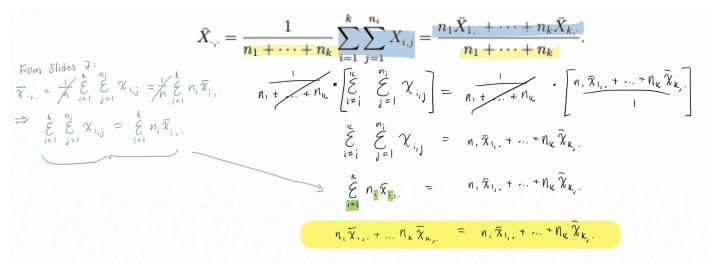
Emma Erdtmann

Daniel Detore

I pledge my honor that I have abided by the Stevens Honor System.

Query 2

Problem 1



Problem 2

$$SSB = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (\bar{X}_{i,\cdot} - \bar{X}_{\cdot,\cdot})^2 = \sum_{i=1}^{k} n_i (\bar{X}_{i,\cdot} - \bar{X}_{\cdot,\cdot})^2.$$
in each group, the #
of observations is constant

Each group & sum of squares is

just
$$\hat{E}_i \left(\bar{X}_{i,\cdot} - \bar{X}_{\cdot,\cdot} \right)^2 = N_i \left(\bar{X}_{i,\cdot} - \bar{X}_{\cdot,\cdot} \right)^2$$
To get the sum of squares between all k groups we get the summation using this
$$\hat{E}_i \left(\bar{X}_{i,\cdot} - \bar{X}_{\cdot,\cdot} \right)^2$$

Problem 3

$$SSE = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (X_{i,j} - \bar{X}_{i,\cdot})^2 = \sum_{i=1}^{k} (n_i - 1) S_i^2.$$
From Slides 7 (19, 20), Group Sampke Mariance
$$S_i^2 = \frac{1}{n_i - 1} \frac{e}{S} \left(\chi_{i,j} - \bar{\chi}_{i,\cdot} \right)^2$$
We can cross multiply and see that
$$S_i^2 \left(n_i - 1 \right) = \frac{e}{S} \left(\chi_{i,j} - \bar{\chi}_{i,\cdot} \right)^2$$
So we can make this substitution into the LHS:
$$SSE = \frac{e}{S} \left(n_i - 1 \right) S_i^2$$

Problem 4

According to lecture 7, $\psi = a_1 \mu_1 + \dots + a_k \mu_k$.

Problem 5

The lecture 7 notes give us the following: $\psi = a_1 \mu_1 + ... + a_k \mu_k$, which we can rewrite as $\psi = \sum\limits_{i=1}^k a_i \mu_i$. The notes also tell us $C_{\psi} = \sum\limits_{i=1}^k a_i \overline{X}_{i,\cdot}$, and we can approximate ψ as $E[C_{\psi}]$. We can simplify this knowing that $\overline{X}_{i,\cdot} \sim N(\mu_i, \sigma^2/n_i)$, i=1,...,k are independent.

$$E[C_{\psi}] = E[\sum_{i=1}^{k} a_{i}\overline{X}_{i,i}] = \sum_{i=1}^{k} a_{i}E[\overline{X}_{i,i}] = \sum_{i=1}^{k} a_{i}\mu_{i} = \psi.$$

Problem 6

According to lecture 7:

$$Var[C_{\psi}] = Var[\sum_{i=1}^{k} a_{i}\overline{X}_{i,}] = \sum_{i=1}^{k} Var[a_{i}\overline{X}_{i,}] = \sum_{i=1}^{k} a_{i}^{2} Var[\overline{X}_{i,}] = \sigma^{2} \sum_{i=1}^{k} a_{i}^{2}/n \equiv \sigma^{2}_{\psi}.$$

This is possible because $\overline{X}_{i} \sim N(\mu_i, \sigma^2/n_i)$, i = 1, ..., k are independent.

Problem 7

 \mathcal{C}_{ψ} is a linear combination of standard normal random variables, therefore $\mathcal{C}_{\psi} \sim N(\psi, \sigma_{\psi}^2)$

Problem 8

i

Under H_0 : $\psi=0$, C_{ψ} follows a standard normal distribution. Therefore $P_{H_0}(|C_{\psi}|>3)=2(1-pnorm(3))=0.002699796.$

ii

Under H_a : $\psi>0$, we must assume that $\mathcal{C}_{\psi}>0$. This is because \mathcal{C}_{ψ} follows a normal distribution, so it is more likely that, even if we subtract 0.5, $\mathcal{C}_{\psi}>0$. Therefore $P(\mathcal{C}_{\psi}>0)-\frac{1}{2}>0$, thus its sign is positive.