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I pledge my honor that I have abided by the Stevens Honor System.

Problem 1

(i)

```
> pbinom(8, 30, .35)
[1] 0.2246957
> pbinom(8, 60, .35)
[1] 0.000146853
> pbinom(8, 90, .35)
[1] 9.635798e-09
> pbinom(8, 120, .35)
[1] 2.41666e-13
```

(ii)

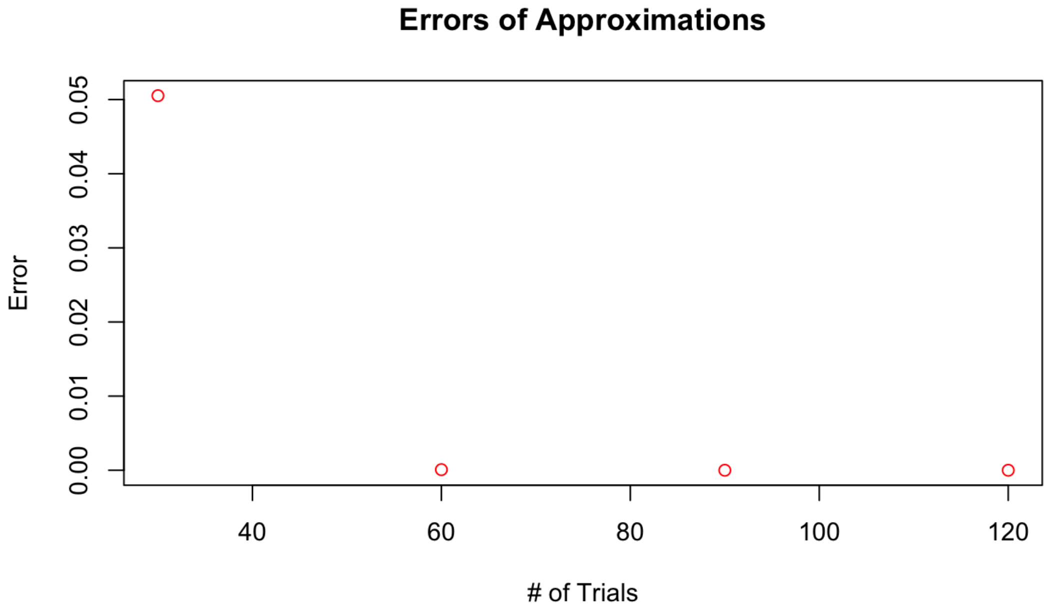
```
> pnorm(8.05, 30*.35, sqrt(30*.35*(1-.35)))
[1] 0.1741711
> pnorm(8.05, 60*.35, sqrt(60*.35*(1-.35)))
[1] 0.0002281969
> pnorm(8.05, 90*.35, sqrt(90*.35*(1-.35)))
[1] 1.095247e-07
> pnorm(8.05, 120*.35, sqrt(120*.35*(1-.35)))
[1] 4.078348e-11
```

(iii)

```
> abs(pbinom(8, 30, .35) - pnorm(8.05, 30*.35, sqrt(30*.35*(1-.35))))
[1] 0.05052456
> abs(pbinom(8, 60, .35) - pnorm(8.05, 60*.35, sqrt(60*.35*(1-.35))))
[1] 8.134398e-05
> abs(pbinom(8, 90, .35) - pnorm(8.05, 90*.35, sqrt(90*.35*(1-.35))))
[1] 9.988888e-08
> abs(pbinom(8, 120, .35) - pnorm(8.05, 120*.35,
sqrt(120*.35*(1-.35))))
[1] 4.054182e-11
```

(iv)

```
> n <- c(30, 60, 90, 120)
> abs_errors <- c(abs(pbinom(8, 30, .35) - pnorm(8.05, 30*.35,
sqrt(30*.35*(1-.35)))),
+               abs(pbinom(8, 60, .35) - pnorm(8.05, 60*.35,
sqrt(60*.35*(1-.35)))),
+               abs(pbinom(8, 90, .35) - pnorm(8.05, 90*.35,
sqrt(90*.35*(1-.35)))),
+               abs(pbinom(8, 120, .35) - pnorm(8.05, 120*.35,
sqrt(120*.35*(1-.35)))))
>
> plot(n, abs_errors, col="red", xlab="# of Trials", ylab = "Error",
main = "Errors of Approximations")
```



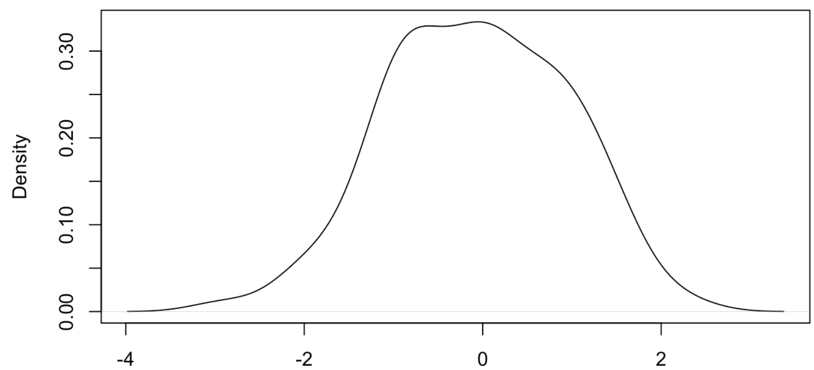
After 30 trials the errors tend to basically zero, so the more trials the less errors.

Problem 2
(i, ii, iii, iv)

```{r}
generate_plots <- function(n) {
  X_values <- c()
  S_values <- c()

  for (i in 1:100) {
    x <- rnorm(n, mean=2, sd=3)
    X_bar <- mean(x)
    s_squared <- var(x)

    X_values <- c(X_values, (X_bar - 2) / sqrt(9/n))
    S_values <- c(S_values, ((n-1) * s_squared) / 9)
  }

  plot(density(X_values), main=paste("Density curve of (X_bar - 2) /
sqrt(9/n) where n =", n))
  plot(density(S_values), main=paste("Density curve of (n-1)S^2 / 9
where n =", n))
  plot(X_values, S_values, main=paste("Scatterplot for both equations
where n =", n), xlab = "(X_bar - 2) / sqrt(9/n)", ylab="(n-1)S^2 /
9")
}
```

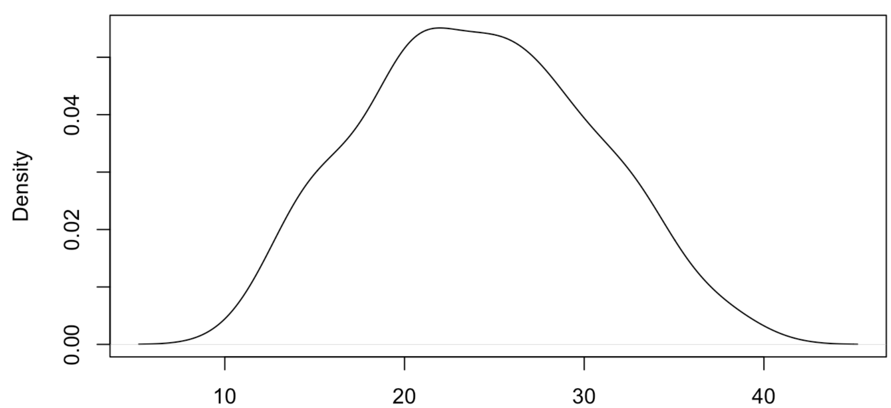
```r
for (n in c(25, 50, 75, 100)) {
  generate_plots(n)
}
```

Density curve of (X_bar - 2) / sqrt(9/n) where n = 25

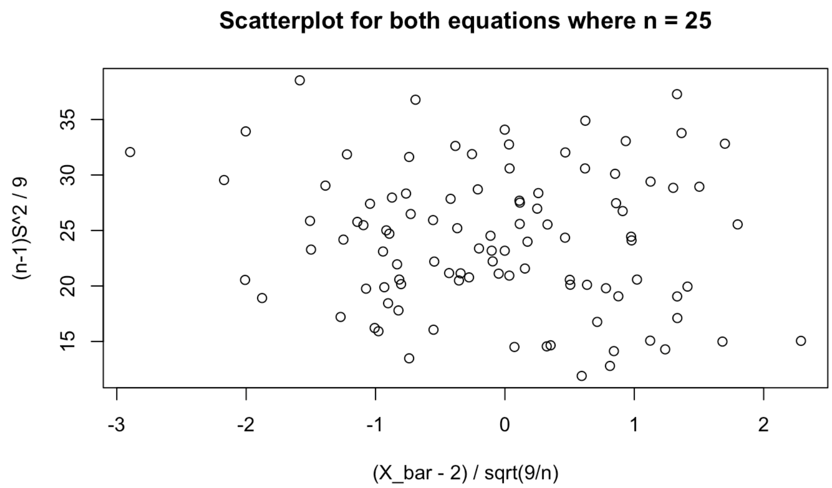


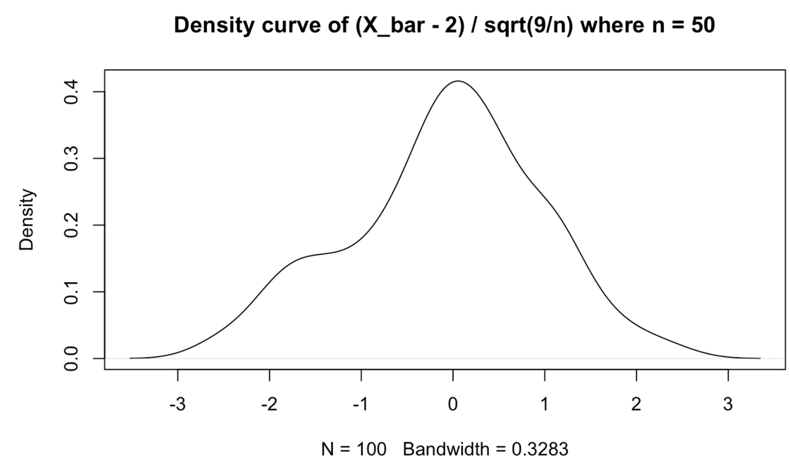
N = 100   Bandwidth = 0.3614

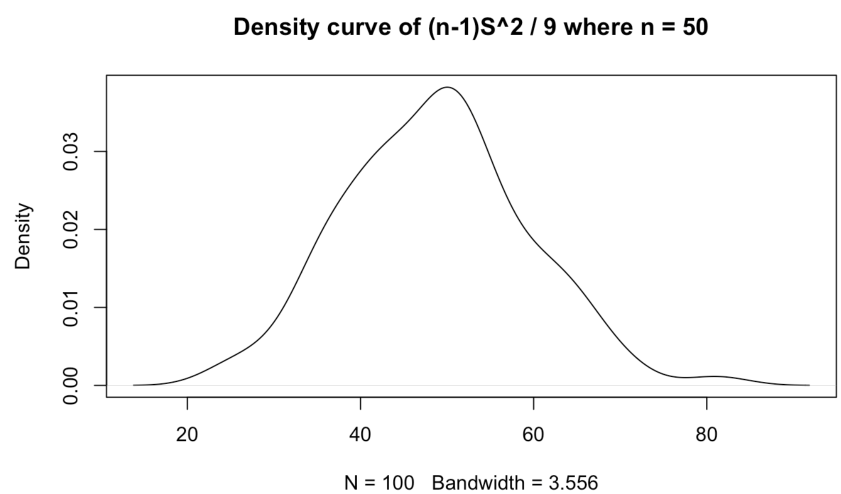
Density curve of (n-1)S^2 / 9 where n = 25

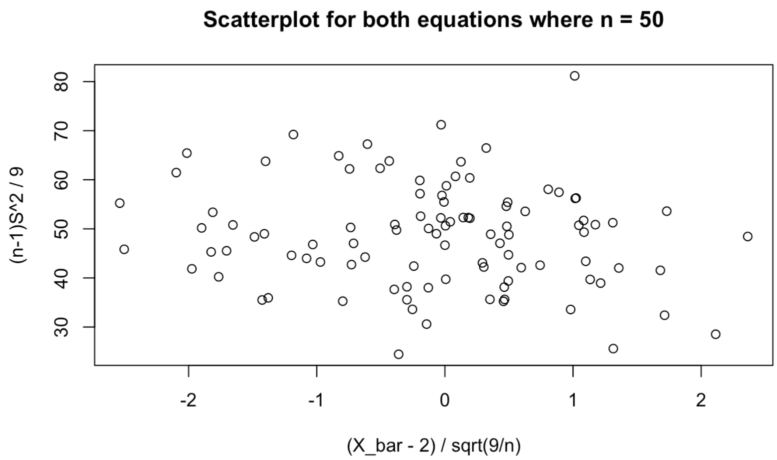


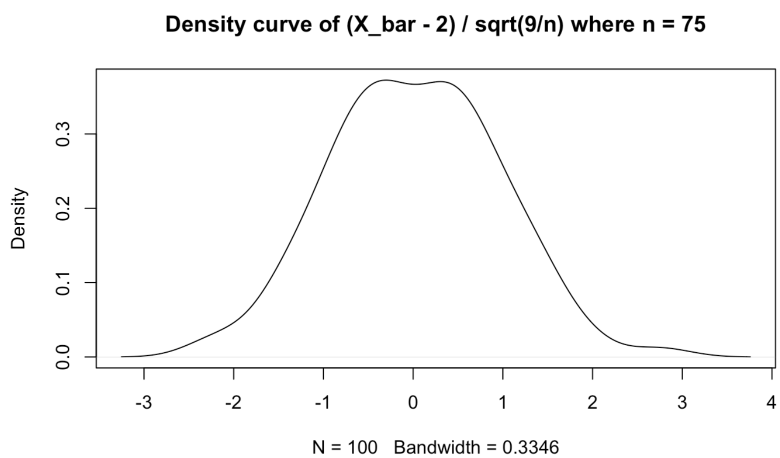
N = 100   Bandwidth = 2.228

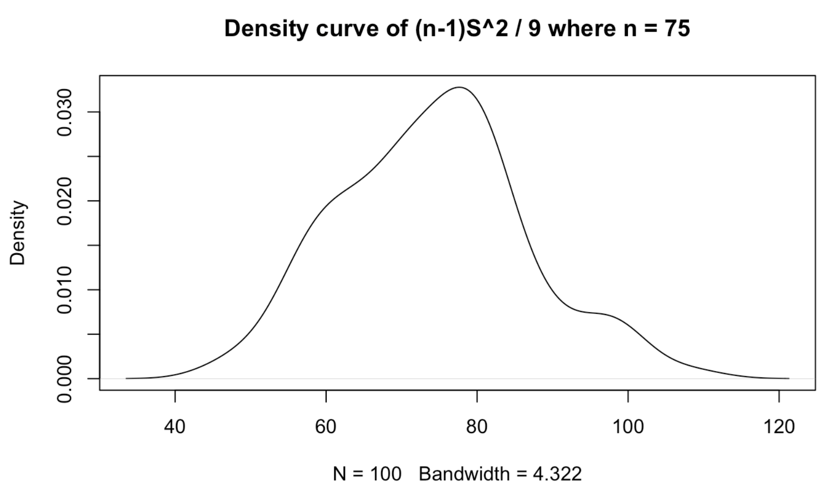

Scatterplot for both equations where n = 25
(n-1)S^2 / 9
15
20
25
30
35
-3
-2
-1
0
1
2
(X_bar - 2) / sqrt(9/n)


Density curve of (X_bar - 2) / sqrt(9/n) where n = 50
Density
0.0
0.1
0.2
0.3
0.4
-3
-2
-1
0
1
2
3
N = 100 Bandwidth = 0.3283

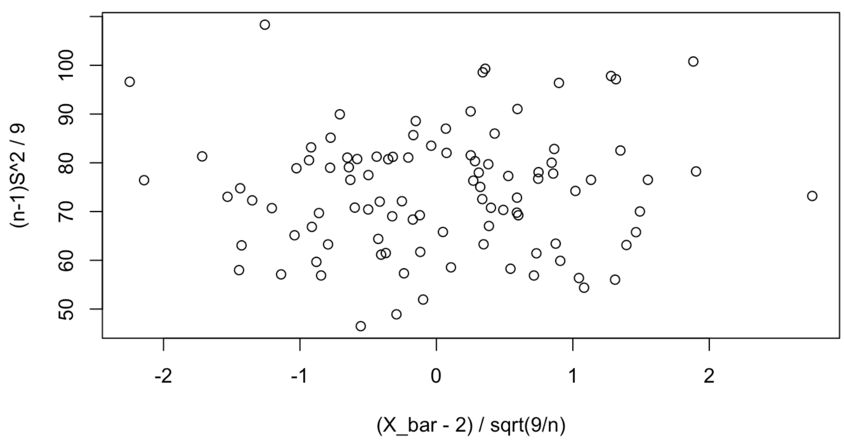

Density curve of (n-1)S^2 / 9 where n = 50
Density
0.00
0.01
0.02
0.03
20
40
60
80
N = 100 Bandwidth = 3.556


Scatterplot for both equations where n = 50
(n-1)S^2 / 9
(X_bar - 2) / sqrt(9/n)

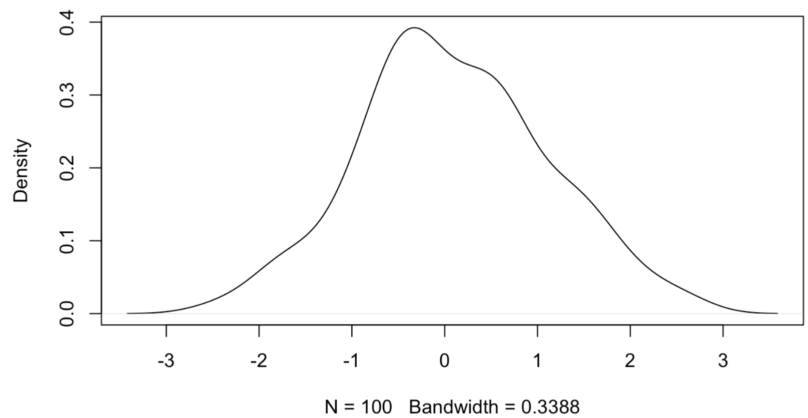

Density curve of (X_bar - 2) / sqrt(9/n) where n = 75
Density
N = 100   Bandwidth = 0.3346


Density curve of (n-1)S^2 / 9 where n = 75
Density
N = 100   Bandwidth = 4.322

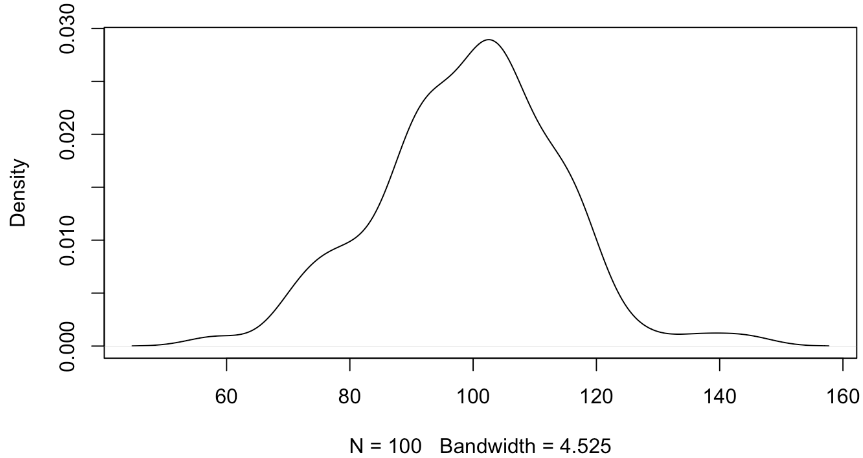
Scatterplot for both equations where n = 75



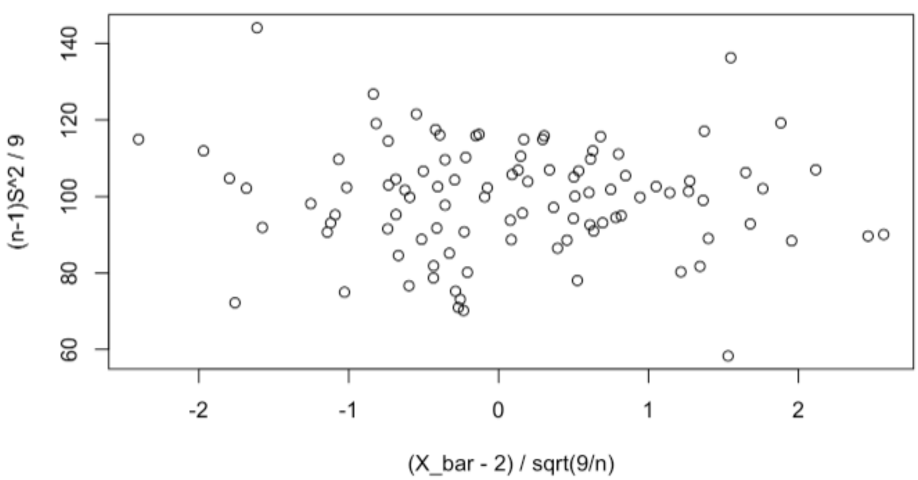
Density curve of (X_bar - 2) / sqrt(9/n) where n = 100



Density curve of (n-1)S^2 / 9 where n = 100



Scatterplot for both equations where n = 100



(v)
The higher the n, the closer the distribution of (x_bar - 2) / sqrt(9 / n) was to normal while conversely, the distribution for the other equation became less normal.

(vi)
As the scatterplots reveal no certain correlation, we can assume the values are independent.

Problem 3

(i)

```
> X <- c(-0.4, 0.15, 0.5, -1.2, -0.5, 0.9, 1.5, -0.7, 1.5, -1, 2.25,
3.5, -1.45, 0.2, 2.75)
> n = length(X)
> Xbar = mean(X)
> a1 <- sum((X - 1)^2) / 2
> p1 <- 1 - pchisq(a1, df=n)

> a₁ = 17.47
> P(> a₂) = 0.2915524
a1 = 17.47   P(Chi-squared > a1) = 0.2915524
```

(ii)

```
> a2 <- sum((X - Xbar)^2) / 2
> p2 <- 1 - pchisq(a2, df=n-1)

> a₂ = 15.83667
> P(> a₂) = 0.3234504
a2 = 15.83667   P(Chi-squared > a2) = 0.3234504
```

(iii)

```
> a3 <- (Xbar - 1) / sqrt(var(X) / n)
> p3 <- 2 * (1 - pt(abs(a3), df=n-1))
```

> a_3 = -1.201627

> P(> $|a_3|$) = 0.2494416

a3 = -1.201627   P(|T| > |a3|) = 0.2494416

Problem 4

(i)

P(X ∈ (−1, 2)):

```
> pnorm(2, mean = -1, sd = 4) - pnorm(-1, mean = -1, sd = 4)
[1] 0.2733726
```

P(Y ∈ (−1, 2)):

```
> pchisq(2, 10) - pchisq(-1, 10)
[1] 0.003659847
```

P(T ∈ (−1, 2)):

```
> pt(2, 11) - pt(-1, 11)
[1] 0.7951977
```

P(F ∈ (−1, 2)):

```
> pf(2, 8, 11) - pf(-1, 8, 11)
[1] 0.8578956
```

(ii)

```
> qs <- c(0.05/2, 0.05, 1 - 0.05, 1 - 0.05/2)
> qs
[1] 0.025 0.050 0.950 0.975
> qnorm(qs, mean = -1, sd = 4)
[1] -8.839856 -7.579415  5.579415  6.839856
> qchisq(qs, 10)
[1]  3.246973  3.940299 18.307038 20.483177
> qt(qs, 11)
[1] -2.200985 -1.795885  1.795885  2.200985
> qf(qs, 8, 11)
[1] 0.2356594 0.3018457 2.9479903 3.6638190
```

Problem 5

(i)

The expected value for a binomial distribution would be the product of each trial's possibilities of succeeding. This would look like $\Sigma_{i=0}^{n}(x_i p_i) = np$. Therefore $E[N \sim B(n,\ p)] = np$.

Generally, $Var[N] = E[N^2] - E[N]^2$. For the binomial distribution,
$Var[X] = np + n(n-1)p^2 - (np)^2 = np - np^2 = np(1-p)$.

(ii)
According to Slide 20 in slide deck 2, if $X \sim N(\mu, \sigma^2)$ then $\frac{X-\mu}{\sigma} \sim N(0,1)$. Multiplying standard normal distributions changes the standard deviation but not the expected value, which means that $E[(\frac{X-\mu}{\sigma})^3] = E[\frac{X-\mu}{\sigma}] = 0$.

(iii)
The student T distribution is defined as $T = \frac{X}{\sqrt{Y/n}}$ where $X \sim N(0,1)$ and $X \sim \chi_n^2$ are independent.

$Y/n \sim \chi_1^2$ where $E[Y/n] = 1$. Therefore $E[\frac{X}{\sqrt{Y/n}}] = 0$.
The student T distribution is symmetric about the y-axis, so $P(T \le t) = 1 - P(T \ge t)$. We know $F^{-1}(\alpha)$ = some x such that $P(T \le x) = \alpha$. Therefore $-F^{-1}(\alpha) = F^{-1}(1-\alpha)$.

Problem 6
(i)
$X + Y \sim N(0, 13)$;
$\mu = -1 + 1 = 0$
$\sigma^2 = 9 + 4 = 13$

$X - Y \sim N(-2,\ 13)$;
$\mu = -1 - 1 = -2$
$\sigma^2 = 9 + 4 = 13$

$W - 3X + 2Y \sim N(7,\ 113)$;
$\mu = 2 - 3(-1) + 2(1) = 7$
$\sigma^2 = 16 + (-3)^2(9) + 2^2(4) = 113$

(ii)
Recall W, X, and Y follow normal distributions and are mutually independent.
$\frac{(X+1)^2}{9} = (\frac{X+1}{3})^2 \sim \chi_1^2$
$\frac{(Y-1)^2}{4} = (\frac{Y-1}{2})^2 \sim \chi_1^2$
$\Rightarrow \frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4} = 2\chi_1^2 \sim \chi_2^2$

$\frac{(W-2)^2}{16} = (\frac{W-1}{16})^2 \sim \chi_1^2$

We already know $\frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4} \sim \chi_2^2$

$\Rightarrow \frac{(W-2)^2}{16} + \frac{(X+1)^2}{9} + \frac{(Y-1)^2}{4} \sim \chi_3^2$

(iii)

Recall W, X, and Y follow normal distributions and are mutually independent.

$\frac{Y-1}{2} \sim N(0, 1)$

$\frac{(X+1)^2}{9} + \frac{(W-2)^2}{16} \sim \chi_2^2$

$\frac{\frac{Y-1}{2}}{\sqrt{[\frac{(X+1)^2}{9} + \frac{(W-2)^2}{16}]/2}} \sim T_2$ because it is of the form $\frac{N(0,1)}{\sqrt{\chi_n^2/n}}$.

$\frac{X+1}{3} \sim N(0, 1)$

$\frac{(Y-1)^2}{4} + \frac{(W-2)^2}{16} \sim \chi_2^2$

$\frac{\frac{X+1}{3}}{\sqrt{[\frac{(Y-1)^2}{4} + \frac{(W-2)^2}{16}]/2}} \sim T_2$ because it is of the form $\frac{N(0,1)}{\sqrt{\chi_n^2/n}}$.

(iv)

$\frac{(Y-1)^2}{4} \sim \chi_1^2 \equiv \frac{\chi_1^2}{1}$

$\frac{1}{2}[\frac{(X+1)^2}{9} + \frac{(W-2)^2}{16}] \sim \frac{\chi_2^2}{2}$

$\frac{(Y-1)^2}{4} / \frac{1}{2}[\frac{(X+1)^2}{9} + \frac{(W-2)^2}{16}] \sim F_{1,2}$ because it is of the form $\frac{\chi_n^2/n}{\chi_m^2/m}$.

$\frac{1}{2}[\frac{(X+1)^2}{9} + \frac{(W-2)^2}{16}] / \frac{(Y-1)^2}{4} \sim F_{2,1}$ because it is of the form $\frac{\chi_n^2/n}{\chi_m^2/m}$.