Written Assignment 5

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CS135-B/LF

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1

1.a

$$f = \{(a,1),(b,1),(c,2)\}$$
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All of these sets of pairs are functions and their domains contain all elements of B, therefore they are all onto functions $f : A \to B$.

1.b

$$f = \{(a,1),(b,1),(c,1)\}$$
$$f = \{(a,2),(b,2),(c,2)\}$$

All of these sets of pairs are functions but their domains do not contain all elements of B, therefore none of them are onto functions $f : A \to B$.

1.c

There are no one-to-one functions $f : A \to B$ because $|A| > |B|$. As such it is impossible to map each element of A uniquely onto an element of B.

2

2.a

We can prove that P is one-to-one by proving that, if two inputs provide the same output, than the inputs must be equal.
Let us define two strings $a, b \in X$. To prove that P is one-to-one, we can find that if $P(a) = P(b)$ then $a = b$. If $P(a) = P(b)$, then $1a = 1b$. Since these strings are equal, removing both of their first character will still result in equal strings. This leaves us with $a = b$. This proves that in all cases where $P(a) = P(b)$ then it must be true that $a = b$. As such, P is a one-to-one function from X to X.

2.b

A simple counterexample is 0. Any bit string that starts with 0 is also a counterexample.

- P's domain and target are both X, the set of all bit strings, as per its definition.
- X contains strings that begin with both 1 and 0.
- P guarantees that the first element of any string in its range will be 1.
- Any element in X that starts with 0 is outside of the range of P.
- P cannot be onto because there are elements in its target that are not in its domain.

3

We need to prove:

$$\left\lfloor \frac{\left\lfloor \frac{x}{3} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{x}{6} \right\rfloor \tag{1}$$

Let us start by making a placeholder for the left side of the equation. Let

$$\left\lfloor \frac{\left\lfloor \frac{x}{3} \right\rfloor}{2} \right\rfloor = \alpha. \tag{2}$$

Table 1, on page 159 of our textbook *Discrete Mathematics and Its Applications*, offers the following biconditional:

$$\lfloor x \rfloor = n \text{ if and only if } n \leq x < n+1 \tag{Table 1.(1a)}$$

We can apply (Table 1.(1a)) to (2) to acquire the following:

$$\alpha \leq \frac{\left\lfloor \frac{x}{3} \right\rfloor}{2} < \alpha + 1$$

We can use algebra and multiply each part of this compound inequality by 2 to remove that denominator:

$$2\alpha \leq \left\lfloor \frac{x}{3} \right\rfloor < 2\alpha + 2$$

Table 1 also offers the following compound inequality:

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1 \tag{Table 1.(2)}$$

We can shave this down to the relevant inequality:

$$\lfloor x \rfloor \leq x \tag{3}$$

Using (3), we can reasonably assume the following:

$$2\alpha \leq \lfloor \frac{x}{3} \rfloor \implies 2\alpha \leq \frac{x}{3} \tag{4}$$

$$\lfloor \frac{x}{3} \rfloor < 2\alpha + 2 \implies \frac{x}{3} < 2\alpha + 2 \tag{5}$$

Using the right sides of (4) and (5), we get the following compound inequality:

$$2\alpha \leq \frac{x}{3} < 2\alpha + 2$$

Dividing all parts by 2 gives us the following:

$$\alpha \leq \frac{x}{6} < \alpha + 1 \tag{6}$$

We can now apply (Table 1.(1a)) to (6):

$$\left\lfloor \frac{x}{6} \right\rfloor = \alpha$$

And we can fill in the value of α:

$$\left\lfloor \frac{x}{6} \right\rfloor = \left\lfloor \frac{\left\lfloor \frac{x}{3} \right\rfloor}{2} \right\rfloor$$

If we switch the sides, we reach (1), which was to be proven:

$$\left\lfloor \frac{\left\lfloor \frac{x}{3} \right\rfloor}{2} \right\rfloor = \left\lfloor \frac{x}{6} \right\rfloor \tag{1}$$