函数语言作业9

10205101530-赵晗瑜

Theorem or_commut : forall P Q : Prop, P ∨ Q -> Q ∨ P.

Proof. (* FILL IN HERE *) Admitted.

【ans】

```
Theorem or_commut:forall P Q:Prop,
  P\/Q->Q\/P.
Proof. intros P Q [HP|HQ].
  - right. apply HP.
  - left. apply HQ.
Qed.
```

【运行结果】

```
Theorem or_commut:forall P Q:Prop,
  P\/Q->Q\/P.
Proof. intros P Q [HP|HQ].
  - right. apply HP.
  - left. apply HQ.
Qed.
```

Theorem not_both_true_and_false : ∀ P : Prop,
¬ (P ∧ ¬P).
Proof.
(* FILL IN HERE *) Admitted.

【ans】

```
Theorem not_both_true_and_false : forall P : Prop,
  ~ (P /\ ~P).
Proof. unfold not. intros P [HP HNA]. apply HNA. apply HP. Qed.
```

【运行结果】

```
(** **** Exercise: 1 star, standard (not_both_true_and_false) *)
Theorem not_both_true_and_false : forall P : Prop,
  ~ (P /\ ~P).
Proof. unfold not. intros P [HP HNA]. apply HNA. apply HP. Qed.
```

Theorem or_distributes_over_and : ∀ P Q R : Prop,
P ∨ (Q ∧ R) -> (P ∨ Q) ∧ (P ∨ R).
Proof.
(* FILL IN HERE *) Admitted.

【ans】

```
Theorem or_distributes_over_and : forall P Q R : Prop,
  P \/ (Q /\ R) -> (P \/ Q) /\ (P \/ R).
Proof.
  intros P Q R [HP | [HQ HR]].
    + split. left. apply HP. left. apply HP.
    + split. right. apply HQ. right. apply HR.
Qed.
```

【运行结果】

```
Theorem or_distributes_over_and : forall P Q R : Prop,
  P \/ (Q /\ R) -> (P \/ Q) /\ (P \/ R).
Proof.
  intros P Q R [HP | [HQ HR]].
    + split. left. apply HP. left. apply HP.
    + split. right. apply HQ. right. apply HR.
Qed.
```

Theorem or_distributes_over_and' : ∀ P Q R : Prop,
(P ∨ Q) ∧ (P ∨ R) -> P ∨ (Q ∧ R).
Proof.
(* FILL IN HERE *) Admitted.

【ans】

```
Theorem or_distributes_over_and' : forall P Q R : Prop,
  (P \/ Q) /\ (P \/ R) -> P \/ (Q /\ R).
Proof.
  (* FILL IN HERE *)
  intros P Q R [[HP | HQ] [HP' | HR]].
    + left. apply HP.
    + left. apply HP.
    + left. apply HP'.
    + right. split. apply HQ. apply HR.
Qed.
```

【运行结果】

```
Theorem or_distributes_over_and' : forall P Q R : Prop,
  (P \/ Q) /\ (P \/ R) -> P \/ (Q /\ R).
Proof.
  (* FILL IN HERE *)
  intros P Q R [[HP | HQ] [HP' | HR]].
    + left. apply HP.
    + left. apply HP.
    + left. apply HP'.
    + right. split. apply HQ. apply HR.
Qed.
```

最后两题可以用iff:

```
Theorem or_distributes_over_and_iff : forall P Q R : Prop,
  P \/ (Q /\ R) <-> (P \/ Q) /\ (P \/ R).
Proof.
  (* FILL IN HERE *)
  intros P Q R. split.
  - intros [HP | [HQ HR]].
    + split; left; assumption.
    + split; right; assumption.
  - intros [[HP | HQ] [HP' | HR]].
    + left; assumption.
    + left; assumption.
    + left; assumption.
    + right. split; assumption.
Qed.
```

```
Theorem or_distributes_over_and_iff : forall P Q R : Prop,
  P \/ (Q /\ R) <-> (P \/ Q) /\ (P \/ R).
Proof.
  (* FILL IN HERE *).
  intros P Q R. split.
  - intros [HP | [HQ HR]].
    + split; left; assumption.
    + split; right; assumption.
  - intros [[HP | HQ] [HP' | HR]].
    + left; assumption.
    + left; assumption.
    + left; assumption.
    + right. split; assumption.
Qed.
```