- This assignment should be submitted via Canvas. Late assignments will not be accepted.

- You should attempt to solve the problems on your own. You are permitted to study with friends and have high-level discussions on the problems; however, *you must write up your own solutions, in your own words.*

- If you do collaborate with any of the other students on any problem, you must list all the collaborators in your submission for each problem.

- Finding solutions to homework problems on the web, or by asking students not enrolled in the class (or the class staff) is strictly prohibited.

- Please explain your answers. There will be penalties for unexplained solutions when the answers are not obvious.

- All answers can be typed or handwritten, and must be readable.

- From the syllabus: penalties will be applied for illegibility (a problem that will not occur for typed submissions). We reserve the right to reject papers that are unacceptably messy. The students are responsible for checking that the uploaded submissions are complete.

- I encourage you to learn how to typeset documents with LATEX . You can download Texmaker at http://www.xm1math.net/texmaker/. It is available for most platforms. Another option is to use online service https://www.overleaf.com/. There is a wealth of information online on how to format documents with LATEX and you can always post a question on Moodle's Technical Forum. Most scholarly articles in Mathematics and Computer Science, and even many books, are typeset with this tool. There is, however, an initial learning curve one needs to climb before getting comfortable with LATEX – hence this great tool is not required for this course.

1. Let $A = \{a, b, c\}$ and $B = \{1, 2\}$.

   (a) List all the onto functions $f : A \to B$.
   (b) List all the functions $f : A \to B$ that are not onto.
   (c) How many 1-1 functions $f : A \to B$ are there? Explain.

When listing a function, please list its pairs alphabetically by the argument value, e.g.,

$$f = \{(a, 2), (b, 1), (c, 1)\}$$

2. Let X be the set of all bit strings, i.e., the set of all strings consisting of only 0s and 1s. For example, "010111", "0", and "1111110" are elements of X. Consider the function $p : X \to X$ defined by

$$P(s) = 1s \quad \text{for each string } s \in X$$

(for two strings a, b, the symbol ab denotes concatenation of a and b)

(a) Is P one-to-one? Prove or give a counterexample.

(b) Is P onto? Prove or give a counterexample.

3. Is the below statement guaranteed to be true? If yes, provide a proof. If not, provide a counterexample.

$$\left\lfloor \frac{\lfloor \frac{x}{3} \rfloor}{2} \right\rfloor = \left\lfloor \frac{x}{6} \right\rfloor$$

Hint. Observe that every real number x can be represented in the form $x = 6n + \epsilon$ where $0 \le \epsilon < 6$. You may also find helpful that $\lfloor x + m \rfloor = \lfloor x \rfloor + m$ for all $x \in \mathbb{R}$ and $m \in \mathbb{Z}$.

4. Consider the function $T : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by

$$T(k, n) = k + \frac{1}{2}(k + n)(k + n + 1)$$

(a) Explain why $T(k, n)$ always generates integer values even though a fraction appears in its formula.

(b) Show that $T(k, n)$ is a one-to-one function.

5. Show that the set $\mathbb{Z} \times \mathbb{Z}$ is countable by providing a complete description of a one-to-one and onto mapping $\mathbb{Z}^+ \to \mathbb{Z} \times \mathbb{Z}$

6. Suppose that Hilbert's Grand Hotel is fully occupied with one guest per room, but due to a water main break all guests in rooms whose numbers are multiples of 3 are asked to leave (i.e., guests in rooms 3, 6, 9, 12 ...). Because many rooms were affected repairs are expected to take a very long time. Show that one can reassign new rooms to the unaffected guests in such a way so that there is space for all guests who were evacuated. You can assume that Mr. Hilbert, the owner of the hotel, is not concerned with profits given the inconvenience caused to his customers.