

1.

$$\begin{array}{c}
 \frac{}{f:A \rightarrow A, x:A \vdash f:A \rightarrow A} \quad \frac{}{f:A \rightarrow A, x:A \vdash f:A \rightarrow A} \quad \frac{}{f:A \rightarrow A, x:A \vdash x:A} \\
 \frac{}{f:A \rightarrow A, x:A \vdash f:A \rightarrow A} \quad \frac{}{f:A \rightarrow A, x:A \vdash f(x):A} \\
 \frac{}{f:A \rightarrow A, x:A \vdash f(f(x)):A} \\
 \frac{}{f:A \rightarrow A \vdash \lambda x^A. f(f(x)):A \rightarrow A} \\
 \frac{}{\vdash \lambda f^{A \rightarrow A} x^A. f(f(x)): (A \rightarrow A) \rightarrow A \rightarrow A}
 \end{array}$$

2.3

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*scratch*
Inductive fruit:Type:=
|apple
|pear.
Inductive food:Type:=
|rice
|steak
|dessert(f:fruit).
(*定义一个函数vegetarian,使得vegetarian f返回true当且仅当f是除了steak之外的食物*)
Definition vegetarian (f:food):bool:=
match f with
|steak=>false
|rice=>true
|dessert f=>true
end.
Check dessert.
Check (dessert apple).
Example test_vegetarian1:(vegetarian steak)=false.
Proof. simpl. reflexivity. Qed.
Example test_vegetarian2:(vegetarian rice)=true.
Proof. simpl. reflexivity. Qed.
Example test_vegetarian3:(vegetarian (dessert apple))=true.
Proof. simpl. reflexivity. Qed.
Example test_vegetarian4:(vegetarian (dessert pear))=true.
Proof. simpl. reflexivity. Qed.

```