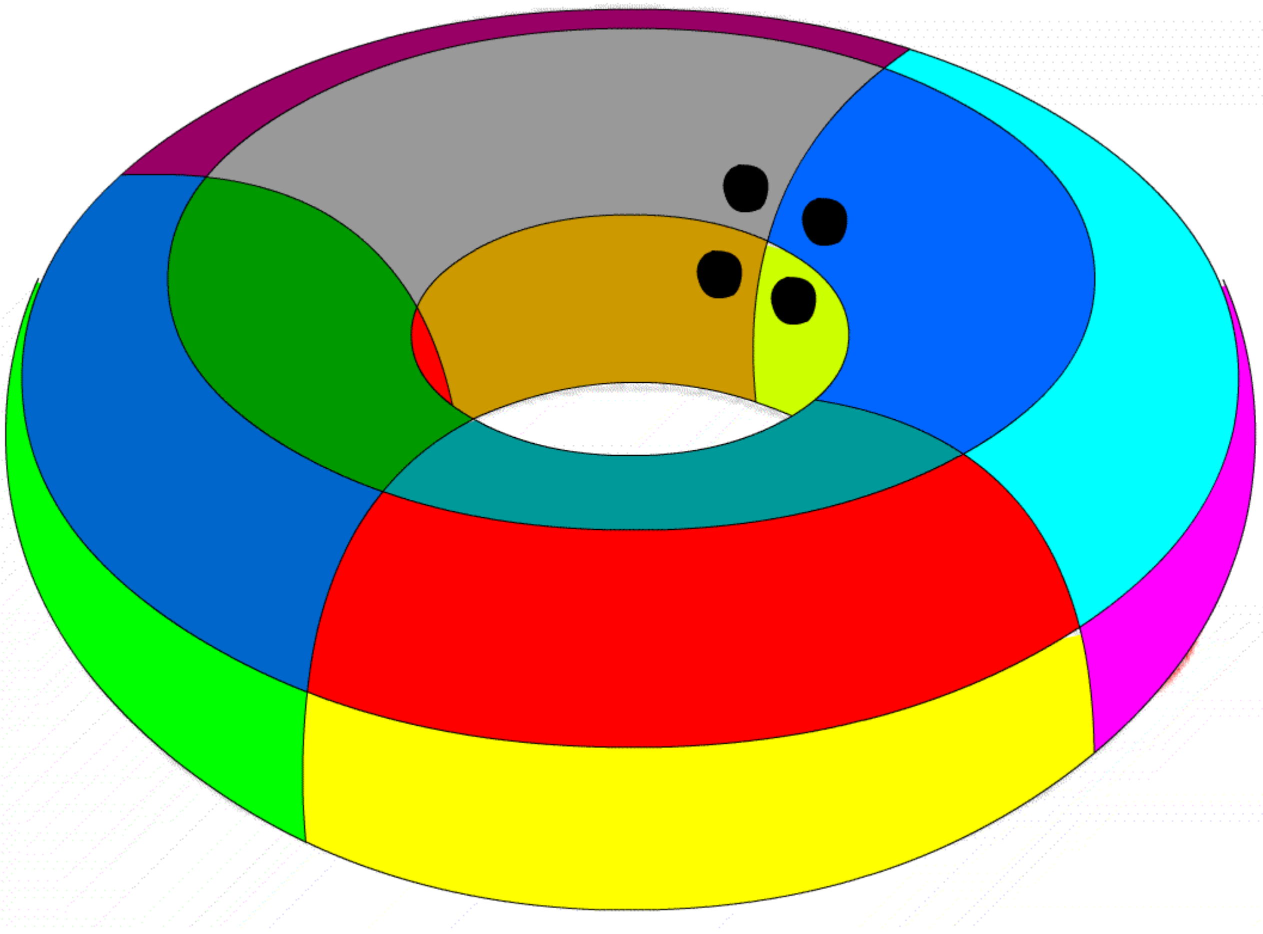


torus

WZ \ YX	00	01	11	10
00	1 m_0	0 m_1	0 m_3	1 m_2
01	0 m_4	0 m_5	0 m_7	0 m_6
11	0 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}
10	1 m_8	0 m_9	0 m_{11}	1 m_{10}

$$F\,(W,Z,Y,X) = \sum m(0, 2, 8, 10)$$
$$= Z'X'$$

MAXTERMS



torus

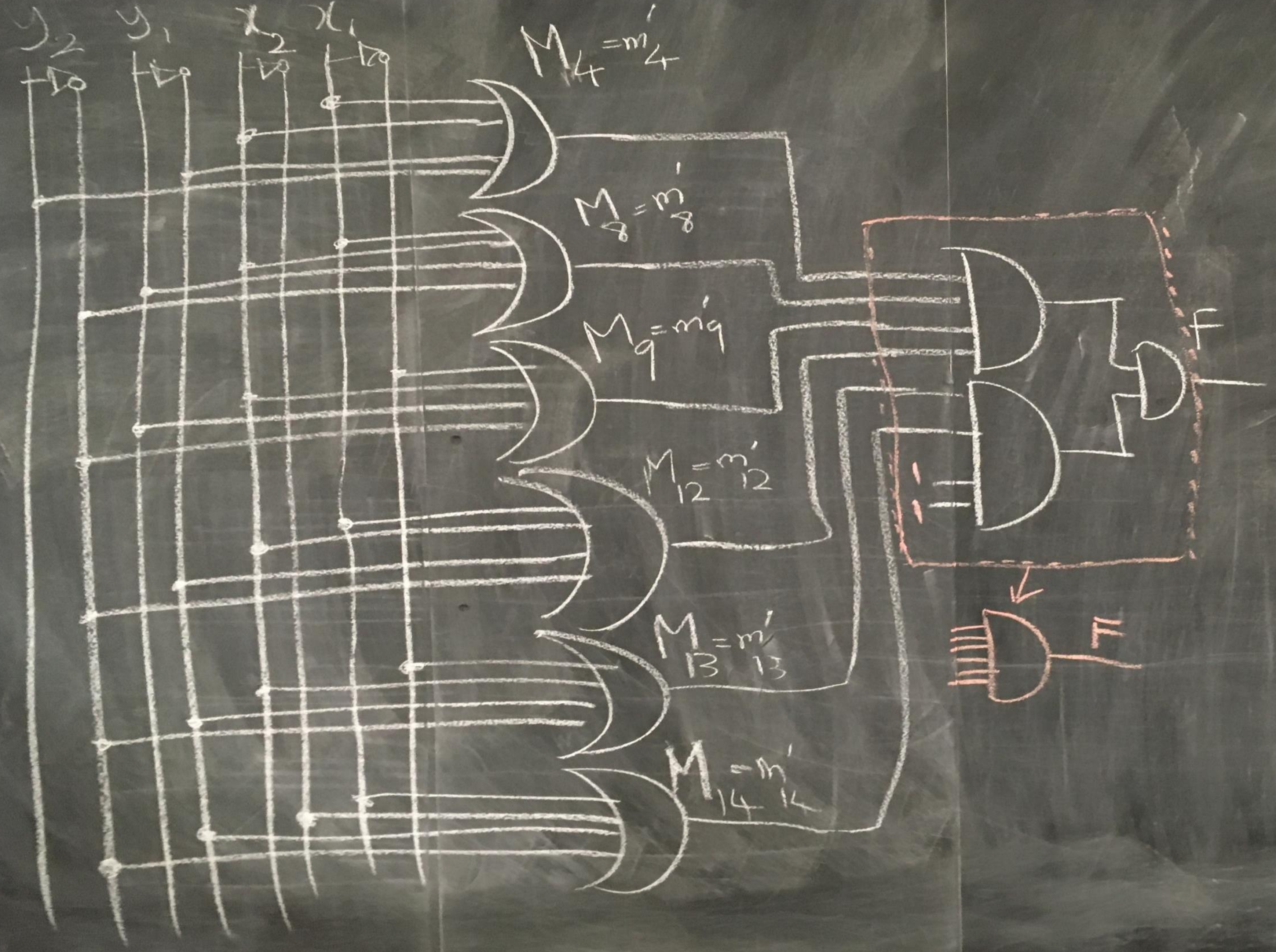
WZ \ YX	00	01	11	10
00	1 m_0	0 m_1	0 m_3	1 m_2
01	0 m_4	0 m_5	0 m_7	0 m_6
11	0 m_{12}	0 m_{13}	0 m_{15}	0 m_{14}
10	1 m_8	0 m_9	0 m_{11}	1 m_{10}

$$
\begin{aligned}
F\ (W,Z,Y,X) &= \sum m(0,\ 2,\ 8,\ 10) \\
&= Z'X' \\
&= \prod M(1,3\text{-}7,9,11\text{-}15) \\
&= (X)'(Z)' \\
&= X'Z'
\end{aligned}
$$

Given two unsigned numbers x and y, design a logic circuit to see

$$x \overset{?}{\geq} y$$

Y2	Y1	X2	X1	$F(Y2,Y1,X2,X1)=\Sigma\ m(0,1,2,3,5,6,7,10,11,15)$	$F(Y2,Y1,X2,X1)=\Pi\ M(4,8,9,12,13,14)$
0	0	0	0	1	1
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	1	1



$$F = \prod M(4,8,9,12,13,14)$$

$$= M_4 M_8 M_9 M_{12} M_{13} M_{14}$$

$$= m'_4 m'_8 m'_9 m'_{12} m'_{13} m'_{14}$$

$= (y'_2 y_1 x'_2 x'_1)'$ $\quad m'_4 \rightarrow y_2 + y'_1 + x_2 + x_1$

$(y_2 y'_1 x'_2 x'_1)'$ $\quad m'_8 \rightarrow y'_2 + y_1 + x_2 + x_1$

$(y_2 y'_1 x'_2 x_1)'$ $\quad m'_9 \rightarrow y'_2 + y_1 + x_2 + x'_1$

$(y_2 y_1 x'_2 x'_1)'$ $\quad m'_{12} \rightarrow y'_2 + y'_1 + x_2 + x_1$

$(y_2 y_1 x'_2 x_1)'$ $\quad m'_{13} \rightarrow y'_2 + y'_1 + x_2 + x'_1$

$(y_2 y_1 x_2 x'_1)'$ $\quad m'_{14} \rightarrow y'_2 + y'_1 + x'_2 + x_1$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$

$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Pi}\ M(4,8,9,12,13,14)$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Sigma}\, m(0,1,2,3,5,6,7,10,11,15)$$
$$=Y'_2Y'_1 +$$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

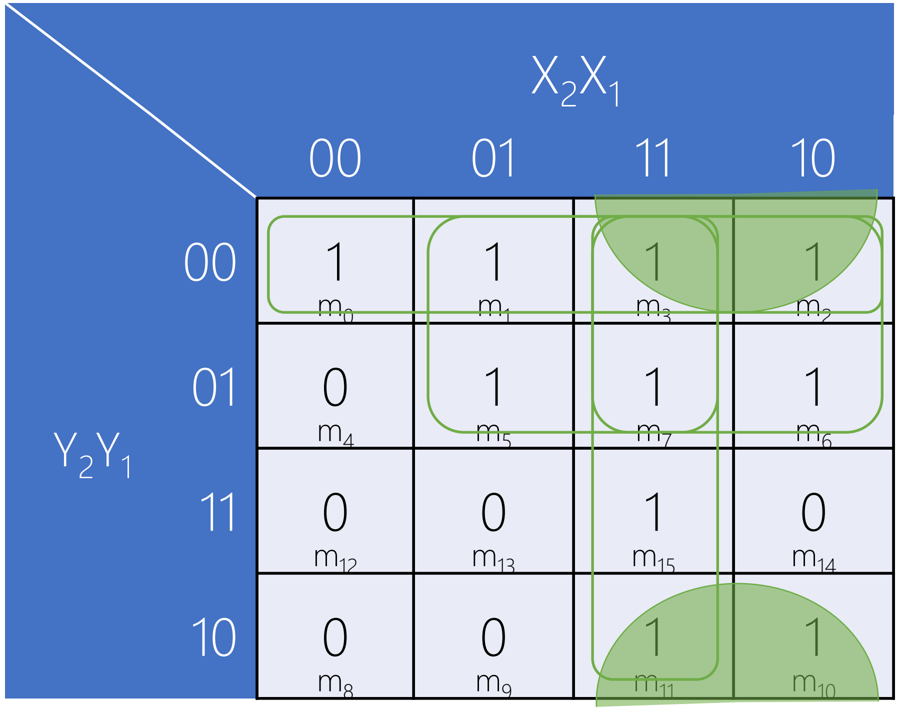
$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1$$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1$$

Y_2Y_1 \ X_2X_1	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1 + Y'_2X_2$$



$$F(Y_2,Y_1,X_2,X_1)=\boldsymbol{\Sigma}\ m(0,1,2,3,5,6,7,10,11,15)$$
$$= Y'_2Y'_1 + X_2X_1 + Y'_2X_1 + Y'_2X_2 + Y'_1X_2$$

Change of Variable:

$X1 \rightarrow X$

$X2 \rightarrow Y$

$Y1 \rightarrow Z$

$Y2 \rightarrow W$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

Change of Variable:

$X1 \rightarrow X$

$X2 \rightarrow Y$

$Y1 \rightarrow Z$

$Y2 \rightarrow W$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

$$F(W,Z,Y,X)=()'$$

MAXTERMS

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

$$F(W,Z,Y,X) = (WY' + \ )'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

$$F(W,Z,Y,X)\ =(WY' + ZY'X' +)'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

$$F(W,Z,Y,X) = (WY' + ZY'X' + WZX')'$$

WZ \ YX	00	01	11	10
00	1 m_0	1 m_1	1 m_3	1 m_2
01	0 m_4	1 m_5	1 m_7	1 m_6
11	0 m_{12}	0 m_{13}	1 m_{15}	0 m_{14}
10	0 m_8	0 m_9	1 m_{11}	1 m_{10}

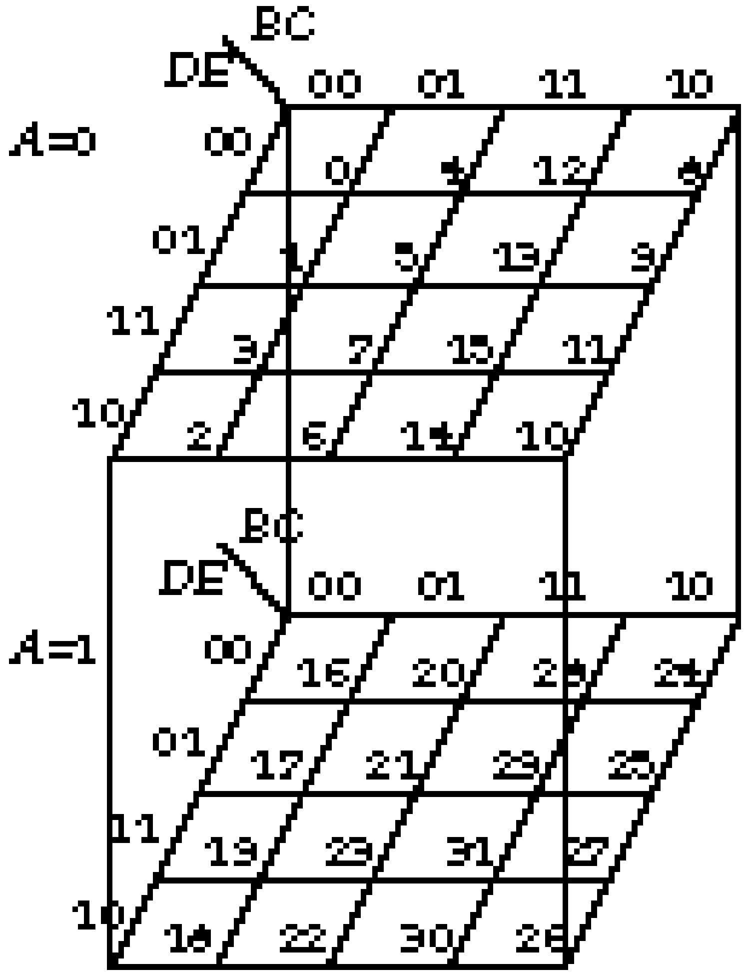
$$F(Y_2,Y_1,X_2,X_1)=\mathbf{\Pi}\ M(4,8,9,12,13,14)$$

$$F(W,Z,Y,X) = (WY' + ZY'X' + WZX')'$$

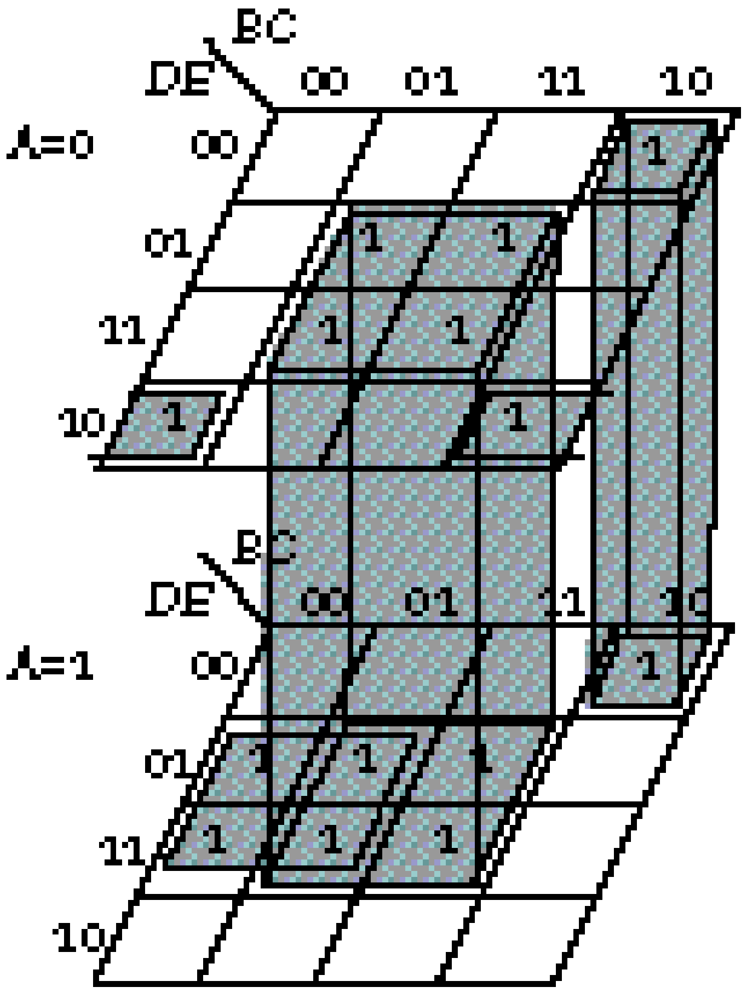
$$= (WY')'\ (ZY'X')'\ (WZX')'$$

$$= (W'+Y)\ (Z'+Y+X)'\ (W'+Z'+X)$$

5-Variable KARNAUGH MAP



BC
DE
00 01 11 10
A=0
00
01
11
10
0 4 12 8
1 5 13 9
3 7 15 11
2 6 14 10
BC
DE
00 01 11 10
A=1
00
01
11
10
16 20 28 24
17 21 29 25
19 23 31 27
18 22 30 26


BC
DE
00
01
11
10
A=0
00
01
11
10
1
1
1
1
1
1
1
BC
DE
00
01
11
10
A=1
00
01
11
10
1
1
1
1
1
1
1
1

n-Variable ~~KARNAUGH MAP~~

n-Variable Quine–McCluskey Algorithm

https://en.wikipedia.org/wiki/Quine%E2%80%93McCluskey_algorithm

1878 ← 1937 ← 1952 ← 1956

Demo
Quine–McCluskey Algorithm

https://www.mathematatik.uni-marburg.de/~thormae/lectures/ti1/code/qmc/

Don’t Care Conditions

In practice, in some applications the function is not specified for certain combinations of the variables.

Z	Y	X	F=if input is positive(2's comp.) then 1 else 0
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Z	Y	X	F=if input is positive(2's comp.) then 1 else 0
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

In math, 0 is not positive neither negative!

Z	Y	X	$F=\sum m(1,2,3)=\prod M(0,4,5,6,7)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Z \ YX	00	01	11	10
0	0 m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F(Z,Y,X) = \sum m(1, 2, 3)$$
$$= Z'X + Z'Y$$

Boolean algebra → $Z'(X+Y)$

MAXTERMS

Z \ YX	00	01	11	10
0	0 (m_0)	1 (m_1)	1 (m_3)	1 (m_2)
1	0 (m_4)	0 (m_5)	0 (m_7)	0 (m_6)

$$
\begin{aligned}
F(Z,Y,X) &= \Pi\, M(0,4,5,6,7) \\
&= (Z + Y'X')' \\
&= Z'\,(Y+X)
\end{aligned}
$$

Z	Y	X	F=if positive(2’s comp.) then 1 if negative 0
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

In math, 0 is not positive neither negative!

Z	Y	X	F=if positive(2’s comp.) then 1 if negative 0
0	0	0	
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Z \ YX	00	01	11	10
0	m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F\,(Z,Y,X) = \sum m(1,\ 2,\ 3) + \sum d(0)$$
$$= Z'X + Z'Y$$

Z \ YX	00	01	11	10
0	1 m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F(Z,Y,X) = \sum m(1, 2, 3) + \sum m(0)$$
$$= Z'$$

In this case, the don’t care condition help to more simplification

MAXTERMS

Z \ YX	00	01	11	10
0	m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F(Z,Y,X) = \prod M(4,5,6,7) + \sum D(0)$$
$$= (Z)'$$
$$= Z'$$

Z \ YX	00	01	11	10
0	0 m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F(Z,Y,X) = \prod M(0,4,5,6,7) + \sum M(0)$$

$$= (Z + Y'X')'$$

$$= Z'\,(Y+X)$$

In this case, the don't care condition does NOT help to more simplification

Don’t Care Conditions

Functions that have unspecified outputs for some input combinations are called *incompletely specified functions*.

Don’t-care conditions can be used on a map to provide further simplification of the Boolean expression.

Don’t Care Conditions

To distinguish the don’t-care condition from 1’s and 0’s, an x is used.

Z \ YX	00	01	11	10
0	x m_0	1 m_1	1 m_3	1 m_2
1	0 m_4	0 m_5	0 m_7	0 m_6

$$F\ (Z,Y,X) = \sum m(1,\ 2,\ 3) + \sum d(0)$$

$$F\ (Z,Y,X) = \prod M(4,5,6,7) + \sum D(0)$$