

**4-1 Recurrence examples**

Give asymptotic upper and lower bounds for  $T(n)$  in each of the following recurrences. Assume that  $T(n)$  is constant for  $n \leq 2$ . Make your bounds as tight as possible, and justify your answers.

a.  $T(n) = 2T(n/2) + n^4.$

b.  $T(n) = T(7n/10) + n.$

c.  $T(n) = 16T(n/4) + n^2.$

d.  $T(n) = 7T(n/3) + n^2.$

e.  $T(n) = 7T(n/2) + n^2.$

f.  $T(n) = 2T(n/4) + \sqrt{n}.$

g.  $T(n) = T(n-2) + n^2.$

### 2.3-7 ★

Describe a  $\Theta(n \lg n)$ -time algorithm that, given a set  $S$  of  $n$  integers and another integer  $x$ , determines whether or not there exist two elements in  $S$  whose sum is exactly  $x$ .

### 4.5-5 ★

Consider the regularity condition  $af(n/b) \leq cf(n)$  for some constant  $c < 1$ , which is part of case 3 of the master theorem. Give an example of constants  $a \geq 1$  and  $b > 1$  and a function  $f(n)$  that satisfies all the conditions in case 3 of the master theorem except the regularity condition.

### 5.3-6

Explain how to implement the algorithm PERMUTE-BY-SORTING to handle the case in which two or more priorities are identical. That is, your algorithm should produce a uniform random permutation, even if two or more priorities are identical.