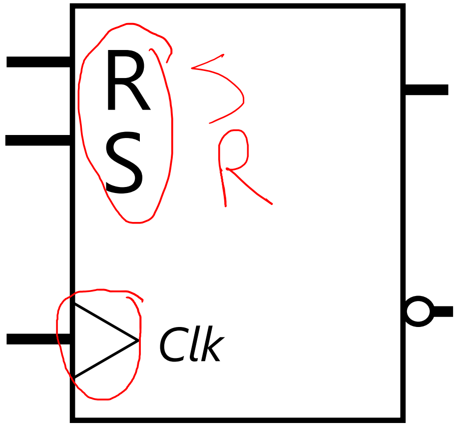
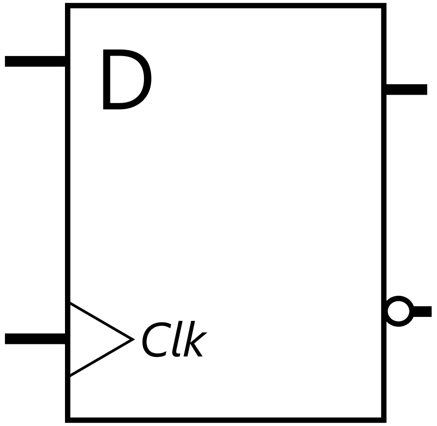
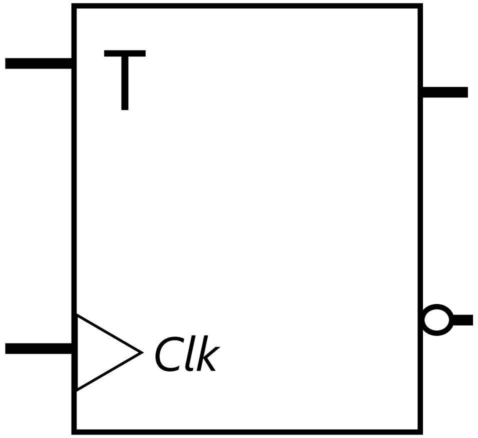
Flip-Flop

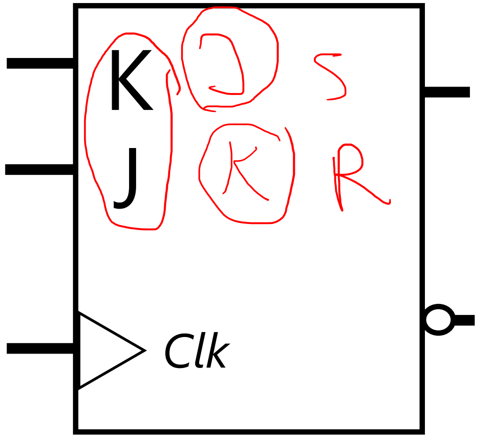
A single edge triggered latch

level edge









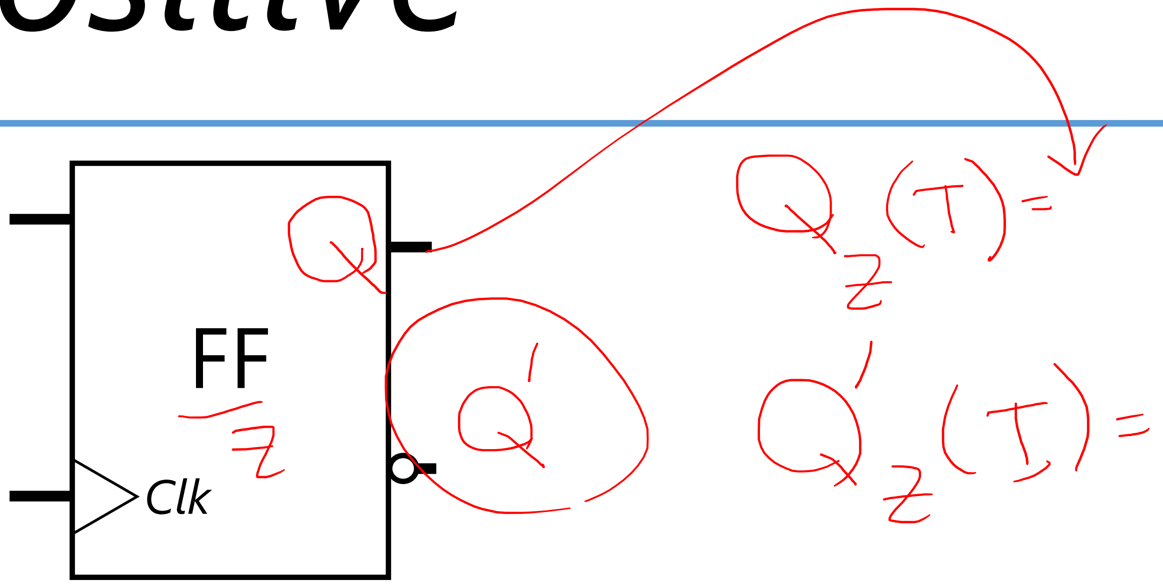
S	R	Q
0	0	Q_t
0	1	0
1	0	1
1	1	X

D	Q
0	0
1	1

T	Q
0	Q_t
1	Q'_t

J (S)	K (R)	Q
0	0	Q_t
0	1	0
1	0	1
1	1	Q'_t

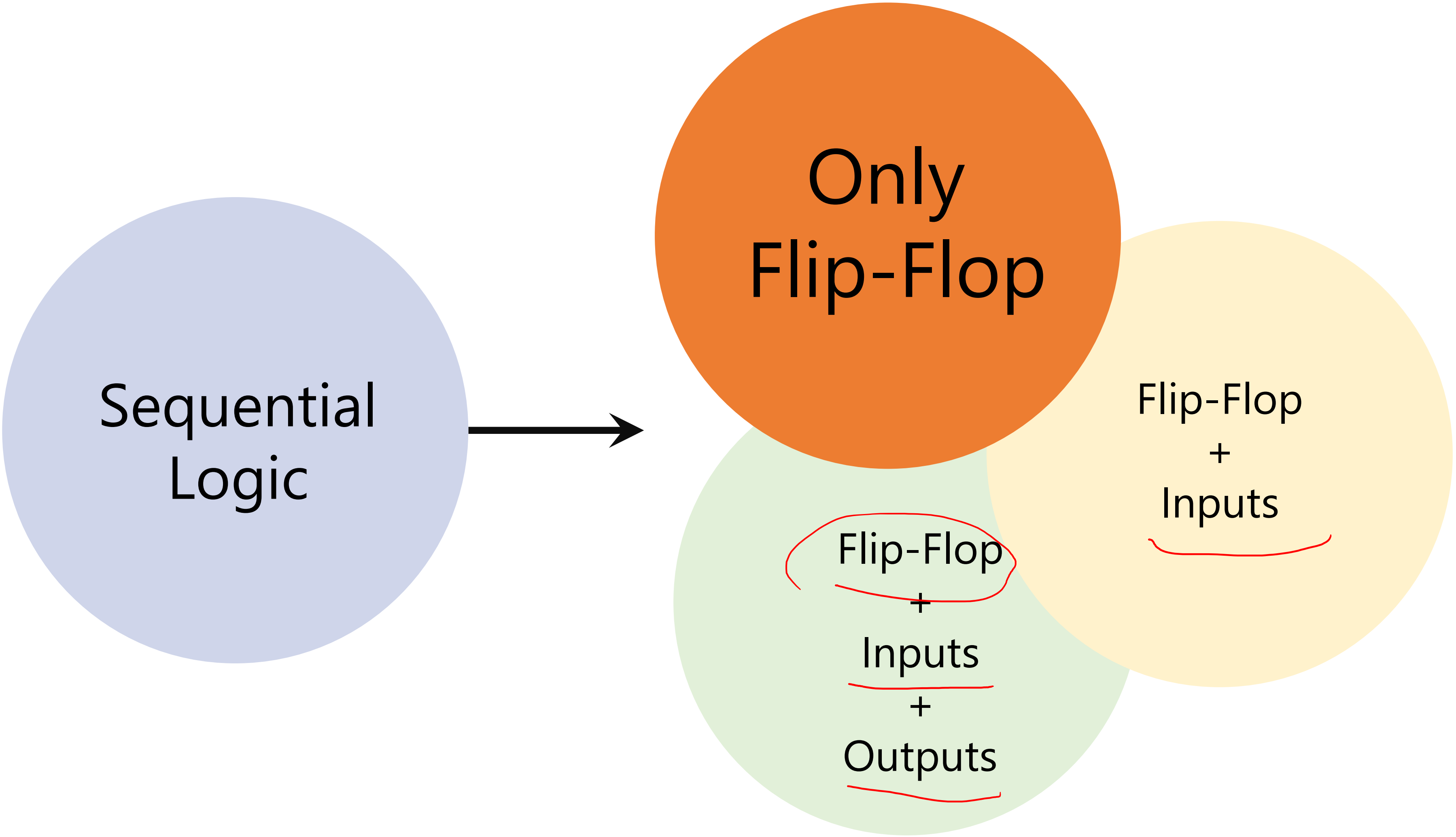
Single Edge
Positive



Analysis: Given a sequential circuit, show the behavior

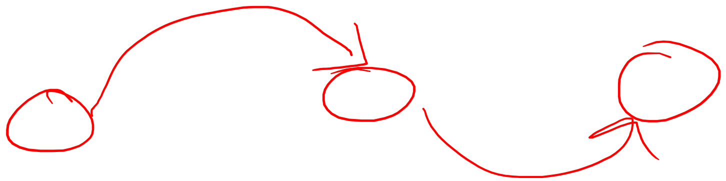
vs.

Design: Given a behavior, build the sequential circuit


Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
Flip-Flop
+
Inputs
+
Outputs

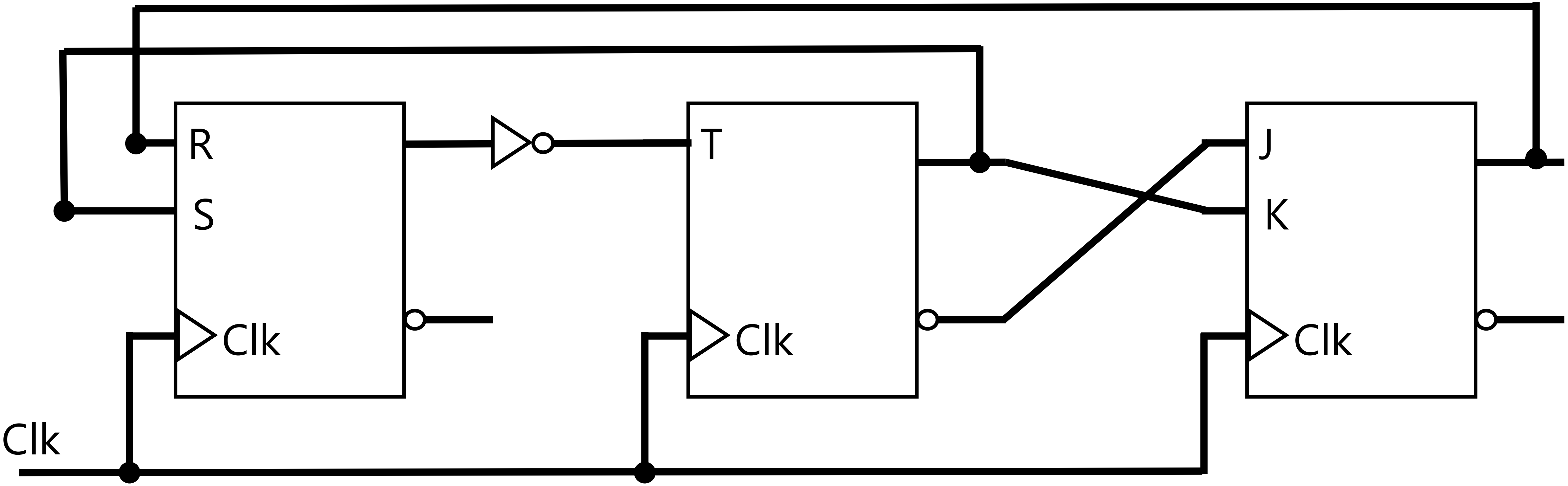
Analysis (Recap)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, ...)
2. What are the state combinations? $2^{\#FF}$
3. Form "State" table:
   a) Columns: for each FF, two columns:
      - one for current state,
      - one for next state
   b) Rows: for each state combination
      - In total: $2^{\#FF}$
4. Fill the state table for next state columns based on:
   a) the current state
   b) the inputs to the FFs ⇒ action



5. Form State Transition Diagram
6. (Optional) Analyze paths and states in state transition diagram

Analysis
by an example



R
S
Clk
T
Clk
J
K
Clk
Clk

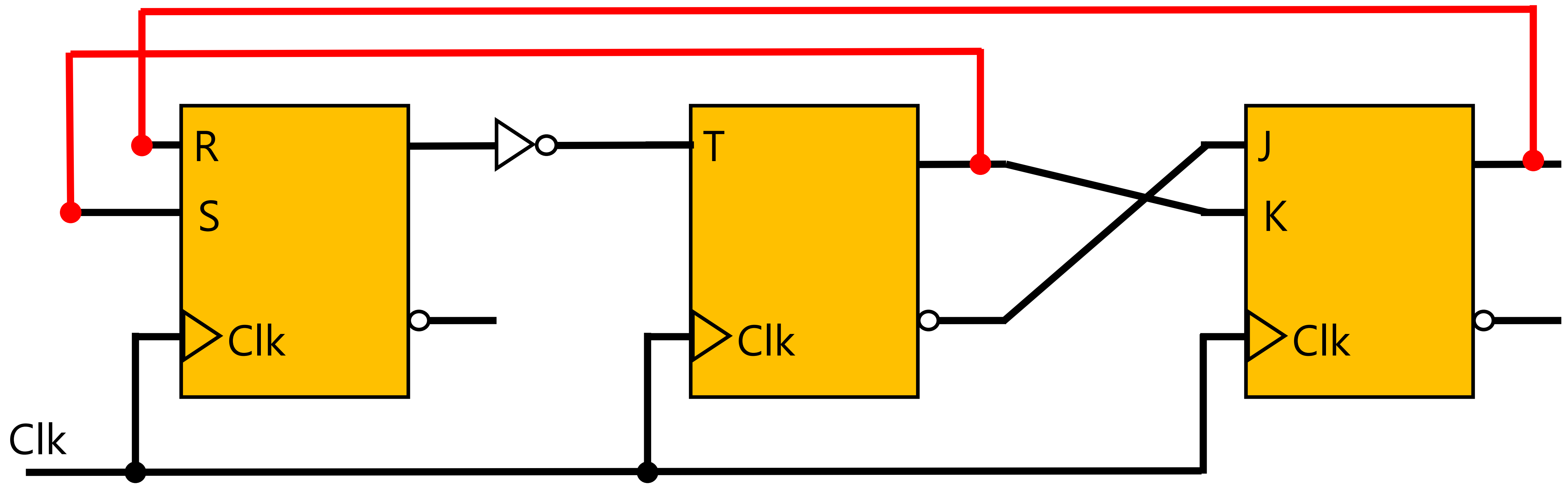
Analysis

0) Is it sequential circuit?

At least one FF → Yes

At least one feedback → Yes

Otherwise → No

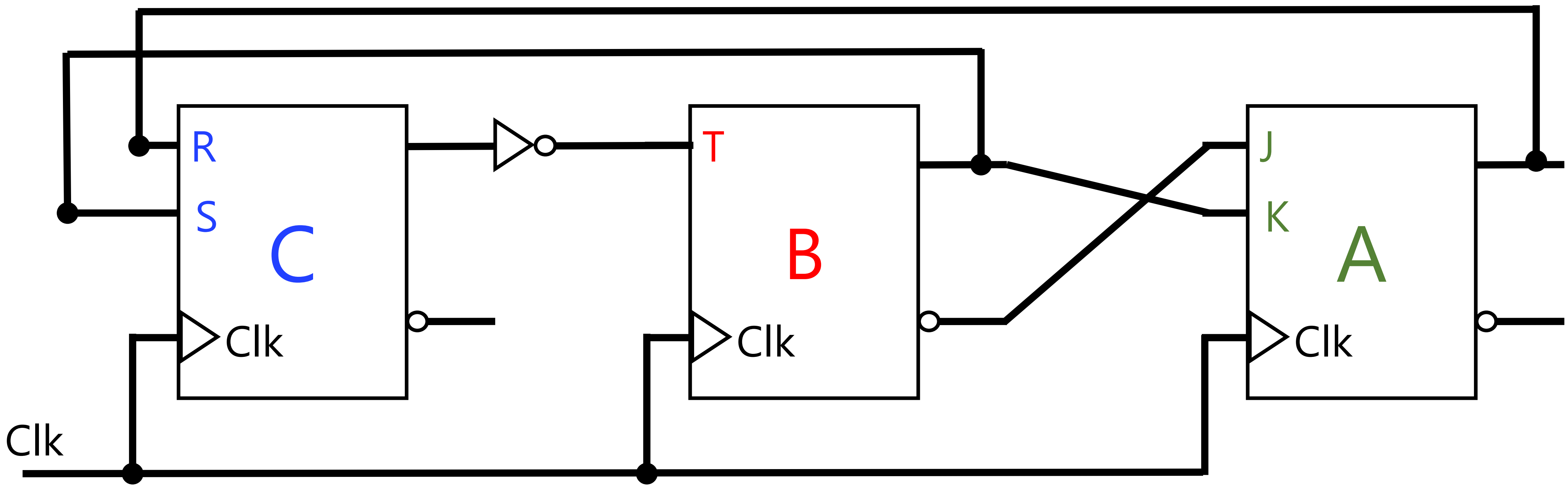

R
S
Clk
T
Clk
J
K
Clk
Clk

Analysis

1) What are the FFs?

1.1. We pick a name for each FF

1.2. We note the type of FF

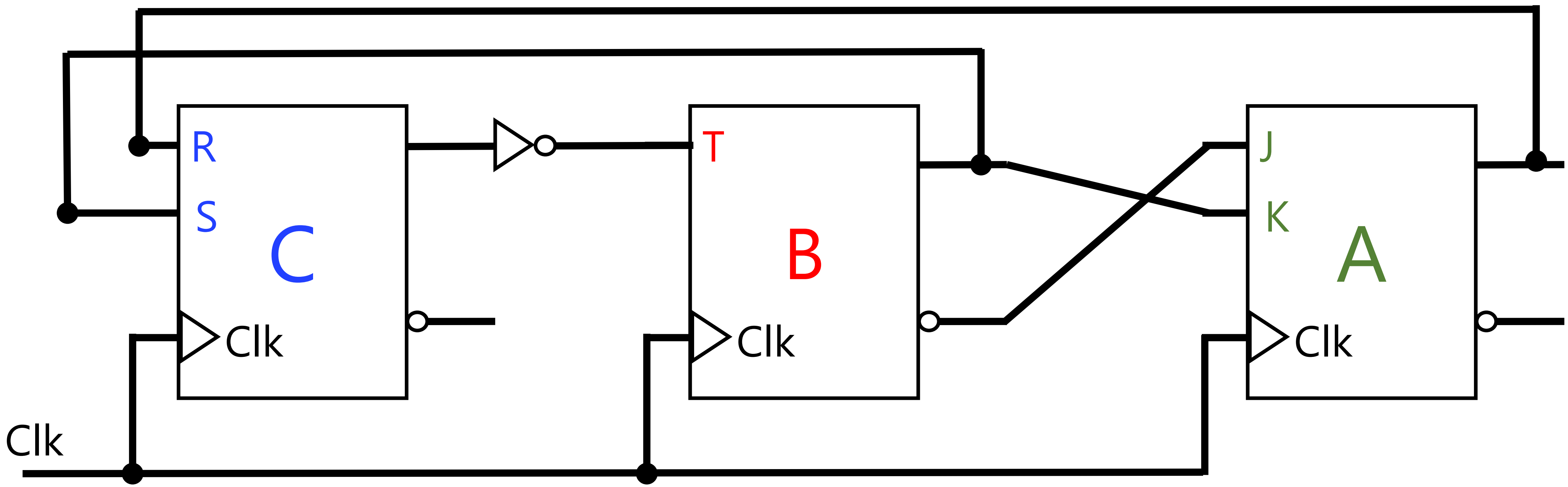

R
S
C
Clk
T
B
Clk
J
K
A
Clk
Clk

Analysis

2) What are the state combinations (possibilities)?

Each FF can have {0,1} states

In total, $2^{\#FFs}$

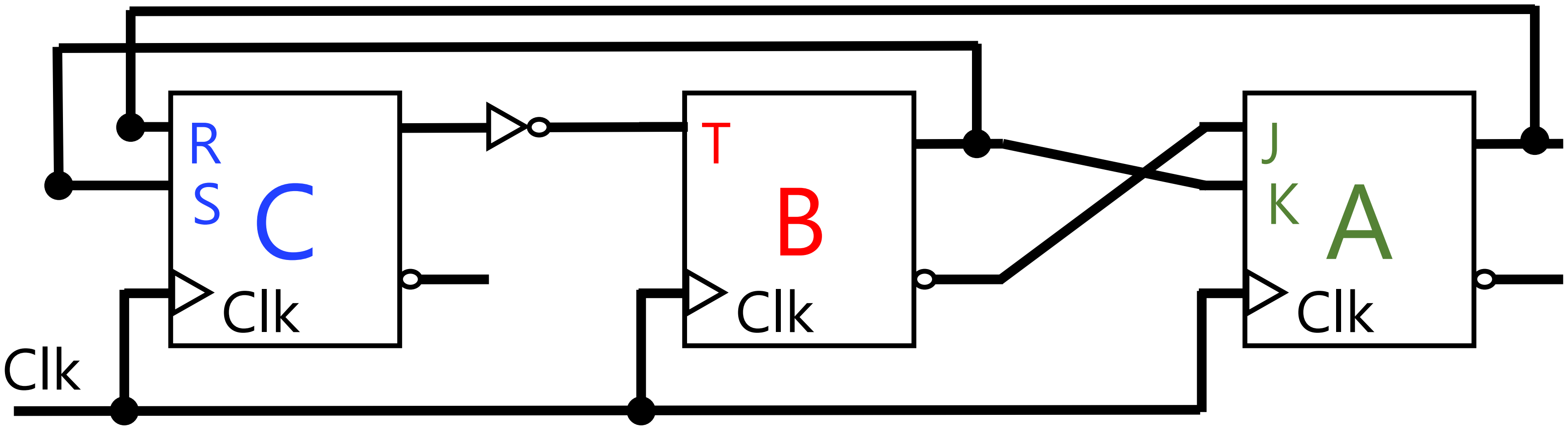


#FFs = 3 → 2^3 = 8 combinations

Analysis

3) Form a ‘State’ Table

3.1. For each FF, one column for current state
3.2. For each FF, one column for next state
3.3. For each combination of current state one row



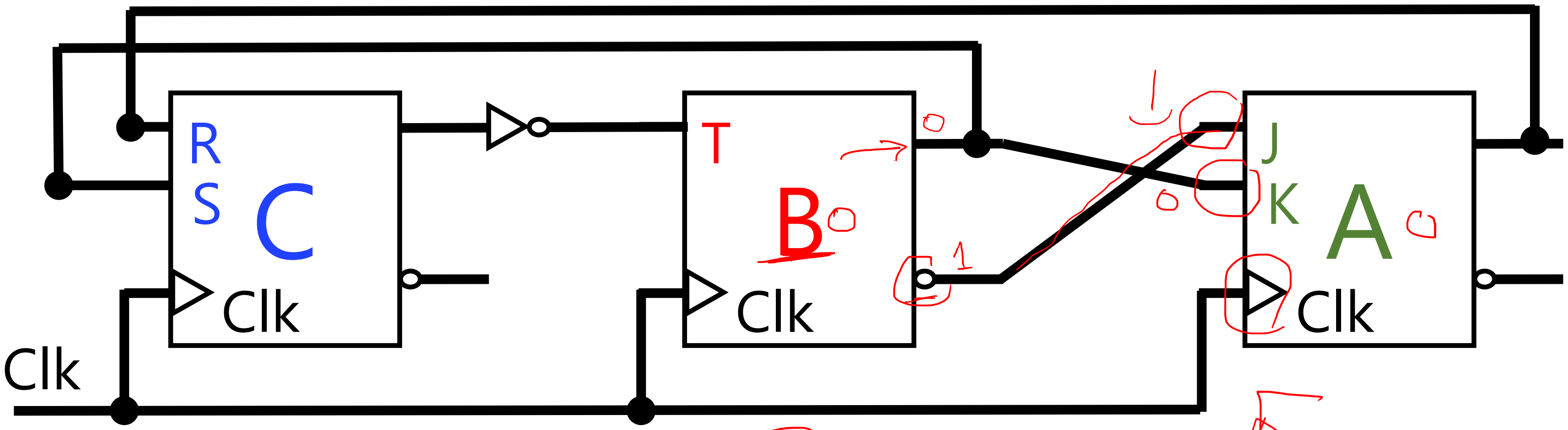
Q(T)			Q(T+1)		
C	B	A	C	B	A

Analysis

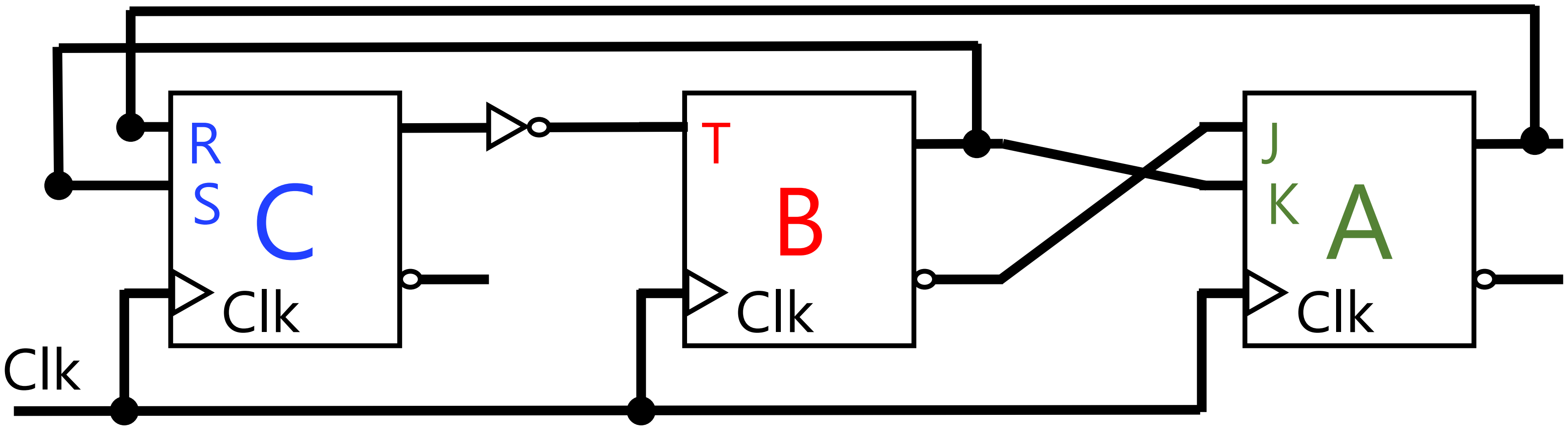
4) Fill the ‘State’ table

For each FF, we determine the next state based on

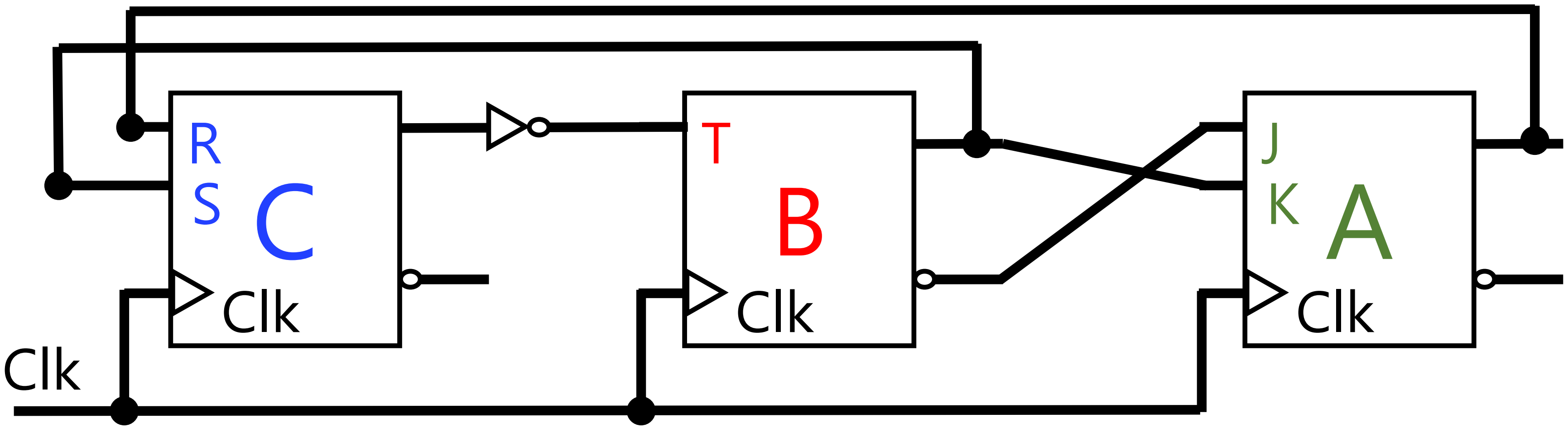
I) current state
II) the current value of inputs to the FF



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0			$Q_A(T)=0$ $J_A=Q'_B(T)=1$ $K_A=Q_B(T)=0$ ----------------- Set Action: 1
0	0	1			
0	1	0			
0	1	1			
1	0	0			



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0		$Q_B(T)=0$ $T_B=Q'_C(T)=1$ -------------- Comp. $(Q_B(T)) = 1$	1
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			

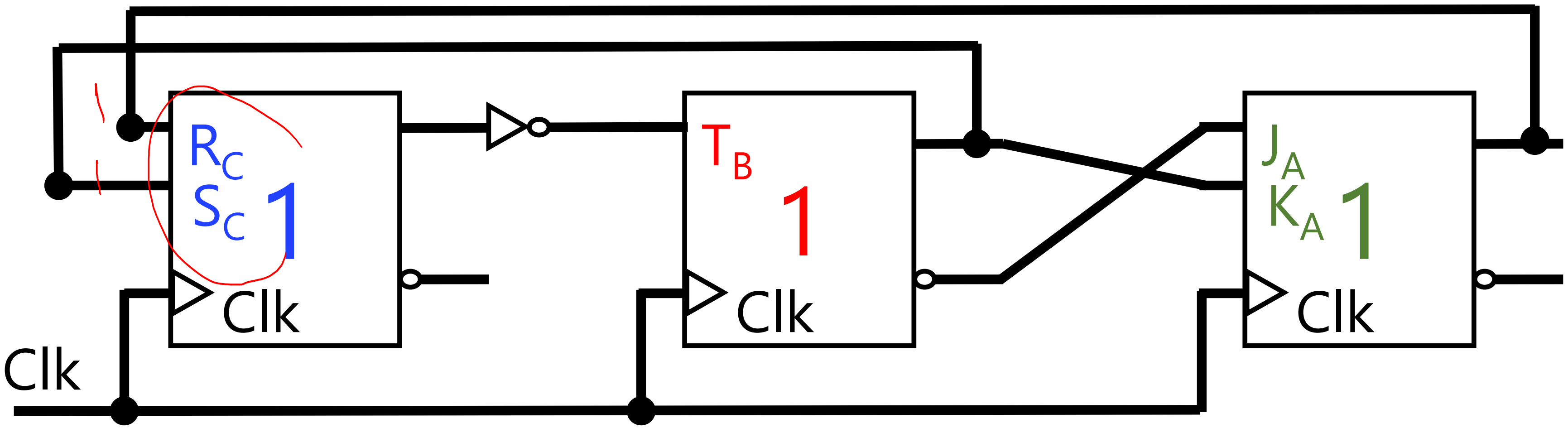


Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	$Q_C(T)=0$ $R_C=Q_A(T)=0$ $S_C=Q_B(T)=0$ --------------- Store $Q_C(T) = 0$	1	1
0	0	1			
0	1	0			
0	1	1			
1	0	0			

Analysis

$$Q_A(T) = A,\ Q'_A(T) = A'$$

For simplicity, the current status of a FF can be assume to be as a binary variable



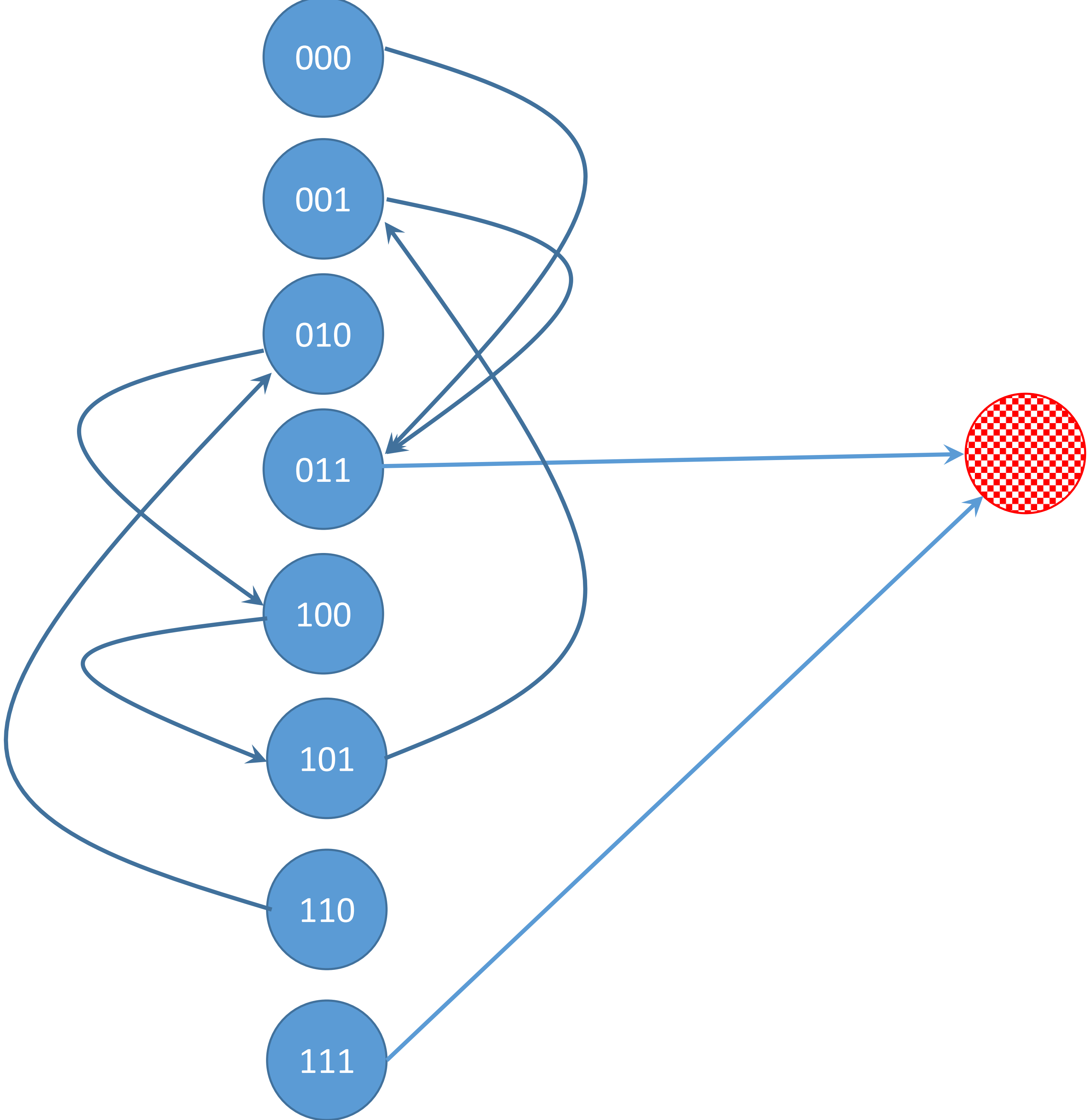
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	X	0	0
1	0	0	1	0	1
1	0	1	0	0	1
1	1	0	0	1	0
1	1	1	X	1	0

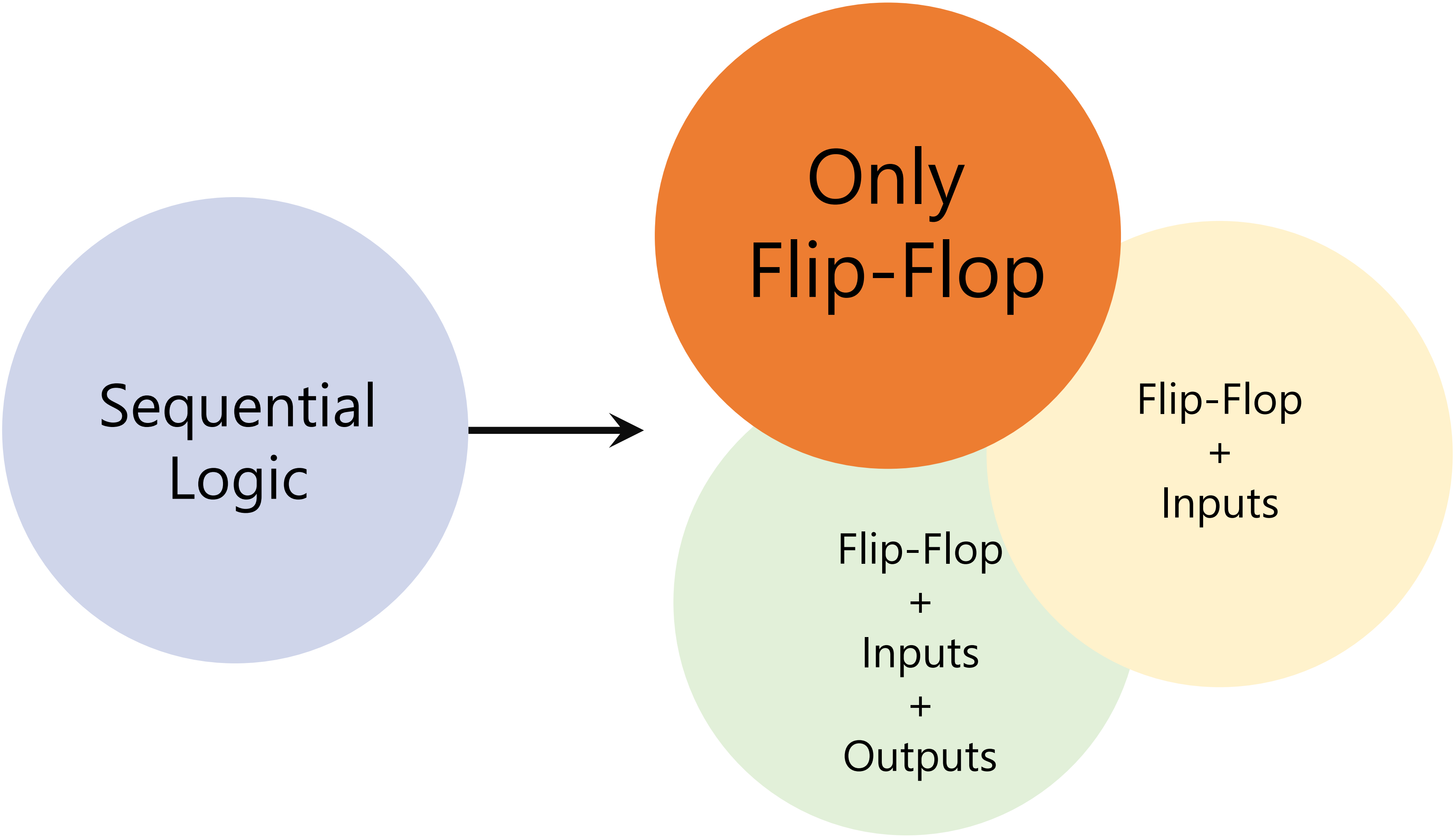
Analysis

5) State Transition Diagram

5.1. for each state combination (each row), a node

5.2. from one state (node) to another state, a directed edge


000
001
010
011
100
101
110
111


Sequential Logic
Only Flip-Flop
Flip-Flop + Inputs
Flip-Flop + Inputs + Outputs

Design

by an example

Counter

Count from 0 to N

Design

0. Do we need combinational logic or sequential logic?

Do we need memory?

Counter

Count from 0 to N

0 → 1 → 2 → 3 → … → N-1 → N

Counter

Count from 0 to N

$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow \ldots \rightarrow N\text{-}1 \rightarrow N$

At each step, we have to see at number we are and then move to next number: $i \rightarrow i+1$

Counter

Count from 0 to N

We need a storage to store current number.

We need a sequential circuit!

Design

1. How many storage (flip-flops)?

0
1

Depends on the storage you need to store the current state in binary system!

Counter

Count from 0 to N

N = 7

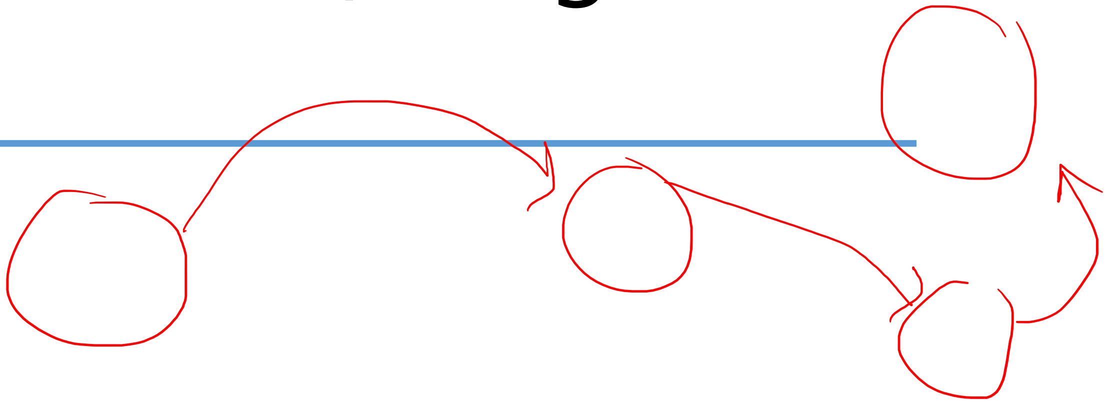
0 → 1 → 2 → 3 → 4 → 5 → 6 → 7

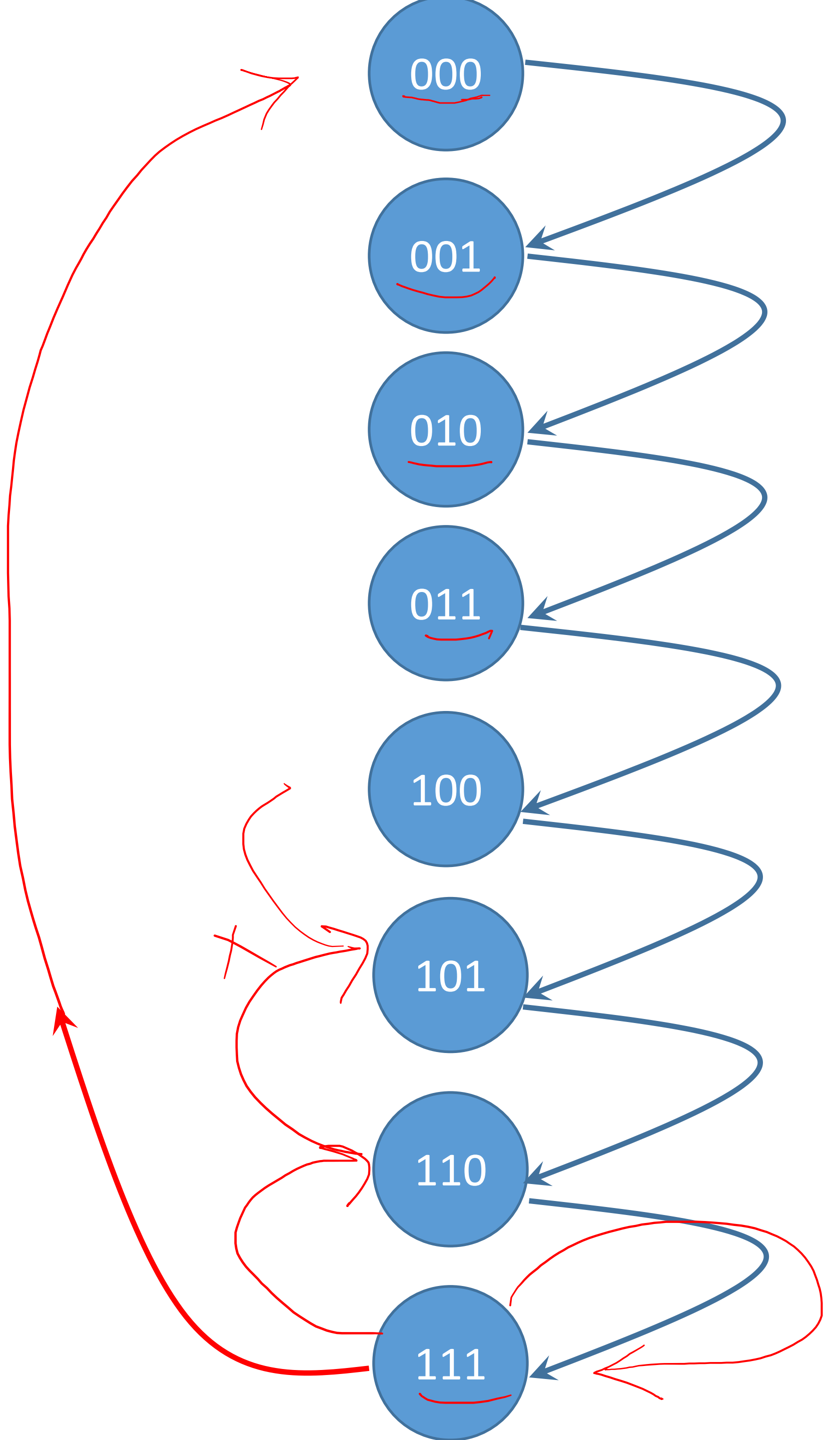
000 → 001 → 010 → 011 → 100 → 101 → 110 → 111

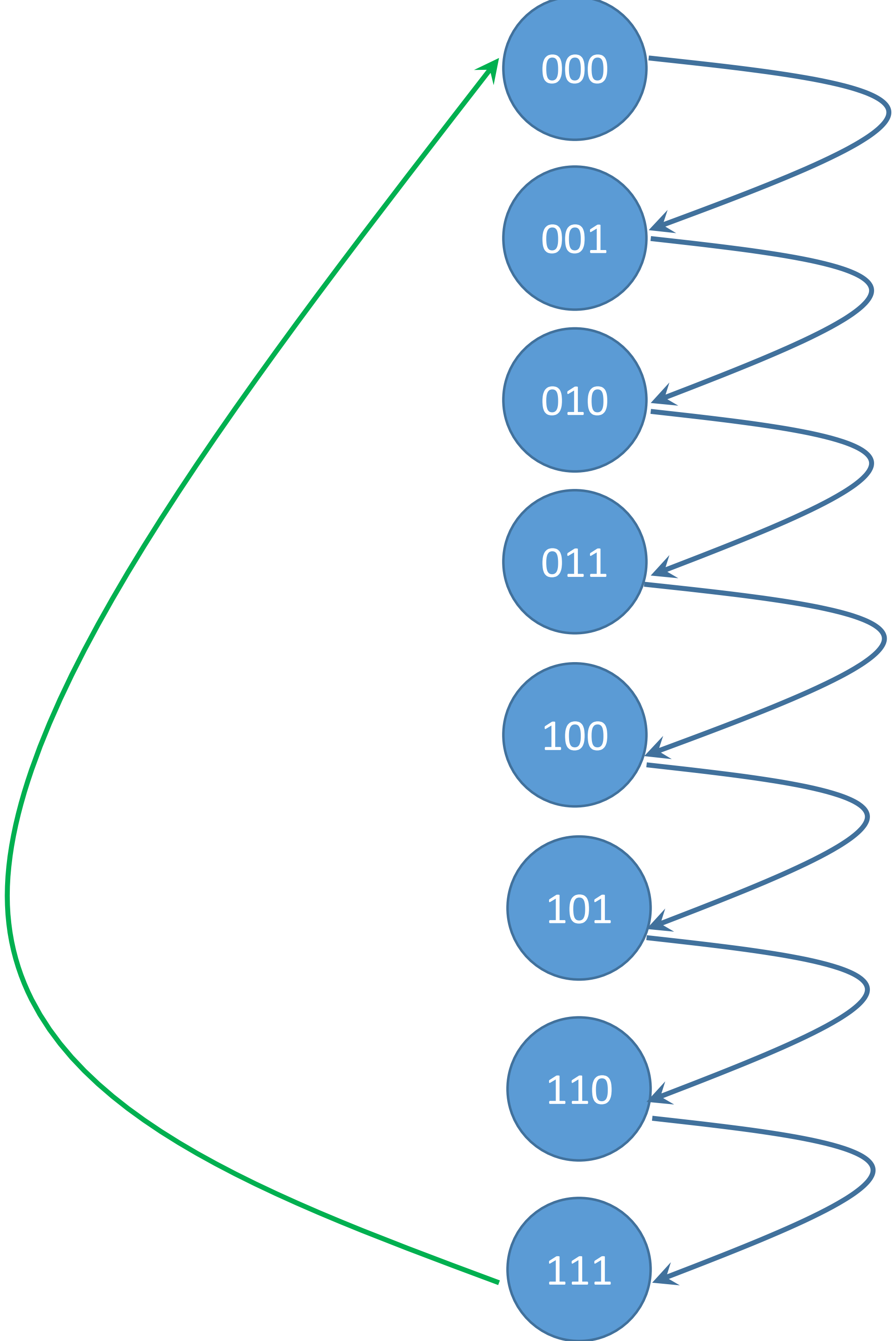
For each intermediate state, we need 3 bits → 3 flip-flops

Design
2. Form the state (transition) diagram

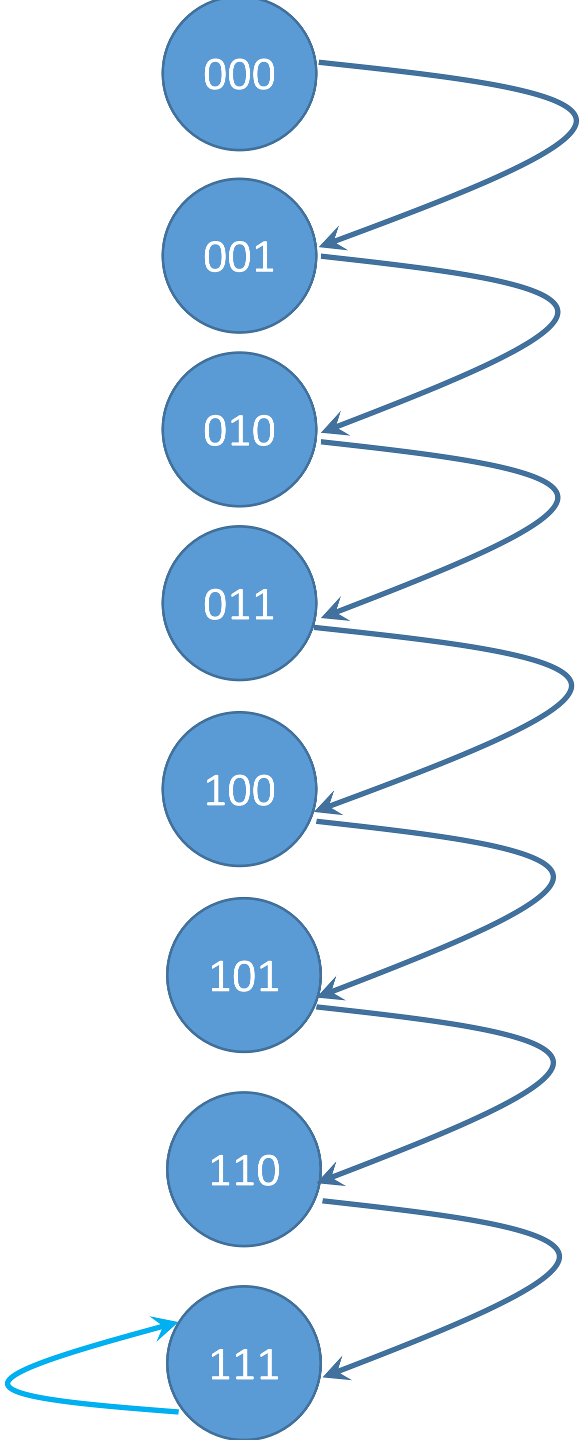
Same as analysis,
- For each state → one node
- For each state transition to next state → a directed edge




000
001
010
011
100
101
110
111



Loop to the beginning!

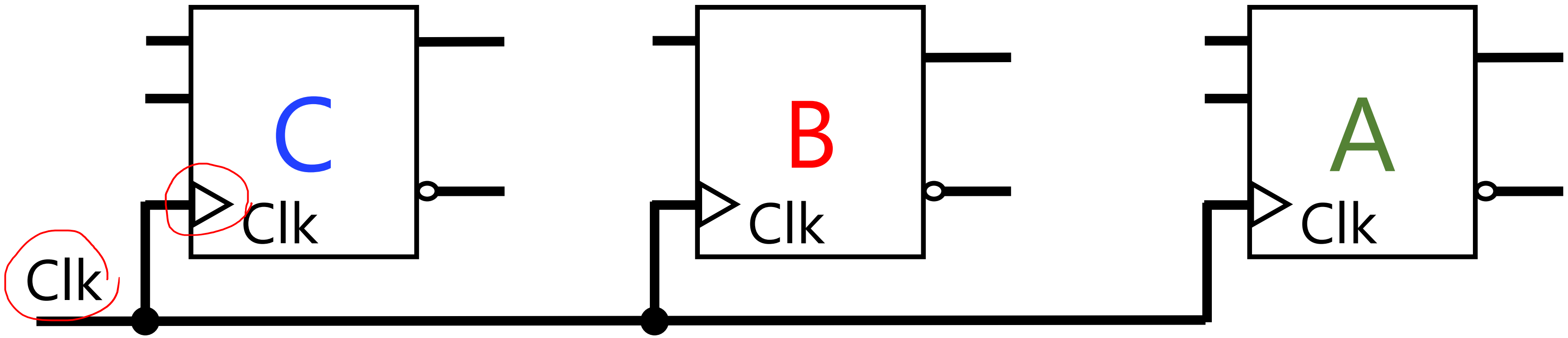


Stuck in 7
Just one time counter!

Design
3. Form the state table

Same as analysis, two columns for each flip-flop (storage unit)

a) One for current state Q(T)
b) One for next state Q(T+1)

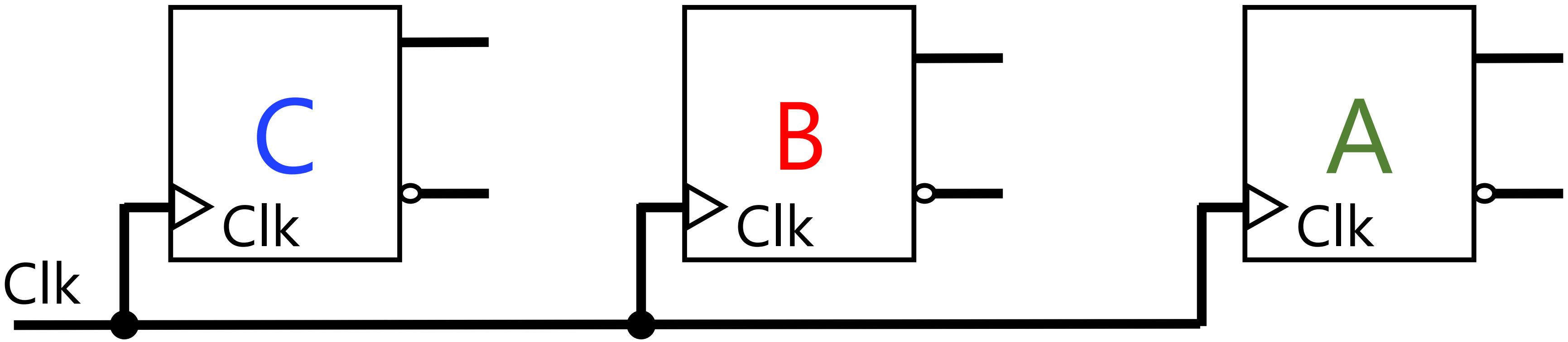


Q(T)			Q(T+1)		
C	B	A	C	B	A

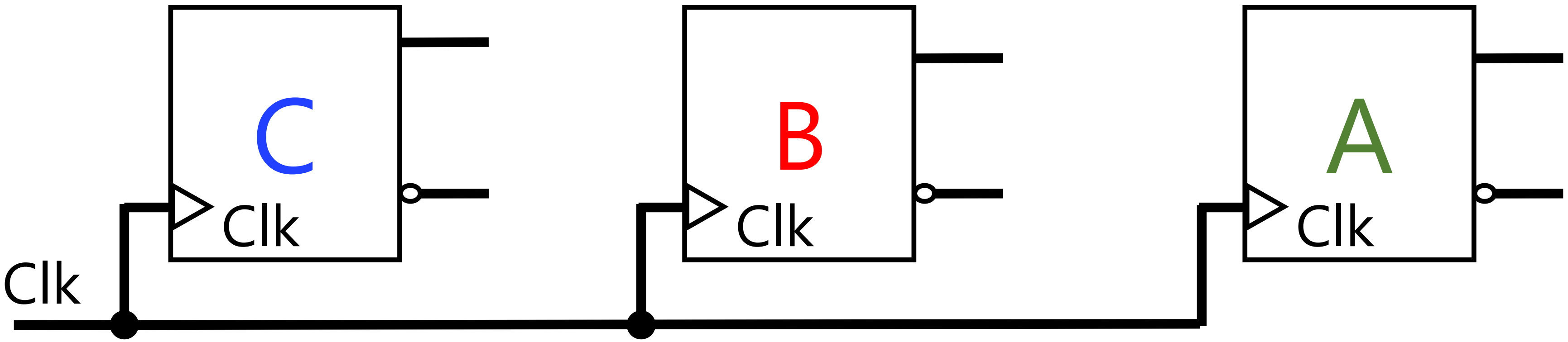
Design

4. Fill the state table

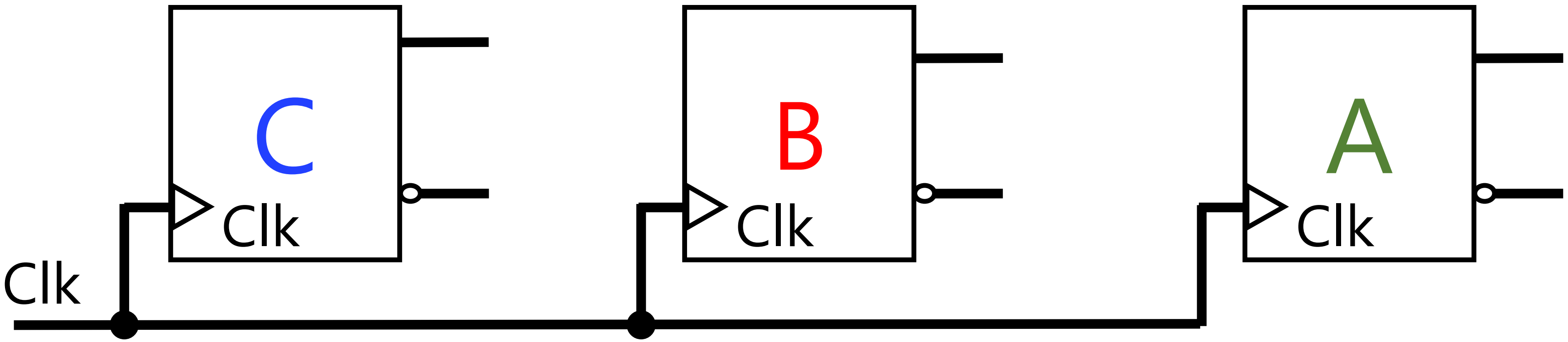
Unlike analysis, here we already know what is going to be the next state Q(T+1) based on current state Q(T)



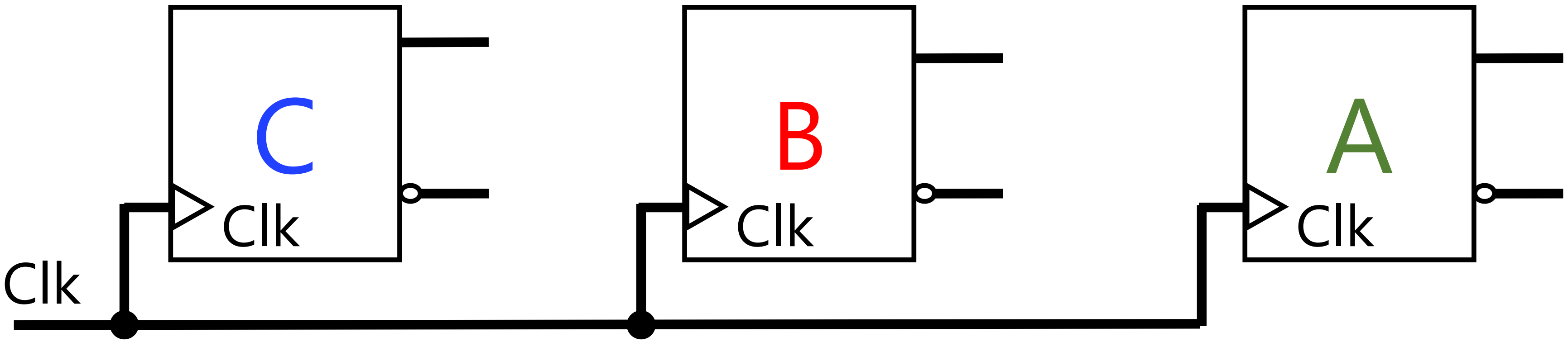
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



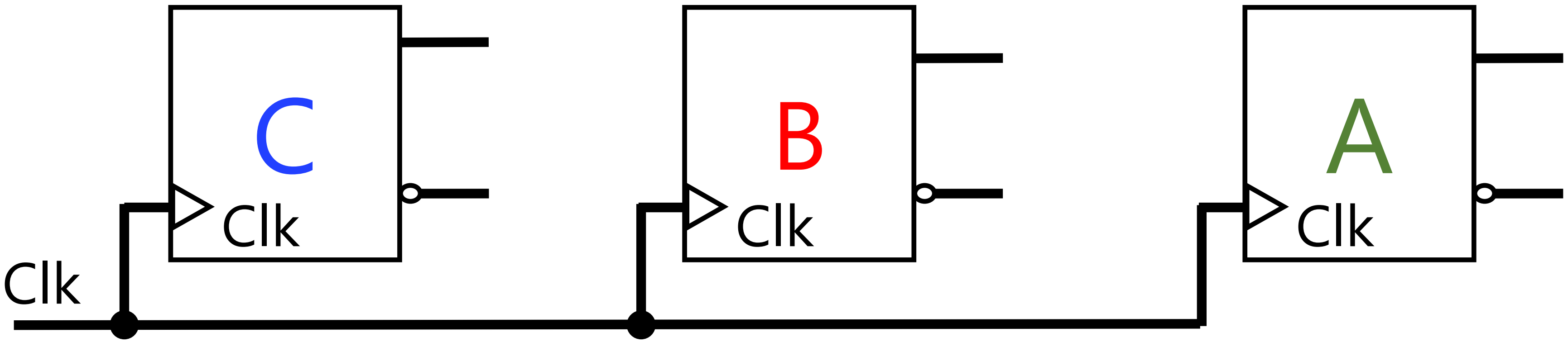
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



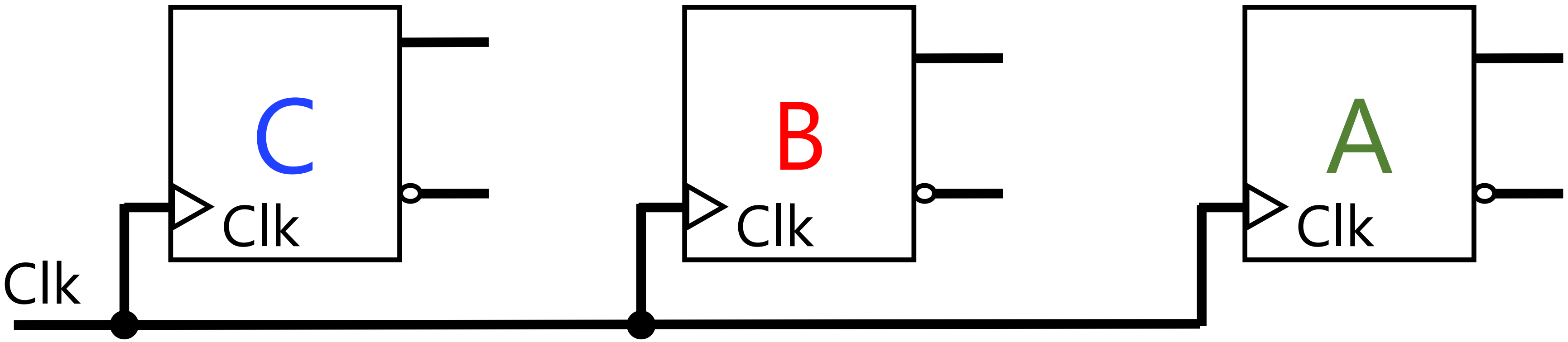
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



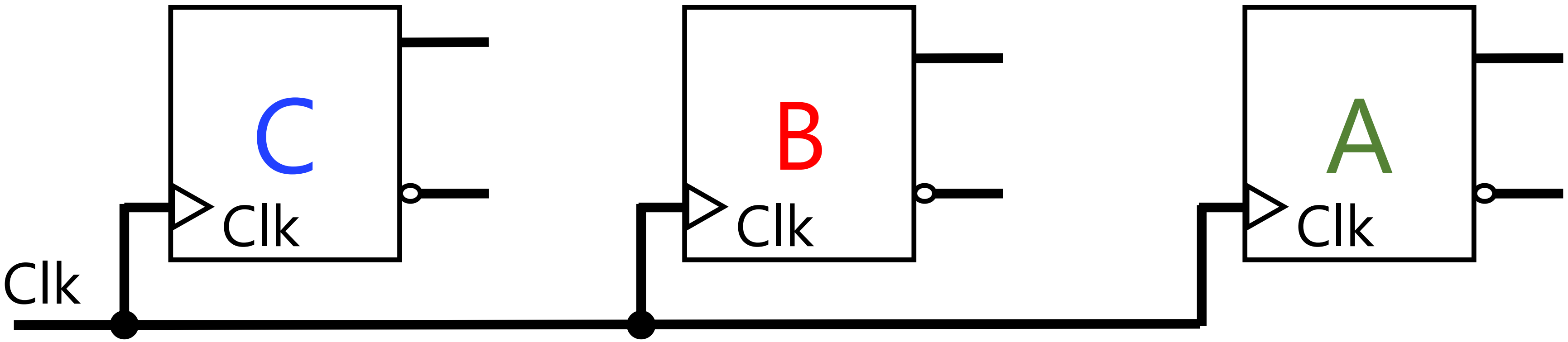
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0			
1	0	1			
1	1	0			
1	1	1			



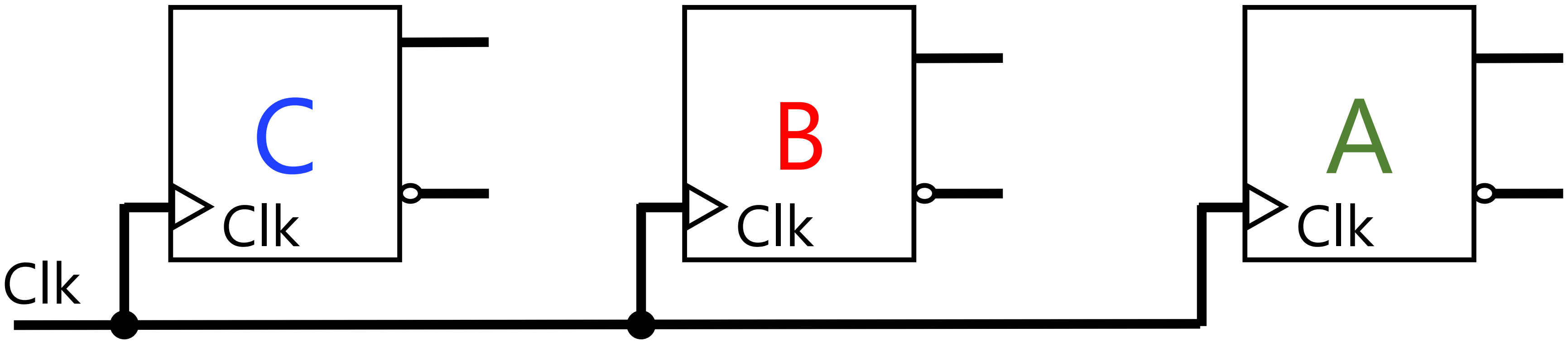
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1			
1	1	0			
1	1	1			



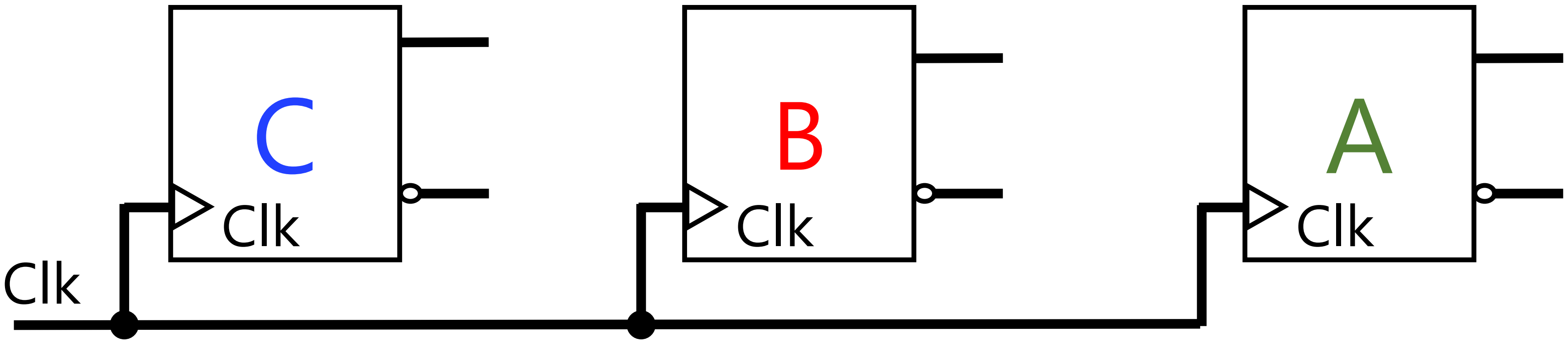
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0			
1	1	1			



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1			

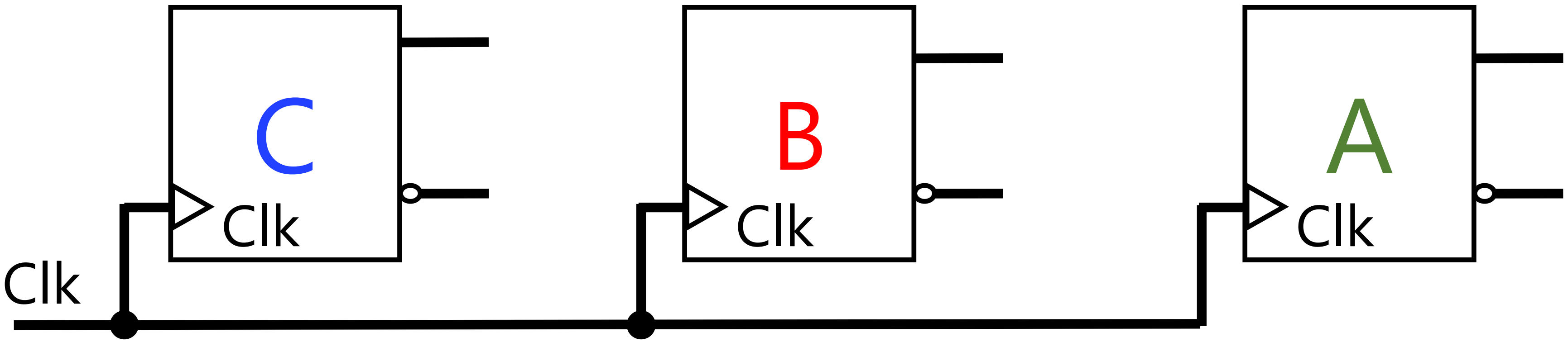


Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	?	?	?



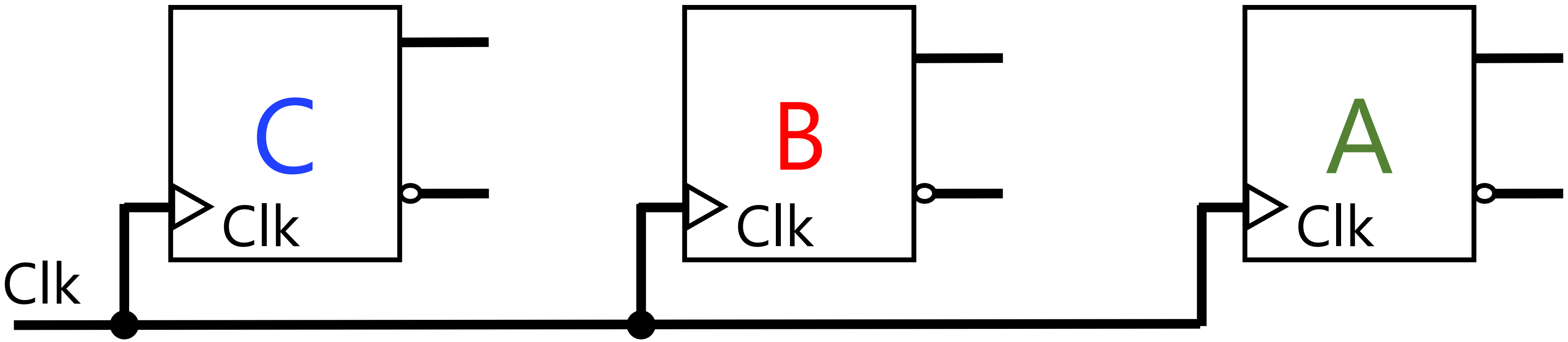
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	[illegible]	1
1	0	1	1	[illegible]	0
1	1	0	1	[illegible]	1
1	1	1	0	0	0

Loop to the beginning!



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1		1
1	0	1	1		0
1	1	0	1		1
1	1	1	1	1	1

Stuck in 7
Just one time counter!



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	[illegible]	1
1	0	1	1	[illegible]	0
1	1	0	1	1	1
1	1	1	0	0	0

Our Design Choice!

Design

5. What type of storage (flip-flop)?

RS, D, T, JK, or Mixed

Design

5. What type of storage (flip-flop)?

RS, D, T, JK, or Mixed

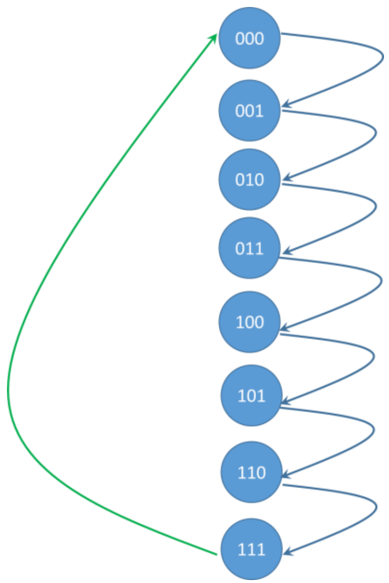
In terms of design, does not matter.

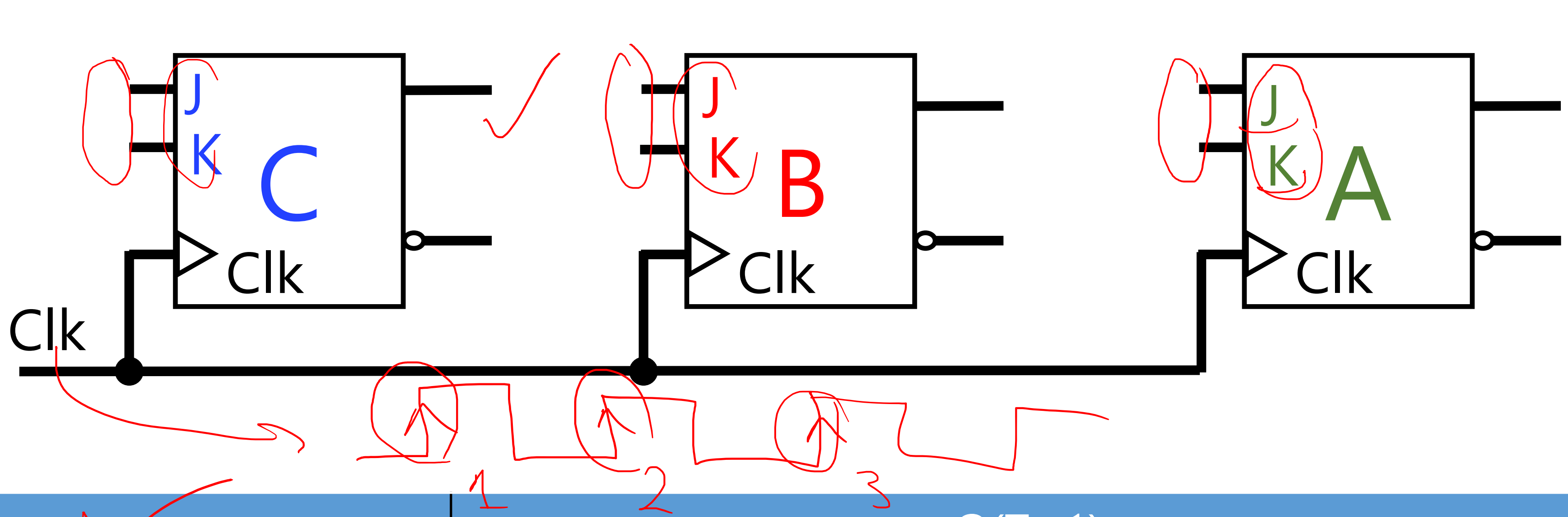
In terms of efficiency, matters!

Counter

Count from 0 to N=7

Let's select JK, the complete FF.





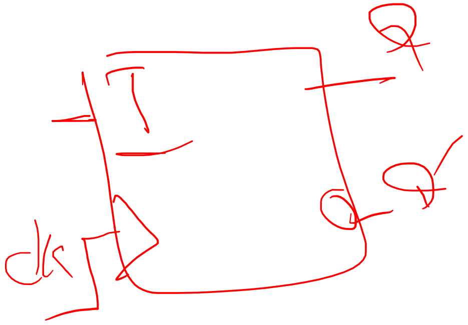
Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

Design

6. Boolean expression for the flip-flops' input?

input equations, aka, *excitation* equations

T_A D_A R_A S_A



T_B D_B R_B S_B

T_C D_C R_C S_C

Counter

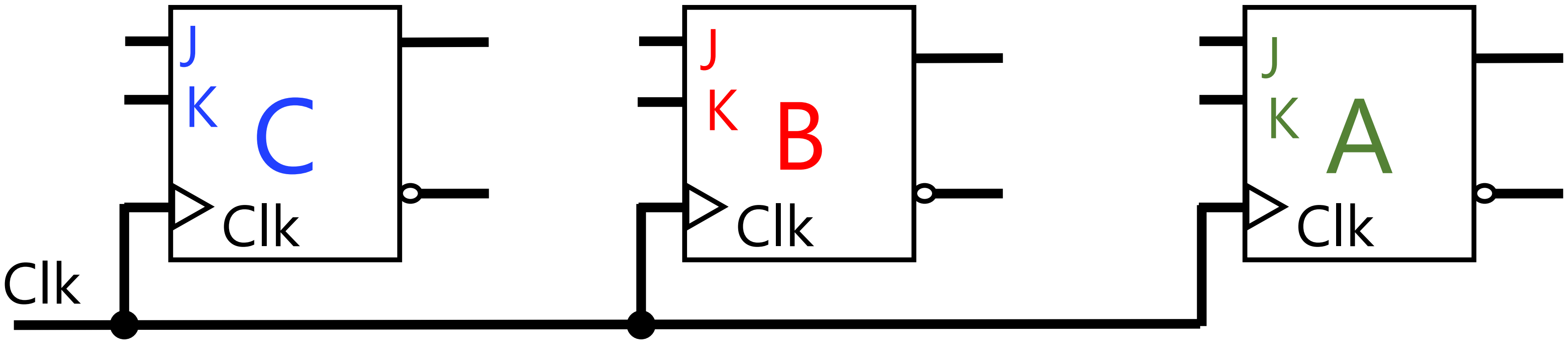
Count from 0 to N=7

A	$J_A=$	$K_A=$
B	$J_B=$	$K_B=$
C	$J_C=$	$K_C=$

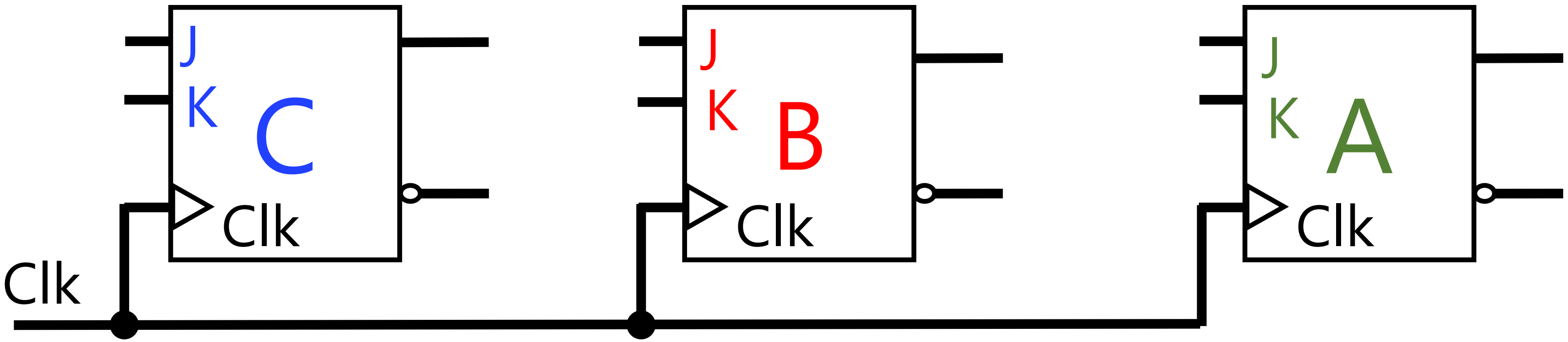
Counter

Count from 0 to N=7

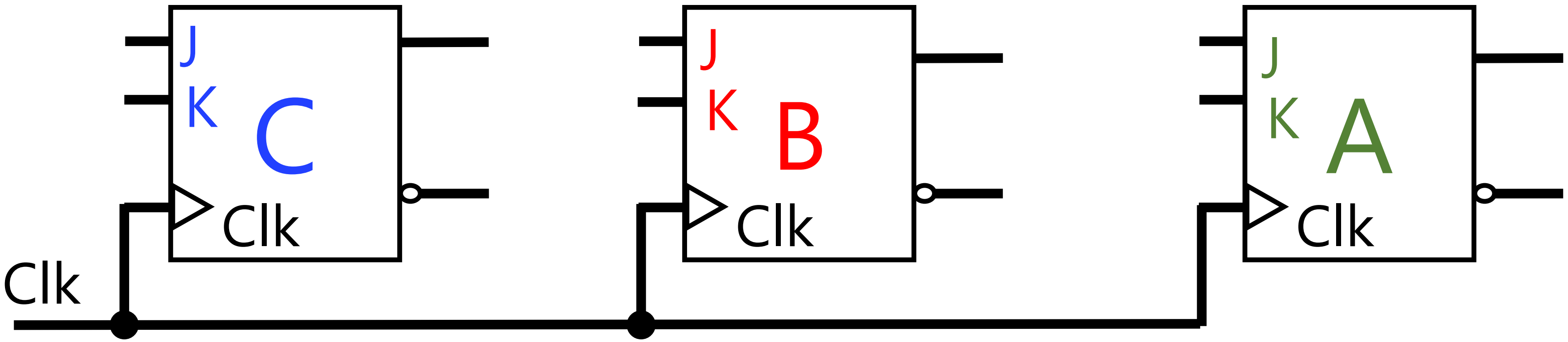
A	$J_A=$	$K_A=$
B	$J_B=$	$K_B=$
C	$J_C=$	$K_C=$



Q(T)			Q(T+1)		
C	B	A	C	B	A
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	0

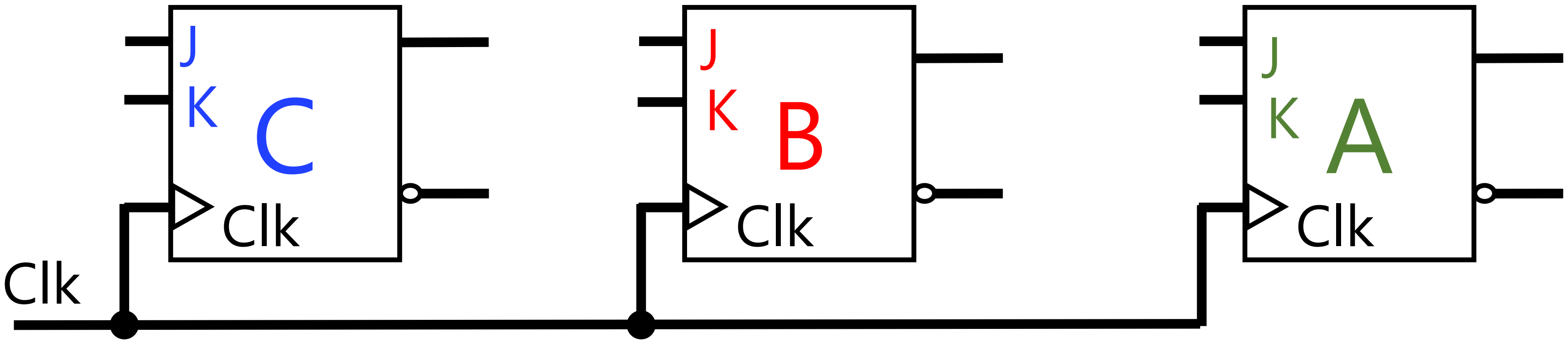


Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1			
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

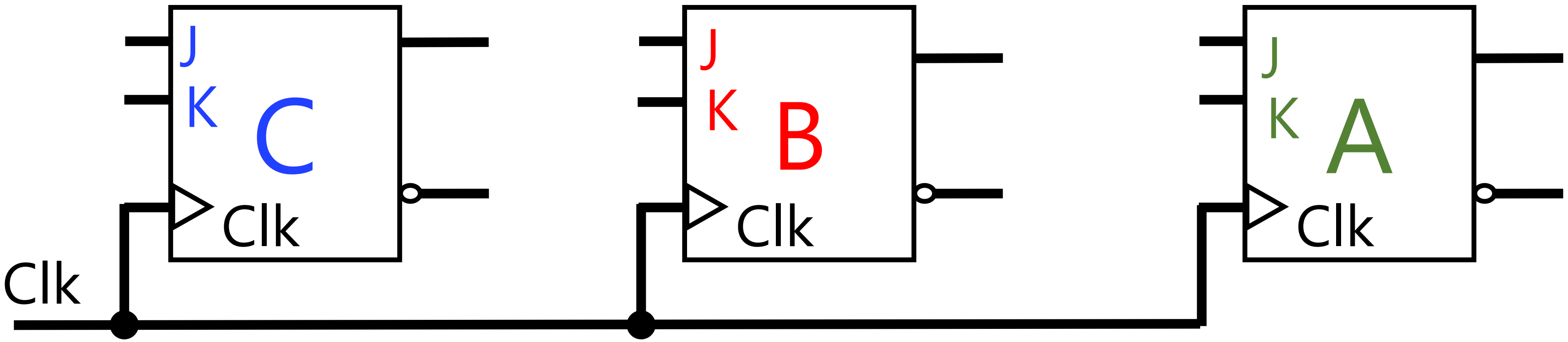


Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set	1	0
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

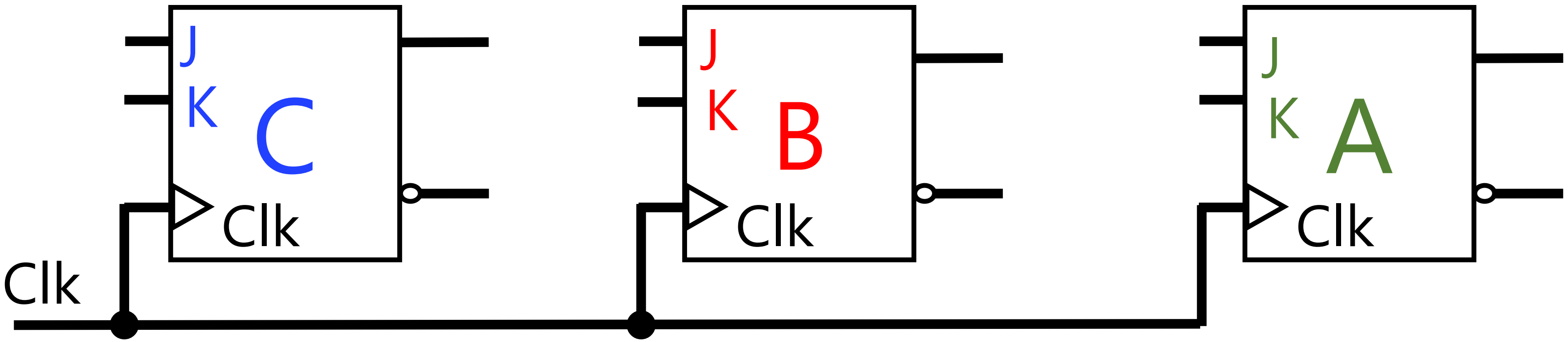
OR



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Comp	1	1
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

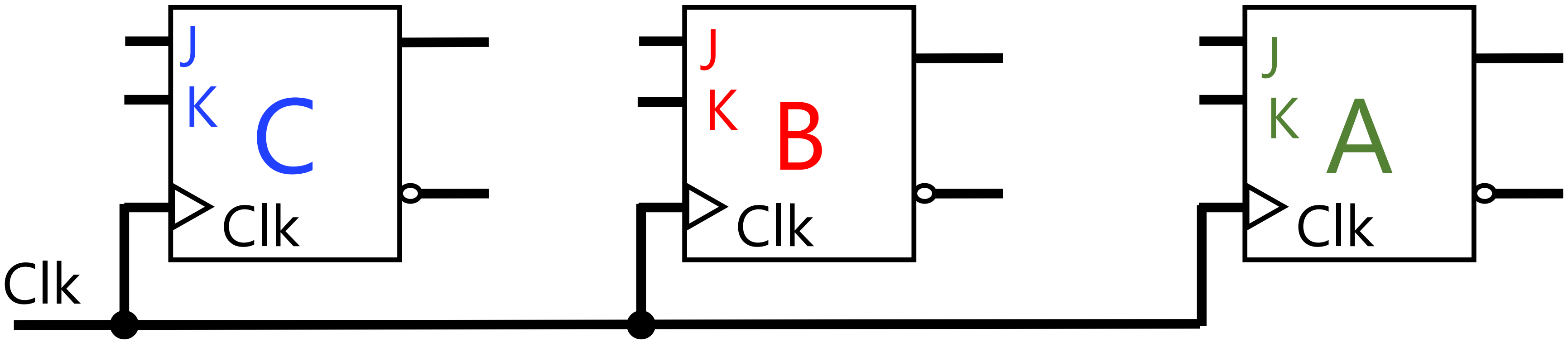


Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	0/1 → x
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

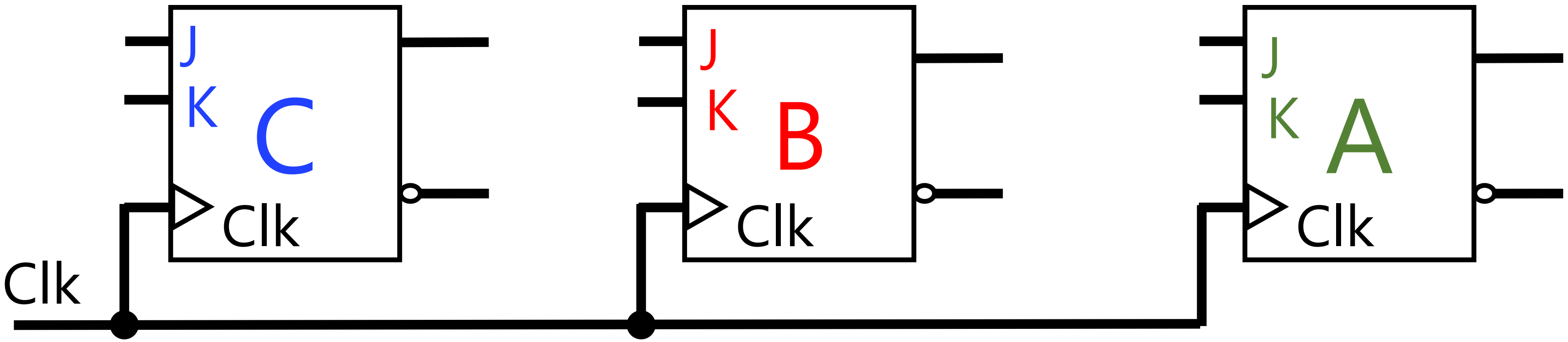


Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Reset	0	1
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

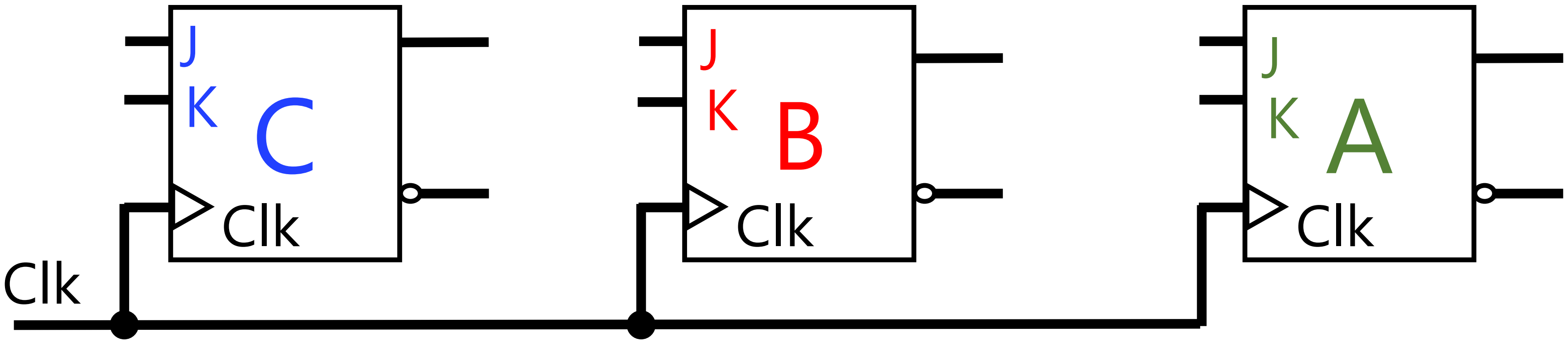
OR



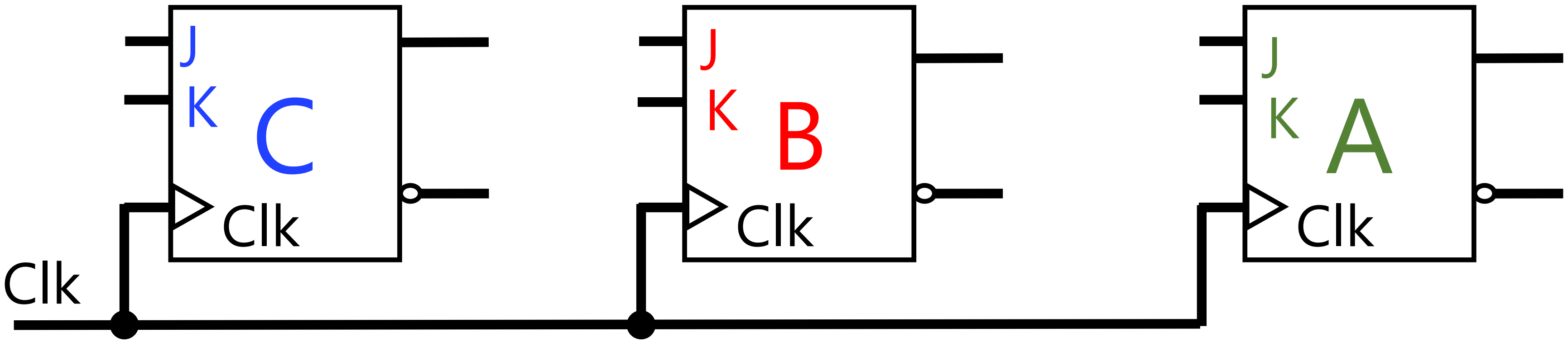
Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Comp.	1	1
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



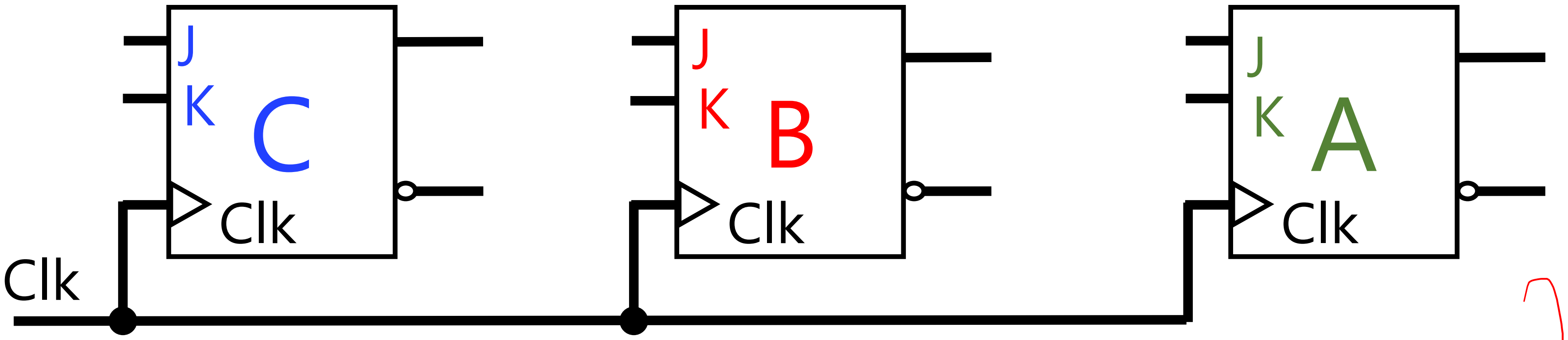
Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Reset/Comp	x	1
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Reset/Comp	x	1
0	1	0	0	1	1	Set/Comp	1	x
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Reset/Comp	x	1
0	1	0	0	1	1	Set/Comp	1	x
0	1	1	1	0	0	Reset/Comp	x	1
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



$J_A = \sum m(0,3)$

m_0 Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_A	K_A
0	0	0	0	0	1	Set/Comp	1	x
0	0	1	0	1	0	Reset/Comp	x	1
0	1	0	0	1	1	Set/Comp	1	x
0	1	1	1	0	0	Reset/Comp	x	1
1	0	0	1	0	1	Set/Comp	1	x
1	0	1	1	1	0	Reset/Comp	x	1
1	1	0	1	1	1	Set/Comp	1	x
1	1	1	0	0	0	Reset/Comp	x	1

Counter

Count from 0 to N=7

Flip-Flops

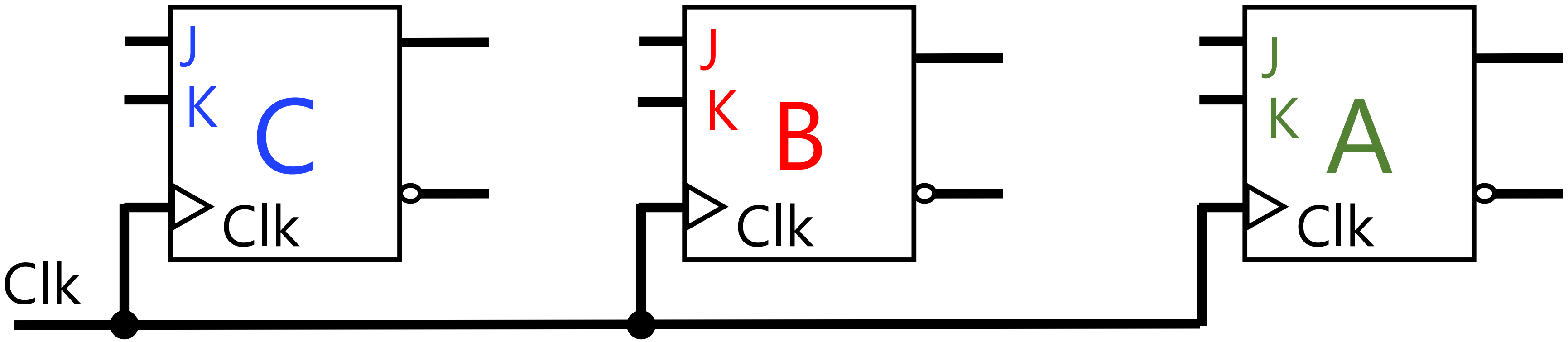
$Q_A(T)$

A	$J_A = F(C,B,A) = \sum(0,2,4,6) + d(1,3,5,7)$	$K_A = F(C,B,A) = \sum(1,3,5,7) + d(0,2,4,6)$
B	$J_B =$	$K_B =$
C	$J_C =$	$K_C =$

Counter

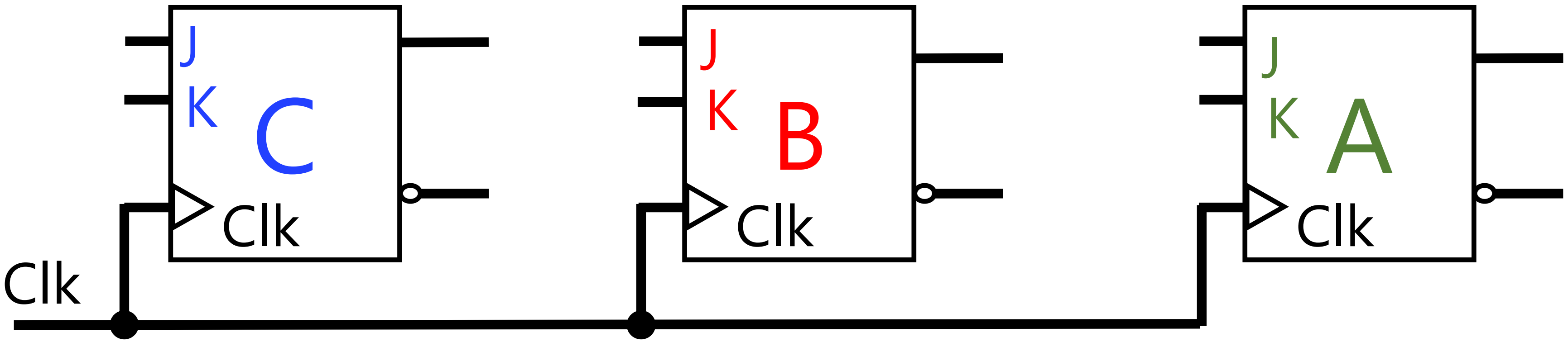
Count from 0 to N=7

A	$J_A=F(C,B,A)=\sum(0,2,4,6)+d(1,3,5,7)$	$K_A= F(C,B,A)=\sum(1,3,5,7)+d(0,2,4,6)$
B	$J_B=$	$K_B=$
C	$J_C=$	$K_C=$

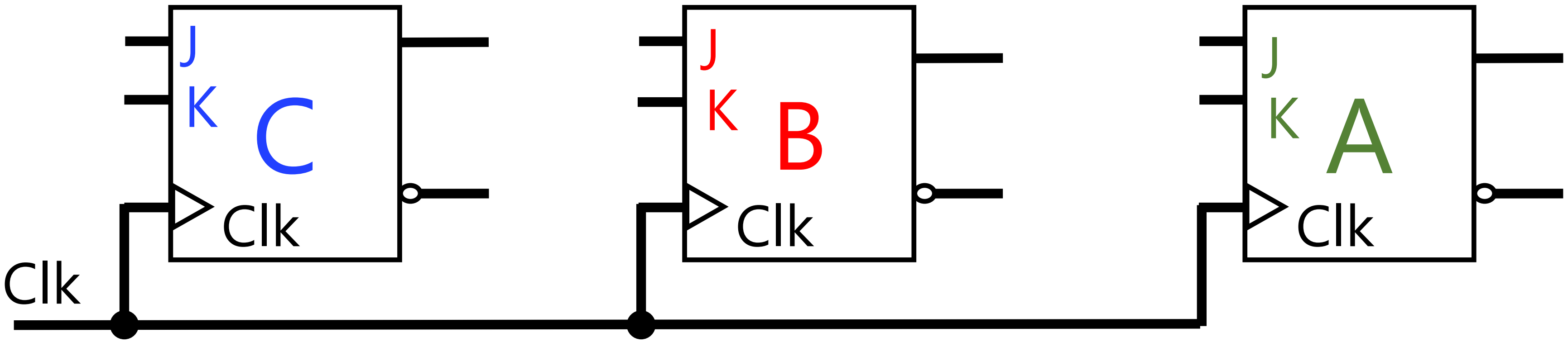


Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_B	K_B
0	0	0	0	0	1	Store	0	0
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			

OR



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_B	K_B
0	0	0	0	1 0	1	~~Reset~~ ~~Set~~	0	1
0	0	1	0	1	0			
0	1	0	0	1	1			
0	1	1	1	0	0			
1	0	0	1	0	1			
1	0	1	1	1	0			
1	1	0	1	1	1			
1	1	1	0	0	0			



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_B	K_B
0	0	0	0	0	1	Store/Reset	0	X
0	0	1	0	1	0	Set/Comp	1	X
0	1	0	0	1	1	Store/Set	X	0
0	1	1	1	0	0	Reset/Comp	X	1
1	0	0	1	0	1	Store/Reset	0	X
1	0	1	1	1	0	Set/Comp	1	X
1	1	0	1	1	1	Store/Set	X	0
1	1	1	0	0	0	Reset/Comp	X	1

Counter

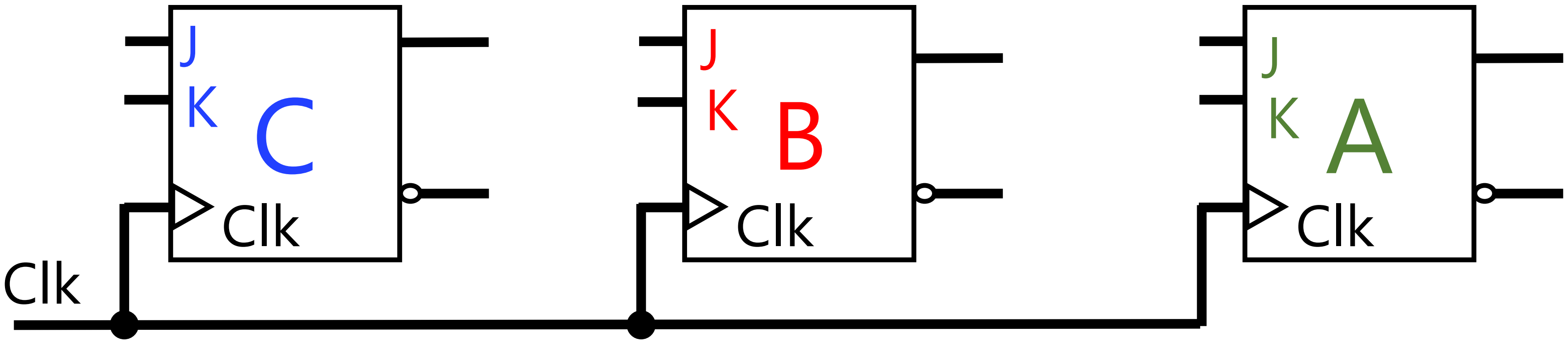
Count from 0 to N=7

A	$J_A=F(C,B,A)=\sum(0,2,4,6)+d(1,3,5,7)$	$K_A= F(C,B,A)=\sum(1,3,5,7)+d(0,2,4,6)$
B	$J_B=F(C,B,A)= \sum(1,5)+d(2,3,6,7)$	$K_B=F(C,B,A)= \sum(3,7)+d(0,1,4,5)$
C	$J_C=$	$K_C=$

Counter

Count from 0 to N=7

A	$J_A=F(C,B,A)=\sum(0,2,4,6)+d(1,3,5,7)$	$K_A= F(C,B,A)=\sum(1,3,5,7)+d(0,2,4,6)$
B	$J_B=F(C,B,A)= \sum(1,5)+d(2,3,6,7)$	$K_B=F(C,B,A)= \sum(3,7)+d(0,1,4,5)$
C	$J_C=$	$K_C=$



Q(T)			Q(T+1)			Not part of state table!		
C	B	A	C	B	A	Action	J_C	K_C
0	0	0	0	0	1	Store/Reset	0	X
0	0	1	0	1	0	Store/Reset	0	X
0	1	0	0	1	1	Store/Reset	0	X
0	1	1	1	0	0	Comp/Set	1	X
1	0	0	1	0	1	Store/Set	X	0
1	0	1	1	1	0	Store/Set	X	0
1	1	0	1	1	1	Store/Set	X	0
1	1	1	0	0	0	Comp/Reset	X	1

Counter

Count from 0 to N=7

3-variable

6 x

A	$J_A=F(C,B,A)=\sum(0,2,4,6)+d(1,3,5,7)$	$K_A= F(C,B,A)=\sum(1,3,5,7)+d(0,2,4,6)$
B	$J_B=F(C,B,A)= \sum(1,5)+d(2,3,6,7)$	$K_B=F(C,B,A)= \sum(3,7)+d(0,1,4,5)$
C	$J_C=F(C,B,A)=\sum(3)+d(4,5,6,7)$	$K_C=F(C,B,A)= \sum(7)+d(0,1,2,3)$

Design

7. Minimization of input (*excitation*) equations

Counter

Count from 0 to N=7

3-Variable K-Map

A	$J_A=F(C,B,A)=\sum(0,2,4,6)+d(1,3,5,7)$	$K_A= F(C,B,A)=\sum(1,3,5,7)+d(0,2,4,6)$
B	$J_B=F(C,B,A)= \sum(1,5)+d(2,3,6,7)$	$K_B=F(C,B,A)= \sum(3,7)+d(0,1,4,5)$
C	$J_C=F(C,B,A)=\sum(3)+d(4,5,6,7)$	$K_C=F(C,B,A)= \sum(7)+d(0,1,2,3)$

C \ BA	00	01	11	10
0	0 m_0	0 m_1	1 m_3	0 m_2
1	X m_4	X m_5	X m_7	X m_6

$J_C = F(C,B,A) = \sum(3) + d(4,5,6,7)$
$J_C = BA$

C \ BA	00	01	11	10
0	X m_0	X m_1	X m_3	X m_2
1	0 m_4	0 m_5	1 m_7	0 m_6

$K_C = F(C,B,A) = \sum(7) + d(0,1,2,3)$
$K_C = BA$

C \ BA	00	01	11	10
0	0 m_0	1 m_1	X m_3	X m_2
1	0 m_4	1 m_5	X m_7	X m_6

$J_B = F(C,B,A) = \sum(1,5) + d(2,3,6,7)$
$J_B = A$

C \ BA	00	01	11	10
0	X m_0	X m_1	0 m_3	$\cancel{1}$ 0 m_2
1	X m_4	X m_5	0 m_7	$\cancel{1}$ 0 m_6

$K_B = F(C,B,A) = \sum(3,7) + d(0,1,4,5)$
$K_B = \cancel{A'}$ A

C \ BA	00	01	11	10
0	1 m_0	X m_1	X m_3	1 m_2
1	1 m_4	X m_5	X m_7	1 m_6

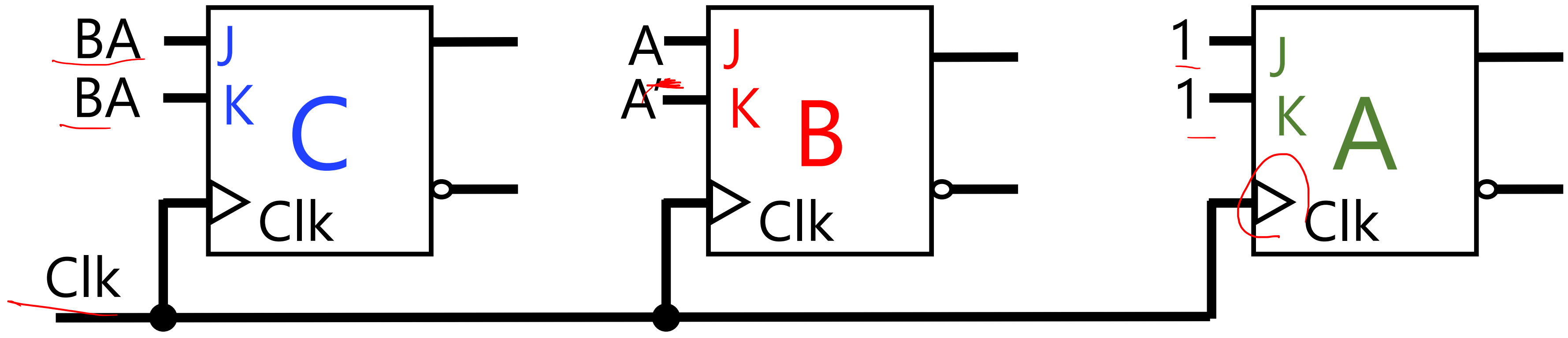
$J_A = F(C,B,A) = \sum(0,2,4,6) + d(1,3,5,7)$
$J_A = 1$

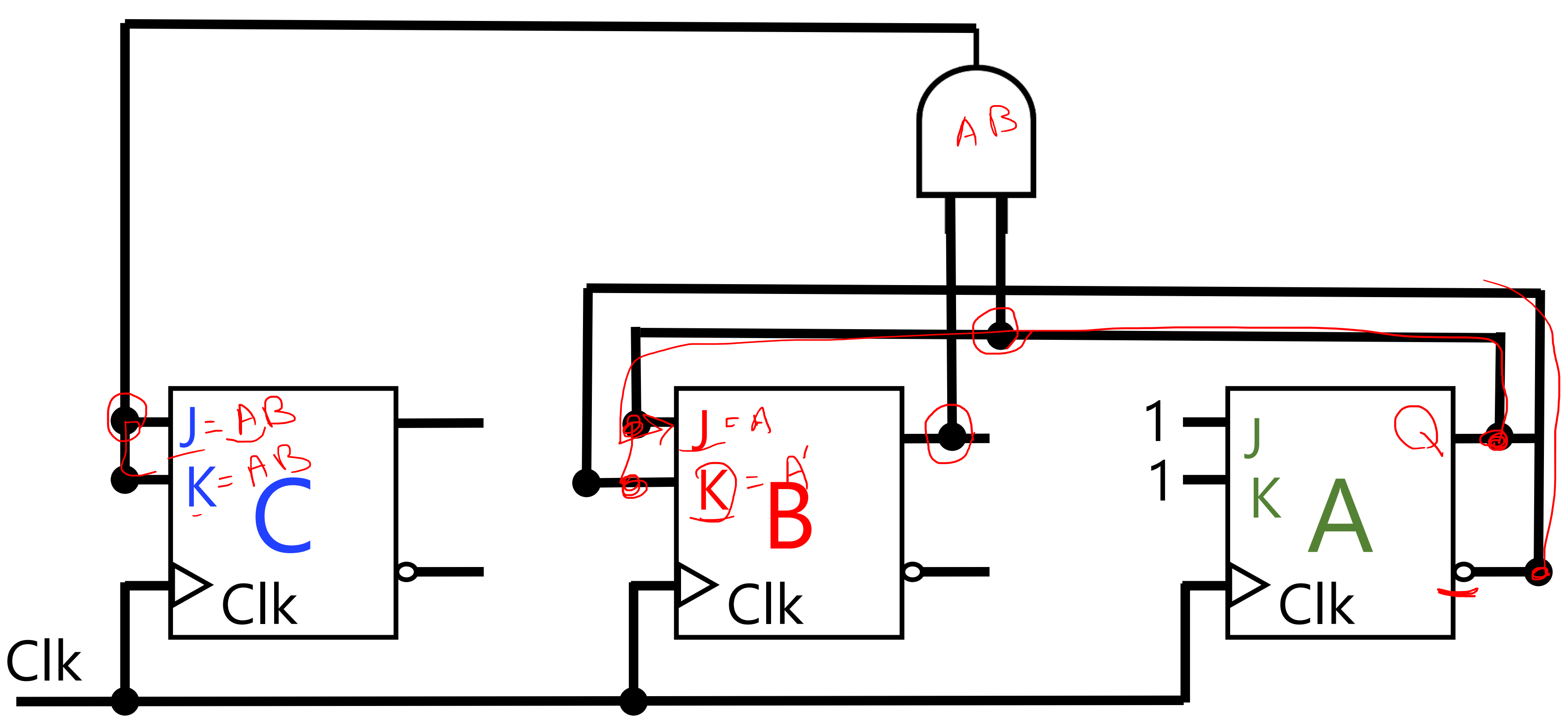
C \ BA	00	01	11	10
0	X m_0	1 m_1	1 m_3	X m_2
1	X m_4	1 m_5	1 m_7	X m_6

$K_A = F(C,B,A) = \sum(1,3,5,7) + d(0,2,4,6)$
$K_A = 1$

Design

8. Draw/Sketch Logic Circuit

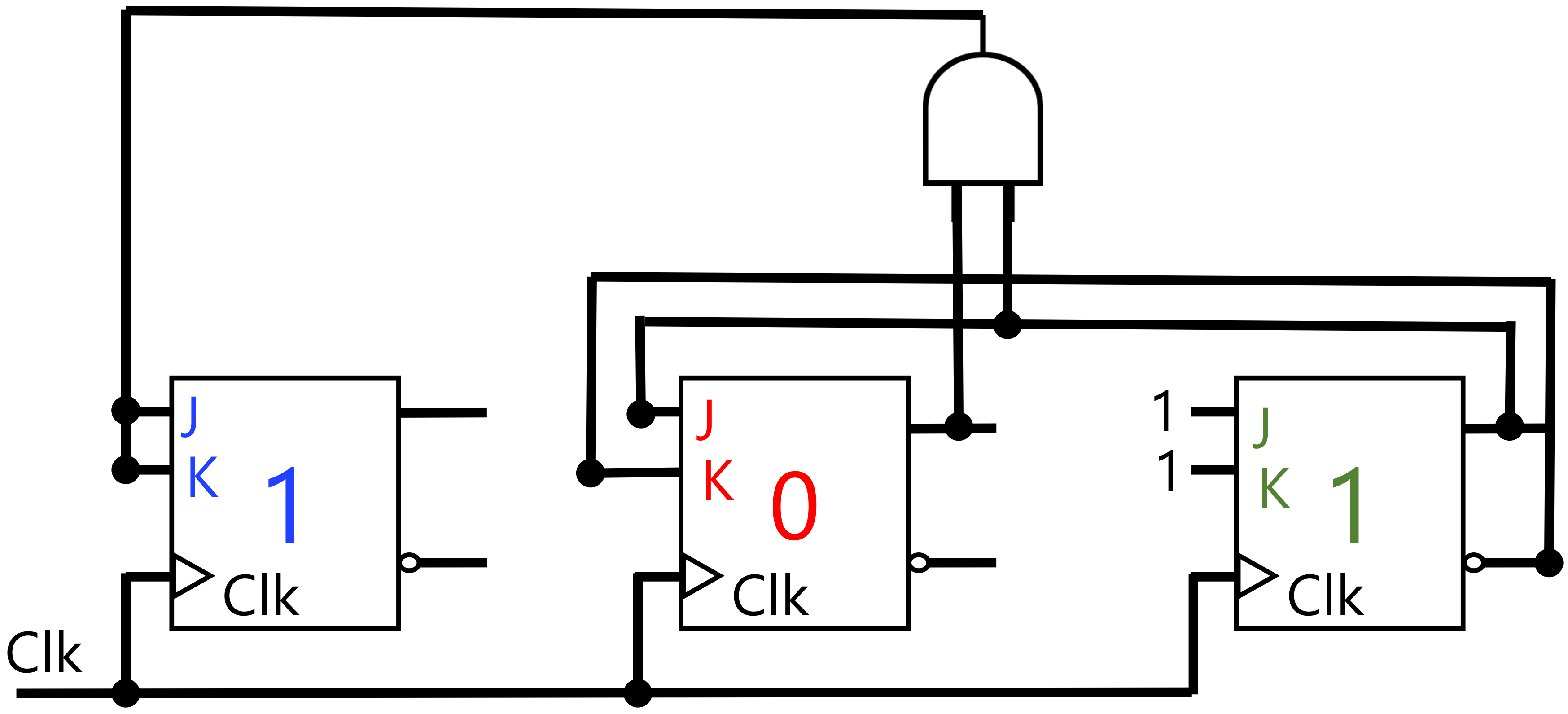

BA
BA
J
K
C
Clk
A
A'
J
K
B
Clk
1
1
J
K
A
Clk
Clk



AB
J = AB
K = AB
C
Clk
J = A
K = A'
B
Clk
1
1
J
K
A
Q
Clk
Clk

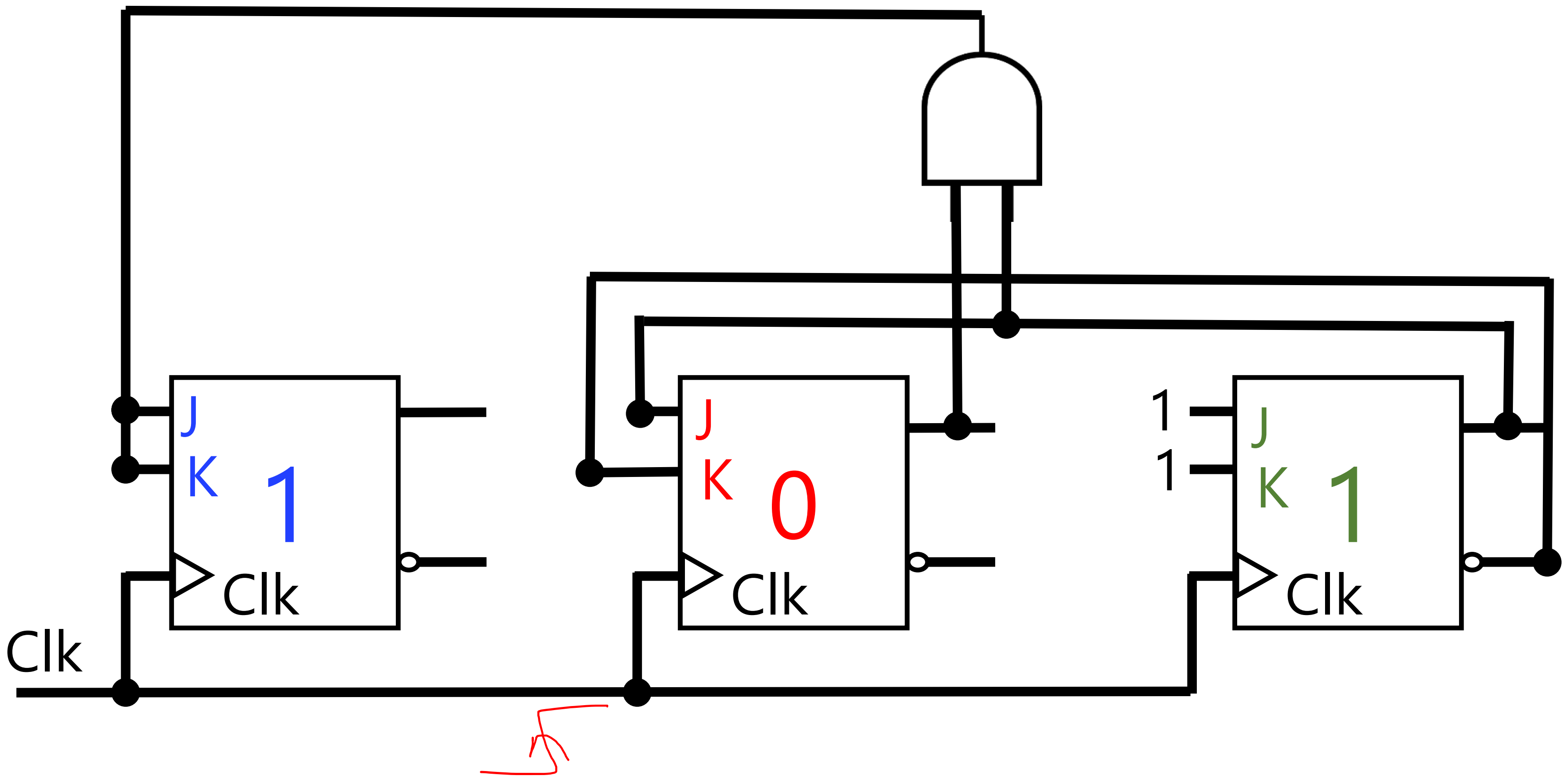
Design

9. (Optional) Test

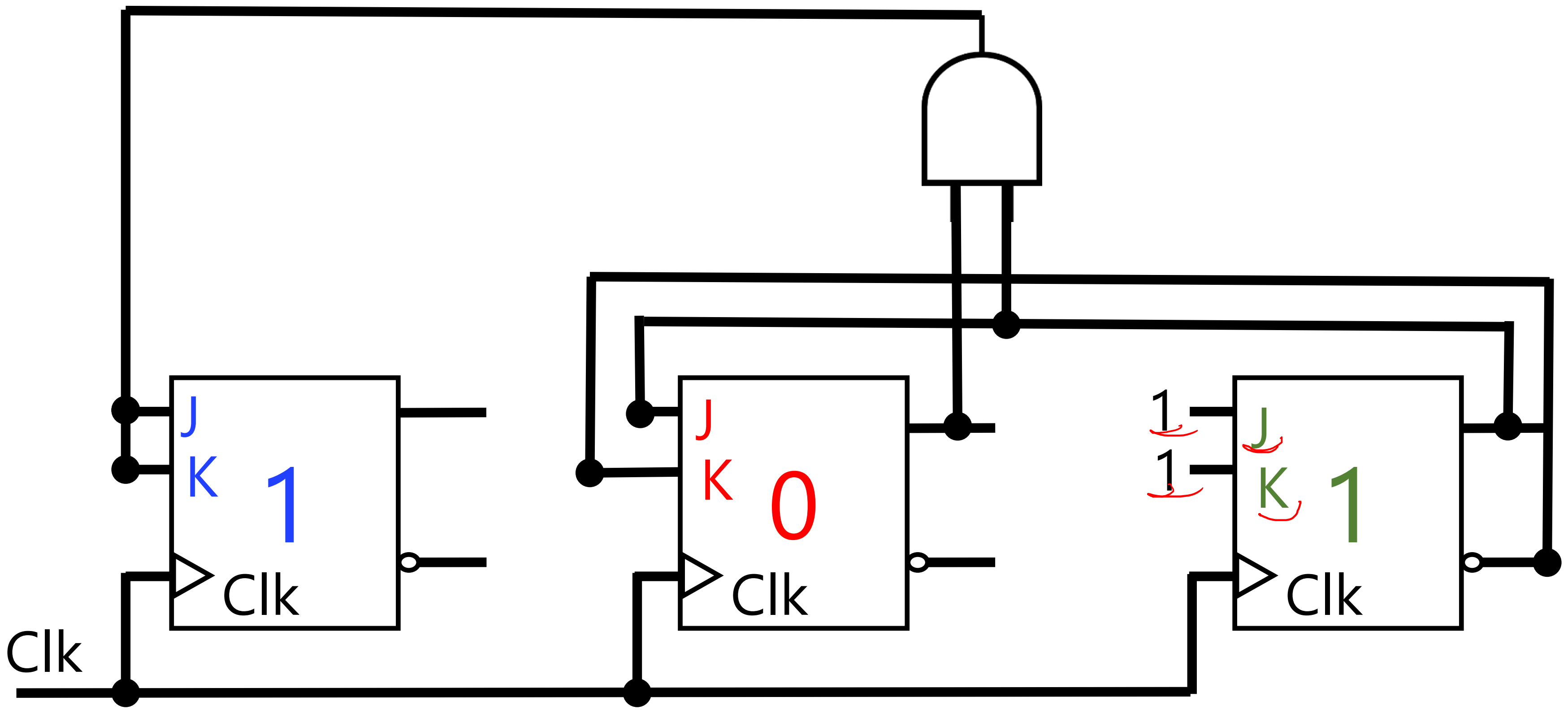


Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	?	?

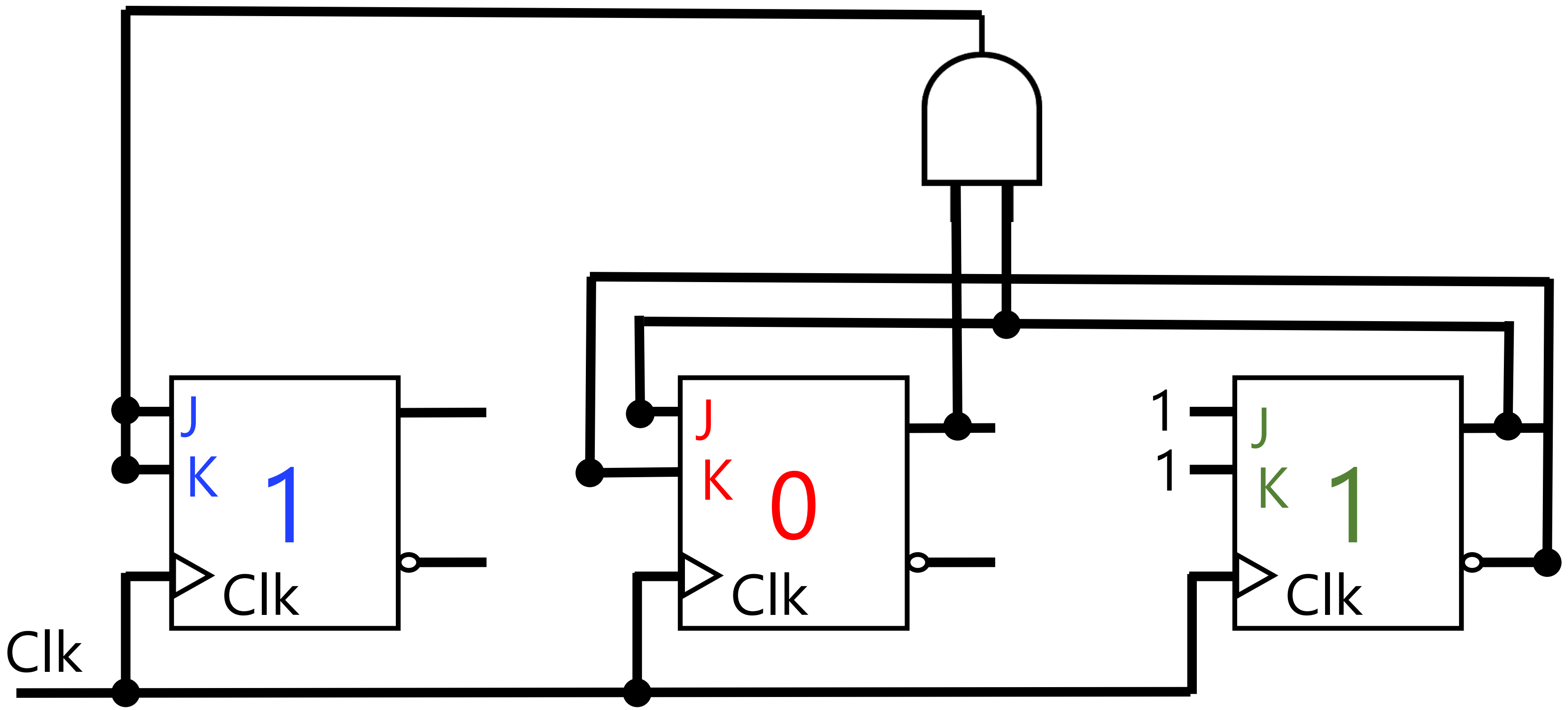
5 → ? 6



Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	?	?
5 → ?					

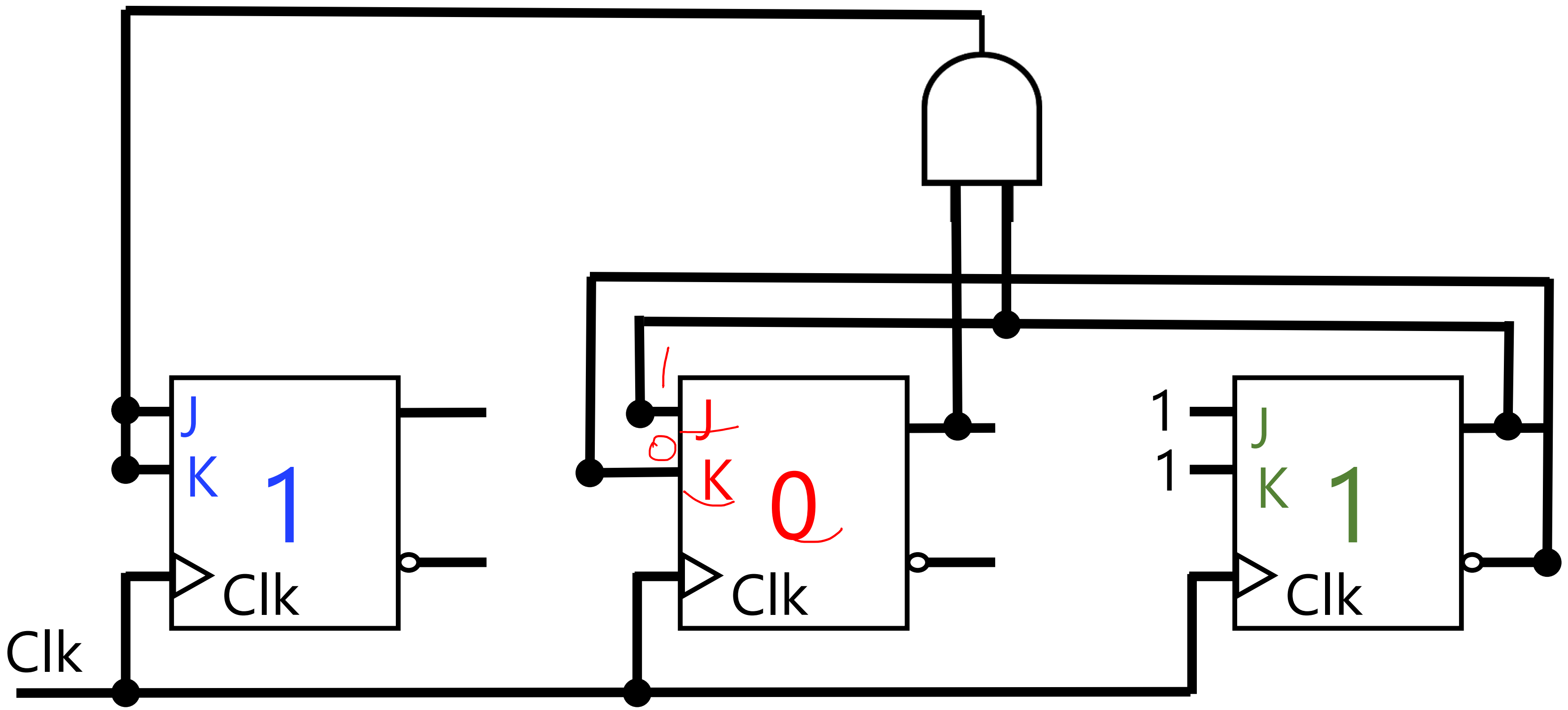


Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	?	A=1, J_A=1, K_A=1 ----------- Comp. → 0
5 → ?					



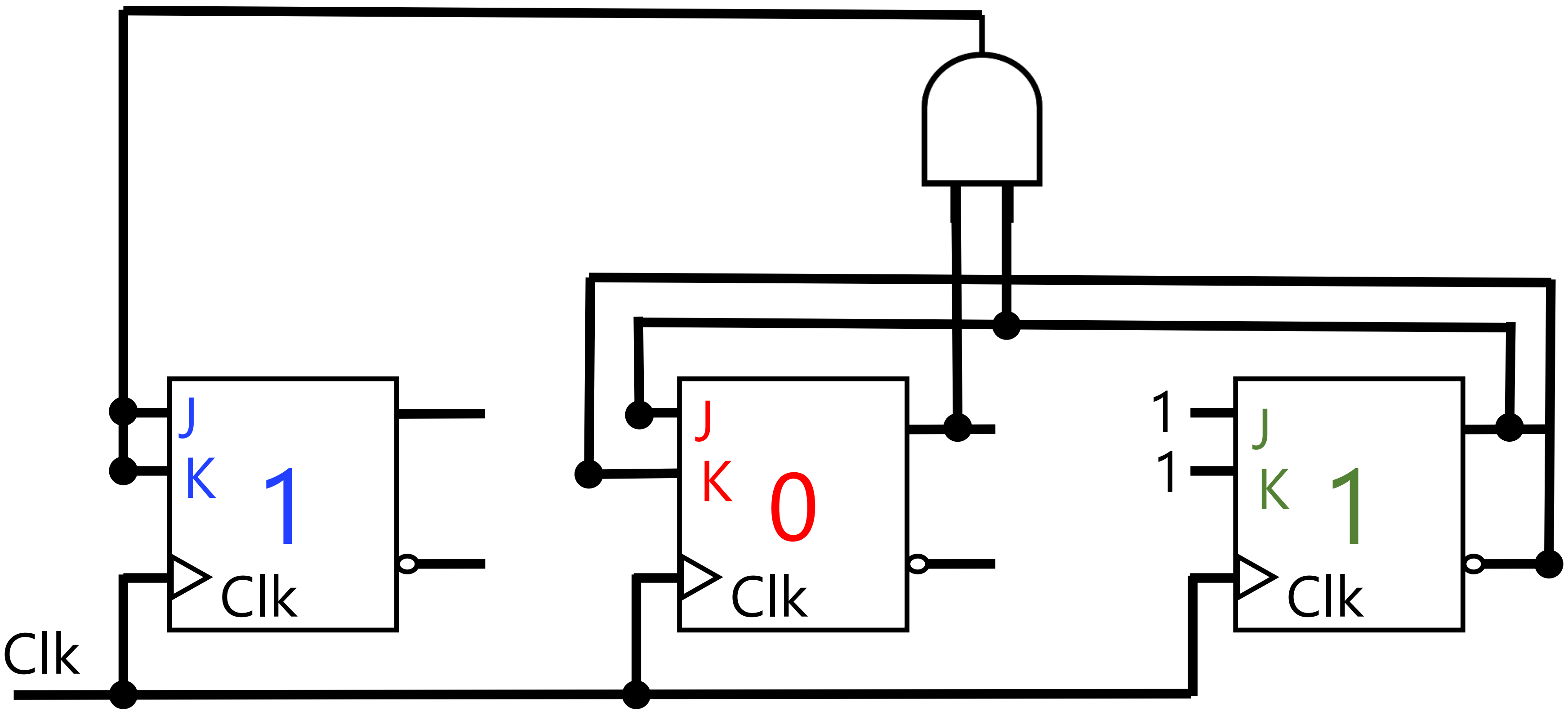
Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	?	0

5 → ?



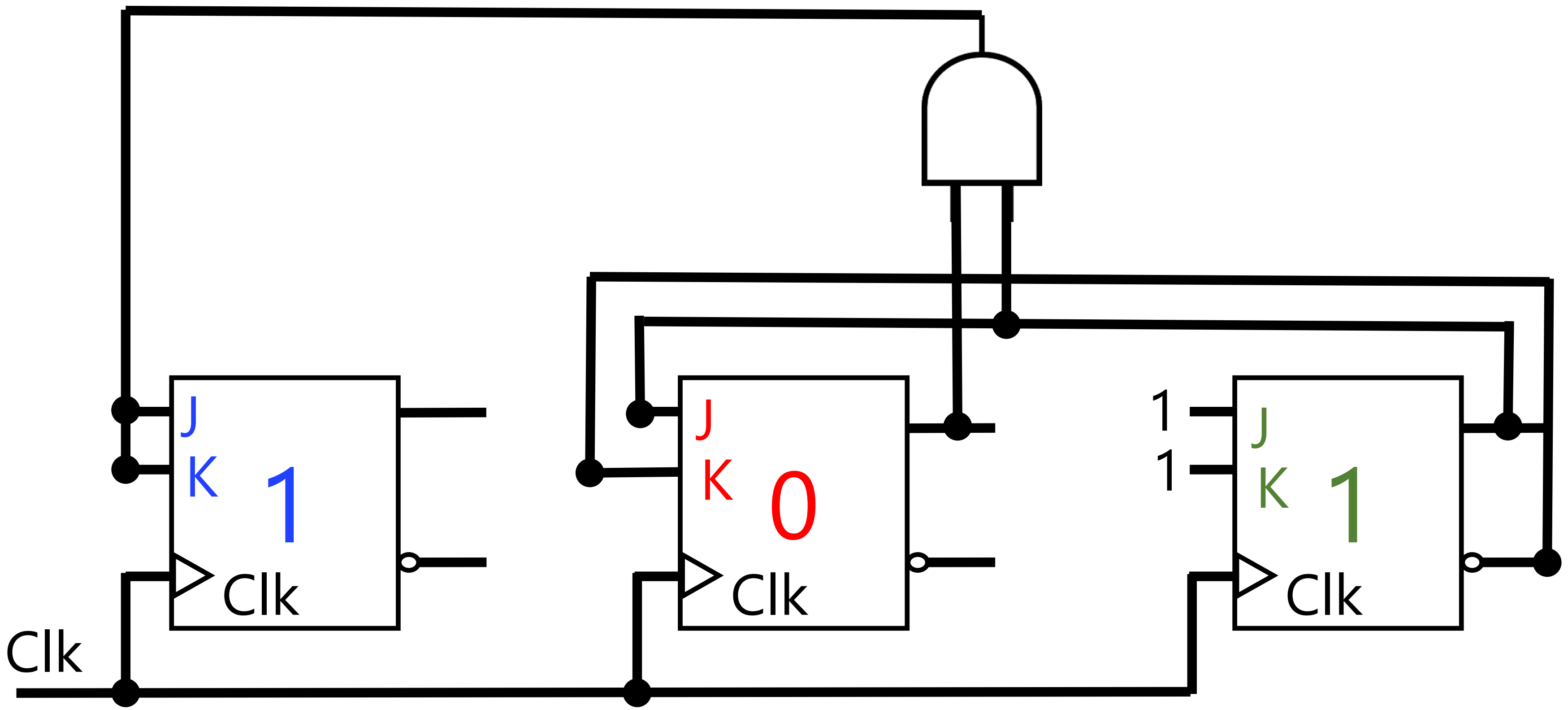
Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	B=0, J_B=A=1, K_B=A'=0 --------- Set → 1	0

5 → ?



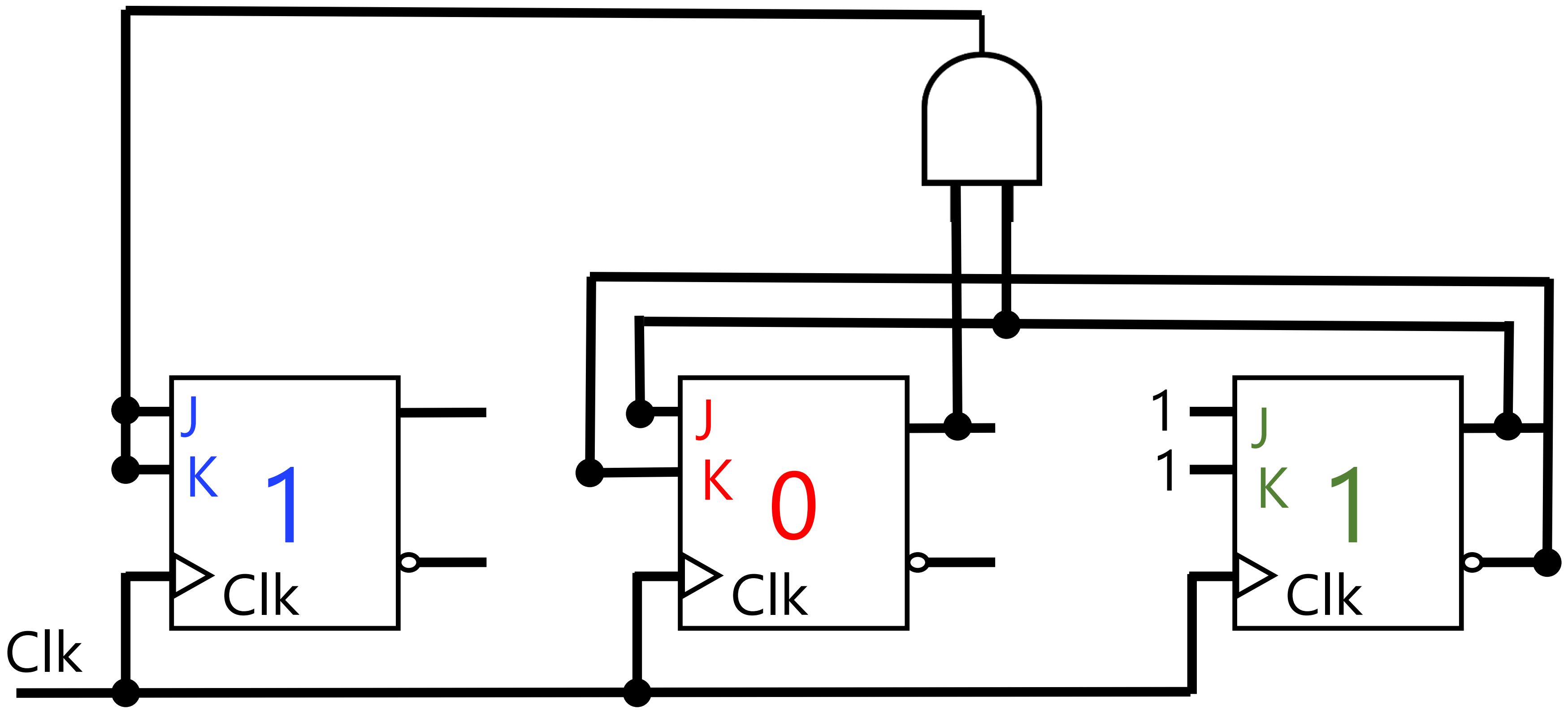
Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	1	0

5 → ?

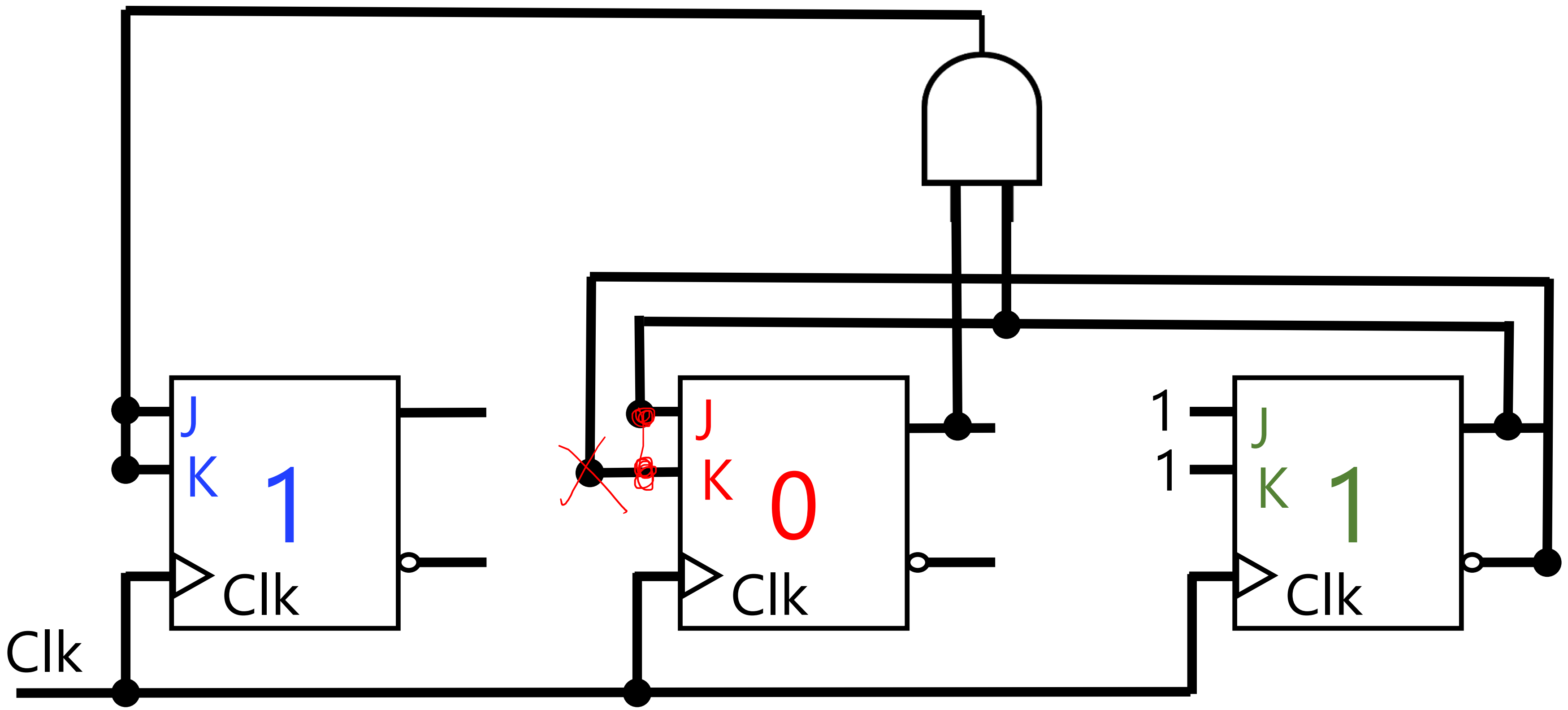


Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	?	1	0

5 → ?



Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	C=1, J_C=BA=01=0 K_C=BA=01=0 --------------- Store → 1	1	0
5 → ?					



Q(T)			Q(T+1)		
C	B	A	C	B	A
1	0	1	1	1	0
0	1	1	1	0	0

5 → 6

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. Form the state (transition) diagram
3. Form the state table
4. Fill the state table

easy

5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
6. Input (*excitation*) equations for each FF
7. Minimization of input (*excitation*) equations
8. Draw/Sketch Logic Circuit
9. (Optional) Test

Design (Recap)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. Form the state (transition) diagram
3. Form the state table
4. Fill the state table
5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
6. Input (*excitation*) equations for each FF
7. Minimization of input (*excitation*) equations
8. Draw/Sketch Logic Circuit
9. (Optional) Test

$0 \rightarrow 7$

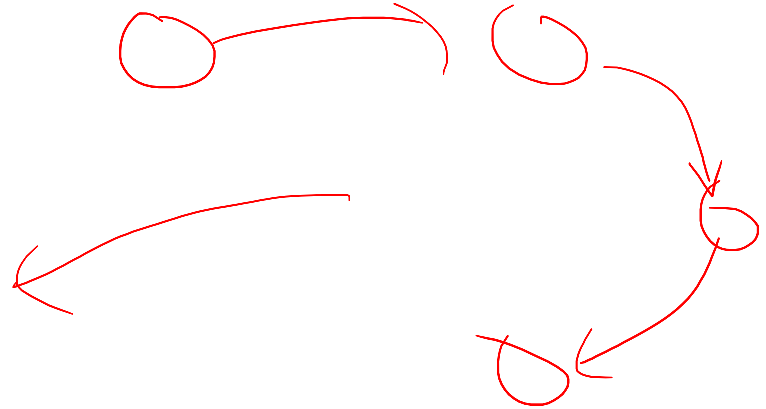
$000 \quad 111 \Rightarrow$ #FF

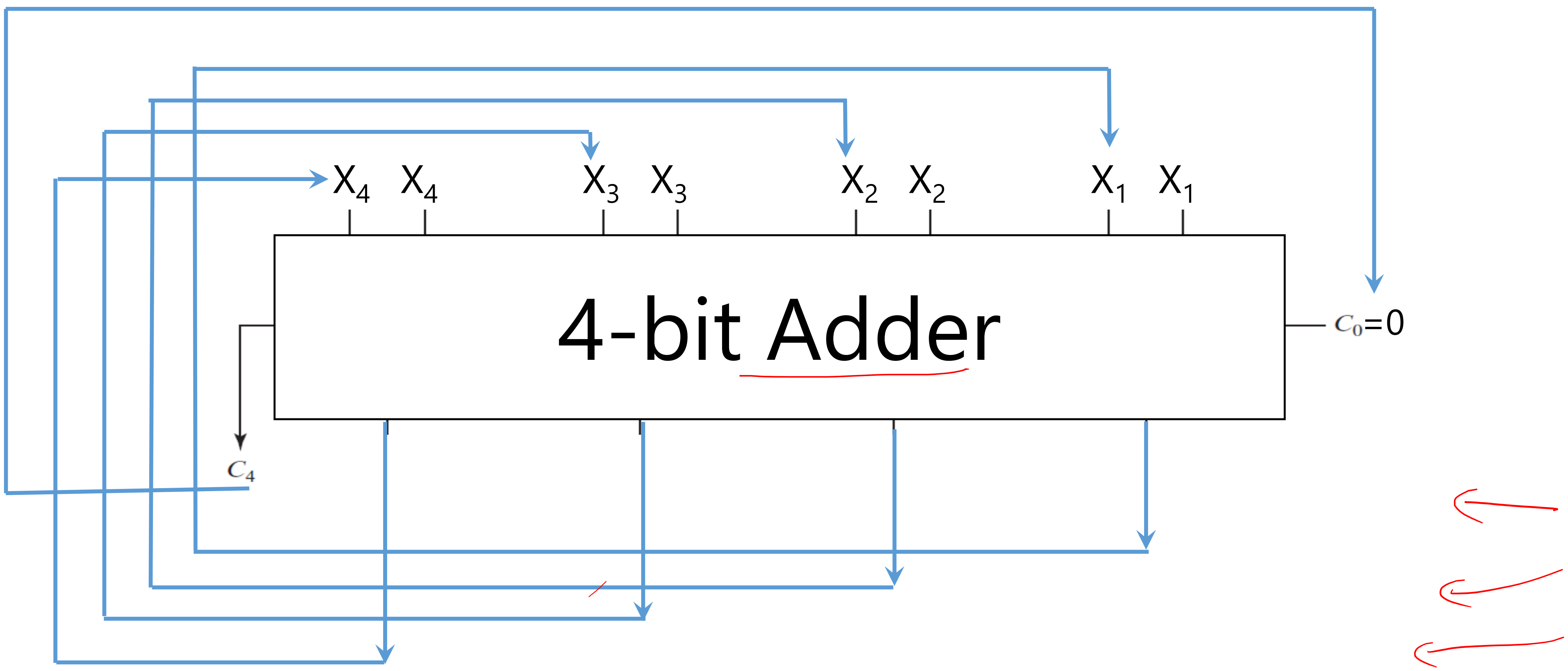
Design (Advanced)

0. Do we need combinational logic or sequential logic? Do we need memory?
1. How many storage (flip-flops)? #FF
2. Form the state (transition) diagram
2.1. State Reduction
3. Form the state table
4. Fill the state table
5. What type of storage (flip-flop)? RS, D, T, JK, or Mixed
6. Input (*excitation*) equations for each FF
7. Minimization of input (*excitation*) equations
8. Draw/Sketch Logic Circuit
9. (Optional) Test

Theory of Automata

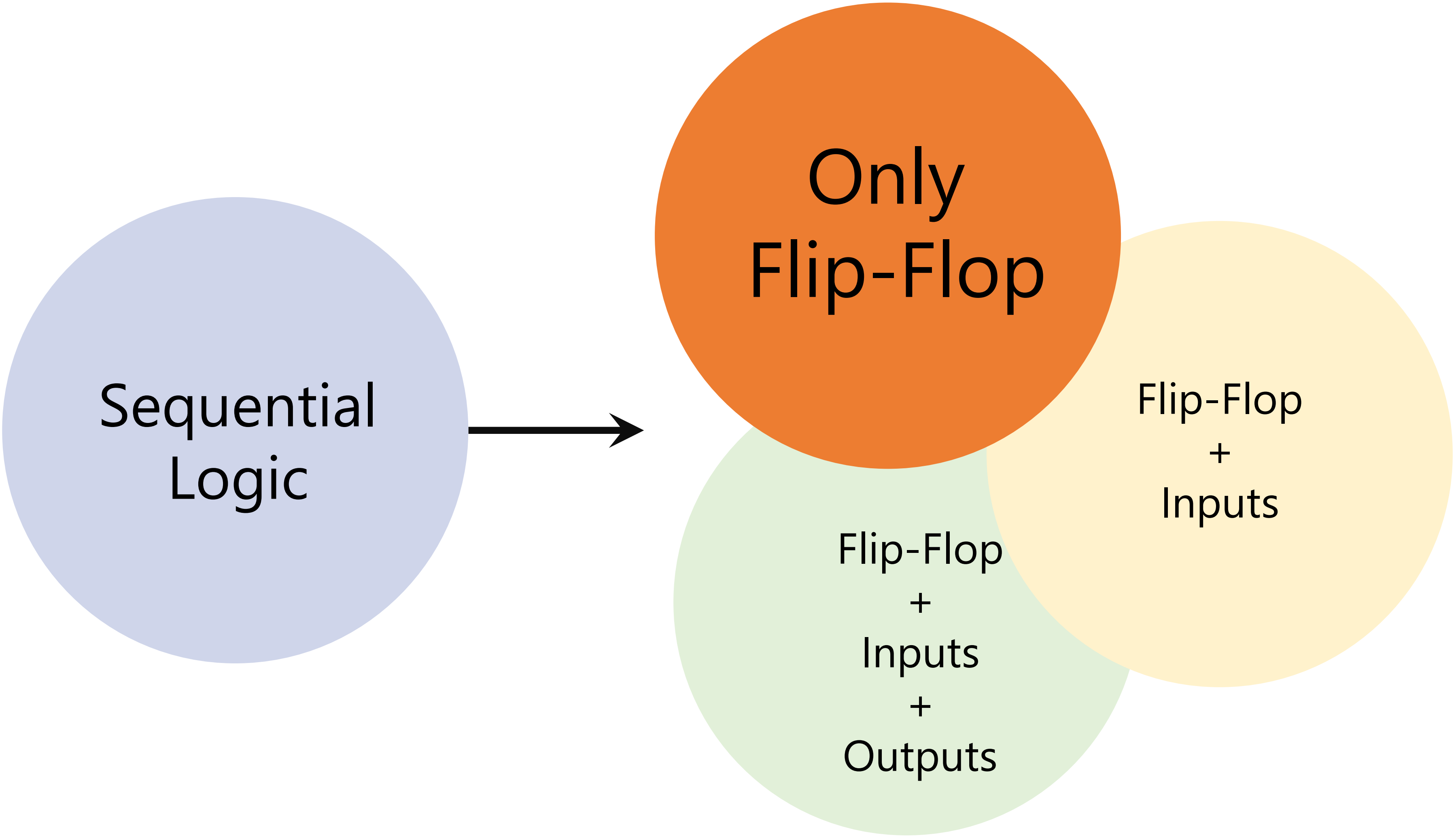
COMP-2140: Computer Languages, Grammars, and Translators

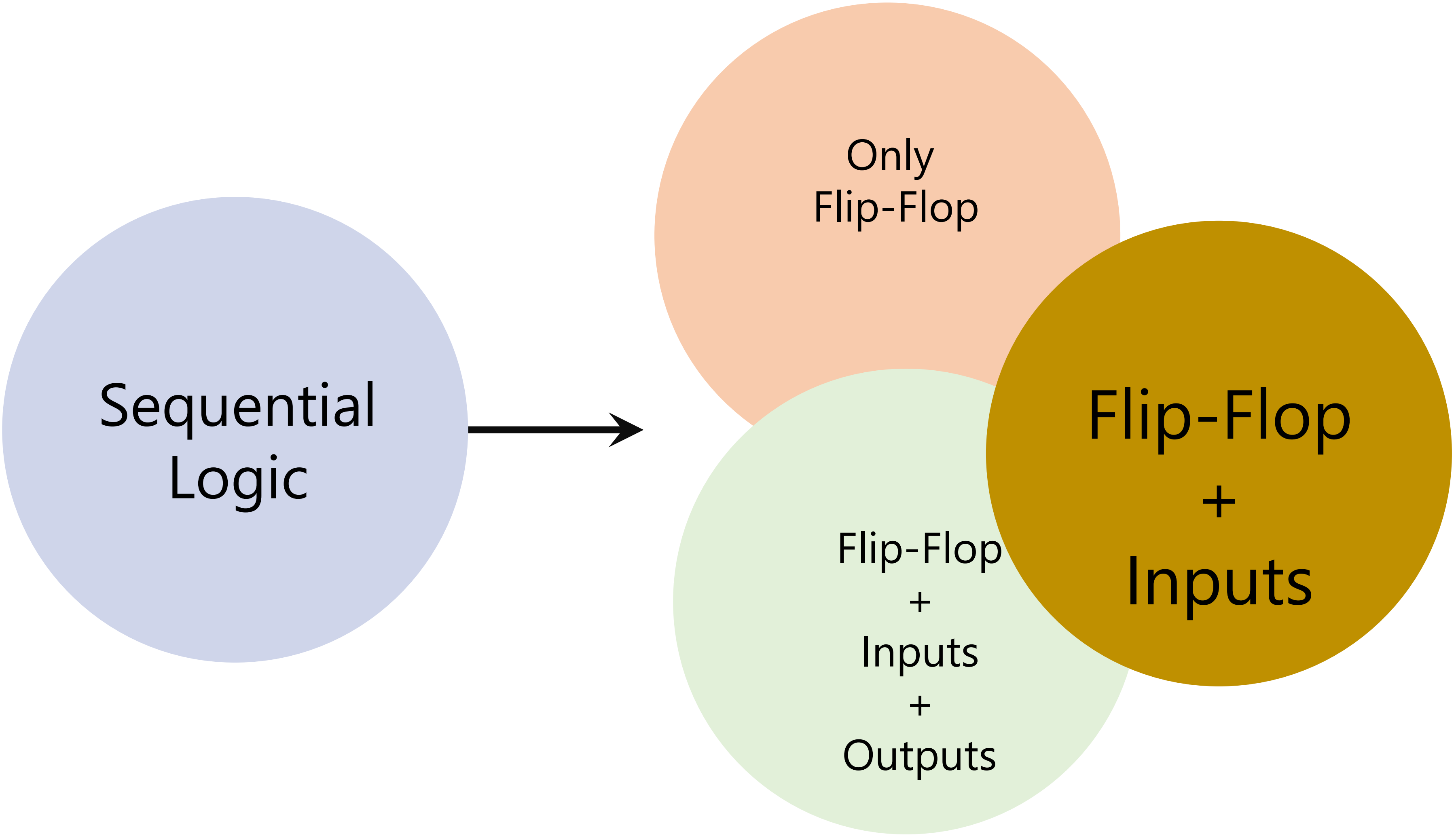


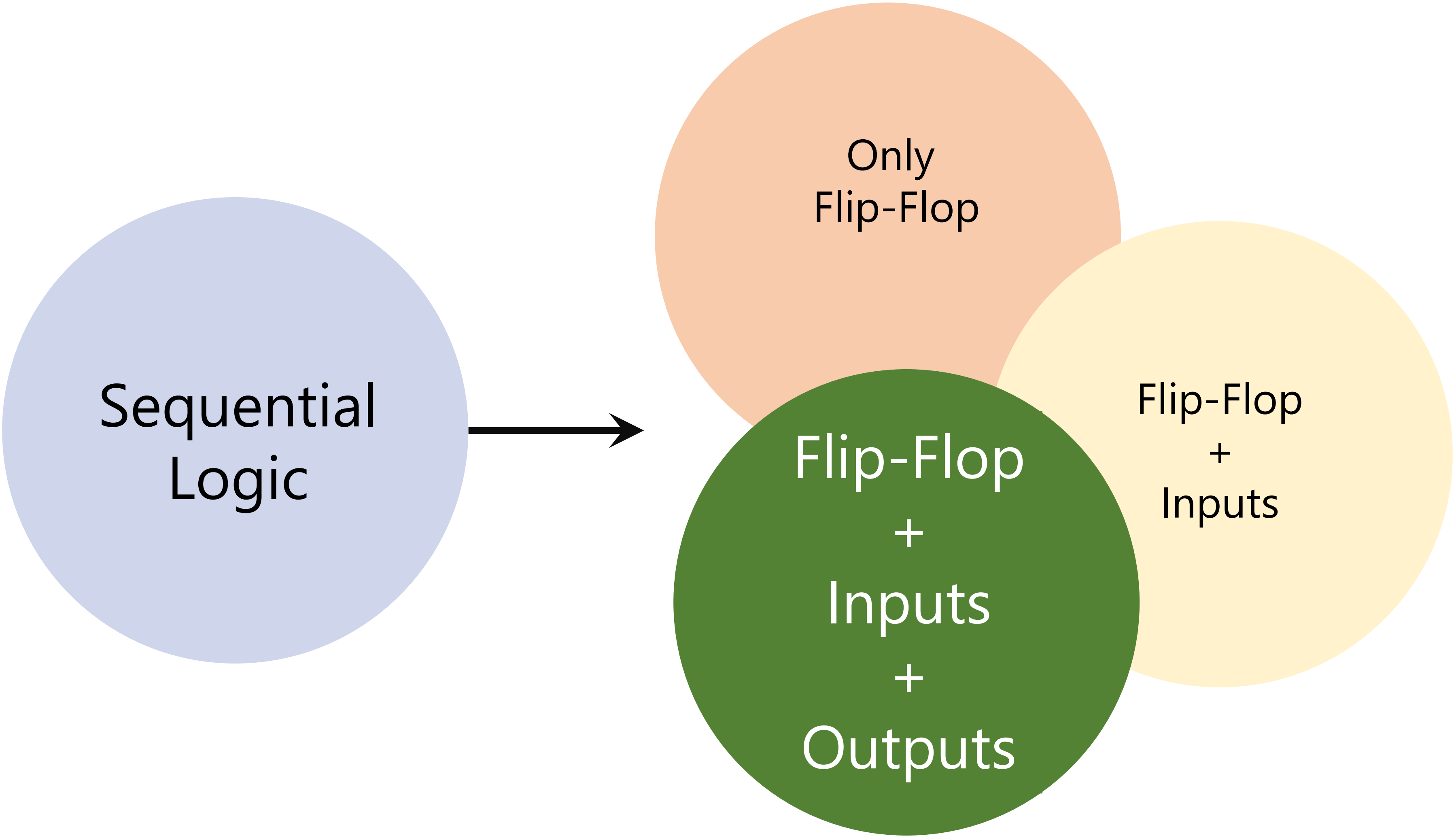


$X \times Y = X + \ldots + X \rightarrow$ When to stop?

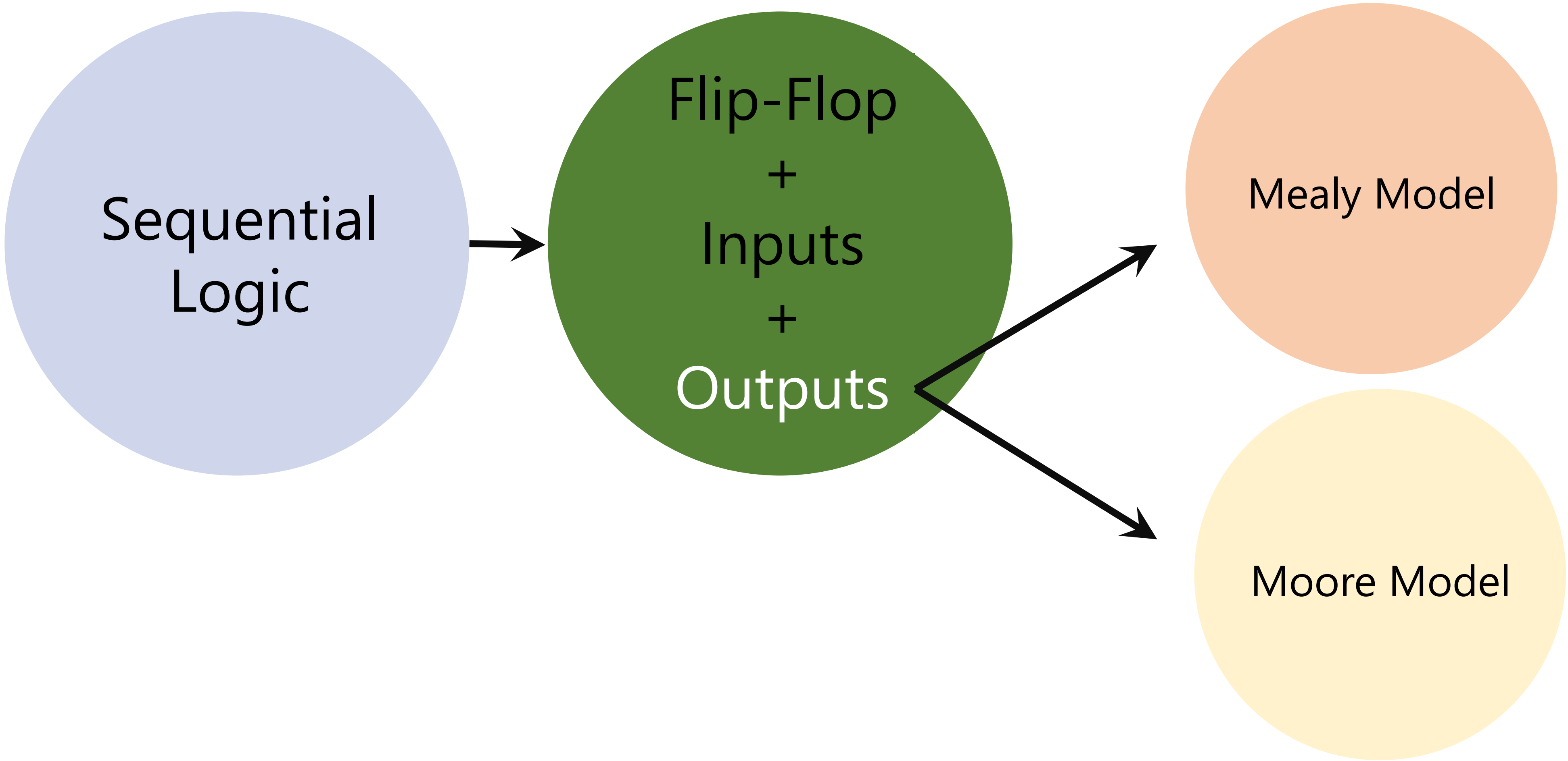
Feedback → Sequential Logic

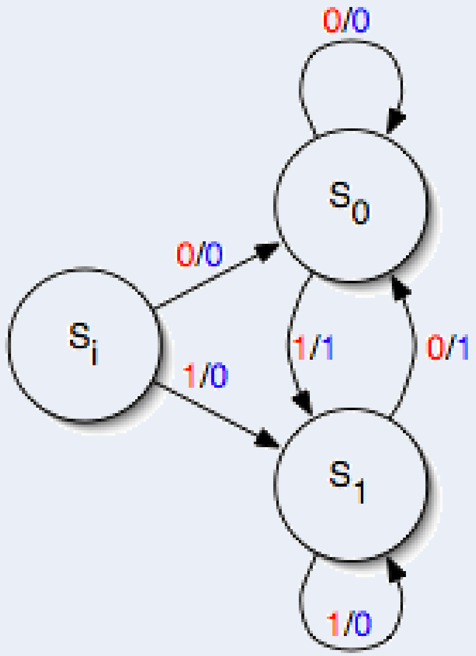

Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
Flip-Flop
+
Inputs
+
Outputs


Sequential Logic
Only Flip-Flop
Flip-Flop + Inputs + Outputs
Flip-Flop + Inputs


Sequential Logic
Only Flip-Flop
Flip-Flop + Inputs
Flip-Flop + Inputs + Outputs

Analysis
vs.
Design


Sequential Logic
Flip-Flop
+
Inputs
+
Outputs
Mealy Model
Moore Model



George H. Mealy

(1927 – 2010)

Mathematician and Computer Scientist

Invented Mealy Machine

Also a pioneer of modular programming

Outputs = Function(Current State, Inputs)

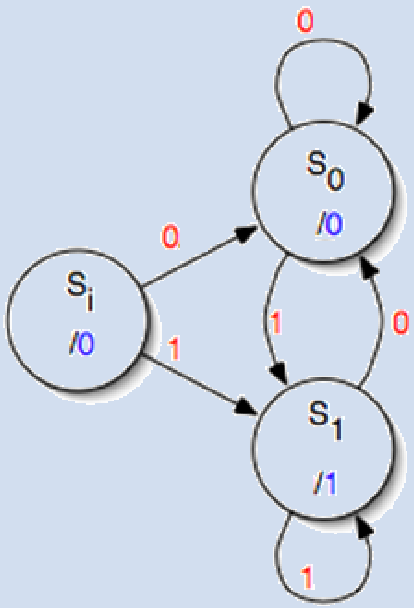
Edward Forrest Moore

(1925 – 2003)

Mathematician and Computer Scientist

Inventor of the Moore Machine

Also an early pioneer of artificial life



Outputs = Function(Current State~~, Inputs~~)

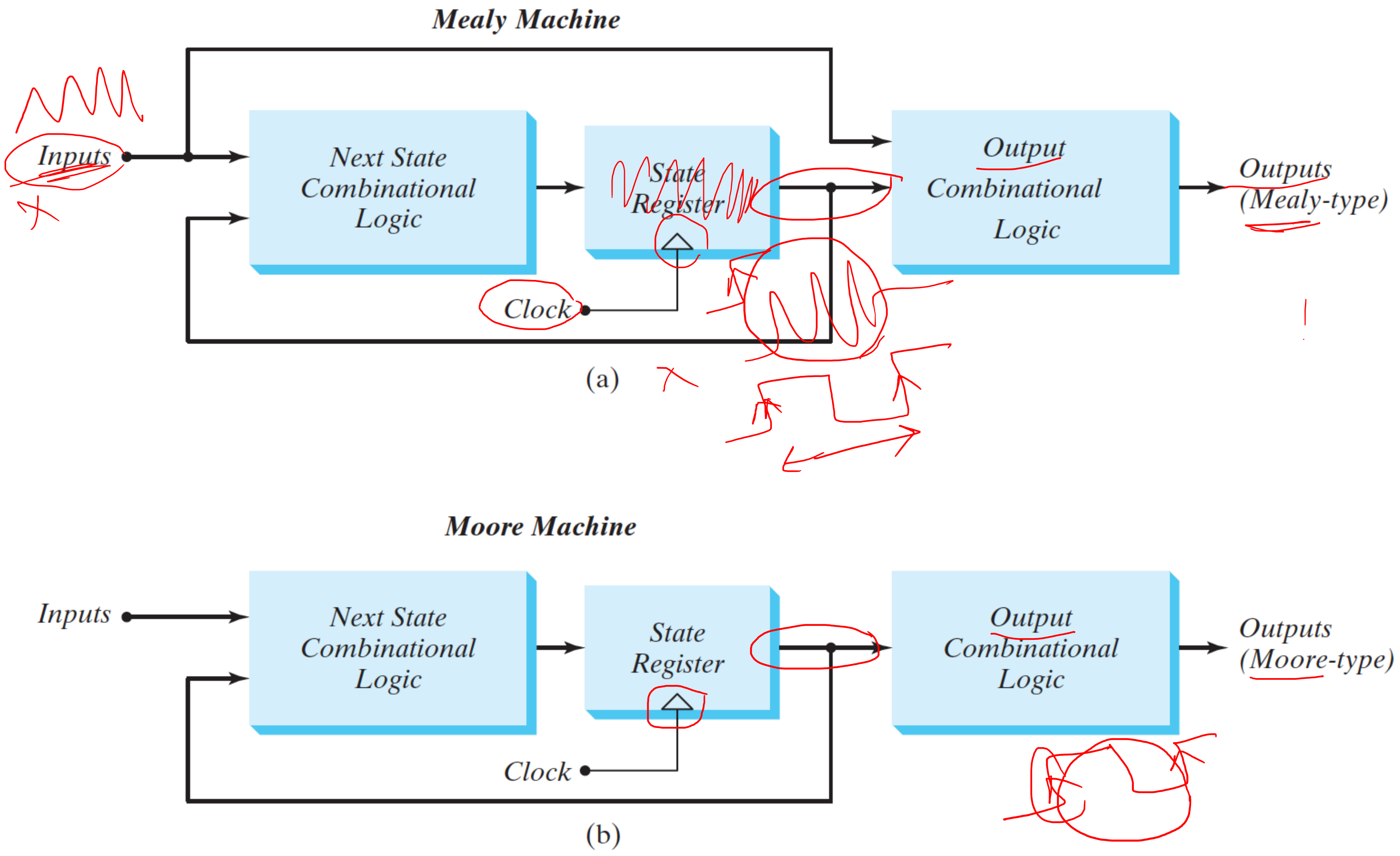


FIGURE 5.21
Block diagrams of Mealy and Moore state machines

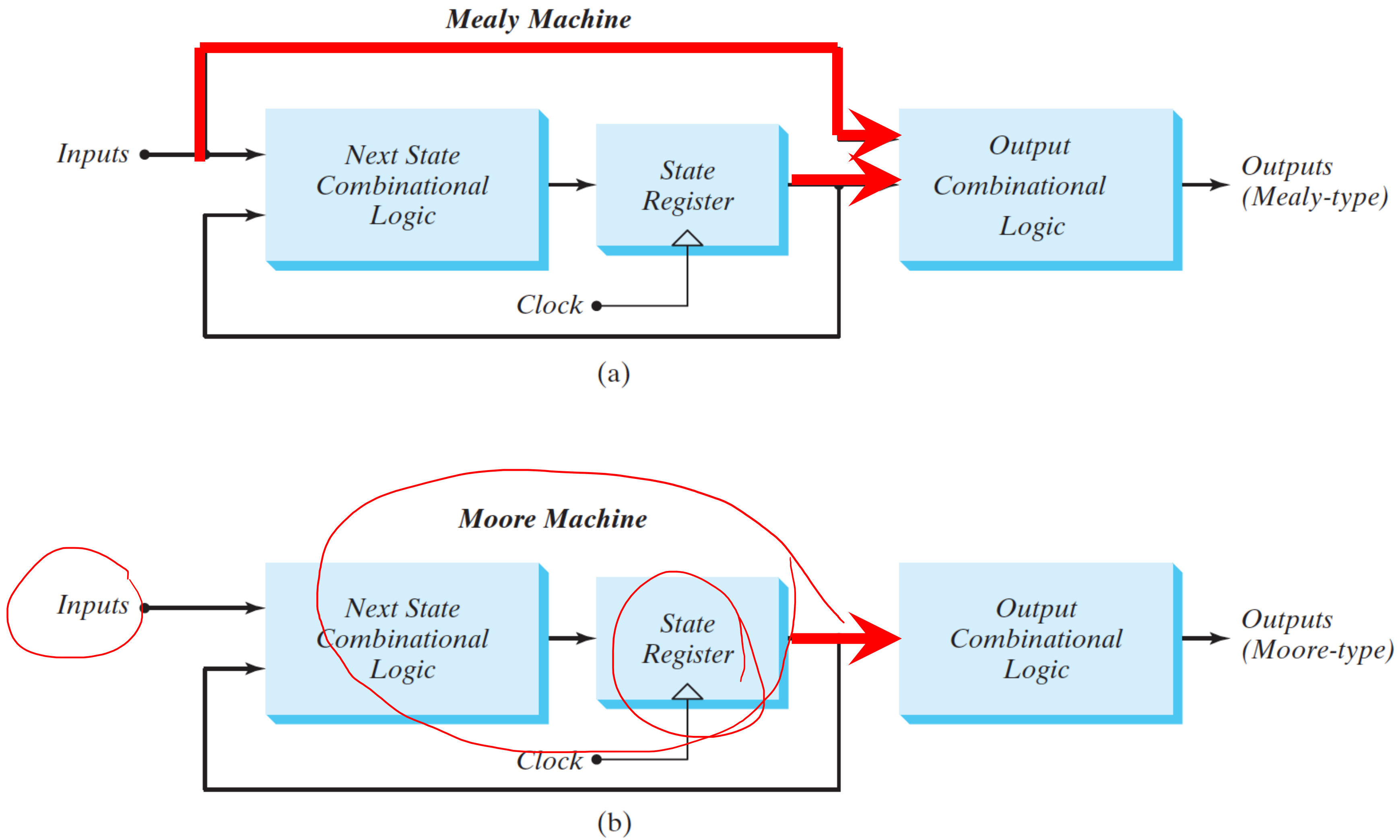
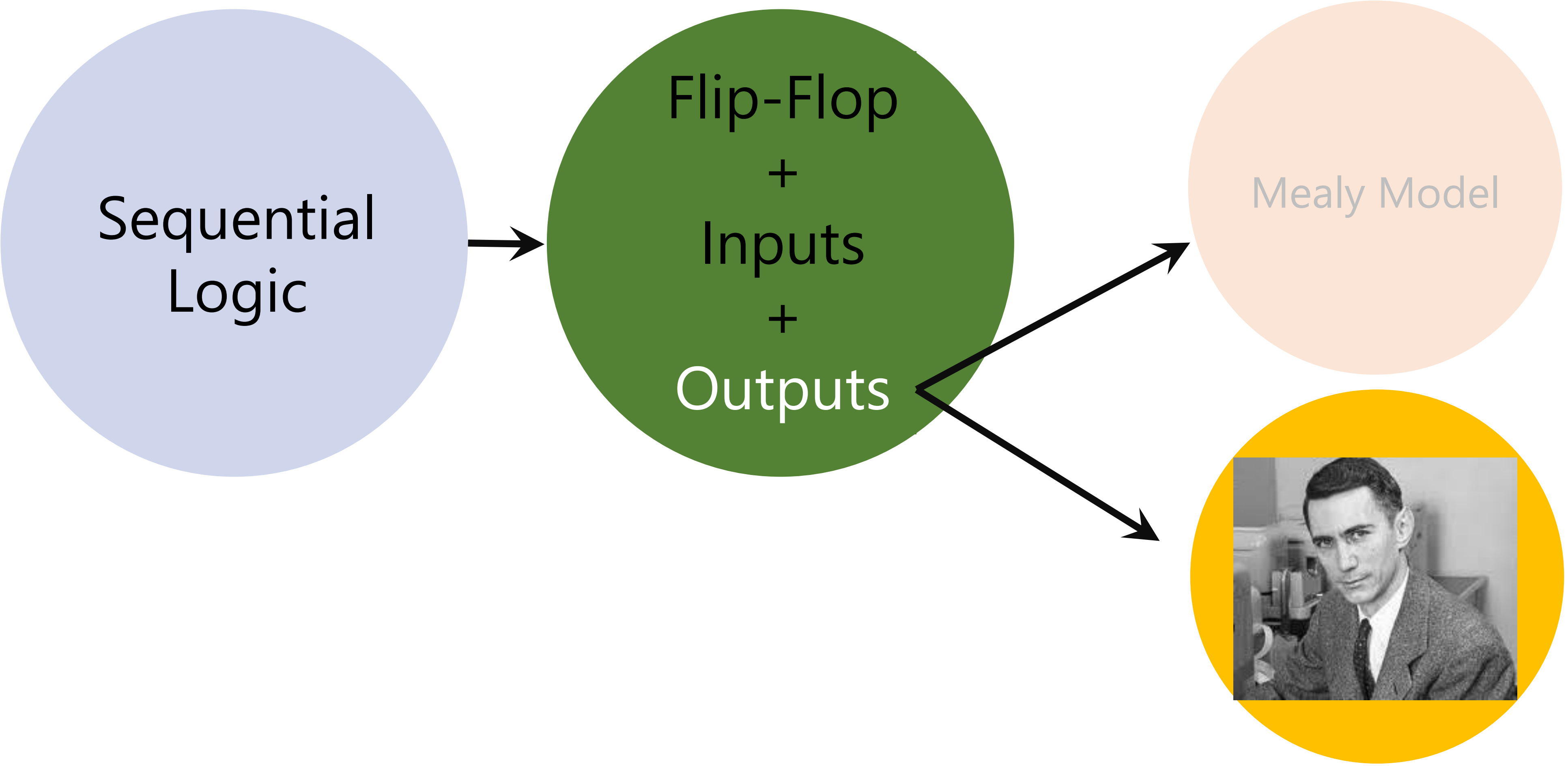


FIGURE 5.21
Block diagrams of Mealy and Moore state machines


Sequential Logic
Flip-Flop
+
Inputs
+
Outputs
Mealy Model

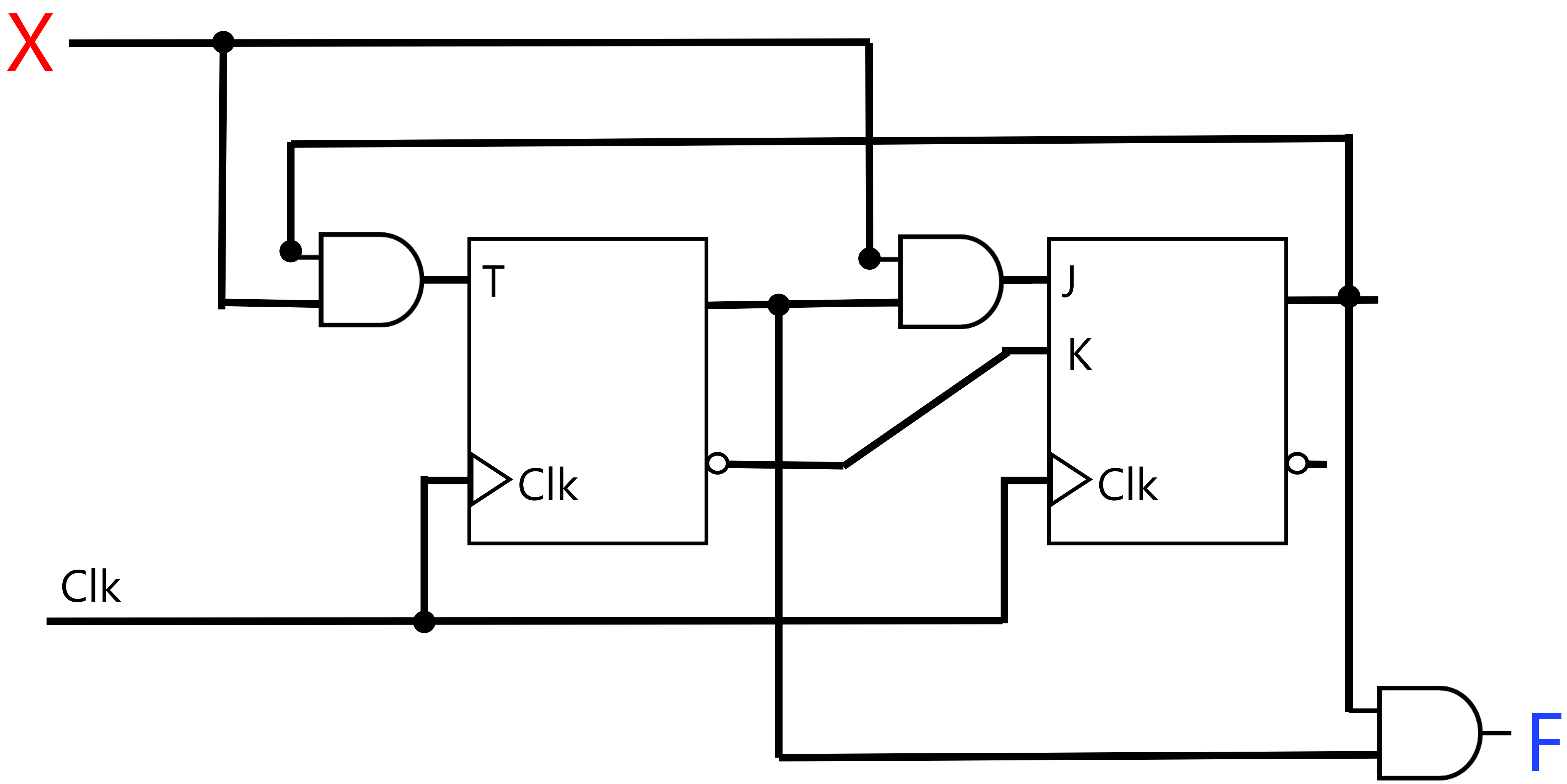
Analysis (Moore model in output)
by an example

Analysis (Recap)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, ...)
2. What are the state combinations? $2^{\#FF}$
3. Form "State" table:
   a) Columns: for each FF, two columns:
      - one for current state,
      - one for next state
   b) Rows: for each state combination
      - In total: $2^{\#FF}$
4. Fill the state table for next state columns based on:
   a) the current state
   b) the inputs to the FFs
5. Form State Transition Diagram
6. (Optional) Analyze paths and states in state transition diagram

Analysis (+ Input + Moore Model Output)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential
1. What are the flip-flops? RS, D, T, JK, or mixed (e.g., 2 JK, 1 RS, …)
2. What are the state combinations? $\cancel{2^{\#FF}}$
3. Form "State" table:
   a) Columns: for each FF, two columns:
      - one for current state,
      - one for next state
   b) Rows: for each state combination
      - In total: $\cancel{2^{\#FF}}$
4. Fill the state table for next state columns based on:
   a) the current state
   b) the inputs to the FFs
5. Form State Transition Diagram
6. (Optional) Analyze paths and states in state transition diagram

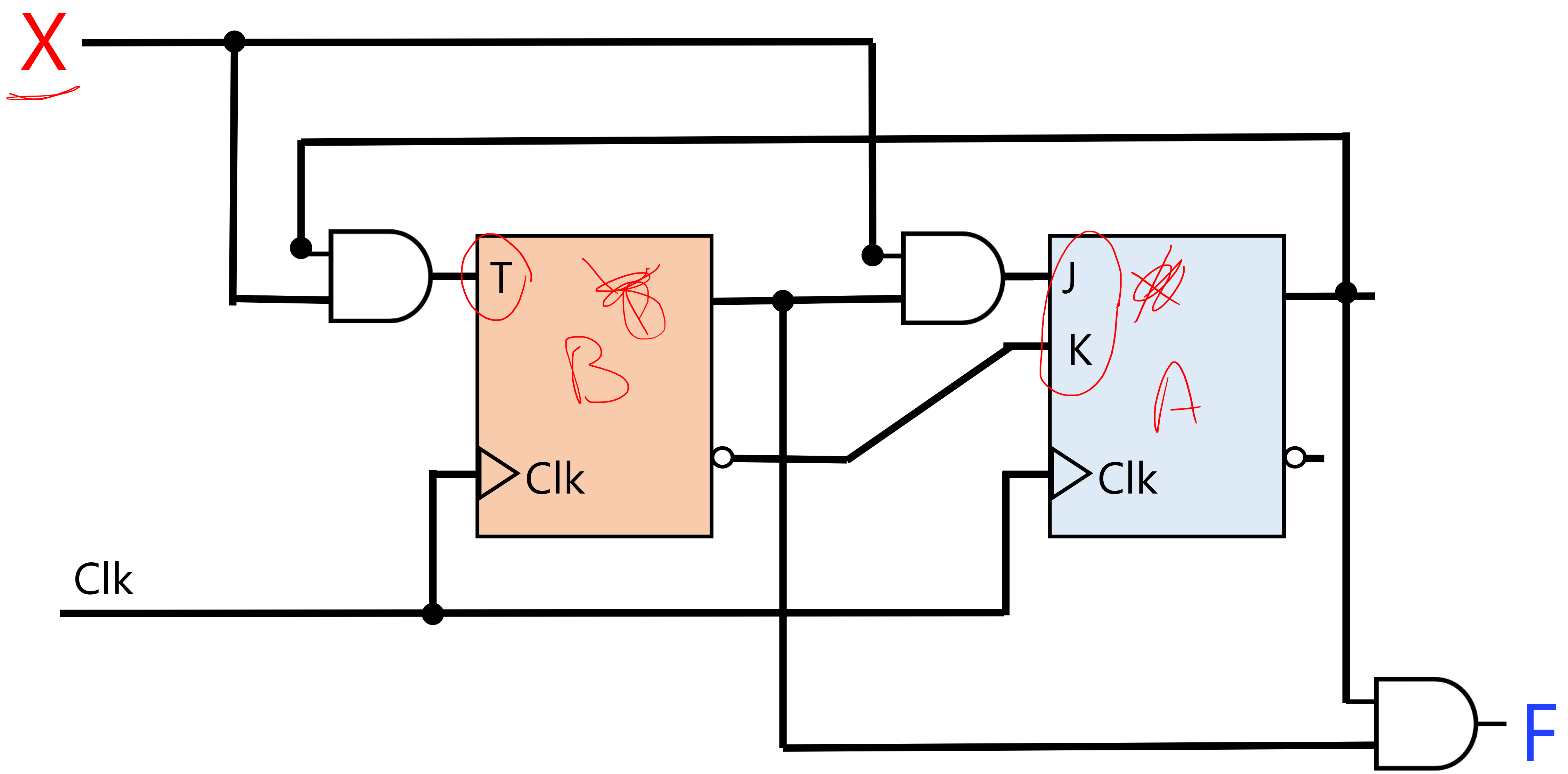

X
T
Clk
J
K
Clk
Clk
F

Analysis (Recap)

0. Is the circuit sequential or combinational? Any FF or feedback → Sequential

Analysis (Recap)

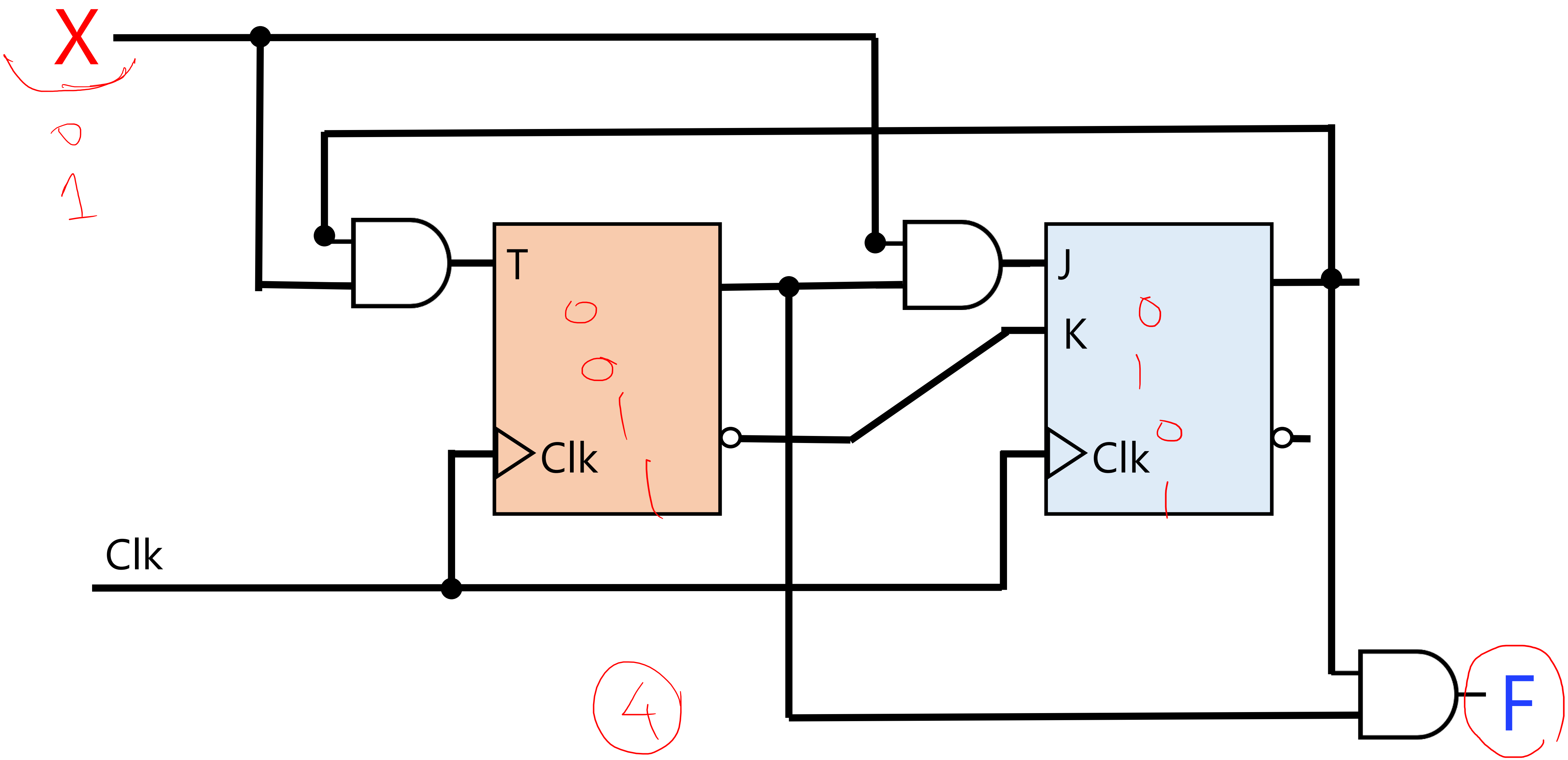
0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops?



X
T
B
J
K
A
Clk
Clk
Clk
F

Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations?



#FFs + #Inputs = 2+1 → 2^3 = 8 combinations

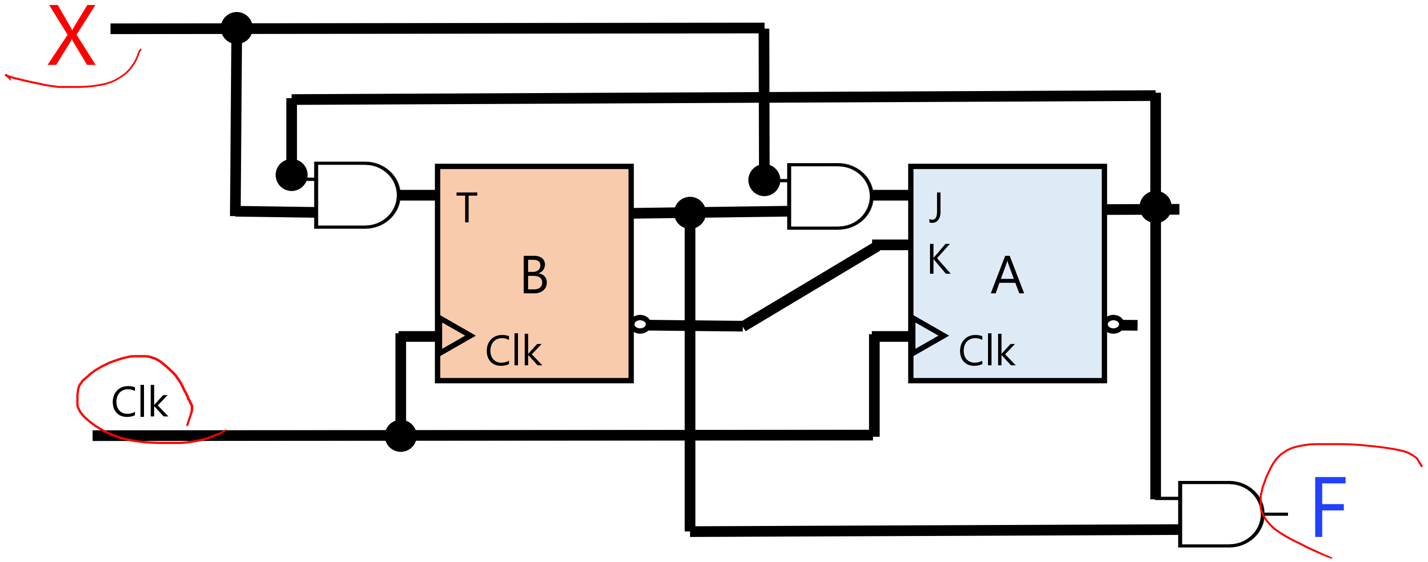
Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form “State” table:

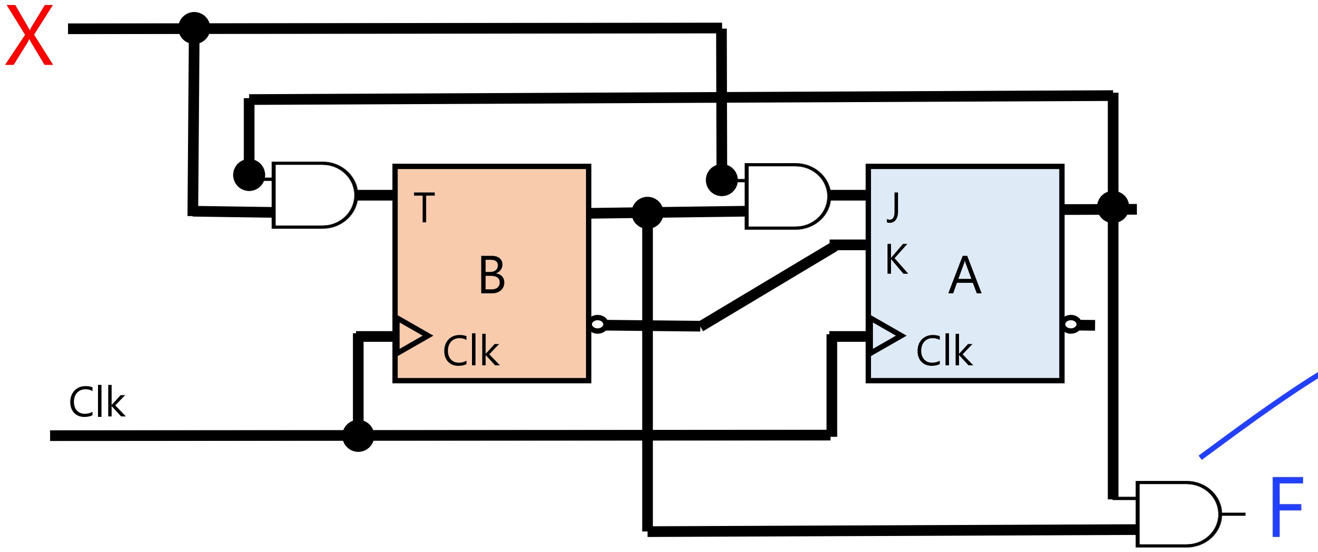
   a) Columns:

   - For each FF, two columns: one for current state, one for next state
   - For each input, one column => left
   - For each output, one column => right

   b) Rows: See item 2



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F
0	0	0			
0	0	1			



Q(T)		Q(T+1) when X=0		Q(T+1) when X=1		Outputs
B	A	B	A	B	A	F

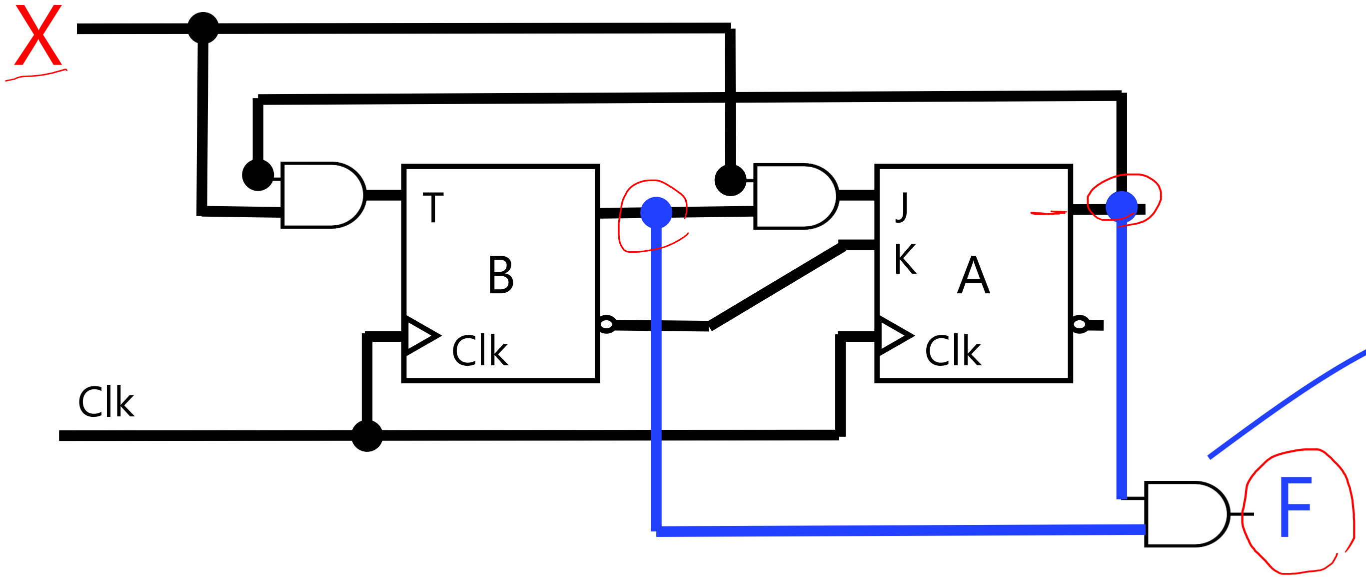
Moore model:
does not depend on X

Alternative State Table

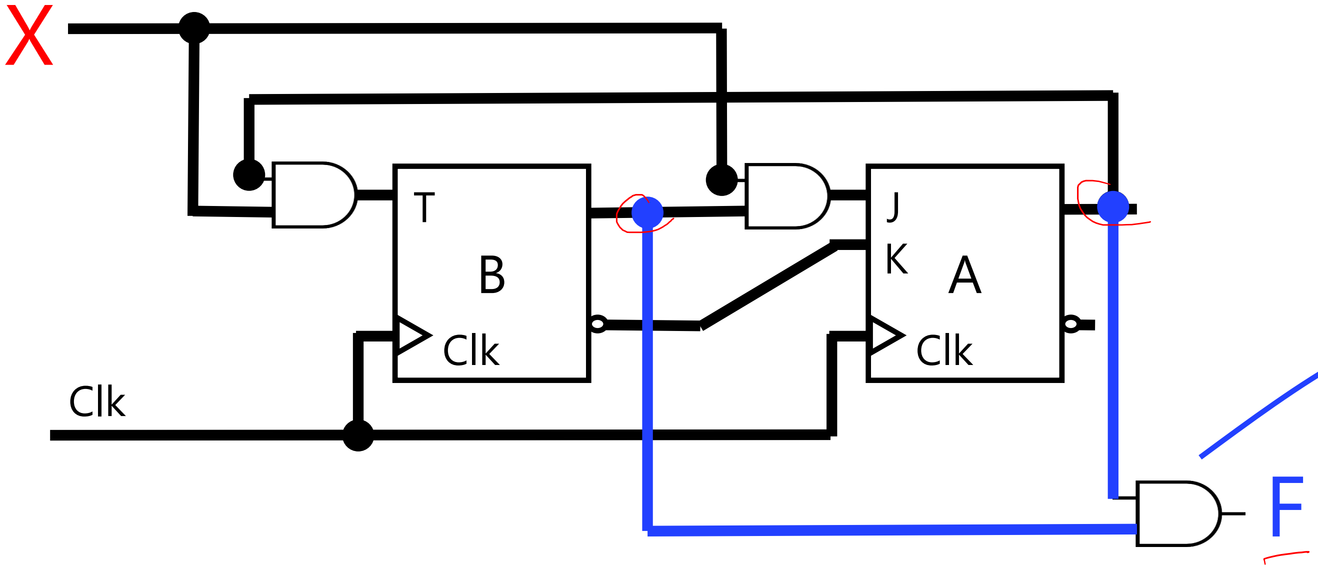
Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form “State” table:
   a) Columns:
      - For each FF, two columns: one for current state, one for next state
      - For each input, one column
      - For each output, one column

   b) Rows: See item 2
4. Fill the state table for
   a) next state columns
   b) the output value

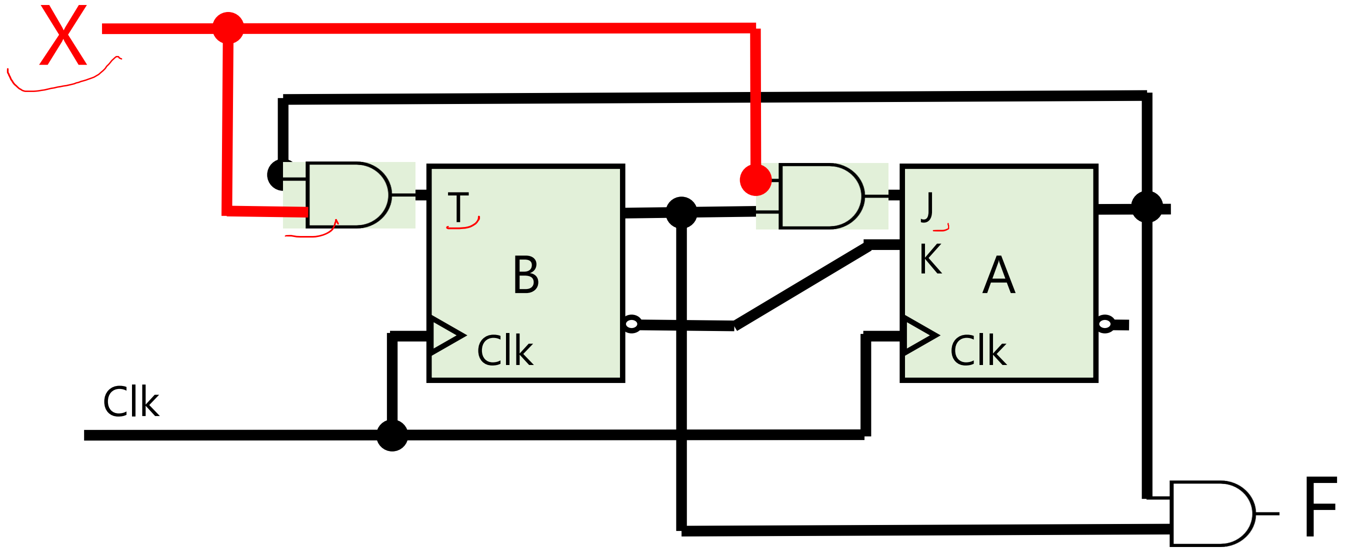


Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

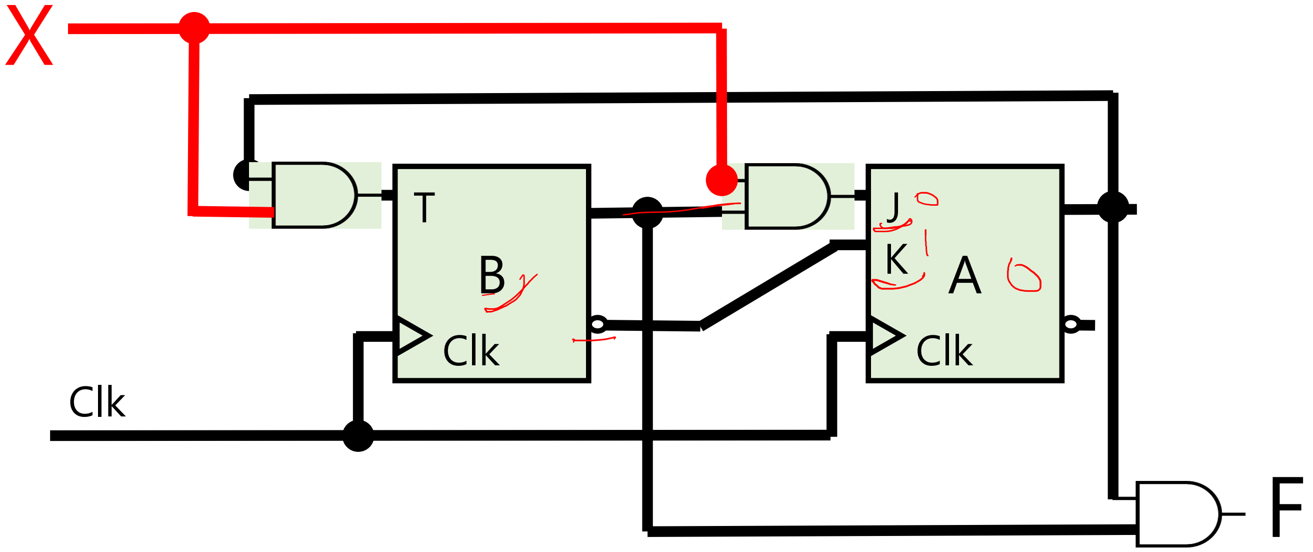


Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0			0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1

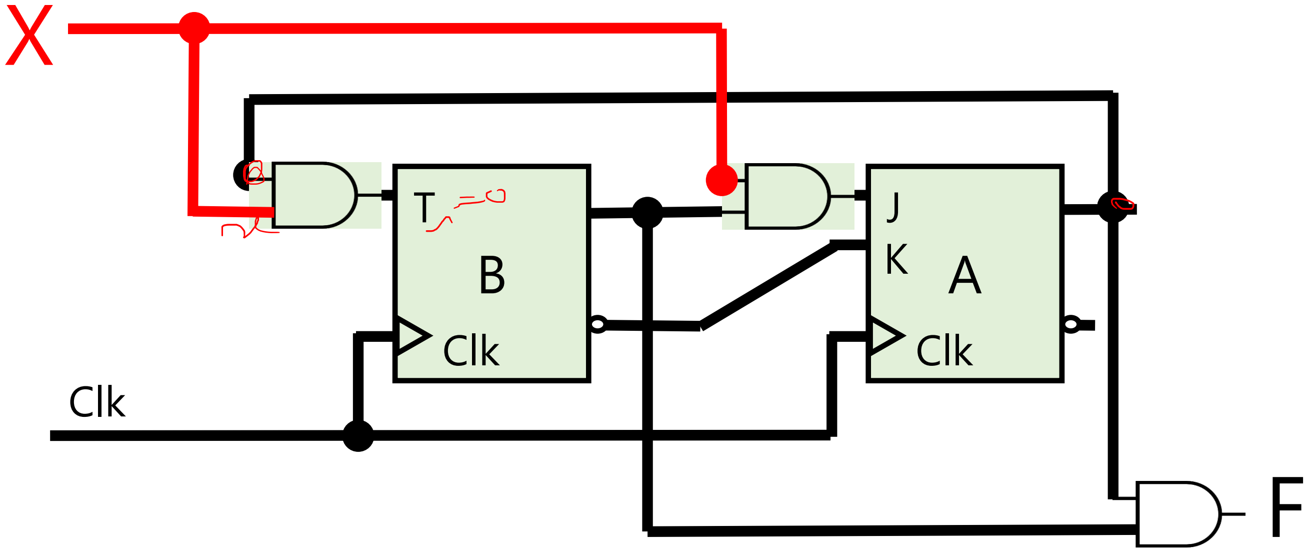
Moore Model
Only depends on current state
X is not involved!



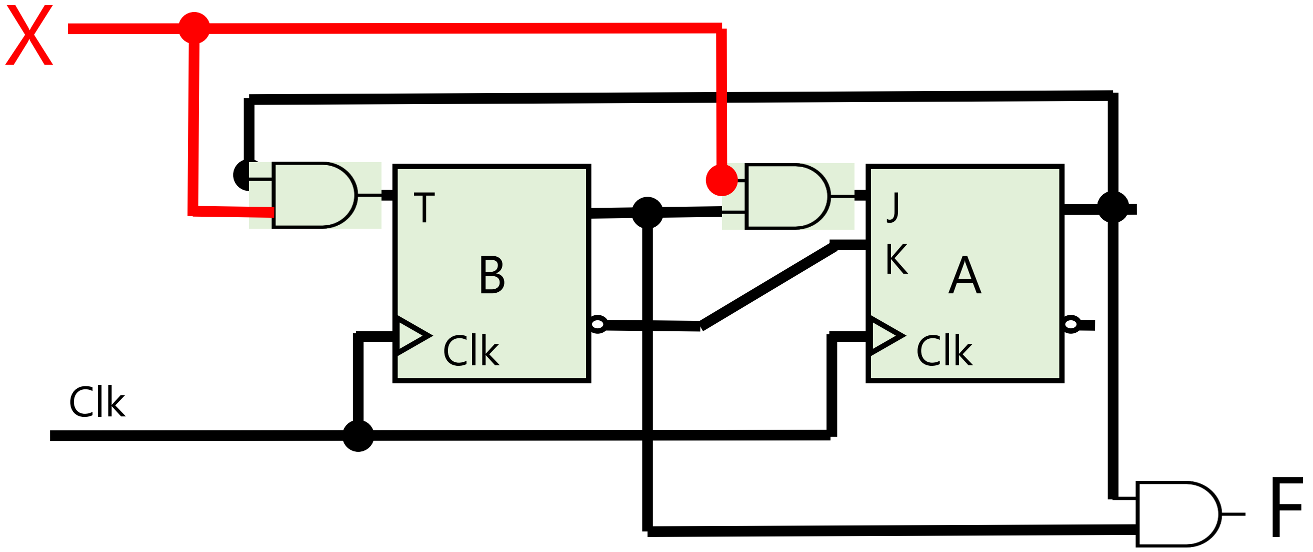
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0			0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1



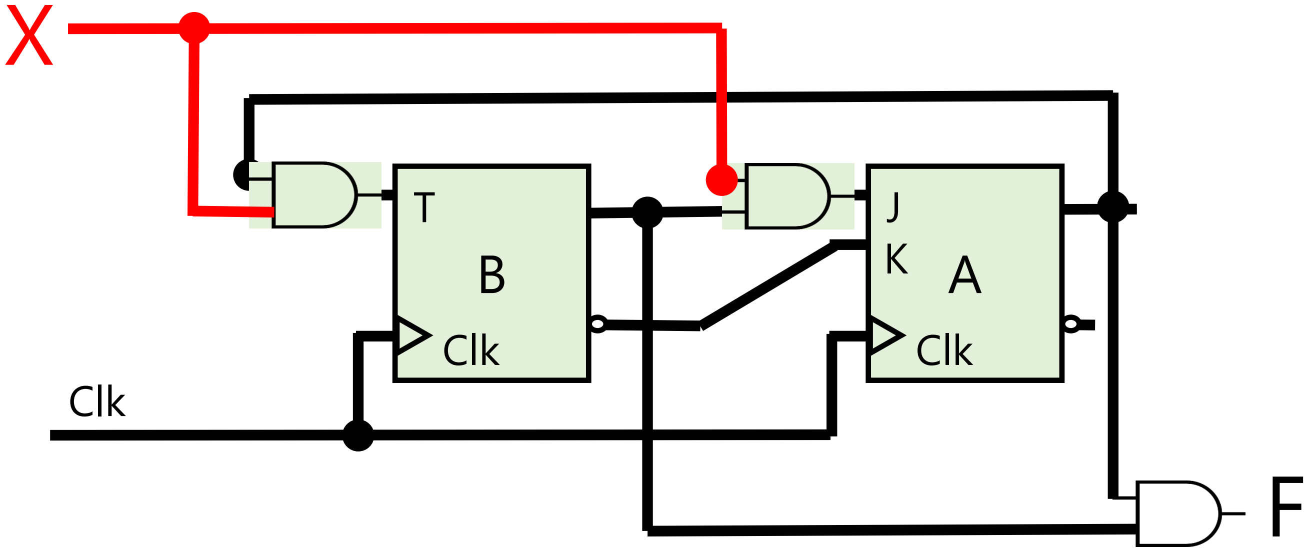
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0		A=0 $J_A=XB=00=0$ $K_A=B'=0'=1$ --------------- Reset → 0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0



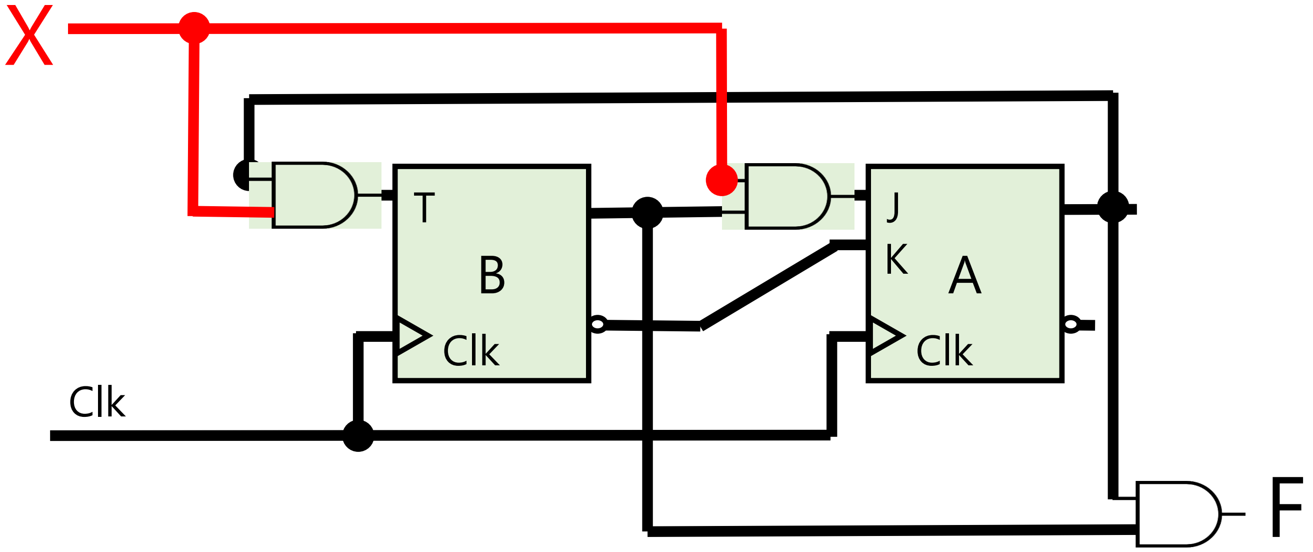
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0	B=0 T_B=XA=00=0 -------------- Store → 0	0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0



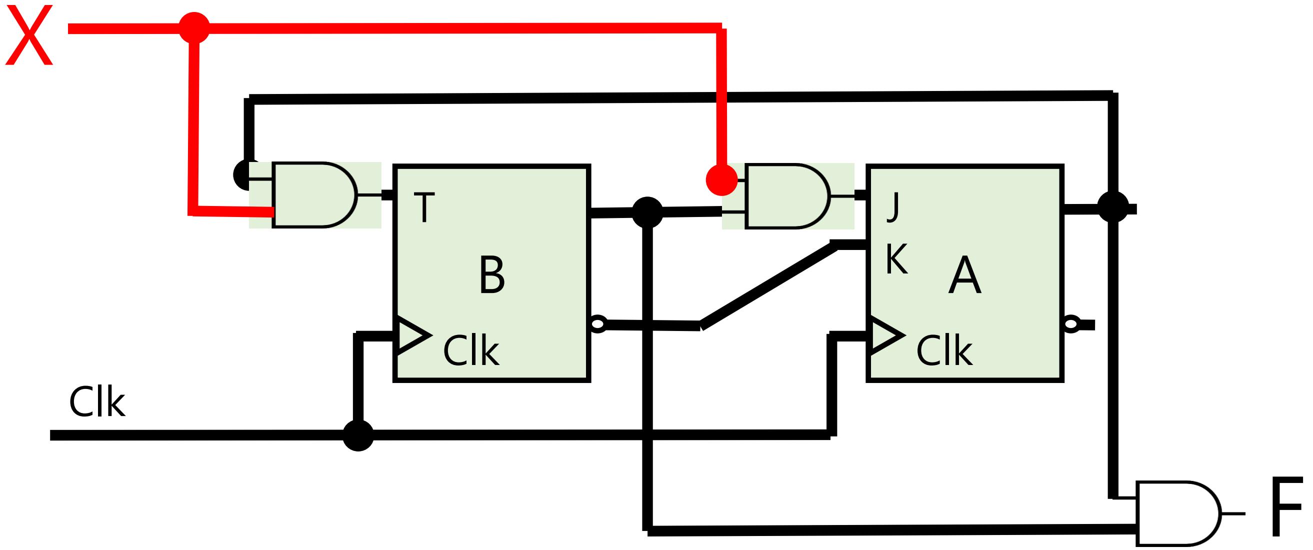
Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0	0	0	0
0	0	1			0
0	1	0			0
0	1	1			1
1	0	0			0
1	0	1			0
1	1	0			0
1	1	1			1



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	?	0	0
1	1	0	1	1	0
1	1	1	0	1	1



Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	B=0 TB=XA=11=1 ---------- Comp. → 1	0	0

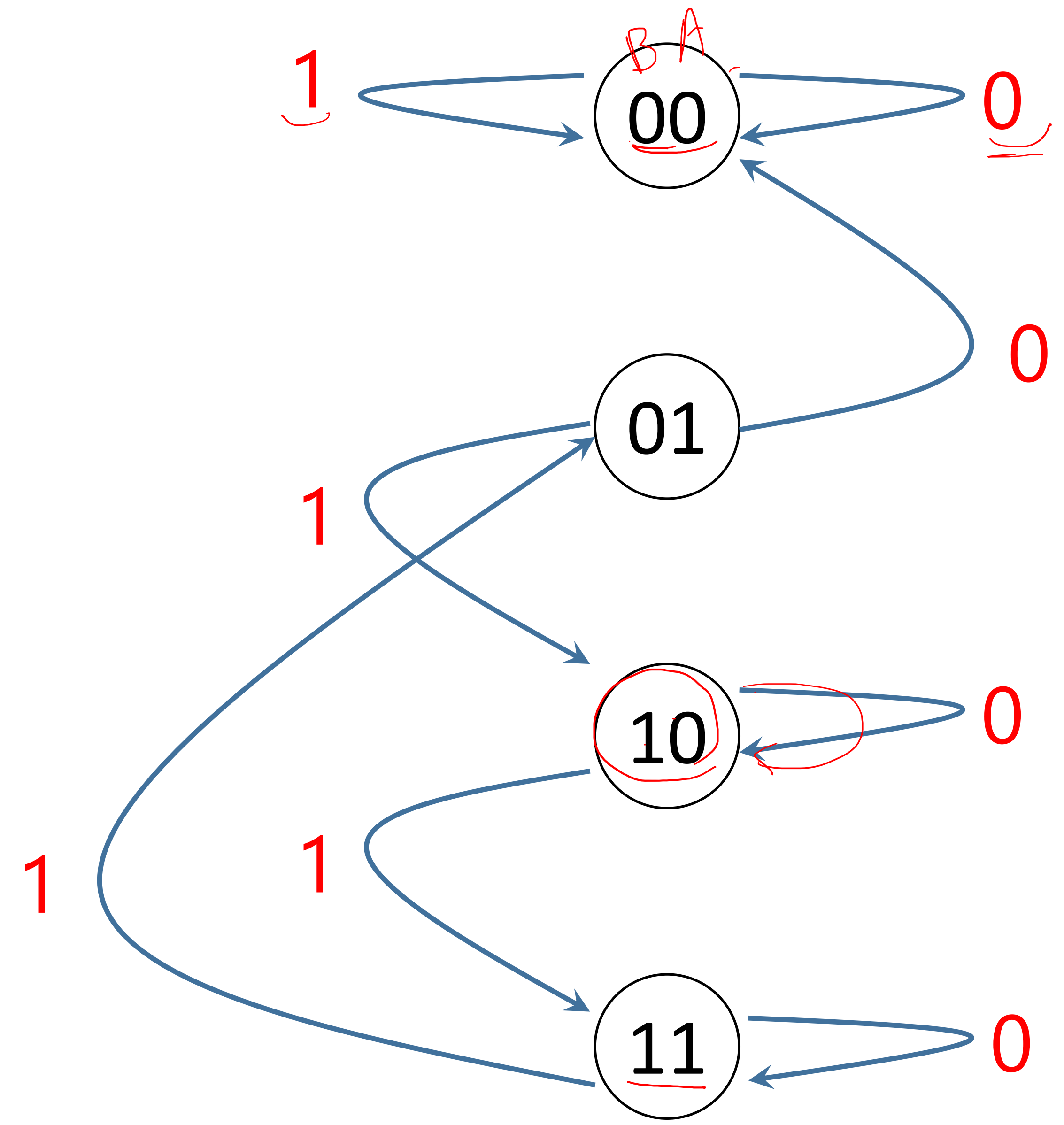


Inputs	Q(T)		Q(T+1)		Outputs
X	B	A	B	A	F=BA
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	0	0
1	1	0	1	1	0
1	1	1	0	1	1

Analysis (Recap)

0. Is the circuit sequential or combinational? Sequential
1. What are the flip-flops? T, JK
2. What are the state combinations? $2^{\#FF} \times 2^{\#inputs} = 2^{\#FF+\#inputs} = 2^3 = 8$
3. Form "State" table:
   a) Columns:
      - For each FF, two columns: one for current state, one for next state
      - For each input, one column
      - For each output, one column

   b) Rows: See item 2
4. Fill the state table for
   a) next state columns
   b) the output value
5. Form state (transition) diagram
   a) nodes for states, directed edges for transitions between states
   b) labels for edges by the value of <u>input</u>
   c) labels for nodes by the value of <u>output</u>

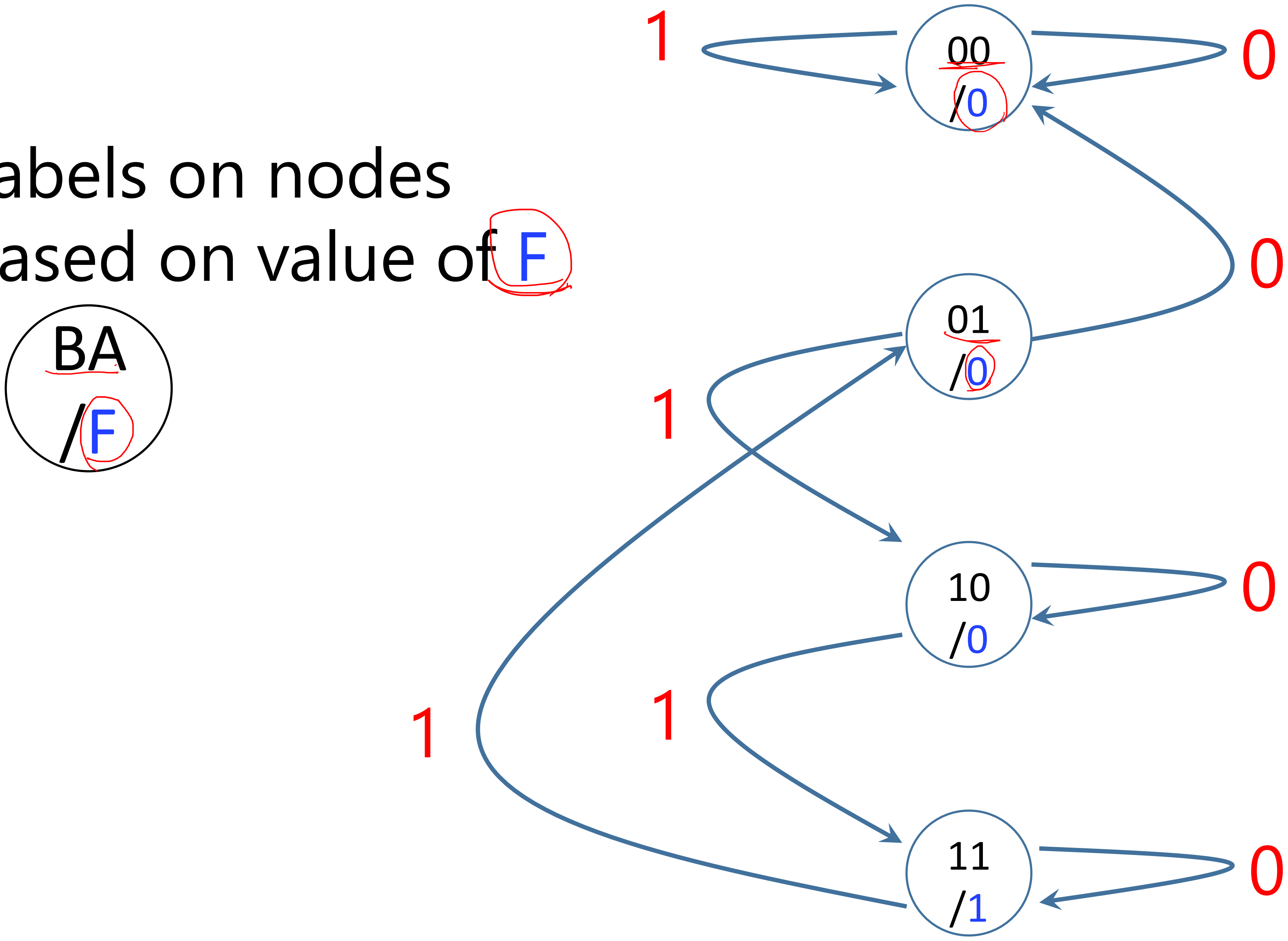


BA

Labels on edges based on value of X

X

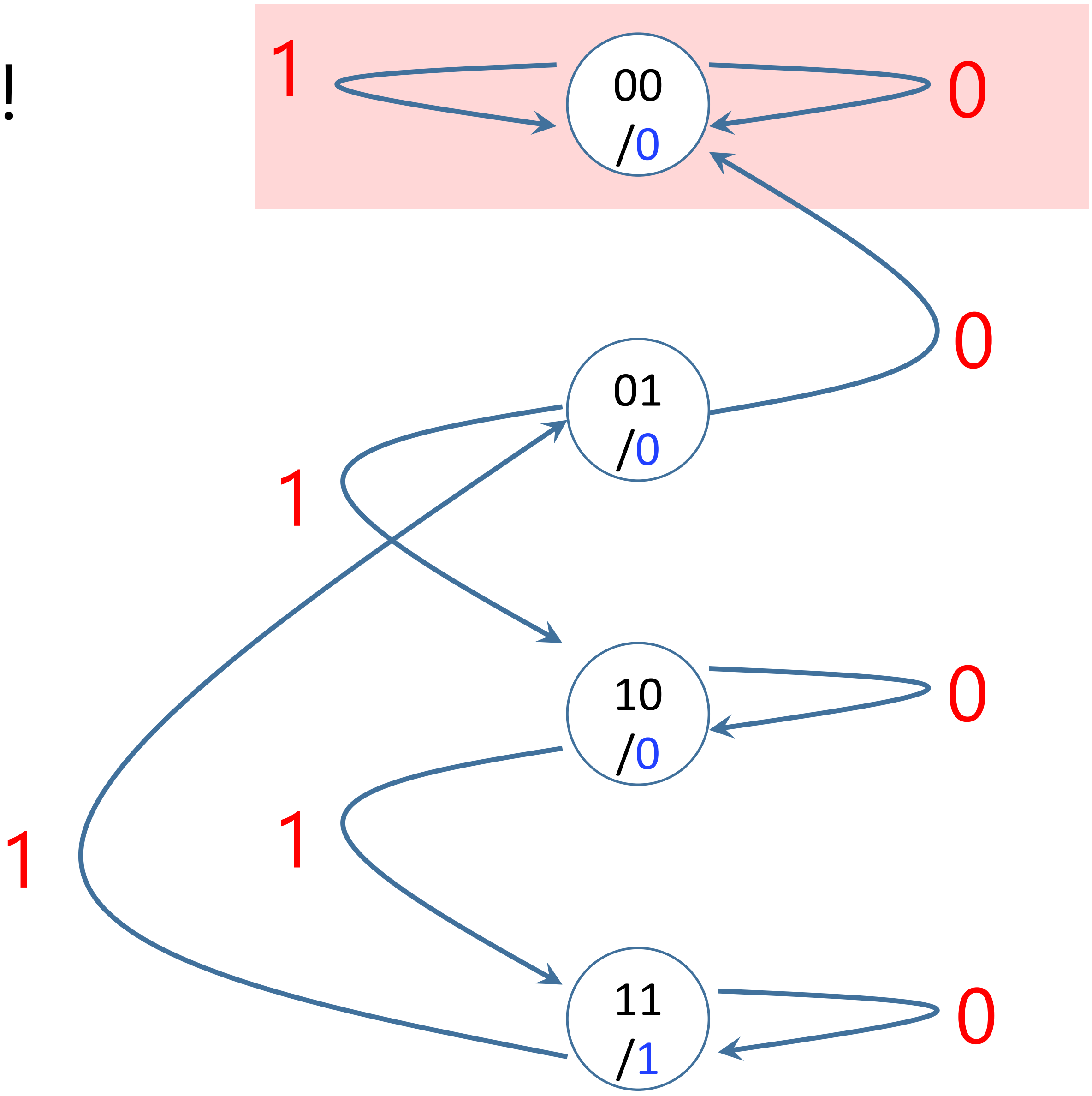
Labels on nodes based on value of F

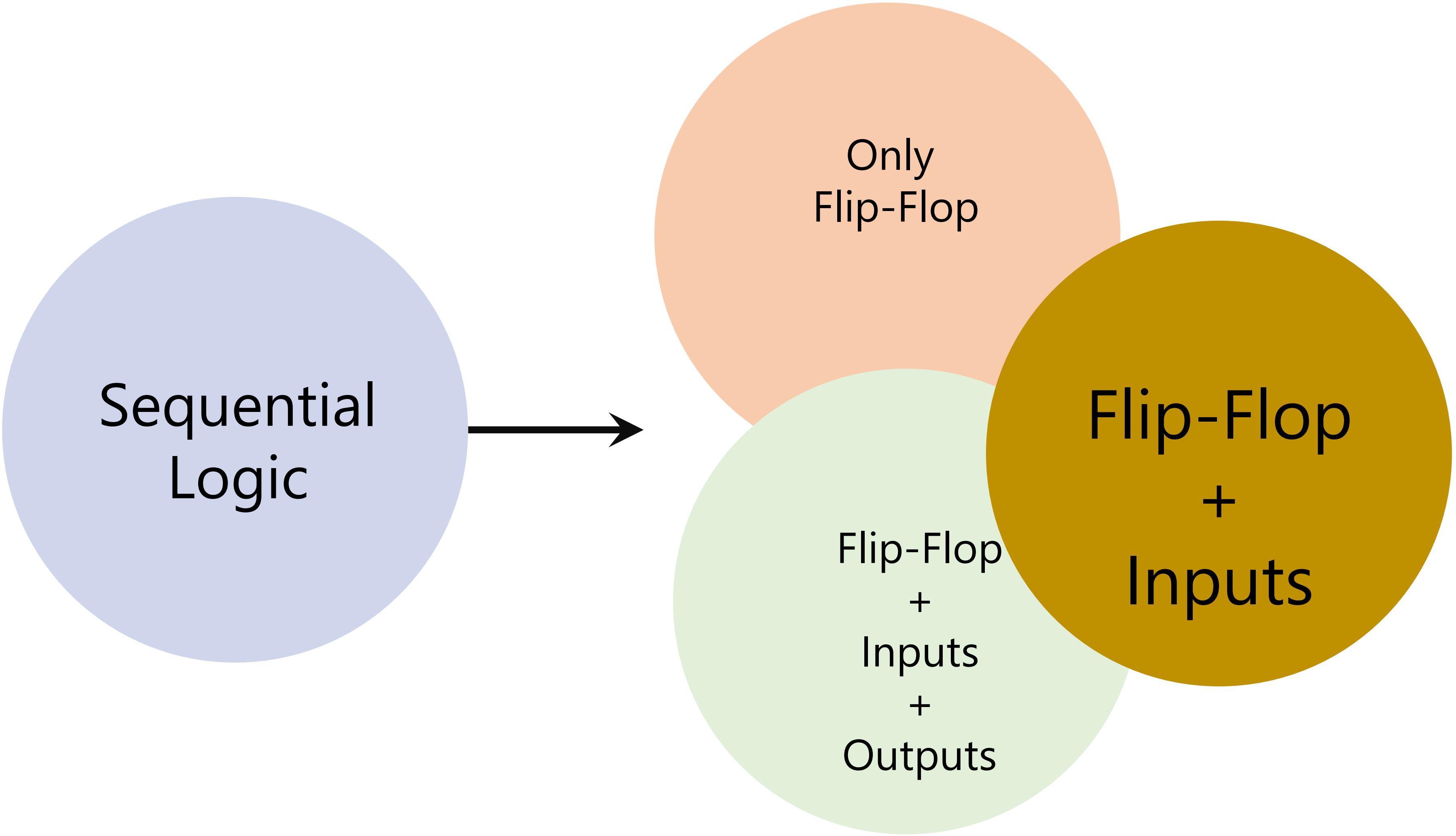


Analysis

6) (*Optional*) Path on State Transitions

Life lock!




Sequential
Logic
Only
Flip-Flop
Flip-Flop
+
Inputs
+
Outputs
Flip-Flop
+
Inputs