Daniel Detore, Emm Erdtmann
I pledge my honor that I have abided by the Stevens Honor System.

Problem 1

```
> alpha <- 0.01
> control <-
c(5.33,5.67,5.33,5.17,5.67,5.67,3.83,5.33,4.67,4.83,5.33,3.83,5.33,5.33,4.33,5.
50,5.17,6.17,4.83,4.33)
> comfort <-
c(5.83,5.00,6.00,5.00,4.17,4.67,5.17,5.50,4.67,4.50,5.17,5.17,4.50,4.67,4.83,5.
83,5.17,4.33,4.67,4.33,4.67,3.67)
> s1 <- var(control); s1
[1] 0.3864724
> s2 <- var(comfort); s2
[1] 0.3282303
> n1 <- length(control); n1
[1] 20
> n2 <- length(comfort); n2
[1] 22
```

(i)
Null hypothesis:

1. $\sigma^2_1 = \sigma^2_2$

Alternative hypotheses:

2. $\sigma^2_1 < \sigma^2_2$ (requires left-tail test)
3. $\sigma^2_1 > \sigma^2_2$ (requires right-tail test)

(ii)

From slide 18 of lecture 5, we know $F = \frac{S_1}{S_2} \sim F_{n_1-1, n_2-1}$

Thus our observed f:

```
> f <- s1/s2; f
[1] 1.177443
```

and our null distribution H_0: $F_{19,21}$

(iii)
Left-tail test:
$pf(f, n_1 - 1, n_2 - 1) = 0.6435875 > \alpha = 0.01$; we cannot reject H_0.

```
> pf(f, n1 - 1, n2 - 1)
[1] 0.6435875
```

Right-tail test:

$1 - pf(f, n_1 - 1, n_2 - 1) = 0.3564125 > \alpha = 0.01$; we cannot reject H_0.

```
> 1-pf(f, n1 - 1, n2 - 1)
[1] 0.3564125
```

(iv)

Left-tail test:

```
> var.test(control, comfort, ratio = 1, alternative = c("less"))

      F test to compare two variances

data:  control and comfort
F = 1.1774, num df = 19, denom df = 21, p-value = 0.6436
alternative hypothesis: true ratio of variances is less than 1
95 percent confidence interval:
 0.000000 2.524242
sample estimates:
ratio of variances
          1.177443
```

Right-tail test:

```
> var.test(control, comfort, ratio = 1, alternative = c("greater"))

      F test to compare two variances

data:  control and comfort
F = 1.1774, num df = 19, denom df = 21, p-value = 0.3564
alternative hypothesis: true ratio of variances is greater than 1
95 percent confidence interval:
 0.5582997       Inf
sample estimates:
ratio of variances
          1.177443
```

(v)

Both tests fail, which means we cannot reject H_0. Therefore $\sigma^2_1 = \sigma^2_2$.

Problem 2

```
> Xbar = mean(control); Xbar
[1] 5.0825
> Ybar = mean(comfort); Ybar
[1] 4.887273
> n1 = length(control); n1
[1] 20
> n2 = length(comfort); n2
[1] 22
```

Since we know $\sigma_1^2 = \sigma_2^2$ but we don't know their actual value, we will do a two-sample T-interval test with equal variances.

(i)
Null hypothesis:

1. $\mu_x = \mu_Y$

Alternative hypotheses:

2. $\mu_x \neq \mu_Y$ (requires two-tail test)
3. $\mu_x < \mu_Y$ (requires left-tail test)
4. $\mu_x > \mu_Y$ (requires right-tail test)

(ii)

We know from slide 10 of Lecture 5 that

$$T \equiv \frac{\bar{X} - \bar{Y}}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim \mathcal{T}_{n_1+n_2-2},$$

so our observed t:

```
> ss1=var(control); ss2=var(comfort); ss1; ss2
[1] 0.3864724
[1] 0.3282303
> ssp <- (((n1 - 1)*ss1) + ((n2 - 1)*ss2)) / (n1+n2-2); ssp
[1] 0.3558953
> t <- (mean(comfort) - mean(control)) / sqrt(ssp * ((1/n1) + (1/n2))); t
[1] -1.059208
```

From slide 8, our null distribution H_0: $\chi^2_{n_1+n_2-2} \equiv \chi^2_{40}$

(iii)
Two-tail test:
$2pt(-\ |t|,\ n1\ +\ n2\ -\ 2)\ =\ 0.2958597\ >\ \alpha\ =\ 0.01$; we cannot reject H_0.

```
> 2*pt(-abs(t), n1+n2-2)
[1] 0.2958597
```

Left-tail test:
$pt(t,\ n1\ +\ n2\ -\ 2)\ =\ 0.1479299\ >\ \alpha\ =\ 0.01$; we cannot reject H_0.

```
> pt(t, n1+n2-2)
[1] 0.1479299
```

Right-tail test:
$1\ -\ pt(t,\ n1\ +\ n2\ -\ 2)\ =\ 0.8520701\ >\ \alpha\ =\ 0.01$; we cannot reject H_0.

```
> 1 - pt(t, n1+n2-2)
```

```
[1] 0.8520701
```

(iv)

Two-tail test:

```
> t.test(comfort, control, alternative = c("two.sided"), paired = FALSE,
var.equal = TRUE)

	Two Sample t-test

data:  comfort and control
t = -1.0592, df = 40, p-value = 0.2959
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5677407  0.1772862
sample estimates:
mean of x mean of y
 4.887273  5.082500
```

Left-tail test:

```
> t.test(comfort, control, alternative = c("less"), paired = FALSE, var.equal =
TRUE)

	Two Sample t-test

data:  comfort and control
t = -1.0592, df = 40, p-value = 0.1479
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
      -Inf 0.1151308
sample estimates:
mean of x mean of y
 4.887273  5.082500
```

Right-tail test:

```
> t.test(comfort, control, alternative = c("greater"), paired = FALSE,
var.equal = TRUE)

	Two Sample t-test

data:  comfort and control
t = -1.0592, df = 40, p-value = 0.8521
alternative hypothesis: true difference in means is greater than 0
95 percent confidence interval:
 -0.5055854        Inf
sample estimates:
mean of x mean of y
 4.887273  5.082500
```

(v)

Even after running all three tests, we cannot reject H_0. This means that $\mu_1 = \mu_2$ at the 0.01 significance level.

Problem 3

```
> Xstar <- ifelse(control > 4.75, 0, 1)
> Ystar <- ifelse(comfort > 4.75, 0, 1)
> Xstar
 [1] 0 0 0 0 0 0 1 0 1 0 0 1 0 0 1 0 0 0 0 1
> Ystar
 [1] 0 0 0 0 1 1 0 0 1 1 0 0 1 1 0 0 0 1 1 1 1 1
> p1 <- sum(Xstar)/length(Xstar); p1
[1] 0.25
> p2 <- sum(Ystar)/length(Ystar); p2
[1] 0.5
> phat <- (sum(Xstar)+sum(Ystar))/(length(Xstar)+length(Ystar)); phat
[1] 0.3809524
```

(i)

Null hypothesis:

4. $p_1 = p_2$

Alternative hypotheses:

5. $p_1 \neq p_2$ (requires two-tail test)
6. $p_1 < p_2$ (requires left-tail test)
7. $p_1 > p_2$ (requires right-tail test)

(ii)

From the slides, we know:

$$Z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \sim \mathcal{N}(0,1)$$

Thus our null distribution $H_0 \sim N(0, 1)$

and our observed z is

```
> z <- (p1 - p2) / sqrt(phat*(1 - phat)*( (1/length(Xstar)) + (1/length(Ystar))
)); z
[1] -1.666266
```

(iii)

Two-tail test:

$2\Phi(-\ |z|) = 0.09566045 > \alpha = 0.01$; we cannot reject H_0.

```
> 2*pnorm(-abs(z))
[1] 0.09566045
```

Left-tail test:

$\Phi(z) = 0.04783022 > \alpha = 0.01$; we cannot reject H_0.

```
> pnorm(z)
[1] 0.04783022
```

Right-tail test:

$\Phi(z) = 0.9521698 > \alpha = 0.01$; we cannot reject H_0.

```
> 1 - pnorm(z)
[1] 0.9521698
```

(iv)

Two-tail test:

```
> prop.test(c(sum(Xstar), sum(Ystar)), c(length(Xstar), length(Ystar)),
alternative = c("two.sided"), correct = FALSE )

      2-sample test for equality of proportions without continuity correction

data:  c(sum(Xstar), sum(Ystar)) out of c(length(Xstar), length(Ystar))
X-squared = 2.7764, df = 1, p-value = 0.09566
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.53225275  0.03225275
sample estimates:
prop 1 prop 2
  0.25   0.50
```

Left-tail test:

```
> prop.test(c(sum(Xstar), sum(Ystar)), c(length(Xstar), length(Ystar)),
alternative = c("less"), correct = FALSE )

      2-sample test for equality of proportions without continuity correction

data:  c(sum(Xstar), sum(Ystar)) out of c(length(Xstar), length(Ystar))
X-squared = 2.7764, df = 1, p-value = 0.04783
alternative hypothesis: less
95 percent confidence interval:
 -1.00000000 -0.01312602
sample estimates:
prop 1 prop 2
  0.25   0.50
```

Right-tail test:

```
> prop.test(c(sum(Xstar), sum(Ystar)), c(length(Xstar), length(Ystar)),
alternative = c("greater"), correct = FALSE )

      2-sample test for equality of proportions without continuity correction

data:  c(sum(Xstar), sum(Ystar)) out of c(length(Xstar), length(Ystar))
X-squared = 2.7764, df = 1, p-value = 0.9522
alternative hypothesis: greater
95 percent confidence interval:
 -0.486874  1.000000
sample estimates:
prop 1 prop 2 
  0.25   0.50 
```

(v)

Even after running all three tests, we cannot reject H_0. This means that $p_1 = p_2$ at the 0.01 significance level.