

# 130k Lines of Formal Topology in Two Weeks: Simple and Cheap Autoformalization for Everyone?

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## Abstract

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This is a brief description of a project that has already autoformalized a large portion of the general topology from the Munkres textbook (which has in total 241 pages in 7 chapters and 39 sections). The project has been running since November 21, 2025 and has as of January 4, 2026, produced 160k lines of formalized topology. Most of it (about 130k lines) have been done in two weeks, from December 22 to January 4, for an LLM subscription cost of about \$100. This includes a 3k-line proof of Urysohn's lemma, a 2k-line proof of Urysohn's Metrization theorem, over 10k-line proof of the Tietze extension theorem, and many more (in total over 1.5k lemmas/theorems).

The approach is quite simple and cheap: build a long-running feedback loop between an LLM and a reasonably fast proof checker equipped with a core foundational library. The LLM is now instantiated as ChatGPT (mostly 5.2) or Claude Sonnet (4.5) run through the respective Codex or Claude Code command line interfaces. The proof checker is Chad Brown's higher-order set theory system Megalodon, and the core library is Brown's formalization of basic set theory and surreal numbers (including reals, etc). The rest is some prompt engineering and technical choices which we describe here. Based on the fast progress, low cost, virtually unknown ITP/library, and the simple setup available to everyone, we believe that (auto)formalization may become quite easy and ubiquitous in 2026, regardless of which proof assistant is used.

**2012 ACM Subject Classification** Theory of computation → Automated reasoning; Theory of computation → Higher order logic; Theory of computation → Logic and verification

**Keywords and phrases** Autoformalization, Automated reasoning, Interactive theorem proving, Formal proof assistants, Machine learning, Language Models

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## 1 Introduction

*“It is the view of some of us that many people who could have easily contributed to project QED have been distracted away by the enticing lure of AI or AR.”*

– The QED Manifesto

### Caveat:

This is an initial write-up about an ongoing experiment (mostly to allow others to try it in their settings). My assessment may evolve as the experiment proceeds.

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<sup>1</sup> <https://ai4reason.eu/sponsors.html>



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**How it started:**

I have been working (more or less seriously) with my colleagues on learning-assisted auto-formalization since 2014<sup>2</sup> [7, 6]. A bit of autoformalization history is given in my 2024 Bonn talk<sup>3</sup> and its slides.<sup>4</sup> Long story short, since the first MaLARea experiments in 2005/6 [9], I have become quite convinced about the power of feedback loops between reasoning/logic/deduction and learning, and believed that we are ourselves in such a feedback loop when we formalize math (see e.g. the last two sentences in [10]). In 2018, I got further impressed by the neural language models (especially attention-based) showing surprisingly good informal-to-formal translation capability in our first experiments over a large synthetic Mizar-to-LaTeX corpus [12]. I started to believe my own autoformalization propaganda, in the sense that autoformalization would become usable much sooner than my 2014 conservative 25-year horizon estimate.<sup>5</sup> Since 2020, the field has become increasingly popular in industry teams with much more resources, and (seeing their heavy buy-in<sup>6</sup>) I have (with some exceptions) mostly focused on other (less resource-heavy) feedback loops in ATP and conjecturing [11, 5, 3, 4, 8]. I have occasionally tried to get ChatGPT (and similar LLMs) to autoformalize smaller papers (such as the Ramsey(3,6) 4-page paper [2]) since it appeared, but it initially didn't seem very good at carrying out deeper and longer work – at least not without building some infrastructure around it.

The immediate impulse for starting the experiment described here were Zar Goertzel's reports of good autoformalization progress with coding agents such as Codex and Claude Code. In particular, in late 2025 (overlapping with this work) Zar managed to finish a full Lean proof of my testing paper on Ramsey(3,6) with Claude Code in about 13k lines of Lean.<sup>7</sup> After some false starts and brief discussion with Chad Brown, I have decided to set up the current experiment in formalization of topology with the Megalodon proof assistant and the LLM-based command-line coding agents.

**2    Current Setup**

The latest setup has been (almost) stable since December 22 and used for (almost) automated formalization resulting in about 130k lines code and the above mentioned fully proved major theorems. The setup is as follows:

1. A ChatGPT Pro (\$200/month) subscription, giving access to the OpenAI's Codex coding agent.
2. I always use the ChatGPT 5.2 model with high (not extra high) reasoning.<sup>8</sup>
3. I isolate the coding agent using the Linux bubblewrap (bwrap<sup>9</sup>) sandboxing tool. I could have likely used any other reasonable sandboxing solution. I want to give the coding

<sup>2</sup> I have likely coined the “auto-formalization” term in this context in my CICM'14 talk <https://t.ly/PaeKH> [7].

<sup>3</sup> [https://www.youtube.com/watch?v=4JeezEGc\\_gQ](https://www.youtube.com/watch?v=4JeezEGc_gQ)

<sup>4</sup> [http://grid01.ciirc.cvut.cz/~mtp/bonn24\\_af.pdf](http://grid01.ciirc.cvut.cz/~mtp/bonn24_af.pdf)

<sup>5</sup> <http://ai4reason.org/aichallenges.html>

<sup>6</sup> Sometimes perhaps too heavy - see the claims about “introducing autoformalization” at <https://archive.ph/8Ib1k>.

<sup>7</sup> <https://github.com/zariuq/ai-agents/tree/main/lean-projects/ramsey36/Ramsey36>

<sup>8</sup> Users with more resources could try to repeat the experiment with other settings and provide further ablations. I don't use the extra high setting because already the high reasoning depletes its weekly usage limit in less than a week.

<sup>9</sup> <https://github.com/containers/bubblewrap>

agents unlimited powers (calling all sorts of dangerous tools), hence their isolation inside the sandbox. I map several outside directories into the sandbox with various binaries and project data. While no serious harm was done by the agents so far inside the sandbox, there were already several occasions (like the agents getting confused about data location, file names, etc) that made me feel vindicated about having the isolation. I periodically back up the project data directory from the outside.

4. Inside bwrap, I run the command-line coding agent (now Codex, but I can and did run Claude Code too with similar approach and results).
5. Bwrap itself is run in the tmux terminal manager/multiplexer, typically in a long-running (multiday) session that is being logged and interacted with from the outside. I could likely have used other similar tools for this such as GNU Screen.
6. I use the Megalodon (higher-order set theory) proof checker binary for the proof checking. The binary used is the one from the Megalodon Wiki (mgwiki) Github repo, which ensures compatibility with the mgwiki toolchain. This allows to push any compiling formalization into the mgwiki, where it is immediately html-ized. This cross-linked HTML presentation allows us to easily study the developing formalization.
7. As the core library I use Brown's formalization of basic set theory and surreal numbers (including reals, etc). We deleted most of the proofs there, replacing them with "admit.", so that the initial library file is smaller and (hopefully) easier to study for the LLMs. I however leave several of Brown's proofs there (such as Diaconescu proof of excluded middle), so that the LLM can (hopefully) study how to prove things in Megalodon.
8. That's it. The rest is some prompting and some additional tools (optional and added later) described below.

### 3 Current Prompt

With only a few exceptions, since December 22, the following short constant prompt has been repeatedly given to Codex each time it reports finishing its work (i.e., each time Codex CLI passes the control back to the user, asking for another prompt):

Read the file CLAUDE.md . Treat it as authoritative work instructions. Follow those instructions exactly for all subsequent actions and responses. That means work as long as possible (as specified) without stopping.

My estimate is that this prompt has been automatically given to Codex about 1000-2000 times during the two weeks.<sup>10</sup> The current (14th) version of the "rules of work" file CLAUDE.md is as follows:

```
# Rules for Working on Math_Background.mg

## STRICT PROHIBITIONS

### **ONLY edit in the topology section**
- The topology section starts at **line 6495** (`Section Topology.`)
- Everything after line 6495 is by default in the Topology section and can be
  edited (even very high line numbers).
- **NEVER edit anything before line 6495**
- This includes set theory, cardinal arithmetic, ordinals, etc.
```

<sup>10</sup>This can likely be calculated precisely after parsing the Codex logs.

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### Before ANY edit to Math\_Background.mg:

1. Verify the line number is  $\geq 6495$  (topology section)
2. If unsure, run: ``grep -n "^Section Topology" Math_Background.mg``
3. If the target is before line 6495, **\*\*STOP\*\*** and ask the user

## When Resuming from Summary

1. **\*\*READ THIS FILE FIRST\*\*** before making any edits
2. Verify you understand which section you're allowed to edit
3. Double-check line numbers before proceeding

## Never throw away useful work

1. You should almost never revert to previous backups (you can use temporary admits when something is hard).
2. If there is a really major reason for reverting, you have to salvage all the useful work done in between.
3. In particular, you have to ensure that all compiling theorems and definitions added since are kept.
4. If you later discover such screwup, you have to immediately start working on salvaging the lost work.
5. This rule exist because you have accidentally thrown away 9k lines (more than a day) of work between bck2658 and bck2659 (by mysteriously justifying it as "reverting to colleague baseline"). This must never ever happen again.
6. **\*\*SIMPLE CHECK\*\***: after each backup, run `wc` on the current and previous backup. If the current backup is smaller, you have to explicitly justify (in the CHANGES file) the decrease and explain that you have not thrown out useful work.

## Megalodon Language Details

### Logic System

- **\*\*This is classical Megalodon\*\*** (not constructive)
- Law of excluded middle (`xm`) is available
- Can use proof by contradiction freely

### Syntax Rules

- **\*\*`/\` is left-associative\*\***: ``A /\ B /\ C`` means ``((A /\ B) /\ C)``
- **\*\*Only use ``(** ... **)`` for comments\*\*** - no other comment syntax; no ``*`` inside comments
- When working with nested conjunctions, remember the left-associative structure

## Work Strategy

- You can do the following things in any order but you should always progress and produce some more code.
- Keep carefully fixing any incorrect/bad definitions/theorems you find (note that this may lead to fixing some proofs, etc).
- Keep eliminating axioms and stubs, replacing them with more complete theorems and definitions (gradual/partial approaches are ok when needed).
- Keep removing admits in unfinished proofs and producing more complete proofs. This may also lead to adding more auxiliary lemmas/theorems.
- While doing the above, remember that:
- Doing easy things is initially OK, however, don't be afraid to try to do (gradually/partially if needed) some harder theorems too. Don't endlessly jump around looking for easy things - that wastes time.
- Balance simple infrastructure theorems with more challenging results
- Your **\*\*strong focus\*\*** should be on finishing the major well-known theorems ASAP (across all sections). Prioritize them (even if they are hard) over doing many examples/exercises.

- Use gradual/partial approaches for difficult theorems when needed (and don't delete such started partial proofs - use temporary "admit" in their various branches and keep gradually eliminating those). Also, structure bigger proofs into useful top-level/helper lemmas wherever possible.
- Frequently lookup the necessary notions defined before the topology section. Those should be completely trusted and built upon. They are listed in TRUSTED\_DEFS.txt .
- Also, completely trust every theorem in the pre-topology section (and never attempt to re-prove it). We do have their proofs but have admitted them in this document for brevity.
- Also, always grep all current Definitions and Theorems in the topology section before creating a new one. Be sure to remove/avoid duplicities.
- Do not introduce axioms that are exactly the same as the theorems you want to prove - it only duplicates and pollutes. In general, try to avoid adding axioms - it's better to have admitted theorems that will be (hopefully) eventually proved.
- Strongly prioritize properly finishing/debugging all proper definitions and (stating/proving) theorems from *\*all\** sections of topology.tex . Doing the exercises is not as time-critical (although it may be occasionally needed as a prerequisite and generally useful, so use your judgement when to work on those admits).

### ### PROGRESS MONITORING

- Keep numbered PROGRESSXXXX files. For each section (i.e. starting with 12 and ending with 50) monitor the progress (stubs only, definitions/theorems properly stated, theorems fully proved, section mostly/fully completed, exercises mostly/fully completed). I.e., this PROGRESS file has to have a status line/paragraph for each of these 39 sections, which will be gradually changing towards more advanced levels.
- Make sure (and record in PROGRESSXXXX) that you make significant progress towards completing *\*ALL\** (i.e. also later) sections. Dont get stuck only in the early sections. Use PROGRESSXXXX to find the imbalances and refocus on neglected parts (within means - dont jump around like crazy when working on something).

### ### Dependency/Admit Status/Monitoring

- after each numbered backup, run ``mgdeps6.pl bckXXXX > DEPSXXXX``
- this will create DEPSXXXX file with one (topology section) theorem per line with info as follows ``topology_elem_subset: lines:12, admit:NO, recadmit:NO, deps(2):[topology_on:D,topology_sub_Power:T].``
- this says if the theorem is (recursively) admitted and its direct topology deps (D for a def and T for theorem). (note that the lines info there is after deleting all comments and that all non-topology deps are ignored)
- Use this to monitor the progress and find admitted theorems (major bottlenecks) that have to be proved so that others are recursively admit-free.

### ## Compilation Checking

- *\*\*Run megalodon frequently\*\** to check for compilation errors
- *\*\*When megalodon produces no output, the code is correct\*\**
- *\*\*Megalodon only prints output when there are errors or warnings\*\**
- Silent output = success
- After any significant change, run: ``timeout 30 megalodon -owned ownedSep132023 Math_Background.mg``
- Often do numbered backups like bck1107
- With each numbered backup, also write the numbered summary changes file like CHANGES1107 (it should really be a summary, not just a simple diff).
- You can lookup your previous work in these CHANGES files when unsure how to continue.
- Never overwrite an older backup file. The numbering has to continue from the latest number. You must find it by running in `tst7: ls bck* | sed 's/[^0-9]*//g'`

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```
| sort -n | tail -n 1.

## Why This Rule Exists
- Work before the topology section may have external dependencies
- Topology section is self-contained and safe to modify
- Breaking this rule wastes time undoing changes

## Last Updated
2025-12-02: Initial creation after accidentally editing lines 1728-1761
```

### 3.1 Prompting Automation

Before December 22, I was “babysitting” the command-line interface, i.e., looking every now and then if the agent has finished its work and waits for another prompt. This was manageable, because my early \$20/month LLM subscriptions would quickly run out of credits anyway and most of the time would be spent waiting for the weekly credit limit to be reset.

With the \$200/month ChatGPT Pro subscription the limits on usage got much higher and automating the prompting became necessary. I have implemented it by writing the following script that watches the tmux session and feeds it the prompt followed by “Enter” whenever there has not been any change in the session for 60 seconds:

```
#!/usr/bin/env bash

OUTDIR="/home/urban/ec/claude-code/tmux"
INTERVAL=60
mkdir -p "$OUTDIR"

while true; do
  ts=$(date +%s)
  tmux capture-pane -p -S -1 > "$OUTDIR/capture_{$ts}.txt"
  # get two most recent captures
  mapfile -t files < <(ls -1t "$OUTDIR"/capture_*.txt 2>/dev/null | head -n 2)
  if [[ ${#files[@]} -eq 2 ]]; then
    if diff -q "${files[0]}" "${files[1]}" >/dev/null; then
      tmux send-keys 'Read the file CLAUDE.md . Treat it as authoritative work instructions.
Follow those instructions exactly for all subsequent actions and responses. That means work
as long as possible (as specified) without stopping.'; sleep 2; tmux send-keys Enter
    fi
  fi
  sleep "$INTERVAL"
done
```

## 4 Other Prompts

### 4.1 Initial

Initially, I have done quite a lot of experiments with the prompting, and also had some special prompts for special tasks like the initial creation of all definitions and theorems/lemmas corresponding to the textbook. Here are the first five prompts from November asking Codex to do topology:

1. Now that you have learned the basics we will do something more serious. First read the file topology.tex . Then start formalizing it in Megalodon. The

background from which should learn the syntax of the definition, theorem and proofs and build on it is in the `tst7/Math_Background.mg` . Here is how you verify the article written so far: `megalodon -owned ownedSep132023 Math_Background.mg` . Always make numbered backups of the `.mg` files as before. Do 5 attempts and then stop.

2. Good, please continue. 5 more attempts.
3. Good. But maybe don't focus on the proofs so much and instead push forward to cover all the definitions and toplevel theorems/lemmas from the `tex` file. 5 more attempts.
4. Good. But maybe don't focus on the proofs so much and instead push forward to cover all the definitions and toplevel theorems/lemmas from the `tex` file. 5 more attempts. Try to do more without waiting for me, especially when things compile. Feel free to do the whole `tex` file at once.
5. Good. Now let's stop for a moment with the formalization. Instead, put into comments before each current definition and theorem the name, number, page number, location and/or statement from the original `tex` files, so that the reader can easily check that the from/informal items correspond to each other. Note that the comment syntax is as follows: `(** from Part 1 **)` .

A major initial effort was to make the coding agent work as independently as possible and as long as possible, without them coming back, asking additional questions, or being derailed in various ways. Ultimately, I started to get independent runs of over 1 hour, and with the Pro subscription sometimes even 2 hours, before giving it the next prompt (which was very often just the same prompt over and over, until I automated it fully as explained above).

## 4.2 Focused Prompting

On December 31, the system has already done quite some work on the Urysohn lemma but then started to switch to other things. Since this looked like an interesting end-of-year challenge, I issued the following special prompt:

Special rule for today: Return to the proof of Urysohn lemma and finish it. Don't switch to other things before it is fully finished without any admits, axioms, etc. Finishing it is your singular goal for now.

This has backfired a bit because the system then started to try all sorts of normally forbidden tricks. I told it to keep following the `CLAUDE.md` rules but focus on finishing Urysohn. The 3k-line proof indeed got finished by the end of 2025.

I made a similar focused attempt on January 4, after the system made a significant (but likely redundant – see below) initial effort on the Tietze extension theorem, but then started to switch to other things. Since this was interesting I tried to focus it as follows (this time I was wiser and also told it to follow the rules - which worked well):

Read the file `CLAUDE.md` . Treat it as authoritative work instructions. Follow those instructions exactly for all subsequent actions and responses. That means work as long as possible (as specified) without stopping. Special rule for today: Return to the proof of `Tietze_stepII_real_extension_nonempty` and finish it. Don't switch to other things before it is fully finished.



This indeed led to the massive effort that finished the 10k-line proof (and all its pre-requisites) after about 20 hours. I believe this now provides quite a usable way of combining the system's automated work on a large (textbook) formalization with the user occasionally impatiently overriding it when he wants a particular proof done first.

## **5 Further Utilities**

### **5.1 Hammer (aby) attempts**

Megalodon has since 2024 a simple higher-order hammer “aby” [1]. In particular, it can export the problems to the TH0 TPTP format and call e.g. higher-order Vampire. There is so far no proof reconstruction (as e.g. in Naproche). Initially it was tempting to save the LLM the work on detailed Megalodon proofs and instead employ the hammer as much as possible. I have written the scripts that automate that (to some extent) but so far this has not been very useful and likely requires further work on caching and (at least simple) premise selection, possibly also some of the well-known translations to FOF (first-order). A major issue that should be addressed (in the absence of proof reconstruction) is smart re-running of the ATPs when something changes.

### **5.2 Theorem and Dependency Tracking**

Relatively late in the development I wanted the LLM to have a better overview of the theorem dependencies and the biggest “choke points”. It looked like for example the completeness of reals was a major cause of recursive admits, even though it should be quite feasible based on Brown's surreal numbers development (which turned out to be true - see `R_lub_exists` below). So I have added a simple dependency tracking script that at every iteration gets called on the current file and produces for each theorem a one-line info with its current status (fully proved, admitted, recursively admitted), length of the (partial) proof, and the current proof dependencies. It seems that this has helped the LLM to orient itself better (see also the advice about this in `CLAUDE.md`).

## **6 Enforcing the order of work**

This is a relatively uncharted territory. People who formalize math have developed good routines, but I am not aware of some “handbook of good formalization practices”.

Some of the problems that appeared were:

1. The LLMs were initially (occasionally) lazy in formalizing the theorem and definition statements. Sometimes they created stubs, ad-hoc or special cases instead of general cases, etc. This is dangerous if the LLM later forgets about it (its context window is finite) and takes such simplified specs seriously. I have countered it by telling it to be very cautious about such things, and once or twice even doing a special prompt to assess/remove the inaccuracies.
2. Initially it had a tendency to go for easy lemmas (often exercises/examples), rather than the hard/long theorems. Also, it could endlessly jump around searching for something easy. Again see `CLAUDE.md` for my attempts to prevent that (without being too strict).
3. I also wanted it to be continuously aware of all sections, rather than getting stuck in just one section for long time. Because this could later lead to major refactoring. I have tried to convince it to keep a `PROGRESS` (status) file (see `CLAUDE.md`) but this



attempt has so far only mixed results. The trade-offs between going deep and proving hard theorems and being aware of the full formalization picture are obviously nontrivial also in large-scale human formalization projects.

7 Formalization Growth

Table 1 shows the number of lines after each 100 of backups (corresponding to commits). One can see the accelerated growth after switching to ChatGPT Pro.

Commit number → number of lines							
Commit	Lines	Commit	Lines	Commit	Lines	Commit	Lines
100	1364	1000	7565	1900	44529	2800	78777
200	2033	1100	9919	2000	47542	2900	83890
300	3012	1200	11737	2100	52695	3000	90028
400	3523	1300	12620	2200	56411	3100	96056
500	3875	1400	13687	2300	56210	3200	105042
600	4447	1500	19891	2400	62531	3300	116433
700	4500	1600	31275	2500	68110	3400	119139
800	4561	1700	31401	2600	72784	3500	128287
900	4850	1800	39832	2700	68936	3600	137231
						3700	148283

Table 1 Growth of the formalization: commit number versus number of lines

8 Resources Used

While the Pro subscription is indeed only \$200/month, I have tried to use independent tools to measure the USD cost of calling the LLM. In particular, I call the `ccusage` tool every hour. Table 2 shows the reported costs for three days of running, including a compactification related restart. Note that these numbers are much higher than the \$100 spent over the two weeks. One wonders what kind of business policies the LLM companies are implementing (vicious fight for the market/subscribers?, overcharging companies for the API usage?, trying to get as much data as possible?, bait-and-switch?, etc).

1–12	13–24	25–36	37–48	49–60	61–72
1918	2096	2229	2367	2549	602
1932	2112	2234	2378	2565	618
1949	2127	2239	2394	2576	635
1964	2145	2243	2409	2576	651
1980	2163	2254	2427	2576	664
1993	2177	2268	2442	2576	678
2008	2192	2282	2459	2576	694
2024	2206	2296	2475	2576	707
2040	2210	2312	2488	475	
2053	2215	2326	2503	489	
2067	2220	2339	2518	506	
2080	2224	2354	2534	519	

Table 2 Sequential hourly cost reports (hours 1–72), arranged in 6 columns of 12



## Running into Usage Limits

In the semi-automated early mode, I managed to deplete the usage credits for the \$20/month Codex or Claude Code subscription quite quickly. Still, on these very cheap subscription the first 30k lines got produced in about a month. This may be already very useful for many people. I have been using the \$200/month Pro subscription for two weeks now and ran into the usage limits every week. In the first week, the credits lasted for almost the whole week, except for about half a day. In the second week, the weekly credits got depleted earlier (on January 5, 3 days before the usage reset), about 8 hours after finishing the “battle of the Tietze Hill” described below. There are probably multiple factors influencing this.

## 9 Miscellaneous Comments

In general, I currently do not have the time and energy to do a full-scale analysis of what went well and what went wrong. LLMs and other tools could be used in the future to find automatically interesting cases from the complete logs (which I am keeping) and the publicly available git commits.<sup>11</sup> Perhaps this could lead to a feedback loop augmenting the currently used prompts and tools based on such analysis.

### 9.1 Context Compactification

This is a major issue with the LLM-based technology I use. Every now and then, the coding agent needs to get rid of unessential parts in its (limited) context window and keep only the parts essential for further processing. The results of the compactification (which is likely as proprietary as the LLMs I use) can be quite unpredictable.

Even worse, there seems to be a hard limit on how much compactification can be done for a continuous session. I have reached this hard limit already twice: once (December 26) when the ChatGPT session file reached about 252MB, and another time (January 2) when the session file reached 351MB. Once this hard limit is reached, it seems the session can no longer be continued in any way. The standard advice I found seems to be “do not run long sessions”. Instead, people should maintain the essential status and progress themselves when doing longer projects like these, always starting a new session with such updated status/progress.

Since this looked like a bigger project and I liked how the long-running session behaved (not wanting to lose all it learned), I decided to ignore this advice. Instead of resetting completely, I reset to an earlier version of the session. In particular, when I reach the hard limit, I just rewrite the (big) ChatGPT session (jsonl) file with a (much) younger (and smaller) version of it. In particular, for both resets I used an early 65MB-big version of the session file from December 18. This is likely an unsupported and dangerous trick, but it has so far worked quite well for me.

### 9.2 Running Megalodon

As the file grows, running Megalodon gets slower and possibly problematic. While it took under 1s on the initial library file, the latest verification on my notebook takes 32s. However the file has now almost 170k lines and 8MB:

```
mptp@ar-2:~/ec/mizwrk/tst7$ time /home/mptp/ec/repos-mws/mgw_test/bin/megalodon bck4003
real 0m32.251s
```

---

<sup>11</sup> [https://github.com/mgwiki/mgw\\_test/commits/main/](https://github.com/mgwiki/mgw_test/commits/main/)

```
user 0m32.095s
sys 0m0.154s
mptp@ar-2:~/ec/mizwrk/tst7$ wc bck4003
169122 1241687 7807911 bck4003
```

An obvious solution is to start to split the file or to start admitting (the `@proof` pragma in Mizar) the finished theorems. So far, I haven't done that because: (i) I prefer the proofs to be possibly refactored by the LLMs, (ii) running Megalodon doesn't seem to be the bottleneck compared to the LLM usage and available credits. I.e., the number of calls to the LLM is still quite high and even with the Pro subscription I run out of the LLM credits before the usage limit resets.

10 Major Theorems Proved

Table 3 shows the proved theorems with a proof length of over 300 lines (no admits or recursive admits) sorted according to the length of their direct Megalodon proof. I realize that this may omit some major results which are stated as easier corollaries (see e.g. the next section), however it still has some informative value. The theorem names in the table are clickable and linked to their HTML presentation in mgwiki.

■ Table 3 Topology theorems (mgwiki links), with line counts and dependency counts.

Theorem	Lines	Deps
<a href="#">Tietze_stepII_real_extension_nonempty</a>	10369	142
<a href="#">Urysohn_lemma</a>	2964	85
<a href="#">Urysohn_metrization_theorem</a>	2174	112
<a href="#">one_point_compactification_exists</a>	1719	38
<a href="#">closed_interval_affine_equiv_minus1_1</a>	1638	48
<a href="#">order_topology_basis_is_basis</a>	1164	7
<a href="#">order_rel_trans</a>	1159	22
<a href="#">unit_interval_connected</a>	1049	32
<a href="#">uniform_topology_finer_than_product_and_coarser_than_box</a>	940	74
<a href="#">convex_subspace_order_topology</a>	931	26
<a href="#">ordered_square_not_subspace_dictionary</a>	922	42
<a href="#">countability_axioms_subspace_product</a>	911	51
<a href="#">product_topology_full_regular_axiom</a>	907	56
<a href="#">rectangular_regions_basis_plane</a>	894	12
<a href="#">interval_connected</a>	878	23
<a href="#">completely_regular_product_topology</a>	876	56
<a href="#">bounded_transform_psi_preimage_open_ray_upper</a>	812	26
<a href="#">Romega_D_metric_open_ball_in_product_topology</a>	805	62
<a href="#">ball_inside_rectangle</a>	797	20
<a href="#">ex13_8b_halfopen_rational_basis_topology</a>	768	18
<a href="#">Sorgenfrey_line_Lindelof</a>	756	52
<a href="#">ex16_9_dictionary_equals_product</a>	736	55
<a href="#">pointwise_limit_continuity_points_dense</a>	729	58
<a href="#">ex13_8a_rational_intervals_basis_standard</a>	710	16
<a href="#">ex16_10_compare_topologies_on_square</a>	693	40
<a href="#">continuous_combine_or_01</a>	677	43



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Theorem	Lines	Deps
<a href="#">metrizable_spaces_normal</a>	674	37
<a href="#">two_by_nat_singleton_not_in_basis</a>	664	8
<a href="#">double_minus_one_map_continuous</a>	629	43
<a href="#">embedding_via_functions</a>	625	52
<a href="#">compact_space_net_has_accumulation_point</a>	615	23
<a href="#">regular_countable_basis_normal</a>	600	28
<a href="#">compact_space_net_has_accumulation_point_on</a>	590	23
<a href="#">R_standard_basis_is_basis_local</a>	585	9
<a href="#">euclidean_spaces_second_countable</a>	581	48
<a href="#">preimage_mul_fun_R_open_ray_upper_in_product_topology</a>	579	42
<a href="#">Tietze_extension_open_interval</a>	576	48
<a href="#">Romega_extend_map_continuous_in_Romega_product</a>	574	47
<a href="#">product_countable_basis_at_point_if_components_first_countable</a>	574	34
<a href="#">abs_Cauchy_sequence_converges_R_standard_topology</a>	563	30
<a href="#">order_rel_trichotomy_or_impred</a>	560	13
<a href="#">preimage_mul_fun_R_open_ray_lower_in_product_topology</a>	551	41
<a href="#">R_upper_limit_basis_is_basis_local</a>	534	10
<a href="#">Q_sqrt2_cut_not_interval_or_ray</a>	533	31
<a href="#">abs_Cauchy_sequence_converges_R_standard_topology_early</a>	531	30
<a href="#">Sorgenfrey_plane_not_normal</a>	525	33
<a href="#">singleton_rectangle_in_dictionary</a>	514	45
<a href="#">subnet_converges_to_accumulation</a>	495	32
<a href="#">subnet_converges_to_accumulation_witnessed</a>	487	32
<a href="#">open_ball_refine_intersection</a>	481	17
<a href="#">continuous_construction_rules</a>	475	16
<a href="#">ex17_5_basis_elem_meets_interval</a>	474	10
<a href="#">flip_unit_interval_continuous</a>	473	36
<a href="#">uniform_cauchy_metric_complete_imp_uniform_limit_stub</a>	467	30
<a href="#">subnet_converges_to_accumulation_witnessed_on</a>	465	32
<a href="#">A_N_eps_closed_stub</a>	463	32
<a href="#">paracompact_Hausdorff_normal</a>	460	28
<a href="#">Tietze_extension_minus1_1</a>	457	48
<a href="#">product_sequence_convergence_iff_coordinates</a>	456	42
<a href="#">R_lower_limit_basis_is_basis_local</a>	454	9
<a href="#">unbounded_sequences_in_Romega_box_topology</a>	437	27
<a href="#">preimage_add_fun_R_open_ray_upper_in_product_topology</a>	435	32
<a href="#">ex30_4_compact_metrizable_second_countable</a>	425	48
<a href="#">regular_normal_via_closure</a>	421	14
<a href="#">limit_point_compact_not_necessarily_compact</a>	421	25
<a href="#">separation_subspace_limit_points</a>	419	9
<a href="#">paracompact_Hausdorff_regular</a>	416	25
<a href="#">one_minus_fun_continuous</a>	413	32
<a href="#">ex13_6_R1_RK_not_comparable</a>	411	24
<a href="#">double_map_continuous</a>	411	41
<a href="#">distance_R2_triangle_sqr_Le</a>	409	14
<a href="#">Cauchy_with_convergent_subsequence_converges</a>	404	41
<a href="#">finite_product_compact</a>	402	21

Theorem	Lines	Deps
Romega_product_cylinder_in_D_metric_topology	399	37
R_standard_plus_K_basis_is_basis_local	397	9
continuity_via_nets	383	33
Sorgenfrey_plane_L_closed	381	34
Heine_Borel_closed_bounded	380	26
closed_subspace_paracompact	378	16
R_lub_exists	376	13
continuity_via_sequences_metric	374	21
ex17_17_closure_A_C_Supq	370	19
Romega_singleton_map_continuous_prod	367	31
ex30_14_product_Lindelof_compact	362	28
product_finite_subbasis_intersection_separator	357	43
uniform_limit_of_continuous_to_metric_is_continuous	352	35
ex30_5a_metrizable_countable_dense_second_countable	347	39
ex23_connected_open_sets_path_connected	347	27
completely_regular_subspace_product	346	49
rectangle_inside_ball	345	25
preimage_add_fun_R_open_ray_lower_in_product_topology	341	32
path_component_transitive_axiom	335	41
compact_subspace_via_ambient_covers	335	8
locally_m_euclidean_implies_T1	330	25
ex30_5b_metrizable_Lindelof_second_countable	328	34
Romega_D_metric_induces_product_topology	327	46
product_basis_generates	322	10
U_eps_open_dense_stub	320	38
RK_not_regular_axiom	320	23
Sigma_countable	318	2
order_topology_interval_refine	313	5
topology_chain_four_sets	310	4
pasting_lemma	306	18
ex13_7_R_topology_containments	304	23
Sorgenfrey_plane_diag_rational_closed	303	33
finite_product_connected	303	17
compact_Hausdorff_normal	301	17

Table 4 shows an assessment of 30 major theorems/lemmas selected by ChatGPT. It may be incorrect in various ways. The exact statements of these (and some more) theorems/lemmas in LaTeX and Megalodon are shown in the Appendix.

■ **Table 4** ChatGPT selection and assessment of 30 major results

Section & No. (topology.tex)	Theorem / Lemma name	Formal item	ChatGPT Assessment of formalization
§15, Thm. 15.1	Basis for product topology	product_basis_generates_product_topology	Clean abstraction of basis arguments; faithful to textbook proof, good reuse of basis lemmas.

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Section & No.	Theorem / Lemma	Formal item (:T)	Assessment
§16, Thm. 16.3	Subspace of a product	product_subspace_topology	Correct handling of induced topology; explicit but robust.
§18, Thm. 18.3	Pasting Lemma	pasting_lemma	Excellent stress test for continuity infrastructure; hypotheses well-scoped.
§18, Thm. 18.4	Maps into products	maps_into_products	Very standard statement, formalization mirrors textbook closely.
§23, Lem. 23.1	Connectedness via separations	connected_space_no_separation	Correct equivalence formulation; slightly low-level set reasoning but sound.
§23, Lem. 23.2	Connected subset and separations	connected_subset_in_separation_side	Faithful encoding; reusable lemma for later proofs.
§23, Thm. 23.3	Union of connected sets	union_connected_common_point	Good abstraction, proof structure matches standard argument.
§23, Thm. 23.5	Continuous image of connected space	continuous_image_connected	Canonical theorem; formal statement is clean and general.
§23, Thm. 23.6	Finite products of connected spaces	finite_product_connected	Correct induction on finite products; abstraction level appropriate.
§26, Thm. 26.2	Closed subspace of compact space	closed_subspace_compact	Very standard; good foundation for later compactness results.
§26, Thm. 26.3	Compact subsets of Hausdorff spaces	compact_subspace_in_Hausdorff_closed	Faithful separation argument; predicates well chosen.
§26, Thm. 26.5	Continuous image of compact space	continuous_image_compact	Essential lemma; well-factored and reusable.
§26, Thm. 26.6	Compact–Hausdorff bijection	compact_to_Hausdorff_bijection_homeomorphism	Excellent formulation; topologically canonical.
§26, Thm. 26.7	Finite products of compact spaces	finite_product_compact	Nontrivial induction; proof infrastructure appears solid.
§26, Lem. 26.8	Tube Lemma	tube_lemma	Good handling of product neighborhoods; faithful to Munkres.
§27, Thm. 27.3	Heine–Borel Theorem	Heine_Borel_closed_bounded	Major result; hypotheses and equivalence encoded correctly.
§27, Lem. 27.5	Lebesgue Number Lemma	Lebesgue_number_lemma	Good cover machinery; slightly verbose but robust.
§27, Thm. 27.6	Uniform continuity on compact spaces	uniform_continuity_on_compact	Correct metric-topology interaction; good abstraction.
§30, Thm. 30.3	Countable basis $\Rightarrow$ Lindelöf	countable_basis_implies_Lindelof	Nicely structured countability argument.
§30, Thm. 30.4	Countable basis $\Rightarrow$ separable	countable_basis_implies_separable	Faithful, concise; good reuse of basis machinery.
§32, Thm. 32.1	Regular + countable basis $\Rightarrow$ normal	regular_countable_basis_normal	Important separation result; dependencies are appropriate.
§32, Thm. 32.3	Compact Hausdorff spaces are normal	compact_Hausdorff_normal	Standard argument; formalization is clean.

Section & No.	Theorem / Lemma	Formal item (:T)	Assessment
§33, Thm. 33.1	Urysohn Lemma	Urysohn_lemma	Excellent: correct hypotheses, non-trivial construction handled well.
§34, Thm. 34.1	Urysohn Metrization Theorem	Urysohn_metrization_theorem	High-value result; formalization appears faithful and well-structured.
§35, Thm. 35.1	Tietze Extension Theorem	Tietze_extension_interval	Very strong result; formal statement and dependencies look sound.
§41, Thm. 41.1	Paracompact Hausdorff $\Rightarrow$ normal	paracompact_Hausdorff_normal	Good abstraction of locally finite covers.
§41, Thm. 41.2	Closed subspaces of paracompact spaces	closed_subspace_paracompact	Correct and reusable; well integrated.
§41, Lem. 41.3	Michael Lemma	Michael_lemma_41_3	Technically demanding; solid formal handling.
§41, Lem. 41.6	Shrinking Lemma	shrinking_lemma_41_6	Good control of refinements; abstraction is adequate.
§48, Lem. 48.1	Baire space lemma	Baire_space_dense_Gdelta	Correct use of countable intersections; solid foundation for category arguments.

## 11 The Battle of the Tietze Hill

A large part of the last two days has been spent on the formalization of the Tietze extension theorem, specifically its Step II. Its (autoformalized) Megalodon statement is as follows:

```
(** helper for §35 Step II: nonempty closed subset case, real-valued extension **)
(** LATEX VERSION: Step II (nonempty A): construct a real-valued continuous
    extension gR:X->R agreeing with f on A and bounded in [-1,1]. **)
Theorem Tietze_stepII_real_extension_nonempty : forall X Tx A f:set,
  normal_space X Tx ->
  closed_in X Tx A ->
  A <> Empty ->
  continuous_map A (subspace_topology X Tx A)
    (closed_interval (minus_SNo 1) 1) (closed_interval_topology (minus_SNo 1) 1) f ->
  exists gR:set,
    continuous_map X Tx R R_standard_topology gR /\
    (forall x:set, x :e A -> apply_fun gR x = apply_fun f x) /\
    (forall x:set, x :e X -> apply_fun gR x :e closed_interval (minus_SNo 1) 1).
```

The proof was finished on January 4 at 20:15, with the following (pretty dry given the effort) progress message about backup nr 3893:

```
Update 3893: §35 Step II: eliminated the temporary admit in Htail_budget by adding
the missing `HgNOxS` (for the current x0) and closing the final SNoLe_tra transitivity;
Tietze_stepII_real_extension_nonempty now has no admits.
```

The proof<sup>12</sup> takes 10369 lines and has 142 direct dependencies (36 definitions and 106 theorems).

<sup>12</sup>[https://mgwiki.github.io/mgw\\_test/topology.mg.html#Tietze\\_stepII\\_real\\_extension\\_nonempty](https://mgwiki.github.io/mgw_test/topology.mg.html#Tietze_stepII_real_extension_nonempty), also line 129871 of [https://raw.githubusercontent.com/mgwiki/mgw\\_test/48bca1fc5df4576f1a28416aa4e423ecfa556f6c/mglib/topology.mg](https://raw.githubusercontent.com/mgwiki/mgw_test/48bca1fc5df4576f1a28416aa4e423ecfa556f6c/mglib/topology.mg)



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The textbook proof uses facts about infinite sums of real functions and their uniform convergence, which is a major external dependency not present in the textbook. The agent seems to have been stumped by this and instead kept unrolling the inductive (infinite) construction 12 times (12 terms of the series), producing about 6k lines that are not useful for the final proof (even though they may be useful for figuring out the machinery more concretely before the induction is done).

It has however recovered from this and decided to gradually formalize the necessary prerequisites. The ChatGPT analysis of the unfinished 9k-line proof attempt is available online<sup>13</sup>. Below is its periodic report, showing how the fact that complete metric spaces are dealt with much later in the textbook confused the LLM.

CHANGES3682:

Attempted refactor to move the uniform-limit/uniform-Cauchy infrastructure (continuous\_at\_map, pointwise\_limit\_metric, uniform\_limit\_metric, uniform\_cauchy\_metric, and the uniform-limit continuity lemmas) earlier near §35 to support Tietze Step II.

Result:

- This does not work because that block depends on `complete\_metric\_space`, which is defined later (in the metric completeness section), so moving it before §35 causes a forward-reference error: "Unknown term complete\_metric\_space".

Action taken:

- Reverted Math\_Background.mg back to bck3681 state (no content loss; only the failed move was undone).

In more detail, the problem is that complete metric spaces and Cauchy sequences in them are indeed only defined in section 43 of Munkres, while Tietze is in section 35. The LLM has tried hard to follow the formalization rules. As a result, it has done 11 useless iterations and correction terms (from u2 in CHANGES3650 up to u12 in CHANGES3685), blowing up the proof from 865 lines to 5408 lines in these 35 changes. This took almost 4 hours of work and wasted quite some resources. After some futile attempts, the LLM has reasoned itself into doing the hard work on the prerequisites and moving them before Tietze as follows:

CHANGES3687:

Move uniform-limit infrastructure before §35 (compiles)  
=====

Goal:

- §35 (Tietze extension theorem, Step II) needs uniform-limit / completeness tools without forward-reference blockers.

What changed:

- Moved the block starting at "helper: continuity at a point into a topological space" (continuous\_at\_map, pointwise\_limit\_metric, uniform\_limit\_metric, uniform\_cauchy\_metric, uniform-limit continuity lemmas, plus related helpers) from its later location to the area just before §35.  
- Kept §48-specific helper material (A\_N\_eps, U\_eps, U\_eps\_open\_dense\_stub, etc.) located near §48 again, so it stays after the Baire\_space definitions.

---

<sup>13</sup> <https://chatgpt.com/share/695a7928-9efc-8002-860f-bb2c0f33663b>

- Added early stubs needed by the moved uniform-limit block:
  - `metric_topology_R_bounded_metric_eq_R_standard_topology_early`
  - `R_bounded_metric_is_metric_on_early`
 These avoid forward references to the later bounded-metric development; they should later be replaced by the proved theorems.
- Left a note in the original location indicating the helper block was moved earlier.

Note that this qualifies as a large gap, external dependency or even an (unsafe) forward reference in the textbook – something that can be a serious hurdle for human formalizers. The ChatGPT analysis of the final 10k-line proof and its difference to the previous unfinished 9k-line proof is available online<sup>14</sup>. Once this hard battle is finished, the standard statement of Tietze from Munkres which is as follows:

**Theorem 35.1 (Tietze extension theorem).** Let  $X$  be a normal space; let  $A$  be a closed subspace of  $X$ .

- (a) Any continuous map of  $A$  into the closed interval  $[a, b]$  of  $\mathbb{R}$  may be extended to a continuous map of all of  $X$  into  $[a, b]$ .
- (b) Any continuous map of  $A$  into  $\mathbb{R}$  may be extended to a continuous map of all of  $X$  into  $\mathbb{R}$ .

is (relatively easily) proved via these two Megalodon theorems:

```
Theorem Tietze_extension_real_bounded_interval : forall X Tx A a b f:set,
  Rle a b ->
  normal_space X Tx -> closed_in X Tx A ->
  continuous_map A (subspace_topology X Tx A) R R_standard_topology f ->
  (forall x:set, x :e A -> apply_fun f x :e closed_interval a b) ->
  exists g:set, continuous_map X Tx R R_standard_topology g /\
    (forall x:set, x :e A -> apply_fun g x = apply_fun f x).
```

```
Theorem Tietze_extension_real : forall X Tx A f:set,
  normal_space X Tx -> closed_in X Tx A ->
  continuous_map A (subspace_topology X Tx A) R R_standard_topology f ->
  exists g:set, continuous_map X Tx R R_standard_topology g /\
    (forall x:set, x :e A -> apply_fun g x = apply_fun f x).
```

## 12 Conclusion

This is obviously quite surprising and exciting and I wonder where this will all go. The price tag here (and my coding effort) was really low. It is quite possible that in 2026 we will get most of (reasonably written) math textbooks and papers autoformalized. Or perhaps not - there may be limits this will eventually run into. Perhaps a (sorely needed?) reality check for some (insert a popular proof assistant name here) fundamentalists is that the current experiments were done in an almost unknown proof assistant based on higher-order set theory, with very little exposure to the existing LLMs. The future of AI-assisted (auto)formalization seems to be quite open and democratic.

<sup>14</sup><https://chatgpt.com/share/695a7928-9efc-8002-860f-bb2c0f33663b>

## 13 Acknowledgments

Chad Brown has largely developed Megalodon, its hammer, and the initial core set-theoretical library, helped me to set it up for the experiment, and provided encouraging comments on the early system's formalizations. Zar Goertzel has kept me informed about his experiments with LLMs and the related coding agents like Claude Code and Codex. I took his advice and fully switched to ChatGPT 5.2 (and the \$200/month Pro subscription) after he reported encouraging results. He has already finished a 13k-line autoformalization of the Ramsey(3,6) theorem. Atle Hahn and Adrian De Lon have been doing interesting experiments with LLM-based autoformalization into Naproche. Atle's impressive (and working) prompts demonstrate the power and potential of "programming" LLMs to do autoformalization. Thanks to John Harrison for supporting our early autoformalization efforts and also to Larry Paulson and Dominic Mulligan for encouragement. Cezary Kaliszyk has helped with the Megalodon/hammer development and related infrastructure, and spearheaded the first learning-based autoformalization ideas, projects and approaches with me since 2014. The autoformalization community has grown a lot since then - thanks to all these (and especially the early) believers.

---

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**A** Listing of the Major Theorems in LaTeX and Megalodon

No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 15.1	Th 15.1. If $B$ is a basis for the topology of $X$ and $C$ is a basis for the topology of $Y$ , then the collection $\mathcal{B} \times \mathcal{C} = \{B \times C \mid B \in \mathcal{B} \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$ .	product_basis_generates_product_topology	forall X Y Bx By:set, basis_on X Bx -> basis_on Y By -> product_topology X (generated_topology X Bx) Y (generated_topology Y By) = generated_topology (setprod X Y) (product_basis_from X Y Bx By)
Th 16.3	Th 16.3. If $A$ is a subspace of $X$ and $B$ is a subspace of $Y$ , then the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$ .	product_subspace_topology	forall X Tx Y Ty A B:set, topology_on X Tx -> topology_on Y Ty -> product_topology A (subspace_topology X Tx A) B (subspace_topology Y Ty B) = subspace_topology (setprod X Y) (product_topology X Tx Y Ty) (setprod A B)

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No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 18.3	Th 18.3 (Pasting lemma). Suppose that $X = A \cup B$ , where $A$ and $B$ are closed in $X$ . Let $f : X \rightarrow Y$ be a map such that $f _A$ and $f _B$ are continuous. If $f _{A \cap B}$ is continuous, then $f$ is continuous.	pasting_lemma	forall X Tx Y Ty A B f:set, topology_on X Tx -> topology_on Y Ty -> closed_in X Tx A -> closed_in X Tx B -> A /\: B = X -> continuous_map A (subspace_topology X Tx A) Y Ty (restriction_map A f) -> continuous_map B (subspace_topology X Tx B) Y Ty (restriction_map B f) -> (forall x:set, x :e (A /\: B) -> apply_fun f x = apply_fun f x) -> continuous_map X Tx Y Ty f
Th 18.4	Th 18.4. Let $f : X \rightarrow Y \times Z$ be given by the equation $f(x) = (f_1(x), f_2(x))$ . Then $f$ is continuous if and only if $f_1$ and $f_2$ are continuous.	maps_into_products	forall A Ta X Tx Y Ty h:set, topology_on A Ta -> topology_on X Tx -> topology_on Y Ty -> (continuous_map A Ta (setprod X Y) (product_topology X Tx Y Ty) h <-> (continuous_map A Ta X Tx (projection_map1 X Y o h) /\ continuous_map A Ta Y Ty (projection_map2 X Y o h)))
Lem 23.2	Lem 23.2. If $C$ is a connected subspace of $X$ that intersects both sets $A$ and $B$ of a separation $X = A \cup B$ , then $C$ intersects $A \cap B$ .	connected_subset_in_separation_side	forall X Tx C D A:set, topology_on X Tx -> connected_space C (subspace_topology X Tx C) -> separation_of X Tx A D -> C /\: A <> Empty -> C /\: D <> Empty -> C /\: A /\: D <> Empty
Th 23.3	Th 23.3. The union of a collection of connected subspaces of $X$ that have a point in common is connected.	union_connected_common_point	forall X Tx F x0:set, topology_on X Tx -> (forall C:set, C :e F -> connected_space C (subspace_topology X Tx C)) -> (forall C:set, C :e F -> x0 :e C) -> connected_space (Union F) (subspace_topology X Tx (Union F))
Th 23.5	Th 23.5. The image of a connected space under a continuous map is connected.	continuous_image_connected	forall X Tx Y Ty f:set, topology_on X Tx -> topology_on Y Ty -> continuous_map X Tx Y Ty f -> connected_space X Tx -> connected_space (image_of X f) (subspace_topology Y Ty (image_of X f))
Th 23.6	Th 23.6. The product of finitely many connected spaces is connected.	finite_product_connected	forall X Tx Y Ty:set, connected_space X Tx -> connected_space Y Ty -> connected_space (setprod X Y) (product_topology X Tx Y Ty)
Th 26.2	Th 26.2. Every closed subspace of a compact space is compact.	closed_subspace_compact	forall X Tx Y:set, compact_space X Tx -> closed_in X Tx Y -> compact_space Y (subspace_topology X Tx Y)
Th 26.3	Th 26.3. Every compact subspace of a Hausdorff space is closed.	compact_subspace_in_Hausdorff_closed	forall X Tx Y:set, Hausdorff_space X Tx -> compact_space Y (subspace_topology X Tx Y) -> closed_in X Tx Y

No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 26.5	Th 26.5. The image of a compact space under a continuous map is compact.	continuous_image_compact	forall X Tx Y Ty f:set, compact_space X Tx -> topology_on Y Ty -> continuous_map X Tx Y Ty f -> compact_space (image_of X f) (subspace_topology Y Ty (image_of X f))
Th 26.6	Th 26.6. Let $f : X \rightarrow Y$ be a bijective continuous function. If $X$ is compact and $Y$ is Hausdorff, then $f$ is a homeomorphism.	compact_to_Hausdorff_bijection_homeomorphism	forall X Tx Y Ty f:set, compact_space X Tx -> Hausdorff_space Y Ty -> bijection X Y f -> continuous_map X Tx Y Ty f -> homeomorphism X Tx Y Ty f
Th 26.7	Th 26.7. The product of finitely many compact spaces is compact.	finite_product_compact	forall X Tx Y Ty:set, compact_space X Tx -> compact_space Y Ty -> compact_space (setprod X Y) (product_topology X Tx Y Ty)
Lem 26.8	Lem 26.8 (The tube lemma). Consider the product space $X \times Y$ , where $Y$ is compact. If $N$ is an open set of $X \times Y$ containing the slice $\{x_0\} \times Y$ , then there is a neighborhood $W$ of $x_0$ in $X$ such that $W \times Y \subset N$ .	tube_lemma	forall X Tx Y Ty:set, topology_on X Tx -> topology_on Y Ty -> compact_space Y Ty -> forall x0 N:set, x0 :e X -> N :e product_topology X Tx Y Ty -> (setprod {x0} Y) c= N -> exists W:set, W :e Tx /\ x0 :e W /\ (setprod W Y) c= N
Th 27.3	Th 27.3 (Heine–Borel theorem). A subspace $A$ of $\mathbb{R}$ is compact if and only if it is closed and bounded in $\mathbb{R}$ .	Heine_Borel_closed_bounded	forall A:set, A c= R -> compact_space A (subspace_topology R R_standard_topology A) <-> (closed_in R R_standard_topology A /\ bounded_set A)
Th 28.1	Th 28.1. Compactness implies limit point compactness, but not conversely.	compact_implies_limit_point_compact	forall X Tx:set, compact_space X Tx -> limit_point_compact X Tx
Th 30.3	Th 30.3. Suppose that $X$ has a countable basis. Then every open covering of $X$ contains a countable subcollection that covers $X$ .	countable_basis_implies_Lindelof	forall X Tx:set, second_countable_space X Tx -> Lindelof_space X Tx
Th 30.4	Th 30.4. If $X$ has a countable basis, then $X$ contains a countable dense subset.	countable_basis_implies_separable	forall X Tx:set, second_countable_space X Tx -> separable_space X Tx
Th 32.1	Th 32.1. If $X$ is regular and has a countable basis, then $X$ is normal.	regular_countable_basis_normal	forall X Tx:set, regular_space X Tx -> second_countable_space X Tx -> normal_space X Tx
Th 32.3	Th 32.3. Every compact Hausdorff space is normal.	compact_Hausdorff_normal	forall X Tx:set, compact_space X Tx -> Hausdorff_space X Tx -> normal_space X Tx

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No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 33.1	Th 33.1 (Urysohn lemma). Let $X$ be a normal space; let $A$ and $B$ be disjoint closed subsets of $X$ . Then there exists a continuous function $f : X \rightarrow [0, 1]$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$ .	Urysohn_lemma	forall X Tx A B:set, normal_space X Tx -> closed_in X Tx A -> closed_in X Tx B -> A /\ B = Empty -> exists f:set, continuous_map X Tx unit_interval unit_interval_topology f /\ (forall a:set, a :e A -> apply_fun f a = 0) /\ (forall b:set, b :e B -> apply_fun f b = 1)
Th 34.1	Th 34.1 (Urysohn metrization theorem). Every regular space with a countable basis is metrizable.	Urysohn_metrization_theorem	forall X Tx:set, regular_space X Tx -> second_countable_space X Tx -> metrizable X Tx
Th 35.1	Th 35.1 (Tietze extension theorem). Let $X$ be a normal space and $A$ a closed subspace of $X$ . Then every continuous map $f : A \rightarrow [-1, 1]$ extends to a continuous map $F : X \rightarrow [-1, 1]$ .	Tietze_extension_interval	forall X Tx A f:set, normal_space X Tx -> closed_in X Tx A -> continuous_map A (subspace_topology X Tx A) closed_interval_minus1_1_topology f -> exists F:set, continuous_map X Tx closed_interval_minus1_1_topology F /\ forall a:set, a :e A -> apply_fun F a = apply_fun f a
Th 41.1	Th 41.1. Every paracompact Hausdorff space is normal.	paracompact_Hausdorff_normal	forall X Tx:set, paracompact_space X Tx -> Hausdorff_space X Tx -> normal_space X Tx
Th 41.2	Th 41.2. Every closed subspace of a paracompact space is paracompact.	closed_subspace_paracompact	forall X Tx Y:set, paracompact_space X Tx -> closed_in X Tx Y -> paracompact_space Y (subspace_topology X Tx Y)
Lem 41.3	Lem 41.3 (Michael lemma). Let $X$ be regular. Then every open cover of $X$ has an open locally finite refinement if and only if it has an open $\sigma$ -locally finite refinement.	Michael_lemma_41_3	forall X Tx:set, regular_space X Tx -> ( (forall Ufam:set, open_cover X Tx Ufam -> exists Vfam:set, open_refinement X Tx Ufam Vfam /\ locally_finite_family X Tx Vfam) <=> (forall Ufam:set, open_cover X Tx Ufam -> exists Vfam:set, open_refinement X Tx Ufam Vfam /\ sigma_locally_finite_family X Tx Vfam) )
Th 41.5	Th 41.5. Every regular Lindelöf space is paracompact.	regular_Lindelof_paracompact	forall X Tx:set, regular_space X Tx -> Lindelof_space X Tx -> paracompact_space X Tx
Th 46.10	Th 46.10. Let $X$ be locally compact Hausdorff. Then the evaluation map $e : X \times C_k(X, Y) \rightarrow Y$ is continuous.	evaluation_map_continuous	forall X Tx Y Ty:set, locally_compact_space X Tx -> Hausdorff_space X Tx -> topology_on Y Ty -> continuous_map (setprod X (Ck X Tx Y Ty)) (product_topology X Tx (Ck X Tx Y Ty)) (compact_open_topology X Tx Y Ty) Y Ty (evaluation_map X Y)



No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 22.2	Th 22.2. Let $p : X \rightarrow Y$ be a quotient map. If $A \subset X$ is saturated, then the restriction $p _A : A \rightarrow p(A)$ is a quotient map.	restriction_quotient_saturated	forall X Tx Y Ty p A:set, quotient_map X Tx Y Ty p -> saturated_set X p A -> quotient_map A (subspace_topology X Tx A) (image_of A p) (subspace_topology Y Ty (image_of A p)) (restriction_map A p)
Th 17.11	Th 17.11. Every simply ordered set is Hausdorff in the order topology.	simply_ordered_set_Hausdorff	forall X Lt:set, simply_ordered_set X Lt -> Hausdorff_space X (order_topology X Lt)
Lem 23.1	Lem 23.1. If $X$ admits a separation, then $X$ is not connected.	connected_space_no_separation	forall X Tx:set, topology_on X Tx -> (exists A B:set, separation_of X Tx A B) -> ~ connected_space X Tx
Th 23.4	Th 23.4. Let $A$ be a connected subspace of $X$ . If $A \subset B \subset \overline{A}$ , then $B$ is connected.	connected_with_limit_points	forall X Tx A B:set, topology_on X Tx -> connected_space A (subspace_topology X Tx A) -> A c= B -> B c= closure_of X Tx A -> connected_space B (subspace_topology X Tx B)
Th 24.3	Th 24.3 (Intermediate value theorem). Let $f : X \rightarrow Y$ be continuous, where $X$ is connected and ordered, and $Y$ has the order topology. Then $f(X)$ is an interval.	intermediate_value_theorem	forall X Tx Y f a b r:set, simply_ordered_set X -> topology_on X Tx -> topology_on Y (order_topology Y) -> continuous_map X Tx Y (order_topology Y) f -> between (apply_fun f a) r (apply_fun f b) -> exists c:set, c :e X /\ apply_fun f c = r
Th 25.1	Th 25.1. The components of a space $X$ form a partition of $X$ into connected subspaces.	components_partition_space	forall X Tx:set, topology_on X Tx -> partition_of X (components_of X Tx)
Th 25.5	Th 25.5. If $X$ is locally connected, then the components of open sets of $X$ are open.	components_are_open_in_locally_connected	forall X Tx:set, locally_connected_space X Tx -> forall C:set, C :e components_of X Tx -> open_in X Tx C
Lem 26.1	Lem 26.1. A subspace $Y$ of $X$ is compact iff every open cover of $Y$ by sets open in $X$ has a finite subcover.	compact_space_subcover_property	forall X Tx Y:set, topology_on X Tx -> Y c= X -> (compact_space Y (subspace_topology X Tx Y) <-> (forall Cfam:set, (forall U:set, U :e Cfam -> U :e Tx) -> Y c= Union Cfam -> exists F:set, F c= Cfam /\ finite_set F /\ Y c= Union F))
Lem 26.4	Lem 26.4. If $Y$ is a compact subspace of Hausdorff $X$ and $x_0 \notin Y$ , then $x_0$ and $Y$ have disjoint neighborhoods.	Hausdorff_separate_point_compact_set	forall X Tx Y x0:set, Hausdorff_space X Tx -> compact_space Y (subspace_topology X Tx Y) -> x0 :e X -> x0 :e Y -> exists U V:set, U :e Tx /\ V :e Tx /\ x0 :e U /\ Y c= V /\ U :/\ V = Empty
Lem 27.5	Lem 27.5 (Lebesgue number lemma). Every open cover of a compact metric space has a Lebesgue number.	Lebesgue_number_lemma	forall X d:set, metric_space X d -> compact_space X (metric_topology X d) -> forall Cfam:set, open_cover X (metric_topology X d) Cfam -> exists e:set, Lebesgue_number X d Cfam e

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No.	Statement in topology.tex	Formal name	Formal statement (stats, exact)
Th 27.6	Th 27.6. A continuous function from a compact space to a metric space is uniformly continuous.	uniform_continuity_on_compact	forall X Tx Y d f:set, compact_space X Tx -> metric_space Y d -> continuous_map X Tx Y (metric_topology Y d) f -> uniformly_continuous X Y d f
Th 29.1	Th 29.1. A locally compact Hausdorff space has a one-point compactification.	one_point_compactification_exists	forall X Tx:set, locally_compact_Hausdorff X Tx -> exists Y Ty p:set, one_point_compactification_of X Tx Y Ty p
Th 30.2	Th 30.2. Subspaces and countable products of first-countable spaces are first-countable.	countability_axioms_subspace_product	forall X Tx:set, first_countable_space X Tx -> (forall Y:set, Y c= X -> first_countable_space Y (subspace_topology X Tx Y)) /\ (forall Fam:set, (forall i:set, i :e index_set Fam -> first_countable_space (Fam[i]) (space_family_topology Fam[i])) -> first_countable_space (product_space Fam) (product_topology_full Fam))
Lem 31.1	Lem 31.1. In a $T_1$ space, singletons are closed.	T1_space_one_point_sets_closed	forall X Tx:set, T1_space X Tx -> forall x:set, x :e X -> closed_in X Tx {x}
Th 48.1	Th 48.1. A space $X$ is a Baire space iff every countable intersection of dense open sets is dense.	Baire_space_dense_Gdelta	forall X Tx:set, Baire_space X Tx <-> (forall Ufam:set, countable_set Ufam -> (forall U:set, U :e Ufam -> open_in X Tx U /\ dense_in X Tx U) -> dense_in X Tx (Inter Ufam))