Written Assignment 3

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1

$\neg r$	Hypothesis	(1)
$q \implies r$	Hypothesis	(2)
$\neg q$	Modus tollens, 1,2	(3)
$\neg q \implies u \wedge s$	Hypothesis	(4)
$u \wedge s$	Modus ponens, 3,4	(5)
s	Simplification, 5	(6)
$p \vee q$	Hypothesis	(7)
p	Disjunctive syllogism, 3, 7	(8)
$p \wedge s$	Conjunction, 6, 8	(9)
$p \wedge s \implies t$	Hypothesis	(10)
t	Modus ponens, 9,10	(11)

2

For simplicity's sake, I will use this key to represent the given propositions:

- p: The dorm is locked.
- q: The phone is on top of the tall bookshelf.
- r: The dorm room is odd-numbered.
- s: The phone is under the pillow.
- t: The dorm has more than 10 floors.
- u: The phone is in the bottom drawer of the desk.

This gives us the following list of statements:

$p \implies \neg q$

$r \implies q$

p

$r \vee s$

$t \implies u$

Using this list, we can deduce the following:

$$
\begin{aligned}
&p && \text{Hypothesis} && (1)\\
&p \implies \neg q && \text{Hypothesis} && (2)\\
&\neg q && \text{Modus ponens, 1, 2} && (3)\\
&r \implies q && \text{Hypothesis} && (4)\\
&\neg r && \text{Modus tollens, 3, 4} && (5)\\
&r \vee s && \text{Hypothesis} && (6)\\
&s && \text{Disjunctive syllogism, 5, 6} && (7)
\end{aligned}
$$

We have now confirmed that s must be true. If we check the key, this means that the phone is under the pillow.

3

Let us start at the contradiction form $x = \neg x$ and derive the given formula from it. I will use E to replace x, and introduce new variables in reverse alphabetical order.

$$
\begin{aligned}
&E \vee \neg E && \text{Original proposition}\\
&\equiv (D \vee E) \wedge (\neg D \vee E) \wedge \neg E && \text{Resolution}\\
&\equiv (B \vee E) \wedge (\neg B \vee D) \wedge (\neg D \vee E) \wedge \neg E && \text{Resolution}\\
&\equiv (C \vee B) \wedge (\neg C \vee E) \wedge (\neg B \vee D) \wedge (\neg D \vee E) \wedge \neg E && \text{Resolution}\\
&\equiv (A \wedge B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee E) \wedge (\neg D \vee E) \wedge \neg E && \text{Resolution}
\end{aligned}
$$

4

To make this argument form invalid, its premises need to be true and its conclusion needs to be false. For simplicity's sake I will make $P(x)$ and $Q(x)$'s domains $\{T, F\}$ and they will return the value I give them.

$$
\begin{aligned}
&\forall x(P(x) \implies Q(x))\\
&\neg P(a)\\
\therefore &\neg Q(a)
\end{aligned}
$$

Because $\forall x(P(x) \implies Q(x))$ places the same x value into both $P(x)$ and $Q(x)$, we can simplify it to $x \implies x$. This statement is a tautology. Using the same logic, $(T \wedge P(a)) \implies Q(a)$ (the statement implied by argument form) will be equivalent to $a \implies a$ which is, again, a tautology. With this I have proven this argument form to be a tautology and, as such, valid.

5

5.a

Although the **argument** put forward is true, this is not a valid argument **form**.
As an example, I will replace the predicate "The sum of alternating digits of x is a multiple of 11" with "I ate an apple for lunch." Then I will replace the predicate "x is a multiple of 11" with "I ate lunch today." Here is what this substitution yields:

If I ate an apple for lunch today, then I ate lunch today.
I did not eat an apple for lunch today.
∴ I did not eat lunch today.

This form is invalid because the premises are true:

- It is true that if I ate an apple for lunch today, then I ate lunch today. I would have eaten the apple for lunch.
- I did not eat an apple for lunch today.

And the conclusion is false:

- I had pancakes for lunch, therefore I *did* have lunch.

This new version of the argument demonstrates that this form, which is denying the antecedent rather than the consequent, is not valid.

5.b

This is another invalid argument form.
Here is a reinterpretation of the argument with different predicates:

$$\begin{aligned} & \text{Every player on the Yankees plays baseball.} \\ & \text{I play baseball.} \\ \therefore\ & \text{I am on the Yankees.} \end{aligned}$$

This form is invalid because the premises are true:

- Every player on the Yankees plays baseball (no matter how well).
- I do play baseball.

And the conclusion is false:

- I am not on the Yankees.

This new version of the argument demonstrates that this form, which is affirming the consequent rather than the antecedent, is not valid.

6

This argument is invalid. It seems to be a confused interpretation of the Resolution rule of inference.
Let us define some placeholder predicate names.

$$\begin{aligned} T(x)&: x \text{ can run 10 km in} < 30 \text{ minutes.} \\ S(x)&: x \text{ is a smoker.} \\ H(x)&: x \text{ can run 100 m in} < 11 \text{ seconds.} \end{aligned}$$

The domain of all three predicates consists of all athletes. Here is the argument rewritten with these placeholders:

$$\begin{aligned} & \neg\exists x(T(x) \land S(x)) \\ & \neg\exists x(S(x) \land H(x)) \\ \therefore & \neg\exists x(T(x) \land H(x)) \end{aligned}$$

Let us find out what happens when we fill in a specific athlete Josh. Josh is not a smoker and can run at 0.75 kilometers per minute. Here is Josh's running speed graphed:

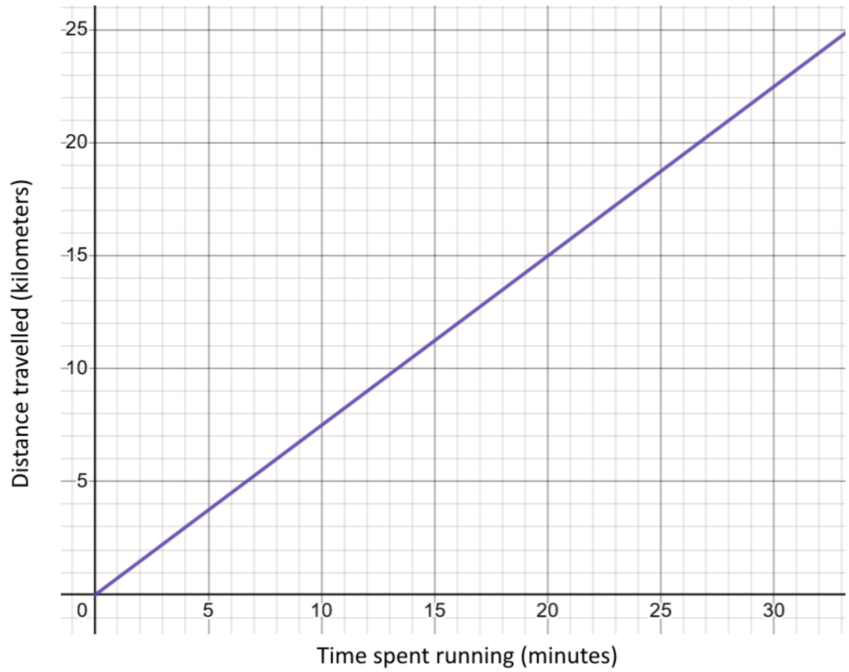

Figure 1: Distance vs. time graph of Josh's running speed.

Now we can compare Josh to Tina, who runs 10 kilometers in 30 minutes, and Hunter who runs 100 meters in 11 seconds.

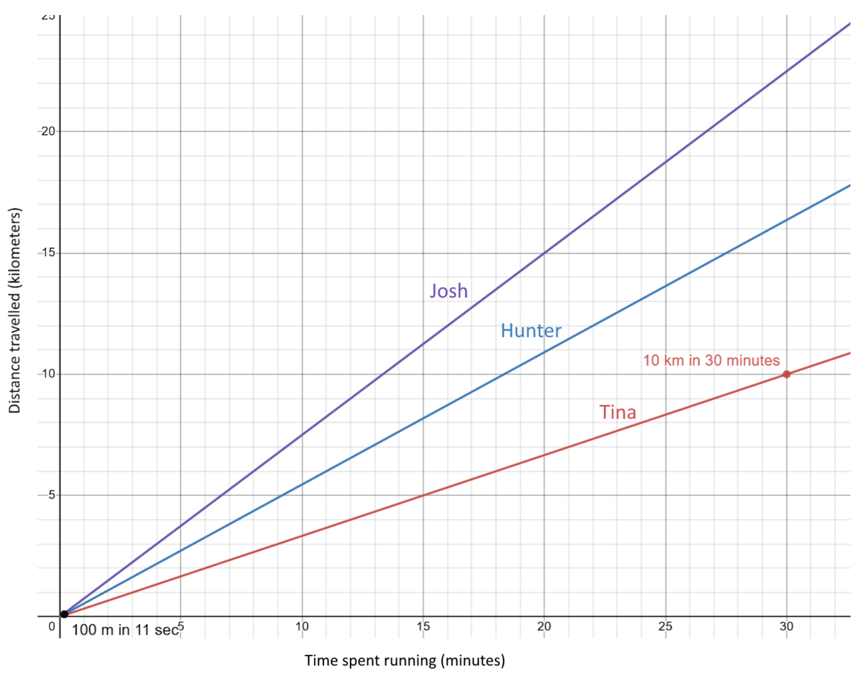

Figure 2: Distance vs. time graph of Tina, Hunter, and Josh's running speeds.

We can see from this diagram that anybody, like Josh, who can run as fast or faster than Hunter can both run 10 km in < 30 minutes and run 100 m in < 11 seconds, regardless of whether they smoke or not. Let us return to the argument form I provided earlier, and place Josh into it:

$$\begin{aligned}
&\neg(T(Josh) \land S(Josh)) \\
&\neg(S(Josh) \land H(Josh)) \\
\therefore &\neg(T(Josh) \land H(Josh))
\end{aligned}$$

And this argument's truth values simplify as such:

$$\begin{aligned}
&\neg(T \land F) \\
&\neg(T \land F) \\
\therefore &\neg(T \land T)
\end{aligned}$$

which becomes

$$\begin{aligned}
&T \\
&T \\
\therefore &F
\end{aligned}$$

which proves this argument form invalid.