

**School of Computer Science**

**Faculty of Science**

**COMP-2650: Computer Architecture I: Digital Design**

**Winter 2021**

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| Assignment# | Date | Title | Due Date | Grade Release Date |
| Lec 06 | Week 06 | **W06: Boolean Algebra & Minimization** | Feb. 23, 2021  Tuesday Midnight [AoE](https://www.timeanddate.com/time/zones/aoe)  Wednesday 7 AM EDT | March. 01, 2021 |

The objectives of the lecture (weekly) assignments are to practice on topics covered in the lectures as well as improving the student’s critical thinking and problem-solving skills in ad hoc topics that are closely related but not covered in the lectures. Lecture assignments also help students with research skills, including the ability to access, retrieve, and evaluate information (information literacy).

**Lecture Assignments Deliverables**

You should answer 4 of the below questions based on your preference using an editor like MS Word, Notepad, and the likes or pen in papers. You have to write and scan the papers clearly and merge them into a single file in the latter case. In the end, you have to submit all your answers in one single pdf file COMP2650\_Lec06\_{UWinID}.pdf containing the following items:

1. Your name, UWinID, and student number
2. The question Id for each answer. Preferably, the questions should be answered in order of increasing Ids. *Please note that if your answers cannot be read, you will lose marks.*
3. Including the questions in your submission pdf file is optional.

*Please follow the naming convention as you lose marks otherwise.* Instead of {UWinID}, use your own UWindsor account name, e.g., mine is [hfani@uwindsor.ca](mailto:hfani@uwindsor.ca), so, my submission would be: COMP2650\_Lec06\_hfani.pdf

**Lecture Assignments**

**(select only 4 questions based on your preference)**

1. Minterm is also called standard product (standard ANDing). Maxterm is also called standard sum (standard ORing). Boolean functions that expressed as a sum of products (SoP) or product of sums (PoS) are said to be in **canonical form** if they are in the forms of minterms or MAXTERMs that include ALL the input binary variables in each term either in normal form or in complement form. For instance, F1(Y, X)=Y’X+Y’X’ is in canonical form because F1 is in the form of m0 + m1 and each minterm includes all input variables. However, although F3(Z,Y,X)=Y’X+ZY is sum of products, it is not in canonical form since the first term (Y’X) does not have the input variable Z or Z’ or the second term (ZY) does not have X or X’. In design process, we start with truth table and write the Boolean function in their canonical forms either in sum of minterms or product of MAXTERMs. Then, through the minimization process, the Boolean function may lose its canonical form in order to have less number of terms or variables instead.

Using truth table, given any two Boolean functions F1 and F2, prove that

* 1. The Boolean function E = F1 + F2 contains the sum of the minterms of F1 and F2 in order to be in sum of products canonical form.
  2. The Boolean function G = F1F2 contains only the minterms that are common to F1 and F2.

1. Obtain the truth table of the following functions, and express each function in sum of products (SoP) and product of sums (PoS) in canonical forms:
   1. (c’ + d)(b + c’)
   2. bd' + acd’ + ab’c + a’c’
2. Express the **complement** of the following functions in sum of minterms form:
   1. F(A,B ,C, D) = m(2, 4, 7, 10, 12, 14) 🡪 F’(A,B ,C, D) = m(?,…)
   2. F(x, y, z) = M(3, 5, 7) 🡪 F’(x, y, z) = m(?,…)
3. By default, means sum of minterms. So, we can drop ‘m’ and is the same as m. Similarly, means product of maxterms and we can drop ‘M’ and is the same as M. Convert each of the following to the other canonical form. Then algebraically simplify them, if possible:
   1. F(x, y, z) = (1, 3, 5)
   2. F(A, B, C, D) = (3, 5, 8, 11)
4. Determine the number of AND and OR gate as well as the number of their inputs (e.g., 2-input AND, 3-input AND, …) for the design of the following Boolean functions. Try to algebraically simplify them, if possible, and compare the original requirement with the simplified version. The design is not needed.
   1. (x, y, z) = (1, 2, 4, 5)
   2. F(A, B, C, D) = (0, 3, 5, 8, 11,13)
5. Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams. Then algebraically simplify the Boolean expression, if possible.

Diagram, schematic

Description automatically generated

1. **By convention,** when writing a Boolean expression as a function of binary variables, e.g., F(Z, Y, X), we assume the left most binary variable (e.g., Z) having the highest significance in writing the index numbers for minterms and MAXTERMs, e.g., m2=Z’YX’ or M3=(Z+Y’+X’). However, we know that AND and OR are commutative. Hence, m2=Z’YX’ = YZ’X’ = m4 or M3= (Z+Y’+X’) = (Y’+X’+Z)= M6. Is this argument correct? Justify your answer.
2. True or False:
   1. If F(X,Y,Z)=X’YZ then F(Z,Y,X)=X’YZ
   2. If F(X,Y,Z)=X’YZ then F(Z,Y,X)=Z’YX
   3. If F(X,Y,Z)=X’YZ then F(Z,Y,X)=YXZ’
   4. If F(X,Y,Z)=m0+m1 then F(Z,Y,X)=m0+m1
   5. If F(X,Y,Z)=m0+m1 then F(Z,Y,X)=m0+m4