Chapter 1

1. In case of an accident, there is a high chance of getting lost. The transportation cost is very high each time. However, if the infrastructure is set once, it will be very easy to use it repeatedly. Time for wireless transmission is negligible as signals travel at the speed of light.

2. Advantages of bursty data communication

   (a) Pulses are made very narrow, so multipaths are resolvable

   (b) The transmission device needs to be switched on for less time.

   Disadvantages

   (a) Bandwidth required is very high

   (b) Peak transmit power can be very high.

3. $P_b = 10^{-12}$
   $\frac{1}{2\overline{\gamma}} = 10^{-12}$
   $\overline{\gamma} = \frac{10^{12}}{2} = 5 \times 10^{11}$ (very high)

4. Geo: 35,786 Km above earth $\Rightarrow RTT = \frac{2\times 35786\times 10^3}{c} = 0.2386s$
   Meo: 8,000- 20,000 Km above earth $\Rightarrow RTT = \frac{2\times 8000\times 10^3}{c} = 0.0533s$
   Leo: 500- 2,000 Km above earth $\Rightarrow RTT = \frac{2\times 500\times 10^3}{c} = 0.0033s$
   Only Leo satellites as delay $= 3.3ms < 30ms$

5.

6. optimum no. of data user = d
   optimum no. of voice user = v
   Three different cases:
   Case 1: d=0, v=6
   $\Rightarrow revenue = 60.80.2 = 0.96$

   Case 2: d=1, v=3
   revenue = [prob. of having one data user]×(revenue of having one data user)
   + [prob. of having two data user]×(revenue of having two data user)
   + [prob. of having one voice user]×(revenue of having one voice user)
   + [prob. of having two voice user]×(revenue of having two voice user)
   + [prob. of having three or more voice user]×(revenue in this case)

$$\Rightarrow \quad 0.5^2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} \times \$1 + 0.5^2 \times \$1 + \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 + \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4 +$$

$$\left[1 - \begin{pmatrix} 6 \\ 1 \end{pmatrix} 0.8 \times 0.2^5 \times \$0.2 - \begin{pmatrix} 6 \\ 2 \end{pmatrix} 0.8^2 \times 0.2^4 \times \$0.4\right] \times \$0.6$$

$$\Rightarrow \$1.35$$

   Case 3: d=2, v=0
   revenue $= 2 \times 0.5 = \$1$
   So the best case is case 2, which is to allocate 60kHz to data and 60kHz to voice.

7.

8. 1. Hand-off becomes a big problem.
   2. Inter-cell interference is very high and should be mitigated to get reasonable SINR.
   3. Infrastructure cost is another problem.

9. Smaller the reuse distance, larger the number of users who can use the same system resource and so capacity (data rate per unit bandwidth) increases.

10. (a) 100 cells, 100 users/cell $\Rightarrow$ 10,000 users

    (b) 100 users/cell $\Rightarrow$ 2500 cells required
    $\frac{100km^2}{Area/cell} = 2500cells \Rightarrow \frac{Area}{cell} = .04km^2$

    (c) From Rappaport or iteration of formula, we get that 100 $\frac{channels}{cell} \Rightarrow 89 \frac{channels}{cell}$ $@P_b = .02$
    Each subscriber generates $\frac{1}{30}$ of an Erlang of traffic.
    Thus, each cell can support $30 \times 89 = 2670$ subscribers
    Macrocell: $2670 \times 100 \Rightarrow 267,000$ subscribers
    Microcell: 6,675,000 subscribers

    (d) Macrocell: \$50 M
    Microcell: \$1.25 B

    (e) Macrocell: \$13.35 M/month $\Rightarrow$ 3.75 months *approx* 4 months to recoop
    Microcell: \$333.75 M/month $\Rightarrow$ 3.75 months *approx* 4 months to recoop

11. One CDPD line : 19.2Kbps
    average $W_{imax} \sim 40Mbps$
    $\therefore$ number of CDPD lines $\sim 2 \times 10^3$

Chapter 2

1.

$$
\begin{aligned}
P_r &= P_t\left[\frac{\sqrt{G_l}\lambda}{4\pi d}\right]^2 \qquad \lambda = c/f_c = 0.06 \\
10^{-3} &= P_t\left[\frac{\lambda}{4\pi 10}\right]^2 \Rightarrow P_t = 4.39KW \\
10^{-3} &= P_t\left[\frac{\lambda}{4\pi 100}\right]^2 \Rightarrow P_t = 438.65KW
\end{aligned}
$$

Attenuation is very high for high frequencies

2. d= 100m
h_t = 10m
h_r = 2m
delay spread = $\tau = \frac{x+x'-l}{c} = 1.33\times$

3. $\Delta\phi = \frac{2\pi(x'+x-l)}{\lambda}$

$$
\begin{aligned}
x'+x-l &= \sqrt{(h_t+h_r)^2+d^2}-\sqrt{(h_t-h_r)^2+d^2} \\
&= d\left[\sqrt{\left(\frac{h_t+h_r}{d}\right)^2+1}-\sqrt{\left(\frac{h_t-h_r}{d}\right)^2+1}\right]
\end{aligned}
$$

$d \gg h_t, h_r,$ we need to keep only first order terms

$$
\begin{aligned}
&\sim d\left\{\left[\frac{1}{2}\sqrt{\left(\frac{h_t+h_r}{d}\right)^2+1}\right]-\left[\frac{1}{2}\sqrt{\left(\frac{h_t-h_r}{d}\right)^2+1}\right]\right\} \\
&= \frac{2(h_t+h_r)}{d} \\
\Delta\phi &\sim \frac{2\pi}{\lambda}\frac{2(h_t+h_r)}{d}
\end{aligned}
$$

4. Signal nulls occur when $\Delta\phi = (2n+1)\pi$

$$\begin{aligned}
\frac{2\pi(x'+x-l)}{\lambda} &= (2n+1)\pi \\
\frac{2\pi}{\lambda}\left[\sqrt{(h_t+h_r)^2+d^2}-\sqrt{(h_t-h_r)^2+d^2}\right] &= \pi(2n+1) \\
\sqrt{(h_t+h_r)^2+d^2}-\sqrt{(h_t-h_r)^2+d^2} &= \frac{\lambda}{2}(2n+1) \\
\text{Let m } = (2n+1) & \\
\sqrt{(h_t+h_r)^2+d^2} &= m\frac{\lambda}{2}+\sqrt{(h_t-h_r)^2+d^2} \\
\text{square both sides} & \\
(h_t+h_r)^2+d^2 &= m^2\frac{\lambda^2}{4}+(h_t-h_r)^2+d^2+m\lambda\sqrt{(h_t-h_r)^2+d^2} \\
x=(h_t+h_r)^2,\ y=(h_t-h_r)^2,\ x-y=4h_th_r & \\
x &= m^2\frac{\lambda^2}{4}+y+m\lambda\sqrt{y+d^2} \\
\Rightarrow d &= \sqrt{\left[\frac{1}{m\lambda}\left(x-m^2\frac{\lambda^2}{4}-y\right)\right]^2-y} \\
d &= \sqrt{\left(\frac{4h_th_r}{(2n+1)\lambda}-\frac{(2n+1)\lambda}{4}\right)^2-(h_t-h_r)^2} \quad ,n\in\mathbb{Z}
\end{aligned}$$

5. $h_t = 20m$
   $h_r = 3m$
   $f_c = 2GHz \quad \lambda = \frac{c}{f_c} = 0.15$
   $d_c = \frac{4h_th_r}{\lambda} = 1600m = 1.6Km$
   This is a good radius for suburban cell radius as user density is low so cells can be kept fairly large. Also, shadowing is less due to fewer obstacles.

6. Think of the building as a plane in $\mathbb{R}^3$
   The length of the normal to the building from the top of Tx antenna $= h_t$
   The length of the normal to the building from the top of Rx antenna $= h_r$
   In this situation the 2 ray model is same as that analyzed in the book.

7. $h(t) = \alpha_1\delta(t-\tau)+\alpha_2\delta(t-(\tau+0.22\mu s))$
   $G_r = G_l = 1$
   $h_t = h_r = 8m$
   $f_c = 900MHz, \lambda = c/f_c = 1/3$
   $R = -1$

$$\begin{aligned}
\text{delay spread} = \frac{x+x'-l}{c} &= 0.022\times 10^{-6}s \\
\Rightarrow \frac{2\sqrt{8^2+\left(\frac{d}{2}\right)^2}-d}{c} &= 0.022\times 10^{-6}s \\
\Rightarrow d = 16.1m & \\
\therefore \tau = \frac{d}{c} = 53.67ns &
\end{aligned}$$

$$\alpha_1 = \left(\frac{\lambda}{4\pi}\frac{\sqrt{G_l}}{l}\right)^2 = 2.71 \times 10^{-6}$$

$$\alpha_2 = \left(\frac{\lambda}{4\pi}\frac{\sqrt{RG_r}}{x+x'}\right)^2 = 1.37 \times 10^{-6}$$

8. A program to plot the figures is shown below. The power versus distance curves and a plot of the phase difference between the two paths is shown on the following page. From the plots it can be seen that as G_r (gain of reflected path) is decreased, the asymptotic behavior of P_r tends toward d^{-2} from d^{-4}, which makes sense since the effect of reflected path is reduced and it is more like having only a LOS path. Also the variation of power before and around dc is reduced because the strength of the reflected path decreases as G_r decreases. Also note that the the received power actually increases with distance up to some point. This is because for very small distances (i.e. d = 1), the reflected path is approximately two times the LOS path, making the phase difference very small. Since R = -1, this causes the two paths to nearly cancel each other out. When the phase difference becomes 180 degrees, the first local maxima is achieved. Additionally, the lengths of both paths are initially dominated by the difference between the antenna heights (which is 35 meters). Thus, the powers of both paths are roughly constant for small values of d, and the dominant factor is the phase difference between the paths.

```
clear all;
close all;
ht=50;
hr=15;
f=900e6;
c=3e8;
lambda=c/f;
GR=[1,.316,.1,.01];
Gl=1;
R=-1;
counter=1;
figure(1);
d=[1:1:100000];
l=(d.^2+(ht-hr)^2).^.5;
r=(d.^2+(ht+hr)^2).^.5;
phd=2*pi/lambda*(r-1);
dc=4*ht*hr/lambda;
dnew=[dc:1:100000];

for counter = 1:1:4,
    Gr=GR(counter);
    Vec=Gl./l+R*Gr./r.*exp(phd*sqrt(-1));
    Pr=(lambda/4/pi)^2*(abs(Vec)).^2;
    subplot(2,2,counter);
    plot(10*log10(d),10*log10(Pr)-10*log10(Pr(1)));
    hold on;
    plot(10*log10(dnew),-20*log10(dnew));
    plot(10*log10(dnew),-40*log10(dnew));
end
hold off
```

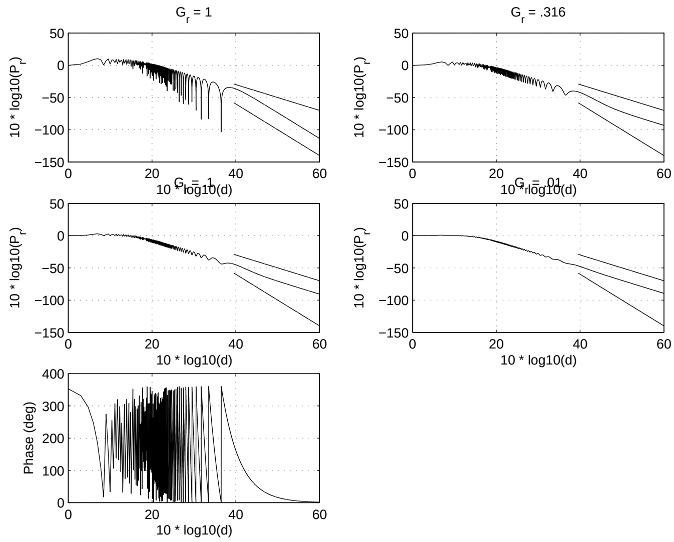

Figure 1: Problem 8

9. As indicated in the text, the power fall off with distance for the 10-ray model is d^{-2} for relatively large distances

10. The delay spread is dictated by the ray reaching last $d = \sqrt{(500/6)^2 + 10^2} = 83.93m$
Total distance = 6d = 503.59m
$\tau_0 = 503.59/c = 1.68\mu s$
L.O.S ray d = 500m
$\tau_0 = 500/c = 1.67\mu s$
$\therefore$ delay spread $= 0.01\mu s$

11. $f_c = 900MHz$
$\lambda = 1/3m$
$G = 1$ radar cross section $20dBm^2 = 10\log_1 0\sigma \Rightarrow \sigma = 100$
d=1 , $s = s' = \sqrt{(0.5d)^2 + (0.5d)^2} = d\sqrt{0.5} = \sqrt{0.5}$
Path loss due to scattering

$$\frac{P_r}{P_t} = \left[\frac{\lambda\sqrt{G\sigma}}{(4\pi)^{3/2}ss'}\right]^2 = 0.0224 = -16.498dB$$

Path loss due to reflection (using 2 ray model)

$$\frac{P_r}{P_t} = \left(\frac{R\sqrt{G}}{s+s'}\right)^2\left(\frac{\lambda}{4\pi}\right)^2 = 3.52\times10^{-4} = -34.54dB$$

$d = 10$ $\quad P_{scattering} = -56.5dB$ $\quad P_{reflection} = -54.54dB$
$d = 100$ $\quad P_{scattering} = -96.5dB$ $\quad P_{reflection} = -74.54dB$
$d = 1000$ $\quad P_{scattering} = -136.5dB$ $\quad P_{reflection} = -94.54dB$
Notice that scattered rays over long distances result in tremendous path loss

12.

$$P_r = P_tK\left(\frac{d_0}{d}\right)^\gamma \quad \rightarrow \text{simplified}$$

$$P_r = P_t\left(\frac{\sqrt{G_l}}{4\pi}\right)^2\left(\frac{\lambda}{d}\right)^2 \quad \rightarrow \text{free space}$$

$\therefore$ when $K = \left(\frac{\sqrt{G_l}}{4\pi}\right)^2$ and $d_0 = \lambda$
The two models are equal.

13. $P_{noise} = -160dBm$
$f_c = 1GHz, d_0 = 1m,\ K = (\lambda/4\pi d_0)^2 = 5.7 \times 10^{-4}, \lambda = 0.3, \gamma = 4$
We want $SNR_{recd} = 20dB = 100$
$\because$ Noise power is 10^{-19}

$$P = P_t K \left(\frac{d_0}{d}\right)^\gamma$$

$$10^{-17} = 10K \left(\frac{0.3}{d}\right)^4$$

$$d \leq 260.7m$$

14. d = distance between cells with reused freq
p = transmit power of all the mobiles

$$\left(\frac{S}{I}\right)_{\text{uplink}} \geq 20dB$$

(a) Min. S/I will result when main user is at A and Interferers are at B
d_A = distance between A and base station #1 = $\sqrt{2}km$ d_B = distance between B and base station #1 = $\sqrt{2}km$

$$\left(\frac{S}{I}\right)_{\text{min}} = \frac{P\left[\frac{G\lambda}{4\pi d_A}\right]^2}{2P\left[\frac{G\lambda}{4\pi d_B}\right]^2} = \frac{d_B^2}{2d_A^2} = \frac{(d_{min}-1)^2}{4} = 100$$

$\Rightarrow d_{min} - 1 = 20km \Rightarrow d_{min} = 21km$ since integer number of cells should be accommodated in distance d $\Rightarrow d_{min} = 22km$

(b)

$$\frac{P_\gamma}{P_u} = k\left[\frac{d_0}{d}\right]^\gamma \Rightarrow \left(\frac{S}{I}\right)_{\text{min}} = \frac{Pk\left[\frac{d_0}{d_A}\right]^\gamma}{2Pk\left[\frac{d_0}{d_B}\right]^\gamma} =$$

$$\frac{1}{2}\left[\frac{d_B}{d_A}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^3 = 100$$

$\Rightarrow d_{min} = 9.27 \Rightarrow$ with the same argument $\Rightarrow d_{min} = 10km$

(c)

$$\left(\frac{S}{I}\right)_{\text{min}} = \frac{k\left[\frac{d_0}{d_A}\right]_A^\gamma}{2k\left[\frac{d_0}{d_B}\right]_B^\gamma} = \frac{(d_{min}-1)^4}{0.04} = 100$$

$\Rightarrow d_{min} = 2.41km \Rightarrow$ with the same argument $d_{min} = 4km$

15. $f_c = 900MHz, h_t = 20m, h_r = 5m, d = 100m$

Large urban city	$PL_{largecity} = 353.52dB$
small urban city	$PL_{smallcity} = 325.99dB$
suburb	$PL_{suburb} = 207.8769dB$
rural area	$PL_{ruralarea/countryside} = 70.9278dB$

As seen , path loss is higher in the presence of multiple reflectors, diffractors and scatterers

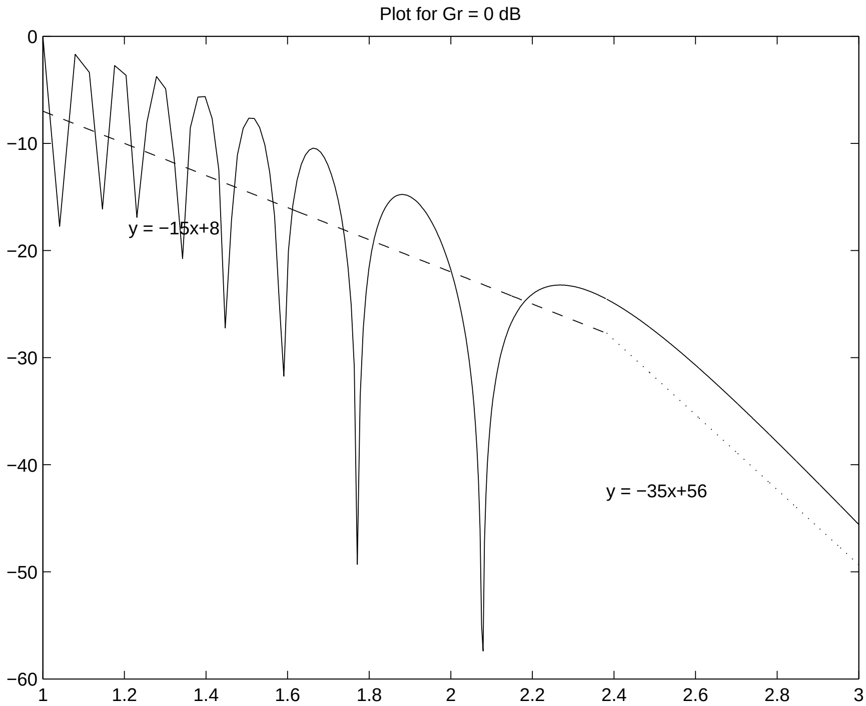

Figure 2: Problem 16

16. Piecewise linear model for 2-path model. See Fig 2

17. $P_r = P_t - P_L(d) - \sum_i^3 FAF_i - \sum_j^2 PAF_j$
FAF =(5,10,6), PAF =(3.4,3.4)

$$P_L(d)K\left(\frac{d_0}{d}\right)_0^\gamma = 10^{-8} = -8dB$$

$$-110 = P_t - 80 - 5 - 10 - 6 - 3.4 - 3.4$$

$$\Rightarrow P_t = -2.2dBm$$

18. (a) $\frac{P_r}{P_t}dB = 10\log_{10} K - 10r\log_1 0\frac{d}{d_0}$
using least squares we get
$10\log_{10} K = -29.42dB$
$\gamma = 4$

(b) PL(2Km) $= 10\log_{10} K - 10r\log_{10} d = -161.76dB$

(c) Receiver power can be assumed to be Gaussian with variance $\sigma^2_{\psi dB}$

$$X \sim N(0, \sigma^2_{\psi dB})$$

$$Prob(X < -10) = Prob\left(\frac{X}{\sigma_{\psi dB}} < \frac{-10}{\sigma_{\psi dB}}\right) = 6.512 \times 10^{-4}$$

19. Assume free space path loss parameters
$f_c = 900MHz \rightarrow \lambda = 1/3m$
$\sigma_{\psi dB} = 6$
$SNR_{recd} = 15dB$
$P_t = 1W$

$g = 3dB$
$P_{noise} = -40dBm \Rightarrow P_{recvd} = -55dB$
Suppose we choose a cell of radius d

$$\begin{aligned}
\mu(d) &= P_{recvd}(\text{due to path loss alone}) \\
&= P_t \left(\frac{\sqrt{G_l}\lambda}{4\pi d}\right)^2 = \frac{1.4 \times 1-^{-3}}{d^2}
\end{aligned}$$

$$\begin{aligned}
\mu_{dB} = 10\log_1 0(\mu(d)) \\
P(P_{recd}(d) > -55) &= 0.9
\end{aligned}$$

$$\begin{aligned}
P\left(\frac{P_{recd}(d) - \mu_{dB}}{\sigma_{\psi dB}} > \frac{-55 - \mu_{dB}}{6}\right) &= 0.9 \\
\Rightarrow \frac{-55 - \mu_{dB}}{6} &= -1.282 \\
\Rightarrow \mu_{dB} &= -47.308 \\
\Rightarrow \mu(d) &= 1.86 \times 10^{-5} \\
\Rightarrow d &= 8.68m
\end{aligned}$$

20. MATLAB CODE

```
Xc = 20;
ss = .01;
y = wgn(1,200*(1/ss));
for i = 1:length(y)
    x(i) = y(i);
    for j = 1:i
        x(i) = x(i)+exp(-(i-j)/Xc)*y(j);
    end
end
```

21. Outage Prob. = Prob. [received $power_{dB} \leqslant Tp_{dB}$]
Tp = 10dB

(a)

$$outageprob. = 1 - Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) = 1 - Q\left(\frac{-5}{8}\right) = Q\left(\frac{5}{8}\right) = 26\%$$

(b) σ_ψ = 4dB, outage prob < 1% $\Rightarrow$

$$Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) > 99\% \Rightarrow \frac{Tp - \mu_\psi}{\sigma_\psi} < -2.33 \Rightarrow$$

$$\mu_\psi \geq 19.32dB$$

(c)

$$\sigma_\psi = 12dB, \frac{Tp - \mu_\psi}{\sigma_\psi} < -6.99 \Rightarrow \mu_\psi \geq 37.8dB$$

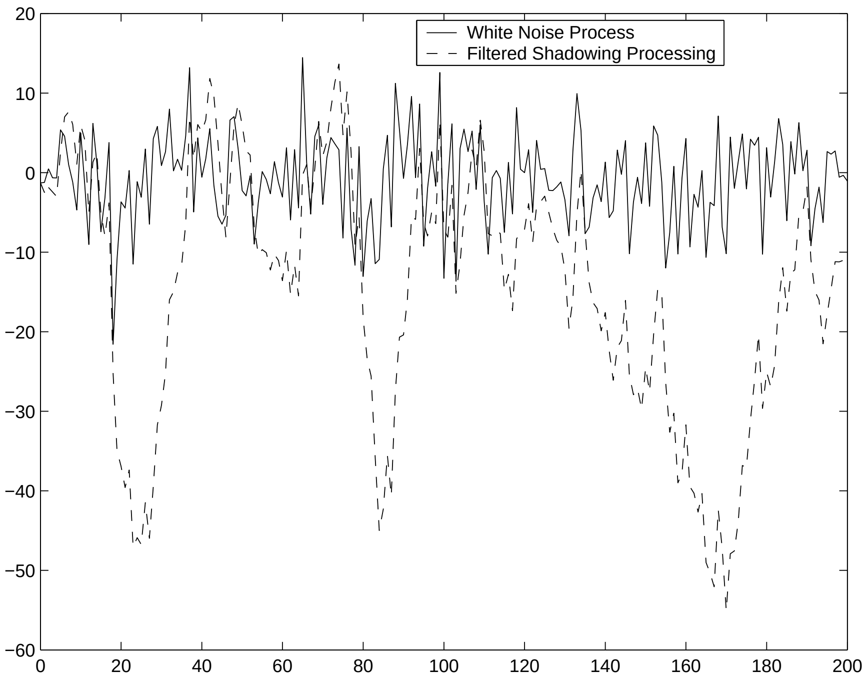

Figure 3: Problem 20

(d) For mitigating the effect of shadowing, we can use macroscopic diversity. The idea in macroscopic diversity is to send the message from different base stations to achieve uncorrelated shadowing. In this way the probability of power outage will be less because both base stations are unlikely to experience an outage at the same time, if they are uncorrelated.

22.

$$C = \frac{2}{R^2} \int_{r=0}^{R} rQ\left(a + b\ln\frac{r}{R}\right) dr$$

To perform integration by parts, we let $du = rdr$ and $v = Q\left(a + b\ln\frac{r}{R}\right)$. Then $u = \frac{1}{2}r^2$ and

$$dv = \frac{\partial}{\partial r}Q\left(a + b\ln\frac{r}{R}\right) = \frac{\partial}{\partial x}Q(x)|_{x=a+b\ln(r/R)}\frac{\partial}{\partial r}\left(a + b\ln\frac{r}{R}\right) = \frac{-1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr. \quad (1)$$

where $k = a + b\ln\frac{r}{R}$. Then we get

$$\begin{aligned}
C &= \frac{2}{R^2}\left[\frac{1}{2}r^2 Q\left(a + b\ln\frac{r}{R}\right)\right]_{r=0}^{R} + \frac{2}{R^2}\int\limits_{r=0}^{R}\frac{1}{2}r^2\frac{1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr && (2)\\
&= Q(a) + \frac{1}{R^2}\int\limits_{r=0}^{R} r^2\frac{1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr && (3)\\
&= Q(a) + \frac{1}{R^2}\int\limits_{r=0}^{R}\frac{1}{\sqrt{2\pi}}R^2\exp\left(\frac{2(k-a)}{b}\right)\exp(-k^2/2)\frac{b}{r}dr && (4)\\
&= Q(a) + \int\limits_{k=-\infty}^{a}\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-k^2}{2} + \frac{2k}{b} - \frac{2a}{b}\right)dk && (5)\\
&= Q(a) + \exp\left(\frac{-2a}{b} + \frac{2}{b^2}\right)\int\limits_{k=-\infty}^{a}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(k - \frac{2}{b})^2\right)dk && (6)\\
&= Q(a) + \exp\left(\frac{2-2ab}{b^2}\right)\left[1 - Q\left(\frac{a - \frac{2}{b}}{1}\right)\right] && (7)\\
&= Q(a) + \exp\left(\frac{2-2ab}{b^2}\right)Q\left(\frac{2-ab}{b}\right) && (8)\\
& && (9)
\end{aligned}$$

Since $Q(-x) = 1 - Q(x)$.

23. $\gamma = 3$
$d_0 = 1$
$k = 0dB$
$\sigma = 4dB$
$R = 100m$
$P_t = 80mW P_{min} = -100dBm = -130dB$

$$\overline{P_\gamma}(R) = P_t K\left(\frac{d_0}{d}\right)^\gamma = 80 \times 10^{-9} = -70.97dB$$

$$a = \frac{P_{min} - \overline{P_\gamma}(R)}{\sigma} = 14.7575$$

$$b = \frac{10\gamma\log_{10}e}{\sigma_{\psi dB}} = 3.2572$$

$$c = Q(a) + e^{\frac{2-2ab}{b^2}}Q\left(\frac{2-2ab}{b}\right) \simeq 1$$

24. $\gamma = 6$
$\sigma = 8$

$$\overline{P_\gamma}(R) = 20 + P_{min}$$

$$a = -20/8 = -2.5$$

$$b = \frac{10 \times 6 \times \log_{10}e}{8} = 20.3871$$

$$c = 0.9938$$

25.

$\gamma/\sigma_{\psi_{\rm dB}}$	2	4	6
4	0.7728	0.8587	0.8977
8	0.6786	0.7728	0.8255
12	0.6302	0.7170	0.7728

Since $\overline{P_r}(r) \geq P_{min}$ for all $r \leq R$, the probability of non-outage is proportional to $Q\left(\frac{-1}{\sigma}\right)$, and thus decreases as a function of σ. Therefore, C decreases as a function of σ. Since the average power at the boundary of the cell is fixed, C increases with γ, because it forces higher transmit power, hence more received power at $r < R$. Due to these forces, we have minimum coverage when $\gamma = 2$ and $\sigma = 12$. By a similar argument, we have maximum coverage when $\gamma = 6$ and $\sigma = 4$. The same can also be seen from this figure:

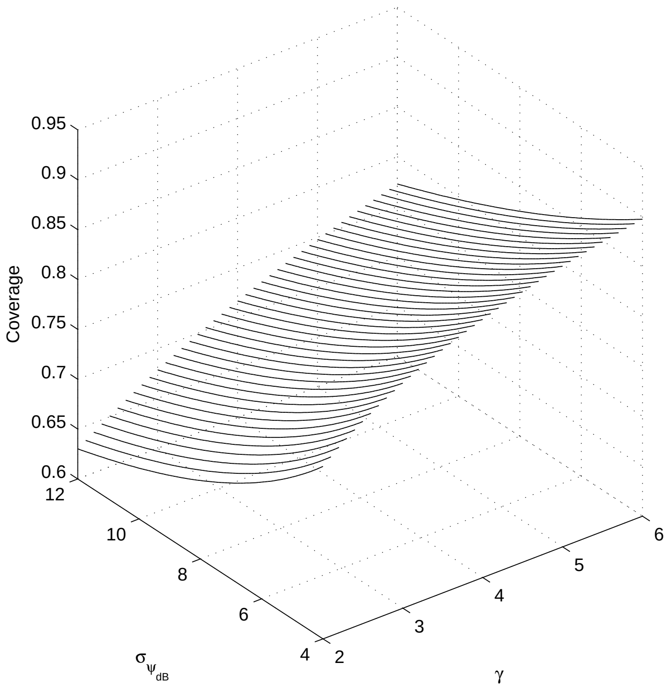

Figure 4: Problem 25

The value of coverage for middle point of typical values i.e. $\gamma = 4$ and $\sigma = 8$ can be seen from the table or the figure to be 0.7728.

Chapter 3

1. $d = vt$
   $r + r' = d + \frac{2h^2}{d}$
   Equivalent low-pass channel impulse response is given by

$$c(\tau, t) = \alpha_0(t)e^{-j\phi_0(t)}\delta(\tau - \tau_0(t)) + \alpha_1(t)e^{-j\phi_1(t)}\delta(\tau - \tau_1(t))$$

   $\alpha_0(t) = \frac{\lambda\sqrt{G_l}}{4\pi d}$ with $d = vt$
   $\phi_0(t) = 2\pi f_c \tau_0(t) - \phi_{D_0}$
   $\tau_0(t) = d/c$
   $\phi_{D_0} = \int_t 2\pi f_{D_0}(t)dt$
   $f_{D_0}(t) = \frac{v}{\lambda}\cos\theta_0(t)$
   $\theta_0(t) = 0 \ \forall t$
   $\alpha_1(t) = \frac{\lambda R\sqrt{G_l}}{4\pi(r+r')} = \frac{\lambda R\sqrt{G_l}}{4\pi(d+\frac{2h^2}{d})}$ with $d = vt$
   $\phi_1(t) = 2\pi f_c \tau_1(t) - \phi_{D_1}$
   $\tau_1(t) = (r + r')/c = (d + \frac{2h^2}{d})/c$
   $\phi_{D_1} = \int_t 2\pi f_{D_1}(t)dt$
   $f_{D_1}(t) = \frac{v}{\lambda}\cos\theta_1(t)$
   $\theta_1(t) = \pi - \arctan\frac{h}{d/2} \ \forall t$

2. For the 2 ray model:

$$\tau_0 = \frac{l}{c}$$

$$\tau_1 = \frac{x + x'}{c}$$

$$\therefore \text{delay spread}(T_m) = \frac{x + x' - l}{c} = \frac{\sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}}{c}$$

   when $d \gg (h_t + h_r)$

$$T_m = \frac{1}{c}\frac{2h_t h_r}{d}$$

$$h_t = 10m,\ h_r = 4m,\ d = 100m$$

$$\therefore T_m = 2.67 \times 10^{-9}s$$

3. Delay for LOS component $= \tau_0 = 23$ ns
   Delay for First Multipath component $= \tau_1 = 48$ ns
   Delay for Second Multipath component $= \tau_2 = 67$ ns

   $\tau_c =$ Delay for the multipath component to which the demodulator synchronizes.

$$T_m = \max_m \tau_m - \tau_c$$

   So, when $\tau_c = \tau_0$, $T_m = 44$ ns. When $\tau_c = \tau_1$, $T_m = 19$ ns.

4. $f_c = 10^9 Hz$
   $\tau_{n,min} = \frac{10}{3\times10^8} s$
   $\therefore \min f_c\tau_n = \frac{10^{10}}{3\times10^8} = 33 \gg 1$

5. Use CDF strategy.

$$F_z(z){=}\ P[x^2{+}y^2 \leq z^2]\ =\ \int\int_{x^2+y^2\leq z^2} \frac{1}{2\pi\sigma^2}e^{\frac{-(x^2+y)^2}{2\sigma^2}}dxdy\ =\ \int_0^{2\pi}\int_0^{z} \frac{1}{2\pi\sigma^2}e\frac{-r^2}{2\sigma^2}rdrd\theta\ = 1 - e^{\frac{-z^2}{2\sigma^2}}(z \geq 0)$$

$$\frac{df_z(z)}{dz}{=}\frac{z}{\sigma^2}e^{-\frac{z^2}{2\sigma^2}} \rightarrow Rayleigh$$

For Power:

$$F_{z^2}(z){=}P[Z \leq \sqrt{z}]\ = 1 - e\frac{-z^2}{2\sigma^2}$$

$$f_z(z){=}\frac{1}{2\sigma^2}e\frac{-z}{2\sigma^2} \rightarrow Exponential$$

6. For Rayleigh fading channel

$$Pr(P_r < P_0) = 1 - e^{-P_0/2\sigma^2}$$

$$2\sigma^2 = -80dBm,\ P_0 = -95dBm,\ Pr(P_r < P_0) = 0.0311$$

$$2\sigma^2 = -80dBm,\ P_0 = -90dBm,\ Pr(P_r < P_0) = 0.0952$$

7. For Rayleigh fading channel

$$P_{outage} = 1 - e^{-P_0/2\sigma^2}$$

$$0.01 = 1 - e^{-P_0/P_r}$$

$$\therefore P_r = -60dBm$$

8. $2\sigma^2$ = -80dBm = 10^{-11}
   Target Power P_0 = -80 dBm = 10^{-11}
   Avg. power in LOS component = s^2 = -80dBm = 10^{-11}

$$Pr[z^2 \leq 10^{-11}] = Pr[z \leq \frac{10^{-5}}{\sqrt{10}}]$$

Let $z_0 = \frac{10^{-5}}{\sqrt{10}}$

$$= \int_0^{z_0} \frac{z}{\sigma^2}e^{-\frac{-(z^2+s^2)}{2\sigma^2}} I_0\left(\frac{zs}{\sigma^2}\right)dz, \quad z \geq 0$$

$$= 0.3457$$

To evaluate this, we use Matlab and $I_0(x)$ = besseli(0,x). Sample Code is given:

```
clear P0 = 1e-11; s2 = 1e-11; sigma2 = (1e-11)/2; z0 =
sqrt(1e-11); ss = z0/1e7; z = [0:ss:z0]; pdf =
(z/sigma2).*exp(-(z.^2+s2)/(2*sigma2)).*besseli(0,z.*(sqrt(s2)/sigma2));
int_pr = sum(pdf)*ss;
```

9. CDF of Ricean distribution is

$$F_Z^{\text{Ricean}}(z) = \int_0^z p_Z^{\text{Ricean}}(z)$$

where

$$p_Z^{\text{Ricean}}(z) = \frac{2z(K+1)}{Pr}\exp\left[-K - \frac{(K+1)z^2}{Pr}\right] I_0\left(2z\sqrt{\frac{K(K+1)}{Pr}}\right), \quad z \geq 0$$

For the Nakagami-m approximation to Ricean distribution, we set the Nakagami m parameter to be $(K+1)^2/(2K+1)$. CDF of Nakagami-m distribution is

$$F_Z^{\text{Nakagami-m}}(z) = \int_0^z p_Z^{\text{Nakagami-m}}(z)$$

where

$$p_Z^{\text{Nakagami-m}}(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)Pr^m}\exp\left[\frac{-mz^2}{Pr}\right], \quad z \geq 0, \quad m \geq 0.5$$

We need to plot the two CDF curves for K = 1,5,10 and Pr =1 (we can choose any value for Pr as it is the same for both the distributions and our aim is to compare them). Sample code is given:

```
z = [0:0.01:3]; K = 10; m = (K+1)^2/(2*K+1); Pr = 1; pdfR =
((2*z*(K+1))/Pr).*exp(-K-((K+1).*(z.^2))/Pr).*besseli(0,(2*sqrt((K*(K+1))/Pr))*z);
pdfN = ((2*m^m*z.^(2*m-1))/(gamma(m)*Pr^m)).*exp(-(m/Pr)*z.^2);
for i = 1:length(z)
    cdfR(i) = 0.01*sum(pdfR(1:i));
    cdfN(i) = 0.01*sum(pdfN(1:i));
end plot(z,cdfR); hold on plot(z,cdfN,'b--'); figure;
plot(z,pdfR); hold on plot(z,pdfN,'b--');
```

As seen from the curves, the Nakagami-m approximation becomes better as K increases. Also, for a fixed value of K and x, prob(γ<x) for x large is always greater for the Ricean distribution. This is seen from the tail behavior of the two distributions in their pdf, where the tail of Nakagami-distribution is always above the Ricean distribution.

10. (a) W = average received power
    Z_i = Shadowing over link i
    P_{ri} = Received power in dBW, which is Gaussian with mean W, variance σ^2

    (b)

$$P_{outage} = P\left[P_{r,1} < T \cap P_{r,2} < T\right] = P\left[P_{r,1} < T\right] P[P_{r,2} < T] \; \textit{since } Z_1, Z_2 \textit{ independent}$$

$$= \left[Q\left(\frac{W-T}{\sigma}\right)\right]^2 = \left[Q\left(\frac{\triangle}{\sigma}\right)\right]^2$$

    (c)

$$P_{out} = \int_{-\infty}^{\infty} P\left[P_{r,1} \leq T \;,\; P_{r,2} < T | Y = y\right] f_y\left(y\right) dy$$

$$P_{r,1}|Y = y \sim N\left(W + by, a^2\sigma^2\right), \; \textit{and} \; \left[P_{r,1}|Y = y\right] \perp \left[P_{r,2}|Y = y\right]$$

$$P_{outage=} \int_{-\infty}^{\infty} \left[Q\left(\frac{W + by - T}{a\sigma}\right)\right]^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$$

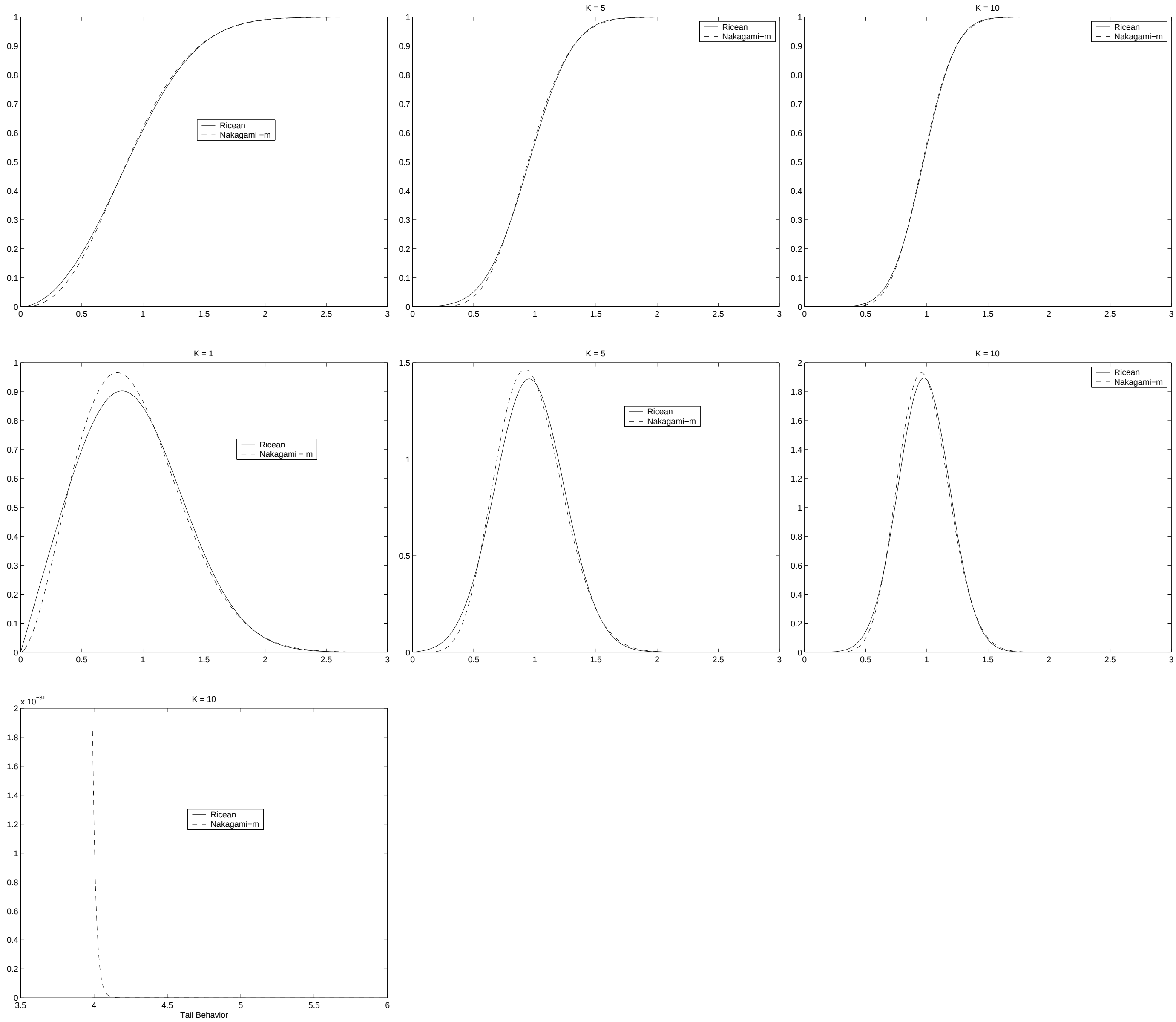

Figure 1: The CDF and PDF for $K = 1, 5, 10$ and the Tail Behavior

let $\frac{y}{\sigma}= u$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{W-T+bu\sigma}{a\sigma} \right) \right]^2 e^{\frac{-u^2}{2}} du \;=\; \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\triangle+by\sigma}{a\sigma} \right) \right]^2 e \frac{-y^2}{2} dy$$

(d) Let a = b = $\frac{1}{\sqrt{2}}$, $\sigma = 8$, $\triangle = 5$. With independent fading we get

$$P_{out} = \left[Q\left(\frac{5}{8} \right) \right]^2 = \; 0.0708.$$

With correlated fading we get $P_{out}=$ 0.1316.
Conclusion : Independent shadowing is prefarable for diversity.

11. There are many acceptable techniques for this problem. Sample code for both the stochastic technique(preferred) and the Jake's technique are included.

Jakes: Summing of appropriate sine waves

```
%Jake's Method
close all; clear all;
%choose N=30
N=30; M=0.5*(N/2-1); Wn(M)=0; beta(M)=0;
%We choose 1000 samples/sec
ritemp(M,2001)=0; rqtemp(M,2001)=0; rialpha(1,2001)=0; fm=[1 10
100]; Wm=2*pi*fm; for i=1:3
    for n=1:1:M
        for t=0:0.001:2
            %Wn(i)=Wm*cos(2*pi*i/N)
            Wn(n)=Wm(i)*cos(2*pi*n/N);
            %beta(i)=pi*i/M
            beta(n)=pi*n/M;
            %ritemp(i,20001)=2*cos(beta(i))*cos(Wn(i)*t)
            %rqtemp(i,20001)=2*sin(beta(i))*cos(Wn(i)*t)
            ritemp(n,1000*t+1)=2*cos(beta(n))*cos(Wn(n)*t);
            rqtemp(n,1000*t+1)=2*sin(beta(n))*cos(Wn(n)*t);
            %Because we choose alpha=0,we get sin(alpha)=0 and cos(alpha)=1
            %rialpha=(cos(Wm*t)/sqrt(2))*2*cos(alpha)=2*cos(Wm*t)/sqrt(2)
            %rqalpha=(cos(Wm*t)/sqrt(2))*2*sin(alpha)=0
            rialpha(1,1000*t+1)=2*cos(Wm(i)*t)/sqrt(2);
        end
    end
    %summarize ritemp(i) and rialpha
    ri=sum(ritemp)+rialpha;
    %summarize rqtemp(i)
    rq=sum(rqtemp);
    %r=sqrt(ri^2+rq^2)
    r=sqrt(ri.^2+rq.^2);
    %find the envelope average
    mean=sum(r)/2001;
    subplot(3,1,i);
```

```
    time=0:0.001:2;
    %plot the figure and shift the envelope average to 0dB
    plot(time,(10*log10(r)-10*log10(mean)));
    titlename=['fd = ' int2str(fm(i)) ' Hz'];
    title(titlename);
    xlabel('time(second)');
    ylabel('Envelope(dB)');
end
```

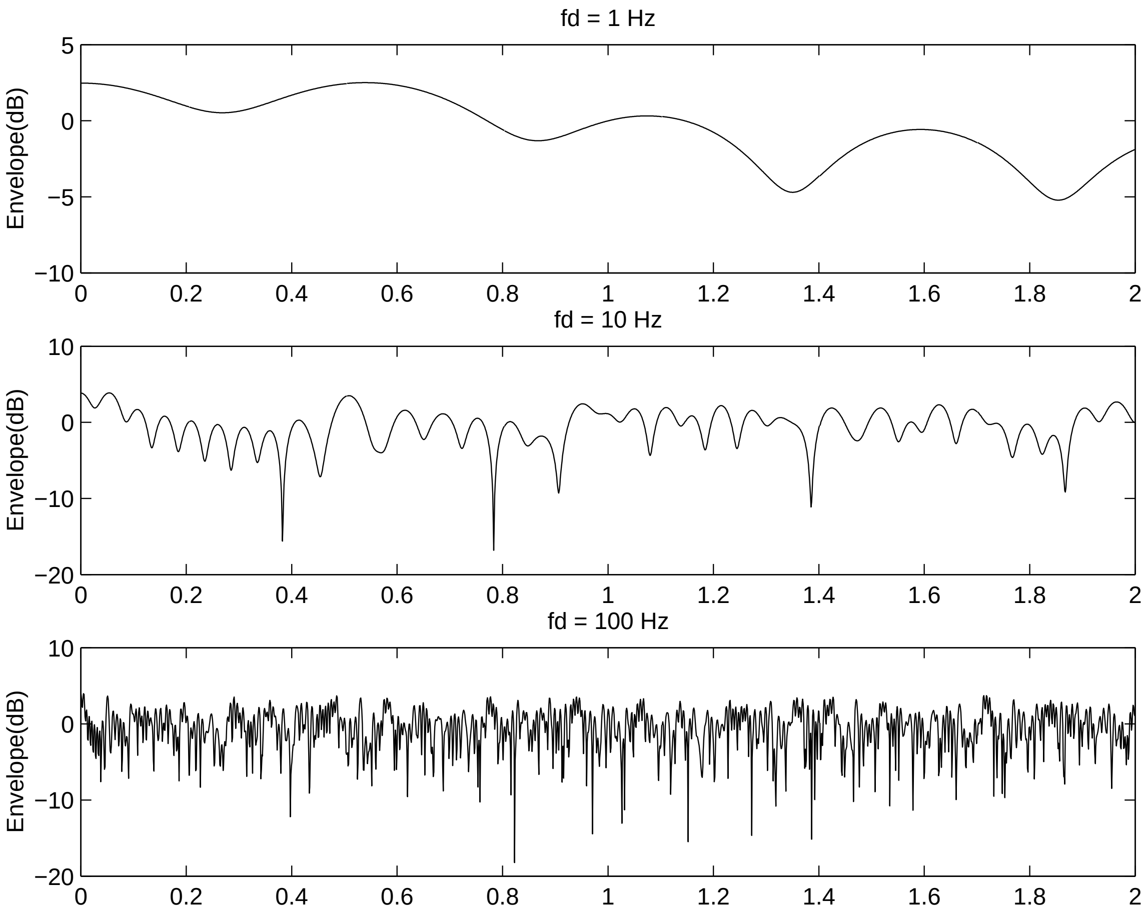

Figure 2: Problem 11

Stochastic: Usually two guassian R.V.'s are filtered by the Doppler Spectrum and summed. Can also just do a Rayleigh distribution with an adequate LPF, although the above technique is prefered.

```
function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
%
% function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
% generates a Rayleigh fading signal for given Doppler frequency f_D,
% during the time perios [0, t], with sampling frequency f_s >= 1000Hz.
%
% Input(s)
%   -- f_D  : [Hz] [1x1 double] Doppler frequency
%   -- t    : simulation time interval length, time interval [0,t]
%   -- f_s  : [Hz] sampling frequency, set to 1000 if smaller.
% Output(s)
%   -- Ts   : [Sec][1xN double] time instances for the Rayleigh signal
%   -- z_dB : [dB] [1xN double] Rayleigh fading signal
%

% Required parameters
if f_s < 1000;
  f_s = 1000;                % [Hz]  Minumum required sampling rate
end;
```

```
N = ceil( t*f_s );           % Number of samples

% Ts contains the time instances at which z_dB is specified
Ts = linspace(0,t,N);

if mod( N, 2) == 1
  N = N+1;                   % Use even number of samples in calculation
end;
f = linspace(-f_s,f_s,N);    % [Hz]  Frequency samples used in calculation

% Generate complex Gaussian samples with line spectra in frequency domain
% Inphase :
Gfi_p = randn(2,N/2); CGfi_p = Gfi_p(1,:)+i*Gfi_p(2,:); CGfi = [
conj(fliplr(CGfi_p)) CGfi_p ];

% Quadrature :
Gfq_p = randn(2,N/2); CGfq_p = Gfq_p(1,:)+i*Gfq_p(2,:); CGfq = [
conj(fliplr(CGfq_p)) CGfq_p ];

% Generate fading spectrum, this is used to shape the Gaussian line spectra
omega_p = 1;   % this makes sure that the average received envelop can be 0dB
S_r = omega_p/4/pi./(f_D*sqrt(1-(f/f_D).^2));

% Take care of samples outside the Doppler frequency range, let them be 0
idx1 = find(f>=f_D); idx2 = find(f<=-f_D); S_r(idx1) = 0;
S_r(idx2) = 0; S_r(idx1(1)) = S_r(idx1(1)-1);
S_r(idx2(length(idx2))) = S_r(idx2(length(idx2))+1);

% Generate r_I(t) and r_Q(t) using inverse FFT:
r_I = N*ifft(CGfi.*sqrt(S_r)); r_Q = -i*N*ifft(CGfq.*sqrt(S_r));

% Finally, generate the Rayleigh distributed signal envelope
z = sqrt(abs(r_I).^2+abs(r_Q).^2); z_dB = 20*log10(z);

% Return correct number of points
z_dB = z_dB(1:length(Ts));
```

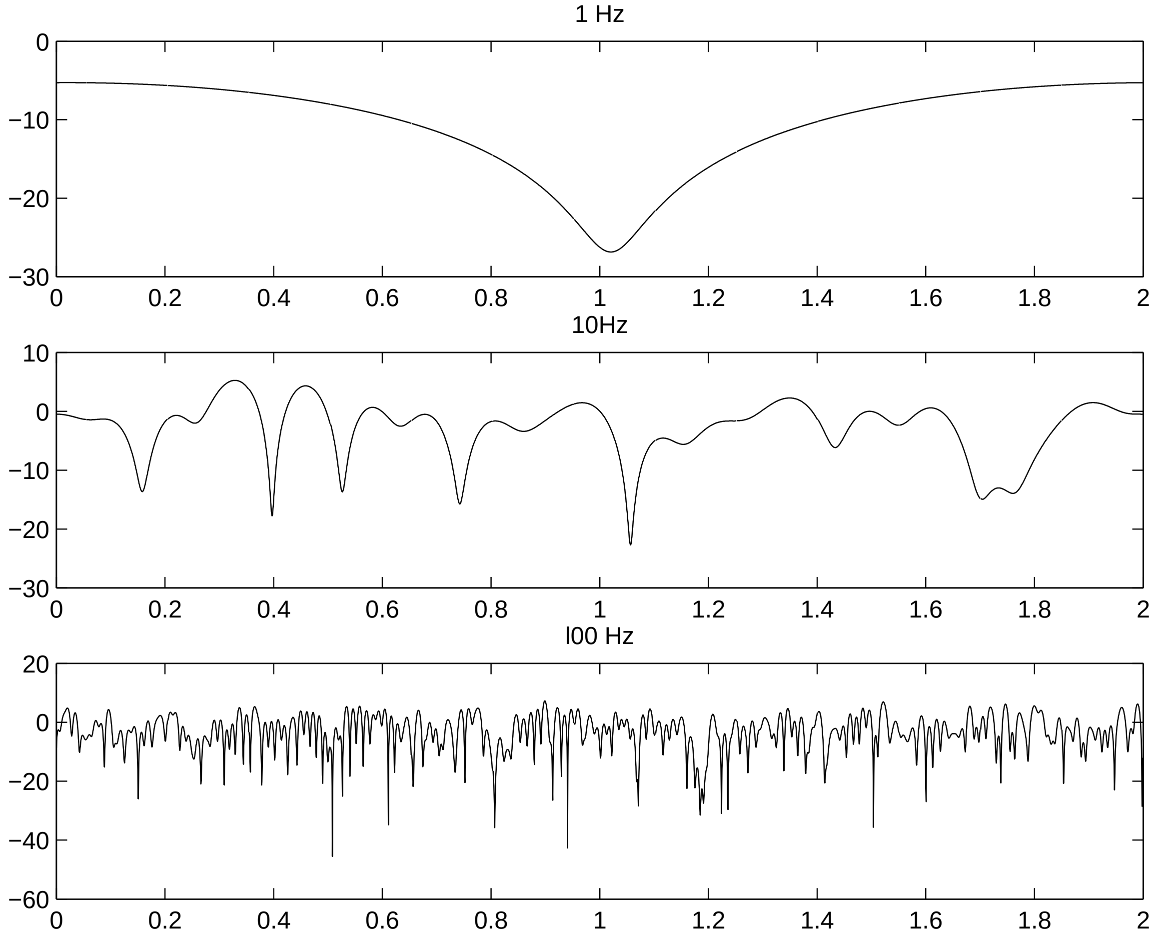

Figure 3: Problem 11

12. $P_r = 30dBm$
    $f_D = 10Hz$

$$P_0 = 0dBm,\ \bar{t}_z = \frac{e^{\rho^2} - 1}{\rho f_D \sqrt{2\pi}} = 0.0013s$$
$$P_0 = 15dBm,\ \bar{t}_z = 0.0072s$$
$$P_0 = 25dBm,\ \bar{t}_z = 0.0264s$$

13. In the reader, we found the level crossing rate below a level by taking an average of the number of times the level was crossed over a large period of time. It is easy to convince that the level crossing rate above a level will have the same expression as eq. 3.44 in reader because to go below a level again, we first need to go above it. There will be some discrepancy at the end points, but for a large enough T it will not effect the result. So we have

$$L_Z(above) = L_Z(below) = \sqrt{2\pi} f_D \rho e^{-\rho^2}$$

And,

$$\overline{t_Z}(above) = \frac{p(z > Z)}{L_Z(above)}$$
$$p(z > Z) = 1 - p(z \leq Z) = 1 - (1 - e^{-\rho^2}) = e^{-\rho^2}$$
$$\overline{t_Z}(above) = \frac{1}{\sqrt{2\pi} f_D \rho}$$

The values of $\overline{t_Z}(above)$ for $f_D = $ 10,50,80 Hz are 0.0224s, 0.0045s, 0.0028s respectively. Notice that as f_D increases, $\overline{t_Z}(above)$ reduces.

14. Note: The problem has been solved for $T_s = 10\mu s$
$P_r = 10dB$
$f_D = 80Hz$

$$\begin{array}{rrl}
R_1: & -\infty \leq \gamma \leq -10dB, & \pi_1 = 9.95 \times 10^{-3} \\
R_2: & -10dB \leq \gamma \leq 0dB, & \pi_2 = 0.085 \\
R_3: & 0dB \leq \gamma \leq 5dB, & \pi_3 = 0.176 \\
R_4: & 5dB \leq \gamma \leq 10dB, & \pi_4 = 0.361 \\
R_5: & 10dB \leq \gamma \leq 15dB, & \pi_5 = 0.325 \\
R_6: & 15dB \leq \gamma \leq 20dB, & \pi_6 = 0.042 \\
R_7: & 20dB \leq \gamma \leq 30dB, & \pi_7 = 4.54 \times 10^{-5} \\
R_8: & 30dB \leq \gamma \leq \infty, & \pi_8 = 3.72 \times 10^{-44}
\end{array}$$

$N_j \rightarrow$ level crossing rate at level A_j

$$\begin{array}{ll}
N_1 = 0, & \rho = \sqrt{\frac{0}{10}} \\
N_2 = 19.85, & \rho = \sqrt{\frac{0.1}{10}} \\
N_3 = 57.38, & \rho = \sqrt{\frac{1}{10}} \\
N_4 = 82.19, & \rho = \sqrt{\frac{10^{0.5}}{10}} \\
N_5 = 73.77, & \rho = \sqrt{\frac{10}{10}} \\
N_6 = 15.09, & \rho = \sqrt{\frac{10^{1.5}}{10}} \\
N_7 = 0.03, & \rho = \sqrt{\frac{10^{2}}{10}} \\
N_8 = 0, & \rho = \sqrt{\frac{10^{3}}{10}}
\end{array}$$

```
MATLAB CODE:
N = [0 19.85 57.38 82.19 73.77 15.09 .03 0];

Pi = [9.95e-3 .085 .176 .361 .325 .042 4.54e-5 3.72e-44];

T = 10e-3;
for i = 1:8
    if i == 1
        p(i,1) = 0;
        p(i,2) = (N(i+1)*T)/Pi(i);
        p(i,3) = 1-(p(i,1)+p(i,2));
    elseif i == 8
        p(i,1) = (N(i)*T)/Pi(i);
        p(i,2) = 0;
        p(i,3) = 1-(p(i,1)+p(i,2));
    else
        p(i,1) = (N(i)*T)/Pi(i);
        p(i,2) = (N(i+1)*T)/Pi(i);
        p(i,3) = 1-(p(i,1)+p(i,2));
```

```
    end
end

% p =
%
%          0    0.0199    0.9801
%     0.0023    0.0068    0.9909
%     0.0033    0.0047    0.9921
%     0.0023    0.0020    0.9957
%     0.0023    0.0005    0.9973
%     0.0036    0.0000    0.9964
%     0.0066         0    0.9934
%          0         0    1.0000
```

15. (a)

$$S(\tau,\rho) = \begin{cases} \alpha_1\delta(\tau) & \rho = 70Hz \\ \alpha_2\delta(\tau - 0.022\mu sec) & \rho = 49.5Hz \\ 0 & else \end{cases}$$

The antenna setup is shown in Fig. 15

From the figure, the distance travelled by the LOS ray is d and the distance travelled by the first multipath component is

$$2\sqrt{\left(\frac{d}{2}\right)^2 + 64}$$

Given this setup, we can plot the arrival of the LOS ray and the multipath ray that bounces off the ground on a time axis as shown in Fig. 15

So we have

$$2\sqrt{\left(\frac{d}{2}\right)^2 + 8^2} - d = 0.022\times 10^{-6} 3\times 10^8$$

$$\Rightarrow 4\left(\frac{d^2}{4} + 8^2\right) = 6.6^2 + d^2 + 2d(6.6)$$

$$\Rightarrow d = 16.1m$$

$f_D = v\cos(\theta)/\lambda$. $v = f_D\lambda/\cos(\theta)$. For the LOS ray, $\theta = 0$ and for the multipath component, $\theta = 45^o$. We can use either of these rays and the corresponding f_D value to get $v = 23.33$m/s.

(b)

$$d_c = \frac{4h_t h_r}{\lambda}$$

$d_c = 768$ m. Since $d \ll d_c$, power fall-off is proportional to d^{-2}.

(c) $T_m = 0.022\mu$s, $B^{-1} = 0.33\mu$s. Since $T_m \ll B^{-1}$, we have flat fading.

16. (a) Outdoor, since delay spread ≈ 10 μsec.
Consider that 10 μsec $\Rightarrow$ d = ct = 3km difference between length of first and last path

(b) Scattering function

$$S(\tau,\rho) = F_{\Delta t}[A_c(\tau,\Delta t)]$$

$= \frac{1}{W} rect\left(\frac{1}{W}\rho\right)$ for $0 \le \tau \le 10\mu sec$

The Scattering function is plotted in Fig. 16

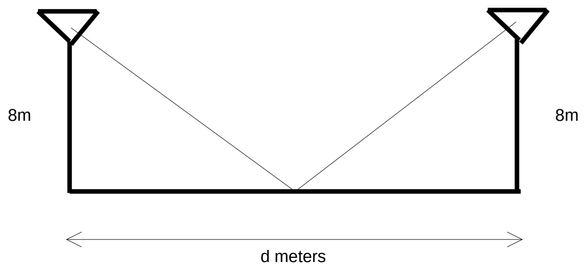

Figure 4: Antenna Setting

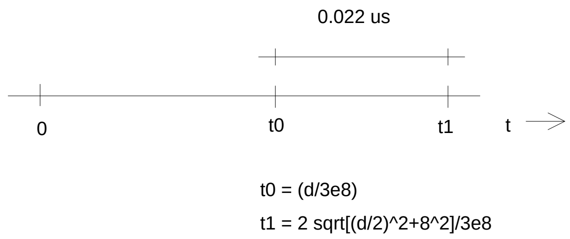

Figure 5: Time Axis for Ray Arrival

(c) Avg Delay Spread $= \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau} = 5\mu$sec

RMS Delay Spread $= \sqrt{\frac{\int_0^\infty (\tau-\mu_{Tm})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}} = 2.89\mu$sec

Doppler Spread $= \frac{W}{2} = 50$ Hz

(d) $\beta_u >$ Coherence BW $\Rightarrow$ Freq. Selective Fading $\approx \frac{1}{T_m} = 10^5 \Rightarrow \beta_u >= 10^5$ kHz
Can also use μ_{Tm} or σ_{Tm} instead of T_m

(e) Rayleigh fading, since receiver power is evenly distributed relative to delay; no dominant LOS path

(f) $t_R = \frac{e^{\rho^2}-1}{\rho F_D\sqrt{2\pi}}$ with $\rho = 1$, $f_D = \frac{W}{2} \rightarrow\ t_r = .0137$ sec

(g) Notice that the fade duration never becomes more than twice the average. So, if we choose our data rate such that a single symbol spans the average fade duration, in the worst case two symbols will span the fade duration. So our code can correct for the lost symbols and we will have error-free transmission. So $\frac{1}{t_R} = 72.94$ symbols/sec

17. (a) $T_m \approx .1\text{msec} = 100\mu\text{sec}$
$B_d \approx .1\text{Hz}$
Answers based on μ_{T_m} or σ_{T_m} are fine too. Notice, that based on the choice of either T_m, μ_{T_m} or σ_{T_m}, the remaining answers will be different too.

(b) $B_c \approx \frac{1}{T_m} = 10\text{kHz}$
$\Delta f > 10\text{kHz}$ for $u_1 \perp u_2$

(c) $(\Delta t)_c = 10\text{s}$

(d) $3\text{kHz} < B_c \Rightarrow$ Flat
$30\text{kHz} > B_c \Rightarrow$ Freq. Selective

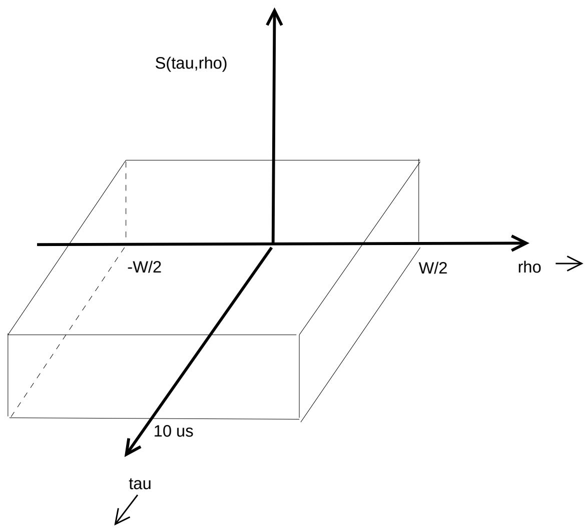

Figure 6: Scattering Function

Chapter 4

1. $C = B\log_2\left(1 + \frac{S}{N_0 B}\right)$
   $C = \frac{\log_2 \; 1+\frac{S}{N_0 B}}{\frac{1}{B}}$
   As $B \to \infty$ by L'Hospital's rule
   $$C = \frac{S}{N_0}\frac{1}{\ln 2}$$

2. $B = 50$ MHz
   $P = 10$ mW
   $N_0 = 2 \times 10^{-9}$ W/Hz
   $N = N_0 B$
   $C = 6.87$ Mbps.

   $P_{\text{new}} = 20$ mW, $C = 13.15$ Mbps (for $x \ll 1$, $\log(1+x) \approx x$)
   $B = 100$ MHz, Notice that both the bandwidth and noise power will increase. So $C = 7$ Mbps.

3. $P_{noise} = 0.1mW$
   $B = 20MHz$

   (a) $C_{user1 \to \text{base station}} = 0.933B = 18.66Mbps$
   (b) $C_{user2 \to \text{base station}} = 3.46B = 69.2Mbps$

4. (a) Ergodic Capacity (with Rcvr CSI only)= $B[\sum_{i=1}^{6} \log_2(1+\gamma_i)p(\gamma_i)] = 2.8831 \times B = 57.66$ Mbps.

   (b) $p_{\text{out}} = Pr(\gamma < \gamma_{\text{min}})$
   $C_o =$ (1-p_{out})$B\log_2(1+\gamma_{min})$
   For
   $\gamma_{\text{min}} > 20$dB, $p_{\text{out}} = 1$, $C_o = 0$
   15dB $< \gamma_{\text{min}} <$ 20dB, $p_{\text{out}} = .9$, $C_o = 0.1B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ 20dB.
   10dB $< \gamma_{\text{min}} <$ 15dB, $p_{\text{out}} = .75$, $C_o = 0.25B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ 15dB.
   5dB $< \gamma_{\text{min}} <$ 10dB, $p_{\text{out}} = .5$, $C_o = 0.5B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ 10dB.
   0dB $< \gamma_{\text{min}} <$ 5dB, $p_{\text{out}} = .35$, $C_o = 0.65B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ 5dB.
   -5dB $< \gamma_{\text{min}} <$ 0dB, $p_{\text{out}} = .1$, $C_o = 0.9B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ 0dB.
   $\gamma_{\text{min}} < -5$dB, $p_{\text{out}} = 0$, $C_o = B\log_2(1+\gamma_{\text{min}})$, max C_o at $\gamma_{\text{min}} \approx$ -5dB.

   Plot is shown in Fig. 1. Maximum at $\gamma_{\text{min}} = 10$dB, p_{out}=0.5 and $C_o = 34.59$ Mbps.

5. (a) We suppose that all channel states are used
   $$\frac{1}{\gamma_0} = 1 + \sum_{i=1}^{4}\frac{1}{\gamma_i}p_i \Rightarrow \;\gamma_0 = 0.8109$$
   $$\frac{1}{\gamma_0} - \frac{1}{\gamma_4} > 0 \therefore true$$
   $$\frac{S(\gamma_i)}{\overline{S}} = \frac{1}{\gamma_0} - \frac{1}{\gamma_i}$$

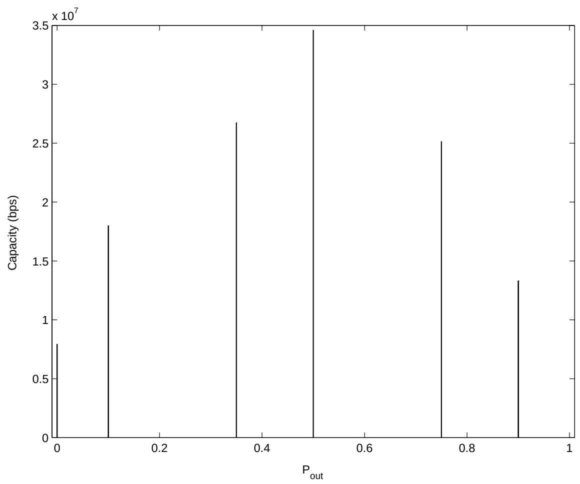

Figure 1: Capacity vs P_{out}

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} 1.2322 & \gamma = \gamma_1 \\ 1.2232 & \gamma = \gamma_2 \\ 1.1332 & \gamma = \gamma_3 \\ 0.2332 & \gamma = \gamma_4 \end{cases}$$

$$\frac{C}{B} = \sum_{i=1}^{4} \log_2\left(\frac{\gamma_i}{\gamma_0}\right) p(\gamma_i) = 5.2853 bps/Hz$$

(b) $\sigma = \frac{1}{E[1/\gamma]} = 4.2882$

$\frac{S(\gamma_i)}{\overline{S}} = \frac{\sigma}{\gamma_i}$

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} 0.0043 & \gamma = \gamma_1 \\ 0.0029 & \gamma = \gamma_2 \\ 0.4288 & \gamma = \gamma_3 \\ 4.2882 & \gamma = \gamma_4 \end{cases}$$

$$\frac{C}{B} = \log_2(1+\sigma) = 2.4028 bps/Hz$$

(c) To have $p_{out} = 0.1$ or 0.01 we will have to use all the sub-channels as leaving any of these will result in a p_{out} of at least 0.2 $\therefore$ truncated channel power control policy and associated spectral efficiency are the same as the zero-outage case in part b .

To have p_{out} that maximizes C with truncated channel inversion, we get

$$\max \frac{C}{B} = 4.1462 bps/Hz \quad p_{out} = 0.5$$

6. (a)

$$SNR_{recvd} = \frac{P_\gamma(d)}{P_{noise}} = \begin{cases} 10dB & w.p.\ 0.4 \\ 5dB & w.p.\ 0.3 \\ 0dB & w.p.\ 0.2 \\ -10dB & w.p.\ 0.1 \end{cases}$$

Assume all channel states are used

$$\frac{1}{\gamma_0} = 1 + \sum_{i=1}^{4} \frac{1}{\gamma_i} p_i \Rightarrow \gamma_0 = 0.4283 > 0.1 \quad \therefore \text{not possible}$$

Now assume only the best 3 channel states are used

$$\frac{0.9}{\gamma_0} = 1 + \sum_{i=1}^{3} \frac{1}{\gamma_i} p_i \Rightarrow \gamma_0 = 0.6742 < 1 \quad \therefore \text{ok!}$$

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} 1.3832 & \gamma = \gamma_1 = 10 \\ 1.1670 & \gamma = \gamma_2 = 3.1623 \\ 0.4832 & \gamma = \gamma_3 = 1 \\ 0 & \gamma = \gamma_4 = 0.1 \end{cases}$$

$$C/B = 2.3389 bps/Hz$$

(b) $\sigma = 0.7491$
$C/B = \log_2(1+\sigma) = 0.8066 bps/Hz$

(c) $\left(\frac{C}{B}\right)_{max} = 2.1510 bps/Hz$ obtained by using the best 2 channel states.
With $p_{out} = 0.1 + 0.2 = 0.3$

7. (a) Maximize capacity given by

$$C = \max_{S(\gamma):\int S(\gamma)p(\gamma)d\gamma = \overline{S}} \int_\gamma B \log\left(1 + \frac{S(\gamma)\gamma}{\overline{S}}\right) p(\gamma) d\gamma.$$

Construct the Lagrangian function

$$\mathcal{L} = \int_\gamma B \log\left(1 + \frac{S(\gamma)\gamma}{\overline{S}}\right) p(\gamma) d\gamma - \lambda \int \frac{S(\gamma)}{\overline{S}} p(\gamma) d\gamma$$

Taking derivative with respect to $S(\gamma)$, (refer to discussion section notes) and setting it to zero, we obtain,

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

Now, the threshold value must satisfy

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma) d\gamma = 1$$

Evaluating this with $p(\gamma) = \frac{1}{10} e^{-\gamma/10}$, we have

$$\begin{aligned} 1 &= \frac{1}{10\gamma_0} \int_{\gamma_0}^{\infty} e^{-\gamma/10} d\gamma - \frac{1}{10} \int_{\gamma_0}^{\infty} \frac{e^{-\gamma/10}}{\gamma} d\gamma & (1) \\ &= \frac{1}{\gamma_0} e^{-\gamma_0/10} - \frac{1}{10} \int_{\frac{\gamma_0}{10}}^{\infty} \frac{e^{-\gamma}}{\gamma} d\gamma & (2) \\ &= \frac{1}{\gamma_0} e^{-\gamma_0/10} - \frac{1}{10} \text{EXPINT}(\gamma_0/10) & (3) \end{aligned}$$

where EXPINT is as defined in matlab. This gives $\gamma_0 = 0.7676$. The power adaptation becomes

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{0.7676} - \frac{1}{\gamma} & \gamma \geq 0.7676 \\ 0 & \gamma < 0.7676 \end{cases}$$

(b) Capacity can be computed as

$$C/B = \frac{1}{10}\int_{0.7676}^{\infty} \log\left(\gamma/0.7676\right) e^{-\gamma/10} d\gamma = 2.0649 \quad \text{nats/sec/Hz.}$$

Note that I computed all capacites in nats/sec/Hz. This is because I took the natural log. In order to get the capacity values in bits/sec/Hz, the capacity numbers simply need to be divided by natural log of 2.

(c) AWGN capacity $C/B = \log(1 + 10) = 2.3979$ nats/sec/Hz.

(d) Capacity when only receiver knows γ

$$C/B = \frac{1}{10}\int_{0}^{\infty} \log\left(1+\gamma\right) e^{-\gamma/10} d\gamma = 2.0150 \quad \text{nats/sec/Hz.}$$

(e) Capacity using channel inversion is ZERO because the channel can not be inverted with finite average power. Threshold for outage probability 0.05 is computed as

$$\frac{1}{10}\int_{\gamma 0}^{\infty} e^{-\gamma/10} d\gamma = 0.95$$

which gives $\gamma_0 = 0.5129$. This gives us the capacity with truncated channel inversion as

$$\begin{aligned} C/B &= \log\left[1 + \frac{1}{\frac{1}{10}\int_{\gamma_0}^{\infty}\frac{1}{\gamma}e^{-\gamma/10}d\gamma}\right] * 0.95 &(4)\\ &= \log\left[1 + \frac{1}{\frac{1}{10}\text{EXPINT}(\gamma_0/10)}\right] * 0.95 &(5)\\ &= 1.5463 \quad \text{nats/s/Hz.} &(6)\end{aligned}$$

(f) Channel Mean=-5 dB = 0.3162. So for perfect channel knowledge at transmitter and receiver we compute $\gamma_0 = 0.22765$ which gives capacity $C/B = 0.36$ nats/sec/Hz.
With AWGN, $C/B = \log(1 + 0.3162) = 0.2748$ nats/sec/Hz.
With channel known only to the receiver $C/B = 0.2510$ nats/sec/Hz.

Capacity with AWGN is always greater than or equal to the capacity when only the reciever knows the channel. This can be shown using Jensen's inequality. However the capacity when the transmitter knows the channel as well and can adapt its power, can be higher than AWGN capacity specially at low SNR. At low SNR, the knowledge of fading helps to use the low SNR more efficiently.

8. (a) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

$$C = B\log\left(1 + \frac{\overline{S}}{N_0B + \overline{I}}\right) = 10.7Kbps.$$

(b) Suppose we transmit at power S_1 when jammer is off and S_2 when jammer is off,

$$C = B\max\left[\log\left(1 + \frac{S_1}{N_oB}\right)0.75 + \log\left(1 + \frac{S_2}{N_oB + \overline{I}}\right)0.25\right]$$

subject to

$$0.75S_1 + 0.25S_2 = \overline{S}.$$

This gives $S_1 = 12.25mW$, $S_2 = 3.25mW$ and $C = 53.21Kbps$.

(c) The jammer should transmit $-x(t)$ to completely cancel off the signal.

$\overline{S} = 10$mW
$N_0 = .001\ \mu$W/Hz
$B = 10$ MHz

Now we compute the SNR's as:

$$\gamma_j = \frac{|H_j|^2\overline{S}}{N_0 B}$$

This gives: $\gamma_1 = \frac{|1|^2 10^{-3}}{0.001\times 10^{-6} 10\times 10^6} = 1$, $\gamma_2 = .25$, $\gamma_3 = 4$, $\gamma_4 = 0.0625$

To compute γ_0 we use the power constraint as:

$$\sum_j \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_j}\right)_+ = 1$$

First assume that $\gamma_0 < 0.0625$, then we have

$$\frac{4}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{.25} + \frac{1}{4} + \frac{1}{.0625}\right)$$

$$\Rightarrow \gamma_0 = .1798 > 0.0625$$

So, our assumption was wrong. Now we assume that $0.0625 < \gamma_0 < .25$, then

$$\frac{3}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{.25} + \frac{1}{4}\right)$$

$$\Rightarrow \gamma_0 = .48 > 0.25$$

So, our assumption was wrong again. Next we assume that $0.25 < \gamma_0 < 1$, then

$$\frac{2}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{4}\right)$$

$$\Rightarrow \gamma_0 = .8889 < 1$$

This time our assumption was right. So we get that only two sub-bands each of bandwidth 10 MHz are used for transmission and the remaining two with lesser SNR's are left unused.

Now, we can find capacity as:

$$C = \sum_{j:\gamma_j \geq \gamma_0} B \log_2\left(\frac{\gamma_j}{\gamma_0}\right)$$

This gives us, C = 23.4 Mbps.

9. Suppose transmit power is P_t, interference power is P_{int} and noise power is P_{noise}.
In the first strategy $C/B = \log_2\left(1 + \frac{P_t}{P_{int}+P_{noise}}\right)$
In the second strategy $C/B = \log_2\left(1 + \frac{P_t - P_{int}}{P_{noise}}\right)$
Assuming that the transmitter transmits -x[k] added to its message always, power remaining for actual messages is $P_t - P_{int}$
The first or second strategy may be better depending on

$$\frac{P_t}{P_{int} + P_{noise}} \gtrless \frac{P_t - P_{int}}{P_{noise}} \Rightarrow P_t - P_{int} - P_{noise} \gtrless 0$$

P_t is generally greater than $P_{int} + P_{noise}$, and so strategy 2 is usually better.

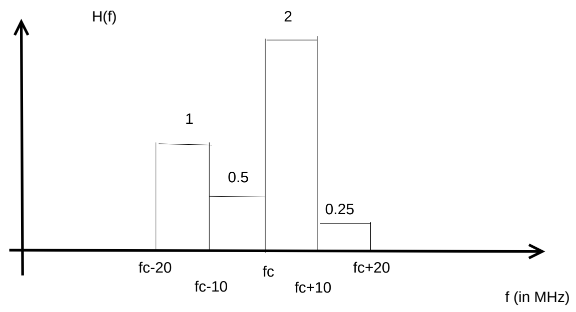

Figure 2: Problem 11

10. We show this for the case of a discrete fading distribution

$$C = \Sigma \log\left(1 + \frac{(1+j)^2 P_j}{N_0 B}\right)$$

$$\mathcal{L} = \sum_i \log\left(1 + \frac{(1+j)^2 P_j}{N_0 B}\right) - d_j\left(\sum_j P_j - P\right)$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_j} &= 0 \\
\Rightarrow \frac{(1+j)^2 P_j}{N_0 B} &= \frac{1}{\lambda}\frac{(1+j)^2}{N_0 B} - 1 \\
\text{let} \gamma_j = \frac{(1+j)^2 P}{N_0 B} & \\
\Rightarrow \frac{P_j}{P} &= \frac{1}{\lambda P} - \frac{1}{\gamma_j} \\
\text{denote} \frac{1}{\gamma_0} = \frac{1}{\lambda P} & \\
\therefore \frac{P_j}{P} &= \frac{1}{\gamma_0} - \frac{1}{\gamma_j} \\
\text{subject to the constraint} & \\
\frac{\Sigma P_j}{P} &= 1
\end{aligned}$$

11. $\overline{S} = 10\text{mW}$
$N_0 = .001\ \mu\text{W/Hz}$
$B = 10$ MHz

Now we compute the SNR's as:

$$\gamma_j = \frac{|H_j|^2 \overline{S}}{N_0 B}$$

This gives: $\gamma_1 = \frac{|1|^2 10^{-3}}{0.001 \times 10^{-6} 10 \times 10^6} = 1$, $\gamma_2 = .25$, $\gamma_3 = 4$, $\gamma_4 = 0.0625$

To compute γ_0 we use the power constraint as:

$$\sum_j \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_j}\right)_+ = 1$$

First assume that $\gamma_0 < 0.0625$, then we have

$$\frac{4}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{.25} + \frac{1}{4} + \frac{1}{.0625}\right)$$

$$\Rightarrow \gamma_0 = .1798 > 0.0625$$

So, our assumption was wrong. Now we assume that $0.0625 < \gamma_0 < .25$, then

$$\frac{3}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{.25} + \frac{1}{4}\right)$$

$$\Rightarrow \gamma_0 = .48 > 0.25$$

So, our assumption was wrong again. Next we assume that $0.25 < \gamma_0 < 1$, then

$$\frac{2}{\gamma_0} = 1 + \left(\frac{1}{1} + \frac{1}{4}\right)$$

$$\Rightarrow \gamma_0 = .8889 < 1$$

This time our assumption was right. So we get that only two sub-bands each of bandwidth 10 MHz are used for transmission and the remaining two with lesser SNR's are left unused.

Now, we can find capacity as:

$$C = \sum_{j:\gamma_j \geq \gamma_0} B \log_2\left(\frac{\gamma_j}{\gamma_0}\right)$$

This gives us, C = 23.4 Mbps.

12. For the case of a discrete number of frequency bands each with a flat frequency response, the problem can be stated as

$$\max_{s.t. \sum_i P(f_i) \leq P} \sum_i \log_2\left(1 + \frac{|H(f_i)|^2 P(f_i)}{N_0}\right)$$

denote $\gamma(f_i) = \frac{|H(f_i)|^2 P(f_i)}{N_0}$

$$L = \sum_i \log_2\left(1 + \gamma(f_i)\frac{P(f_i)}{P}\right) + \lambda(\sum P(f_i))$$

denote $x_i = \frac{P(f_i)}{P}$, the problem is similar to problem 10

$$\Rightarrow x_i^\star = \frac{1}{\gamma_0} - \frac{1}{\gamma(f_i)}$$

where γ_0 is found from the constraints

$$\sum_i \left(\frac{1}{\gamma_0} - \frac{1}{\gamma(f_i)}\right) = 1 \text{ and } \frac{1}{\gamma_0} - \frac{1}{\gamma(f_i)} \geq 0 \forall i$$

13. (a) C=13.98Mbps

MATLAB

```
Gammabar = [1 .5 .125];
ss = .001;
P = 30e-3;
N0 = .001e-6;

Bc = 4e6;
Pnoise = N0*Bc;
hsquare = [ss:ss:10*max(Gammabar)];
gamma = hsquare*(P/Pnoise);

for i = 1:length(Gammabar)
    pgamma(i,:) = (1/Gammabar(i))*exp(-hsquare/Gammabar(i));
end

gamma0v = [1:.01:2];
for j = 1:length(gamma0v)
    gamma0 = gamma0v(j);
    sumP(j) = 0;
    for i = 1:length(Gammabar)
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(i,c:length(gamma));
        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sumP(j) + sum(Pj_by_P.*pgammac)*ss;
    end
end
[b,c] = min(abs((sumP-1)));
gamma0ch = gamma0v(c);

C = 0;
for i = 1:length(Gammabar)
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(i,c:length(gamma));
    C = C + Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end
```

(b) C=13.27Mbps

MATLAB

```
Gammabarv = [1 .5 .125];
ss = .001;
Pt = 30e-3;
N0 = .001e-6;

Bc = 4e6;
Pnoise = N0*Bc;
```

```
P = Pt/3;
for k = 1:length(Gammabarv)
    Gammabar = Gammabarv(k);
    hsquare = [ss:ss:10*Gammabar];
    gamma = hsquare*(P/Pnoise);
    pgamma = (1/Gammabar)*exp(-hsquare/Gammabar);
    gamma0v = [.01:.01:1];
    for j = 1:length(gamma0v)
        gamma0 = gamma0v(j);
        a = gamma.*(gamma>gamma0);
        [b,c] = max(a>0);
        gammac = a(find(a));
        pgammac = pgamma(c:length(gamma));
        Pj_by_P = (1/gamma0)-(1./gammac);
        sumP(j) = sum(Pj_by_P.*pgammac)*ss;
    end
    [b,c] = min(abs((sumP-1)));
    gamma0ch = gamma0v(c);
    a = gamma.*(gamma>gamma0ch);
    [b,c] = max(a>0);
    gammac = a(find(a));
    pgammac = pgamma(c:length(gamma));
    C(k) = Bc*ss*sum(log2(gammac/gamma0ch).*pgammac);
end Ctot = sum(C);
```

Chapter 6

1. (a) For sinc pulse, $B = \frac{1}{2T_s} \Rightarrow T_s = \frac{1}{2B} = 5 \times 10^{-5} s$

   (b) $SNR = \frac{P_b}{N_0 B} = 10$
   Since 4-QAM is multilevel signalling
   $SNR = \frac{P_b}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{2E_s}{N_0 B} \quad (\because BT_s = \frac{1}{2})$
   $\therefore$ SNR per symbol $= \frac{E_s}{N_0} = 5$
   SNR per bit $= \frac{E_b}{N_0} = 2.5$ (a symbol has 2 bits in 4QAM)

   (c) SNR per symbol remains the same as before $= \frac{E_s}{N_0} = 5$
   SNR per bit is halved as now there are 4 bits in a symbol $\frac{E_b}{N_0} = 1.25$

2. $p_0 = 0.3, p_1 = 0.7$

   (a)

$$\begin{aligned} P_e &= Pr(0 \text{ detected, 1 sent — 1 sent})p(1 \text{ sent}) + Pr(1 \text{ detected, 0 sent — 0 sent})p(0 \text{ sent}) \\ &= 0.7Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) + 0.3Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \end{aligned}$$

   $d_{min} = 2A$

$$= Q\left(\sqrt{\frac{2A^2}{N_0}}\right)$$

   (b)

$$p(\hat{m} = 0|m = 1)p(m = 1) = p(\hat{m} = 1|m = 0)p(m = 0)$$

$$0.7Q\left(\frac{A + a}{\sqrt{\frac{N_0}{2}}}\right) = 0.3Q\left(\frac{A - a}{\sqrt{\frac{N_0}{2}}}\right) \quad , a > 0$$

   Solving gives us 'a' for a given A and N_0

   (c)

$$p(\hat{m} = 0|m = 1)p(m = 1) = p(\hat{m} = 1|m = 0)p(m = 0)$$

$$0.7Q\left(\frac{A}{\sqrt{\frac{N_0}{2}}}\right) = 0.3Q\left(\frac{B}{\sqrt{\frac{N_0}{2}}}\right) \quad , a > 0$$

   Clearly $A > B$, for a given A we can find B

   (d) Take $\frac{E_b}{N_0} = \frac{A^2}{N_0} = 10$
   In part a) $P_e = 3.87 \times 10^{-6}$
   In part b) a=0.0203 $P_e = 3.53 \times 10^{-6}$
   In part c) B=0.9587 $P_e = 5.42 \times 10^{-6}$
   Clearly part (b) is the best way to decode.

```
MATLAB CODE:
A = 1;
N0 = .1;
a = [0:.00001:1];
t1 = .7*Q(A/sqrt(N0/2));
```

```
t2=.3*Q(a/sqrt(N0/2));
diff = abs(t1-t2);
[c,d] = min(diff);
a(d)
c
```

3. $s(t) = \pm g(t)\cos 2\pi f_c t$
   $r = \hat{r}\cos\Delta\phi$
   where $\hat{r}$ is the signal after the sampler if there was no phase offset. Once again, the threshold that minimizes P_e is 0 as $(\cos\Delta\phi)$ acts as a scaling factor for both +1 and -1 levels. P_e however increases as numerator is reduced due to multiplication by $\cos\Delta\phi$

$$P_e = Q\left(\frac{d_{min}\cos\Delta\phi}{\sqrt{2N_0}}\right)$$

4.

$$\begin{aligned}
A_c^2\int_0^{T_b}\cos^2 2\pi f_c t dt &= A_c^2\int_0^{T_b}\frac{1+\cos 4\pi f_c t}{2} \\
&= A_c^2\left[\frac{T_b}{2} + \underbrace{\frac{\sin(4\pi f_c T_b)}{8\pi f_c}}_{\to 0 \text{ as } f_c \gg 1}\right] \\
&= \frac{A_c^2 T_b}{2} = 1 \\
x(t) &= 1 + n(t)
\end{aligned}$$

Let prob 1 sent $=p_1$ and prob 0 sent $=p_0$

$$\begin{aligned}
P_e &= \frac{1}{6}[1.p_1 + 0.p_0] + \frac{2}{6}[0.p_1 + 0.p_0] + \frac{2}{6}[0.p_1 + 0.p_0] + \\
&\quad \frac{1}{6}[0.p_1 + 1.p_0] \\
&= \frac{1}{6}[p_1 + p_0] = \frac{1}{6} \quad (\because p_1 + p_0 = 1 \text{ always })
\end{aligned}$$

5. We will use the approximation $P_e \sim$ (average number of nearest neighbors).$Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$
   where number of nearest neighbors = total number of points taht share decision boundary

   (a) 12 inner points have 5 neighbors
       4 outer points have 3 neighbors
       avg number of neighbors = 4.5
       $P_e = 4.5Q\left(\frac{2a}{\sqrt{2N_0}}\right)$
   (b) 16QAM, $P_e = 4\left(1-\frac{1}{4}\right)Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 3Q\left(\frac{2a}{\sqrt{2N_0}}\right)$
   (c) $P_e \sim \frac{2\times3+3\times2}{5}Q\left(\frac{2a}{\sqrt{2N_0}}\right) = 2.4Q\left(\frac{2a}{\sqrt{2N_0}}\right)$
   (d) $P_e \sim \frac{1\times4+4\times3+4\times2}{9}Q\left(\frac{3a}{\sqrt{2N_0}}\right) = 2.67Q\left(\frac{3a}{\sqrt{2N_0}}\right)$

6.

$$P_{s,\text{exact}} = 1 - \left(1 - \frac{2(\sqrt{M}-1)}{\sqrt{M}}Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)\right)^2$$

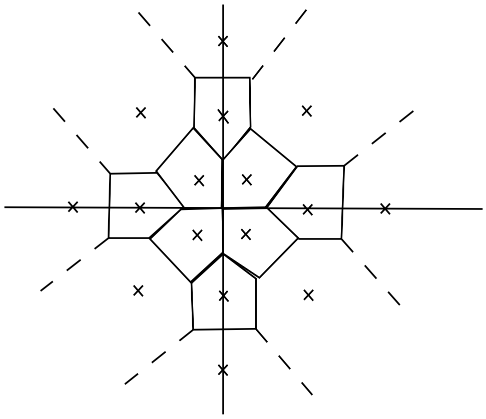

Figure 1: Problem 5

$$P_{s,\text{approx}} = \frac{4(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$$

approximation is better for high SNRs as then the multiplication factor is not important and P_e is dictated by the coefficient of the Q function which are same.

```
MATLAB CODE:
gamma_db = [0:.01:25];
gamma = 10.^(gamma_db/10);
M = 16;

Ps_exact=1-exp(2*log((1-((2*(sqrt(M)-1))/(sqrt(M)))*Q(sqrt((3*gamma)/(M-1))))));
Ps_approx = ((4*(sqrt(M)-1))/sqrt(M))*Q(sqrt((3*gamma)/(M-1)));
semilogy(gamma_db, Ps_exact);
hold on
semilogy(gamma_db,Ps_approx,'b:');
```

7. See figure. The approximation error decreases with SNR because the approximate formula is based on nearest neighbor approximation which becomes more realistic at higher SNR. The nearest neighbor bound over-estimates the error rate because it over-counts the probability that the transmitted signal is mistaken for something other than its nearest neighbors. At high SNR, this is very unlikely and this over-counting becomes negligible.

8. (a)

$$I_x(a) = \int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt$$

since the integral converges we can interchange integral and derivative for a¿0

$$\begin{aligned}\frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{-te^{-at^2}}{x^2+t^2}dt \\ x^2 I_x(a) - \frac{\partial I_x(a)}{\partial a} &= \int_0^\infty \frac{(x^2+t^2)e^{-at^2}}{x^2+t^2}dt = \int_0^\infty e^{-at^2}dt = \frac{1}{2}\sqrt{\frac{\pi}{a}}\end{aligned}$$

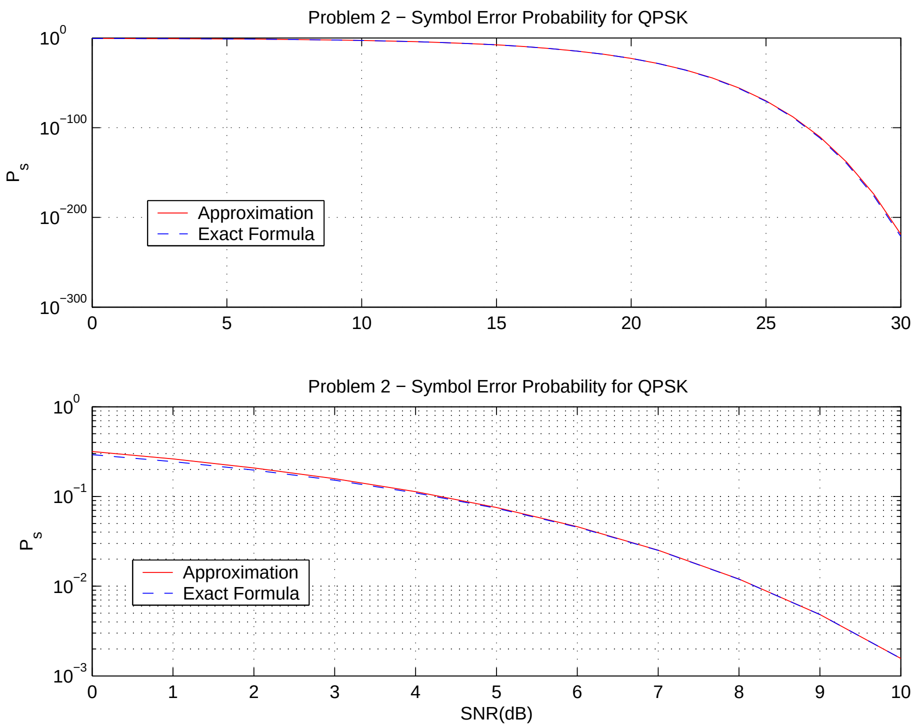

Figure 2: Problem 7

(b) Let $I_x(a) = y$, we get

$$
\begin{aligned}
y' - x^2 y &= -\frac{1}{2}\sqrt{\frac{\pi}{a}} \\
\text{comparing with} & \\
y' + P(a)y &= Q(a) \\
P(a) = -x^2 \quad &, \quad Q(a) = -\frac{1}{2}\sqrt{\frac{\pi}{a}} \\
I.F. = e^{\int P(u)u} = e^{-x^2 a} & \\
\therefore e^{-x^2 a} y &= \int -\frac{1}{2}\sqrt{\frac{\pi}{a}} e^{-x^2 u} du \\
\text{solving we get} & \\
y &= \frac{\pi}{2x} e^{ax^2} erfc(x\sqrt{a})
\end{aligned}
$$

(c)

$$
\begin{aligned}
erfc(x\sqrt{a}) &= I_x(a)\frac{2x}{\pi}e^{-ax^2} = \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \\
a = 1 & \\
erfc(x) &= \frac{2x}{\pi}e^{-ax^2}\int_0^\infty \frac{e^{-at^2}}{x^2+t^2}dt \\
&= \frac{2}{\pi}\int_0^{\pi/2} e^{-x^2/sin^2\theta} d\theta \\
Q(x) = \frac{1}{2}erfc(x/\sqrt{2}) &= \frac{1}{\pi}\int_0^{\pi/2} e^{-x^2/2sin^2\theta} d\theta
\end{aligned}
$$

9. $\overline{P} = 100W$
$N_0 = 4W, \quad SNR = 25$
$P_e = Q(\sqrt{2\gamma}) = Q(\sqrt{50}) = 7.687 \times 10^{-13}$
data requires $P_e \sim 10^{-6}$
voice requires $P_e \sim 10^{-3}$
so it can be used for both.
with fading $P_e = \frac{1}{4\overline{\gamma}_b} = 0.01$
So the system can't be used for data at all. It can be used for very low quality voice.

10. $T_s = 15\mu sec$
at 1mph $T_c = \frac{1}{B_d} = \frac{1}{v/\lambda} = 0.74s \gg T_s$
$\therefore$ outage probability is a good measure.
at 10 mph $T_c = 0.074s \gg T_s$ $\therefore$ outage probability is a good measure.
at 100 mph $T_c = 0.0074s = 7400\mu s > 15\mu s$ outage or outage combined with average prob of error can be a good measure.

11.

$$\begin{aligned} M_\gamma(s) &= \int_0^\infty e^{s\gamma} p(\gamma) d\gamma \\ &= \int_0^\infty e^{s\gamma} \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} d\gamma \\ &= \frac{1}{1 - \overline{\gamma} s} \end{aligned}$$

12. (a) When there is path loss alone, $d = \sqrt{100^2 + 500^2} = 100\sqrt{6} \times 10^3$
$P_e = \frac{1}{2} e^{-\gamma_b} \Rightarrow \gamma_b = 13.1224$
$\frac{P_\gamma}{N_0 B} = 13.1224 \Rightarrow P_\gamma = 1.3122 \times 10^{-14}$
$\frac{P_\gamma}{P_t} = \left[\frac{\sqrt{G}\lambda}{4\pi d}\right]^2 \Rightarrow 4.8488W$

(b)

$$x = 1.3122 \times 10^{-14} = -138.82dB$$

$$P_{\gamma,dB} \sim N(\mu P_\gamma, 8), \sigma_{dB} = 8$$

$$\begin{aligned} P(P_{\gamma,dB} \geq x) &= 0.9 \\ P\left(\frac{P_{\gamma,dB} - \mu P_\gamma}{8} \geq \frac{x - \mu P_\gamma}{8}\right) &= 0.9 \\ \Rightarrow Q\left(\frac{x - \mu P_\gamma}{8}\right) &= 0.9 \\ \Rightarrow \frac{x - \mu P_\gamma}{8} &= -1.2816 \end{aligned}$$

$$\Rightarrow \mu P_\gamma = -128.5672dB = 1.39 \times 10^{-13}$$

13. (a) Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos C$ with a,b = $\sqrt{E_s}$, c = d_{min}, C = $\Theta = 22.5$
c = $d_{min} = \sqrt{2E_s(1 - \cos 22.5)} = .39\sqrt{E_s}$
Can also use formula from reader

(b) $P_s = \alpha_m Q\left(\sqrt{\beta_m \gamma_s}\right) = 2Q\left(\sqrt{\frac{{d_{min}}^2}{2N_o}}\right) = 2Q(\sqrt{.076\gamma_s})$
$\alpha_m = 2, \ \beta_m = .076$

(c) $\overline{P_e} = \int\limits_0^\infty P_s(\gamma_s) f(\gamma_s) d\gamma_s$

$= \int\limits_0^\infty \alpha_m Q(\sqrt{\beta_m \gamma_s}) f(\gamma_s) d\gamma_s$

Using alternative Q form

$= \frac{\alpha_m}{\pi} \int\limits_0^{\frac{\pi}{2}} \left(1 + \frac{g\overline{\gamma_s}}{(\sin\phi)^2}\right)^{-1} d\phi$ with g $= \frac{\beta_m}{2}$

$= \frac{\alpha_m}{2}\left[1 - \sqrt{\frac{g\overline{\gamma_s}}{1+g\overline{\gamma_s}}}\right] = 1 - \sqrt{\frac{.038\overline{\gamma_s}}{1+.038\overline{\gamma_s}}} = \frac{1}{.076\overline{\gamma_s}}$, where we have used an integral table to evaluate the integral

(d) $P_d = \frac{P_s}{4}$

(e) BPSK: $\overline{P}_b = \frac{1}{4\overline{\gamma}_b} = 10^{-3}$, $\Rightarrow \overline{\gamma_b} = 250$, 16PSK: From above get $\overline{\gamma_s} = 3289.5$

Penalty $= \frac{3289.5}{250} = 11.2$dB

Also will accept $\gamma_b(16PSK) = 822 \Rightarrow= 5.2dB$

14.

$$\begin{aligned}
\overline{P}_b &= \int_0^\infty P_b(\gamma)p(\gamma)d\gamma \\
P_b(\gamma) = \frac{1}{2}e^{-\gamma} & \\
\overline{P}_b &= \frac{1}{2}\int_0^\infty e^{-\gamma b} p_\gamma(\gamma) d\gamma = \frac{1}{2}\mathcal{M} \\
\text{But from 6.65} & \\
\mathcal{M}_\gamma(s) &= \left(1 - \frac{s\gamma}{m}\right)^{-m} \\
\therefore \overline{P}_b &= \frac{1}{2}\left(1 + \frac{\gamma}{m}\right)^{-m} \\
\text{For} \quad M = 4, \quad \gamma = 10 & \\
\overline{P}_b &= 3.33 \times 10^{-3}
\end{aligned}$$

15.
```
%Script used to plot the average probability of bit error for BPSK modulation in
%Nakagami fading m = 1, 2, 4.
%Initializations
avg_SNR = [0:0.1:20]; gamma_b_bar = 10.^(avg_SNR/10); m = [1 2 4];
line = ['-k', '-r', '-b']

for i = 1:size(m,2)
   for j = 1:size(gamma_b_bar, 2)
        Pb_bar(i,j) = (1/pi)*quad8('nakag_MGF',0,pi/2,[],[],gamma_b_bar(j),m(i),1);
   end
   figure(1);
   semilogy(avg_SNR, Pb_bar(i,:), line(i));
   hold on;
end

xlabel('Average SNR ( gamma_b ) in dB'); ylabel('Average bit error
probability ( P_b ) '); title('Plots of P_b for BPSK modulation in
Nagakami fading for m = 1, 2, 4'); legend('m = 1', 'm = 2', 'm =
4');
```

```
function out = nakag_MGF(phi, gamma_b_bar, m, g);
%This function calculates the m-Nakagami MGF function for the specified values of phi.
%phi can be a vector. Gamma_b_bar is the average SNR per bit, m is the Nakagami parameter
%and g is given by Pb(gamma_b) = aQ(sqrt(2*g*gamma_b)).

out = (1 + gamma_b_bar./(m*(sin(phi).^2)) ).^(-m);
```

SNR = 10dB

M	BER
1	2.33×10^{-2}
2	5.53×10^{-3}
4	1.03×10^{-3}

16. For DPSK in Rayleigh fading, $\overline{P_b} = \frac{1}{2\overline{\gamma_b}} \Rightarrow \overline{\gamma_b} = 500$
$N_oB = 3\times10^{-12} mW \Rightarrow P_{target} = \overline{\gamma}_b N_0 B = 1.5\times10^{-9} mW$ = -88.24 dBm

Now, consider shadowing:
$P_{out} = P[P_r < P_{target}] = P[\Psi < P_{target} - \overline{P_r}] = \Phi\left(\frac{P_{target}-\overline{P_r}}{\sigma}\right)$
$\Rightarrow \Phi^{-1}(.01) = 2.327 = \frac{P_{target}-\overline{P_r}}{\sigma}$
$\overline{P_r} = -74.28$ dBm $= 3.73\times10^{-8}$ mW $= P_t\left(\frac{\lambda}{4\pi d}\right)^2$
$\Rightarrow$ d = 1372.4 m

17. (a)

$$\gamma_1 = \begin{cases} 0 & \text{w.p. } 1/3 \\ 30 & \text{w.p. } 2/3 \end{cases}$$

$$\gamma_2 = \begin{cases} 5 & \text{w.p. } 1/2 \\ 10 & \text{w.p. } 1/2 \end{cases}$$

In MRC, $\gamma_\Sigma = \gamma_1 + \gamma_2$. So,

$$\gamma_\Sigma = \begin{cases} 5 & \text{w.p. } 1/6 \\ 10 & \text{w.p. } 1/6 \\ 35 & \text{w.p. } 1/3 \\ 40 & \text{w.p. } 1/3 \end{cases}$$

(b) Optimal Strategy is water-filling with power adaptation:

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma}, & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

Notice that we will denote γ_Σ by γ only hereon to lighten notation. We first assume $\gamma_0 < 5$,

$$\sum_{i=1}^{4}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right)p_i = 1$$

$$\Rightarrow \frac{1}{\gamma_0} = 1 + \sum_{i=1}^{4}\frac{p_i}{\gamma_i}$$

$$\Rightarrow \gamma_0 = 0.9365 < 5$$

So we found the correct value of γ_0.

$$C = B \sum_{i=1}^{4} \log_2 \left(\frac{\gamma_i}{\gamma_0} \right) p_i$$

C = 451.91 Kbps

(c) Without, receiver knowledge, the capacity of the channel is given by:

$$C = B \sum_{i=1}^{4} \log_2 (1 + \gamma_i) p_i$$

C = 451.66 Kbps

Notice that we have denote γ_Σ by γ to lighten notation.

18. (a)

$$\begin{aligned} s(k) &= s(k-1) \\ z(k-1) &= g_{k-1} s(k-1) + n(k-1) \\ z(k) &= g_k s(k) + n(k) \end{aligned}$$

From equation 5.63 , the input to the phase comparator is

$$\begin{aligned} z(k) z^\star(k-1) &= g_k g_(k-1)^\star s(k) s^\star(k-1) + g_k s(k-1) n_{k-1}^\star + \\ & \quad g_(k-1)^\star s^\star(k-1) n_k + n_k n_{k-1}^\star \end{aligned}$$

but $s(k) = s(k-1)$

$$s(k) s^\star(k-1) = |s_k|^2 = 1 \quad \text{(normalized)}$$

(b)

$$\begin{aligned} \widetilde{n}_k &= s_{k-1}^\star n_k \\ \widetilde{n}_{k-1} &= s_{k-1}^\star n_{k-1} \\ \widetilde{z} &= g_k g_{k-1}^\star + g_k \widetilde{n}_{k-1}^\star + g_{k-1}^\star \widetilde{n}_k \\ \phi_x(s) = \frac{p_1 p_2}{(s-p_1)(s-p_2)} &= \frac{A}{s-p_1} + \frac{B}{s-p_2} \\ A = (s-p_1)\phi_x(s)|_{s=p_1} &= \frac{p_1 p_2}{p_1 - p_2} \\ B = (s-p_2)\phi_x(s)|_{s=p_2} &= \frac{p_1 p_2}{p_2 - p_1} \end{aligned}$$

(c) Relevant part of the pdf

$$\phi_x(s) = \frac{p_1 p_2}{(p_2 - p_1)(s - p_2)}$$

$$\therefore p_x(x) = \frac{p_1 p_2}{(p_2 - p_1)} \mathcal{L}^{-1} \left(\frac{1}{(s-p_2)} \right) = \frac{p_1 p_2}{(p_2 - p_1)} e^{p_2 x} \quad , x < 0$$

(d)

$$P_b = prob(x < 0) = \frac{p_1 p_2}{(p_2 - p_1)} \int_{-\infty}^{0} e^{p_2 x} dx = -\frac{p_1}{p_2 - p_1}$$

(e)

$$p_2 - p_1 = \frac{1}{2N_0[\overline{\gamma}_b(1-\rho_c)+1]} + \frac{1}{2N_0[\overline{\gamma}_b(1+\rho_c)+1]} = \frac{\overline{\gamma}_b + 1}{N_0[\overline{\gamma}_b(1-\rho_c)+1][\overline{\gamma}_b(1+\rho_c)+1]}$$

$$\therefore \overline{P}_b = \frac{\overline{\gamma}_b(1-\rho_c)+1}{2(\overline{\gamma}_b+1)}$$

(f) $\rho_c = 1$

$$\therefore \overline{P}_b = \frac{1}{2(\overline{\gamma}_b+1)}$$

19. $\overline{\gamma}_b$ 0 to 60dB
$\rho_c = J_0(2\pi B_D T)$ with $B_D T = 0.01, 0.001, 0.0001$
where J_0 is 0 order Bessel function of 1^{st} kind.

$$\overline{P}_b = \frac{1}{2}\left[\frac{1+\overline{\gamma}_b(1-\rho_c)}{1+\overline{\gamma}_b}\right]$$

when $B_D T = 0.01$, floor can be seen about $\overline{\gamma}_b = 40dB$
when $B_D T = 0.001$, floor can be seen about $\overline{\gamma}_b = 60dB$
when $B_D T = 0.0001$, floor can be seen between $\overline{\gamma}_b = 0$ to $60dB$

20. Data rate = 40 Kbps
Since DQPSK has 2 bits per symbol. $\therefore T_s = \frac{2}{40\times10^3} = 5\times10^{-5}sec$

DQPSK
Gaussian Doppler power spectrum, $\rho_c = e^{-(\pi B_D T)^2}$
$B_D = 80Hz$
Rician fading $K = 2$
$\rho_c = 0.9998$

$$\overline{P}_{floor} = \frac{1}{2}\left[1 - \sqrt{\frac{(\rho_c/\sqrt{2})^2}{1-(\rho_c/\sqrt{2})^2}}\right]\exp\left[-\frac{(2-\sqrt{2})K/2}{1-\rho_c/\sqrt{2}}\right] = 2.138\times10^{-5}$$

21. ISI:

Formula based approach:

$$P_{floor} = \left(\frac{\sigma T_m}{T_s}\right)^2$$

Since its Rayleigh fading, we can assume that $\sigma_{T_m} \approx \mu_{T_m} = 100$ns
$P_{floor} \leq 10^{-4}$
which gives us

$$\left(\frac{\sigma T_m}{T_s}\right)^2 \leq 10^{-4}$$

$$T_s \geq \frac{\sigma_{T_m}}{\sqrt{P_{floor}}} = 10\mu sec$$

So, $T_s \geq 10\mu s$. $T_b \geq 5\mu s$. $R_b \leq 200$ Kbps.

Thumb - Rule approach:

μ_t = 100 nsec will determine ISI. As long as $T_s \gg \mu_T$, ISI will be negligible. Let $T_s \geq 10\ \mu_T$. Then $R \leq \frac{2bits}{symbol} \frac{1}{T_s} \frac{symbols}{sec}$ = 2Mbps

Doppler:

B_D = 80 Hz

$$P_{floor} = 10^{-4} \geq \frac{1}{2}\left[1 - \sqrt{\frac{\left(\rho_c/\sqrt{2}\right)^2}{1-\left(\rho_c/\sqrt{2}\right)^2}}\right]$$

$$\Rightarrow \rho_c \geq 0.9999$$

But ρ_c for uniform scattering is $J_0(2\pi B_D T_s)$, so

$$\rho_c = J_0(2\pi B_D T_s) = 1 - (\pi f_D T_s)^2 \geq 0.9999$$

$$\Rightarrow T_s \leq 39.79\mu s$$

$T_b \leq 19.89\mu s$. $R_b \geq 50.26$ Kbps.

Combining the two, we have 50.26 Kbps $\leq R_b \leq$ 200 Kbps (or 2 Mbps).

22. From figure 6.5
with $P_b = 10^{-3}$, $\quad d = \theta_{T_m}/T_s$, $\quad \theta_{T_m} = 3\mu s$
BPSK

$$d = 5 \times 10^{-2}$$

$$T_s = 60\mu sec$$

$$R = 1/T_s = 16.7Kbps$$

QPSK

$$d = 4 \times 10^{-2}$$

$$T_s = 75\mu sec$$

$$R = 2/T_s = 26.7Kbps$$

MSK

$$d = 3 \times 10^{-2}$$

$$T_s = 100\mu sec$$

$$R = 2/T_s = 20Kbps$$

Chapter 7

1. $P_s = 10^{-3}$
   QPSK, $P_s = 2Q(\sqrt{\gamma_s}) \leq 10^{-3}$, $\gamma_s \geq \gamma_0 = 10.8276$.

$$P_{out}(\gamma_0) = \prod_{i=1}^{M}\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_i}}\right)$$

   $\overline{\gamma}_1 = 10$, $\overline{\gamma}_2 = 31.6228$, $\overline{\gamma}_3 = 100$.

   $M = 1$
   $P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right) = 0.6613$

   $M = 2$
   $P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_2}}\right) = 0.1917$

   $M = 3$
   $P_{out} = \left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_1}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_2}}\right)\left(1 - e^{-\frac{\gamma_0}{\overline{\gamma}_3}}\right) = 0.0197$

2. $p_{\gamma_\Sigma}(\gamma) = \frac{M}{\overline{\gamma}}\left[1 - e^{-\gamma/\overline{\gamma}}\right]^{M-1} e^{-\gamma/\overline{\gamma}}$
   $\overline{\gamma} = 10dB = 10$
   as we increase M, the mass in the pdf keeps on shifting to higher values of γ and so we have higher values of γ and hence lower probability of error.

   MATLAB CODE

```
gamma = [0:.1:60];
gamma_bar = 10;
M = [1 2 4 8 10];
fori=1:length(M)
    pgamma(i,:) = (M(i)/gamma_bar)*(1-exp(-gamma/gamma_bar)).^...
                  (M(i)-1).*(exp(-gamma/gamma_bar));
end
```

3.

$$\begin{aligned}
\overline{P}_b &= \int_0^\infty \frac{1}{2} e^{-\gamma} p_{\gamma_\Sigma}(\gamma) d\gamma \\
&= \int_0^\infty \frac{1}{2} e^{-\gamma} \frac{M}{\overline{\gamma}}\left[1 - e^{-\gamma/\overline{\gamma}}\right]^{M-1} e^{-\gamma/\overline{\gamma}} d\gamma \\
&= \frac{M}{2\overline{\gamma}} \int_0^\infty e^{-(1+1/\overline{\gamma})\gamma}\left[1 - e^{-\gamma/\overline{\gamma}}\right]^{M-1} d\gamma \\
&= \frac{M}{2\overline{\gamma}} \sum_{n=0}^{M-1} \begin{pmatrix} M-1 \\ n \end{pmatrix} (-1)^n e^{-(1+1/\overline{\gamma})\gamma} d\gamma \\
&= \frac{M}{2} \sum_{n=0}^{M-1} \begin{pmatrix} M-1 \\ n \end{pmatrix} (-1)^n \frac{1}{1+n+\overline{\gamma}} = \text{desired expression}
\end{aligned}$$

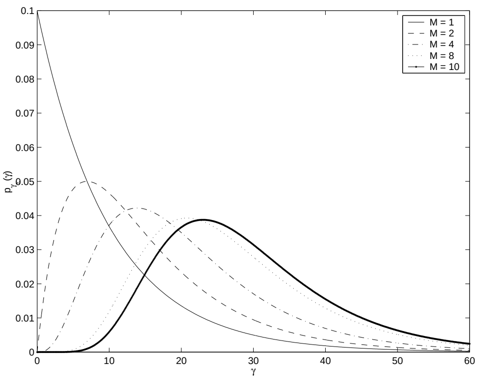

Figure 1: Problem 2

4.

$$p_{\gamma_\Sigma}(\gamma) = \begin{cases} Pr\{\gamma_2 < \gamma_\tau, \gamma_1 < \gamma\} & \gamma < \gamma_\tau \\ Pr\{\gamma_\tau \le \gamma_1 \le \gamma\} + Pr\{\gamma_2 < \gamma_\tau, \gamma_1 < \gamma\} & \gamma > \gamma_\tau \end{cases}$$

If the distribution is iid this reduces to

$$p_{\gamma_\Sigma}(\gamma) = \begin{cases} P_{\gamma_1}(\gamma) P_{\gamma_2}(\gamma_\tau) & \gamma < \gamma_\tau \\ Pr\{\gamma_\tau \le \gamma_1 \le \gamma\} + P_{\gamma_1}(\gamma) P_{\gamma_2}(\gamma_\tau) & \gamma > \gamma_\tau \end{cases}$$

5.

$$\overline{P}_b = \int_0^\infty \frac{1}{2} e^{-\gamma} p_{\gamma_\Sigma}(\gamma) d\gamma$$

$$p_{\gamma_\Sigma}(\gamma) = \begin{cases} \left(1 - e^{-\gamma_T/\overline{\gamma}}\right) \frac{1}{\overline{\gamma}} e^{-\gamma_r/\overline{\gamma}} & \gamma < \gamma_T \\ \left(2 - e^{-\gamma_T/\overline{\gamma}}\right) \frac{1}{\overline{\gamma}} e^{-\gamma_r/\overline{\gamma}} & \gamma > \gamma_T \end{cases}$$

$$\begin{aligned} \overline{P}_b &= \frac{1}{2\overline{\gamma}} \left(1 - e^{-\gamma_T/\overline{\gamma}}\right) \int_0^{\gamma_T} e^{-\gamma/\overline{\gamma}} e^{-\gamma} d\gamma + \frac{1}{2\overline{\gamma}} \left(2 - e^{-\gamma_r/\overline{\gamma}}\right) \int_{\gamma_T}^\infty e^{-\gamma/\overline{\gamma}} e^{-\gamma} d\gamma \\ &= \frac{1}{2(\overline{\gamma}+1)} \left(1 - e^{-\gamma_T/\overline{\gamma}} + e^{-\gamma_T} e^{-\gamma_T/\overline{\gamma}}\right) \end{aligned}$$

6.

	$\overline{P}_b$	$\overline{P}_b(10dB)$	$\overline{P}_b(20dB)$
no diversity	$\frac{1}{2(\overline{\gamma}+1)}$	0.0455	0.0050
SC(M=2)	$\frac{M}{2} \sum_{m=0}^{M-1} (-1)^m \frac{\begin{pmatrix} M-1 \\ m \end{pmatrix}}{1+m+\overline{\gamma}}$	0.0076	9.7×10^{-5}
SSC	$\frac{1}{2(\overline{\gamma}+1)} \left(1 - e^{-\gamma_T/\overline{\gamma}} + e^{-\gamma_T} e^{-\gamma_T/\overline{\gamma}}\right)$	0.0129	2.7×10^{-4}

As SNR increases SSC approaches SC

7. See

```
MATLAB CODE:
gammab_dB = [0:.1:20];
gammab = 10.^(gammab_dB/10);
M= 2;
```

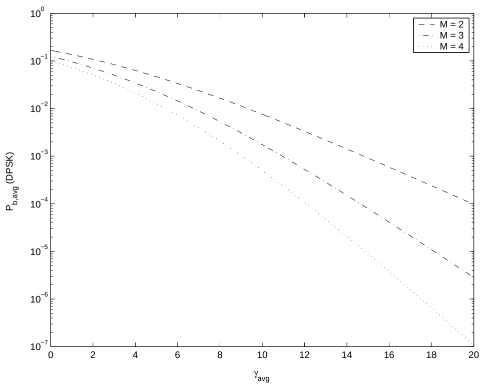

Figure 2: Problem 7

```
for j = 1:length(gammab)
    Pbs(j) = 0
    for m = 0:M-1
        f = factorial(M-1)/(factorial(m)*factorial(M-1-m));
        Pbs(j) = Pbs(j) + (M/2)*((-1)^m)*f*(1/(1+m+gammab(j)));
    end
end
semilogy(gammab_dB,Pbs,'b--')
hold on

M = 3;
for j = 1:length(gammab)
    Pbs(j) = 0
    for m = 0:M-1
        f = factorial(M-1)/(factorial(m)*factorial(M-1-m));
        Pbs(j) = Pbs(j) + (M/2)*((-1)^m)*f*(1/(1+m+gammab(j)));
    end
end
semilogy(gammab_dB,Pbs,'b-.');
hold on

M = 4;
for j = 1:length(gammab)
    Pbs(j) = 0
    for m = 0:M-1
        f = factorial(M-1)/(factorial(m)*factorial(M-1-m));
        Pbs(j) = Pbs(j) + (M/2)*((-1)^m)*f*(1/(1+m+gammab(j)));
    end
end
semilogy(gammab_dB,Pbs,'b:');
hold on
```

8.

$$\gamma_\Sigma = \frac{1}{N_0}\frac{\left(\sum_{i=1}^M a_i\gamma_i\right)^2}{\sum_{i=1}^M a_i^2} \le \frac{1}{N_0}\frac{\sum a_i^2 \sum \gamma_i^2}{\sum a_i^2} = \frac{\sum \gamma_i^2}{N_0}$$

Where the inequality above follows from Cauchy-Schwartz condition. Equality holds if $a_i = c\gamma_i$ where c is a constant

9. (a) $\gamma_i = 10$ dB $= 10$, $1 \le i \le N$
   $N = 1$, $\gamma = 10$, $M = 4$
   $P_b = .2e^{-1.5\frac{\gamma}{(M-1)}} = .2e^{-15/3} = 0.0013.$

   (b) In MRC, $\gamma_\Sigma = \gamma_1 + \gamma_2 + \ldots + \gamma_N$.
   So $\gamma_\Sigma = 10N$

$$P_b = .2e^{-1.5\frac{\gamma_\Sigma}{(M-1)}} = .2e^{-5N} \le 10^{-6}$$
$$\Rightarrow N \ge 2.4412$$

   So, take $N = 3$, $P_b = 6.12 \times 10^{-8} \le 10^{-6}$.

10. Denote $N(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$, $Q'(x) = -N(x)$

$$\begin{aligned}
\overline{P}_b &= \int_0^\infty Q(\sqrt{2\gamma})dP(\gamma) \\
Q(\infty) = 0, \quad P(0) = 0 & \\
\frac{d}{d\gamma}Q(\sqrt{2\gamma}) &= -N(\sqrt{2\gamma})\frac{\sqrt{2}}{2\sqrt{\gamma}} = -\frac{1}{\sqrt{2\pi}}e^{-\gamma}\frac{1}{2\sqrt{\gamma}} \\
\overline{P}_b &= \int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-\gamma}\frac{1}{2\sqrt{\gamma}}P(\gamma)d\gamma \\
P(\gamma) &= 1 - e^{-\gamma/\overline{\gamma}}\sum_{k=1}^{M}\frac{(\gamma/\overline{\gamma})^{k-1}}{(k-1)!} \\
1 \quad \int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-\gamma}\frac{1}{2\sqrt{\gamma}}d\gamma &= \frac{1}{2} \\
2 \quad \int_0^\infty \frac{1}{\sqrt{2\pi}}e^{-\gamma}\frac{1}{2\sqrt{\gamma}}e^{-\gamma/\overline{\gamma}}\sum_{k=1}^{M}\frac{(\gamma/\overline{\gamma})^{k-1}}{(k-1)!}d\gamma &= \sum_{k=1}^{M}\frac{1}{(k-1)!}\left[\frac{1}{2\sqrt{\pi}}\int_0^\infty e^{-\gamma \quad 1+\frac{1}{\overline{\gamma}}}\ \gamma^{-1/2}\left(\frac{\gamma}{\overline{\gamma}}\right)^{k-1}d\gamma\right] \\
\text{Denote } A = \left(1+\frac{1}{\overline{\gamma}}\right)^{-1/2} & \\
&= \sum_{m=0}^{M-1}\frac{1}{m!}\frac{1}{2\sqrt{pi}}\int_0^\infty e^{-\gamma/A^2}\gamma^{-1/2}\left(\frac{\gamma}{\overline{\gamma}}\right)^m d\gamma \\
\text{let } \gamma/A^2 = u & \\
&= \sum_{m=0}^{M-1}\frac{1}{m!}\frac{1}{2\sqrt{pi}}\int_0^\infty e^{-u}\frac{u^{-1/2}}{A}\left(\frac{uA^2}{\overline{\gamma}}\right)^m A^2du \\
&= \frac{A}{2} + \sum_{m=1}^{M-1}\left(\begin{array}{c}2m-1\\ m\end{array}\right)\frac{A^{2m}}{2^{2m}}\frac{A}{\overline{\gamma}^m} \\
\overline{P}_b &= \frac{1-A}{2} - \sum_{m=1}^{M-1}\left(\begin{array}{c}2m-1\\ m\end{array}\right)\frac{A^{2m+1}}{2^{2m}\overline{\gamma}^m}
\end{aligned}$$

11.

$$\text{Denote} N(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad Q'(x) = [1-\phi(x)]' = -N(x)$$

$$\overline{P_b} = \int_0^\infty Q(\sqrt{2\gamma}) dP(\gamma) \quad = \quad \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\gamma} \frac{1}{\sqrt{2\gamma}} P(\gamma) d\gamma$$

$$\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\gamma} \frac{1}{\sqrt{2\gamma}} d\gamma = \frac{1}{\sqrt{\pi}} \Gamma\left(\frac{1}{2}\right) = \frac{1}{2} \tag{1}$$

$$\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\gamma} \frac{1}{\sqrt{2\gamma}} e^{-2\gamma/\overline{\gamma}} d\gamma = \frac{1}{2\sqrt{1+\frac{2}{\overline{\gamma}}}} \tag{2}$$

$$\int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-\gamma} \frac{1}{\sqrt{2\gamma}} \sqrt{\frac{\pi\gamma}{\overline{\gamma}}} e^{-\gamma/\overline{\gamma}} \left(1 - 2Q\left(\sqrt{\frac{2\gamma}{\overline{\gamma}}}\right)\right) d\gamma = \frac{1}{2\sqrt{\overline{\gamma}}} \frac{1}{B\sqrt{A\overline{\gamma}}} \tag{3}$$

$$\text{where } A = 1 + \frac{2}{\overline{\gamma}}, \quad B = 1 + \frac{1}{\overline{\gamma}}$$

$$\text{overall } \overline{P}_b = \frac{1}{2}\left[1 - \sqrt{1 - \frac{1}{(1+\overline{\gamma})^2}}\right]$$

12.

	$\overline{P}_b$	$\overline{P}_b(10dB)$	$\overline{P}_b(20dB)$
no diversity	$\frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}_b}{1+\overline{\gamma}_b}}\right]$	0.0233	0.0025
two branch SC	$\int Q(\sqrt{2\gamma}) p_{\gamma_\Sigma} d\gamma$	0.0030	3.67×10^{-5}
two branch SSC	$\int Q(\sqrt{2\gamma}) p_{\gamma_\Sigma} d\gamma$	0.0057	1.186×10^{-4}
two branch EGC	$\int Q(\sqrt{2\gamma}) p_{\gamma_\Sigma} d\gamma$	0.0021	2.45×10^{-5}
two branch MRC	$\int Q(\sqrt{2\gamma}) p_{\gamma_\Sigma} d\gamma$	0.0016	0.84×10^{-5}

As the branch SNR increases the performance of all diversity combining schemes approaches the same.

```
MATLAB CODE:
gammatv = [.01:.1:10];
gammab = 100;
gamma = [0:.01:50*gammab];
for i = 1:length(gammatv)
    gammat = gammatv(i);
    gamma1 = [0:.01:gammat];
    gamma2 = [gammat+.01:.01:50*gammab];
    tointeg1 = Q(sqrt(2*gamma1)).*((1/gammab)*(1-exp(-gammat/gammab)).*exp(-gamma1/gammab));
    tointeg2 = Q(sqrt(2*gamma2)).*((1/gammab)*(2-exp(-gammat/gammab)).*exp(-gamma2/gammab));
    anssum(i) = sum(tointeg1)*.01+sum(tointeg2)*.01;
end
```

13.
```
gammab_dB = [10];
gammab = 10.^(gammab_dB/10);
Gamma=sqrt(gammab./(gammab+1));
pb_mrc =(((1-Gamma)/2).^2).*(((1+Gamma)/2).^0+2*((1+Gamma)/2).^1);
pb_egc = .5*(1-sqrt(1-(1./(1+gammab)).^2));
```

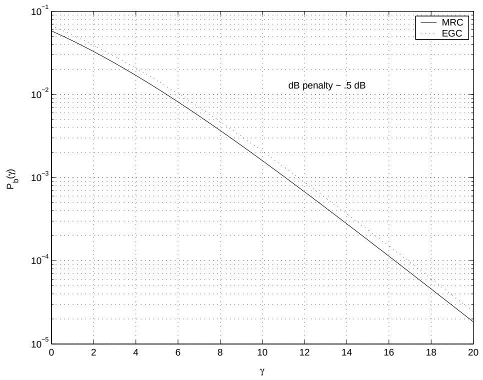

Figure 3: Problem 13

14. $10^{-3} = P_b = Q(\sqrt{2\gamma_b}) \Rightarrow 4.75, \overline{\gamma} = 10$
MRC $P_{out} = 1 - e^{-\gamma_0/\overline{\gamma}} \sum_{k=1}^{M} \frac{(\gamma_0/\overline{\gamma})^{k-1}}{(k-1)!} = 0.0827$
ECG $P_{out} = 1 - e^{-2\gamma_R} - \sqrt{\pi\gamma_R} e^{-\gamma_R}(1 - 2Q(\sqrt{2\gamma_R})) = 0.1041 > P_{out,MRC}$

15. $\overline{P}_{b,MRC} = 0.0016 < 0.0021 \overline{P}_{b,EGC}$

16. If each branch has $\overline{\gamma} = 10dB$ Rayleigh
$\gamma_\Sigma =$ overall recvd SNR $= \frac{\gamma_1+\gamma_2}{2} \sim \frac{\gamma e^{-\gamma/(\overline{\gamma}/2)}}{(\overline{\gamma}/2)^2} \gamma \geq 0$
BPSK

$$\overline{P}_b = \int_0^\infty Q(\sqrt{2\gamma}) p_{\gamma_\Sigma} d\gamma = 0.0055$$

17. $p(\gamma)$ where $\int_0^\infty p(\gamma) e^{-x\gamma} d\gamma = \frac{0.01\overline{\gamma}}{\sqrt{x}}$
we will use MGF approach

$$\overline{P}_b = \frac{1}{\pi} \int_0^{\pi/2} \Pi_{i=1}^2 M_{\gamma_i}\left(-\frac{1}{\sin^2\phi}\right) d\phi$$

$$= \frac{1}{\pi} \int_0^{\pi/2} (0.01\overline{\gamma}\sin\phi)^2 d\phi$$

$$= \frac{(0.01\overline{\gamma})^2}{4} = 0.0025$$

18.

$$\overline{P}_b = \left(\frac{1-\pi}{2}\right)^3 \sum_{m=0}^{2} \left(\begin{array}{c} l+m \\ m \end{array}\right) \left(\frac{1+\pi}{2}\right)^m; \quad \pi = \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}$$

Nakagami-2 fading

$$M_\gamma\left(-\frac{1}{\sin^2\phi}\right) = \left(1 + \frac{\overline{\gamma}}{2\sin^2\phi}\right)^{-2}$$

$$\overline{P}_b = \frac{1}{\pi} \int_0^{\pi/2} \left(M_\gamma\left(-\frac{1}{\sin^2\phi}\right)\right)^3 d\phi, \overline{\gamma} = 10^{1.5} = 5.12 \times 10^{-9}$$

```
MATLAB CODE:

gammab = 10^(1.5);
Gamma = sqrt(gammab./(gammab+1));
```

```
sumf = 0;

for m = 0:2
    f = factorial(2+m)/(factorial(2)*factorial(m));
    sumf = sumf+f*((1+Gamma)/2)^m;
end
pb_rayleigh = ((1-Gamma)/2)^3*sumf;
phi = [0.001:.001:pi/2];
sumvec = (1+(gammab./(2*(sin(phi).^2)))).^(-6);
pb_nakagami = (1/pi)*sum(sumvec)*.001;
```

19.

$$\overline{P}_b = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\overline{\gamma}}{2\sin^2\phi}\right)^{-2}\left(1+\frac{\overline{\gamma}}{\sin^2\phi}\right)^{-1}d\phi$$

```
gammab_dB = [5:.1:20];
gammabvec = 10.^(gammab_dB/10);

for i = 1:length(gammabvec)
    gammab = gammabvec(i);
    phi = [0.001:.001:pi/2];
    sumvec = ((1+(gammab./(2*(sin(phi).^2)))).^(-2)).*((1+...
             (gammab./(1*(sin(phi).^2)))).^(-1));
    pb_nakagami(i) = (1/pi)*sum(sumvec)*.001;
end
```

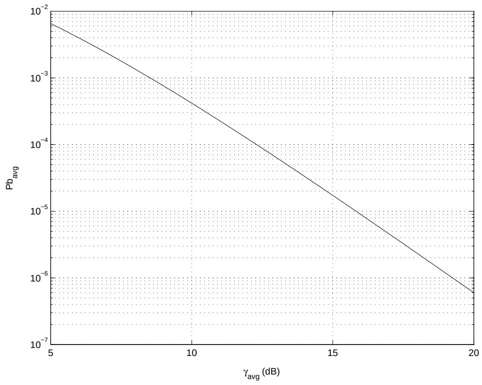

Figure 4: Problem 19

20.

$$P_b = \frac{2}{3}Q\left(\sqrt{2\gamma_b(3)}\sin\left(\frac{\pi}{8}\right)\right)$$

$$\alpha = 2/3, \quad g = 3\sin^2\left(\frac{\pi}{8}\right)$$

$$M_\gamma\left(-\frac{g}{\sin^2\phi}\right) = \left(1+\frac{g\overline{\gamma}}{\sin^2\phi}\right)^{-1}$$

$$\overline{P}_b = \frac{\alpha}{\pi}\int_0^{\pi/2}\left(1+\frac{g\overline{\gamma}}{\sin^2\phi}\right)^{-M}d\phi$$

```
MATLAB CODE:
M = [1 2 4 8];
alpha = 2/3; g = 3*sin(pi/8)^2;

gammab_dB = [5:.1:20];
gammabvec = 10.^(gammab_dB/10);

for k = 1:length(M)
    for i = 1:length(gammabvec)
        gammab = gammabvec(i);
        phi = [0.001:.001:pi/2];
        sumvec = ((1+((g*gammab)./(1*(sin(phi).^2)))).^(-M(k)));
        pb_nakagami(k,i) = (alpha/pi)*sum(sumvec)*.001;
    end
end
```

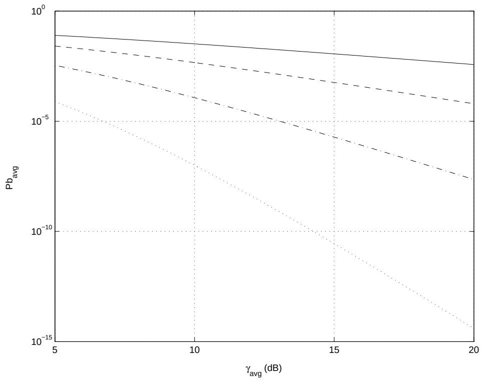

Figure 5: Problem 20

21.

$$
\begin{aligned}
Q(z) &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left[-\frac{z^2}{\sin^2\phi}\right] d\phi \quad , z>0 \\
Q^2(z) &= \frac{1}{\pi}\int_0^{\pi/4} \exp\left[-\frac{z^2}{2\sin^2\phi}\right] d\phi \quad , z>0 \\
P_s(\gamma_s) &= \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\pi/2} \exp\left[-\frac{g\gamma_s}{\sin^2\phi}\right] d\phi - \\
&\quad \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}^2\right)\int_0^{\pi/4} \exp\left[-\frac{g\gamma_s}{\sin^2\phi}\right] d\phi \\
\overline{P}_s &= \int_0^{\infty} P_s(\gamma_\Sigma) p_{\gamma_\Sigma}(\gamma_\Sigma) d\gamma_\Sigma \\
&= \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\pi/2}\int_0^{\infty} exp\left(\frac{g\gamma_\Sigma}{\sin^2\phi}\right) p_{\gamma_\Sigma}(\gamma) d\gamma_\Sigma d\phi - \\
&\quad \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\pi/4}\int_0^{\infty} exp\left(\frac{g\gamma_\Sigma}{\sin^2\phi}\right) p_{\gamma_\Sigma}(\gamma) d\gamma_\Sigma d\phi \\
\text{But } \gamma_\Sigma = \gamma_1+\gamma_2+\ldots+\gamma_M = \Sigma\gamma_i & \\
&= \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)\int_0^{\pi/2} \Pi_{i=1}^M M_{\gamma_i}\left(-\frac{g}{\sin^2\phi}\right) d\phi - \\
&\quad \frac{4}{\pi}\left(1-\frac{1}{\sqrt{M}}\right)^2\int_0^{\pi/4} \Pi_{i=1}^M M_{\gamma_i}\left(-\frac{g}{\sin^2\phi}\right) d\phi
\end{aligned}
$$

22. Rayleigh: $M_{\gamma_s}(s) = (1-s\overline{\gamma}_s)^{-1}$
Rician: $M_{\gamma_s}(s) = \frac{1+k}{1+k-s\overline{\gamma}_s}\exp\left(\frac{k_s\overline{\gamma}_s}{1+k-s\overline{\gamma}_s}\right)$
MPSK

$$
\overline{P}_s = \int_0^{(M-1)\pi/M} M\gamma_s\left(-\frac{g}{\sin^2\phi}\right) d\phi \rightarrow \text{no diversity}
$$

Three branch diversity

$$
\begin{aligned}
\overline{P}_s &= \frac{1}{\pi}\int_0^{(M-1)\pi/M}\left(1+\frac{g\overline{\gamma}}{\sin^2\phi}\right)^{-1}\left[\frac{(1+k)\sin^2\phi}{(1+k)\sin^2\phi+g\overline{\gamma}_s}\exp\left(-\frac{k\overline{\gamma}_s g}{(1+k)\sin^2\phi+g\overline{\gamma}_s}\right)\right]^2 d\phi \\
g = \sin^2\left(\frac{\pi}{16}\right) & \\
&= 0.1670
\end{aligned}
$$

MQAM:
Formula derived in previous problem with $g = \frac{1.5}{16-1} = \frac{1.5}{15}$
$\overline{P}_s = 0.0553$

```
MATLAB CODE:

gammab_dB = 10;
gammab = 10.^(gammab_dB/10);
K = 2;
```

```
g = sin(pi/16)^2;
phi = [0.001:.001:pi*(15/16)];

sumvec=((1+((g*gammab)./(sin(phi).^2))).^(-1)).*((((...
        (1+K)*sin(phi).^2)./((1+K)*sin(phi).^2+...
         g*gammab)).*exp(-(K*gammab*g)./((1+K)*sin(phi).^2+g*gammab))).^2);
pb_mrc_psk = (1/pi)*sum(sumvec)*.001;

g = 1.5/(16-1);
phi1 = [0.001:.001:pi/2];
phi2 = [0.001:.001:pi/4];

sumvec1=((1+((g*gammab)./(sin(phi1).^2))).^...
        (-1)).*(((((1+K)*sin(phi1).^2)./((1+K)*...
        sin(phi1).^2+g*gammab)).*exp(-(K*gammab*g)./((...
        1+K)*sin(phi1).^2+g*gammab))).^2);
sumvec2=((1+((g*gammab)./(sin(phi2).^2))).^(-1)).*((((...
         (1+K)*sin(phi2).^2)./((1+K)*sin(phi2).^2+...
        g*gammab)).*exp(-(K*gammab*g)./((1+K)*sin(phi2).^2+g*gammab))).^2);
pb_mrc_qam = (4/pi)*(1-(1/sqrt(16)))*sum(sumvec1)*.001 - ...
             (4/pi)*(1-(1/sqrt(16)))^2*sum(sumvec2)*.001;
```

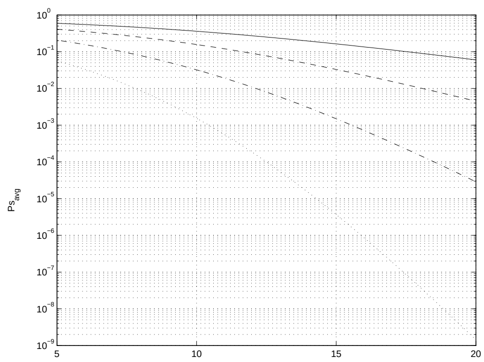

Figure 6: Problem 22

23. MATLAB CODE:

```
M = [1 2 4 8];
alpha = 2/3;
g = 1.5/(16-1);

gammab_dB = [5:.1:20];
gammabvec = 10.^(gammab_dB/10);

for k = 1:length(M)
    for i = 1:length(gammabvec)
        gammab = gammabvec(i);
        phi1 = [0.001:.001:pi/2];
```

```
        phi2 = [0.001:.001:pi/4];
        sumvec1 = ((1+((g*gammab)./(1*(sin(phi1).^2)))).^(-M(k)));
        sumvec2 = ((1+((g*gammab)./(1*(sin(phi2).^2)))).^(-M(k)));
        pb_mrc_qam(k,i) = (4/pi)*(1-(1/sqrt(16)))*sum(sumvec1)*.001 - ...
                          (4/pi)*(1-(1/sqrt(16)))^2*sum(sumvec2)*.001;
    end
end
```

Chapter 10

1. (a)

$$\begin{aligned}
\overline{(AA^H)^T} &= \overline{(A^H)^T.A^T} \\
&= \overline{\overline{(A^T)}^T}\,\overline{A^T} \\
&= AA^H \\
\therefore (AA^H)^H &= AA^H
\end{aligned}$$

For AA^H, $\quad \lambda = \overline{\lambda}$, i.e. eigen-values are real

$$AA^H = Q\Lambda Q^H$$

(b) $X^H AA^H X = (X^H A)(X^H A)^H = \|X^H A\| \geq 0$
$\therefore AA^H$ is positive semidefinite.

(c) $I_M + AA^H = I_M + Q\Lambda Q^H = Q(I+\Lambda)Q^H$
A^H positive semidefinite $\Rightarrow \lambda_i \geq 0 \forall i$
$\therefore 1 + \lambda_i > 0 \forall i$
$\therefore I_M + AA^H$ positive definite

(d)

$$\begin{aligned}
\det[I_M + AA^H] &= \det[I_M + Q\Lambda Q^H] \\
&= \det[Q(I_M + \Lambda_M)Q^H] \\
&= \det[I_M + \Lambda_M] \\
&= \Pi_{i=1}^{Rank(A)}(1+\lambda_i)
\end{aligned}$$

$$\begin{aligned}
\det[I_N + A^H A] &= \det[I_N + \widetilde{Q}\Lambda\widetilde{Q}^H] \\
&= \det[\widetilde{Q}(I_N + \Lambda_N)\widetilde{Q}^H] \\
&= \det[I_N + \Lambda_N] \\
&= \Pi_{i=1}^{Rank(A)}(1+\lambda_i)
\end{aligned}$$

$\because AA^H \quad$ and $\quad A^H A$ have the same eigen-value
$\therefore \det[I_M + AA^H] = \det[I_N + A^H A]$

2. $H = U\Sigma V^T$

$$U = \begin{bmatrix} -0.4793 & 0.8685 & -0.1298 \\ -0.5896 & -0.4272 & -0.6855 \\ -0.6508 & -0.2513 & 0.7164 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 1.7034 & 0 & 0 \\ 0 & 0.7152 & 0 \\ 0 & 0 & 0.1302 \end{bmatrix}$$

$$V = \begin{bmatrix} -0.3458 & 0.6849 & 0.4263 \\ -0.5708 & 0.2191 & 0.0708 \\ -0.7116 & -0.6109 & 0.0145 \\ -0.2198 & 0.3311 & -0.9017 \end{bmatrix}$$

3. $H = U\Sigma V^T$
   Let
$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
$$\therefore H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. Check the rank of each matrix
   rank$(H_I) = 3$
   $\therefore$ multiplexing gain $= 3$
   rank$(H_2) = 4$
   $\therefore$ multiplexing gain $= 4$

5.
$$C = \sum_{i=1}^{R_H} \log_2 \left(1 + \frac{\lambda_i \rho}{M_t}\right)$$
   Constraint $\sum V_i = \rho \sum \lambda_i =$ constant
$$\therefore \frac{\partial C}{\partial \lambda_i} = \frac{\rho}{M_t \ln 2} \frac{1}{(1 + \frac{\lambda_i \rho}{M_t})} - \frac{\rho}{M_t \ln 2} \frac{1}{(1 + \frac{\lambda_i \rho}{M_t})} = 0$$
$$\Rightarrow \lambda_i = \lambda_j$$
   $\therefore$ when all R_H singular values are equal, this capacity is maximized.

6. (a) Any method to show H $\approx U\Lambda$V is acceptable. For example:
   D $= \begin{bmatrix} .13 & .08 & .11 \\ .05 & .09 & .14 \\ .23 & .13 & .10 \end{bmatrix}$ where : $d_{ij} = \left|\frac{H_{ij} - H}{H_{ij}}\right| \times 100$

   (b) precoding filter M $= V^{-1}$
   shaping filter F $= U^{-1}$
   F $= \begin{bmatrix} -.5195 & -.3460 & -.7813 \\ -.0251 & -.9078 & .4188 \\ -.8540 & .2373 & .4629 \end{bmatrix}$
   M $= \begin{bmatrix} -.2407 & -.8894 & .3887 \\ -.4727 & -.2423 & -.8473 \\ -.8478 & .3876 & .3622 \end{bmatrix}$
   Thus $\overline{Y}$ = F(H)M$\overline{X}$ + F$\overline{N}$ $= U^* U \Lambda \mathrm{V} V^* \overline{X} + U^* \overline{N}$
   $= \Lambda \overline{X} + U^* \overline{N}$

   (c) $\frac{P_i}{P} = \frac{1}{\gamma_o} - \frac{1}{\gamma_i}$ for $\frac{1}{\gamma_i} > \frac{1}{\gamma_o}$, 0 else
   $\gamma_i = \frac{{\lambda_i}^2 P}{N_o B} = 94.5$ for i = 1, 6.86 for i = 2, .68 for i = 3
   Assume $\gamma_2 > \gamma_0 > \gamma_3$ since $\gamma_3 = .68$ is clearly too small for data transmission

$\sum \frac{P_i}{P} = 1 \Rightarrow \frac{2}{\gamma_0} - \frac{1}{\gamma_1} - \frac{1}{\gamma_2} = 1 \Rightarrow \gamma_0 = 1.73$
$\frac{P_1}{P} = .5676$ $\frac{P_2}{P} = .4324$
C $= B\left[\log_2\left(1+\gamma_1\frac{P_1}{P}\right) + \log_2\left(1+\gamma_2\frac{P_2}{P}\right)\right]$
$= 775.9$ kbps

(d) With equal weight beamforming, the beamforming vector is given by $c = \frac{1}{\sqrt{(3)}}[1\ 1\ 1]$. The SNR is then given by:

$$SNR = \frac{c^H H^H H c}{N_0 B} = (.78)(100) = 78. \tag{1}$$

This gives a capacity of 630.35 kbps. The SNR achieved with beamforming is smaller than the best channel in part (c). If we had chosen c to equal the eigenvector corresponding to the best eigenvalue, then the SNR with beamforming would be equal to the largest SNR in part(c). The beamforming SNR for the given c is greater than the two smallest eigenvalues in part(c) because the channel matrix has one large eigenvalue and two very small eigenvalues.

7. $C = \max B \log_2 \det[I_{M\gamma} + HR_X H^H]$
$R_X : T_\gamma(R_X) = \rho$ If the channel is known to the transmitter, it will perform an SVD decomposition of H as

$$H = U\Sigma V$$

$$HR_X H^H = (U\Sigma V)R_X(U\Sigma V)^H$$

By Hadamard's inequality we have that for $A \in \Re^{n\times n}$

$$\det(A) \leq \Pi_{i=1}^n A_{ii}$$

with equality iff A is diagonal.
We choose R_X to be diagonal, say $= \Omega$ then

$$\det(I_{MR} + HR_X H^H) = \det(I + \Omega\Sigma^2)$$

$$\therefore C = \max_{\sum_i \rho_i \leq \rho} B\sigma_i \log_2(1 + \lambda_i\rho_i)$$

where $\sqrt{\lambda_i}$ are the singular values.

8. The capacity of the channel is found by the decomposition of the channel into R_H parallel channels, where R_H is the rank of the channel matric H.

$$C = \max_{\rho_i : \sum_i \rho_i \leq \rho} \sum_i B \log_2(1 + \lambda_i \rho_i)$$

where $\sqrt{\lambda_i}$ are the R_H non-zero singular values of the channel matrix H and ρ is the SNR constraint.

$$\gamma_i = \lambda_i \rho$$

Then the optimal power allocation is given as

$$\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} & \gamma_i \geq \gamma_0 \\ 0 & \gamma_i < \gamma_0 \end{cases} \tag{2}$$

for some cut-off value γ_0. The resulting capacity is given as

$$C = \sum_{i:\gamma_i \geq \gamma_0} B \log_2(\gamma_i/\gamma_0)$$

For

$$H = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$R_H = 3$, $\gamma_1 = 80$, $\gamma_2 = 40$, $\gamma_3 = 40$. We first assume that γ_0 is less than the minimum γ_i which is 40.

$$\gamma_0 = \frac{3}{1 + \sum_{i=1}^{3} \frac{1}{\gamma_i}}$$

which gives $\gamma_0 = 2.8236 < \min_i \gamma_i$, hence the assumption was correct.

$$\frac{C}{B} = 12.4732\,\text{bits/sec/Hz}$$

For

$$H = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \end{bmatrix}$$

$R_H = 4$, $\gamma_1 = 40$, $\gamma_2 = 40$, $\gamma_3 = 40$, $\gamma_4 = 40$. We first assume that γ_0 is less than the minimum γ_i which is 40.

$$\gamma_0 = \frac{4}{1 + \sum_{i=1}^{4} \frac{1}{\gamma_i}}$$

which gives $\gamma_0 = 3.6780 < \min_i \gamma_i$, hence the assumption was correct.

$$\frac{C}{B} = 13.7720\,\text{bits/sec/Hz}$$

9. $H = \begin{bmatrix} h_{11} & \dots & h_{1M_t} \\ . & \dots & . \\ . & \dots & . \\ . & \dots & . \\ h_{M_r1} & \dots & h_{M_rM_t} \end{bmatrix}_{M_r \times M_t}$

Denote $G = HH^T$

$$
\begin{aligned}
\lim_{M_t \to \infty} \frac{1}{M_t} G_{ii} &= \lim_{M_t \to \infty} \frac{1}{M_t} [h_{i1} \ldots h_{iM_t}] \begin{bmatrix} \overline{h_{i1}} \\ . \\ . \\ . \\ \overline{h_{iM_t}} \end{bmatrix} \\
&= \lim_{M_t \to \infty} \frac{1}{M_t} \sum_{j=1}^{M_t} \|h_{ij}\|^2 \\
&= E_j \|h_{ij}\|^2 \\
&= \sigma^2 \\
&= 1 \quad \forall i \\
\lim_{M_t \to \infty, i \neq j} \frac{1}{M_t} G_{ij} &= \lim_{M_t \to \infty} \frac{1}{M_t} [h_{i1} \ldots h_{iM_t}] \begin{bmatrix} \overline{h_{j1}} \\ . \\ . \\ . \\ \overline{h_{jM_t}} \end{bmatrix} \\
&= \lim_{M_t \to \infty} \frac{1}{M_t} \sum_{k=1}^{M_t} h_{ik} \overline{h_{jk}} \\
&= E_k h_{ik} \overline{h_{jk}} \\
&= E_k(h_{ik}) E_k(\overline{h_{jk}}) \\
&= 0 \quad \forall i, j, i \neq j \\
\therefore \lim_{M \to \infty} \frac{1}{M} HH^T &= I_M \\
\therefore \lim_{M \to \infty} B \log_2 \det \left[I_M + \frac{\rho}{M} HH^T \right] &= B \log_2 \det [I_M + \rho I_M] \\
&= B \log_2 [1 + \rho] \det I_M \\
&= MB \log_2 [1 + \rho]
\end{aligned}
$$

10. We find the capacity by randomly generating 10^3 channel instantiations and then averaging over it. We assume that distribution is uniform over the instantiations.

    MATLAB CODE

```
clear;
clc;
Mt = 1;
Mr = 1;
rho_dB = [0:25];

rho = 10.^(rho_dB/10);
for k = 1:length(rho)
    for i = 1:100
        H = wgn(Mr,Mt,0,'dBW','complex');
        [F, L, M] = svd(H);
        for j = 1:min(Mt,Mr)
```

```
            sigma(j) = L(j,j);
        end
        sigma_used = sigma(1:rank(H));
        gamma = rho(k)*sigma_used;
        %% Now we do water filling\
        gammatemp = gamma;
        gammatemp1 = gammatemp;
        gamma0 = 1e3;
        while gamma0 > gammatemp1(length(gammatemp1));
            gammatemp1 = gammatemp;
            gamma0 = length(gammatemp1)/(1+sum(1./gammatemp1));
            gammatemp = gammatemp(1:length(gammatemp)-1);
        end
        C(i) = sum(log2(gammatemp1./gamma0));
    end
    Cergodic(k) = mean(C);
end
```

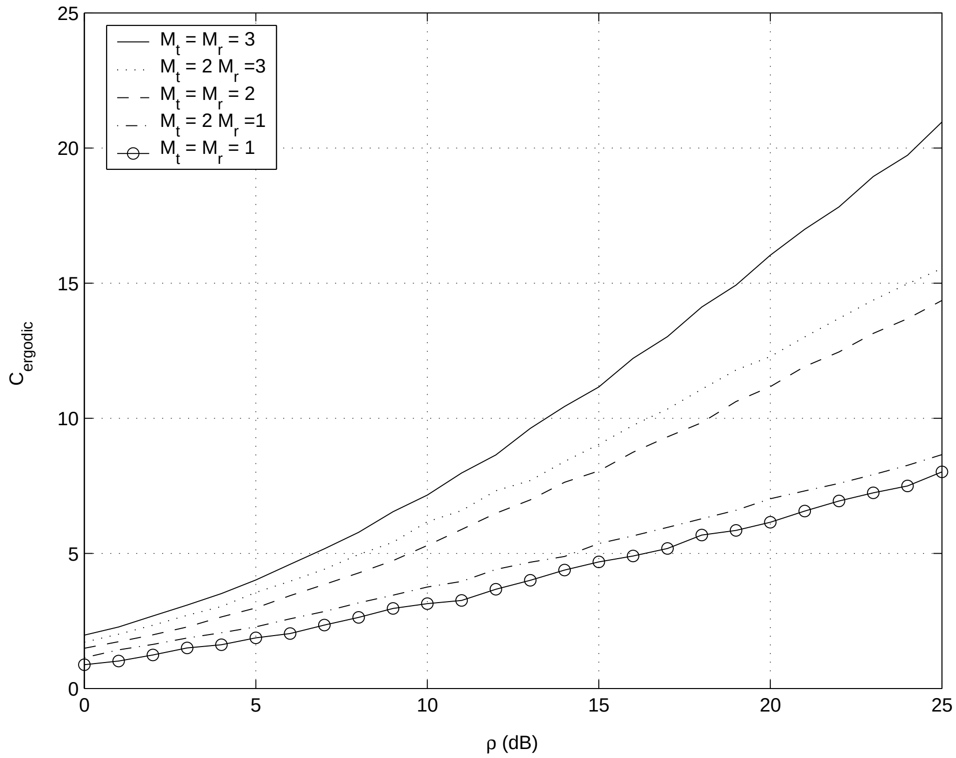

Figure 1: Problem 10

11. We find the capacity by randomly generating 10^4 channel instantiations and then averaging over it. We assume that distribution is uniform over the instantiations.

```
MATLAB CODE

clear;
clc;
Mt = 1;
```

```
Mr = 1;
rho_dB = [0:30];

rho = 10.^(rho_dB/10);
for k = 1:length(rho)
    for i = 1:1000
        H = wgn(Mr,Mt,0,'dBW','complex');
        [F, L, M] = svd(H);
        for j = 1:min(Mt,Mr)
            sigma(j) = L(j,j);
        end
        sigma_used = sigma(1:rank(H));
        gamma = rho(k)*sigma_used;
        C(i) = sum(log2(1+gamma/Mt));
    end
    Cout(k) = mean(C);
    pout = sum(C<Cout(k))/length(C);
    while pout > .01
        Cout(k) = Cout(k)-.1;
        pout = sum(C<Cout(k))/length(C);
    end
    if Cout(k)<0;
        Cout(k) = 0;
    end
end
```

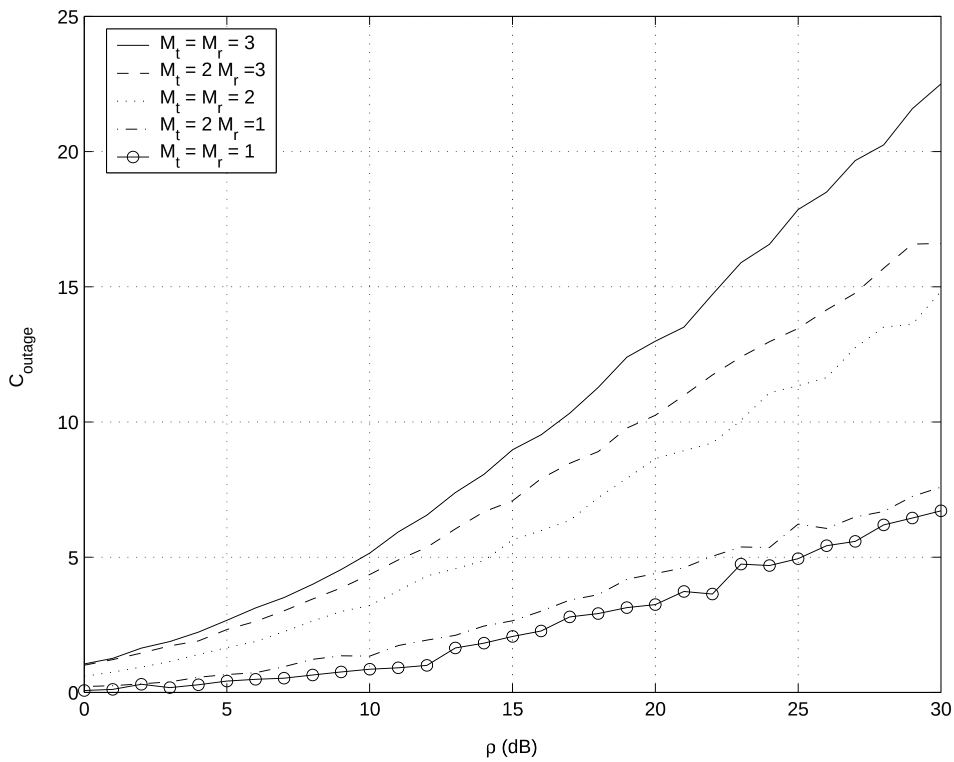

Figure 2: Problem 11

12.

$$\begin{aligned} P\left(u^{\star} n < X\right) &= P\left(\sum_{i=1}^{M_r} u_i n_i < X\right) \\ &= \sum_{i=1}^{M_r} u_i P\left(n_i < X\right) \\ &= P\left(n_i < X\right) \end{aligned}$$

$\therefore$ the statistics of $u^{\star}n$ are the same as the statistics of each of these elements

13.

$$\begin{aligned} \Sigma_x &= \|u^{\star} H v x\| \\ &= \|u^{\star} H v\|^2 \|x\|^2 \\ &= v^H H^H {u^{\star}}^H u^{\star} H v \|x\|^2 \\ &= v^H H^H H v \|x\|^2 \\ &= v^H Q^H Q v \|x\|^2 \\ &\leq \lambda_{max} \|x\|^2 \end{aligned}$$

with equality when u, v are the principal left and right singular vectors of the channel matrix H

$$\therefore SNR_{max} = \lambda_{max} \frac{\|x\|^2}{N} = \lambda_{max}\rho$$

14.

$$H = \begin{bmatrix} 0.1 & 0.5 & 0.9 \\ 0.3 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.7 \end{bmatrix}$$

When both the transmitter and the receiver know the channel, for beamforming, u and v correspond to the principal singular vectors (or the singular vectors corresponding to the maximum singular value of H). Notice that the singular values of H are the square root of the eigen values of HH^H (Wishart Matrix).

Using Matlab, we get that the maximum singular value of H is 1.4480 and the singular vectors corresponding to this value are

$$u_{\text{opt}} = \begin{bmatrix} -0.7101 \\ -0.4641 \\ -0.5294 \end{bmatrix}$$

and

$$v_{\text{opt}} = \begin{bmatrix} -0.1818 \\ -0.4190 \\ -0.8896 \end{bmatrix}$$

It is easy to check that $u_{\text{opt}}^T u_{\text{opt}} = 1$ and $v_{\text{opt}}^T v_{\text{opt}} = 1$ and that

$$u_{\text{opt}}^T * H * v_{\text{opt}} = 1.4480$$

Since, during beamforming from eq. 10.17 in reader,

$$y = (u^T H v)x + u^T n$$

and for a given transmit SNR of ρ, the received SNR is given as

$$\text{SNR}_{\text{rcvd}} = \rho(u_{\text{opt}}^T H v_{\text{opt}})^2$$

since, u_{opt} has norm 1, noise power is not increased. For, $\rho = 1$, SNR is simply $(1.4480)^2 = 2.0968$.

When the channel is not known to the transmitter, it allocates equal power to all the antennas and so the precoding vector (or the optimal weights) at the transmitter is given as

$$v_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Define

$$h = H v_2$$

So, eq. 10.17 in the reader can be written as

$$y = (u^T h)x + u^T n$$

To maximize SNR we need to find a u_2 of norm 1 such that $(u'h)$ is maximized.

Using Matlab, we get that the maximum singular value of h is 1.2477 and the singular vector corresponding to this value is

$$u_2 = \begin{bmatrix} 0.6941 \\ 0.5090 \\ 0.5090 \end{bmatrix}$$

It is easy to check that $u_2^T u_2 = 1$ and that

$$u_2^T * h = 1.2477$$

Alternatively, from MRC concept we know that:

$$u_2 = \frac{h^H}{||h||} = \begin{bmatrix} 0.6941 \\ 0.5090 \\ 0.5090 \end{bmatrix}$$

where $||h||$ is the L_2 norm of h.

For a given transmit SNR of ρ, the received SNR is given as

$$\text{SNR}_{\text{rcvd}} = \rho(u_2^T H v_2)^2$$

since, u_2 has norm 1, noise power is not increased. For, $\rho = 1$, SNR is simply $(1.2477)^2 = 1.5567$.

15. (a) $\rho = 10$ dB $= 10$

$$P_e = \rho^{-d}$$

So to have $P_e \leq 10^{-3}$, we should have $d \geq 3$, or at least $d = 3$. Solving the equation that relates diversity gain d to multiplexing gain r at high SNR's we get

$$d = (M_r - r)(M_t - r)$$

$$\Rightarrow 3 = (8-r)(4-r)$$

Solving for r we get

$$r = 3.35 \,\text{or}\, 8.64$$

We have that $r \leq \min\{M_r, M_t\}$, so $r \leq 4$ and so $r = 3.35$. But we know that r has to be an integer. So, we take the nearest integer which is smaller than the calculated value of r, which gives us $\boxed{r{=}3}$.

No credits for this part:

If we are allowed to assume that equations 10.23 and 10.24 hold at finite SNR's too and we are given that we can use base 2 for logarithms, we can find the data rate as

$$R = r\log_2(\rho) = 9.96 \,\text{bits/s/Hz}$$

(b) With, $r = 3$, we can find d as

$$d = (M_r - r)(M_t - r) = (8-3)(4-3) = 5$$

For this value of d,

$$P_e = \rho^{-d} = 10^{-5}$$

16. According to SVD of h
$\sqrt{\lambda} = 1.242$
$\therefore C/B = \log_2(1+\lambda\rho) = \log_2(1+1.242^2.10) = 4.038 bps/Hz$

17.

$$\mathbf{H} = \begin{bmatrix} .3 & .5 \\ .7 & .2 \end{bmatrix} = \begin{bmatrix} -.5946 & .8041 \\ -.8041 & .5946 \end{bmatrix} \begin{bmatrix} .8713 & 0 \\ 0 & .3328 \end{bmatrix} \begin{bmatrix} -.8507 & .5757 \\ -.5757 & -.8507 \end{bmatrix}$$

P= 10mW
$N_0 = 10^{-9}$ W/Hz
$B = 100$ KHz

(a) When $\mathbf{H}$ is known both at the transmitter and at the receiver, the transmitter will use the optimal precoding filter and the receiver will use the optimal shaping filter to decompose the MIMO channel into 2 parallel channels. We can then do water-filling over the two parallel channels available to get capacity.

Finding the γ_i's

$$\gamma_1 = \frac{\lambda_1^2 P}{N_0 B} = 75.92$$

$$\gamma_2 = \frac{\lambda_2^2 P}{N_0 B} = 11.08$$

Finding γ_0

Now, we have to find the cutoff value γ_0. First assume that γ_0 is less than both γ_1 and γ_2. Then

$$\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_1}\right) + \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_2}\right) = 1$$

$$\Rightarrow \frac{2}{\gamma_0} = 1 + \frac{1}{\gamma_1} + \frac{1}{\gamma_2}$$

$$\Rightarrow \gamma_0 = \frac{1}{1 + \frac{1}{\gamma_1} + \frac{1}{\gamma_2}} = 1.81$$

which is less than both γ_1 and γ_2 values so our assumption was correct.

Finding capacity

Now we can use the capacity expression as

$$C = \sum_{i=1}^{2} B \log_2\left(\frac{\gamma_i}{\gamma_0}\right) = 800 \text{ Kbps}$$

(b) **Total is**

Essentially we have two parallel channels after the precoding filter and the shaping filter are used at the transmitter and receiver respectively.

$$M(\gamma) = 1 + \gamma K \frac{S(\gamma)}{\bar{S}}$$

Finding K

$$K = \frac{-1.5}{\ln(5P_b)} = .283$$

$\gamma_K = \gamma_0/K$.

Finding γ_0 or γ_K

We now find the cut-off γ_0. First assume that $\gamma_0 < \{\gamma_1, \gamma_2\}$. Notice that γ_1 and γ_2 have already been calculated in part (a) as $\gamma_1 = 75.92$ and $\gamma_2 = 11.08$.

$$\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_1 K}\right) + \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_2 K}\right) = 1$$

$$\Rightarrow \frac{2}{\gamma_0} = 1 + \frac{1}{\gamma_1 K} + \frac{1}{\gamma_2 K}$$

$$\Rightarrow \gamma_0 = \frac{1}{1 + \frac{1}{K}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right)} = 1.4649$$

which is less than both γ_1 and γ_2 values, so our assumption was correct.

$$\gamma_K = \gamma_0/K = 5.1742$$

Finding Rate R

Therefore the total rate, R is given as

$$R = B\left[\log_2\left(\frac{\gamma_1}{\gamma_K}\right) + \log_2\left(\frac{\gamma_2}{\gamma_K}\right)\right]$$

$$\Rightarrow R = B4.97$$

This gives that $\boxed{R\text{=497.36 Kbps}}$ (Obviously less than ergodic capacity).

(c) Since now we use beamforming to get diversity only, the transmitter and the receiver use the principal left and right eigen vectors of the **Wishart Matrix** $\mathbf{HH}^H$.

Once this is done the SNR at the combiner output is simply $\lambda_{\max}\rho$, where $\lambda_{\max}$ is the maximum eigen value of the **Wishart Matrix** $\mathbf{HH}^H$ and ρ is $\frac{P}{N_0 B}$

Finding γ_s

As given in the question, $\lambda_{\max}$ is 0.7592 and ρ was calculated to be 100. So we get that $\boxed{\gamma_s = 75.92}$.

Finding P_b

When using BPSK, $\gamma_s = \gamma_b$. Now we can use the expression for P_b for BPSK

$$P_b = Q\left(\sqrt{2\gamma_b}\right) = Q\left(\sqrt{2 \times 75.92}\right) = \begin{cases} 0 & \text{Using the approx. given in the Ques.} \\ 3.4 \times 10^{-35} & \text{Using Matlab} \end{cases}$$

Credit is given for either value.

Finding Rate R

Since we are using BPSK and are given that $B = 1/T_b$, we get the rate using BPSK to be $\boxed{R\text{=100 Kbps}}$.

Comparing with previous part

Comparing with part (b), we can see that the rate R decreases by 397.36 Kbps and the P_b improves as P_b is now $3.4 \times 10^{-35} \sim 0$ whereas earlier it was 10^{-3}.

(d) Therefore we see that we can tradeoff rate for robustness of the system. If we are willing to decrease the rate at which we transmit, we can get more diversity advantage i.e. one strong channel which gives a much less value of P_b.

18. (a)
```
clear;
clc;
Mt = 4;
Mr = Mt;
rho_dB = [0:20];

rho = 10.^(rho_dB/10);
for k = 1:length(rho)
    for i = 1:1000
        H = wgn(Mr,Mt,0,'dBW','complex');
        [F, L, M] = svd(H);
        for j = 1:min(Mt,Mr)
```

```
                sigma(j) = L(j,j);
            end
            sigma_used = sigma(1:rank(H));
            gamma = rho(k)*sigma_used;
            %% Now we do water filling\
            gammatemp = gamma;
            gammatemp1 = gammatemp;
            gamma0 = 1e3;
            while gamma0 > gammatemp1(length(gammatemp1));
                gammatemp1 = gammatemp;
                gamma0 = length(gammatemp1)/(1+sum(1./gammatemp1));
                gammatemp = gammatemp(1:length(gammatemp)-1);
            end
            C(i) = sum(log2(gammatemp1./gamma0));
        end
        Cergodic(k) = mean(C);
    end
```

(b)

```
clear;
clc;
Mt = 4;
Mr = Mt;
rho_dB = [0:20];

rho = 10.^(rho_dB/10);
for k = 1:length(rho)
    for i = 1:1000
        H = wgn(Mr,Mt,0,'dBW','complex');
        [F, L, M] = svd(H);
        for j = 1:min(Mt,Mr)
            sigma(j) = L(j,j);
        end
        sigma_used = sigma(1:rank(H));
        gamma = rho(k)*sigma_used;
        C(i) = sum(log2(1+gamma/Mt));
    end
    Cout(k) = mean(C);
end
```

19. using Matlab we get $C_{out} = 7.8320$

```
MATLAB CODE

clear;
clc;
Mt = 4;
Mr = Mt;
rho_dB = 10;
rho = 10.^(rho_dB/10);

for k = 1:length(rho)
    for i = 1:1000
```

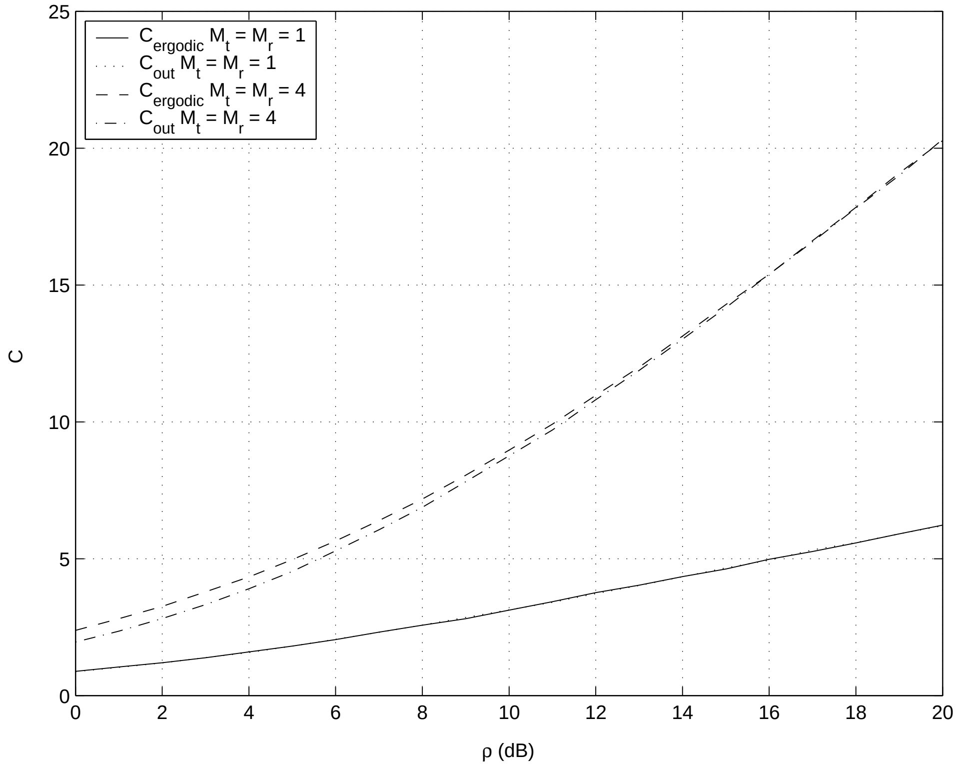

Figure 3: Problem 18

```
        H = wgn(Mr,Mt,0,'dBW','complex');
        [F, L, M] = svd(H);
        for j = 1:min(Mt,Mr)
            sigma(j) = L(j,j);
        end
        sigma_used = sigma(1:rank(H));
        gamma = rho(k)*sigma_used;
        C(i) = sum(log2(1+gamma/Mt));
    end
    Cout(k) = mean(C);
    pout = sum(C<Cout(k))/length(C);
    while pout > .1
        Cout(k) = Cout(k)-.01;
        pout = sum(C<Cout(k))/length(C);
    end
    if Cout(k)<0;
        Cout(k) = 0;
    end
end
```

20. As μ increases, the span of cdf becomes narrower and so capacity starts converging to a single number.

```
MATLAB CODE

clear;
clc;
```

```
Mt = 8;
Mr = Mt;
rho_dB = 10;
rho = 10.^(rho_dB/10);

for i = 1:1000
    H = wgn(Mr,Mt,0,'dBW','complex');
    [F, L, M] = svd(H);
    for j = 1:min(Mt,Mr)
        sigma(j) = L(j,j);
    end
    sigma_used = sigma(1:rank(H));
    gamma = rho*sigma_used;
    C(i) = sum(log2(1+gamma/Mt));
end
[f,x] = ecdf(C);
```

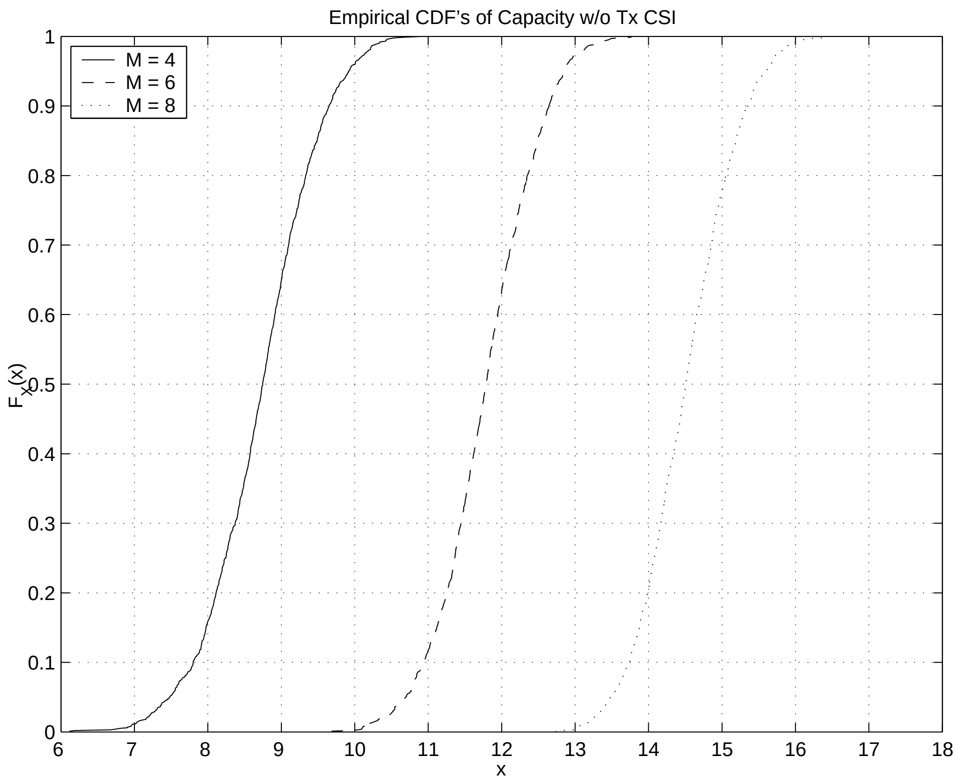

Figure 4: Problem 20

Chapter 11

1. See Fig 1

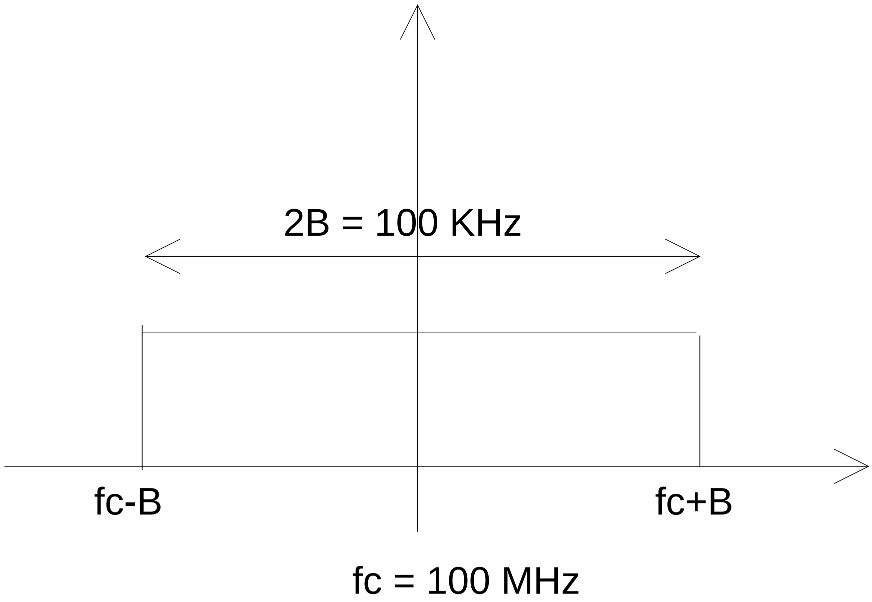

Figure 1: Band of interest.

$B = 50$ KHz, $f_c = 100$ MHz

$$H_{\text{eq}}(f) = \frac{1}{H(f)} = f$$

Noise PSD $= N_0$ W/Hz. Using this we get

$$\begin{aligned}
\text{Noise Power} &= \int_{f_c-B}^{f_c+B} N_0 |H_{\text{eq}}(f)|^2 df && (1)\\
&= N_0 \int_{f_c-B}^{f_c+B} f^2 df && (2)\\
&= N_0 \frac{f^3}{3}\Big]_{(f_c-B)}^{(f_c+B)} && (3)\\
&= \frac{N_0}{3}(f_c+B)^3 - (f_c-B)^3 && (4)\\
&= 10^{21} N_0 \,\text{W} && (5)
\end{aligned}$$

Without the equalizer, the noise power will be $2BN_0 = 10^5 N_0$ W. As seen from the noise power values, there is tremendous noise enhancement and so the equalizer will **not** improve system performance.

2. (a) For the first channel:
ISI power over a bit time $= A^2 T_b/T_b = A^2$ For the 2nd channel:
ISI power over a bit time $= \frac{A^2}{T_b} \sum_{n=1}^{\infty} \int_{nT_b}^{(n+1)T_b} e^{-t/T_m} dt = 2e^{-1/2} A^2$

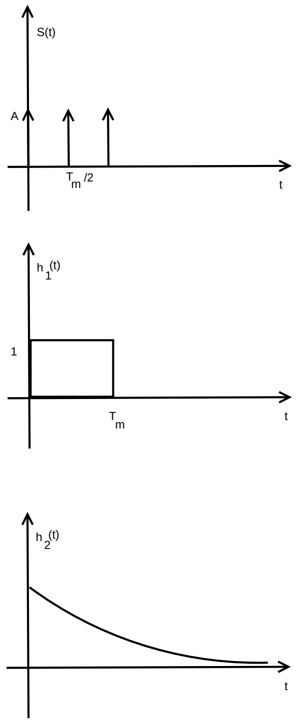

Figure 2: Problem 2a

(b) No ISI: pulse interval $= 11/2\mu s = 5.5\mu s$
$\therefore$ Data rate $= 1/5.5\mu s = 181.8Kbps$
If baseband signal =100KHz: pulse width $= 10\mu s$
Data rate $= 2/10\mu s + 10\mu s = 100Kbps$

3. (a)

$$h(t) = \begin{cases} e^{-\frac{t}{\tau}} & t \geq 0 \\ 0 & o.w. \end{cases} \tag{6}$$

$\tau = 6\ \mu$ sec

$$H_{\text{eq}}(f) = \frac{1}{H(f)}$$

$$\begin{aligned} H(f) &= \int_0^{\infty} e^{-\frac{t}{\tau}} e^{-j2\pi ft} dt && (7) \\ &= \frac{1}{\frac{1}{\tau} + j2\pi f} && (8) \end{aligned}$$

Hence,

$$H_{\text{eq}}(f) = \frac{1}{\tau} + j2\pi f$$

(b)

$$\frac{\text{SNR}_{\text{eq}}}{\text{SNR}_{\text{ISI}}} = \frac{\frac{\int_{-B}^{B} S_x(f)|H(f)|^2|H_{eq}(f)|^2 df}{\int_{-B}^{B} N_0|H_{eq}(f)|^2 df}}{\frac{\int_{-B}^{B} S_x(f)|H(f)|^2 df}{2BN_0}}$$

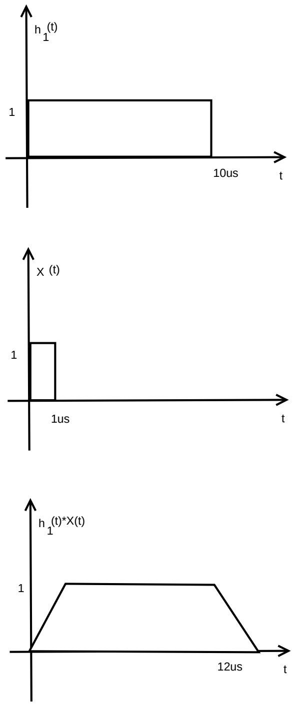

Figure 3: Problem 2b

Assume $S_x\left(f\right) = S$, $-B \le f \le B \Rightarrow$

$$\frac{\dfrac{2BS}{N_0\ \frac{2B}{\tau^2}+\frac{8\pi^2}{3}B^3}}{\dfrac{S\int_{-B}^{B}|H(f)|^2 df}{2BN_0}} = \frac{\dfrac{2B}{\frac{1}{\tau^2}+\frac{4\pi^2}{3}B^2}}{1.617\times 10^{-6}} = 0.9364 = -0.28 \text{ dB}$$

(c)

$$\begin{aligned} h\left[n\right] &= 1\ +\ e^{\frac{-T_s}{\tau}}\delta\left[n-1\right] + e^{\frac{-2T_s}{\tau}}\delta\left[n-2\right] + ... \\ H(z) &= 1 + e^{\frac{-T_s}{\tau}}z^{-1} + e^{\frac{-2T_s}{\tau}}z^{-2} + e^{\frac{-3T_s}{\tau}}z^{-3} + ... \\ &= \sum_{n=0}^{\infty}\left(e^{-\frac{T_s}{\tau}}z^{-1}\right)^n = \frac{z}{z-e^{-\frac{T_s}{\tau}}} = \frac{1}{1-e^{-\frac{T_s}{\tau}}z^{-1}} \end{aligned}$$

$\Rightarrow H_{eq}(z) = \frac{1}{H(z)+N_0}$. Now, we need to use some approximation to come up with the filter tap coefficient values. If we assume $N_0 \approxeq 0$ (the zero-forcing assumption), we get $H_{eq}(z) = 1 - e^{-\frac{T_s}{\tau}}z^{-1}$. Thus, a two tap filter is sufficient. For $T_s = \frac{1}{30}$ ms, we have a_0=1, $a_1 = -0.0039$ as the tap coefficient values. Any other reasonable way is also accepted.

4. $\omega_i = c_i$ where $\{c_i\}$ is the inverse Z- transform of 1/F(z)
Show that this choice of tap weights minimizes

$$\left|\frac{1}{F(z)} - \left(\omega_{-N}z^N + \ldots + \omega_N z^{-N}\right)\right|^2 \quad \ldots(1)$$

at z = $e^{j\omega}$
If F(z) is of length 2 and monic, say $F(z) = 1 - a_1 z$ then

$$\frac{1}{F(z)} = 1 + a_1 z^{-1} + a_1^2 z^{-2} + \ldots \quad \text{where } c_1 = a_1,\ c_2 = a_1^2,\ a_1 < 1$$

It is easy to see that the coefficients become smaller and smaller. So if we had the opportunity to cancel any (2N+1) coefficients we will cancel the ones that are closest to z^0. Hence we get that $\omega_i = c_i$ minimizes (1). The result can be similarly proved for length of F(z) greate than 2 or non-monic.

5. (a) $H_{eq}(f) = \frac{1}{H(f)}$ for ZF equalizer

$$H_{eq}(f) = \begin{cases} 1 & f_c - 20MHz \leq f < f_c - 10MHz \\ 2 & f_c - 10MHz \leq f < f_c \\ 0.5 & f_c \leq f < f_c + 10MHz \\ 4 & f_c + 10MHz \leq f < f_c + 20MHz \\ 0 & o.w. \end{cases} \tag{9}$$

(b) S=10mW Signal power
$N = N_0[1^2 \times 10MHz + 2^2 \times 10MHz + 0.5^2 \times 10MHz + 4^2 \times 10MHz] = 0.2125mW$
$\therefore SNR = 47.0588 = 16.73dB$

(c) $T_s = 0.0125\mu sec$
$P_b \leq 0.2e^{-1.5\gamma/M-1}$ or $M \leq 1 + \frac{1.5SNR}{-ln(5P_b)}$
for $P_b = 10^{-3}$ $M \leq 14.3228$ ($M \geq 4$ thus using the formula is reasonable)
$R = \frac{\log_2 M}{T_s} = 307.2193Mbps$

(d) We use M=4 non overlapping subchannels, each with B=10MHz bandwidth
1: $f_c - 20MHz \leq f < f_c - 10MHz \quad \alpha_1 = 1$
2: $f_c - 10MHz \leq f < f_c \quad \alpha_2 = 0.5$
3: $f_c \leq f < f_c + 10MHz \quad \alpha_3 = 2$
4: $f_c + 10MHz \leq f < f_c + 20MHz \quad \alpha_4 = 0.25$
Power optimization: $\gamma_i = \frac{P\alpha_i^2}{N_0 B}$ for $i = 1, 2, 3, 4$
$\gamma_1 = 1000 \quad \gamma_2 = 250 \quad \gamma_3 = 4000 \quad \gamma_4 = 62.5$
for $P_b = 10^{-3}$ $K = 0.2831$

$$\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{K\gamma_i} & \gamma_i \geq \gamma_0/K \\ 0 & \gamma_i \leq \gamma_0/K \end{cases} \tag{10}$$

We can see that all subchannels will be used and

$$P_1 = 2.6523 \quad P_2 = 2.5464 \quad P_3 = 2.6788 \quad P_4 = 2.1225$$

and

$$\gamma_0 = 3.7207$$

thus $R = 2B\sum_{i=1}^{4} \log(K\gamma_i/\gamma_0) = 419.9711Mbps$

6. (a)

$$\mathfrak{F}\{f(t)\} = \begin{cases} T & |f| < 1/T \\ 0 & o.w. \end{cases}$$

$$\begin{aligned} F_Z(f) &= \frac{1}{T_S}\sum_{n=-\infty}^{\infty} F\left(f + \frac{n}{T_s}\right) \\ &= 1 \end{aligned}$$

$\therefore$ folded spectrum of f(t) is flat.

(b)

$$\begin{aligned}
y_k &= y(kT+t_0) \\
&= \sum_{i=-\infty}^{\infty} X_i f(kT+t_0-iT) \\
&= \sum_{i=-N}^{N} X_i f\left((k-i)T+t_0\right) \\
&= X_k sinc(t_0) + \underbrace{\sum_{i=-N+k, i\neq k}^{N+k} X_i f\left((k-i)T+t_0\right)}_{ISI}
\end{aligned}$$

(c)

$$\begin{aligned}
ISI &= \sum_{i=-N+k, i\neq k}^{N+k} X_i \frac{\sin\left(\pi(k-i)+t_0/T\pi\right)}{\pi(k-i)+t_0/T\pi} \\
&= sin(\pi t_0/T) \sum_{i=-N, i\neq 0}^{N} \frac{1}{\pi t_0/T-\pi i} \\
&= sin(\pi t_0/T) \sum_{i=1}^{N} \left[\frac{-1}{\pi t_0/T-\pi i} + \frac{1}{\pi t_0/T-\pi i}\right] \\
&= \frac{2}{\pi} sin(\pi t_0/T) \sum_{n=1}^{N} \frac{n}{n^2-t_0^2/T^2}
\end{aligned}$$

Thus, ISI $\rightarrow \infty$ as N $\rightarrow \infty$

7. $g_m(t) = g^\star(-t) = g(t) = sinc(t/T_S), |t| < T_s$
Noise whitening filter : $\frac{1}{G_m^\star(1/z^\star)}$

8. $J_{min} = 1 - \sum_{j=-\infty}^{\infty} c_j f_{-j}$

$$\begin{aligned} B(z) &= C(z)F(z) \\ &= \frac{F(z)F^\star(z^{-1})}{F(z)F^\star(z^{-1}) + N_0} \\ &= \frac{X(z)}{X(z)+N_0} \\ \therefore b_0 &= \frac{1}{2\pi j}\oint \frac{B(z)}{z}dz \\ &= \frac{1}{2\pi j}\oint \frac{X(z)}{z[X(z)+N_0]}dz \\ &= \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{X(e^{j\omega T})}{X(e^{j\omega T})+N_0}d\omega \end{aligned}$$

$$\therefore J_{min} = 1 - \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{X(e^{j\omega T})}{X(e^{j\omega T})+N_0}d\omega$$

$$\begin{aligned} &= \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{N_0}{X(e^{j\omega T})+N_0}d\omega \\ &= \frac{T}{2\pi}\int_{-\pi/T}^{\pi/T} \frac{N_0}{T^{-1}\sum_{n=-\infty}^{\infty}|H(\omega+2\pi n/T)|^2+N_0}d\omega \\ &= T_s\int_{-0.5T_s}^{-0.5T_s} \frac{N_0}{F_\Sigma(f)+N_0}df \end{aligned}$$

9.

$$\begin{aligned} V_W J &= \left(\frac{\partial J}{\partial w_0}, \ldots, \frac{\partial J}{\partial w_N}\right) \\ J &= w^T M_v w^\star - 2\Re\{V_d w^\star\} + 1 \end{aligned}$$

$$\therefore \frac{\partial J}{\partial \overline{w}} = 2M_v w^T - 2V_d$$

$$\begin{aligned} \frac{\partial J}{\partial \overline{w}} = 0 &\Rightarrow 2M_v w^T = 2V_d \\ &\Rightarrow w_{opt} = \left(M_v^T\right)^{-1} V_d^H \end{aligned}$$

10.

$$\begin{aligned} J_{min} &= T_s\int_{-0.5T_s}^{0.5T_s} \frac{N_0}{F_\Sigma(f)+N_0}df \\ \because \frac{N_0}{F_\Sigma(f)+N_0} &\geq 0 \quad \therefore J_{min} \geq 0 \\ \frac{N_0}{F_\Sigma(f)+N_0} &\leq \frac{N_0}{N_0} = 1 \\ \therefore J_{min} &\leq T_s\int_{-0.5T_s}^{0.5T_s} 1df = 1 \\ \therefore 0 \leq \quad J_{min} &\leq 1 \end{aligned}$$

11.

$$\begin{aligned}
F_\Sigma(f) &= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} F\left(f+\frac{n}{T_s}\right) \\
&= \frac{1}{T_s}\sum_{n=-\infty}^{\infty} 1 + 0.5e^{-j2\pi\ f+\frac{n}{T_s}} + 0.3e^{-j4\pi\ f+\frac{n}{T_s}}
\end{aligned}$$

MMSE equalizer :

$$J_{min} = T_s \int_{-0.5/T_s}^{0.5/T_s} \frac{N_0}{F_\Sigma(f)+N_0} df$$

DF equalizer :

$$J_{min} = exp\left\{ T_s \int_{-0.5/T_s}^{0.5/T_s} \ln\left[\frac{N_0}{F_\Sigma(f)+N_0}\right]\right\} df$$

12. (a) G(f) is a sinc(), so theoretically infinite. But 2/T is also acceptable (Null to Null bandwidth)
    (b) $\tau \gg T$ is more likely since $T = 10^{-9} sec$
    As long as $\tau > T_b$, get ISI and so, frequency selective fading
    (c) Require $T_b = T_m + T \Rightarrow R = \frac{1}{T_m+T} = 49997.5bps$
    (d) $H_{eq}(z) = \frac{1}{F(z)}$ for ZF equalizer
    $\Rightarrow H_{eq}(z) = \frac{1}{d_0+d_1z^{-1}+d_2z^{-2}}$
    Long division yields the first 2 taps as

$$w_0 = 1/\alpha_0$$
$$w_1 = -\alpha_1/\alpha_0^2$$

13. (a)

$$H_{zf}(f) = \frac{1}{H(f)} = \begin{cases} 1 & 0 \le f \le 10\text{KHz} \\ 2 & 10\text{KHz} \le f \le 20\text{KHz} \\ 3 & 20\text{KHz} \le f \le 30\text{KHz} \\ 4 & 30\text{KHz} \le f \le 40\text{KHz} \\ 5 & 40\text{KHz} \le f \le 50\text{KHz} \end{cases}$$

    (b) The noise spectrum at the output of the filter is given by $N(f) = N_0|H_{eq}(f)|^2$, and the noise power is given by the integral of $N(f)$ from -50 kHz to 50 kHz:

$$\begin{aligned}
N = \int_{f=-50kHz}^{50kHz} N(f)df &= 2N_0 \int_{f=0kHz}^{50kHz} |H_{eq}(f)|^2 df \\
&= 2N_0(1+4+9+16+25)(10kHz) \\
&= 1.1\text{mW}
\end{aligned}$$

    (c) The noise spectrum at the output of the filter is given by $N(f) = \frac{N_0}{(H(f)+\alpha)^2}$, and the noise power is given by the integral of $N(f)$ from -50 kHz to 50 kHz. For $\alpha = .5$ we get

$$N = 2N_0(.44+1+1.44+1.78+2.04))(10kHz) = 0.134 \text{ mW}$$

    For $\alpha = 1$ we get

$$N = 2N_0(.25+.44+.56+.64+.69))(10kHz) = 0.0516 \text{ mW}$$

(d) As α increases, the frequency response $H_{eq}(f)$ decreases for all f. Thus, the noise power decreases, but the signal power decreases as well. The factor α should be chosen to balance maximizing the SNR and minimizing distortion, which also depends on the spectrum of the input signal (which is not given here).

(e) As $\alpha \to \infty$, the noise power goes to 0 because $H_{eq}(f) \to 0$ for all f. However, the signal power also goes to zero.

14. The equalizer must be retrained because the channel de-correlates. In fact it has to be retrained at least every channel correlation time.
Benefits of training

(a) Use detected data to adjust the equalizer coefficients. Can work without training information

(b) eliminate ISI.

15. $N = 4$

LMS-DFE: $2N+1$ operations/iteration $\Rightarrow$ 9 operations/iteration

RLS: $2.5(N)^2 + 4.5N$ operations/iteration $\Rightarrow$ 58 operations/iteration

Each iteration, one bit sent. The bit time is different for LMS-DFE/RLS, T_b (LMS-DFE) $<T_b$ (RLS). But time to convergence is faster for RLS.

Case 1: $f_D = 100$ Hz $\Rightarrow (\Delta t_c) \equiv 10$ msec, must retrain every 5 msec.

LMS-DFE: R $= \frac{10^7}{9}$ - $\frac{1000 \text{ bits}}{5 \text{ msecs}} = 911$ Kbps

RLS: R $= \frac{10^7}{58}$- $\frac{50 \text{ bits}}{5 \text{ msec}} = 162$ Kbps

Case2: $f_D = 1000$ Hz $\Rightarrow$ retrain every 0.5 msec

$R_{\text{LMS-DFE}} = 0$ bps

$R_{\text{RLS}} = 72.4$ Kbps

16. In the adaptive method, we start with some initial value of tap coefficients $\mathbf{W_0}$ and then use the steepest descent method

$$\mathbf{W_{k+1}} = \mathbf{W_K} - \mathbf{\Delta G_K} \quad \ldots (\mathbf{1})$$

where Δ is some small positive number and $\mathbf{G_K}$ is the gradient of MSE $= \text{E}|\hat{d}_k - \hat{\hat{d}}_k|^2$ is $\mathbf{RW_k} - \mathbf{p}$ (Notice that 11.37 was a solution of gradient =0 , $\therefore \mathbf{RW} = \mathbf{p}$)

$$\therefore \mathbf{G_K} = \mathbf{RW_k} - \mathbf{p} = -\mathbf{E}[\varepsilon_\mathbf{k} \mathbf{Y_k}^\star]$$

where $\mathbf{Y_k} = [\mathbf{y_{k+L}} \ldots \mathbf{y_{K-L}}]^\mathbf{T}$ and $\varepsilon_k = \hat{\hat{d}}_k - \hat{d}_k$
Approximately (1) can be rewritten as

$$\mathbf{W_{k+1}} = \mathbf{W_k} + \mathbf{\Delta} \varepsilon_\mathbf{k} \mathbf{Y_k}^\star$$

Chapter 15

1. City has 10 macro-cells
   each cell has 100 users
   $\therefore$ total number of users = 1000
   Cells are of size 1 sqkm
   maximum distance traveled to traverse = $\sqrt{2}$km
   $\therefore$ time $= \frac{\sqrt{2}}{30} = 169.7s$
   In the new setup
   number of cells = 10^5 microcells
   total number of users = 1000×100^2users
   time = $\frac{\sqrt{2}\times 10}{30\times 10^3}$ =1.69s
   $\therefore$ number of users increases by 10000 and handoff time reduces by 1/100

2. See Fig 1

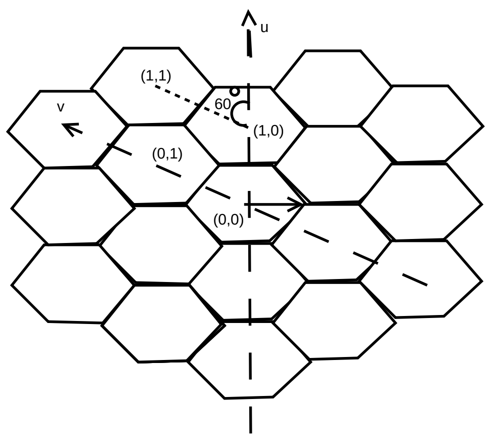

Figure 1: Problem 2

$$D^2 = (j20)^2 + (i20)^2 - 2(j20)(i20)\cos(2\pi/3)$$
$$\Rightarrow D = 2a\sqrt{i^2 + j^2 + ij} = \sqrt{3}R\sqrt{i^2 + j^2 + ij}$$

3. diamond shaped cells, R= 100m
   $D_{min} = 600m$
   $D = 2KR$
   $K = \frac{D}{2R} = \frac{600}{2\times 100} = 3$
   $N = K^2 = 9$

   (a) number of cells per cluster = N = 9
   (b) number of channels per cell = total number/N = 450/9 = 50

4. (a) R=1km
   D=6km
   N = $\frac{A_{cluster}}{A_{cell}} = \frac{\sqrt{3}D^2/2}{3\sqrt{3}R^2/2} = \frac{1}{3}(D/R)^2 = \frac{1}{3}6^2 = 12$
   number of cells per cluster = N = 12

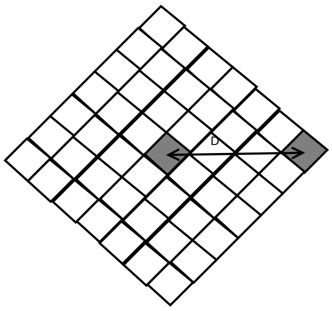

Figure 2: Problem 3

(b) number of channels in each cell = 1200/12 = 100

(c) $\sqrt{i^2 + j^2 + ij} = 2\sqrt{3} \quad \Rightarrow \quad i = 2, \quad j = 2$

5. R=10m
   D=60m
   $\gamma_I = 2$
   $\gamma_0 = 4$
   $M = 4$ for diamond shaped cells

$$SIR_a = \frac{R^{-\gamma_I}}{MD^{-\gamma_0}} = \frac{R^{-2}}{4D^{-4}} = 32400$$

$$SIR_b = \frac{R^{-4}}{4D^{-4}} = 324$$

$$SIR_c = \frac{R^{-2}}{4D^{-2}} = 9$$

$$SIR_a > SIR_b > SIR_c$$

6. $\gamma = 2$
   $BPSK$
   $P_b = 10^{-6} \rightarrow P_b = Q(\sqrt{2\gamma_b}) \Rightarrow \gamma_b = SIR_0 = 4.7534$
   $B = 50MHz$
   each user$100KHz = B_s$
   $SIR = \frac{1}{M}\left(\frac{D}{R}\right)^\gamma$ M=6 for hexagonal cells
   $a_1 = 0.167$
   $a_2 = 3$
   $N > \frac{1}{a_2}\left(\frac{SIR_0}{a_1}\right)^{2/\gamma} \Rightarrow N \geq 9.4879 \therefore N = 10$
   $C_u = 50$

7. $G = 100$
   $\xi = 1$
   $\lambda = 1.5$
   With no sectorization

$$SIR = \frac{1}{\frac{\xi}{3G}(N_c - 1)(1 + \lambda)} = 4.7534$$

$$N_c = \lfloor 26.2450 \rfloor = 26$$

With sectorization, interference is reduced by a factor of 3

$$N_c = \lfloor 76.7349 \rfloor = 76$$

8. SINR $= \frac{G}{\sum_{i=1}^{N_c-1} X_i + N}$
$\alpha = p(X_i = 1)$
$N \sim G(0.247N_c, 0.078N_c)$
$P_out = p(SIR < SIR_0)$

(a) $P_{out} = p\left(\frac{G}{\sum_{i=1}^{N_c-1} X_i+N} < SIR_0\right) = p\left(\sum_{i=1}^{N_c-1} X_i + N > \frac{G}{SIR_0}\right)$

(b) $X = \sum_{i=1}^{N_c-1} X_i$ then X $\sim Bin(\alpha, N_c - 1)$
$p(x + N > G/SIR_0) = \sum_{N=0}^{N_c-1} p(n + N > G/SIR_0|x = n)p(x = n)$

$$p(x = n) = \binom{N_c - 1}{n} \alpha^n (1-\alpha)^{N_c-1-n}$$

$$\begin{aligned} p(x + N > G/SIR_0) &= \sum_{N=0}^{N_c-1} p(N > G/SIR_0 - n|x = n)p(x = n) \\ &= \sum_{N=0}^{N_c-1} p\left(\frac{N - 0.247N_c}{\sqrt{0.078N_c}} > \frac{\frac{G}{SIR_0} - n - 0.247N_c}{\sqrt{0.078N_c}}|x = n\right) \\ &= \sum_{N=0}^{N_c-1} Q\left(\frac{\frac{G}{SIR_0} - n - 0.247N_c}{\sqrt{0.078N_c}}\right) p(x = n) \end{aligned}$$

(c) $N_c = 35$
$\alpha = 0.5$
$SIR_0 = 5$
$G = 150$
$p = 0.0973$

MATLAB

```
for i = 1:length(n)
    pn(i) = (factorial(Nc-1)./(factorial(n(i)).*factorial(Nc-1-n(i))))...
              *alpha.^n(i)*(1-alpha).^(Nc-1-n(i));
end

sump = 0; for i = 1:length(n)
    f = ((G/sir0)-n(i)-.247*Nc)/(sqrt(.078*Nc));
    sump = sump + .5*erfc((f)/sqrt(2))*pn(i);
end
```

(d) If x can be approximated as Gaussian then
x $\sim G((N_c - 1)\alpha, (N_c - 1)\alpha(1 - \alpha))$
$x + N \sim G(0.247N_c + (N_c - 1)\alpha, 0.078N_c + (N_c - 1)\alpha(1 - \alpha))$

$$p(x + N > G/SIR_0) = Q\left(\frac{\frac{G}{SIR_0} - (0.247N_c + (N_c - 1)\alpha)}{\sqrt{0.078N_c + (N_c - 1)\alpha(1 - \alpha)}}\right)$$

(e) p= 0.0969 (very accurate approximation!)

9. define

$$\gamma_k = \frac{g_k P_k}{n_k + \rho \sum_{k \neq j} g_{kj} p_j} \qquad k, j \in \{1, \dots K\}$$

where,
g_k is channel power gain from user k to his base station n_k is thermal noise power at user k's base station
ρ is interference reduction factor ($\rho \sim 1/G$)
g_{kj} is channel power gain from j^{th} interfering transmitter to user k's base station
p_k is user k's Tx power
p_j is user j's Tx power
define a matrix F such that

$$F_{kj} = \begin{cases} 0 & k = j \\ \frac{\gamma_k^\star g_{kj} \rho}{g_k} & k \neq j \end{cases}$$

$k, j \in \{1, \ldots K\}$

$$u = \left(\frac{\gamma_1^\star n_1}{g_1}, \frac{\gamma_2^\star n_2}{g_2}, \ldots, \frac{\gamma_K^\star n_K}{g_K} \right)$$

If Perron Ferbinius eigenvalue of F is less than 1 , then a power control policy exists. The optimal power control policy is given to be $P^\star = (I - F)^{-1} u$

10. `Matlab`

```
D = 2:.01:10;
R = 1;
gamma = 2;
Pdes = R^(-gamma);

for i = 1:length(D)
    Pint = 6*(.2*(D(i)-R)^(-gamma)+.2*(D(i)-R/2)^(-gamma)+.2*(D(i))^(-gamma)...
           +.2*(D(i)+R/2)^(-gamma)+.2*(D(i)+R)^(-gamma));
    Pintbest = 6*((D(i)+R)^(-gamma));
    Pintworst = 6*((D(i)-R)^(-gamma));
    ASE(i) = log(1+Pdes/Pint)/(pi*(.5*D(i))^2);
    ASEbest(i) = log(1+Pdes/Pintbest)/(pi*(.5*D(i))^2);
    ASEworst(i) = log(1+Pdes/Pintworst)/(pi*(.5*D(i))^2);
end
```

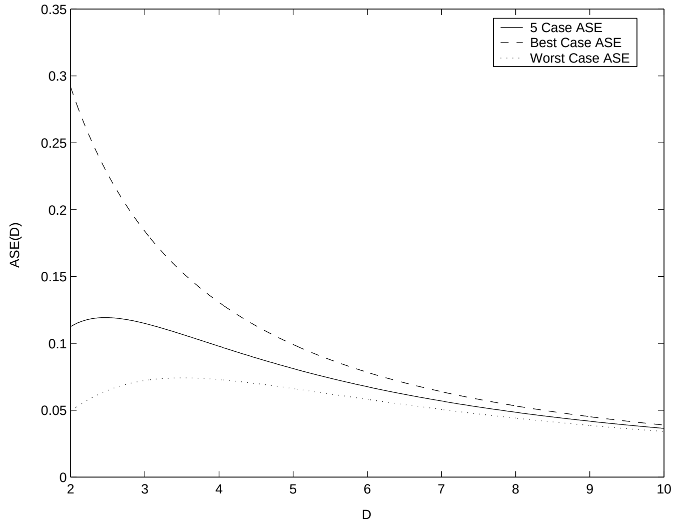

Figure 3: Problem 10

11. $P_t = 5W$
    $B = 100KHz$
    $N_0 = 10^{-16}W/Hz$
    $P_r = P_t K \left(\frac{d_0}{d}\right)^3 \qquad d_0 = 1, K = 100$

    (a) D=2R
    2 users share the band available
    Each user gets 50KHz

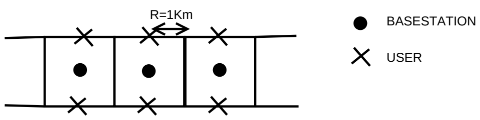

Figure 4: Problem 11a

    (b) $\overline{P}_b = \frac{1}{4\overline{\gamma}_b} \Rightarrow 10^{-3} = \frac{1}{4\overline{\gamma}_b} \Rightarrow \gamma_b = 250$
    If D(n) = 2nR, number of users that share band = 2(2(n-1)+1)
    $\therefore$ each user gets $\frac{100KHz}{2(2n-1)} = B_u(n)$
    interference is only from first tier

$$SIR(n) = \frac{P_t K \left(\frac{d_0}{R}\right)^3}{\frac{N_0}{2}B_u(n) + 2\left(P_t K \left(\frac{d_0}{\sqrt{R^2+D(n)^2}}\right)^3\right)} > 25$$

    using Matlab , $\qquad n = 4, \quad SIR = 261.9253, \quad D = 8R$

```
Matlab

Pt = 5;
R = 1000;
sigma_2 = 1e-16;
n = 1;
```

```
D = 2*n*R;

Bu = (100/(2*(2*n-1)))*1e3;
K = 100;
d0 = 1;
Pdes = Pt*K*(d0/R)^3;
Pint = 2*(Pt*K*(d0/sqrt(R^2+D^2))^3);
Npower = sigma_2*Bu;

sir = Pdes/(Npower+Pint);
while sir < 250
    n = n+1;
    D = 2*n*R;
    Bu = (100/(2*(2*n-1)))*1e3;
    K = 100;
    d0 = 1;
    Pdes = Pt*K*(d0/R)^3;
    Pint = 2*(Pt*K*(d0/sqrt(R^2+D^2))^3);
    Npower = sigma_2*Bu;
    sir = Pdes/(Npower+Pint);
end
```

(c) $ASE = \frac{(R_1+R_2)/B}{2km \times 2km}$
$R_1 = R_2 = B_u(1)\log(1 + SIR(1))$
$B_u(1) = 50KHz, \quad SIR(1) = 5.5899$
$ASE = 0.6801 bps/Hz/km^2$

12. B=100KHz
$N_0 = 10^{-9} W/Hz$
$K = 10$
$P = 10mW$ per user

(a) $0 \leq \alpha \leq 1$ α is channel gain between cells.
See Matlab
If α is large, interference can be decoded and subtracted easily so capacity grows with α as high SNR's (beyond an α value) .
For low SNR values (α less than a value) c decreases with increase in α as interference is increased which cannot be easily decoded due to low SNR.

```
MATLAB CODE:

B = 100e3;
sigma_2 = 1e-9;
P = 10e-3;
K = 10;
ss = .001;

alpha = 0:.01:1;
theta = 0:ss:1;
for i = 1:length(alpha)
    capvec = log2(1+(K*P*(1+2*alpha(i)*cos(2*pi*theta)).^2)/(sigma_2*B));
    C(i) = (1/K)*sum(capvec)*ss;
```

```
end
```

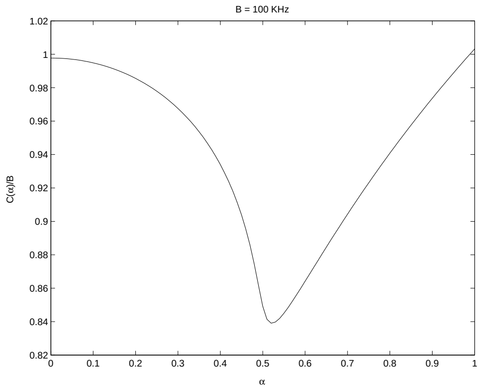

Figure 5: Problem 12a

(b) C(K)↓ as K↑ because as the number of mobile per cell increases system resources get shared more and so per user capacity C(K) has to fall.

```
MATLAB CODE:

BB = 100e3;
sigma_2 = 1e-9;
P = 10e-3;
K = 1:.1:30;
ss = .001;
alpha = .5;
theta = 0:ss:1;
for i = 1:length(K)
    capvec = log2(1+(K(i)*P*(1+2*alpha*cos(2*pi*theta)).^2)/(sigma_2*B));
    C(i) = (1/K(i))*sum(capvec)*ss;
end
```

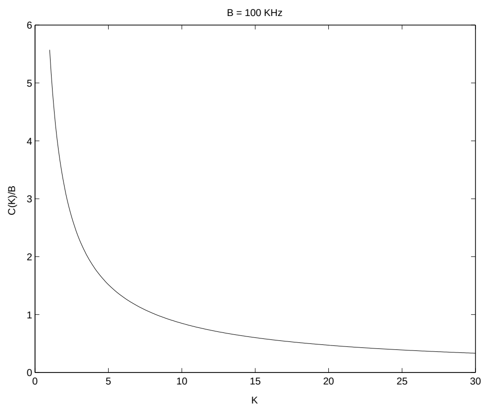

Figure 6: Problem 12b

(c) as transmit power P ↑, capacity C ↑ but gets saturated after a while as the system becomes interference limited.

```
MATLAB CODE:
```

```
B = 100e3;
sigma_2 = 1e-9;
P = [0:.1:100]*1e-3;
K = 10;
ss = .001;
alpha = .5;
theta = 0:ss:1;
for i = 1:length(P)
    capvec = log2(1+(K*P(i)*(1+2*alpha*cos(2*pi*theta)).^2)/(sigma_2*B));
    C(i) = (1/K)*sum(capvec)*ss;
end
```

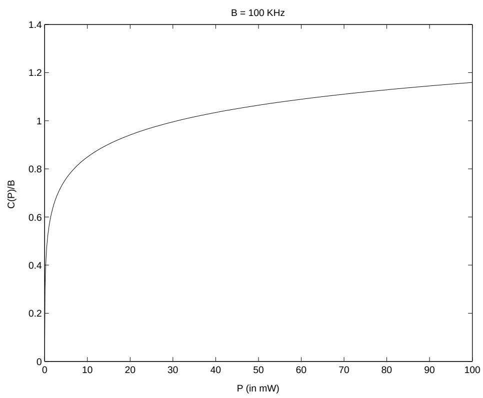

Figure 7: Problem 12c