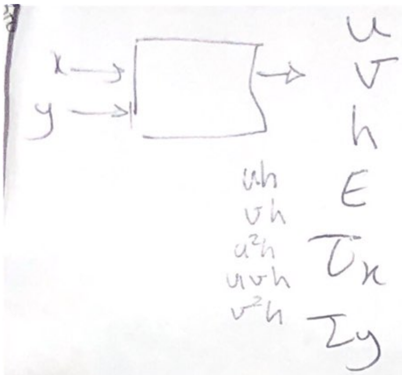
vertically averaged

$$\boxed{\bar{u} = \frac{1}{h}\int_B^E u\,dz}$$

$$\frac{\partial \bar{u}h}{\partial x} + \frac{\partial \bar{v}h}{\partial y} = 0$$

$$\frac{\partial \bar{u}^2 h}{\partial x} + \frac{\partial \bar{u}\bar{v}h}{\partial y} = -gh\frac{\partial E}{\partial x} + 1/\rho\left[\frac{\partial \bar{\tau}_{xx}h}{\partial x} + \frac{\partial \bar{\tau}_{yx}h}{\partial y}\right]$$

$$\frac{\partial \bar{u}\bar{v}h}{\partial x} + \frac{\partial \bar{v}^2 h}{\partial y} = -gh\frac{\partial E}{\partial y} + 1/\rho\left[\frac{\partial \bar{\tau}_{xy}h}{\partial x} + \frac{\partial \bar{\tau}_{yy}h}{\partial y}\right]$$



$$\frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} = 0$$

$$\frac{\partial u^2h}{\partial x} + \frac{\partial uvh}{\partial y} + gh\frac{\partial E}{\partial x} - \frac{1}{\rho}\left[\frac{\partial \tau_x}{\partial x}\right] = 0$$

$$\frac{\partial uvh}{\partial x} + \frac{\partial v^2h}{\partial y} + gh\frac{\partial E}{\partial y} - \frac{1}{\rho}\left[\frac{\partial \tau_y}{\partial y}\right] = 0$$

```
x = sn.Variable("x")
y = sn.Variable("y")
u = sn.Functional("u", [x,y], 4x[150,100,80,50], "tanh")
v =  "      "  ("v", [x,y],  "              ,  " )
E, τx, τy =  "      "         "
```

```
uh_x, vh_y = diff((u*h),x), diff((v*h),y)
u2h_x, v2h_y = diff((u**2*h),x), diff((v**2*h),y)
E_x, E_y = diff(E,x), diff(E,y)
uvh_x, uvh_y = diff((u*v*h),x), diff((u*v*h),y)
τx_x, τy_y = diff(τx,x), diff(τy,y)
```

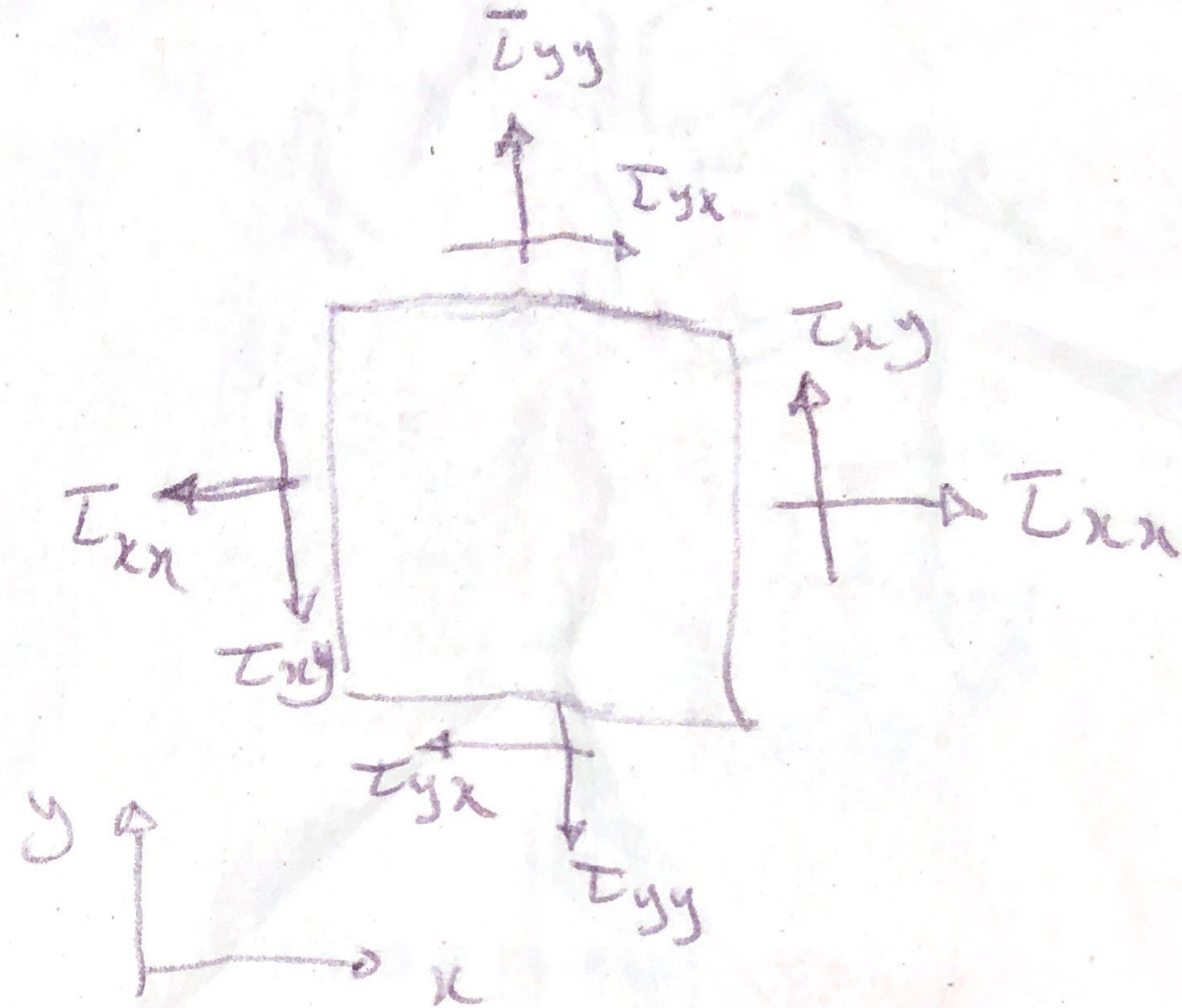
$L_1 = uh\_x + vh\_y \; (0)$

$L_2 = u^2h\_x + uvh\_y + (9.81 \times h) \times (E\_x) - \frac{\tau_{x\_x}}{\rho} \; (0)$

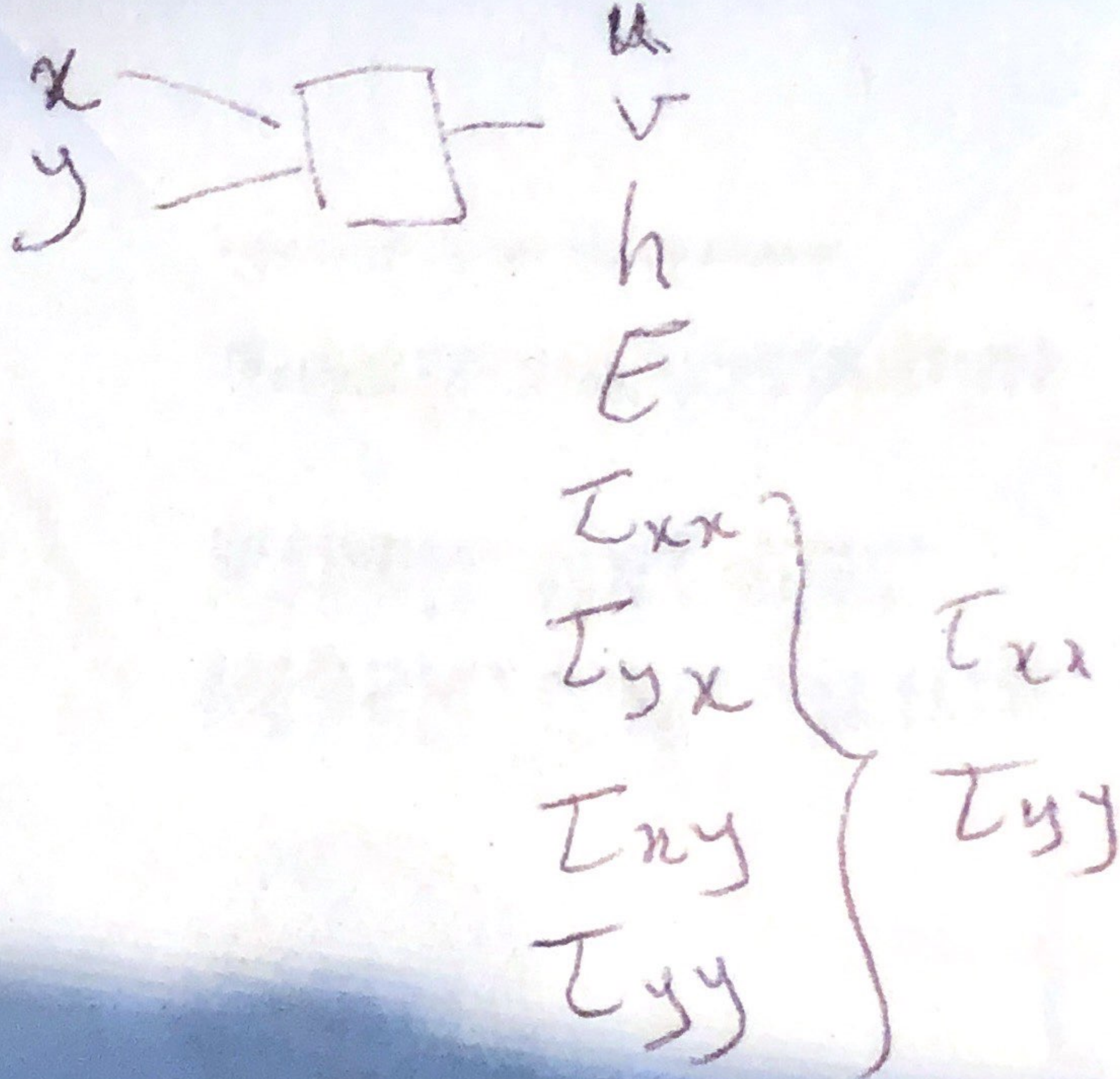
$L_3 = uvh\_x + v2h\_y + (9.81 \times h) \times (E\_y) - \frac{\tau_{y\_y}}{\rho} \; (0)$

$L_4 = u$, $L_5 = v$, $L_6 = h$, $L_7 = E$, $L_8 = \tau_x$, $L_{\cancel{8}9} = \tau_y$

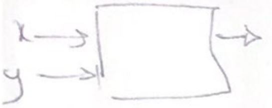
```
m = sn.sciModel([x,y], [L1,L2...,L9], "mse", "Adam")

m.train([x_tr, y_tr],
        [0, 90, 0, u_tr, v_tr, h_tr, E_tr, Tx_tr, Ty_tr],
        [batch_size = 64, epochs = 10000)
```



τ_{yy}
τ_{yx}
τ_{xy}
τ_{xx}
τ_{xx}
τ_{xy}
τ_{yx}
τ_{yy}
y
x



x
y
u
v
h
E
τ_{xx}
τ_{yx}
τ_{xy}
τ_{yy}
τ_{xx}
τ_{yy}



u
v
h
E
τ_x
τ_y

uh
vh
u^2h
uvh
v^2h

$$\frac{\partial uh}{\partial x} + \frac{\partial vh}{\partial y} = 0$$

$$\frac{\partial u^2h}{\partial x} + \frac{\partial uvh}{\partial y} + gh\frac{\partial E}{\partial x} - 1/\rho \left[\frac{\partial \tau_x}{\partial x}\right] = 0$$

$$\frac{\partial uvh}{\partial x} + \frac{\partial v^2h}{\partial y} + gh\frac{\partial E}{\partial y} - 1/\rho \left[\frac{\partial \tau_y}{\partial y}\right] = 0$$

```
x = sn.Variable("x")
y = sn.Variable("y")
u = sn.Functional("u", [x,y], 4*[150,100,80,50], "tanh")
v =  〃  〃  ("v", [x,y],  〃  , 〃)
E, τx, τy = 〃  〃  〃
```

```
uh_x, vh_y = diff((u*h),x), diff((v*h),y)
u2h_x, v2h_y = diff((u**2*h),x), diff((v**2*h),y)
E_x, E_y = diff(E,x), diff(E,y)
uvh_x, uvh_y = diff((u*v*h),x), diff((u*v*h),y)
τx_x, τy_y = diff(τx,x), diff(τy,y)
```

L_1 = uh_x + vh_y (0)

L_2 = u²h_x + uvh_y + (9.81*h)*(E_x) − ~~1/ρ~~ $\frac{\tau_{x\_x}}{\rho}$ (0)

L_3 = uvh_x + v2h_y + (9.81*h)*(E_y) − $\frac{\tau_{y\_y}}{\rho}$ (0)

L_4 = u, L_5 = v, L_6 = h, L_7 = E, L_8 = τ_x, L_{9} = τ_y

```
m = sn.sciModel([x,y], [L1,L2...,L9], "mse", "Adam")

m.train([x_tr, y_tr],
        [0, 0, 0, u_tr, v_tr, h_tr, E_tr, Tx_tr, Ty_tr],
        [batch_size = 64, epochs = 10000)
```

$$\rho\left(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}\right) + \rho g_x$$

$$\rho\left(\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}\right) + \rho g_y$$

$$u_t + \lambda_1 (u u_x + v u_y) = -p_x + \lambda_2 (u_{xx} + u_{yy})$$

$$v_t + \lambda_1 (u v_x + v v_y) = -p_y + \lambda_2 (v_{xx} + v_{yy})$$

~~RANS~~ N-S (Neglectig body forces)

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial z} + \frac{\tau_{xz}}{\partial x} + \frac{\tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$ $\nabla . V$ (convective Derivatie)

$\lambda = -2/3 \, \mu$ (second velocity coeff.)

$$\tau_{xx} = \lambda(\nabla . V) + 2\mu \frac{\partial u}{\partial x}$$

$$\tau_{yy} = \lambda(\nabla . V) + 2\mu \frac{\partial v}{\partial y}$$

$$\tau_{zz} = \lambda(\nabla . V) + 2\mu \frac{\partial w}{\partial z}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} + v\frac{\partial (h)}{\partial y} + h\frac{\partial (v)}{\partial y} = 0$$

$$h\frac{\partial u}{\partial t} + u\frac{\partial h}{\partial t} + u^2\frac{\partial h}{\partial x} + 2hu\frac{\partial u}{\partial x} + gu\frac{\partial u}{\partial x} + uv\frac{\partial h}{\partial y} + hv\frac{\partial u}{\partial y}$$

$$+ hu\frac{\partial v}{\partial y} = gh(S_{ox} - S_{fx})$$

$$h\frac{\partial v}{\partial t} + v\frac{\partial h}{\partial t} + uv\frac{\partial h}{\partial x} + hv\frac{\partial u}{\partial x} + hu\frac{\partial v}{\partial x} + v^2\frac{\partial h}{\partial y} + 2hv\frac{\partial v}{\partial y}$$

$$+ gh\frac{\partial h}{\partial y} = gh(S_{oy} - S_{fy})$$

$\cancel{-\frac{1}{\rho}\frac{\partial P}{\partial x}}$

$x \leftrightarrow s$
$y \leftrightarrow n$

$$\boxed{\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{1}{1-N}\frac{\partial u}{\partial s} - \frac{v}{(1-N)R} \\
\frac{\partial v}{\partial y} &= \frac{\partial v}{\partial n} \\
\frac{\partial w}{\partial z} &= \frac{\partial w}{\partial z}
\end{aligned}}$$

ortho
curvilinear
system

$$\boxed{\begin{aligned}
u\frac{\partial u}{\partial x} &= \frac{u}{1-N}\frac{\partial u}{\partial s} - \frac{uv}{(1-N)R} \\
v\frac{\partial u}{\partial y} &= v\frac{\partial u}{\partial n} \\
w\frac{\partial u}{\partial z} &= w\frac{\partial u}{\partial z}
\end{aligned}}$$

$$\boxed{\begin{aligned}
-\frac{1}{\rho}\frac{\partial P}{\partial x} &= -\frac{g}{1-N}\frac{\partial E}{\partial s} \\
\frac{\partial \tau_{xx}}{\partial x} &= \frac{1}{1-N}\frac{\partial \tau_{ss}}{\partial s} - \frac{2\tau_{ns}}{(1-N)R} \\
\frac{\partial \tau_{yx}}{\partial y} &= \frac{\partial \tau_{ns}}{\partial n} \\
\frac{\partial \tau_{zx}}{\partial z} &= \frac{\partial \tau_{zs}}{\partial z}
\end{aligned}}$$

$$\boxed{\begin{array}{l} x \to s \\ y \to n \end{array}}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} = -g\frac{\partial \epsilon}{\partial x} + \frac{1}{\rho}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right]$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z} = -g\frac{\partial \epsilon}{\partial y} + \frac{1}{\rho}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right]$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial z} - g = 0$$