Written Assignment 4

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1

1.a

32 is not a square square-free number. It is divisible by 4, which is 2^2.

1.b

This is an invalid way to use the superset operation. John is trying to compare an integer with a set, which is not possible. He should have written:

$$P(\{17, 19, 22, 26, 30\}, \{30\})$$

2

The given set U is the set of all perfect square numbers.

2.a

Yes. One such set is $B = \{0, 1, 4, 9, 16, 25, 36, 49\}$. $|B| = 8$, so $|B| > 7$.

2.b

Yes. One such set is $B = \{64\}$. In this case, $B \subseteq U$ and $10 \leq 64 \leq 100$.

3

3.a

- Area 1 = $\overline{A} \cap \overline{B} \cap C$
- Area 2 = $\overline{A} \cap B \cap \overline{C}$
- Area 3 = $A \cap \overline{B} \cap \overline{C}$
- Area 4 = $\overline{A} \cap \overline{B} \cap \overline{C}$
- Area 5 = $\overline{A} \cap B \cap C$
- Area 6 = $A \cap \overline{B} \cap C$
- Area 7 = $A \cap B \cap \overline{C}$
- Area 8 = $A \cap B \cap C$

3.b

There would be 16 areas. The amount of combinations of A, B, C, and D would be the length of the power set $P(\{A, B, C, D\})$. The power set of a set with 4 elements has $2^4 = 16$ elements.

3.c

The simple way is to find all combinations of $A \cap B \cap C \cap D$ possible with 0-4 of the sets replaced with its compliment.

4

4.a

$$S(\{\sqrt{2}, \sqrt{3}, \sqrt{5}\}) = \{\sqrt{2}, \sqrt{3}, \sqrt{5}, \{\sqrt{2}, \sqrt{3}, \sqrt{5}\}\}$$

4.b

$$\begin{aligned} S(\emptyset) &= \emptyset \cup \{\emptyset\} \\ &= \{\emptyset\} \end{aligned}$$

$$\begin{aligned} S(S(\emptyset)) &= \{\emptyset\} \cup \{\{\emptyset\}\} \\ &= \{\emptyset, \{\emptyset\}\} \end{aligned}$$

$$\begin{aligned} S(S(S(\emptyset))) &= \{\emptyset, \{\emptyset\}\} \cup \{\{\emptyset, \{\emptyset\}\}\} \\ &= \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \\ &= \{\{\}, \{\{\}\}, \{\{\}, \{\{\}\}\}\} \end{aligned}$$

4.c

$|S(P)| = |P| + 1$. For instance, if $P = \{5, 6, 7\}$, then $S(P) = \{5, 6, 7, \{5, 6, 7\}\}$.

5

This formula is an erroneous extension of the inclusion-exclusion principle as it is applied to two sets. When including a third set, you need to include elements shared by all three and then remove elements that may only be common to two sets of the three, to avoid double-counting these elements. As such, to find the cardinality of the union of sets A, B, and C, you would need to use this longer formula:

$$|A \cup B \cup C| = |A| + |B| + |C| + |A \cap B \cap C| - |A \cap B| - |B \cap C| - |A \cap C|$$

6

$(A \setminus B) \setminus C$	Original statement
$(A \cap \overline{B}) \setminus C$	Set Difference Law
$(A \cap \overline{B}) \cap \overline{C}$	Set Difference Identity
$A \cap (\overline{B} \cap \overline{C})$	Associativity
$A \setminus \overline{(\overline{B} \cap \overline{C})}$	Set Difference Identity
$A \setminus (\overline{\overline{B}} \cup \overline{\overline{C}})$	De Morgan's Law
$A \setminus (B \cup \overline{\overline{C}})$	Complementation Law
$A \setminus (B \cup C)$	Complementation Law