CHAPTER 1

# Introduction

## 1.1 Introduction

Logic is one of the oldest intellectual disciplines in human history. It dates back to Aristotle. It has been studied through the centuries by people like Leibniz, Boole, Russell, Turing, and many others. And it is still a subject of active investigation today.

We use Logic in just about everything we do. We use the language of Logic to state observations, to define concepts, and to formalize theories. We use logical reasoning to derive conclusions from these bits of information. We use logical proofs to convince others of our conclusions.

And we are not alone! Logic is increasingly being used by computers - to prove mathematical theorems, to validate engineering designs, to diagnose failures, to encode and analyze laws and regulations and business rules.

Logic is also becoming more common at the interface between man and machine, in "logic-enabled" computer systems, where users can view and edit logical sentences. Think, for example, about email readers that allow users to write rules to manage incoming mail messages - deleting some, moving others to various mailboxes, and so forth based on properties of those messages. In the business world, eCommerce systems allow companies to encode price rules based on the product, the customer, the date, and so forth.

Moreover, Logic is sometimes used not just by users in communicating with computer systems but by software engineers in building those systems (using a programming methodology known as *logic programming*).

This chapter is an overview of Logic as presented in this book. We start with a discussion of possible worlds and illustrate the notion in an application area known as Sorority World. We then give an informal introduction to the key elements of Logic - logical sentences, logical entailment, and logical proofs. We then talk about the value of using a formal language for expressing logical information instead of natural language. Finally, we discuss the automation of logical reasoning and some of the computer applications that this makes possible.

## 1.2 Possible Worlds

Consider the interpersonal relations of a small sorority. There are just four members - Abby, Bess, Cody, and Dana. Some of the girls like each other, but some do not.

Figure 1 shows one set of possibilities. The checkmark in the first row here means that Abby likes Cody, while the absence of a checkmark means that Abby does not like the other girls (including herself). Bess likes Cody too. Cody likes everyone but herself. And Dana also likes the popular Cody.

| | Abby | Bess | Cody | Dana |
|---|---|---|---|---|
| **Abby** | | | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | | ✓ | |

Figure 1 - One state of Sorority World

Of course, this is not the only possible state of affairs. Figure 2 shows another possible world. In this world, every girl likes exactly two other girls, and every girl is liked by just two girls.

| | Abby | Bess | Cody | Dana |
|---|---|---|---|---|
| **Abby** | ✓ | | ✓ | |
| **Bess** | | ✓ | | ✓ |
| **Cody** | ✓ | | ✓ | |
| **Dana** | | ✓ | | ✓ |

Figure 2 - Another state of Sorority World

As it turns out, there are quite a few possibilities. Given four girls, there are sixteen possible instances of the *likes* relation - Abby likes Abby, Abby likes Bess, Abby likes Cody, Abby likes Dana, Bess likes Abby, and so forth. Each of these sixteen can be either true or false. There are $2^{16}$ (65,536) possible combinations of these true-false possibilities, and so there are $2^{16}$ possible worlds.

Let's assume that we do not know the likes and dislikes of the girls ourselves but we have informants who are willing to tell us about them. Each informant knows a little about the likes and dislikes of the girls, but no one knows everything.

Here is where Logic comes in. By writing *logical sentences*, each informant can express exactly what he or she knows - no more, no less. For our part, we can use the sentences we have been told to draw conclusions that are *logically entailed* by those sentences. And we can use *logical proofs* to explain our conclusions to others. The next three sections describe these aspects of Logic in greater detail.

## 1.3 Logical Sentences

Figure 1 shows some logical sentences pertaining to our sorority world. The first sentence is straightforward; it tells us directly that Dana likes Cody. The second and third sentences tell us what is not true without saying what is true. The fourth sentence says that one condition holds or another but does not say which. The fifth sentence gives a general fact about the girls Abby likes.

The sixth sentence expresses a general fact about Cody's likes. The last sentence says something about everyone.

*Dana likes Cody.*
*Abby does not like Dana.*
*Dana does not like Abby.*
*Bess likes Cody or Dana.*
*Abby likes everyone that Bess likes.*
*Cody likes everyone who likes her.*
*Nobody likes herself*

Figure 3 - Logical sentences describing Sorority World.

Sentences like these constrain the possible ways the world could be. Each sentence divides the set of possible worlds into two subsets, those in which the sentence is true and those in which the sentence is false. Believing a sentence is tantamount to believing that the world is in the first set. Given two sentences, we know the world must be in the intersection of the set of worlds in which the first sentence is true and the set of worlds in which the second sentence is true. Ideally, when we have enough sentences, we know exactly how things stand.

Effective communication requires a language that allows us to express what we know, no more and no less. If we know the state of the world, then we should write enough sentences to communicate this to others. If we do not know which of various ways the world could be, we need a language that allows us to express only what we know. The beauty of Logic is that it gives us a means to express incomplete information when that is all we have and to express complete information when full information is available.

## 1.4 Logical Entailment

Logical sentences can sometimes pinpoint a specific world from among many possible worlds. However, this is not always the case. Sometimes, a collection of sentences only partially constrains the world. For example, there are four different worlds that satisfy the sentences in Figure 3, viz. the ones shown in figure 4.

| | **Abby** | **Bess** | **Cody** | **Dana** |
|---|---|---|---|---|
| **Abby** | | | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | | ✓ | |

| | **Abby** | **Bess** | **Cody** | **Dana** |
|---|---|---|---|---|
| **Abby** | | ✓ | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | | ✓ | |

| | **Abby** | **Bess** | **Cody** | **Dana** |
|---|---|---|---|---|
| **Abby** | | | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | ✓ | ✓ | |

| | **Abby** | **Bess** | **Cody** | **Dana** |
|---|---|---|---|---|
| **Abby** | | ✓ | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | ✓ | ✓ | |

Figure 4 - Four states of Sorority World.

Even though a set of sentences does not determine a unique world, it is often the case that some sentences are true in every world that satisfies the given sentences. A sentence of this sort is said to be a *logical conclusion* from the given sentences. Said the other way around, a set of sentences *logically entails* a conclusion if and only if every world that satisfies the sentences also satisfies the conclusion.

What can we conclude from the bits of information in Figure 3? Quite a bit, as it turns out. For example, it must be the case that Bess likes Cody. Also, Bess does not like Dana. There are also some general conclusions that must be true. For example, in this world with just four girls, we can conclude that everybody likes somebody. Also, everyone is liked by somebody.

*Bess likes Cody.*
*Bess does not like Dana.*
*Everybody likes somebody.*
*Everybody is liked by somebody.*

Figure 5 - Conclusions about Sorority World

One way to check whether a set of sentences logically entails a conclusion is to examine the set of all worlds in which the given sentences are true. For example, in our case, we notice that, in every world that satisfies our sentences, Bess likes Cody, so the statement that Bess likes Cody is a logical conclusion from our set of sentences.

## 1.5 Logical Proofs

Unfortunately, determining logical entailment by checking all possible worlds is impractical in general. There are usually many, many possible worlds; and in some cases there can be infinitely many.

The alternative is *logical reasoning*, viz. the application of reasoning rules to derive logical conclusions and produce *logical proofs*, i.e. sequences of reasoning steps that leads from *premises* to *conclusions*.

The concept of proof, in order to be meaningful, requires that we be able to recognize certain reasoning steps as immediately obvious. In other words, we need to be familiar with the reasoning "atoms" out of which complex proof "molecules" are built.

One of Aristotle's great contributions to philosophy was his recognition that what makes a step of a proof immediately obvious is its form rather than its content. It does not matter whether you are talking about blocks or stocks or sorority girls. What matters is the structure of the facts with which you are working. Such patterns are called *rules of inference*.

As an example, consider the reasoning step shown below. We know that all Accords are Hondas, and we know that all Hondas are Japanese cars. Consequently, we can conclude that all Accords are Japanese cars.

*All Accords are Hondas.*
*All Hondas are Japanese.*
*Therefore, all Accords are Japanese.*

Now consider another example. We know that all borogoves are slithy toves, and we know that all slithy toves are mimsy. Consequently, we can conclude that all borogoves are mimsy. What's more, in order to reach this conclusion, we do not need to know anything about borogoves or slithy toves or what it means to be mimsy.

*All borogoves are slithy toves.*
*All slithy toves are mimsy.*
*Therefore, all borogoves are mimsy.*

What is interesting about these examples is that they share the same reasoning structure, viz. the pattern shown below.

*All x are y.*
*All y are z.*
*Therefore, all x are z.*

The existence of such reasoning patterns is fundamental in Logic but raises important questions. Which patterns are correct? Are there many such patterns or just a few?

Let us consider the first of these questions. Obviously, there are patterns that are just plain wrong in the sense that they can lead to incorrect conclusions. Consider, as an example, the (faulty) reasoning pattern shown below.

*All x are y.*
*Some y are z.*
*Therefore, some x are z.*

Now let us take a look at an instance of this pattern. If we replace *x* by *Toyotas* and *y* by *cars* and *z* by *made in America*, we get the following line of argument, leading to a conclusion that happens to be correct.

*All Toyotas are cars.*
*Some cars are made in America.*
*Therefore, some Toyotas are made in America.*

On the other hand, if we replace *x* by *Toyotas* and *y* by *cars* and *z* by *Porsches*, we get a line of argument leading to a conclusion that is questionable.

*All Toyotas are cars.*
*Some cars are Porsches.*
*Therefore, some Toyotas are Porsches.*

What distinguishes a correct pattern from one that is incorrect is that it must *always* lead to correct conclusions, i.e. they must be correct so long as the premises on which they are based are correct. As we will see, this is the defining criterion for what we call *deduction*.

Now, it is noteworthy that there are patterns of reasoning that are sometimes useful but do not satisfy this strict criterion. There is inductive reasoning, abductive reasoning, reasoning by analogy, and so forth.

*Induction* is reasoning from the particular to the general. The example shown below illustrates this. If we see enough cases in which something is true and we never see a case in which it is false, we tend to conclude that it is always true.

*I have seen 1000 black ravens.*
*I have never seen a raven that is not black.*
*Therefore, every raven is black.*
Now try red Hondas.

*Abduction* is reasoning from effects to possible causes. Many things can cause an observed result. We often tend to infer a cause even when our enumeration of possible causes is incomplete.

> *If there is no fuel, the car will not start.*
> *If there is no spark, the car will not start.*
> *There is spark.*
> *The car will not start.*
> *Therefore, there is no fuel.*
> What if the car is in a vacuum chamber?

Reasoning by *analogy* is reasoning in which we infer a conclusion based on similarity of two situations, as in the following example.

> *The flow in a pipe is proportional to its diameter.*
> *Wires are like pipes.*
> *Therefore, the current in a wire is proportional to diameter.*
> Now try price.

Of all types of reasoning, deduction is the only one that *guarantees* its conclusions in all cases, it produces only those conclusions that are logically entailed by one's premises. For this reason, in what follows, we concentrate entirely on deduction and leave these other forms of reasoning to others.

## 1.6 Formalization

So far, we have illustrated everything with sentences in English. While natural language works well in many circumstances, it is not without its problems. Natural language sentences can be complex; they can be ambiguous; and failing to understand the meaning of a sentence can lead to errors in reasoning.

Even very simple sentences can be troublesome. Here we see two grammatically legal sentences. They are the same in all but the last word, but their structure is entirely different. In the first, the main verb is *blossoms*, while in the second *blossoms* is a noun and the main verb is *sank*.

> *The cherry blossoms in the Spring.*
> *The cherry blossoms in the Spring sank.*

As another example of grammatical complexity, consider the following excerpt taken from the University of Michigan lease agreement. The sentence in this case is sufficiently long and the grammatical structure sufficiently complex that people must often read it several times to understand precisely what it says.

> *The University may terminate this lease when the Lessee, having made application and executed this lease in advance of enrollment, is not eligible to enroll or fails to enroll in the University or leaves the University at any time prior to the expiration of this lease, or for violation of any provisions of this lease, or for violation of any University regulation relative to resident Halls, or for health reasons, by providing the student with written notice of this termination 30 days prior to the effective date of termination, unless life, limb, or property would be jeopardized, the Lessee engages in the sales of purchase of controlled substances in violation of federal, state or local law, or the Lessee is no longer enrolled as a student, or the Lessee engages in the use or possession of firearms, explosives, inflammable liquids, fireworks, or other dangerous weapons within the building, or turns in a false alarm, in which cases a maximum of 24 hours notice would be sufficient.*

As an example of ambiguity, suppose I were to write the sentence *There's a girl in the room with a telescope*. See Figure 6 for two possible meanings of this sentence. Am I saying that there is a girl in a room containing a telescope? Or am I saying that there is a girl in the room and she is holding a telescope?

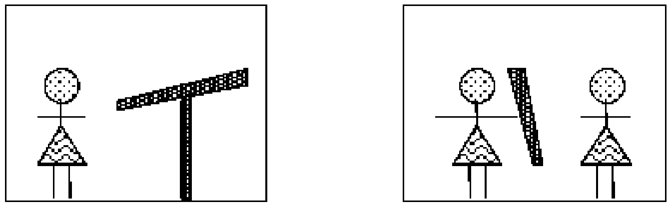

Figure 6 - *There's a girl in the room with a telescope*.

Such complexities and ambiguities can sometimes be humorous if they lead to interpretations the author did not intend. See the examples below for some infamous newspaper headlines with multiple interpretations. Using a formal language eliminates such unintentional ambiguities (and, for better or worse, avoids any unintentional humor as well).

**Crowds Rushing to See Pope Trample 6 to Death**
Journal Star, Peoria, 1980

**Scientists Grow Frog Eyes and Ears**
The Daily Camera, Boulder, 2000

**British Left Waffles On Falkland Islands**

**Food Stamp Recipients Turn to Plastic**
The Miami Herald, 1991

**Indian Ocean Talks**
The Plain Dealer, 1977

**Fried Chicken Cooked in Microwave Wins Trip**
The Oregonian, Portland, 1981

As an illustration of errors that arise in reasoning with sentences in natural language, consider the following examples. In the first, we use the transitivity of the *better* relation to derive a conclusion about the relative quality of champagne and soda from the relative quality of champagne and beer and the relative quality or beer and soda. So far so good.

*Champagne is better than beer.*
*Beer is better than soda.*
*Therefore, champagne is better than soda.*

Now, consider what happens when we apply the same transitivity rule in the case illustrated below. The form of the argument is the same as before, but the conclusion is somewhat less believable. The problem in this case is that the use of *nothing* here is syntactically similar to the use of *beer* in the preceding example, but in English it means something entirely different.

*Bad sex is better than nothing.*
*Nothing is better than good sex.*
*Therefore, bad sex is better than good sex.*

Logic eliminates these difficulties through the use of a formal language for encoding information. Given the syntax and semantics of this formal language, we can give a precise definition for the notion of logical conclusion. Moreover, we can establish precise reasoning rules that produce all and only logical conclusions.

In this regard, there is a strong analogy between the methods of Formal Logic and those of high school algebra. To illustrate this analogy, consider the following algebra problem.

> *Xavier is three times as old as Yolanda. Xavier's age and Yolanda's age add up to twelve. How old are Xavier and Yolanda?*

Typically, the first step in solving such a problem is to express the information in the form of equations. If we let $x$ represent the age of Xavier and $y$ represent the age of Yolanda, we can capture the essential information of the problem as shown below.

$$x - 3y = 0$$
$$x + y = 12$$

Using the methods of algebra, we can then manipulate these expressions to solve the problem. First we subtract the second equation from the first.

$$x - 3y = 0$$
$$x + y = 12$$
$$-4y = -12$$

Next, we divide each side of the resulting equation by -4 to get a value for $y$. Then substituting back into one of the preceding equations, we get a value for $x$.

$$x = 9$$
$$y = 3$$

Now, consider the following logic problem.

> *If Mary loves Pat, then Mary loves Quincy. If it is Monday and raining, then Mary loves Pat or Quincy. If it is Monday and raining, does Mary love Quincy?*

As with the algebra problem, the first step is formalization. Let $p$ represent the possibility that Mary loves Pat; let $q$ represent the possibility that Mary loves Quincy; let $m$ represent the possibility that it is Monday; and let $r$ represent the possibility that it is raining.

With these abbreviations, we can represent the essential information of this problem with the following logical sentences. The first says that *p implies q*, i.e. if Mary loves Pat, then Mary loves Quincy. The second says that *m and r implies p or q*, i.e. if it is Monday and raining, then Mary loves Pat or Mary loves Quincy.

$$p \Rightarrow q$$
$$m \wedge r \Rightarrow p \vee q$$

As with Algebra, Formal Logic defines certain operations that we can use to manipulate expressions. The operation shown below is a variant of what is called *Propositional Resolution*. The expressions above the line are the premises of the rule, and the expression below is the conclusion.

$$\begin{array}{ll} p_1 \wedge \ldots \wedge p_k & \Rightarrow q_1 \vee \ldots \vee q_l \\ r_1 \wedge \ldots \wedge r_m & \Rightarrow s_1 \vee \ldots \vee s_n \\ \hline p_1 \wedge \ldots \wedge p_k \wedge r_1 \wedge \ldots \wedge r_m & \Rightarrow q_1 \vee \ldots \vee q_l \vee s_1 \vee \ldots \vee s_n \end{array}$$

There are two elaborations of this operation. (1) If a proposition on the left hand side of one sentence is the same as a proposition on the right hand side of the other sentence, it is okay to drop the two symbols, with the proviso that *only one* such pair may be dropped. (2) If a constant is repeated on the same side of a single sentence, all but one of the occurrences can be deleted.

We can use this operation to solve the problem of Mary's love life. Looking at the two premises above, we notice that $p$ occurs on the left-hand side of one sentence and the right-hand side of the other. Consequently, we can cancel the $p$ and thereby derive the conclusion that, if is Monday and raining, then Mary loves Quincy or Mary loves Quincy.

$$\begin{array}{c} p \Rightarrow q \\ m \wedge r \Rightarrow p \vee q \\ \hline m \wedge r \Rightarrow q \vee q \end{array}$$

Dropping the repeated symbol on the right hand side, we arrive at the conclusion that, if it is Monday and raining, then Mary loves Quincy.

$$\begin{array}{c} m \wedge r \Rightarrow q \vee q \\ \hline m \wedge r \Rightarrow q \end{array}$$

This example is interesting in that it showcases our formal language for encoding logical information. As with algebra, we use symbols to represent relevant aspects of the world in question, and we use operators to connect these symbols in order to express information about the things those symbols represent.

The example also introduces one of the most important operations in Formal Logic, viz. Resolution (in this case a restricted form of Resolution). Resolution has the property of being *complete* for an important class of logic problems, i.e. it is the *only* operation necessary to solve any problem in the class.

## 1.7 Automation

The existence of a formal language for representing information and the existence of a corresponding set of mechanical manipulation rules together have an important consequence, viz. the possibility of *automated reasoning* using digital computers.

The idea is simple. We use our formal representation to encode the premises of a problem as data structures in a computer, and we program the computer to apply our mechanical rules in a systematic way. The rules are applied until the desired conclusion is attained or until it is determined that the desired conclusion cannot be attained. (Unfortunately, in some cases, this determination cannot be made; and the procedure never halts. Nevertheless, as discussed in later chapters, the idea is basically sound.)

Although the prospect of automated reasoning has achieved practical realization only in the last few decades, it is interesting to note that the concept itself is not new. In fact, the idea of building machines capable of logical reasoning has a long tradition.

One of the first individuals to give voice to this idea was Leibnitz. He conceived of "a universal algebra by which all knowledge, including moral and metaphysical truths, can some day be brought within a single deductive system". Having already perfected a mechanical calculator for arithmetic, he argued that, with this universal algebra, it would be possible to build a machine capable of rendering the consequences of such a system mechanically.

Boole gave substance to this dream in the 1800s with the invention of Boolean algebra and with the creation of a machine capable of computing accordingly.

The early twentieth century brought additional advances in Logic, notably the invention of the predicate calculus by Russell and Whitehead and the proof of the corresponding completeness and incompleteness theorems by Godel in the 1930s.

The advent of the digital computer in the 1940s gave increased attention to the prospects for automated reasoning. Research in artificial intelligence led to the development of efficient algorithms for logical reasoning, highlighted by Robinson's invention of resolution theorem proving in the 1960s.

Today, the prospect of automated reasoning has moved from the realm of possibility to that of practicality, with the creation of *logic technology* in the form of automated reasoning systems, such as Vampire, Prover9, the Prolog Technology Theorem Prover, Epilog, and others.

The emergence of this technology has led to the application of logic technology in a wide variety of areas. The following paragraphs outline some of these uses.

*Mathematics*. Automated reasoning programs can be used to check proofs and, in some cases, to produce proofs or portions of proofs.

*Engineering*. Engineers can use the language of Logic to write specifications for their products and to encode their designs. Automated reasoning tools can be used to simulate designs and in some cases validate that these designs meet their specification. Such tools can also be used to diagnose failures and to develop testing programs.

*Database Systems*. By conceptualizing database tables as sets of simple sentences, it is possible to use Logic in support of database systems. For example, the language of Logic can be used to define virtual views of data in terms of explicitly stored tables, and it can be used to encode constraints on databases. Automated reasoning techniques can be used to compute new tables, to detect problems, and to optimize queries.

*Data Integration* The language of Logic can be used to relate the vocabulary and structure of disparate data sources, and automated reasoning techniques can be used to integrate the data in these sources.

*Logical Spreadsheets*. Logical spreadsheets generalize traditional spreadsheets to include logical constraints as well as traditional arithmetic formulas. Examples of such constraints abound. For example, in scheduling applications, we might have timing constraints or restrictions on who can reserve which rooms. In the domain of travel reservations, we might have constraints on adults and infants. In academic program sheets, we might have constraints on how many courses of varying types that students must take.

*Law and Business*. The language of Logic can be used to encode regulations and business rules, and automated reasoning techniques can be used to analyze such regulations for inconsistency and overlap.

## 1.8 Reading Guide

Although Logic is a single field of study, there is more than one logic in this field. In the three main units of this book, we look at three different types of logic, each more sophisticated than the one before.

*Propositional Logic* is the logic of propositions. Symbols in the language represent "conditions" in the world, and complex sentences in the language express interrelationships among these conditions. The primary operators are Boolean connectives, such as *and*, *or*, and *not*.

*Relational Logic* expands upon Propositional Logic by providing a means for explicitly talking about individual objects and their interrelationships (not just monolithic conditions). In order to do so, we expand our language to include object constants and relation constants, variables and quantifiers.

*Herbrand Logic* takes us one step further by providing a means for describing worlds with infinitely many objects. The resulting logic is much more powerful than Propositional Logic and Relational Logic. Unfortunately, as we shall see, many of the nice computational properties of the first two logics are lost as a result.

Despite their differences, there are many commonalities among these logics. In particular, in each case, there is a language with a formal syntax and a precise semantics; there is a notion of logical entailment; and there are legal rules for manipulating expressions in the language.

These similarities allow us to compare the logics and to gain an appreciation of the fundamental tradeoff between expressiveness and computational complexity. On the one hand, the introduction of additional linguistic complexity makes it possible to say things that cannot be said in more restricted languages. On the other hand, the introduction of additional linguistic flexibility has adverse effects on computability. As we proceed though the material, our attention will range from the completely computable case of Propositional Logic to a variant that is not at all computable.

One final comment. In the hopes of preventing difficulties, it is worth pointing out a potential source of confusion. This book exists in the *meta* world. It contains sentences about sentences; it contains proofs about proofs. In some places, we use similar mathematical symbology both for sentences *in* Logic and sentences *about* Logic. Wherever possible, we try to be clear about this distinction, but the potential for confusion remains. Unfortunately, this comes with the territory. We are using Logic to study Logic. It is our most powerful intellectual tool.

## Recap

*Logic* is the study of information encoded in the form of logical sentences. Each logical sentence divides the set of all possible world into two subsets - the set of worlds in which the sentence is true and the set of worlds in which the set of sentences is false. A set of premises *logically entails* a conclusion if and only if the conclusion is true in every world in which all of the premises are true. *Deduction* is a form of symbolic reasoning that produces conclusions that are logically entailed by premises (distinguishing it from other forms of reasoning, such as *induction*, *abduction*, and *analogical reasoning*). A *proof* is a sequence of simple, more-or-less obvious deductive steps that justifies a conclusion that may not be immediately obvious from given premises. In Logic, we usually encode logical information as sentences in formal languages; and we use rules of inference appropriate to these languages. Such formal representations and methods are useful for us to use ourselves. Moreover, they allow us to automate the process of deduction, though the computability of such implementations varies with the complexity of the sentences involved.

## Exercises

Exercise 1.1: Consider the state of the Sorority World depicted below.

| | Abby | Bess | Cody | Dana |
|---|---|---|---|---|
| **Abby** | | ✓ | ✓ | |
| **Bess** | | | ✓ | |
| **Cody** | ✓ | ✓ | | ✓ |
| **Dana** | | ✓ | ✓ | |

For each of the following sentences, say whether or not it is true in this state of the world.

(*a*) *Abby likes Dana.*
(*b*) *Dana does not like Abby.*
(*c*) *Abby likes Cody or Dana.*
(*d*) *Abby likes someone who likes her.*
(*e*) *Somebody likes everybody.*

Exercise 1.2: Come up with a table of likes and dislikes for the Sorority World that makes *all* of the following sentences true. Note that there is more than one such table.

*Dana likes Cody.*
*Abby does not like Dana.*
*Bess likes Cody or Dana.*
*Abby likes everyone whom Bess likes.*
*Cody likes everyone who likes her.*
*Nobody likes herself.*

Exercise 1.3: Consider a set of Sorority World premises that are true in the four states of Sorority World shown in Figure 4. For each of the following sentences, say whether or not it is logically entailed by these premises.

(*a*) *Abby likes Bess or Bess likes Abby.*
(*b*) *Somebody likes herself.*
(*c*) *Everybody likes somebody.*

Exercise 1.4: Say whether or not the following reasoning patterns are logically correct.

(*a*) *All x are z. All y are z. Therefore, some x are y.*
(*b*) *Some x are y. All y are z. Therefore, some x are z.*
(*c*) *All x are y. Some y are z. Therefore, some x are z.*