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CS 115-B/LF
Written Assignment 1

1.

A	B	A ⇒ B	¬(A ⇒ B)	¬B	A ∧ ¬B
T	T	T	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	F

For all combinations of A and B both propositions have the same truth, therefore ¬(A ⇒ B) ≡ A ∧ ¬B.

A	B	C	(B ⇒ C)	A ⇒ (B ⇒ C)	(A ⇒ B)	(A ⇒ B) ⇒ C
F	F	F	T	T	T	F
F	F	T	T	T	T	T
F	T	F	F	T	T	F
F	T	T	T	T	T	T
T	F	F	T	T	F	T
T	F	T	T	T	F	T
T	T	F	F	F	T	F
T	T	T	T	T	T	T

There are two combinations of A, B, and C for which the two propositions have different values. Therefore A ⇒ (B ⇒ C) and (A ⇒ B) ⇒ C are not logically equivalent.

A	B	C	A ∨ B	A ∨ B ⇒ C	A ⇒ C	B ⇒ C	(A ⇒ C) ∧ (B ⇒ C)
F	F	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	T	F	T	F	T	F	F
F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	T	T	T	T	T	T	T

A ∨ B ⇒ C and (A ⇒ C) ∧ (B ⇒ C) have the same value for all combinations of A, B, and C. Therefore these two propositions are logically equivalent.

2.

p	q	¬p	¬q	¬q → ¬p	(¬q → ¬p) ∧ p	((¬q → ¬p) ∧ p) → q
F	F	T	T	T	F	T
F	T	T	F	T	F	T
T	F	F	T	F	F	T
T	T	F	F	T	T	T

The proposition ((¬q → ¬p) ∧ p) → q outputs true no matter what p and q are set equal to; therefore, it is a tautology.

3.

a) If Argentina wins the Sunday’s game then they will win the World Cup.
b) If the sensor recognizes your fingerprint then the lock will open.
c) For a graph on n vertices, if it has n - 1 edges then it is connected.
d) If a number is divisible by 49 then it is divisible by 7.

4.

a) The equivalency is true.

$C \lor (\neg A \land \neg B) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$	
$C \lor (\neg A \land \neg B)$	Original proposition
$(C \lor \neg A) \land (C \lor \neg B)$	Distributive law
$(\neg A \lor C) \land (C \lor \neg B)$	Commutative law
$(\neg A \lor C) \land (\neg B \lor C)$	Commutative law
$(A \Rightarrow C) \land (\neg B \lor C)$	Implication as a disjunction / Conditional identity
$(A \Rightarrow C) \land (B \Rightarrow C)$	Implication as a disjunction / Conditional identity
$((A \Rightarrow C) \land (B \Rightarrow C))$	Associativity

b) The equivalency is true.

$\neg A \lor \neg B \lor \neg C \lor D \equiv (A \land B \land C) \Rightarrow D$	
$\neg A \lor \neg B \lor \neg C \lor D$	Original proposition
$(\neg A \lor \neg B) \lor \neg C \lor D$	Associativity
$\neg(A \land B) \lor \neg C \lor D$	De Morgan's law
$(\neg(A \land B) \lor \neg C) \lor D$	Associativity
$\neg((A \land B) \land C) \lor D$	De Morgan's law
$\neg(A \land B \land C) \lor D$	Associativity
$(A \land B \land C) \Rightarrow D$	Implication as a disjunction / Conditional identity

5.

A ∧ ¬B ⇒ C	Original proposition
(A ∧ ¬B) ⇒ C	Associativity
¬(A ∧ ¬B) ∨ C	Implication as a disjunction / Conditional identity
¬A ∨ ¬¬B ∨ C	De Morgan's law
¬A ∨ B ∨ C	Double negation law

The original proposition must be rewritten because it uses the implication operator. The final proposition is logically equivalent to the original but it does not use anything other than the provided atoms, ¬, ∧, and ∨.

A ∨ ¬B ⇒ C ∨ B	Original proposition
(A ∨ ¬B) ⇒ C ∨ B	Associativity
(A ∨ ¬B) ⇒ (C ∨ B)	Associativity
¬(A ∨ ¬B) ∨ (C ∨ B)	Implication as a disjunction / Conditional identity
¬A ∧ ¬¬B ∨ (C ∨ B)	De Morgan's law
¬A ∧ ¬¬B ∨ C ∨ B	Associativity
¬A ∧ ¬B ∨ C ∨ B	Double negation law
¬A ∧ ¬B ∨ C ∨ B	Commutative law

The original proposition must be rewritten because it uses the implication operator. The final proposition is logically equivalent to the original but it does not use anything other than the provided atoms, ¬, ∧, and ∨.

(A ⇒ C) ⇔ (B ⇒ C)	Original proposition
(¬A ∨ C) ⇔ (B ⇒ C)	Implication as a disjunction / Conditional identity
(¬A ∨ C) ⇔ (¬B ∨ C)	Implication as a disjunction / Conditional identity
((¬A ∨ C) ⇒ (¬B ∨ C)) ∧ ((¬B ∨ C) ⇒ (¬A ∨ C))	Biconditional identity
(¬(¬A ∨ C) ∨ (¬B ∨ C)) ∧ ((¬B ∨ C) ⇒ (¬A ∨ C))	Implication as a disjunction / Conditional identity
(¬(¬A ∨ C) ∨ (¬B ∨ C)) ∧ (¬(¬B ∨ C) ∨ (¬A ∨ C))	Implication as a disjunction / Conditional identity
(¬¬A ∧ ¬C ∨ (¬B ∨ C)) ∧ (¬(¬B ∨ C) ∨ (¬A ∨ C))	De Morgan's law
(¬¬A ∧ ¬C ∨ (¬B ∨ C)) ∧ (¬¬B ∧ ¬C ∨ (¬A ∨ C))	De Morgan's law
(A ∧ ¬C ∨ (¬B ∨ C)) ∧ (¬¬B ∧ ¬C ∨ (¬A ∨ C))	Double negation law
(A ∧ ¬C ∨ (¬B ∨ C)) ∧ (B ∧ ¬C ∨ (¬A ∨ C))	Double negation law
(A ∧ ¬C ∨ ¬B ∨ C) ∧ (B ∧ ¬C ∨ (¬A ∨ C))	Associativity
(A ∧ ¬C ∨ ¬B ∨ C) ∧ (B ∧ ¬C ∨ ¬A ∨ C)	Associativity

The original proposition must be rewritten because it uses the implication and biconditional operators. The final proposition is logically equivalent to the original but it does not use anything other than the provided atoms, ¬, ∧, ∨, and parentheses. The included parentheses are necessary because there is no simpler way to ensure that the operations happen in the order required to maintain logical equivalence.

6.

The information in its simplest form:

- Adam says Clara did it.
- James says James did not do it.
- Clara says Daniel did it.
- Daniel says Daniel did not do it.

The rules we are given:

- Only one student opened the cage.
- Only one student is telling the truth,
    - which means the other 3 students are lying.

Here is what every possible instance entails:

Truthful student	Adam	James	Clara	Daniel
Adjusted information	Clara did it. James did it. Daniel did not do it. Daniel did it.	Clara did not do it. James did not do it. Daniel did not do it. Daniel did it.	Clara did not do it. James did it. Daniel did it. Daniel did it.	Clara did not do it. James did it. Daniel did not do it. Daniel did not do it.
Possible?	No	No	No	Yes
Why?	Only one student opened the cage. This situation blames 3 students.	Daniel cannot be both guilty and innocent.	Only one student opened the cage. This situation blames 2 students.	Only one student was blamed.

James must have opened the cage. Daniel was telling the truth. This must be the case because this situation is the only one where all of the conditions are met. Only one student, Daniel, tells the truth and only one student, James, is blamed.