# 华东师范大学期期末试卷（A）

2019 — 2020 学年第 一 学期

课程名称：计算机逻辑基础

学生姓名：

学　　号：

专　　业：

年级/班级：

课程性质：公共必修、公共选修、专业必修、**专业选修**

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## 一、简答题（共 3 题，满分 15 分）

1. Please give the definition of soundness and completeness of a given logic system. (5 分)

2. Please list all ways you know to determine the equivalence of two propositional logic formulas. (5 分)

3. Please explain what is Hoare logic and its functionality in program verification. (5 分)

## 二、 计算题（共 4 题，满分 45 分）

1. Given the following formula, draw its corresponding parse tree and then transform it into CNF (Conjunctive Normal Form).

$\neg(p \to q \wedge \neg r) \to \neg p \vee q$　(5 分)

2. Given the following formula, draw its corresponding parse tree and point out all the free variables.

$\forall x\big(P(x,y) \vee Q(x)\big) \to \neg\exists y P(x,y) \vee Q(x)$　(5 分)

3. Let D be a domain of persons, S={Peter,Ann,Father,Mother} be a signature where Peter, Ann are constant symbols, Father and Mother be two predicate symbols with arity 2. Father(x,y) means x is the father of y, Mother(x,y) means x is the mother of y. Please translate the following sentences into predicate logic:

(1) Everybody has a father and a mother. (3 分)

(2) Whoever has a mother has a father. (3 分)

(3) Nobody's grandmother is anybody's father. (3 分)

(4) Peter is a grandfather. (3 分)

(5) Peter's daughter is Ann's mother. (3 分)

F(P, X) ∧ M(X, Ann)

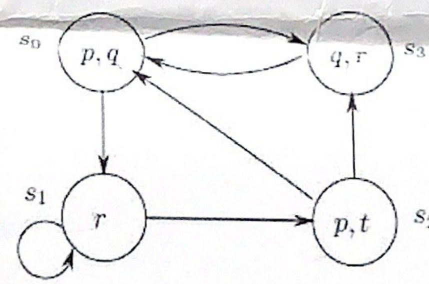
Find a model $M=(D,Father^M,Mother^M,Peter^M,Ann^M)$ which makes all the above formulas hold under it. (5 分)

4. Consider the model M in the following figure. Check whether $M, s_0 \vDash \phi$ hold for the following CTL formulas. If it holds, explain the reason; otherwise, find a counter example.

(1) $AG(p \rightarrow AF(q))$ (5 分)

(2) $EX(AG(q \rightarrow p))$ (5 分)

(3) $AF(A(r\ U\ p))$ (5 分)




三、证明题（共 3 题，满分 40 分）

$$\frac{\phi \quad \psi}{\phi \wedge \psi}\wedge i \qquad \frac{\phi \wedge \psi}{\phi}\wedge e_1 \qquad \frac{\phi \wedge \psi}{\psi}\wedge e_2 \qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\vee e \qquad \frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \rightarrow \psi}\rightarrow i$$

$$\frac{\phi}{\phi \vee \psi}\vee i_1 \qquad \frac{\psi}{\phi \vee \psi}\vee i_2 \qquad \frac{\phi \quad \phi \rightarrow \psi}{\psi}\rightarrow e$$

$$\frac{\phi \quad \neg\phi}{\bot}\neg e \qquad \frac{\bot}{\phi}\bot e \qquad \frac{\neg\neg\phi}{\phi}\neg\neg e \qquad \frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\neg i \qquad \frac{\boxed{\begin{array}{c}\neg\phi\\ \vdots\\ \bot\end{array}}}{\phi}\text{PBC}$$

$$\frac{\phi \rightarrow \psi \quad \neg\psi}{\neg\phi}\text{MT} \qquad \frac{\phi}{\neg\neg\phi}\neg\neg i \qquad \frac{}{\phi \vee \neg\phi}\text{LEM}$$

$$\frac{}{t=t}=\mathrm{i} \qquad \frac{t_1=t_2 \quad \phi[t_1/x]}{\phi[t_2/x]}=\mathrm{e}. \qquad \frac{\boxed{\begin{array}{l} x_0 \\ \quad\vdots \\ \quad \phi[x_0/x] \end{array}}}{\forall x\,\phi}\forall x\,\mathrm{i}. \qquad \frac{\exists x\,\phi \quad \boxed{\begin{array}{l} x_0 \ \phi[x_0/x] \\ \quad\vdots \\ \quad \chi \end{array}}}{\chi}\exists\mathrm{e}.$$

$$\frac{\forall x\,\phi}{\phi[t/x]}\forall x\,\mathrm{e}. \qquad \frac{\phi[t/x]}{\exists x\phi}\exists x\,\mathrm{i}.$$

1. Prove the following sequents using the above reduction rules:

(1) $p \wedge q \vdash \neg(\neg p \vee \neg q)$ (5 分)

(2) $p \rightarrow q, r \rightarrow s \vdash p \vee r \rightarrow q \vee s$ (5 分)

(3) $\exists x \exists y (H(x,y) \vee H(y,x)), \neg\exists x H(x,x) \vdash \exists x \exists y \neg(x=y)$ (10 分)

2. Prove the validity of the semantic entailment:

$\forall x P(x) \vee \forall x Q(x) \vDash \forall x (P(x) \vee Q(x))$ (10 分)

3. Verify the following program Multi is partial correct w.r.t. the pre-condition $y \geq 0$ and the post-condition $z = x * y$. That is $\vdash_{par} [y \geq 0] Multi [z = x * y]$.

Code of Multi:

$y \geq 0$

```
a=0;
z=0;
while (a!=y) {
    z=z+x;
    a=a+1;
}
```

(10 分) $z=x\cdot y$

$z=a*x$.