Assignment 6

- Verify the following Hoare triple is partial correct.
- $\{n > 0 \wedge \forall i.\, 0 \le i < n \rightarrow a[i] \in \mathbb{N}\}$
  Sort
  $\{\forall i, j.\, 0 \le i \le j < n \rightarrow a[i] \le a[j]\}$
- The code of Sort is as follows

```
1   i = 0;
2   while (i < n-1) {
3       j = 0;
4       while (j < n-i-1) {
5           if (a[j] > a[j+1]) {
6               temp = a[j];
7               a[j] = a[j+1];
8               a[j+1] = temp;
9           }
10          j = j+1;
11      }
12      i = i+1;
13  }
```

外层循环的 invariant:

$i \le n-1 \wedge \forall p, q.\, n-i-1 \le p \le q \le n \rightarrow a[p] \le a[q]$

外层循环的 invariant:

$j \le n-1 \wedge \forall p.\, 0 \le p \le j \rightarrow a[p] \le a[j]$

$(|n>0 \wedge \forall i.\, 0 \le i < n \rightarrow a[i] \in \quad |)$

$(|\cdots \wedge \ 0 \le n-1 \wedge \forall p, q.\, n-0-1 \le p \le q < n \rightarrow a[p] \le a[q]|)$ implied

$i := 0;$

$(|\cdots \wedge i \le n-1 \wedge \forall p, q.\, n-i-1 \le p \le q < n \rightarrow a[p] \le a[q]|)$ Assignment

while (j<n-1) {

(1... ∧ i<n-1 ∧ i≤n-1 ∧ ∀p,q. n-i-1 ≤ p ≤ q < n → a[p] ≤ a[q] 1) Invariant HYP ∧ Guard

(1... ∧ i<n-1 ∧ i≤n-1 ∧ ∀p,q. n-i-1 ≤ p ≤ q < n → a[p] ≤ a[q]

∧ 0 ≤ n-i-1 ∧ ∀p. 0 ≤ p ≤ 0 → a[p] ≤ a[0] ∧ 0 < n-i-1 1) Implied

j = 0;

(1... ∧ j ≤ n-i-1 ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j] ∧ j < n-i-1 1) Assignment

while (j<n-i-1) {

(1... ∧ j < n-i-1 ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j] 1) Invariant HYP ∧ Guard

if (a[j] > a[j+1]) {

(1... ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j] ∧ a[j] > a[j+1] 1) If-Statement

temp = a[j]

(1... ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ temp ∧ temp > a[j+1] 1) Assignment

a[j] = a[j+1]

(1... ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ temp ∧ temp > a[j] 1) Assignment

a[j+1] = temp

(1... ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j+1] ∧ a[j+1] > a[j] 1) Assignment

(1... ∧ ∀p. 0 ≤ p ≤ j+1 → a[p] ≤ a[j+1] 1) Implied

} else

(1... ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j] ∧ ¬(a[j] > a[j+1]) 1) If-statement

(1... ∧ ∀p. 0 ≤ p ≤ j+1 → a[p] ≤ a[j+1] 1) Implied

(1... j+1 < n-i ∧ ∀p. 0 ≤ p ≤ j+1 → a[p] ≤ a[j+1] 1) If-Statement

j = j+1;

(1... j < n-i ∧ ∀p. 0 ≤ p ≤ j → a[p] ≤ a[j] 1) Assignment

}

(1... ∧ ¬(i<n-1) ∧ i≤n-1) ∧ ∀p,q. n-i-1 ≤ p ≤ q < n → a[p] ≤ a[q] 1) Partial-While

$(\ | \ldots \wedge \forall p, q.\ 0 \le p \le q < n \to a[p] \le a[q]\ |)$ Implied

$(\ | \ \forall i, j\ .\ 0 \le i \le j < n \to\ a[i] \le a[j]\ |)$ Implied