CS 135 Written Assignment 3

Daniel Detore

February 17, 2024

1

$\neg r$	Hypothesis	(1)
$q \implies r$	Hypothesis	(2)
$\neg q$	Modus tollens, 1,2	(3)
$\neg q \implies u \wedge s$	Hypothesis	(4)
$u \wedge s$	Modus ponens, 3,4	(5)
s	Simplification, 5	(6)
$p \vee q$	Hypothesis	(7)
p	Disjunctive syllogism, 3, 7	(8)
$p \wedge s$	Conjunction, 6, 8	(9)
$p \wedge s \implies t$	Hypothesis	(10)
t	Modus ponens, 9,10	(11)

2

For simplicity's sake, I will use this key to represent the given propositions:

- p: The dorm is locked.
- q: The phone is on top of the tall bookshelf.
- r: The dorm room is locked.
- s: The phone is under the pillow.
- t: The dorm has more than 10 floors.
- u: The phone is in the bottom drawer of the desk.

This gives us the following list of statements:

$p \implies \neg q$

$r \implies q$

p

$r \vee s$

$t \implies u$

Using this list, we can deduce the following:

$$
\begin{array}{llr}
p & \text{Hypothesis} & (1) \\
p \implies \neg q & \text{Hypothesis} & (2) \\
\neg q & \text{Modus ponens 1, 2} & (3) \\
r \implies q & \text{Hypothesis} & (4) \\
\neg r & \text{Modus tollens 3, 4} & (5) \\
r \vee s & \text{Hypothesis} & (6) \\
s & \text{Disjunctive syllogism 5, 6} & (7)
\end{array}
$$

We have now confirmed that s must be true. If we check the key, this means that the phone is under the pillow.

3

Let us start at the contradiction form $x = \neg x$ and derive the given formula from it. I will use E to replace x, and introduce new variables in reverse alphabetical order.

$$
\begin{array}{lr}
E \vee \neg E & \text{Original proposition} \\
\equiv (D \vee E) \wedge (\neg D \vee E) \wedge \neg E & \text{Resolution} \\
\equiv (B \vee E) \wedge (\neg B \vee D) \wedge (\neg D \vee E) \wedge \neg E & \text{Resolution} \\
\equiv (C \vee B) \wedge (\neg C \vee E) \wedge (\neg B \vee D) \wedge (\neg D \vee E) \wedge \neg E & \text{Resolution} \\
\equiv (A \wedge B) \wedge (\neg A \vee C) \wedge (\neg C \vee E) \wedge (\neg B \vee D) \wedge (\neg D \vee E) \wedge \neg E & \text{Resolution} \\
\equiv (A \wedge B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee E) \wedge (\neg D \vee E) \wedge \neg E & \text{Commutative law}
\end{array}
$$

4

To make this argument form invalid, its premises need to be true and its conclusion needs to be false. For simplicity's sake I will make $P(x)$ and $Q(x)$'s domains $\{T, F\}$ and they will return the value I give them.

$$
\begin{array}{l}
\forall x(P(x) \implies Q(x)) \\
\neg P(a) \\
\therefore \neg Q(a)
\end{array}
$$

Because $\forall x(P(x) \implies Q(x))$ places the same x value into both $P(x)$ and $Q(x)$, we can simplify it to $x \implies x$. This statement is a tautology. Using the same logic, $P(a) \implies Q(a)$ (the statement implied by argument form) will be equivalent to $a \implies a$ which is, again, a tautology. Since I have proven all of the statements to be tautologies, this argument form is always true.

5

6

This argument is invalid. If we look past the logic at what argument is being made, the conclusion is always false.