Written Assignment 9

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CS 135-B/LF

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1

Modular arithmetic in this section is done by my code from Labs 10 and 11.

```
(define (mult-inv a b)
    (let ((x (cadr (pulverize a b))))
        (if (> x 0)
            x
            (+ x b))))
(define (mod-exp b e m)
    (cond
        ((= e 0) 1)
        ((= 0 (modulo e 2)) (modulo (expt (mod-exp b (/ e 2) m) 2) m))
        (else (modulo (* b (mod-exp b (- e 1) m)) m))))
```

1.a

"MOVE" $\rightarrow$ 1214, 2104
$1214^{19} \bmod 7387 = 2097$
$2104^{19} \bmod 7387 = 4767$
Alice will send Bob 2097 and 4767.

1.b

Given $n = pq = 83 \cdot 89$, let $\phi = (p-1)(q-1) = 82 \cdot 88 = 7216$.
Let $d = 1899$ i.e. the multiplicative inverse of $e \bmod \phi$. d is Bob's private key. Given ciphertexts 2097 and 4767, we raise both to d and find that value mod n.
$2097^{1899} \bmod 1214 = 1214$
$4767^{1899} \bmod 2104 = 2104$
1214, 2104 $\rightarrow$ "MOVE"

2

We have $e = 23$ and $N = 3233$. To find d, we need to find ϕ because d is the multiplicative inverse of $e \bmod \phi$. If we can find the factors p, q of N, we know $\phi = (p-1)(q-1)$. Since N is small, we can brute force p and q.

3

No. According to page S-61 of the textbook, a complete graph $G = (V, E)$ with $|V| = n$ vertices has $|E| = n(n-1)/2$ edges. To make $|E|$ prime, one of n or $n-1$ must equal 2 and the other must be prime as the 2 is divided out and results in a prime $|E|$; otherwise $|E|$ will have 2 as a factor and be composite. Since we have the bound $n > 3$, the smallest $n = 4$ and $4 \neq 2$ and $4 - 1 = 3 \neq 2$. Therefore there is no $n > 3$ than can satisfy the conditions to make $|E|$ prime. ■

4

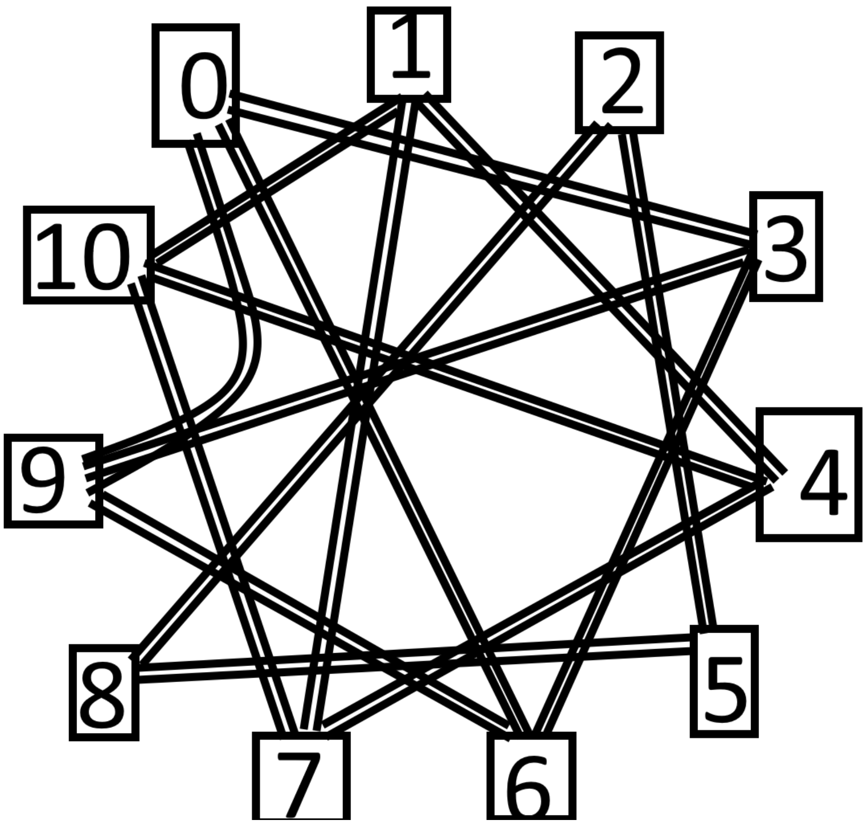

Figure 1: Undirected graph G.

$V = (0, 3), (0, 6), (0, 9), (1, 4), (1, 7), (1, 10), (2, 5), (2, 8), (3, 0), (3, 6), (3, 9), (4, 1), (4, 7), (4, 10), (5, 2), (10, 4), (10, 7)$

5

5.a

For the Handshake Theorem to work, there must be an even number of vertices with an odd degree.

5.b

As long as you are starting and ending at a vertex with an odd degree, this path is possible. Otherwise it is not guaranteed as there might be no way to pass through a vertex.

6

6.a

Any integer between between 0 and n.

6.b

Any integer greater than n.