CS 135 Discrete Structures Spring 2024

WRITTEN HOMEWORK ASSIGNMENT No. 3

Due Saturday 02/17/2024 11:59 pm

Please adhere to the following rules:

- This assignment should be submitted via Canvas. Late assignments will not be accepted.
- You should attempt to solve the problems on your own. You are permitted to study with friends but you can have only high-level discussions about the problems. *You must write up your own solutions, in your own words.*
- Using large language models such as ChatGPT to find solutions, copying solutions from the Internet or asking students not enrolled in the class (or the class staff) is strictly prohibited.
- Please explain your answers. There will be penalties for unexplained solutions when the answers are not obvious.
- All submissions should be neat, preferably submitted either as Word or PDF files. We reserve the right to reject handwritten submissions if they are unacceptably messy or too difficult to read.
- I encourage you to learn how to typeset documents with LATEX . You can download Texmaker at http://www.xm1math.net/texmaker/. It is available for most platforms. Another option is to use online service https://www.overleaf.com/. There is a wealth of information online on how to format documents with LATEX and you can always post a question on Moodle's Technical Forum. Most scholarly articles in Mathematics and Computer Science, and even many books, are typeset with this tool. There is, however, an initial learning curve one needs to climb before getting comfortable with LATEX – hence this great tool is not required for this course.

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1. Use laws of inference for Propositional Logic to deduce the conclusion from the premises. Give a reason for each step.

$$\begin{array}{l} p \lor q \\ q \to r \\ p \land s \to t \\ \neg r \\ \neg q \to u \land s \\ \therefore t \end{array}$$

2. One day you forgot to take your phone, and your roommate (knowing that you're taking CS 135) sent you an email challenging you to find the phone by solving the following puzzle. Apparently, he hid the phone somewhere in your dorm room.

   (a) If the dorm room is locked, then the phone is not on top of the tall bookshelf.
   (b) If the dorm room is odd-numbered, then the phone is on top of the tall bookshelf.
   (c) The dorm room is locked.
   (d) The dorm room is odd numbered or the phone is under the pillow.
   (e) If the dorm has more than 10 floors, then the phone is in the bottom drawer of the desk.

   Present your argument as a sequence of statements in which every statement has been either assumed in the problem or is deduced from the previous statements by applying rules of inference. Specify the rules you used in each deduction (for your convenience these rules are been provided at the end of this assignment)

3. One way to show that a given formula ϕ is not satisfiable is to show that if ϕ were true, then using laws of inference one could derive a contradiction of the form $x \land \neg x$ from ϕ. Using only the resolution rule show that the following formula is not satisfiable by deriving from it a contradiction of the above type.

$$(A \lor B) \land (\neg A \lor C) \land (\neg B \lor D) \land (\neg C \lor E) \land (\neg D \lor E) \land \neg E$$

4. Determine whether the following argument form is valid or invalid

$$\begin{array}{l} \forall_x (P(x) \to Q(x)) \\ \neg P(a) \\ - - - - - - - - \\ \therefore \neg Q(a) \end{array}$$

   If you think this form is valid, give a thorough explanation why. If you think it is invalid, provide a simple, 3-sentence example where the argument form is followed from two premises that are true and yet a false conclusion is obtained.

5. Recall that an *argument* comes from an *argument form* by substituting particular predicates and variable values in the argument form. It is possible to start with a valid argument form (e.g., Universal Modus Ponens, Universal Modus Tollens, Universal Hypothetical Syllogism) but substitute into it values that make the premises false. In such cases you may arrive at a false conclusion despite using a valid argument form. ($\leftarrow$ this is just a general observation that is *not* illustrated in this problem). Answer two questions about each of the below arguments: a) whether it follows a valid argument form, b) if yes, state the involved inference law and explain why the form is followed by providing suitable predicates; if no, explain a plausible situation where the two premises are true

but the conclusion is false.

(a) All numbers whose alternating sum of digits is a multiple of 11 are themselves multiples of 11.

The alternating sum of digits of 7581 is not a multiple of 11.

$\therefore$ 7581 is not a multiple of 11.

Note: The alternating sum of digits of a number $d_1 d_2 \dots d_k$ is

$$d_1 - d_2 + d_3 - d_4 + \cdots$$

Hence for 9581 we get 9 - 5 + 8 - 1 = 4 + 7 = 11.

(b) All soccer players can easily run 10 kilometers.

Adam can run 10 kilometers with ease.

$\therefore$ Adam is a soccer player.

6. Determine whether the argument form of the below argument is valid or invalid. If the argument form is valid, represent the statements as predicates over a certain domain and state the law that was applied to obtain the conclusion. If the argument form is invalid, provide a diagram depicting a model (i.e., a world) in which the premises are true, but the conclusion is false.

No athletes who can run 10 kilometers below 30 minutes are smokers.

No athletes who smoke can run 100 meters below 11 seconds.

$\therefore$ No athletes who run 10 kilometers below 30 minutes can run 100 meters below 11 seconds.

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

Rule of Inference	***Name***
$\forall x(P(x) \to Q(x))$ $P(a)$, where a is a particular element in the domain — $\therefore Q(a)$	Universal Modus Ponens
$\forall x(P(x) \to Q(x))$ $\neg Q(a)$, where a is a particular element in the domain — $\therefore \neg P(a)$	Universal Moddus Tollens
$\forall x(P(x) \to Q(x))$ $\forall x(Q(x) \to R(x))$ — $\therefore \forall x(P(x) \to R(x))$	Universal Hypothetical Syllogism (a.k.a Transitivity Law)