Written Assignment 3

CS135-B/LF

February 17, 2024

1

$\neg r$	Hypothesis	(1)
$q \implies r$	Hypothesis	(2)
$\neg q$	Modus tollens, 1,2	(3)
$\neg q \implies u \wedge s$	Hypothesis	(4)
$u \wedge s$	Modus ponens, 3,4	(5)
s	Simplification, 5	(6)
$p \vee q$	Hypothesis	(7)
p	Disjunctive syllogism, 3, 7	(8)
$p \wedge s$	Conjunction, 6, 8	(9)
$p \wedge s \implies t$	Hypothesis	(10)
t	Modus ponens, 9,10	(11)

2

For simplicity's sake, I will use this key to represent the given propositions:

- p: The dorm is locked.
- q: The phone is on top of the tall bookshelf.
- r: The dorm room is odd-numbered.
- s: The phone is under the pillow.
- t: The dorm has more than 10 floors.
- u: The phone is in the bottom drawer of the desk.

This gives us the following list of statements:

$p \implies \neg q$

$r \implies q$

p

$r \vee s$

$t \implies u$

Using this list, we can deduce the following:

p	Hypothesis	(1)
$p \implies \neg q$	Hypothesis	(2)
$\neg q$	Modus ponens, 1, 2	(3)
$r \implies q$	Hypothesis	(4)
$\neg r$	Modus tollens, 3, 4	(5)
$r \lor s$	Hypothesis	(6)
s	Disjunctive syllogism, 5, 6	(7)

We have now confirmed that s must be true. If we check the key, this means that the phone is under the pillow.

3

Let us start at the contradiction form $x = \neg x$ and derive the given formula from it. I will use E to replace x, and introduce new variables in reverse alphabetical order.

$E \lor \neg E$	Original proposition
$\equiv (D \lor E) \land (\neg D \lor E) \land \neg E$	Resolution
$\equiv (B \lor E) \land (\neg B \lor D) \land (\neg D \lor E) \land \neg E$	Resolution
$\equiv (C \lor B) \land (\neg C \lor E) \land (\neg B \lor D) \land (\neg D \lor E) \land \neg E$	Resolution
$\equiv (A \land B) \land (\neg A \lor C) \land (\neg B \lor D) \land (\neg C \lor E) \land (\neg D \lor E) \land \neg E$	Resolution

4

4.a

Although the **argument** put forward is true, this is not a valid argument **form**.
As an example, I will replace the predicate "The sum of alternating digits of x is a multiple of 11" with "I ate an apple for lunch." Then I will replace the predicate "x is a multiple of 11" with "I ate lunch today." Here is what this substitution yields:

> If I ate an apple for lunch today, then I ate lunch today.
> I did not eat an apple for lunch today.
> $\therefore$ I did not eat lunch today.

This form is invalid because the premises are true:

- It is true that if I ate an apple for lunch today, then I ate lunch today. I would have eaten the apple for lunch.
- I did not eat an apple for lunch today.

And the conclusion is false:

- I had pancakes for lunch, therefore I *did* have lunch.

This new version of the argument demonstrates that this form, which is denying the antecedent rather than the consequent, is not valid.

4.b

This is another invalid argument form.
Here is a reinterpretation of the argument with different predicates:

Every player on the Yankees plays baseball.
I play baseball.
$\therefore$ I am on the Yankees.

This form is invalid because the premises are true:

- Every player on the Yankees plays baseball (no matter how well).
- I do play baseball.

And the conclusion is false:

- I am not on the Yankees.

This new version of the argument demonstrates that this form, which is affirming the consequent rather than the antecedent, is not valid.