

# Final Exam (Tentative)

1

| Date Time                 | Location                      |
|---------------------------|-------------------------------|
| Saturday, 10-Dec, 7:00 PM | Odette Building (OB) 104, 112 |

# Last Week × Q4Me

2

Book vs. Slides

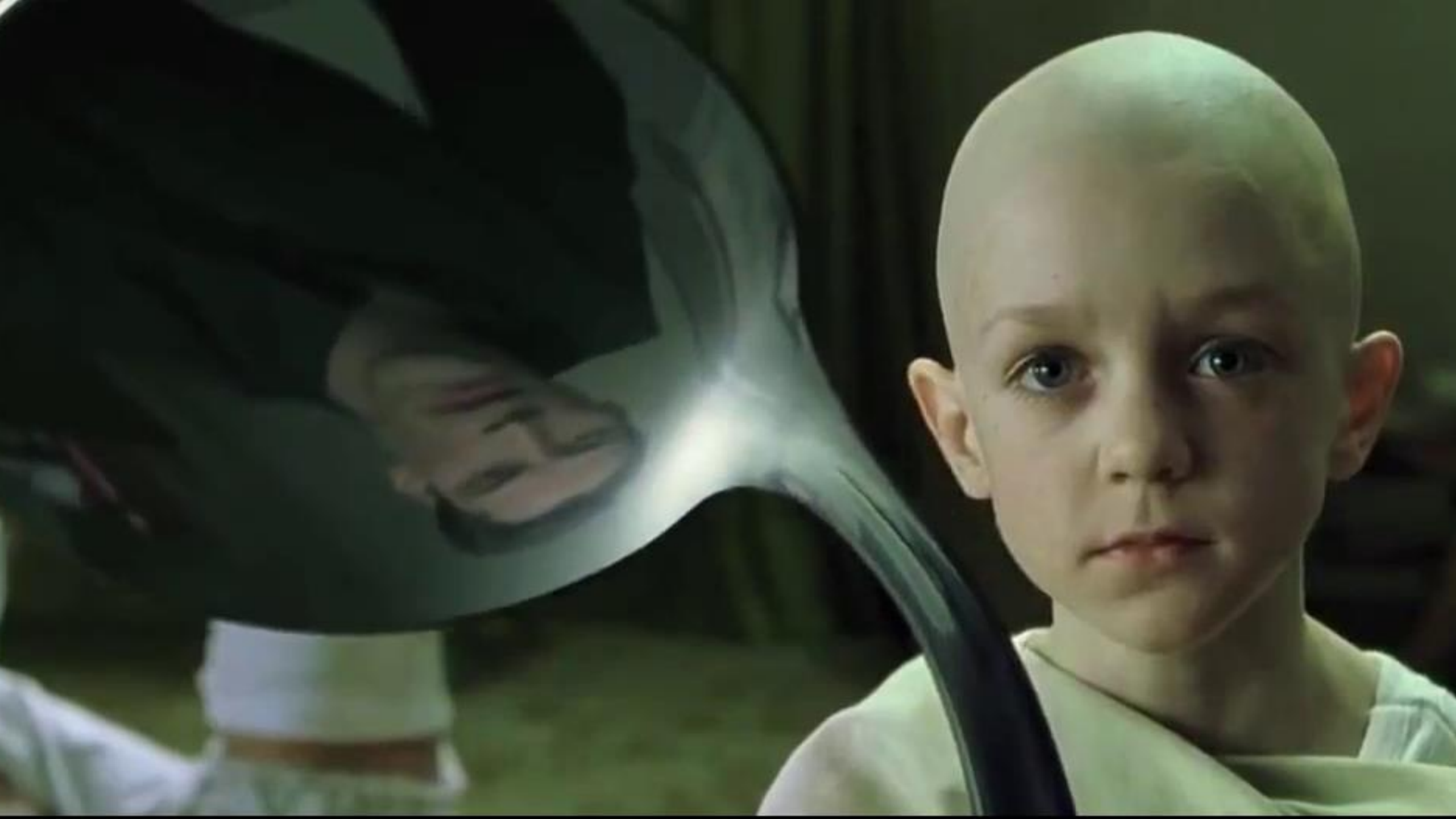
CH02, CH05 (2<sup>nd</sup> Ed.), CH05 (1<sup>st</sup> Ed.)

} Lab

?

} Last Weeks

?



# Today

4



Data Modeling  
in  
RDBMS

Real World Entity

Conceptual Level | Entity-Relationship Model (E/R) Level

Conceptual Level | Logical Level | Relational Model

Conceptual Level | Logical Level → Physical Level | SQL

Conceptual Level | Logical Level | Computable Entity

Welcome | Relational Algebra | Unary Op | Binary Op

# Relational

Edgar Frank “Ted” Codd, IBM, 1969, 1970

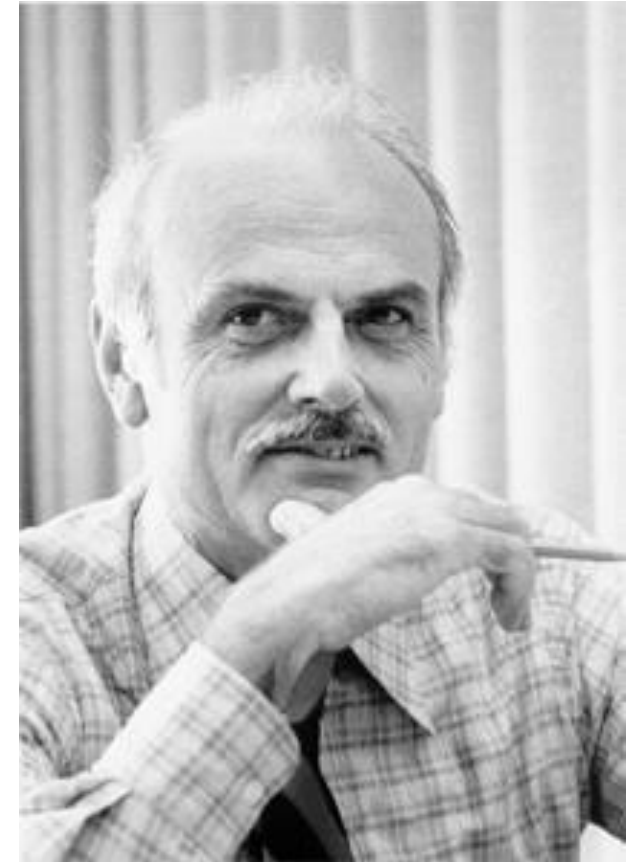
## ***Information Retrieval***

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## **A Relational Model of Data for Large Shared Data Banks**

E. F. Codd

*IBM Research Laboratory, San Jose, California*



# Relational Model × Algebra

Given relational (table) schema filled with actual data instances (rows):  
Operations to **SELECT** Information **FROM** Relations

Query in Natural Language → Query in Math Formula

# Relational Model × Algebra

7

Who made 'Pulp Fiction'?

$\pi$

Director.FirstName  
Director.LastName

$(\sigma$

Movie.Title='PulpFiction'  
Movie.Id=MovieDirector.MovieId  
Director.Id=MovieDirector.DirectorId

(Movie × MovieDirector × Director))

AND  
AND

# Relational Model $\times$ Algebra

Given relational (table) schema filled with actual data instances (rows):  
Operations to **SELECT** Information **FROM** Relations

## Unary Operation

$\pi(R)$  : Project

$\sigma(R)$  : Select

$\rho(R)$  : Rename

## Binary Operation (Set Theory)

$R1 \cup R2$ : Union

$R1 \cap R2$ : Intersection

$R1 \setminus R2$  : Set Difference

$R1 \times R2$ : Cartesian Product



# Algebra × Project ( $\pi$ )

$\pi$ , pi, is used to select a subset of attributes (columns) from a relation

$$A = \pi_{\langle \text{attribute list} \rangle}(R)$$



Vertical  
Filtering

R is a relation

$\langle \text{attribute list} \rangle$  subset of attributes of R

A is a relation including all tuples in R with only attributes in list

# Algebra × Selection ( $\sigma$ )

$\sigma$ , sigma, is used to select a subset of tuples from a relation based on a condition ( $\theta$ ) over relation's attributes.

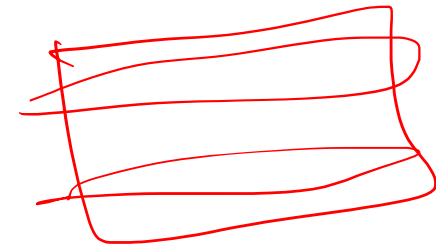
$$A = \sigma_{\theta}(R)$$

Handwritten annotations: A red arrow points from the  $\theta$  in the text above to the  $\theta$  in the formula. Another red arrow points from the  $\theta$  in the formula to a red 'T' below it. A third red arrow points from the  $R$  in the formula to the text 'R is a relation' below.

R is a relation

$\theta$  is a Boolean expression on the attributes of R

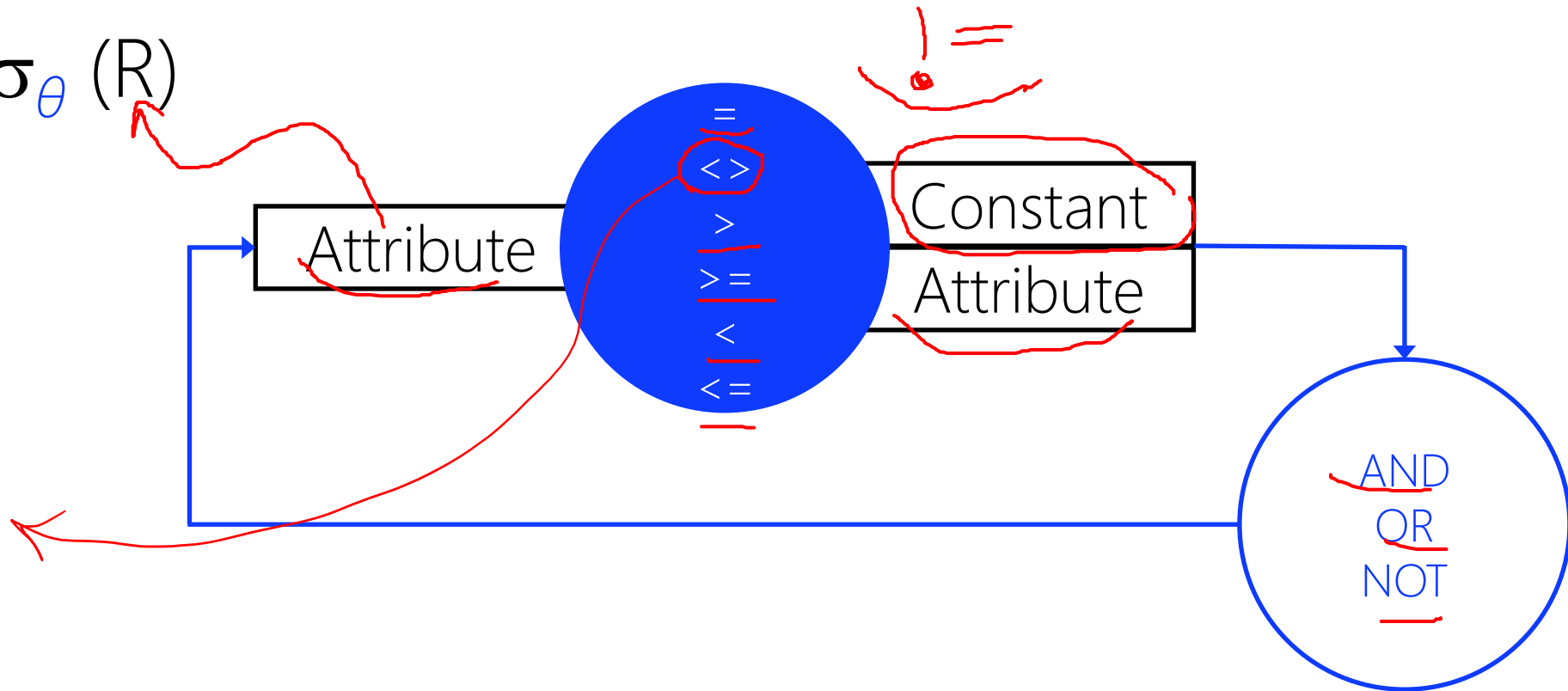
A is a relation including tuples that make  $\theta$  true



# Algebra × Selection ( $\sigma$ )

$\theta$  can be made up of number of Boolean clauses

$$A = \sigma_{\theta} (R)$$



# Algebra × Selection ( $\sigma$ )

Operation Precedence for Logical Operations

() > NOT > AND = OR

Commutative Law

$A \text{ AND } B = B \text{ AND } A$  (we say A commutes with B under AND)

$A \text{ OR } B = B \text{ OR } A$  (we say A commutes with B under OR)

Associative Law

$A \text{ AND } (B \text{ AND } C) = (A \text{ AND } B) \text{ AND } C = A \text{ AND } B \text{ AND } C$

$A \text{ OR } (B \text{ OR } C) = (A \text{ OR } B) \text{ OR } C = A \text{ OR } B \text{ OR } C$

Distributive Law

$A \text{ AND } (B \text{ OR } C) = (A \text{ AND } B) \text{ OR } (A \text{ AND } C)$

$A \text{ OR } (B \text{ AND } C) = (A \text{ OR } B) \text{ AND } (A \text{ OR } C)$

de Morgan's Theorem\*\*

$\text{NOT } (A \text{ AND } B) = \text{NOT}(A) \text{ OR } \text{NOT}(B)$

$\text{NOT } (A \text{ OR } B) = \text{NOT}(A) \text{ AND } \text{NOT}(B)$

$$(A \text{ OR } B)' = A' \text{ AND } B'$$

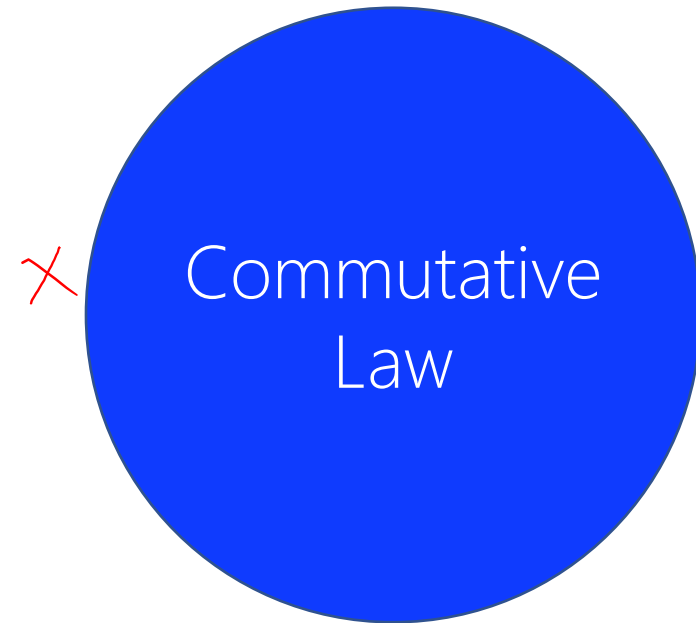
# Algebra × Selection ( $\sigma$ )

$$\sigma_{\theta}(\sigma_{\theta'}(R))$$

$$\sigma_{\theta \text{ AND } \theta'}(R)$$

$$\sigma_{\theta' \text{ AND } \theta}(R)$$

$$\sigma_{\theta'}(\sigma_{\theta}(R))$$



Corollary:  $\sigma_{\theta}(\sigma_{\theta'}(\sigma_{\theta''}(\sigma_{\theta'''}(R))) = \sigma_{\theta \text{ AND } \theta' \text{ AND } \theta'' \dots \text{ AND } \theta'''(R)$

# Algebra × Rename ( $\rho$ )

$\rho$ , rho, is used to rename a relation or its attributes or both

$$A = \rho \langle R'(a'/a, b'/b, \dots) \rangle (R)$$

R is a relation

$R'$  is the new name for R(a, b, ...)

$a'$  is the new name for attribute a of R

$b'$  is the new name for attribute b of R

...

# Relational Model $\times$ Algebra

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Given relational (table) schema filled with actual data instances (rows):

Operations to SELECT Information FROM Relations

Operations to write query

## Unary Operation

$\pi(R)$  : Project

$\sigma(R)$  : Select

$\rho(R)$  : Rename

## Binary Operation (Set Theory)

$R1 \cup R2$ : Union

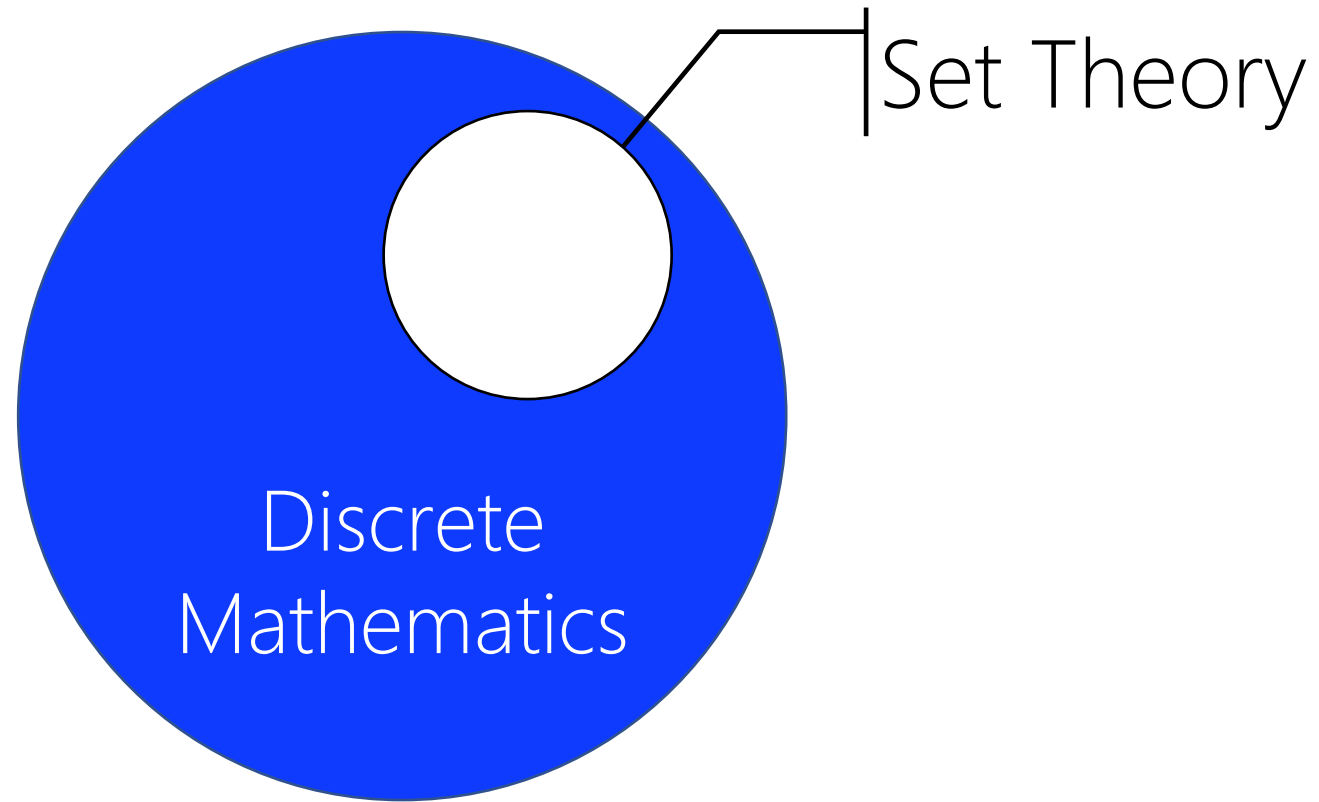
$R1 \cap R2$ : Intersection

$R1 \setminus R2$  : Set Difference

$R1 \times R2$ : Cartesian Product

# Algebra $\times$ Binary Operations

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# Algebra × Union ( $\cup$ )

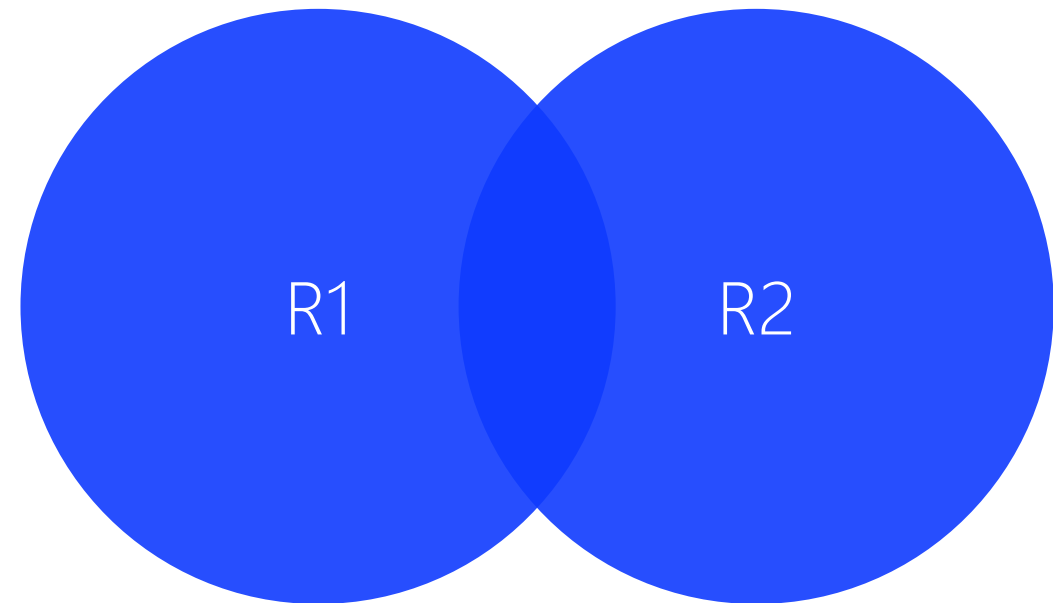
$\cup$ , union, is used to include tuples exist in either relations

$$A = R1 \cup R2$$

R1 and R2 are relations

A has all tuples from R1 and R2

No duplicate!



Venn Diagram

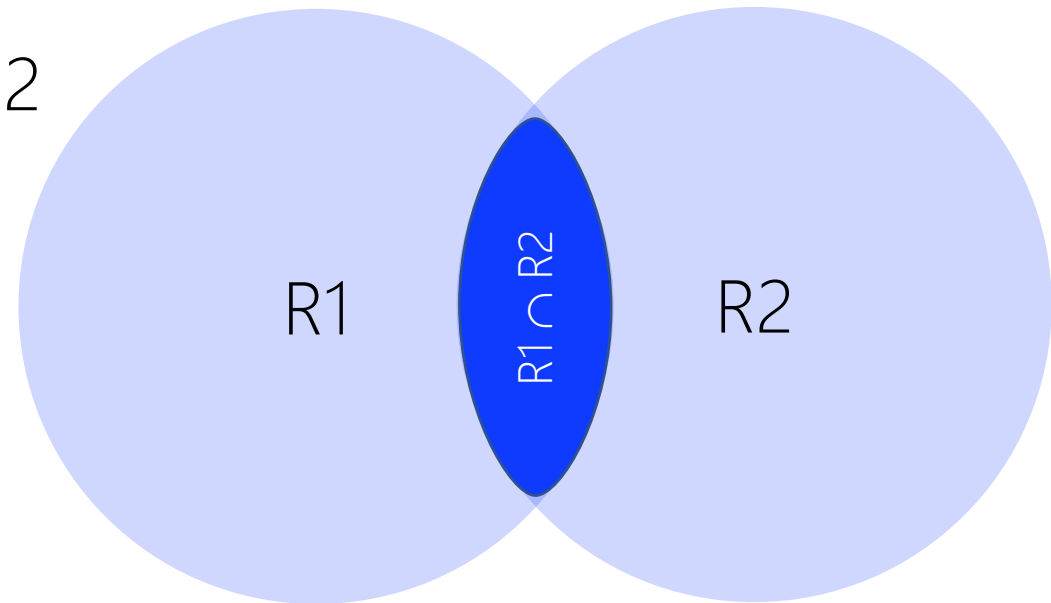
# Algebra × Intersection ( $\cap$ )

$\cap$ , intersection, is used to include tuples exist in both relations

$$A = R1 \cap R2$$

R1 and R2 are relations

A has tuples exist both in R1 and R2



# Algebra × Set Difference ( $\setminus$ )

$\setminus$ , minus, is used to exclude tuples from a relation

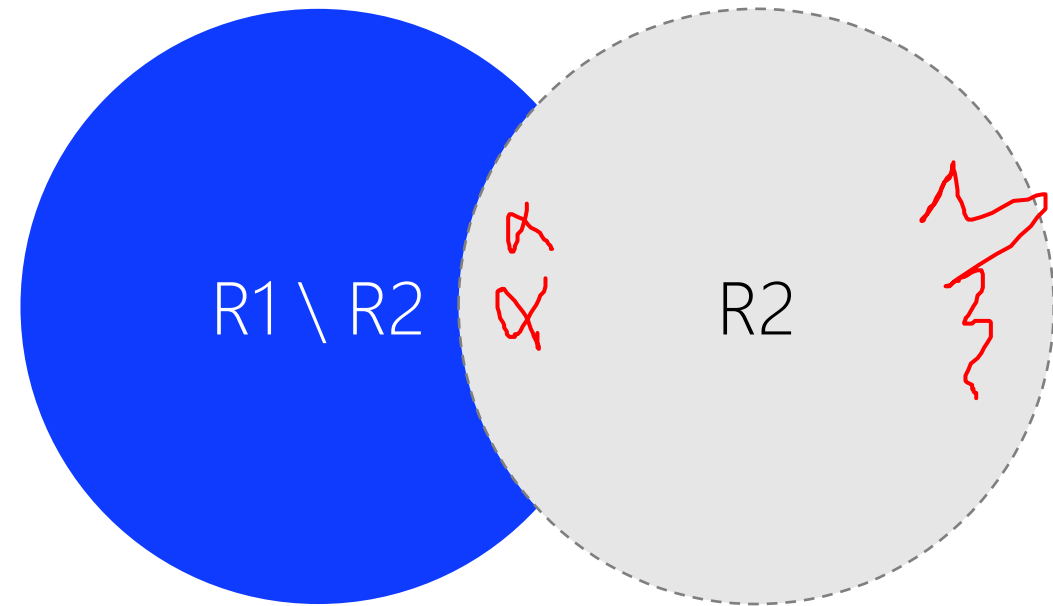
$$A = R1 \setminus R2$$

$$R1(a) = \{2\}$$

$$R2(b)$$

R1 and R2 are relations

A has tuples in R1 but not in R2



✗ Commutative Law

$$R1 \setminus R2 \neq R2 \setminus R1$$

✗ Associative Law

$$R1 \setminus (R2 \setminus R3) \neq (R1 \setminus R2) \setminus R3 \text{ (Why?)}$$

$$2 - (3 - 5) \neq (2 - 3) - 5$$

# Algebra × Set Compatibility

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| Director |           |           |               |              |             |            |
|----------|-----------|-----------|---------------|--------------|-------------|------------|
| Id       | FirstName | LastName  | DateOfBirth   | PlaceOfBirth | BestMovieId | MovieCount |
| 1        | Stanley   | Kubrick   | Jul. 26, 1928 | USA          | 1           | 13         |
| 2        | Alfred    | Hitchcock | Aug. 13, 1899 | England      | 203         | 47         |
| 3        | Clint     | Eastwood  | May 31, 1930  | USA          | 803         | 35         |

| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

Director and Movie do not share same

- Number of attributes
- Datatype in attributes
- Semantics in attributes

$A = \text{Director} \cup \text{Movie}$

$A = \text{Director} \cap \text{Movie}$

$A = \text{Director} \setminus \text{Movie}$

# Algebra × Set Compatibility

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| Director |           |           |               |              |             |            |
|----------|-----------|-----------|---------------|--------------|-------------|------------|
| Id       | FirstName | LastName  | DateOfBirth   | PlaceOfBirth | BestMovieId | MovieCount |
| 1        | Stanley   | Kubrick   | Jul. 26, 1928 | USA          | 1           | 13         |
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| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

Find movies which are NOT the best movies of any directors?

$$A = \text{Movie} \setminus \text{Director}$$

# Algebra × Set Compatibility

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| BestMovieId |
|-------------|
| 1           |
| 203         |
| 803         |

| <u>Id</u> |
|-----------|
| 1         |
| 2         |

Find movies which are NOT the best movies of any directors?

$$A = (\pi_{\text{Id}}(\text{Movie})) \setminus (\pi_{\text{BestMovieId}}(\text{Director}))$$

# Relational Model $\times$ Algebra

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Given relational (table) schema filled with actual data instances (rows):  
Operations to SELECT Information FROM Relations  
Operations to write query

## Unary Operation

$\pi(R)$  : Project

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$\rho(R)$  : Rename

## Binary Operation (Set Theory)

$R1 \cup R2$ : Union

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$R1 \setminus R2$  : Set Difference

  $R1 \times R2$ : Cartesian Product

# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$\underline{A} = \underline{R1} \times \underline{R2}$$



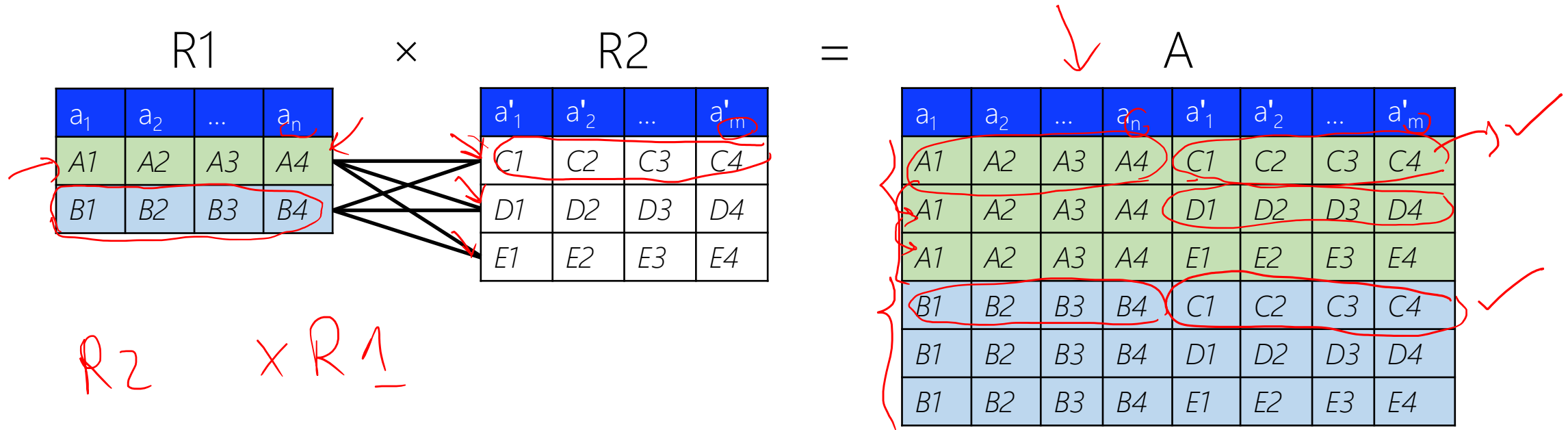
$R1(\underline{a_1, a_2, \dots, a_n})$  and  $R2(a'_1, a'_2, \dots, a'_m)$  are relations

- A is a relation with all attributes in R1 & R2:  $A(a_1, a_3, \dots, a_n, a'_1, a'_2, \dots, a'_m)$
- In A, each tuple of R1 is paired with all tuples of R2



# Algebra $\times$ Cartesian Product ( $\times$ )

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# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$A = R1 \times R2$$

# attributes in A, #attributes in R1 + #attributes in R2

# tuples in A,  $|A| = |R1| \times |R2|$

~~✓~~ Commutative Law

$$R1 \times R2 = R2 \times R1$$

~~✓~~ Associative Law

$$R1 \times (R2 \times R3) = (R1 \times R2) \times R3 = R1 \times R2 \times R3$$

# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$A = R1 \times R2$$

R1 and R2 do not have to be set compatible

Any two relations can be paired!

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Any two relations can be paired!

# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$A = R1 \times R2$$

R1 and R2 do not have to be set compatible

Any two relations can be paired.

But if you want to pair two or more relations, which ones?

# Algebra $\times$ Cartesian Product ( $\times$ )

| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

| User |          |          |
|------|----------|----------|
| Id   | Username | Password |
| 1    | fani     | ***      |
| 2    | cjason   | ***      |
| 3    | h_f492   | ***      |

=

SP = h fani  
 " " cjason  
 " " h\_f492

What meaningful queries can be answered by pairing these two relations?

# Algebra $\times$ Cartesian Product ( $\times$ )



| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

| MovieGenre |         |
|------------|---------|
| MovieId    | GenreId |
| 1          | 1       |
| 1          | 3       |
| 2          | 6       |
| 2          | 7       |
| 2          | 2       |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

What meaningful queries can be answered by pairing these three relations?

# Algebra $\times$ Cartesian Product ( $\times$ )

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| Movie $\times$ MovieGenre |                       |          |             |         |         |
|---------------------------|-----------------------|----------|-------------|---------|---------|
| Id                        | Title                 | Language | RunningTime | MovieId | GenreId |
| 1                         | 2001: A Space Odyssey | English  | 142         | 1       | 1       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 1       | 3       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 6       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 7       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 2       |
| 2                         | Rosemary's Baby       | English  | 136         | 1       | 1       |
| 2                         | Rosemary's Baby       | English  | 136         | 1       | 3       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 6       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 7       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 2       |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

# Algebra $\times$ Cartesian Product ( $\times$ )

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| Movie $\times$ MovieGenre |                       |          |             |         |         |
|---------------------------|-----------------------|----------|-------------|---------|---------|
| Id                        | Title                 | Language | RunningTime | MovieId | GenreId |
| 1                         | 2001: A Space Odyssey | English  | 142         | 1       | 1       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 1       | 3       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 6       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 7       |
| 1                         | 2001: A Space Odyssey | English  | 142         | 2       | 2       |
| 2                         | Rosemary's Baby       | English  | 136         | 1       | 1       |
| 2                         | Rosemary's Baby       | English  | 136         | 1       | 3       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 6       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 7       |
| 2                         | Rosemary's Baby       | English  | 136         | 2       | 2       |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |



# Algebra $\times$ Cartesian Product ( $\times$ )

| Id | Title                 | Language | RunningTime | Movielid | Genrelid |
|----|-----------------------|----------|-------------|----------|----------|
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1        |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3        |
| 2  | Rosemary's Baby       | English  | 136         | 2        | 6        |
| 2  | Rosemary's Baby       | English  | 136         | 2        | 7        |
| 2  | Rosemary's Baby       | English  | 136         | 2        | 2        |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

$\sigma$   
 $\text{Id} = \text{Movielid}$  AND  
 $\text{Title} = '2001'$   
 (Movie  $\times$  MovieGenre)

# Algebra $\times$ Cartesian Product ( $\times$ )

| <u>Id</u> | Title                 | Language | RunningTime | <u>Movielid</u> | <u>Genrelid</u> |
|-----------|-----------------------|----------|-------------|-----------------|-----------------|
| 1         | 2001: A Space Odyssey | English  | 142         | 1               | 1               |
| 1         | 2001: A Space Odyssey | English  | 142         | 1               | 3               |

| Genre     |           |
|-----------|-----------|
| <u>Id</u> | Title     |
| 1         | Sci-fi    |
| 2         | Action    |
| 3         | Adventure |
| 4         | Comedy    |
| 5         | Crime     |
| 6         | Drama     |
| 7         | Horror    |

$\sigma$  Id=Movielid AND (Movie  $\times$  MovieGenre)  
~~Title='2001: A Space Odyssey'~~  
 Id = 1

# Algebra $\times$ Cartesian Product ( $\times$ )

| Id | Title                 | Language | RunningTime | MovieId | GenreId | Id | Title     |
|----|-----------------------|----------|-------------|---------|---------|----|-----------|
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 2  | Action    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 3  | Adventure |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 4  | Comedy    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 5  | Crime     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 6  | Drama     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 7  | Horror    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 2  | Action    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 3  | Adventure |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 4  | Comedy    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 5  | Crime     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 6  | Drama     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 7  | Horror    |

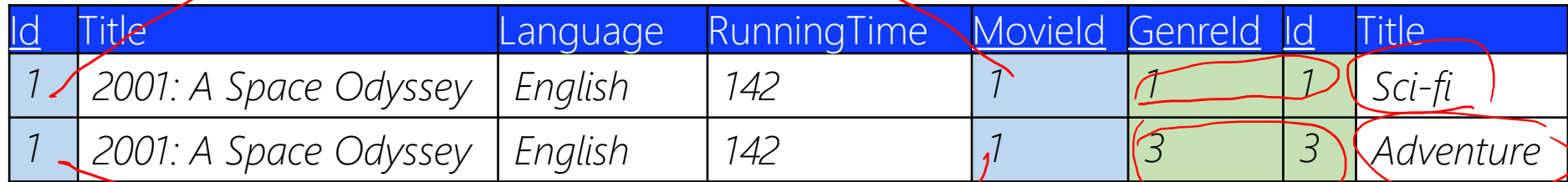
$(\sigma_{\text{Id}=\text{MovieId} \text{ AND } \text{Title}='2001: A Space Odyssey'}$ 
 $(\text{Movie} \times \text{MovieGenre}) \times \text{Genre})$

# Algebra $\times$ Cartesian Product ( $\times$ )

| Id | Title                 | Language | RunningTime | Movielid | GenreId | Id | Title     |
|----|-----------------------|----------|-------------|----------|---------|----|-----------|
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 2  | Action    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 3  | Adventure |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 4  | Comedy    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 5  | Crime     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 6  | Drama     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1       | 7  | Horror    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 2  | Action    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 3  | Adventure |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 4  | Comedy    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 5  | Crime     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 6  | Drama     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3       | 7  | Horror    |

$(\sigma_{\text{Id=Movielid AND Title='2001: A Space Odyssey'}}$ 
 $(\text{Movie} \times \text{MovieGenre}) \times \text{Genre})$

# Algebra $\times$ Cartesian Product ( $\times$ )



| Id | Title                 | Language | RunningTime | Movielid | Genrelid | Id | Title     |
|----|-----------------------|----------|-------------|----------|----------|----|-----------|
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 1        | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1        | 3        | 3  | Adventure |

$$A = \sigma_{\text{Id}=\text{Genrelid}}(\sigma_{\text{Id}=\text{Movielid} \text{ AND Title='2001: A Space Odyssey'}}(\text{Movie} \times \text{MovieGenre})) \times \text{Genre}$$

# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$A = R1 \times R2$$

R1 and R2 do not have to be set compatible

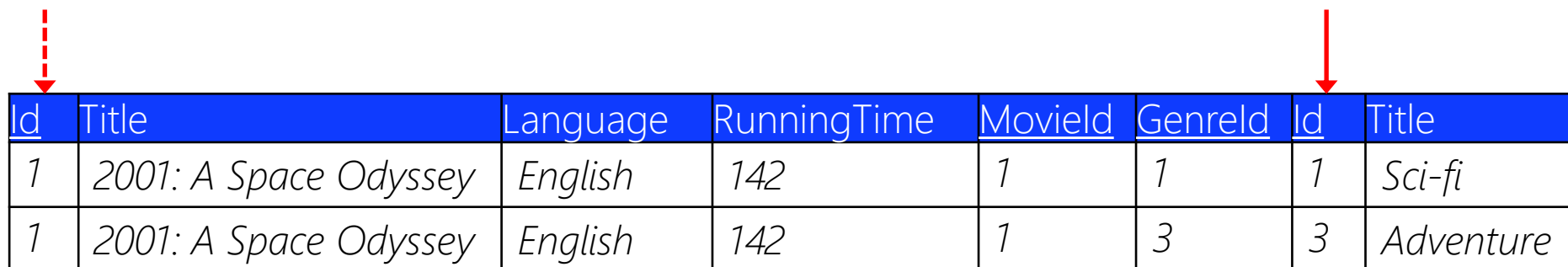
Any two relations can be paired.

But if you want to pair two or more relations, which ones?

Those have relationship in E/R diagram

Those have linked by foreign keys (FK) in relational model

# Algebra $\times$ Cartesian Product ( $\times$ )



| <u>Id</u> | Title                 | Language | RunningTime | <u>MovieId</u> | <u>GenreId</u> | <u>Id</u> | Title     |
|-----------|-----------------------|----------|-------------|----------------|----------------|-----------|-----------|
| 1         | 2001: A Space Odyssey | English  | 142         | 1              | 1              | 1         | Sci-fi    |
| 1         | 2001: A Space Odyssey | English  | 142         | 1              | 3              | 3         | Adventure |

$$A = \sigma_{Id=GenreId}(\sigma_{Id=MovieId \text{ AND } Title='2001: A Space Odyssey'}(Movie \times MovieGenre)) \times Genre$$

Ambiguous name reference! Solution?

# Algebra $\times$ Cartesian Product ( $\times$ )

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| <u>Id</u> | Title                 | Language | RunningTime | <u>MovieId</u> | <u>GenreId</u> | <u>Id</u> | Title     |
|-----------|-----------------------|----------|-------------|----------------|----------------|-----------|-----------|
| 1         | 2001: A Space Odyssey | English  | 142         | 1              | 1              | 1         | Sci-fi    |
| 1         | 2001: A Space Odyssey | English  | 142         | 1              | 3              | 3         | Adventure |

$$A = \sigma_{\text{Genre.Id}=\text{GenreId}}(\sigma_{\text{Movie.Id} \neq \text{MovieId} \text{ AND Title='2001: A Space Odyssey'}}(\text{Movie} \times \text{MovieGenre})) \times \text{Genre}$$

Ambiguous name reference! Solution?

•, dot, namespace operator



# Algebra $\times$ Cartesian Product ( $\times$ )

| <u>MId</u> | Title                 | Language | RunningTime | <u>MovieId</u> | <u>GenreId</u> | <u>Id</u> | Title     |
|------------|-----------------------|----------|-------------|----------------|----------------|-----------|-----------|
| 1          | 2001: A Space Odyssey | English  | 142         | 1              | 1              | 1         | Sci-fi    |
| 1          | 2001: A Space Odyssey | English  | 142         | 1              | 3              | 3         | Adventure |

$$A = \sigma_{\text{Id=GenreId}}(\sigma_{\substack{\text{MId=MovieId} \\ \text{Title='2001: A Space Odyssey'}}}(\rho_{R(\text{MId/Id})}(\text{Movie})) \times \text{MovieGenre}) \times \text{Genre})$$

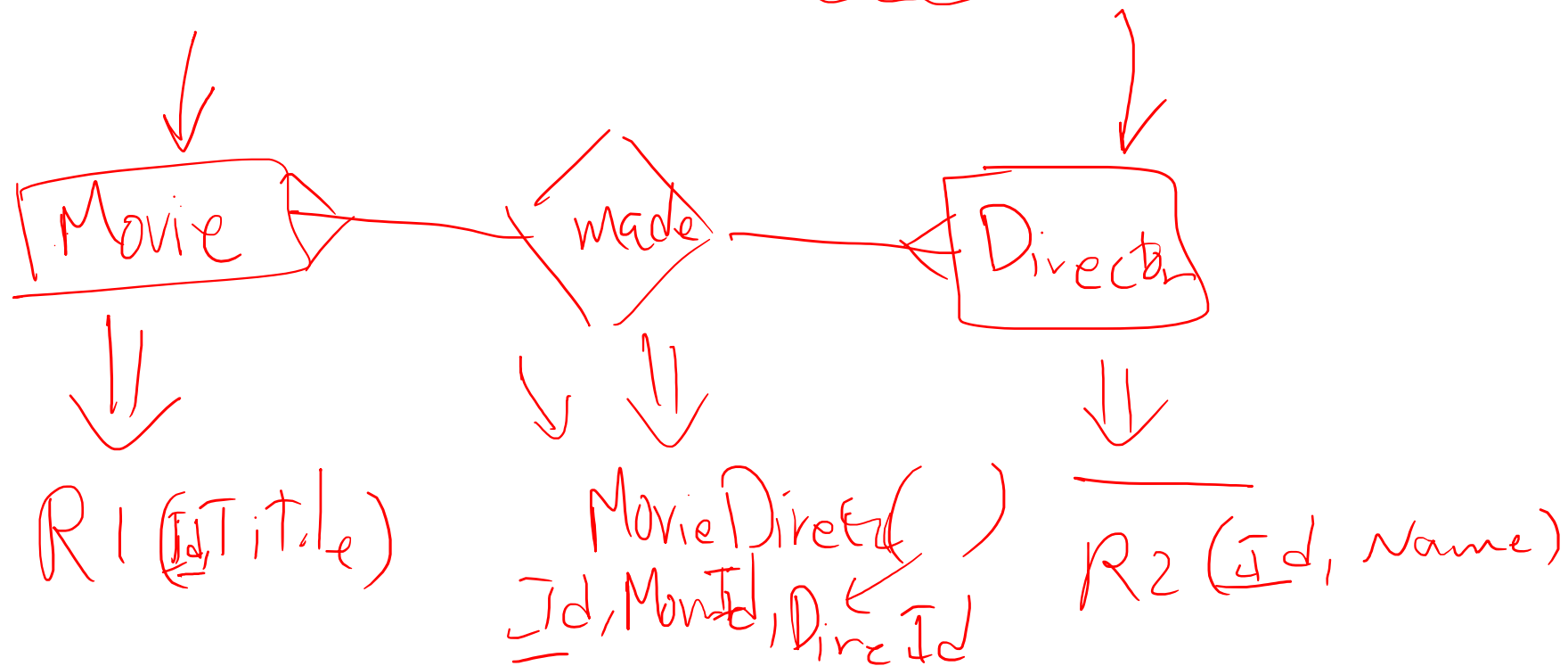

Ambiguous name reference! Solution?

$\rho$ , rho, rename operator

# Relational Model × Algebra


42

Who made 'Pulp Fiction'?



# Relational Model × Algebra

Who made 'Pulp Fiction'?

- i) Find the movie in Movie relation
  - ii) Find directors in Director relation
  - iii) Find who made a movie in MovieDirector relation
- 

# Relational Model $\times$ Algebra

44

Who made 'Pulp Fiction'?

$\sigma_{\text{DirectorId} = \text{MovieDirector.DirectorId}} (\sigma_{\text{Movie.Title} = \text{'Pulp Fiction'} \text{ AND } \text{Movie.Id} = \text{MovieDirector.MovieId}} (\text{Movie} \times \text{MovieDirector})) \times \text{Director}$

*Handwritten notes:*

- A red arrow points to the  $\sigma$  operator, with the word "Name" written next to it.
- The  $\sigma$  operator is circled in red.
- The condition  $\text{Movie.Title} = \text{'Pulp Fiction'}$  is underlined in red.
- The condition  $\text{Movie.Id} = \text{MovieDirector.MovieId}$  is underlined in red.
- The  $\times$  operator between  $(\text{Movie} \times \text{MovieDirector})$  and  $\text{Director}$  is underlined in red.
- A large red bracket spans the entire expression.
- A red arrow points to the  $\text{Movie.Id}$  attribute in the join condition.

# Relational Model $\times$ Algebra

45

Who made 'Pulp Fiction'?

$\sigma$  Director.Id=MovieDirector.DirectorId  $(\sigma$  Movie.Title='PulpFiction' AND Movie.Id=MovieDirector.MovieId) (Movie  $\times$  MovieDirector))  $\times$  Director



Selection ( $\sigma$ ) is commutative

# Relational Model $\times$ Algebra

Who made 'Pulp Fiction'?

$$\begin{aligned}
 & \sigma_{\text{Director.Id} = \text{MovieDirector.DirectorId}} \left( \sigma_{\text{Movie.Title} = \text{'Pulp Fiction'} \text{ AND } \text{Movie.Id} = \text{MovieDirector.MovieId}} (\text{Movie} \times \text{MovieDirector}) \right) \times \text{Director} \\
 & = \\
 & \sigma_{\text{Movie.Title} = \text{'Pulp Fiction'} \text{ AND } \text{Movie.Id} = \text{MovieDirector.MovieId} \text{ AND } \text{Director.Id} = \text{MovieDirector.DirectorId}} (\text{Movie} \times \text{MovieDirector} \times \text{Director})
 \end{aligned}$$

# Relational Model × Algebra

47

Who acted in 'Pulp Fiction'?



# Relational Model × Algebra

48

Who acted in 'Pulp Fiction'?

$\pi$

Actor.FirstName  
Actor.LastName

$(\sigma$

Movie.Title = 'Pulp Fiction' AND  
MovieId = StarIn.MovieId AND  
ActorId = StarIn.ActorId

(Movie × StarIn × Actor))



# Algebra $\times$ Cartesian Product ( $\times$ )

$\times$ , product, is used to pair two relations

$$A = R1 \times R2$$

P.S.

- I) Product is the most important operator in relational model
- II) Product is the most common operator in relational model
- III) Product is the most expensive operator in relational model

# Algebra × Complete Set of Operators

50

|                   |                   |
|-------------------|-------------------|
| $\pi(R)$          | Project           |
| $\sigma(R)$       | Select            |
| $\rho(R)$         | Rename            |
| $R1 \cup R2$      | Union             |
| $R1 \setminus R2$ | Set Difference    |
| $R1 \times R2$    | Cartesian Product |

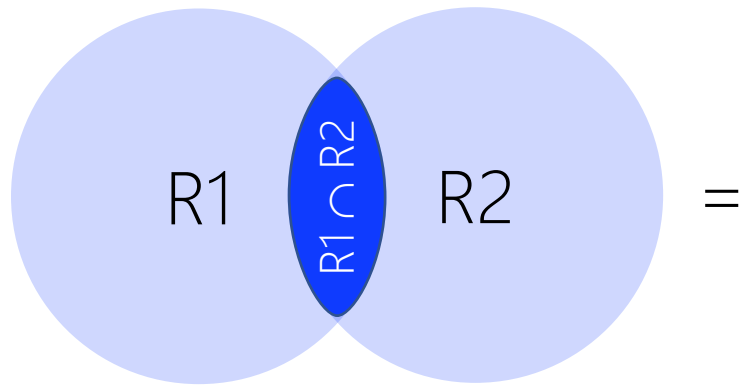
Any other relational algebra expression can be expressed by a combination of these operations.

$R1 \cap R2$

No Need for Intersection!

# Algebra $\times$ Complete Set of Operators

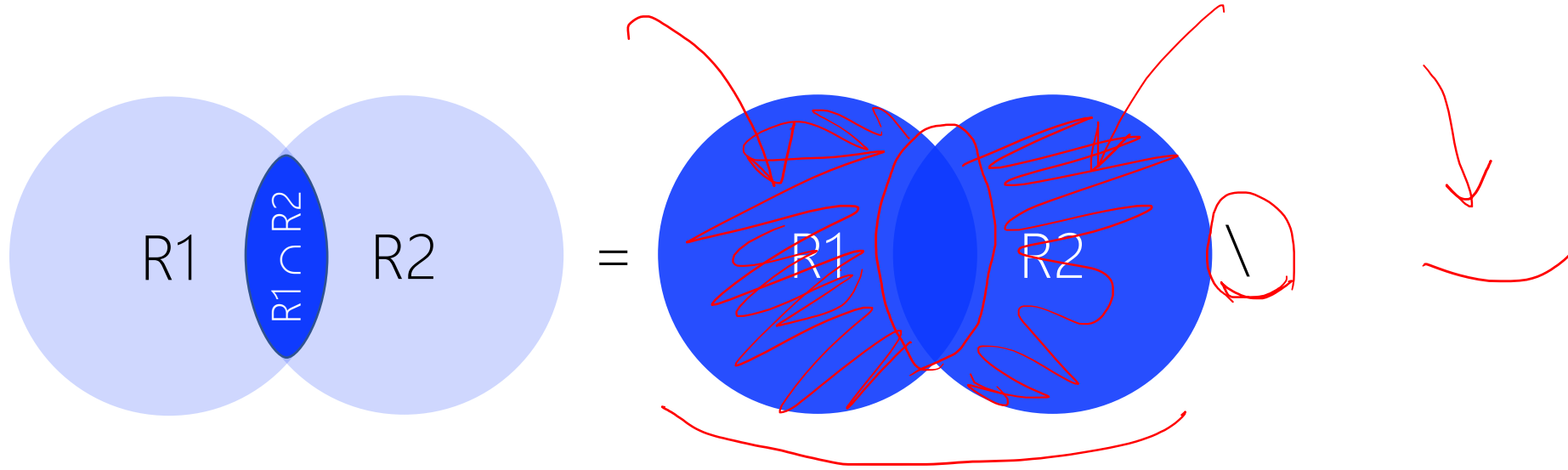
51



$R1 \cap R2 =$  What mixture of other operators?

# Algebra × Complete Set of Operators

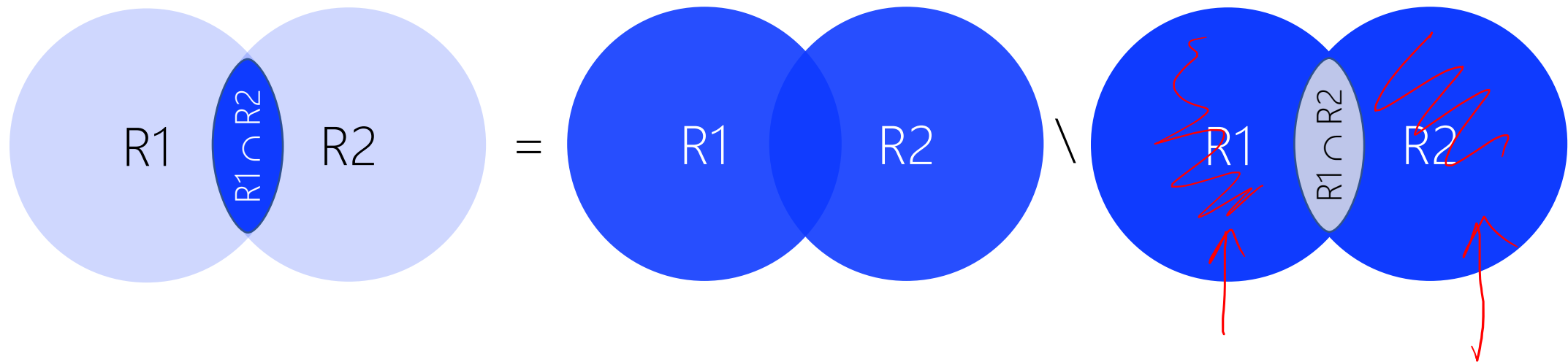
52



$$R1 \cap R2 = (\underline{R1 \cup R2}) \setminus ?$$

# Algebra × Complete Set of Operators

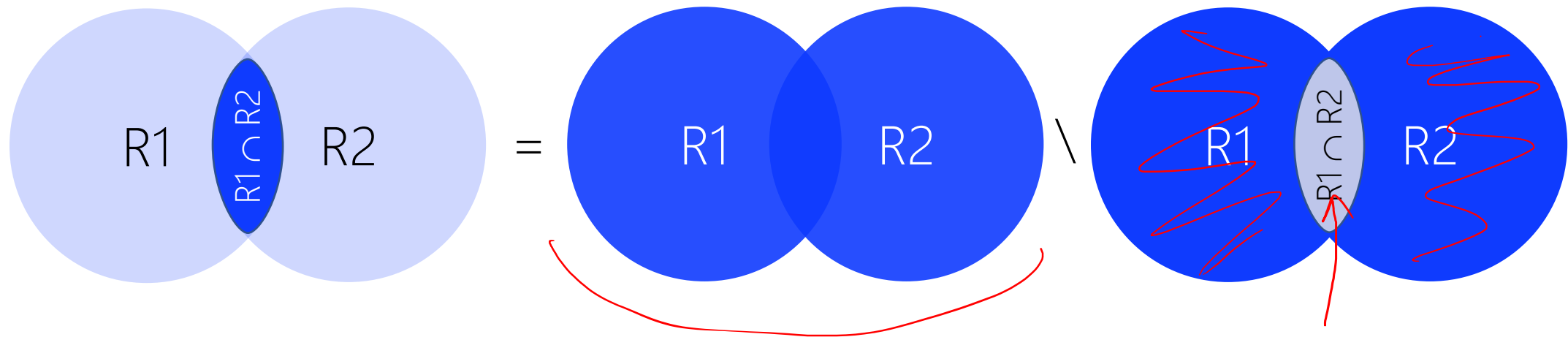
53



$$R1 \cap R2 = (R1 \cup R2) \setminus ?$$

# Algebra × Complete Set of Operators

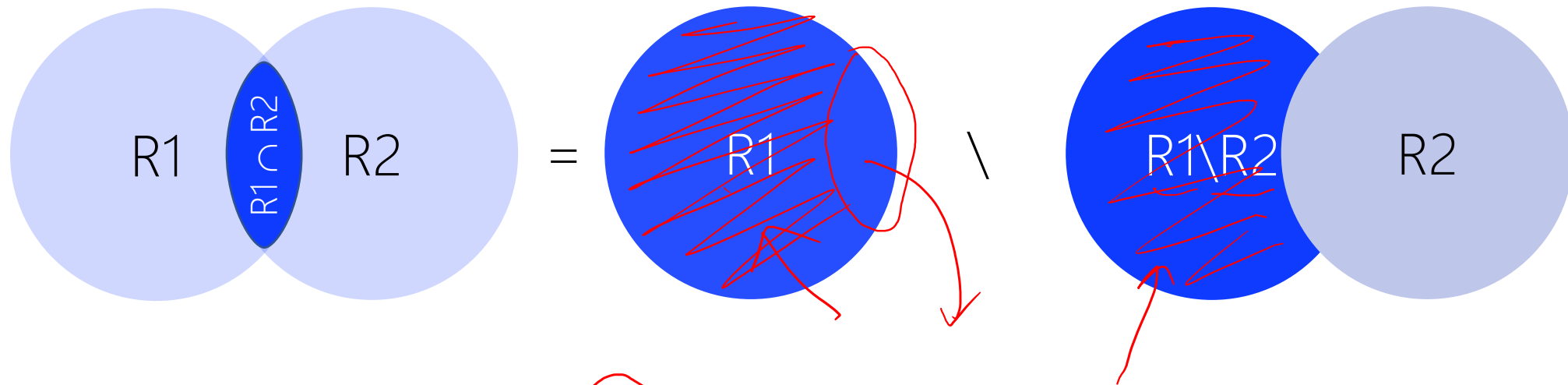
54



$$R1 \cap R2 = (R1 \cup R2) \setminus ((R1 \setminus R2) \cup (R2 \setminus R1))$$

# Algebra × Complete Set of Operators

55



$$R1 \cap R2 = R1 \setminus (R1 \setminus R2)$$





# Algebra $\times$ $\theta$ -Join

$\bowtie_{\theta}$   $\theta$ -join, is product ( $\times$ ) of relations followed by selection ( $\sigma$ )

$$R1 \bowtie_{\theta} R2 = \sigma_{\theta}(R1 \times R2)$$

# Algebra $\times$ $\theta$ -Join

Movies and their genres?

$\sigma_{\text{Genre.Id}=\text{GenreId}}(\sigma_{\text{Movie.Id}=\text{MovieId}}(\text{Movie} \times \text{MovieGenre})) \times \text{Genre}$

# Algebra $\times$ $\theta$ -Join

Movies and their directors?

$\sigma_{\text{Genre.Id}=\text{GenreId}}(\sigma_{\text{Movie.Id}=\text{MovieId}}(\text{Movie} \times \text{MovieGenre})) \times \text{Genre}$

Movie  $\bowtie_{\text{Movie.Id}=\text{MovieId}}$  MovieGenre

# Algebra $\times$ $\theta$ -Join

Movies and their directors?

$$\sigma_{\text{Genre.Id}=\text{GenreId}}(\sigma_{\text{Movie.Id}=\text{MovieId}}(\text{Movie} \times \text{MovieGenre})) \times \text{Genre}$$

$$(\text{Movie} \bowtie_{\text{Movie.Id}=\text{MovieId}} \text{MovieGenre}) \bowtie_{\text{Genre.Id}=\text{GenreId}} \text{Genre}$$

# Algebra $\times$ $\theta$ -Join

$\bowtie$ , natural join, is product ( $\times$ ) of relations followed by selection ( $\sigma$ )

$$\underbrace{R_1 \bowtie R_2}_{\theta} = \underbrace{\sigma}_{\theta}(\underbrace{R_1 \times R_2})$$

✓ Commutative?

✓ Associative?


# Algebra $\times$ $\theta$ -Join

$\bowtie$ , natural join, is product ( $\times$ ) of relations followed by selection ( $\sigma$ )

$$R1 \bowtie_{\theta} R2 = \sigma_{\theta} (R1 \times R2)$$

Commutative:  $R1 \bowtie_{\theta} R2 = R2 \bowtie_{\theta} R1$

Associative:  $(R1 \bowtie_{\theta} R2) \bowtie_{\theta'} R3 \neq R1 \bowtie_{\theta} (R2 \bowtie_{\theta'} R3)$  (Why?)



# Algebra $\times$ $\theta$ -Join

Movies and their genres?

$$\left\{ \begin{array}{l}
 \sigma_{\text{Movie.Id}=\text{Movielid} \text{ AND } \text{Genre.Id}=\text{Genrelid}} (\text{Movie} \times \text{MovieGenre} \times \text{Genre}) \\
 \sigma_{\text{Movie.Id}=\text{Movielid} \text{ AND } \text{Genre.Id}=\text{Genrelid}} (\text{Movie} \times \text{Genre} \times \text{MovieGenre}) \\
 \sigma_{\text{Movie.Id}=\text{Movielid} \text{ AND } \text{Genre.Id}=\text{Genrelid}} (\text{Genre} \times \text{Movie} \times \text{MovieGenre}) \\
 \sigma_{\text{Genre.Id}=\text{Genrelid} \text{ AND } \text{Movie.Id}=\text{Movielid}} (\text{Movie} \times \text{MovieGenre} \times \text{Genre})
 \end{array} \right.$$

Commutative and Associative law for AND and Product ( $\times$ )

# Algebra $\times$ $\theta$ -Join

Movies and their genres?

$\sigma_{\text{Movie.Id}=\text{Movielid} \text{ AND } \text{Genre.Id}=\text{Genrelid}} (\text{Movie} \times \text{MovieGenre} \times \text{Genre})$

$(\text{Movie} \bowtie_{\text{Movie.Id}=\text{Movielid}} \text{MovieGenre}) \bowtie_{\text{Genre.Id}=\text{Genrelid}} \text{Genre}$  ✓  
 $(\text{MovieGenre} \bowtie_{\text{Movie.Id}=\text{Movielid}} \text{Movie}) \bowtie_{\text{Genre.Id}=\text{Genrelid}} \text{Genre}$  ✓

✗  $\text{MovieGenre} \bowtie_{\text{Movie.Id}=\text{Movielid}} (\text{Movie} \bowtie_{\text{Genre.Id}=\text{Genrelid}} \text{Genre})$   
 ✗  $(\text{Movie} \bowtie_{\text{Genre.Id}=\text{Genrelid}} \text{MovieGenre}) \bowtie_{\text{Movie.Id}=\text{Movielid}} \text{Genre}$



# Algebra $\times$ Natural Join $\times$

$\bowtie$ , *natural join*, is  $\theta$ -Join equating all shared attributes (same name)

$$\underbrace{R1} \bowtie \underbrace{R2} = R1 \bowtie_{\theta} R2 = \sigma_{\theta} (R1 \times R2)$$

where  $\theta$ :  $R1.\underline{a} = R2.\underline{a}$  **AND**  $R1.\underline{b} = R2.\underline{b}$  **AND** ... **AND**  $R1.\underline{z} = R2.\underline{z}$

$R1 \times R2$

# Algebra × Natural Join

| Director |           |           |               |              |             |            |
|----------|-----------|-----------|---------------|--------------|-------------|------------|
| Id       | FirstName | LastName  | DateOfBirth   | PlaceOfBirth | BestMovieId | MovieCount |
| 1        | Stanley   | Kubrick   | Jul. 26, 1928 | USA          | 1           | 13         |
| 2        | Alfred    | Hitchcock | Aug. 13, 1899 | England      | 203         | 47         |
| 3        | Clint     | Eastwood  | May 31, 1930  | USA          | 803         | 35         |

| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

What are directors' best movie name?

# Algebra × Natural Join

| A1        |           |                  |
|-----------|-----------|------------------|
| FirstName | LastName  | BestMovieId → Id |
| Stanley   | Kubrick   | 1                |
| Alfred    | Hitchcock | 203              |
| Clint     | Eastwood  | 803              |

| Movie |                       |          |             |
|-------|-----------------------|----------|-------------|
| Id    | Title                 | Language | RunningTime |
| 1     | 2001: A Space Odyssey | English  | 142         |
| 2     | Rosemary's Baby       | English  | 136         |

What are directors' best movie name?

$$A1 = \rho_{\text{Movie}(\text{Id}/\text{BestMovieId})} (\pi_{\text{FirstName, LastName, BestMovieId}} (\text{Movie}))$$



# Algebra × Left Outer Join ( $\bowtie$ )

| A         |           |                  |      |                       |          |             |
|-----------|-----------|------------------|------|-----------------------|----------|-------------|
| FirstName | LastName  | BestMovieId → Id | Id   | Title                 | Language | RunningTime |
| Stanley   | Kubrick   | 1                | 1    | 2001: A Space Odyssey | English  | 142         |
| Alfred    | Hitchcock | <del>202</del>   | NULL | NULL                  | NULL     | NULL        |
| Clint     | Eastwood  | <del>802</del>   | NULL | NULL                  | NULL     | NULL        |
|           |           |                  | 2    | Rosemary's Baby       | English  | 136         |

What are directors' best movie name if any?

$$\begin{aligned}
 A &= A1 \bowtie \text{Movie} \\
 &= (A1 \bowtie_{A1.Id=Movie.Id} \text{Movie}) \cup (?) \\
 &= (\sigma_{A1.Id=Movie.Id} (A1 \times \text{Movie})) \cup (?)
 \end{aligned}$$

# Algebra $\times$ Right Outer Join ( $\bowtie$ )

70

| A         |           |                              |    |                       |          |             |
|-----------|-----------|------------------------------|----|-----------------------|----------|-------------|
| FirstName | LastName  | BestMovieId $\rightarrow$ Id | Id | Title                 | Language | RunningTime |
| Stanley   | Kubrick   | 1                            | 1  | 2001: A Space Odyssey | English  | 142         |
| Alfred    | Hitchcock | 203                          |    |                       |          |             |
| Clint     | Eastwood  | 803                          |    |                       |          |             |
| NULL      | NULL      | NULL                         | 2  | Rosemary's Baby       | English  | 136         |

List all movies and identify whether each one is the best of its director?

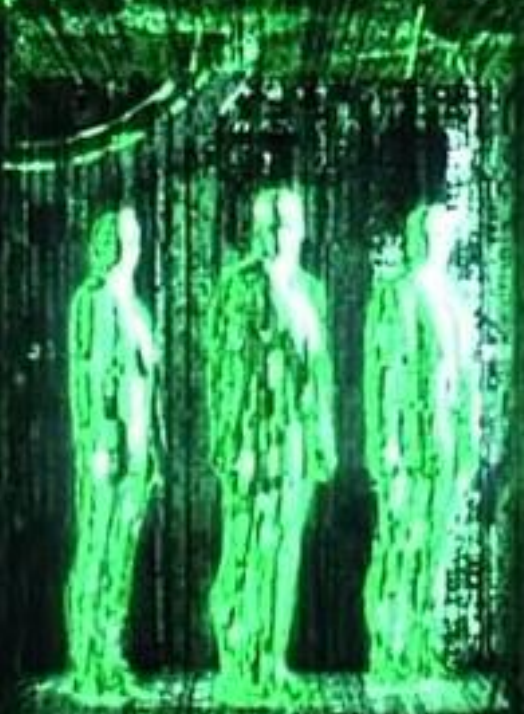
$$\begin{aligned} A &= A1 \bowtie \text{Movie} \\ &= (A1 \bowtie_{A1.Id=Movie.Id} \text{Movie}) \cup (?) \\ &= (\sigma_{A1.Id=Movie.Id} (A1 \times \text{Movie})) \cup (?) \end{aligned}$$

# Algebra $\times$ Full Outer Join ( $\bowtie$ )

| A         |           |                  |      |                       |          |             |
|-----------|-----------|------------------|------|-----------------------|----------|-------------|
| FirstName | LastName  | BestMovieId → Id | Id   | Title                 | Language | RunningTime |
| Stanley   | Kubrick   | 1                | 1    | 2001: A Space Odyssey | English  | 142         |
| Alfred    | Hitchcock | 203              | NULL | NULL                  | NULL     | NULL        |
| Clint     | Eastwood  | 803              | NULL | NULL                  | NULL     | NULL        |
| NULL      | NULL      | NULL             | 2    | Rosemary's Baby       | English  | 136         |

$$\begin{aligned}
 A &= A1 \bowtie \text{Movie} \\
 &= A1 \bowtie \text{Movie} \cup (A1 \bowtie \text{Movie})
 \end{aligned}$$





|   |   |   |   |   |   |   |   |   |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |     |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|-----|



# Algebra × More

Relational algebra has more complex operators such as:

$R1 \triangleright R2$      Antijoin

$R1 / R2$      Division

→ [https://en.wikipedia.org/wiki/Relational\\_algebra](https://en.wikipedia.org/wiki/Relational_algebra)

# Algebra × More × Division (/)

/, division, is used to find tuples (rows) in R1 which matched with ALL tuples (rows) in R2 :

$$A = R1 / R2$$

R1 and R2 are relations

A is a relation with all attributes in R1



# Algebra $\times$ More $\times$ Division ( $/$ )

More  $\times$  Gen

What movie(s) belong to ALL genres?

| A1 |                       |          |             |         |         |    |           |
|----|-----------------------|----------|-------------|---------|---------|----|-----------|
| Id | Title                 | Language | RunningTime | MovieId | GenreId | Id | Title     |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 1       | 1  | Sci-fi    |
| 1  | 2001: A Space Odyssey | English  | 142         | 1       | 3       | 3  | Adventure |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 6       | 6  | Drama     |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 7       | 7  | Horror    |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 2       | 2  | Action    |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 1       | 1  | Sci-fi    |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 3       | 3  | Adventure |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 4       | 4  | Comedy    |
| 2  | Rosemary's Baby       | English  | 136         | 2       | 5       | 5  | Crime     |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

$A1 = \sigma_{\text{Movie.Id=MovieId AND Genre.Id=GenreId}} (\text{Movie} \times \text{MovieGenre} \times \text{Genre})$

# Algebra $\times$ More $\times$ Division ( / )

What movie(s) belong to ALL genres?

| A2                    |    |           |
|-----------------------|----|-----------|
| MovieTitle            | Id | Title     |
| 2001: A Space Odyssey | 1  | Sci-fi    |
| 2001: A Space Odyssey | 3  | Adventure |
| Rosemary's Baby       | 6  | Drama     |
| Rosemary's Baby       | 7  | Horror    |
| Rosemary's Baby       | 2  | Action    |
| Rosemary's Baby       | 1  | Sci-fi    |
| Rosemary's Baby       | 3  | Adventure |
| Rosemary's Baby       | 4  | Comedy    |
| Rosemary's Baby       | 5  | Crime     |

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

$$A2 = \rho_{A1(\text{MovieTitle}/\text{Movie.Title})}(\pi_{\text{Movie.Title, Genre.Id, Genre.Title}}(A1))$$

# Algebra × More × Division ( / )

What movie(s) belong to ALL genres?

| A2                    |    |           |
|-----------------------|----|-----------|
| MovieTitle            | Id | Title     |
| 2001: A Space Odyssey | 1  | Sci-fi    |
| 2001: A Space Odyssey | 3  | Adventure |
| Rosemary's Baby       | 6  | Drama     |
| Rosemary's Baby       | 7  | Horror    |
| Rosemary's Baby       | 2  | Action    |
| Rosemary's Baby       | 1  | Sci-fi    |
| Rosemary's Baby       | 3  | Adventure |
| Rosemary's Baby       | 4  | Comedy    |
| Rosemary's Baby       | 5  | Crime     |

$$A = A2 / \text{Genre}$$

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

# Algebra × More × Division ( / )

What movie(s) belong to ALL genres?

| A2                    |    |           |
|-----------------------|----|-----------|
| MovieTitle            | Id | Title     |
| 2001: A Space Odyssey | 1  | Sci-fi    |
| 2001: A Space Odyssey | 3  | Adventure |
| Rosemary's Baby       | 6  | Drama     |
| Rosemary's Baby       | 7  | Horror    |
| Rosemary's Baby       | 2  | Action    |
| Rosemary's Baby       | 1  | Sci-fi    |
| Rosemary's Baby       | 3  | Adventure |
| Rosemary's Baby       | 4  | Comedy    |
| Rosemary's Baby       | 5  | Crime     |

$$A = A2 / \text{Genre}$$

$$= A2 \{ \pi, \sigma, \rho, \cup, \setminus, \times \} \text{Genre}$$

| Genre |           |
|-------|-----------|
| Id    | Title     |
| 1     | Sci-fi    |
| 2     | Action    |
| 3     | Adventure |
| 4     | Comedy    |
| 5     | Crime     |
| 6     | Drama     |
| 7     | Horror    |

# Algebra × More × Division ( / )

Has been a movie won ALL Oscar awards?  
Students who should graduate? (passed ALL the courses)  
Girls who have collaborated with ALL boys in a class?  
Boys who have collaborated with ALL girls in a class?


$$A \text{ (Fully Experienced)} = R1 \text{ (Reality)} / R2 \text{ (All Possibilities)}$$

# Algebra × More × Division ( / )

Has been a movie won ALL Oscar awards?

Students who should graduate? (passed ALL the courses)

Girls who have collaborated with ALL boys in a class?

Boys who have collaborated with ALL girls in a class?

$$\text{Fully Experienced} = \text{Reality} - (\text{All Possibilities} - \text{Reality})$$

Yet to Experience

[https://en.wikipedia.org/wiki/Relational\\_algebra#Division\\_\(%C3%B7\)](https://en.wikipedia.org/wiki/Relational_algebra#Division_(%C3%B7))



# Algebra × Extensions

Relational algebra accepts some extensions to support SQL from physical level such as:

Outerjoin

Aggregation Functions (SUM, AVG, MAX, MIN)

Grouping

Sorting

We will cover them in more details when explaining SQL.