HW 3

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Problem 3.1. Let $f(x) = tan(x)$, and define $g(x)$ as the following:

$$\begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases} \tag{1}$$

on which domain A and codomain B $g(x)$ an inverse function for $f(x)$?

Proof by contradiction

Let $f(x) = tan(x)$ and $x = 45$.

Assume $g(x) = f(x)^{-1}$.

Let $y = f(x) = f(45) = 1$.

$g(y) = g(1) = 1$.

Since $g(y) \neq f(x)$, $g(x)$ is not $f(x)^{-1}$ ■

Problem 3.2. Take a clock and replace all the numbers on the face with numbers $1 \leq n \leq 19$, such that if you sum all the numbers up you get 30. For example, a valid clock face would be [4, 3, 6, 3, 1, 1, 1, 2, 1, 3, 3, 2] starting with the first number in the 12 position, and going around clockwise from there. Prove that for any valid sequence on the clock, that there must exist a consecutive sequence that adds to exactly 10.

Assume that there is a wrapping list of integers from 1-19 (inclusive) that add up to 30, without any sequence that add up to 10. Since the lowest sequence that doesn't add up to 10 is $= [1, 3, 4, 1, 3, 4, 1, 3, 4, 1, 3, 4]$ and the sum of the following list of integers is 32 which greater than 30.
Since lowering any integer in this list results in at least 1 sequence that adds

to 10, there is no list of integers that can add up to 30 that won't result in sequence of 10. ■

Problem 3.3. Let $a, b, \in \mathbb{Z}_+$ and $gcd(a, b) = 1$. Prove that the decimal expansion of $\frac{a}{b}$ must either terminate or will loop infinitely.

Assume there are 2 numbers a, b (positive integers $gcd(a, b) = 1$) $s.t.$
a and b do not terminate or loop infinitely. π does not terminate or loop infinitely, and there are no 2 numbers that divide into π.
$\therefore$ there are no 2 numbers that will not terminate and not loop infinitely because there are no 2 numbers that divide into π. ■

Problem 3.4. Let $\mathbb{Z}/n$ be the quotient set of $\mathbb{Z}$ under the relation $x \equiv y(mod\ n)$ (a.k.a a complete system of incongruent residues).
Suppose that:

$$\mathbb{Z}/n = \{[x_1], [x_2], ..., [x_n]\}. \tag{2}$$

If $gcd(k, n) = 1$, prove that:

$$\mathbb{Z}/n = \{[x_1], [x_2], ..., [x_n]\} = \{[kx_1], [kx_2], ..., [kx_n]\}. \tag{3}$$

False.

Proof by contradiction

Since $\mathbb{Z}/n$ is a complete system of incongruent residues, the remainders in the set has to all be different from each other.

Let $n = 4, \quad z = ...8, 9, 10, 11...$

$$\mathbb{Z}/n = \{0, 1, 2, 3\}$$

Let $k = 3$.

$$gcd(k, n) = gcd(3, 4) = 1$$
$$\{k(0), k(1), k(2), k(3)\} = \{0, 3, 6, 9\}$$
$$\{0, 1, 2, 3\} \neq \{0, 3, 6, 9\}$$

There is a contradiction, $\therefore$ it is false. ■