DESIGN

a design algorithm for any digital units (logic circuits), given truth table

1. minterm

aka. Standard Product

X' vs. X

1 ← 0

1

1 binary variable appear either:

- in its normal form X, or
- in its complement form X'

m_0	X'
m_1	X

YX vs. YX' vs. $Y'X$ vs. $Y'X'$

2 binary variables appear either in one of these forms:

m_0 00	$Y'X'$
m_1	$Y'X$
m_2 10	YX'
m_3 11	YX

ZYX vs. ZYX’ vs. …

3 binary variables appear either in one of these forms: how many?

ZYX vs. ZYX’ vs. …

3 binary variables appear either in one of these forms: how many?
Each variable can take 2 forms (normal and complement)
We have 3 variables, $2 \times 2 \times 2 = 2^3 = 8$

m_0	$Z'Y'X'$
m_1	$Z'Y'X$
m_2	$Z'YX'$
m_3	$Z'YX$
m_4	$ZY'X'$
m_5	$ZY'X$
m_6	ZYX'
m_7	ZYX

$$A_{n-1} \cdots A_2 A_1 A_0 \text{ vs. } A_{n-1} \cdots A_2 A_1 A'_0 \cdots$$

n binary variables appear either in one of these forms: how many?

Each variable can take 2 forms (normal and complement)

We have n variables, $2 \times 2 \times 2 \times \cdots \times 2 = 2^n$

m_0	$A'_{n-1}\cdots A'_2A'_1A'_0$
m_1	$A'_{n-1}\cdots A'_2A'_1A_0$
m_2	$A'_{n-1}\cdots A'_2A_1A'_0$
$\vdots$	$\vdots$
$\vdots$	$\vdots$
m_{2^n-3}	$A_{n-1}\cdots A_2A'_1A_0$
m_{2^n-2}	$A_{n-1}\cdots A_2A_1A'_0$
m_{2^n-1}	$A_{n-1}\cdots A_2A_1A_0$

2. TRUTH TABLE

en.wikipedia.org/wiki/Truth_table

X	F = F(X) = ?
0	?
1	?

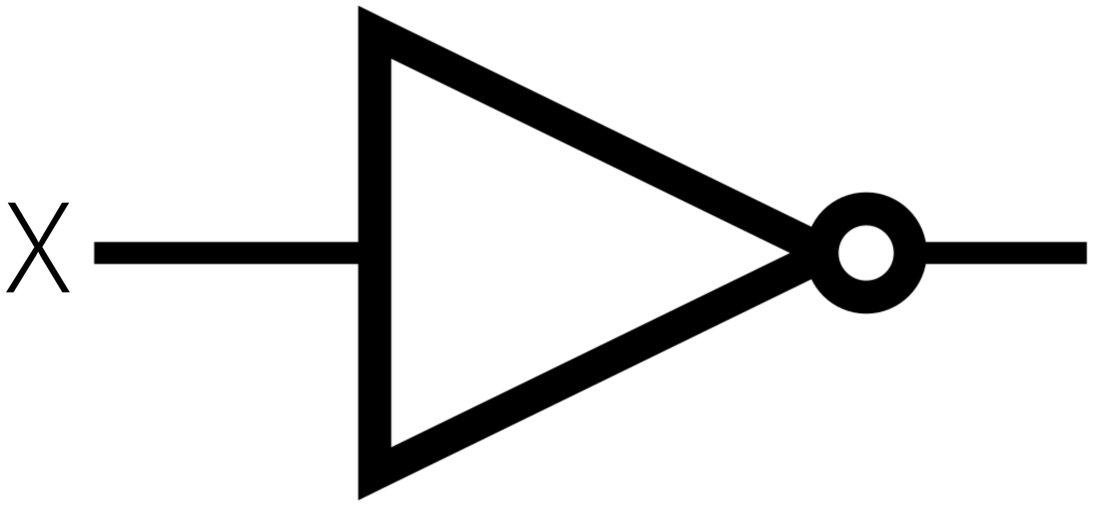
X	F = F(X) = 0
0	0
1	0

X	$F = F(X) = X'$
0	1
1	0

m_0	X'
m_1	X

X	$F = F(X) = X' = m_0$
0	1
1	0

X	$F = F(X) = X' = m_0$
0	1
1	0



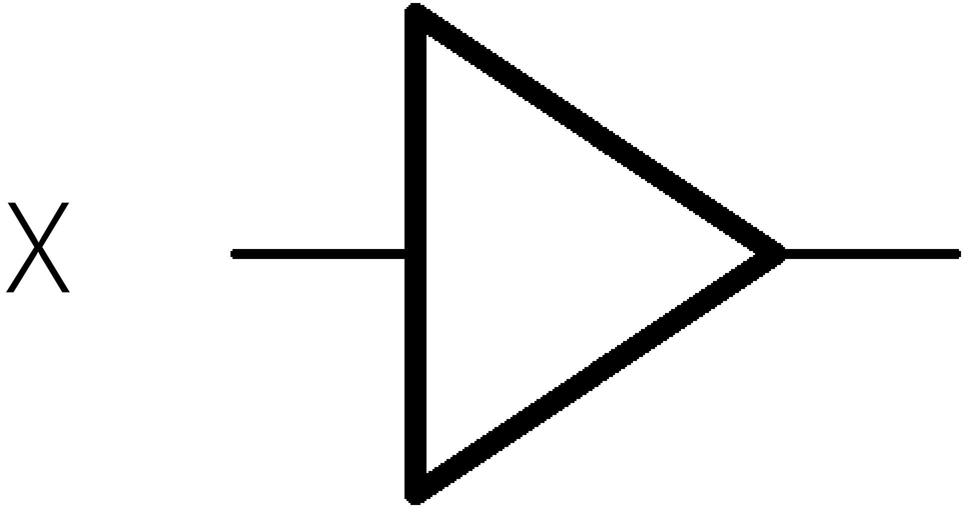
$F = X' = m_0$

X	F = F(X) = X
0	0
1	1

X	$F = F(X) = X = m_1$
0	0
1	1

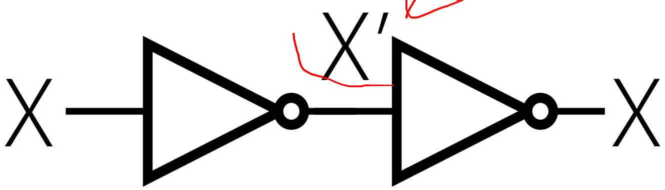
X	F = F(X) = X = m1
0	0
1	1

$f(x) = x$



$F = X = m_1$

$(x')' = x$



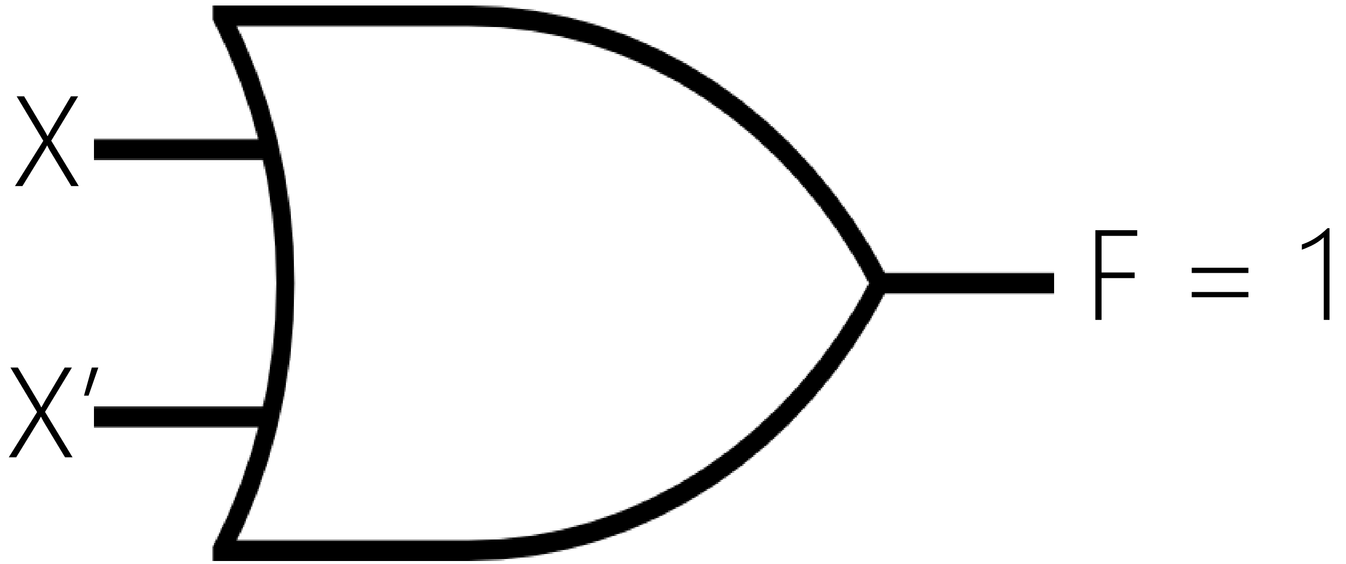
Digital Buffer

X	F = F(X)
0	1
1	1

X	$F = F(X) = X' = m_0$
0	1
1	1

X	$F = F(X) = X' + X = m_0 + m_1$
0	1
1	1

1



X′	X	X′+X
1	0	1
0	1	1

$$F = X + X' = 1$$

TRUTH TABLE $\leftrightarrow$ minterm

Y	X	F = F(Y,X) = ?
0	0	?
0	1	?
1	0	?
1	1	?

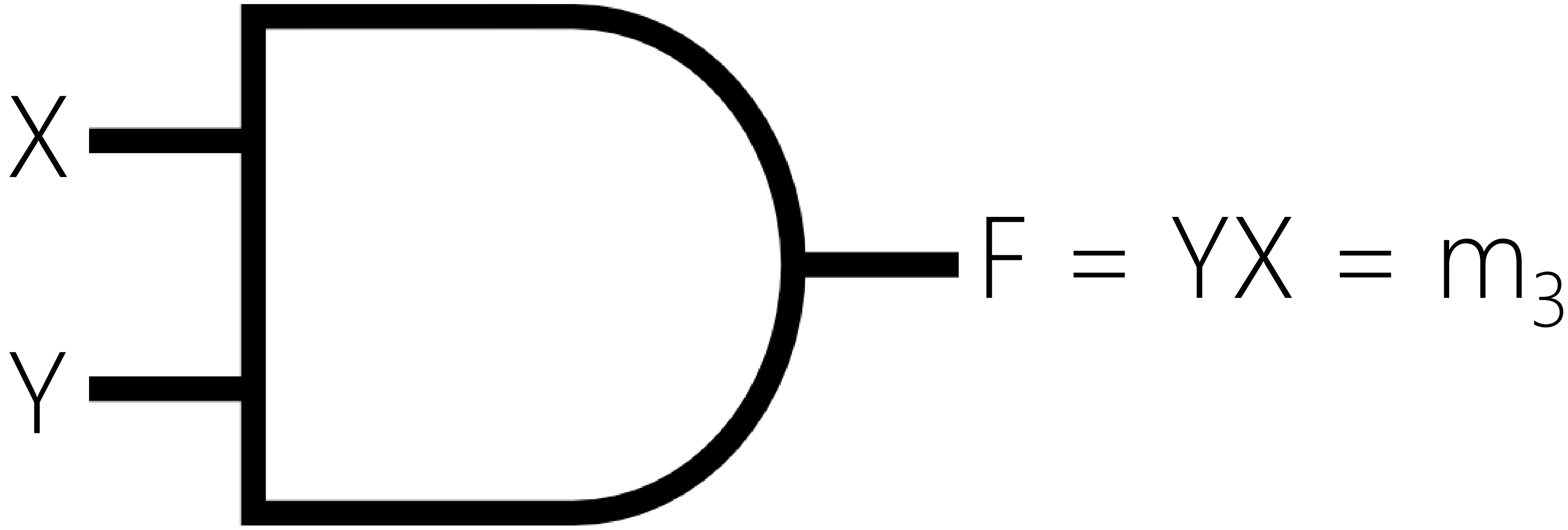
Y	X	$F = F(Y,X) = 0$
0	0	0
0	1	0
1	0	0
1	1	0

Y	X	F = F(Y,X) = YX
0	0	0
0	1	0
1	0	0
1	1	1

m_0	$Y'X'$
m_1	$Y'X$
m_2	YX'
m_3 → 11	1 YX

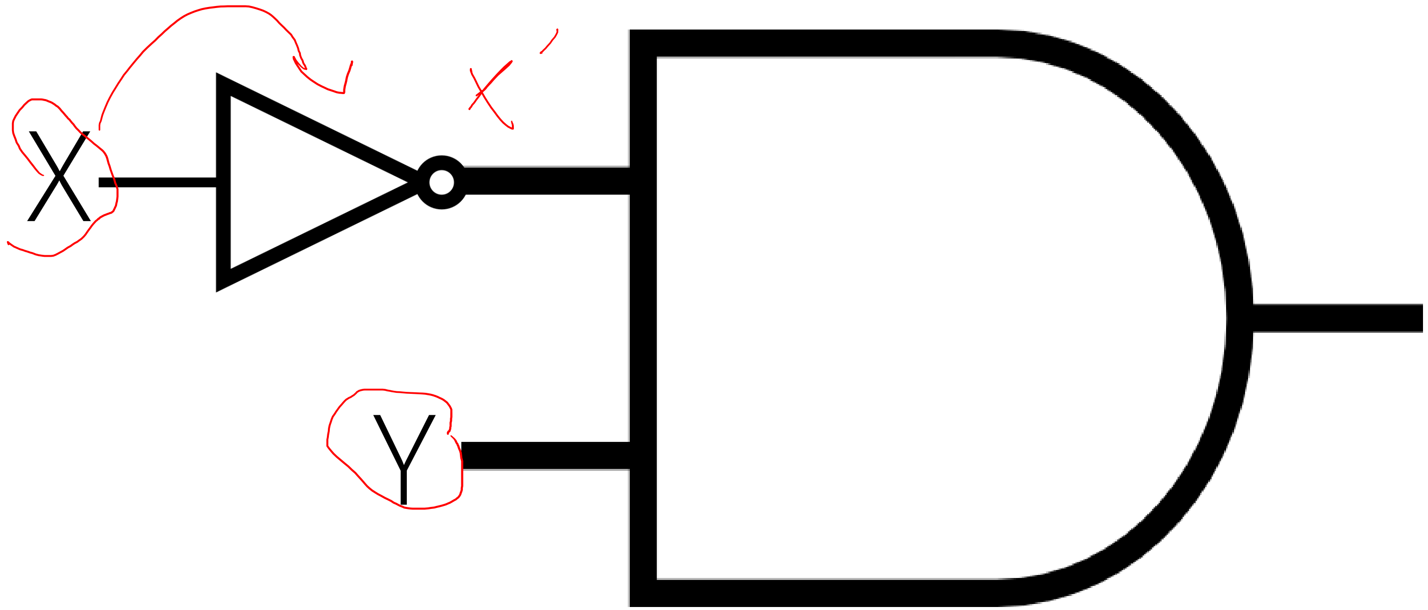
Y	X	$F = F(Y,X) = YX = m_3$
0	0	0
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = YX = m_3$
0	0	0
0	1	0
1	0	0
1	1	1


X
Y
$F = YX = m_3$

Y	X	$F = F(Y,X) = YX' = m_2$
0	0	0
0	1	0
1	0	2 $\rightarrow$ 1
1	1	0

Y	X	$F = F(Y,X) = YX' = m_2$
0	0	0
0	1	0
1	0	1
1	1	0

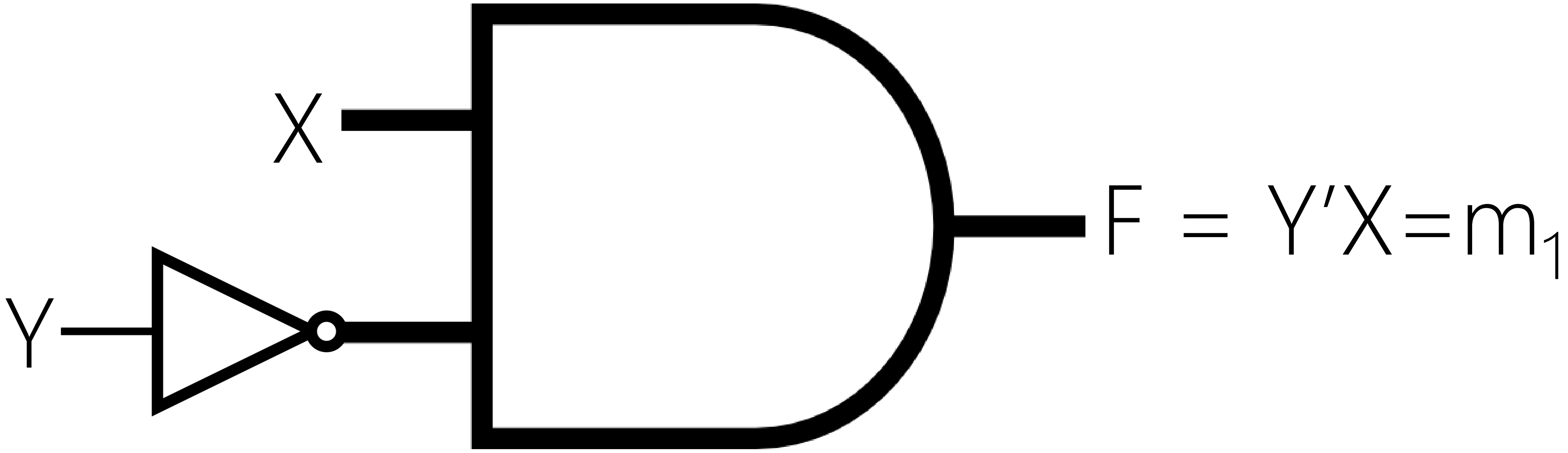


$F = YX' = m_2$

Y	X	$F = F(Y,X) = Y'X = m_1$
0	0	0
0	1	1
1	0	0
1	1	0

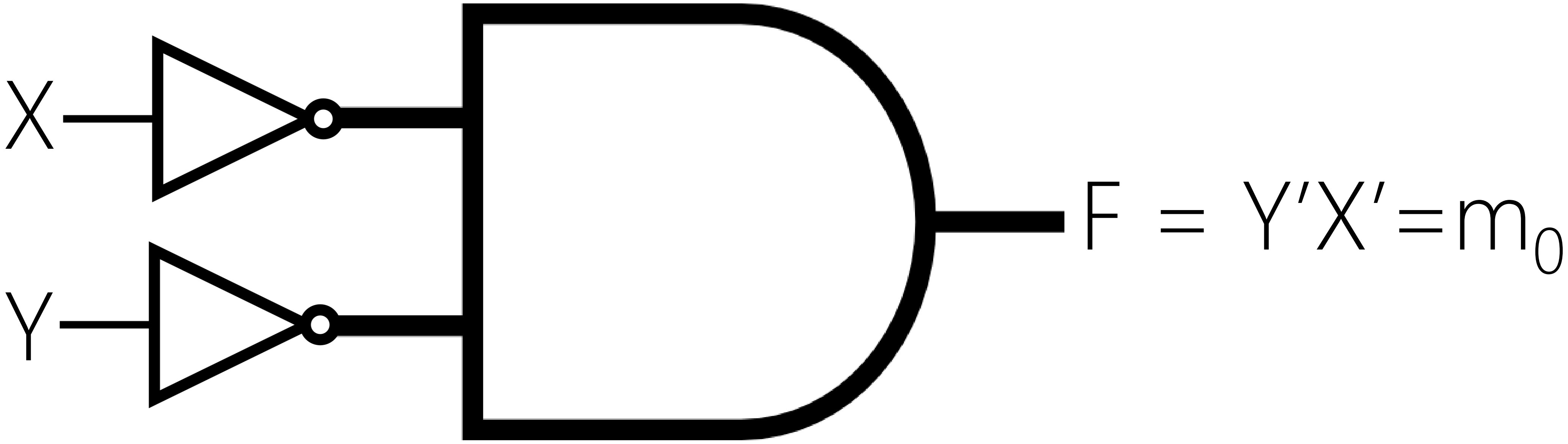
m_1

Y	X	$F = F(Y,X) = Y'X = m_1$
0	0	0
0	1	1
1	0	0
1	1	0


X
Y
F = Y'X=m1

Y	X	$F = F(Y,X) = Y'X' = m_0$
0	0	1
0	1	0
1	0	0
1	1	0

Y	X	$F = F(Y,X) = Y'X' = m_0$
0	0	1
0	1	0
1	0	0
1	1	0


X
Y
F = Y'X'=m0

Y	X	F = F(Y,X) = ?
0	0	1
0	1	1
1	0	0
1	1	0

OR

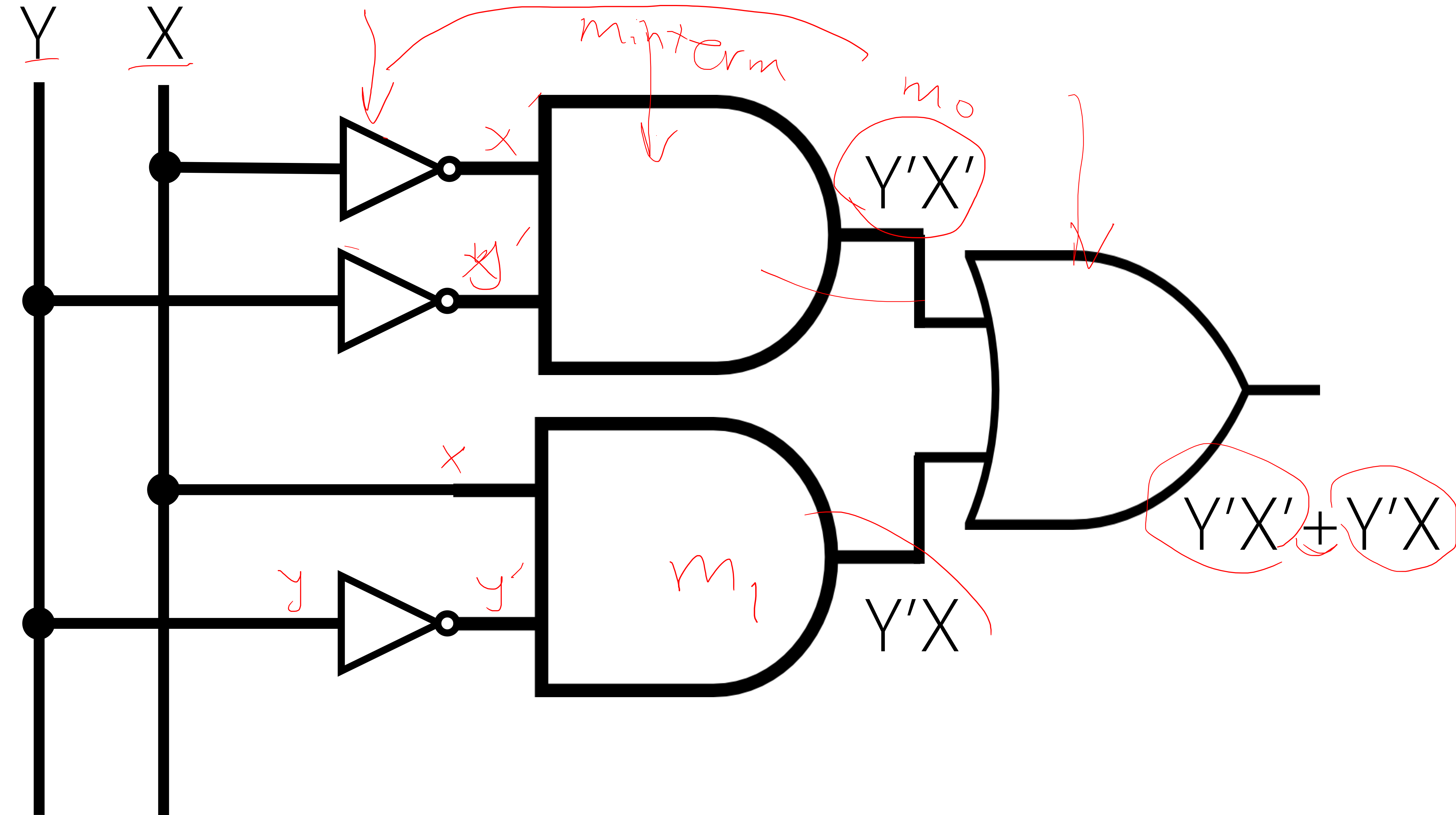
Y	X	F = F(Y,X) = Y'X'
0	0	1
0	1	1
1	0	0
1	1	0

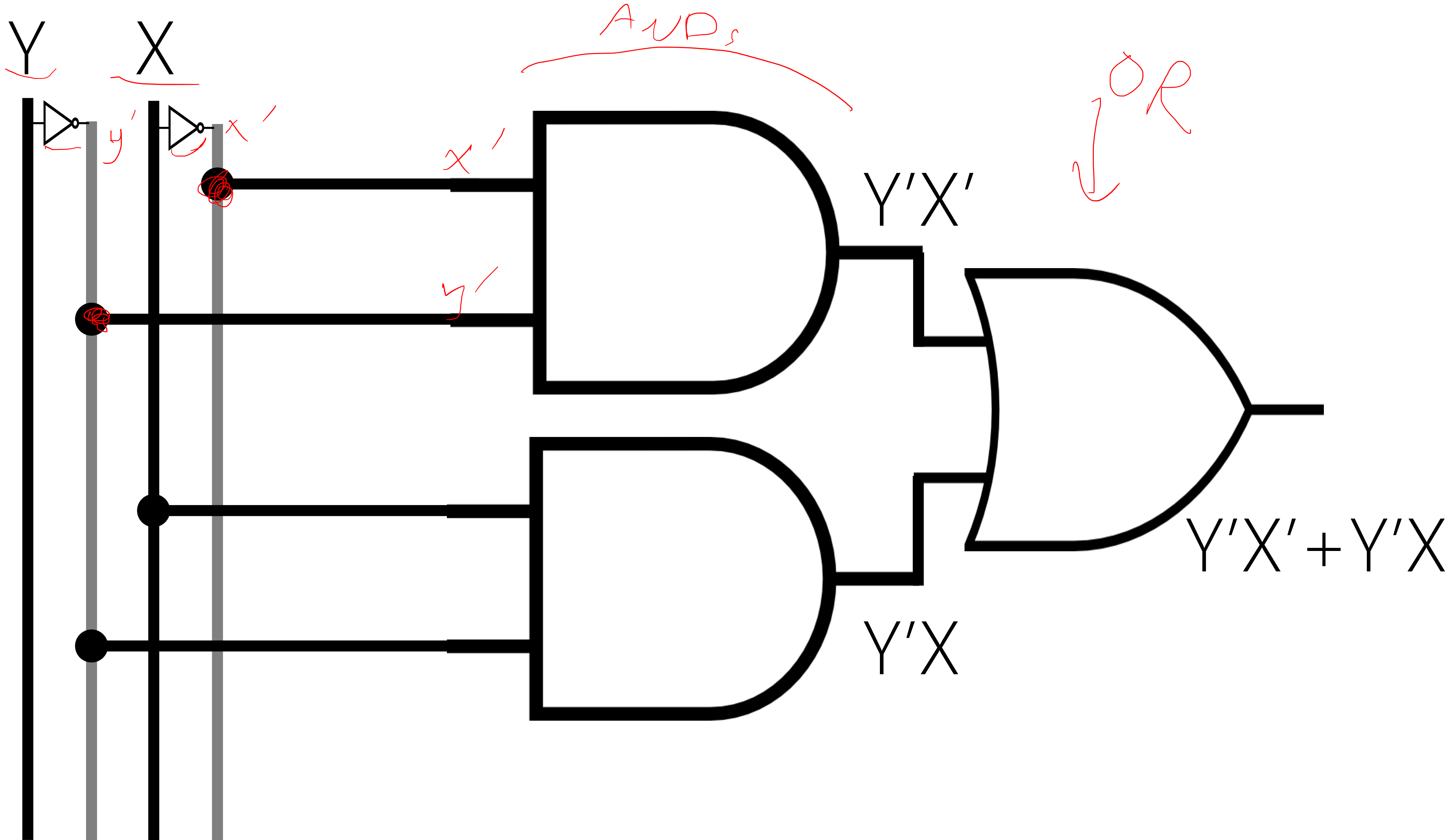
Y	X	$F = F(Y,X) = Y'X' + Y'X$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = m_0+m_1$
0	0	1
0	1	1
1	0	0
1	1	0

Y	X	$F = F(Y,X) = \sum m(0,1)$
0	0	1
0	1	1
1	0	0
1	1	0

$m_0 + m_1$,


Y
X
minterm
m0
Y'X'
x
y'
x
y
y'
m1
Y'X
Y'X'+Y'X



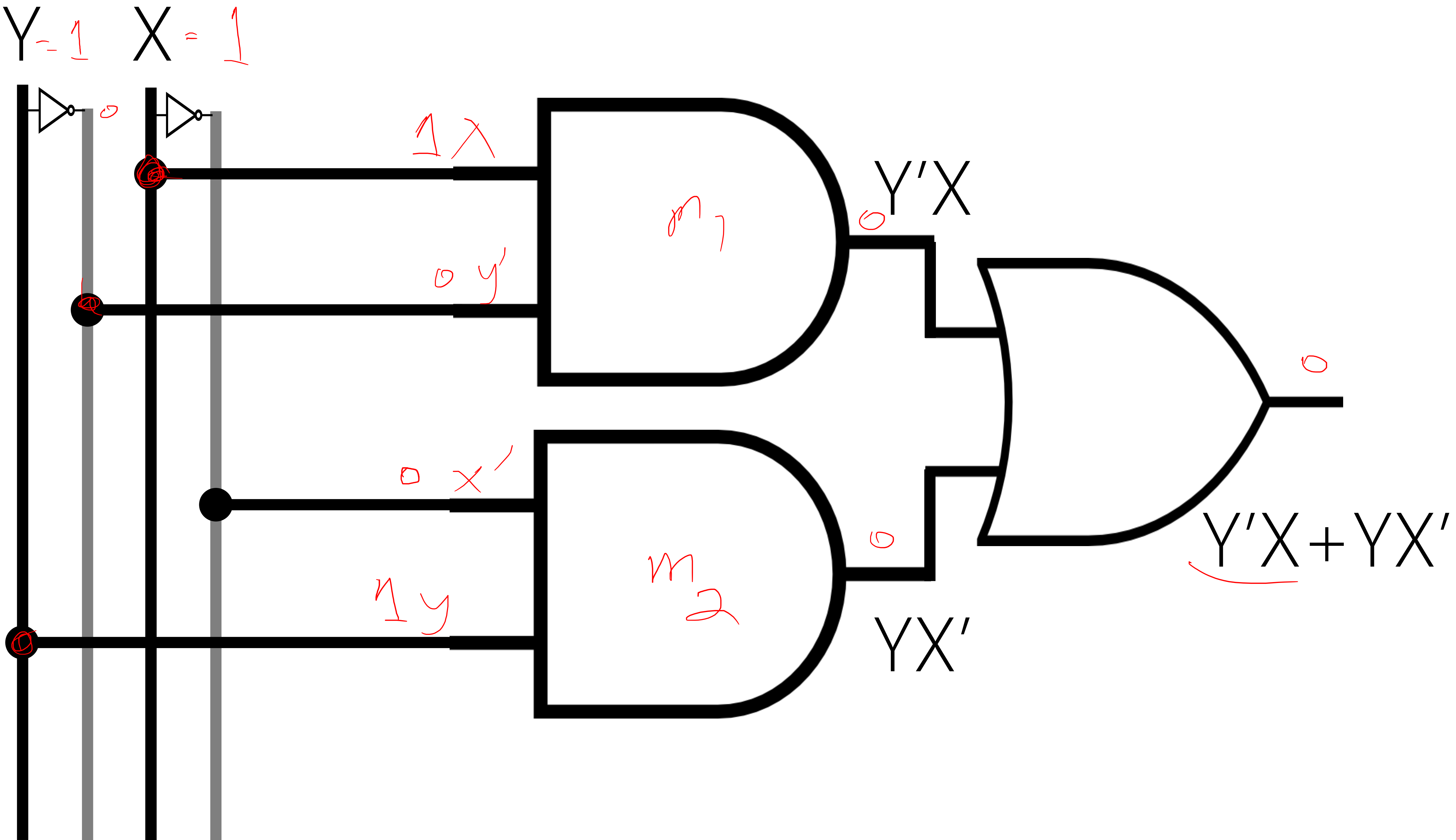
Y
X
ANDs
OR
y'
x'
x'
y'
Y'X'
Y'X
Y'X'+Y'X

Y	X		F = F(Y,X) = ?
0	0		0
0	1	m_1 +	1
1	0	m_2	1
1	1		0

Y	X	F = F(Y,X) = Y’X
0	0	0
0	1	1
1	0	1
1	1	0

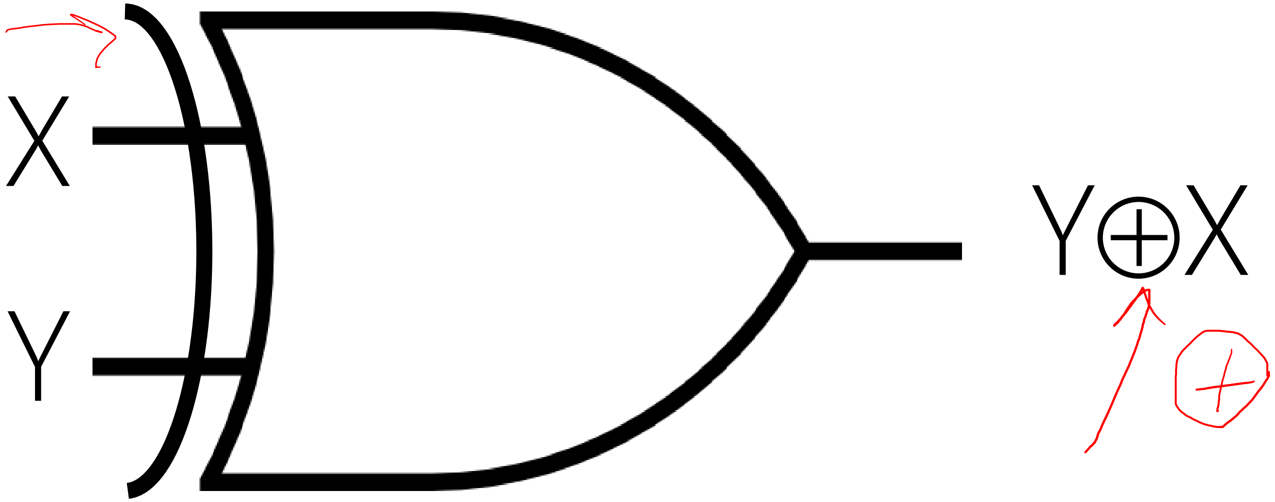
Y	X	$F = F(Y,X) = m_1+m_2$
0	0	0
0	1	1
1	0	1
1	1	0

Y	X	$F = F(Y,X) = \sum m(1,2)$
0	0	0
0	1	1
1	0	1
1	1	0



Y = 1
X = 1
0
1 X
0 y'
m1
0
Y'X
0 x'
1 y
m2
0
YX'
0
Y'X+YX'

Exclusive-OR (XOR) ≟ OR



$\oplus$

inequality

Y	X	OR	$F = F(Y,X) = Y'X + YX' = m_1 + m_2$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	0

$m_0 + m_3$

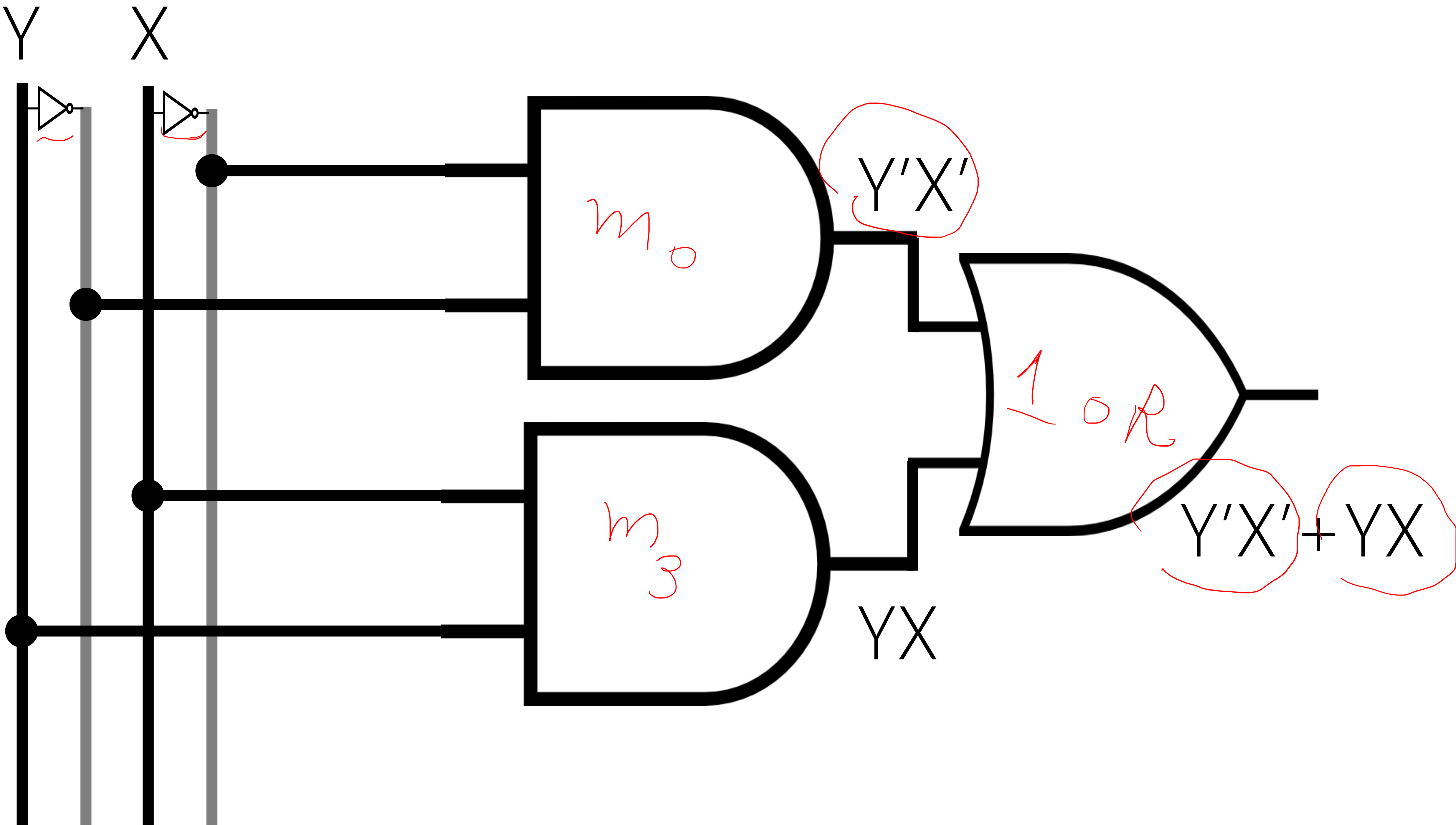
Y	X	F = F(Y,X) = ?
0	0	$m_0 \rightarrow 1$ $\sum m(0,3)$
0	1	0
1	0	0
1	1	m_3 1

Y	X	F = F(Y,X) = Y'X'
0	0	1
0	1	0
1	0	0
1	1	1

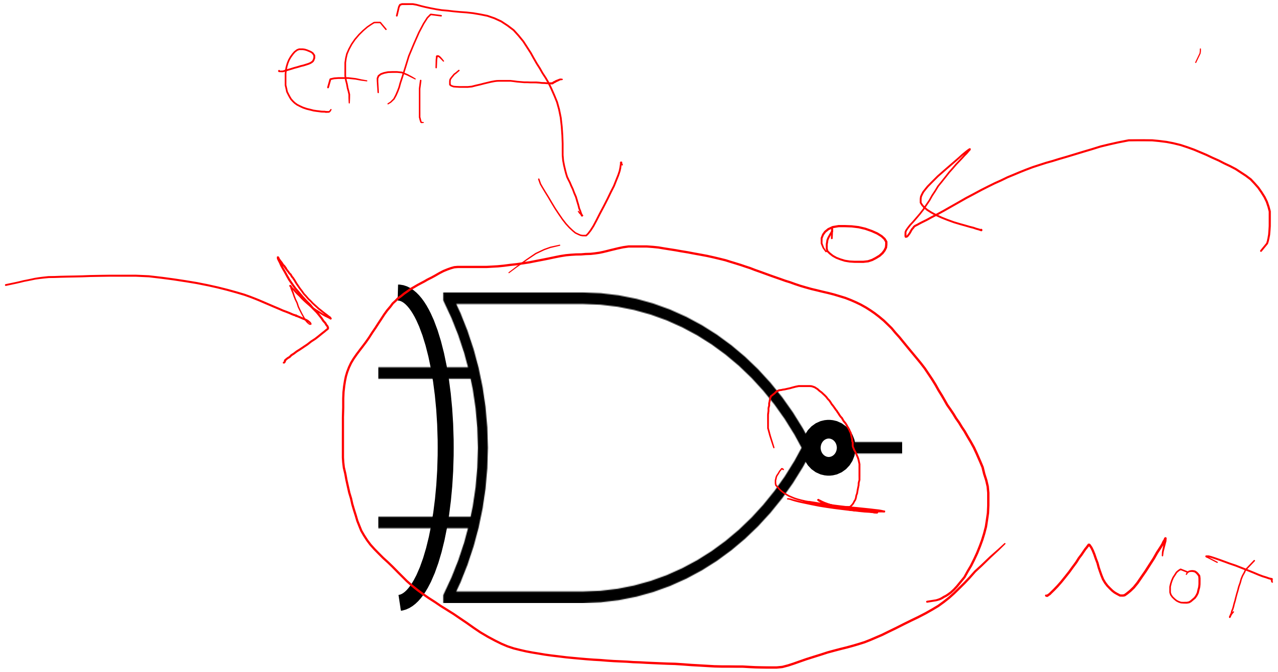
Y	X	$F = F(Y,X) = Y'X' + YX$
0	0	1
0	1	0
1	0	0
1	1	1

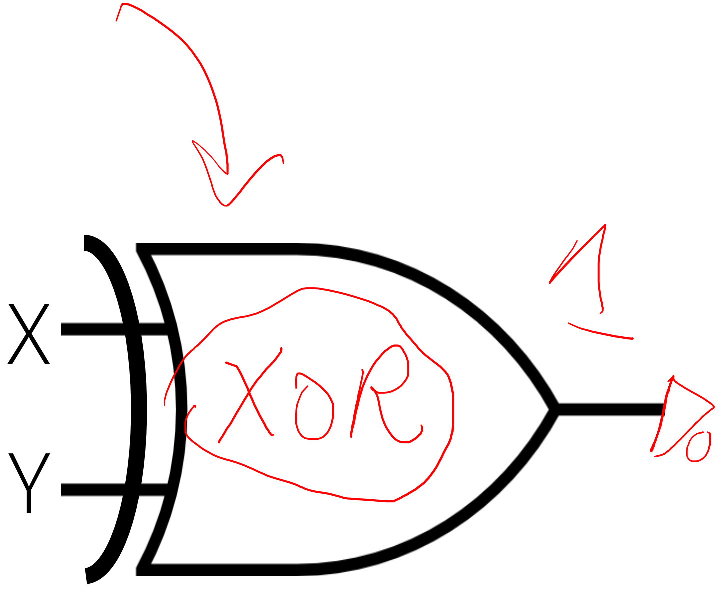
Y	X	$F = F(Y,X) = m_0+m_3$
0	0	1
0	1	0
1	0	0
1	1	1

Y	X	$F = F(Y,X) = \sum m(0,3)$
0	0	1
0	1	0
1	0	0
1	1	1



Y
X
Y'X'
m0
1 OR
m3
YX
Y'X'+YX

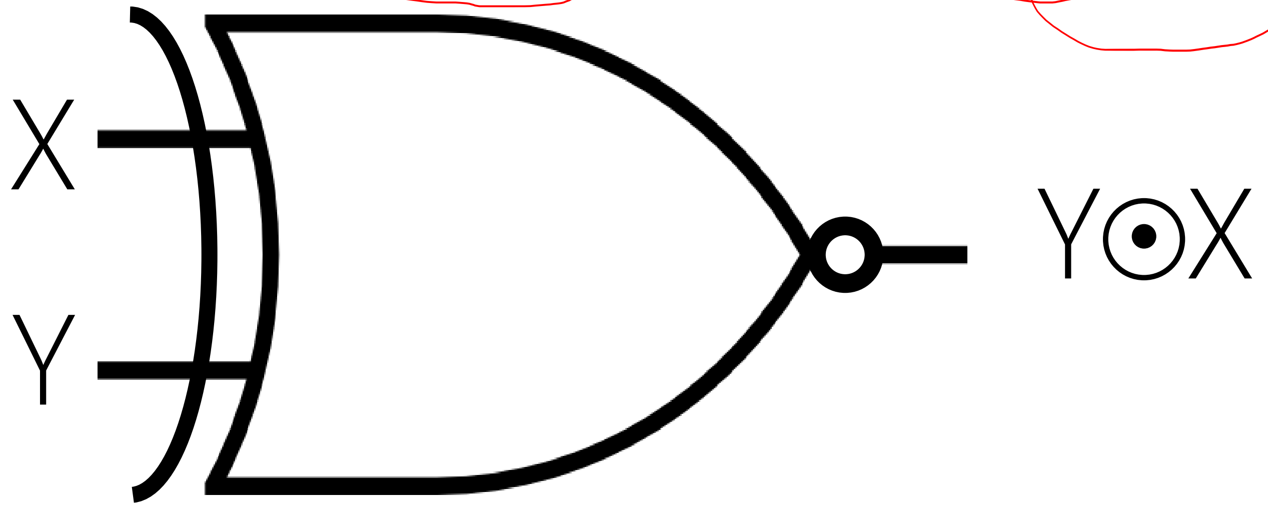




Y	X	$F = F(Y,X) = Y'X'+YX = m_0+m_3$	$F = F(Y,X) = Y'X+YX' = m_1+m_2$
0	0	1	0
0	1	0	1
1	0	0	1
1	1	1	0

NOT Exclusive-OR (NXOR → XNOR)

equality



Y	X	F = F(Y,X) = Y'X'+YX = m_0+m_3
0	0	1
0	1	0
1	0	0
1	1	1

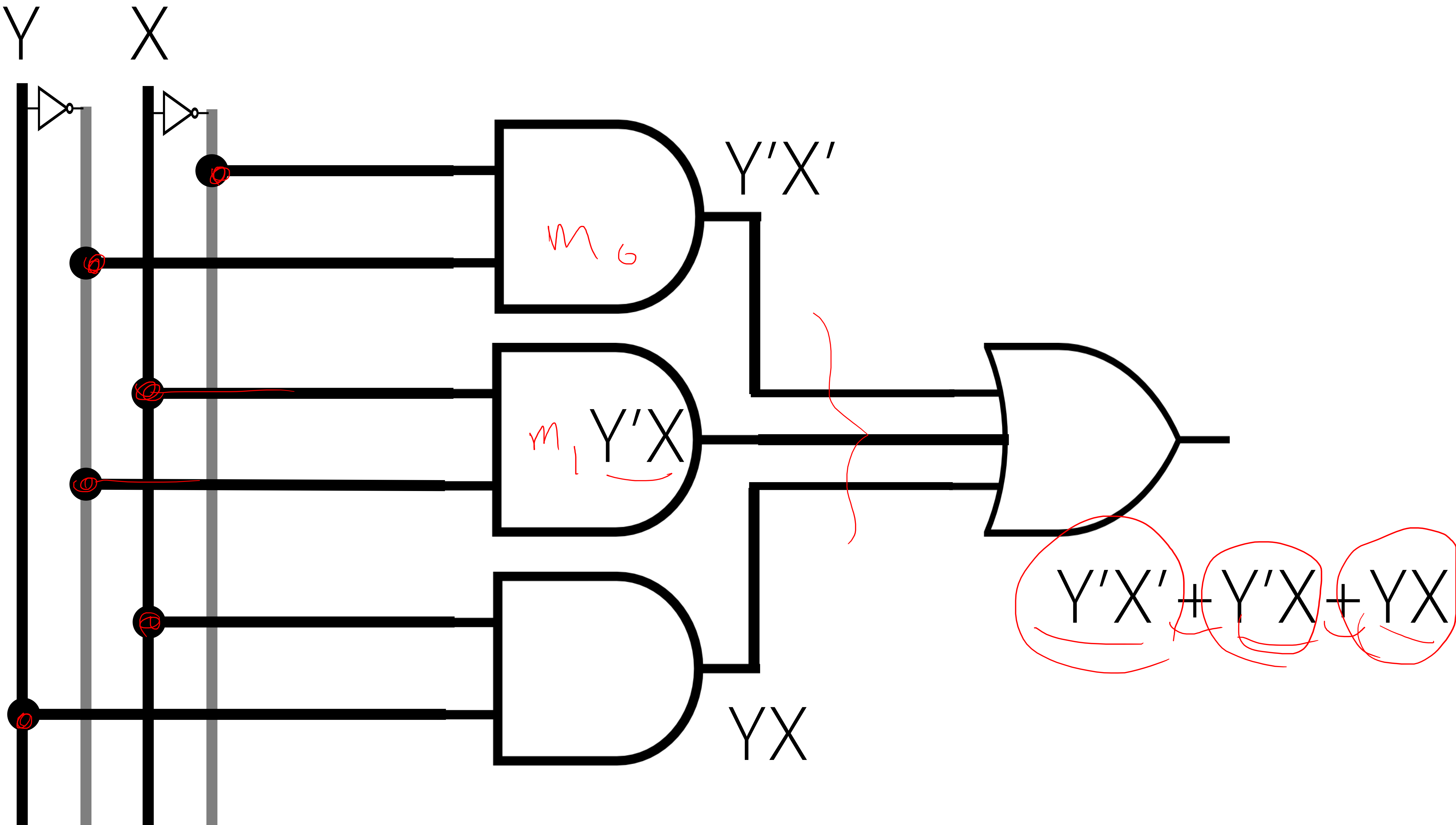
Y	X	F = F(Y,X) = ?
0	0	$m_0 \xrightarrow{AND}$ 1
0	1	$m_1 \xrightarrow{AND}$ 1 +
1	0	0
1	1	$m_3 \xrightarrow{AND}$ 1

Y	X	F = F(Y,X) = Y'X'
0	0	1
0	1	1
1	0	0
1	1	1

Y	X	$F = F(Y,X) = Y'X' + Y'X$
0	0	1
0	1	1
1	0	0
1	1	1

Y	X	F = F(Y,X) = Y’X’+Y’X+YX
0	0	1
0	1	1
1	0	0
1	1	1

Y	X	$F=F(Y,X)=m_0+m_1+m_3$ $=\sum m(0,1,3)$
0	0	1
0	1	1
1	0	0
1	1	1


Y
X
Y'X'
m0
m1
Y'X
YX
Y'X'+Y'X+YX

1. Truth table
2. → Rows ⇒ 1
3. for each row ⇒ m_i

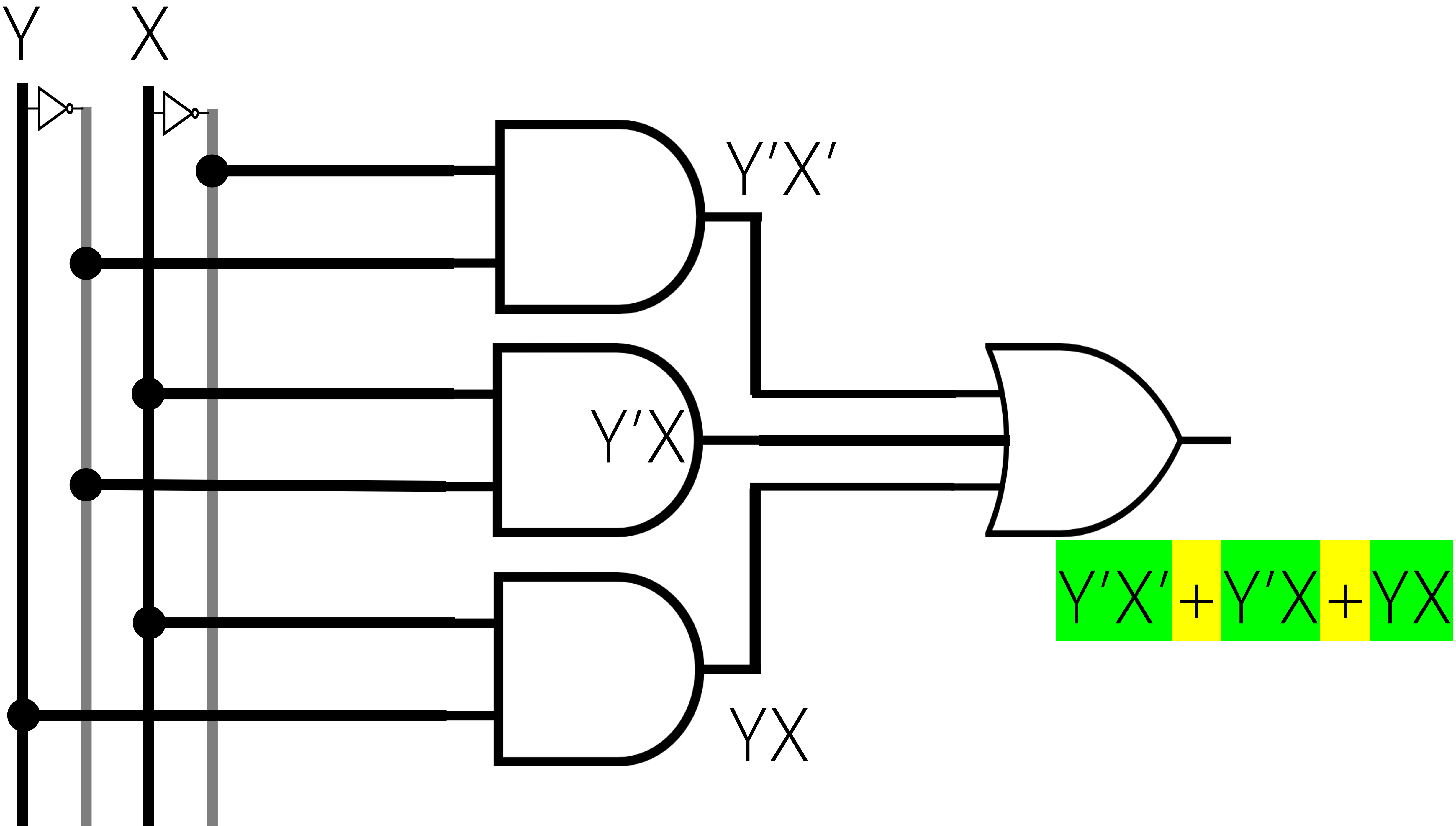
N ⇒ m_i

SUM OF PRODUCTS (SOP)

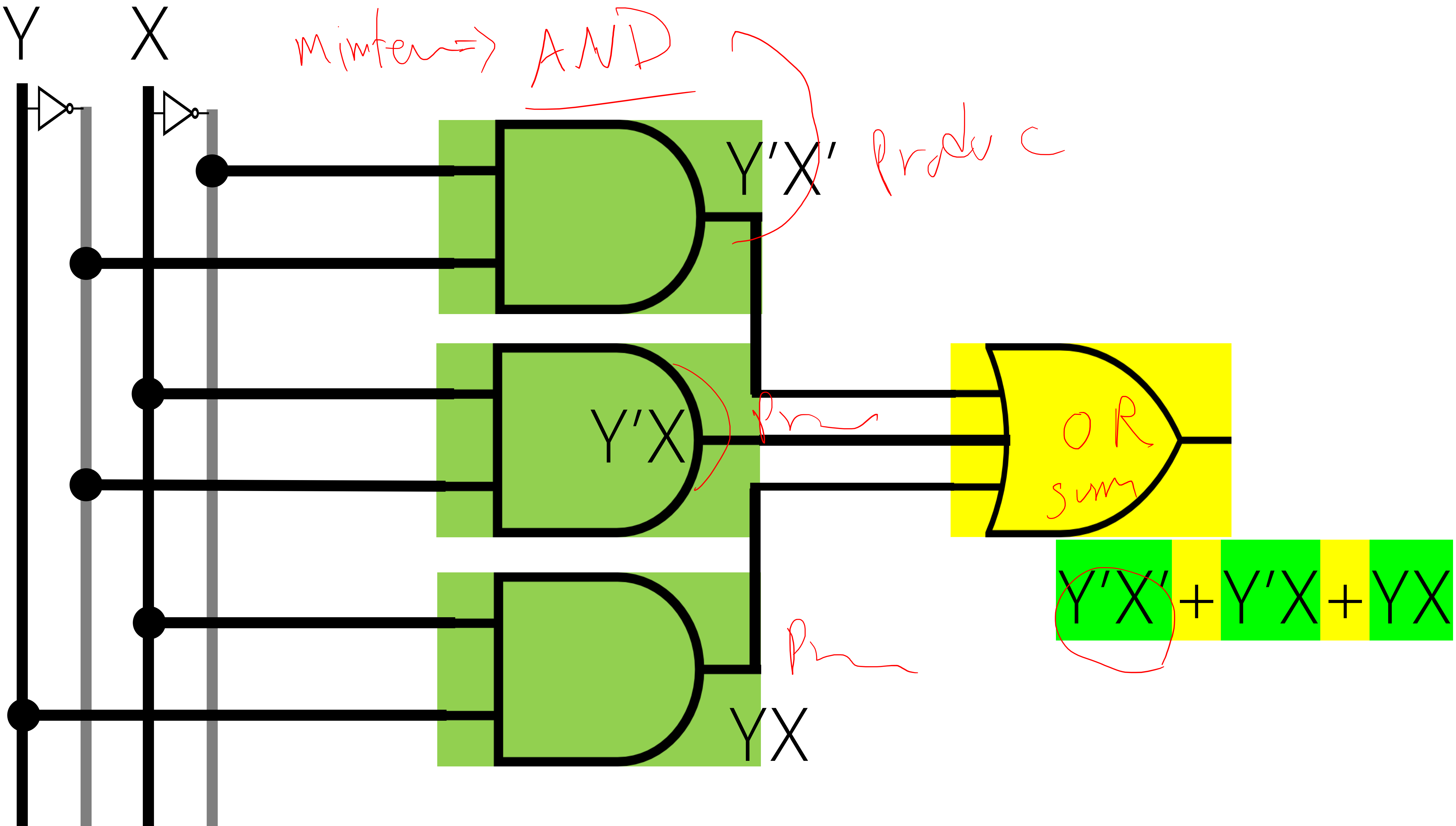
4. +

OR

$XY = X \cdot Y = X * Y$


Y
X
Y'X'
Y'X
YX
Y'X' + Y'X + YX

2 LEVELS
AND → OR



Y
X
minterm → AND
Y'X'
Produc
Y'X
Pr
YX
Pr
OR
sum
Y'X' + Y'X + YX

0 2 4 6 ...

Given 3 inputs, design a circuit to determine if there is even number of 1

TRUTH TABLE $\leftrightarrow$ minterm

INPUTS/BINARY VARIABLES

OUTPUTS/BOOLEAN FUNCTIONS

C Z	B Y	A X	F(Z,Y,X)=?
X 0	Y 0	Z 0	⇒ 1 ?
0	0	1	⇒ 0 ?
0	1	0	⇒ 0 ?
0	1	1	⇒ 1 ?
1	0	0	⇒ 0 ?
1	0	1	⇒ 1 ?
1	1	0	⇒ 1 ?
1	1	1	0 ?

Z	Y	X	F(Z,Y,X)=?
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=Z'Y'X'$
0	0	0	1 m_0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

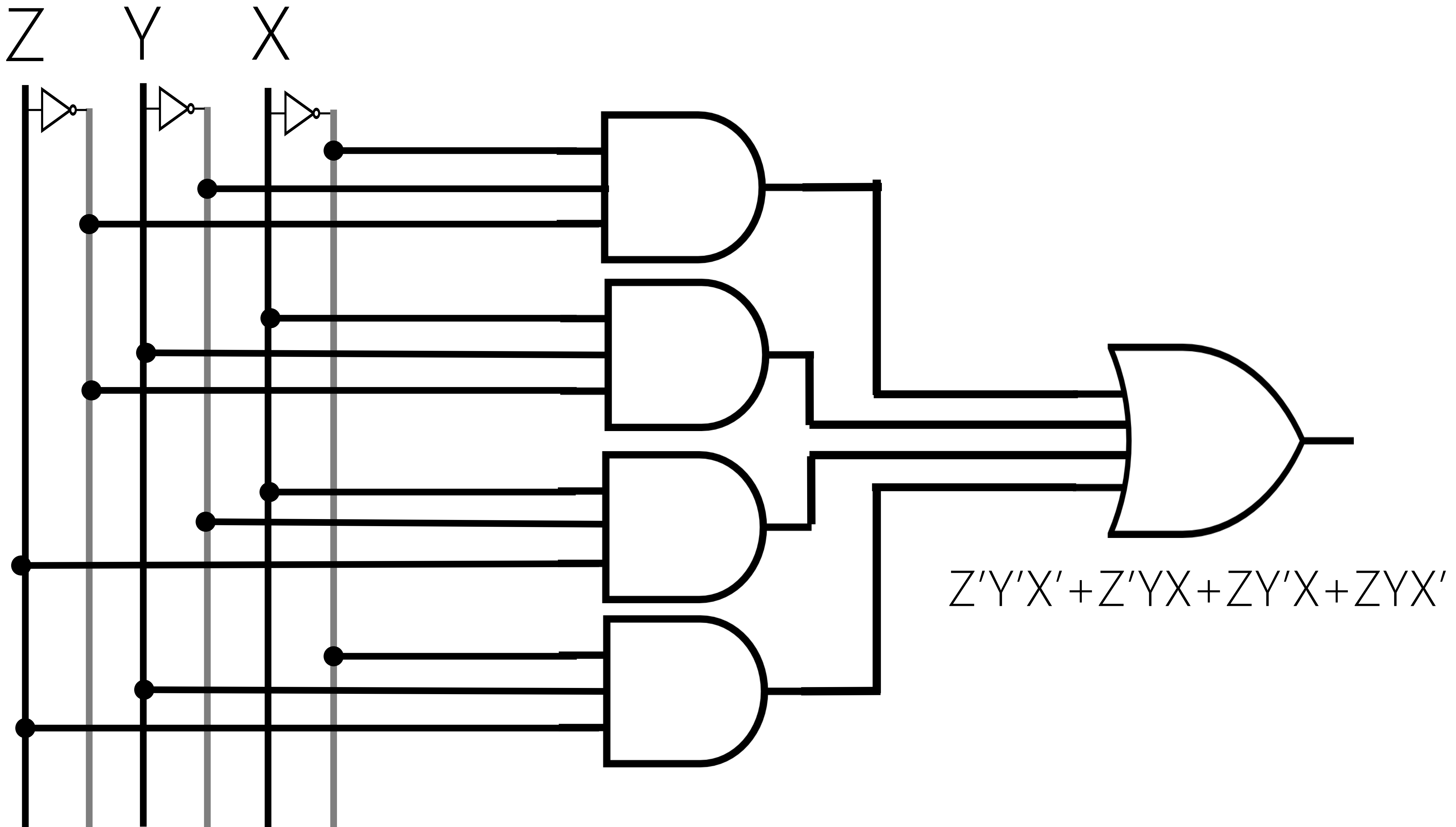
m_3

Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX+ZY'X$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

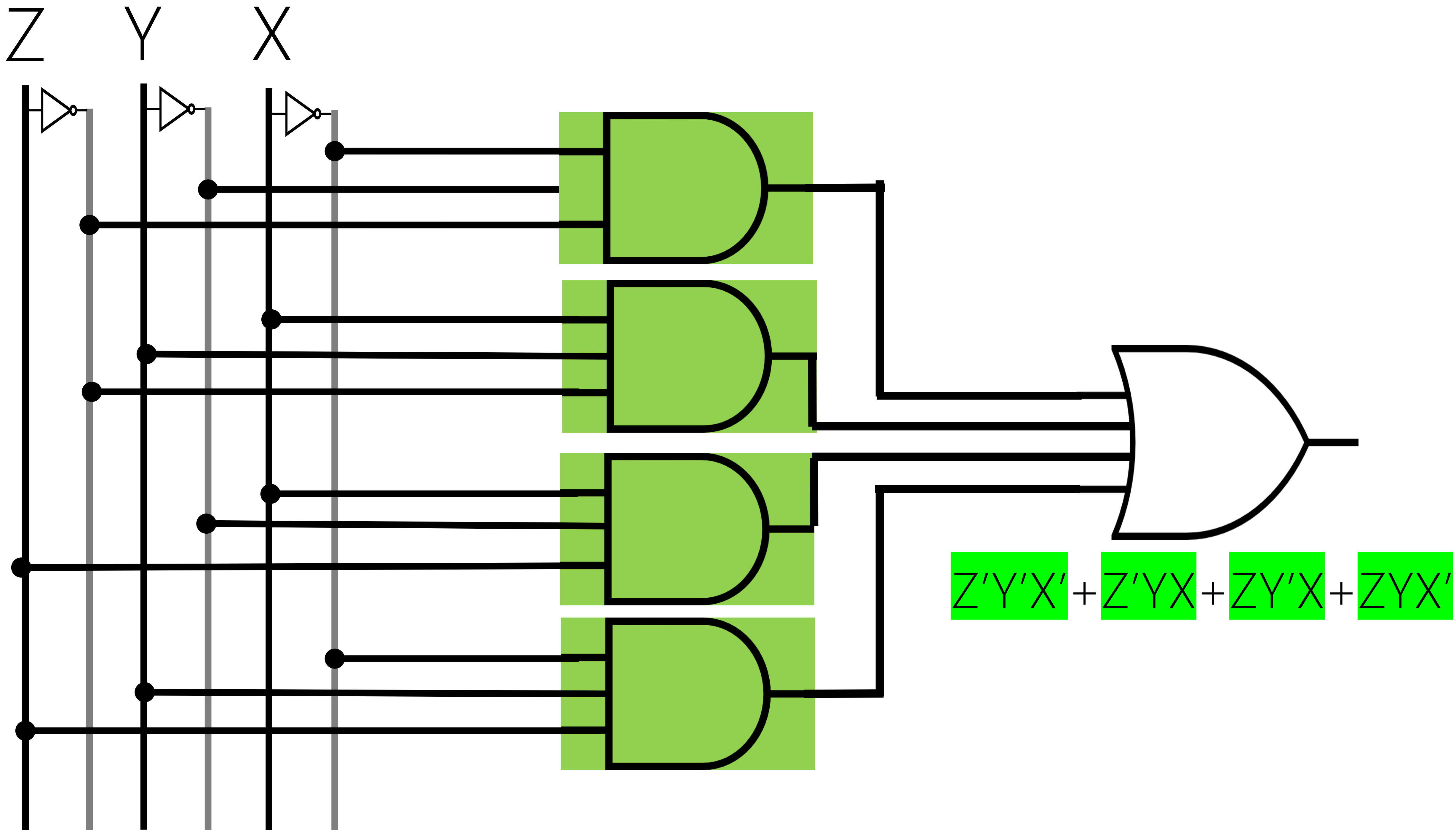
Z	Y	X	$F(Z,Y,X)=Z'Y'X'+Z'YX+ZY'X+ZYX'$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

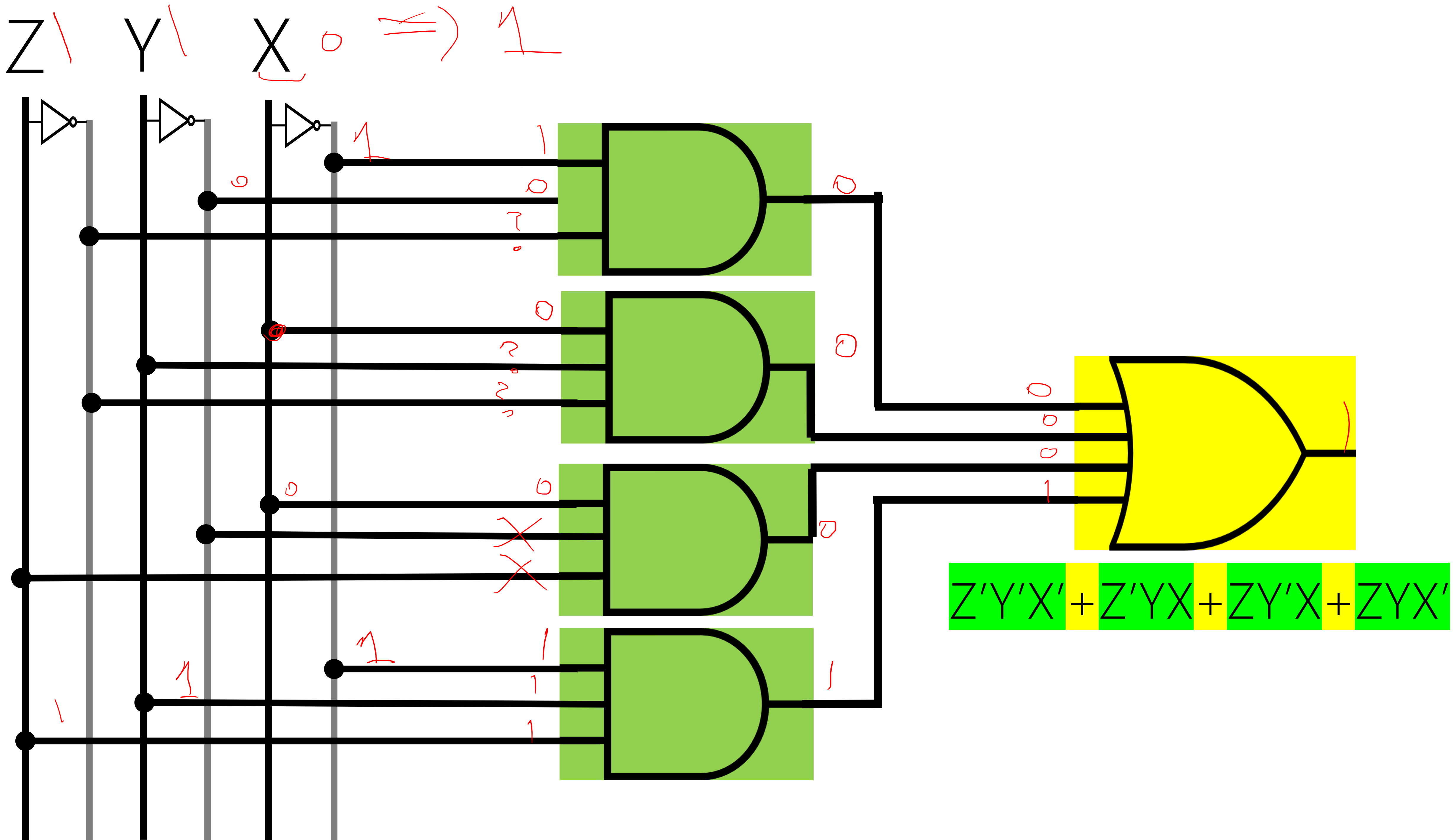
Z	Y	X	$F(Z,Y,X)=m_0+m_3+m_5+m_6$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Z	Y	X	$F(Z,Y,X)=m_0+m_3+m_5+m_6=\sum m(0,3,5,6)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0


Z Y X
Z'Y'X'+Z'YX+ZY'X+ZYX'

SUM OF PRODUCTS (SOP)
2 LEVELS AND-OR


Z
Y
X
Z'Y'X' + Z'YX + ZY'X + ZYX'



Z
Y
X
Z'Y'X' + Z'YX + ZY'X + ZYX'

SHOW THE REMAINDER (MOD)
NUMBER % 3 = ?

TRUTH TABLE $\leftrightarrow$ minterm

INPUTS/BINARY VARIABLES	OUTPUTS/BOOLEAN FUNCTIONS

WHAT IS THE RANGE OF NUMBERS?

WHAT IS THE RANGE OF NUMBERS?

$[0, 15]_{10}$

HOW MANY INPUT BINARY VARIABLES?

$$[0, 15]_{10} = [0,1111]_2 = [0000,1111]_2$$

$3 \% 3 = 0$

0011

D C B A

W Z Y X

	W	Z	Y	X
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
	0	1	0	1
	0	1	1	0
	0	1	1	1
	1	0	0	0
	1	0	0	1
	1	0	1	0
	1	0	1	1
	1	1	0	0
	1	1	0	1
14	1	1	1	0
15	1	1	1	1

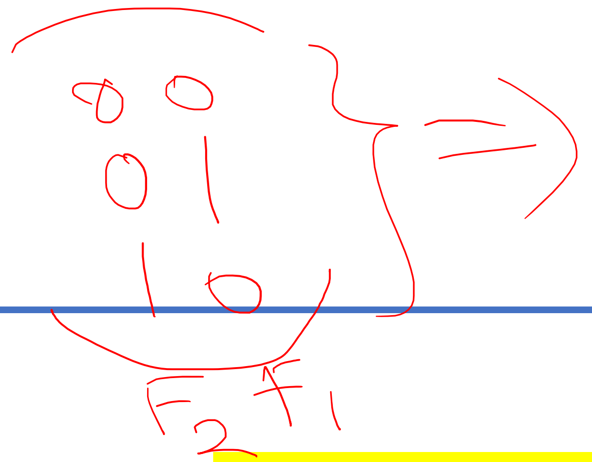
WHAT IS THE RANGE OF OUTPUT?

WHAT IS THE RANGE OF OUTPUT?

The remainder of any number divided by 3 is 0, 1, 2

WHAT IS THE RANGE OF OUTPUT?

$[0, 2]_{10}$



HOW MANY BOOLEAN FUNCTION?

$$[0, 2]_{10} = [0, 10]_2 = [00, 10]_2$$

W	Z	Y	X %3	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

W	Z	Y	X	F_1	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

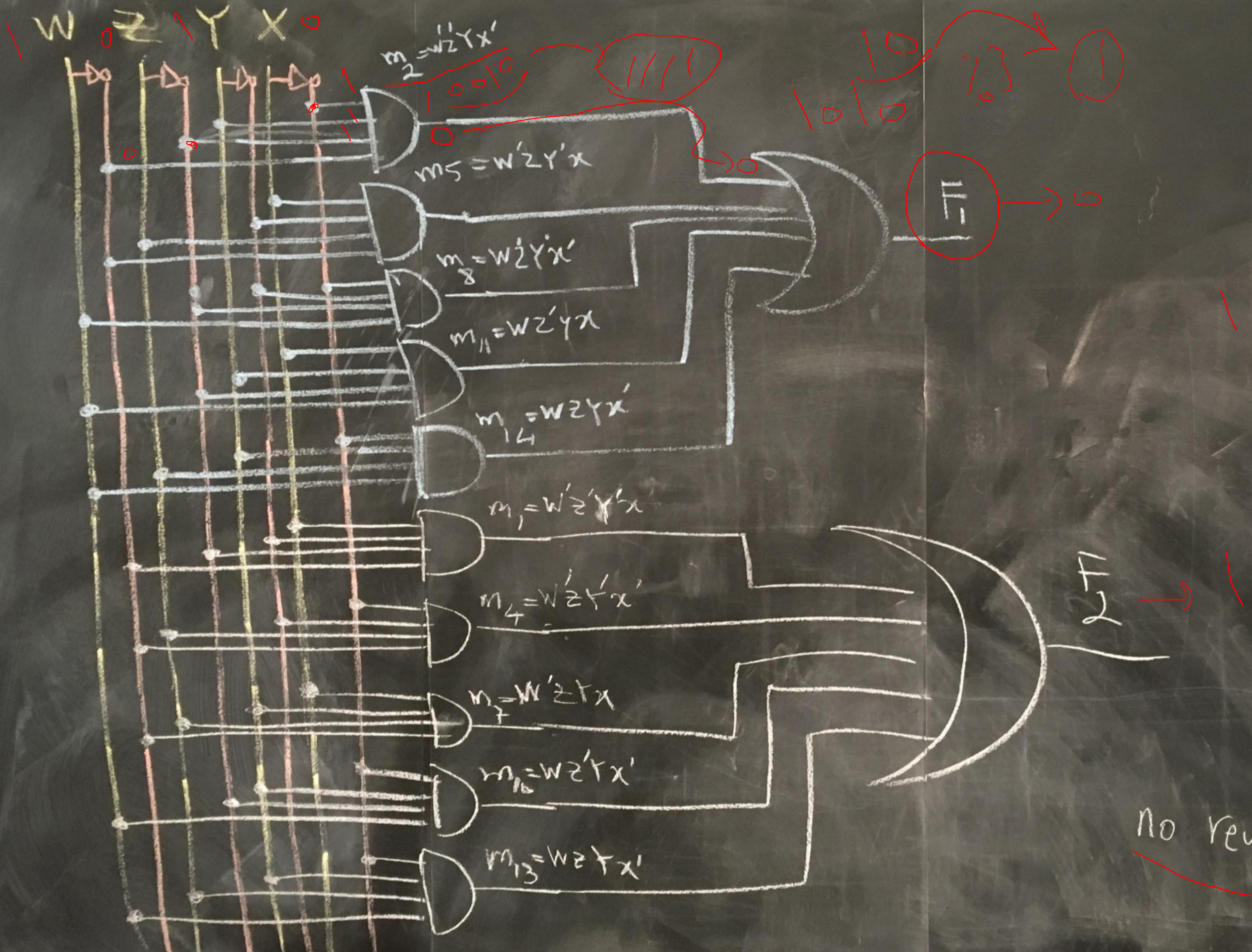
阿记 = 0

minterms

W	Z	Y	X	$F_1=m_2+m_5+m_8+m_{11}+m_{14}$	F_2
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1 8	0	0	0	m_8 1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	m_{11} 1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	m_{14} 1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1=\sum m(2,5,8,11,14)$	$F_2=m_1+m_4+m_7+m_{10}+m_{13}$
0	0	0	0	0	0
0	0	0	1	0	m_1 1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	m_4 1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0

W	Z	Y	X	$F_1 = \sum m(2,5,8,11,14)$	$F_2 = \sum m(1,4,7,10,13)$
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	1	0
0	0	1	1	0	0
0	1	0	0	0	1
0	1	0	1	1	0
0	1	1	0	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	0	1
1	1	1	0	1	0
1	1	1	1	0	0



$F_1 = \sum m(2,5,8,11,14)$

$F_2 = \sum m(1,4,7,10,13)$

$F_3 = \sum m(4,11)$

$1010 \Rightarrow 01$

no reuse for minterms