

计算机逻辑作业3

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$$\neg \forall (P(x) \rightarrow \neg Q(x)) \vdash \neg (\exists (P(x) \wedge Q(x)))$$

1	$\forall x (P(x) \rightarrow \neg Q(x))$	premise
2	$\exists x (P(x) \wedge Q(x))$	assumption
3	$x_0 \quad P(x_0) \wedge Q(x_0)$	assumption
4	$P(x_0)$	$\wedge E \ 3$
5	$Q(x_0)$	$\wedge E \ 3$
6	$P(x_0) \rightarrow \neg Q(x_0)$	$\forall x \in 2$
7	$\neg Q(x_0)$	$\rightarrow E \ 4, 6$
8	\perp	$\neg E \ 5, 7$
9	\perp	$\exists x \in 2, 3-8$
10	$\neg (\exists x (P(x) \wedge Q(x)))$	$\neg i \ 2-9$

$$\text{二. } \forall (P(x) \leftrightarrow x = b) \vdash P(b) \wedge \forall x \forall y (P(x) \wedge P(y) \rightarrow x = y)$$

1	$\forall(P(x) \rightarrow x = b)$	<i>premise</i>
2	$\forall(x = b \rightarrow P(x))$	<i>premise</i>
3	$b = b$	$= i$
4	$b = b \rightarrow P(b)$	$\forall x e 2$
5	$P(b)$	$\rightarrow e 3, 4$
6	$x_0 \quad P(x_0) \rightarrow x_0 = b$	$\forall x e 1$
7	$y_0 \quad P(y_0) \rightarrow y_0 = b$	$\forall y e 1$
8	$P(x_0) \wedge P(y_0)$	<i>assumption</i>
9	$P(x_0)$	$\wedge e_1 8$
10	$x_0 = b$	$\rightarrow e 6, 9$
11	$P(y_0)$	$\wedge e_2 8$
12	$y_0 = b$	$\rightarrow e 7, 11$
13	$x_0 = y_0$	$= e 10, 12$
14	$P(x_0) \wedge P(y_0) \rightarrow x_0 = y_0$	$\rightarrow i 8 - 13$
15	$\forall y(P(x_0) \wedge P(y) \rightarrow x_0 = y)$	$\forall y i 7 - 14$
16	$\forall x \forall y(P(x) \wedge P(y) \rightarrow x = y)$	$\forall x i 6 - 15$
17	$P(b) \wedge \forall x \forall y(P(x) \wedge P(y) \rightarrow x = y)$	$\wedge i 5, 16$