

lg: An R package for Local Gaussian Approximations

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Abstract The package **lg** for the R programming language provides implementations of recent methodological advances on applications of the local Gaussian correlation. This includes estimation of the local Gaussian correlation itself, multivariate density estimation, conditional density estimation, various tests for independence and conditional independence, as well as a graphical module for creating dependence maps. This paper describes the **lg** package, its principles and its practical use.

Introduction

Tjøstheim and Hufthammer (2013) propose the *local Gaussian correlation* (LGC) as a new measure of statistical dependence between two stochastic variables X_1 and X_2 , which has the following important property yet unrivaled in the literature: It can separate between positive and negative nonlinear dependence while still reducing to the ordinary Pearson's correlation coefficient if X_1 and X_2 are jointly normally distributed. The R-package **localgauss** (Berentsen et al., 2014) provides two important functions in this context; one that calculates the sample LGC based on observed values of (X_1, X_2) , and one that uses the estimated LGC to perform a local test of independence between X_1 and X_2 as described in detail by Berentsen and Tjøstheim (2014).

We have lately seen a number of new applications of the LGC that the **localgauss**-package does not support, however. Støve et al. (2014) use the LGC to test for financial contagion across markets during crises. Otneim and Tjøstheim (2017) present a procedure for estimating multivariate density functions via the LGC, which Otneim and Tjøstheim (2018) modify in order to compute estimates of conditional density functions. Lacal and Tjøstheim (2017) present a test for serial independence within a time series, which Lacal and Tjøstheim (2018) extend in order to include a test for cross-correlation between two time series. Finally, Otneim and Tjøstheim (2019) develop the local Gaussian *partial* correlation (LGPC) as a measure of conditional dependence, and a corresponding test for conditional independence.

This paper describes the **lg**-package (Otneim, 2019), which provides a unified framework to implement all these methods, as well as a tool for visualizing the LGC and LGPC as dependence maps (and conditional dependence maps). Jordanger and Tjøstheim (2017) use the LGC in spectral analysis of time series, but those methods have their own computational ecosystem in the **localgaussSpec**-package, Jordanger (2018).

In Section 2.2 we provide a brief introduction to the LGC as well as the methods and applications referred to above. In Section 2.3 we describe the core function in the **lg**-package and move on to demonstrate the implementation of various applications in Section 2.4. We conclude this paper in Section 2.5 by demonstrating the graphical capabilities of the **lg**-package.

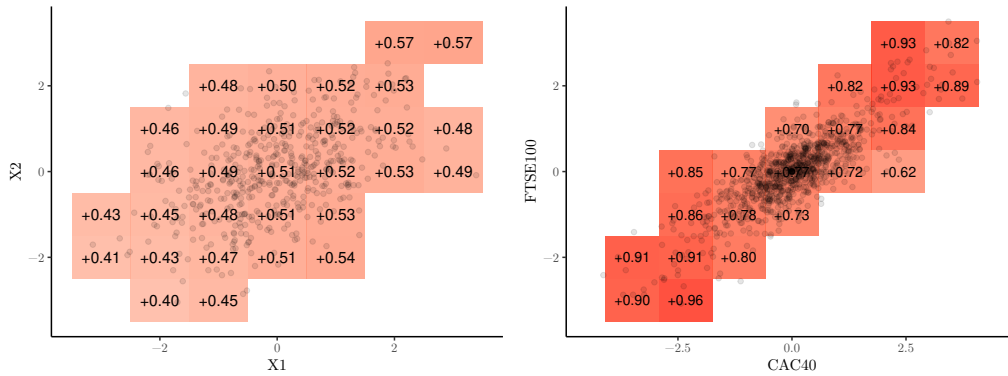
Statistical background

Consider a random vector X having the unknown probability density function $f_X(x)$. Based on a random sample X_1, \dots, X_n we wish to produce an estimate $\hat{f}_X(x)$ of $f_X(x)$, to which end the statistical literature provides an abundance of methods. One may, for example, assume that $f_X \in F_\theta$, where $F_\theta = \{f(x; \theta), \theta \in \Theta\}$ is a family of density functions indexed by some parameter θ living in a parameter space Θ , and estimate θ by means of the maximum likelihood method. On the other hand, we may estimate $f_X(\cdot)$ directly from the data using the classical kernel density estimator

$$\hat{f}_X(x) = \frac{1}{nb} \sum_{i=1}^n K\left(\frac{X_i - x}{b}\right),$$

where K is a symmetric density function (the kernel) and b is a tuning parameter (the bandwidth) controlling the smoothness of $\hat{f}_X(\cdot)$. This requires very little prior assumptions. There is a massive literature on density estimation that include countless variations of the parametric and nonparametric approaches, as well as many other clever and creative techniques.

Hjort and Jones (1996) provide one such idea. They consider a parametric family F_θ , but instead of searching for a single parameter value θ_0 for which $f_X(x) = f(x; \theta_0)$ (or approximately so), they rather assert that different members of F_θ may approximate f_X , locally, in different parts of its domain.



(a) Estimated local Gaussian correlations between two jointly normal variables having correlation equal to 0.5, from the CAC40 and FTSE100 stock indices, based on 1000 observations. (b) Local Gaussian correlation between daily returns 1000 consecutive trading days.

Figure 1: Two dependence maps

In other words, they seek to estimate a parameter *function* $\theta_0(x)$ for which $f_X(x) = f(x; \theta_0(x))$ (or approximately so), and do this by pointwise maximizing a *local likelihood function*:

$$\begin{aligned}\hat{\theta}(x) &= \arg \max_{\theta \in \Theta} L_n(\theta, x) \\ &= \arg \max_{\theta \in \Theta} \frac{1}{nb} \sum_{i=1}^n K\left(\frac{X_i - x}{b}\right) \log f(X_i; \theta) - \int \frac{1}{b} K\left(\frac{y - x}{b}\right) f(y; \theta) dy,\end{aligned}\quad (1)$$

where, again, K is a symmetric density function and b is a bandwidth parameter that controls the smoothness of the estimate.

Tjøstheim and Hufthammer (2013) consider the bivariate case $X = (X_1, X_2)$ and take F_θ to be the family of bivariate normal distribution consisting of densities on the form

$$\begin{aligned}f(x; \theta) &= \psi(x_1, x_2; \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) \\ &= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \\ &\quad \times \exp\left\{-\frac{1}{2(1-\rho^2)}\left(\frac{(x_1 - \mu_1)^2}{\sigma_1^2} - 2\rho\frac{(x_1 - \mu_1)(x_2 - \mu_2)}{\sigma_1\sigma_2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2}\right)\right\}.\end{aligned}\quad (2)$$

Using a sample $\{X_{1i}, X_{2i}\}, i = 1, \dots, n$, they estimate

$$\hat{\theta}(x) = (\hat{\mu}_1(x), \hat{\mu}_2(x), \hat{\sigma}_1(x), \hat{\sigma}_2(x), \hat{\rho}(x))$$

and take special interest in the estimated correlation function $\hat{\rho}(x)$ (i.e. the LGC) because it serves as an attractive local measure of statistical dependence between X_1 and X_2 . They show that the LGC reveals many types of nonlinear statistical dependence not seen by the ordinary (global) Pearson's correlation coefficient. Furthermore, the LGC distinguishes between positive and negative dependence and reduces to Pearson's ρ if X_1 and X_2 are jointly normal. We refer to Tjøstheim and Hufthammer (2013) for a detailed treatment of the theoretical foundations of the LGC as well as several examples, and rather present two simple illustrations at this point in order to demonstrate the concept.

In Figure 1 we have plotted the estimated LGC for two bivariate data sets; 1000 simulated observations from a binormal distribution having correlation equal to 0.5, and the daily return on the CAC40 and FTSE100 stock indices on 1000 consecutive trading days starting on May 5th 2014. In the first panel, we see that the estimated local correlation coincides with the global Pearson's ρ except for the estimation error. In the second panel we see clearly that the local correlation, and thus, as we will see, the dependence, is stronger in the lower left and upper right tails than in central parts of the distribution. This phenomenon is of course well known in the financial literature, but can now also be easily measured and quantitatively interpreted by means of the LGC. One may obtain these particular estimates from the older `localgauss`-package (as well as the `lg`-package of course), but the plotting routine that was used to produce these figures is included in the `lg`-package and will be described in more detail in Section 2.5.

Taking the LGC as a measure of dependence opens up for a number of possibilities to construct statistical tests. Berentsen and Tjøstheim (2014) derive from the representation (2) that $\rho(x) \equiv 0$ implies that X_1 and X_2 are independent (the converse statement holds if $\mu_i(x_1, x_2) = \mu_i(x_i)$ and $\sigma_i(x_1, x_2) = \sigma_i(x_i)$ for $i = 1, 2$, and in general if the observations have been suitably transformed according to a procedure presented later in this section). It follows then that departures from $\hat{\rho}(x) \equiv 0$ may be taken as evidence against X_1 and X_2 being statistically independent. Berentsen and Tjøstheim (2014) formalize this notion by testing whether $\rho(x) \equiv 0$ for all $x \in S \subset \mathbb{R}^2$ using the test statistic

$$T_{n,b} = \int_S h(\hat{\rho}(x)) dF_n(x) \quad (3)$$

for some function h , for example $h(x) = x^2$ or $h(x) = |x|$. Critical values may be obtained by permutations of the data under the null hypothesis. We demonstrate the implementation of this test using the **lg**-package in Section 2.4.2.

Consider next the stationary time series $\{X_t\}$. The autocorrelation $\rho_k = \rho(X_t, X_{t-k})$ is a fundamental descriptive and analytical concept, but is, again, only capable to completely capture *linear* serial dependence relationships. Lacal and Tjøstheim (2017) seek to remedy this by rather calculating the LGC to measure the *local* dependence between X_t and X_{t-k} , which in turn leads to a test for serial independence in the natural way. In fact, this work is mainly a theoretical exercise in order to accommodate dependent data. Testing for serial independence between X_t and X_{t-k} using observations $\{X_t, X_{t-k}\}_{t=1}^T$ leads to the same test statistic and bootstrap procedure as the test for independence between X_1 and X_2 that we described above, and we present its implementation in Section 2.4.2 as well.

Lacal and Tjøstheim (2018) extend this problem to test for serial cross dependence between two time series $\{X_t, Y_t\}$ by measuring the LGC between X_t and Y_{t-k} . Indeed, departures from $\hat{\rho}(x, y) \equiv 0$ is again taken as evidence against independence, and the test statistic (3) provides an aggregate measure of just that in the specified region S . This time, however, we can not just obtain permutations of the data in order to produce replicates of the test statistic under the null hypothesis. Lacal and Tjøstheim (2018) accomplish this by rather suggesting two block bootstrap procedures using fixed and random block sizes. This test is implemented in the **lg**-package, as we demonstrate in Section 2.4.2.

Støve et al. (2014) measure and test for financial contagion using the LGC. They define contagion by "a significant increase in cross-market linkages after a shock to one country" (Forbes and Rigobon, 2002) and employ the LGC to quantify such linkage. Contrary to the independence tests above, Støve et al. (2014) estimate the LGC on a grid $\{x_1, x_2\}_k, k = 1, \dots, K$ along the diagonal $D = \{(x_1, x_2) : x_1 = x_2\}$ before and after some critical event in the financial markets, i.e. in the crisis (C) and non-crisis (NC) periods. They compare the two estimates using the test statistic

$$T_{n,b}^D = \sum_{k=1}^K \{\hat{\rho}_C(x_k, x_k) - \hat{\rho}_{NC}(x_k, x_k)\} w(x_k, x_k),$$

where $w(\cdot, \cdot)$ is a weight function that serve the same purpose as the integration area S in (3). In this case, a standard bootstrap will suffice in order to produce approximate replicates of $T_{n,b}^D$ under the null hypothesis of no financial contagion, and we demonstrate the implementation of this test using the **lg**-package in Section 2.4.3.

Although Hjort and Jones (1996) provide a general framework for locally parametric density estimation using any parametric family, having any number of variables, it is evident that the method may struggle in multivariate applications much in the same way as the kernel density estimator does. This is a consequence of the curse of dimensionality, the effect of which is sought remedied by an algorithm provided by Otneim and Tjøstheim (2017). The idea is to locally fit the p -variate normal distribution $\psi(\mu, \Sigma)$, where μ is the vector of p expectations, and Σ is the $p \times p$ covariance matrix (to which the correlation matrix R corresponds), but under the following structural simplifications:

$$\mu_i(x) = \mu_i(x_1, \dots, x_p) \stackrel{\text{def}}{=} \mu_i(x_i) \quad (4)$$

$$\sigma_i(x) = \sigma_i(x_1, \dots, x_p) \stackrel{\text{def}}{=} \sigma_i(x_i) \quad (5)$$

$$\rho_{ij}(x) = \rho_{ij}(x_1, \dots, x_p) \stackrel{\text{def}}{=} \rho_{ij}(x_i, x_j). \quad (6)$$

These local parameters are estimated by univariate marginal locally normal fits (eqs. 4 and 5), and pairwise bivariate locally normal fits (eq. 6), and in the latter case the estimates $\hat{\mu}_i(x_i)$, $\hat{\mu}_j(x_j)$, $\hat{\sigma}_i(x_i)$ and $\hat{\sigma}_j(x_j)$ are kept fixed.

Denote by $F_i(x_i), i = 1, \dots, p$ the marginal distribution functions of the stochastic vector X , and by

$\hat{F}_i(x_i) = n^{-1} \sum_{i=1}^n 1(X_i \leq x_i)$ their empirical counterparts. [Otneim and Tjøstheim \(2017\)](#) argue that estimating the density $f_Z(z)$ of the vector $Z = \{\Phi^{-1}(F_i(X_i))\}_{i=1,\dots,p}$, in practice approximated by

$$\hat{Z} = \{\Phi^{-1}(\hat{F}_i(X_i))\}_{i=1,\dots,p}, \quad (7)$$

where $\Psi(\cdot)$ is the univariate standard normal cdf, will lead to better density estimates in many situations. In that case, one may simplify even further and fix

$$\mu_i(z_i) \stackrel{\text{def}}{=} 0 \text{ and } \sigma_i(z_i) \stackrel{\text{def}}{=} 1, \quad i = 1, \dots, p, \quad (8)$$

so that the only parameter functions left to estimate are the pairwise local Gaussian correlations $R(z) = \{\rho_{ij}(z_i, z_j)\}_{i < j}$, where we use the notation Z , z_i and z_j to signify that the estimation is performed on the (approximate) standard normal scale (or Z -scale). One then estimates the joint density $f_Z(z)$ of Z as

$$\hat{f}_Z(z) = \psi(z; \mu = 0, \sigma = 1, R = \hat{R}(z)), \quad (9)$$

and then substitute f_Z for \hat{f}_Z in the following relation obtained by [Otneim and Tjøstheim \(2017\)](#) in order to estimate the unknown density f_X :

$$f(x) = f_Z(\Phi^{-1}(F_1(x_1)), \dots, \Phi^{-1}(F_p(x_p))) \times \prod_{i=1}^p \frac{f_i(x_i)}{\phi(\Phi^{-1}(F_i(x_i)))}, \quad (10)$$

where $\phi(\cdot)$ is the standard normal pdf. This estimator is implemented the **lg**-package as described in Section 2.4.1.

One particular feature enjoyed by the jointly normally distributed vector X is that, for any partitioning $(X^{(1)}, X^{(2)})$, the conditional distribution of $X^{(1)} | X^{(2)} = x^{(2)}$ is also normal. In fact, if $X \sim \mathcal{N}(\mu, \Sigma)$, and μ and Σ is partitioned according to $(X^{(1)}, X^{(2)})$ as

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

then $X^{(1)} | X^{(2)} = x^{(2)} \sim \mathcal{N}(\mu^*, \Sigma^*)$, where

$$\mu^* = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x^{(2)} - \mu_2) \quad (11)$$

$$\Sigma^* = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}, \quad (12)$$

see e.g. [Johnson and Wichern \(2007, chapter 4\)](#). [Otneim and Tjøstheim \(2018\)](#) demonstrate that this property may be translated into a corresponding local argument without modification. If the joint density $f_X(\cdot)$ is locally Gaussian, i.e. if

$$f_X(x) = \psi(x, \mu(x), \Sigma(x)),$$

then $X^{(1)} | X^{(2)} = x^{(2)}$ is also locally Gaussian with local parameters given by equations (11) and (12), except that all quantities are x -dependent. If we use the transformation technique described above together with simplification (8), the local versions of equations (11) and (12) simplify to

$$\mu^*(z) = R_{12}(z)R_{22}(z)^{-1}z^{(2)}, \quad (13)$$

$$\Sigma^*(z) = R_{11}(z) - R_{12}(z)R_{22}(z)^{-1}R_{21}(z), \quad (14)$$

where we, again, switch to z -notation in order to make it clear that these quantities are estimated on the standard normal z -scale. An estimator $\hat{f}_{X^{(1)}|X^{(2)}}(\cdot|\cdot)$ of the conditional density $f_{X^{(1)}|X^{(2)}}(\cdot|\cdot)$ follows immediately from an expression corresponding to (10), and the **lg**-package provides functions for implementing this estimator in R as described in Section 2.4.1.

Finally, we refer to [Otneim and Tjøstheim \(2019\)](#), who take the local version of the conditional covariance matrix (12) (or (14) in the transformed case) as a measure for conditional dependence, and thus as an instrument to test for conditional independence. Specifically, consider the stochastic vector $X = (X^{(1)}, X^{(2)}, X^{(3)})$, where $X^{(1)}$ and $X^{(2)}$ are scalars and $X^{(3)}$ may be a vector. $X^{(1)}$ is conditionally independent from $X^{(2)}$ given $X^{(3)}$, written $X^{(1)} \perp X^{(2)} | X^{(3)}$ if the stochastic variables $X^{(1)} | X^{(3)}$ and $X^{(2)} | X^{(3)}$ are independent, or, equivalently, if the joint conditional density of $X^{(1)}$ and $X^{(2)}$ given $X^{(3)}$

can be written as the product

$$f_{X^{(1)}, X^{(2)} | X^{(3)}}(x^{(1)}, x^{(2)} | x^{(3)}) = f_{X^{(1)} | X^{(3)}}(x^{(1)} | x^{(3)}) \times f_{X^{(2)} | X^{(3)}}(x^{(2)} | x^{(3)}). \quad (15)$$

In this case, denote by $\alpha(z)$ the off-diagonal element in the 2×2 local correlation matrix $R^*(z)$ (which derives directly from $\Sigma^*(z)$ as given in (14)). If X has a local Gaussian distribution, the conditional independence (15) is equivalent to $\alpha(z) \equiv 0$, departures from which we take as evidence against conditional dependence between $X^{(1)}$ and $X^{(2)}$ given $X^{(3)}$, and this discrepancy is quantified using the test functional

$$T_{n,b}^{CI} = \int h(\hat{\alpha}(z)) dF_n(z). \quad (16)$$

Otneim and Tjøstheim (2019) describe a bootstrap procedure for generating replicates of $T_{n,b}^{CI}$ under the null hypothesis. In Section 2.4.4 we demonstrate how the **lg**-package may be used to extract estimates of the local partial correlation and perform tests for conditional independence according to this scheme.

The lg-object

The local Gaussian correlation may be used to perform a number of statistical analyses, as is evident from the preceding section. The practitioner must first, however, make three quite specific modelling choices; namely (i) to choose an estimation method, i.e. the level of simplification in multivariate applications, (ii) to determine whether the data should be transformed towards marginal standard normality before estimating the LGC, and (iii) to choose a set of bandwidths, or at least a method for calculating bandwidths. The architecture of the **lg**-packages requires the user to make these choices *before* endeavoring further into specific applications by imposing a strict, two step procedure:

1. Create an lg-object.
2. Apply relevant analysis functions to the lg-object.

In the following we assume that one has a *data set* x loaded into the R workspace, which must be an $n \times p$ matrix (one column per variable, one row per observation), possibly including NA's which will be excluded, or a data frame having the same dimensions. The fundamental syntax for creating an lg-object is `lg_object <- lg_main(x)`, and we will in this section explain how the modelling decisions (i)-(iii) can be encoded into the lg-object.

Selecting the estimation method

Given a data set x having n rows and $p \geq 2$ columns, the user must choose between four distinct estimation methods, and specify this choice by using the argument `est_method` to the `lg_main()`-function. We look at the built-in bivariate data set 'faithful' and load the **lg**-package in order to demonstrate the implementation:

```
R> x <- faithful
R> library(lg)
```

1. A full locally Gaussian fit for bivariate data. If $p = 2$, we may fit the bivariate normal $\psi(x, \theta)$ locally to $f(x)$, and by a "full local fit" we mean that we jointly estimate the five parameters

$$\theta(x) = (\mu_1(x_1, x_2), \mu_2(x_1, x_2), \sigma_1(x_1, x_2), \sigma_2(x_1, x_2), \rho(x_1, x_2))$$

by optimizing the local likelihood function (1) in the grid point $x = (x_1, x_2)$. To use this estimation method in the subsequent analysis, specify `est_method = "5par"` in the call to `lg_main()`:

```
R> lg_object <- lg_main(x, est_method = "5par")
```

The resulting `lg_object` is a list of class `lg`, and we may confirm that the assignment has been carried out correctly by inspecting its `est_method`-element:

```
R> lg_object$est_method
```

```
[1] "5par"
```

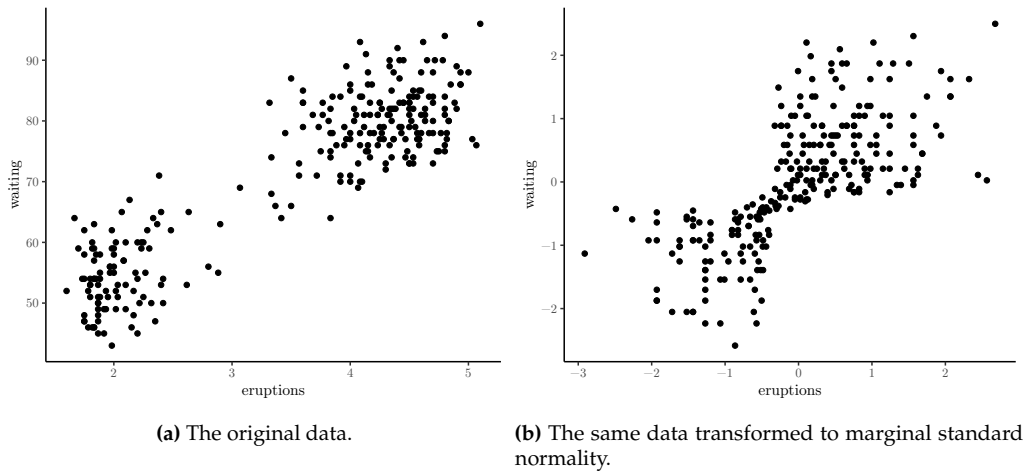


Figure 2: The same data on different scales.

Note that the full locally Gaussian fit for raw data is not available if the number of variables p is greater than 2. The `lg_main()`-function will check for this and throw an error if $p > 2$ and `est_method = "5par"`.

2. A simplified locally Gaussian fit for multivariate data. As described in the preceding section, we may construct a simplified estimation procedure for calculating the LGC in two steps, which in principle works for any number of dimensions (including $p = 2$):

1. Calculate $\mu_i(x_i)$ and $\sigma_i(x_i)$, $i = 1, \dots, p$ by fitting the univariate normal distribution locally to each *marginal* density $f_i(x_i)$ of $f(x)$.
2. Keep $\hat{\mu}_i(x)$ and $\hat{\sigma}_i(x_i)$, $i = 1, \dots, p$ from step 1 fixed when estimating $\rho_{ij}(x_i, x_j)$, $1 \leq i < j \leq p$ by fitting the bivariate normal distribution to each *pair* of variables X_i and X_j .

To use this method, create the `lg`-object by running the following line:

```
R> lg_object2 <- lg_main(x, est_method = "5par_marginals_fixed")
```

3. A simplified locally Gaussian fit for marginally standard normal data. This estimation method is applicable for marginally standard normal data, or data that have been transformed to approximate marginal standard normality by e.g. the transformation (7). In that case, we fix the marginal expectation functions and standard deviation functions to the constant values 0 and 1 respectively, rather than estimating them. To use this estimation method, create the `lg`-object by running

```
R> lg_object3 <- lg_main(x, est_method = "1par")
```

Note that the function call above will issue a warning if the option for transforming the data to marginal standard normality is not at the same time set to `TRUE`, see the next sub-section for details.

4. A full locally Gaussian fit for trivariate data. If $p = 3$ and we choose to transform the data to marginal standard normality (see the next sub-section), the transformed density $f_Z(\cdot)$ in (9) may be estimated by jointly estimating the three local correlations $\rho_{12}(z_1, z_2, z_3)$, $\rho_{13}(z_1, z_2, z_3)$ and $\rho_{23}(z_1, z_2, z_3)$. This estimation method was introduced recently by Otneim and Tjøstheim (2019) in order to increase power of their conditional independence test, but it can be used in any application described in this paper that consider trivariate data. To use this estimation method, create the `lg`-object by running

```
R> lg_object4 <- lg_main(x, est_method = "trivariate")
```

Data transformation

Next, the user must determine if the local Gaussian correlation should be estimated directly on the raw data or on the marginally normal pseudo observations (7). This is carried out by using the logical `transform_to_marginal_normality`-argument in `lg_main`, for example:

```
R> lg_object <- lg_main(x, transform_to_marginal_normality = TRUE)
```


The resulting `lg_object` now includes the element `transform_to_marginal_normality` set according to the input, and if this is `TRUE`, it also includes the `transformed_data` and a function `trans_new()` that may be used later to apply the same transformation to e.g. grid points. If the transformation option is set to `FALSE`, the `transformed_data` element contains the input data x , and `trans_new()` is nothing more than the identity mapping for points in \mathbb{R}^p . See Figure 2 for two plots that demonstrate the effect of the data transformation on the example data.

Bandwidth selection

Finally, the user must specify a set of bandwidths, or a method for calculating them. Given that the different estimation methods described in Section 2.3.1 require different sets of bandwidths (i.e joint, marginal and/or pairwise), the easiest approach for the user is to leave the selection and formatting of the bandwidths to the `lg_main()`-function.

The bandwidth plays a slightly different role in local likelihood estimation than elsewhere in the nonparametric literature. It controls the *level of localization* and thus only indirectly the *smoothness* of the estimates. Indeed, if we concentrate on the univariate case for the moment and assume that the (single) bandwidth b is small, we see from the local likelihood function (1) that only the very few observations closest to a fixed grid point x_0 will have significant weight when determining the local parameter estimates $\hat{\theta}_0(x)$ at that point. Moving on to another nearby point x_1 may then lead to a fairly different estimate $\hat{\theta}(x_1)$ because the set of observations having weight in this point is very different. This may again lead to rougher parameter estimates $\hat{\theta}(x)$ and in turn also to rougher density estimates $f(x, \hat{\theta}(x))$.

If the bandwidth b grows large, on the other hand, all observations receive similar weights, and furthermore: the local likelihood function (1) becomes approximately proportional to the ordinary (global) likelihood function $L_n(\theta) = \sum_{i=1}^n \log f(X_i; \theta)$. In other words, the local parameter estimates $\hat{\theta}(x)$ are smoothed towards the constant maximum likelihood estimates $\hat{\theta}_{ML}$, and the estimated density $f(x; \hat{\theta}(x))$ towards the maximum likelihood estimate $f(x; \hat{\theta}_{ML})$. This means that the bandwidth may be chosen to reflect the goodness-of-fit of $f(x; \theta)$ to the true density $f(x)$.

In the multivariate applications referred to in this paper, the bandwidth b in (1) is a diagonal matrix, and $1/b$ is naturally taken to represent its inverse.

We have in practice seen two automatic bandwidth selectors employed in the applications referred to in Section 2.2: a cross-validation procedure, which is slow, but accurate with respect to density estimation, and a plug-in bandwidth that is quick, but cruder. We use the argument `bw_method` to the `lg_main()`-function in order to choose between the two.

1. Choosing bandwidths by cross-validation. The functional

$$CV(b) = -\frac{1}{n} \sum_{i=1}^n \log f\left(X_i; \hat{\theta}^{(-i)}(X_i)\right),$$

where $\hat{\theta}^{(-i)}(x)$ is the parameter estimate obtained after deleting observation X_i from the data, is proportional to a quantity that estimates the Kullback-Leibler distance between $f(\cdot, \hat{\theta}(\cdot))$ and the true density $f(\cdot)$, see Berentsen and Tjøstheim (2014). The cross-validated bandwidth b_{CV} is hence given by

$$b_{CV} = \arg \min_b CV(b).$$

If we, for example, wish to use the simplified estimation procedure on the transformed data, we need bandwidths for the marginal estimates of the local means and standard deviations, as well as a 2×2 diagonal bandwidth matrix for each pair of variables. This is accomplished by the following call to `lg_main()`:

```
R> lg_object <- lg_main(x,
R+   est_method = "5par_marginals_fixed",
R+   transform_to_marginal_normality = TRUE,
R+   bw_method = "cv")
```

The `lg_object` now contains the necessary bandwidths for this configuration, as can be seen by inspecting the contents of its `bw`-element:

```
R> lg_object$bw

$ marginal
[1] 0.9989327 0.9875333
```

```

$marginal_convergence
[1] 0 0

$joint
  x1 x2      bw1      bw2 convergence
1  1  2 0.2946889 0.331971          0

```

This is itself a list, containing the crucial elements `marginal` for the p marginal bandwidths, and `joint` that contains the $p(p-1)/2$ bandwidth matrices, one for each pair of variables (which in this bivariate example just *one* variable pair, (x_1, x_2)). The convergence flags stem from the `optim()` and `optimize()` functions that we use to obtain the minimizer of $CV(\cdot)$, and 0 indicates successful convergence.

2. Using plug-in bandwidths. Obtaining cross-validated bandwidths is unfortunately fairly slow on a standard computer. For sample sizes in the 500-1000 range, the process may take several minutes, which is unfeasible when embarking on analyses that require e.g. resampling. We have therefore implemented a quick plug-in bandwidth selector as well, that may suffice in many practical situations, especially at the initial or exploratory stage.

Otneim and Tjøstheim (2017) show that the simplified version of the local Gaussian fit have the same convergence rates as the corresponding nonparametric kernel density estimator, for which Silverman (1986) derives the plug-in formula $b = 1.08 \cdot sd(x) \cdot n^{-1/5}$. By specifying `bw_method = "plugin"`, the `lg_main()`-function will select the bandwidths correspondingly, except that the exponent changes to $-1/6$ for joint bandwidths, and the proportionality constant is by default set to 1.75. The latter number is the result of regressing b_{CV} on $n^{-1/6}$ in a large simulation experiment covering various data generating processes (Otneim, 2016). We see the effect of switching to plugin bandwidths in the code below:

```

R> lg_object <- lg_main(x,
R+                       est_method = "5par_marginals_fixed",
R+                       transform_to_marginal_normality = TRUE,
R+                       bw_method = "plugin")
R> lg_object$bw

$marginal
[1] 0.5703274 0.5703274

$marginal_convergence
[1] NA NA

$joint
  x1 x2      bw1      bw2 convergence
1  1  2 0.6875061 0.6875061          NA

```

Summary of the initialization function

In the sub-section above we present the three most important arguments to `lg_main()`. Each of them allows the user to configure one of the three crucial modelling choices. Let us complete this treatment by covering some possibilities to make further adjustments to those choices.

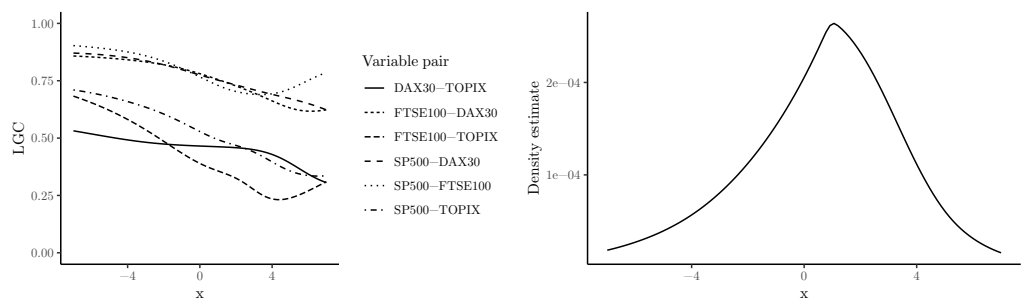
1. The user may supply the bandwidths directly to `lg_main()` by passing them to the `bw`-argument. They have to be in the correct format though, which is a list containing the vector `$marginal` if `est_method = 5par_marginals_fixed`, and always a data frame `$joint` specifying all variable pairs in the `x1` and `x2` columns and the corresponding bandwidths in the `bw1` and `bw2` columns. The function `bw_simple()` will assist in creating bandwidth objects.
2. If `bw_method = "plugin"` the user may change the proportionality constant and exponent in the plugin formula for the joint and, if applicable, the marginal bandwidths. See Table 1 for the necessary argument names.
3. If `bw_method = "cv"`, the user may change the numerical tolerance in the optimization of $CV(b)$. See Table 1 for the necessary argument names.

Statistical inference using the lg-package

We proceed in this section to demonstrate how to implement each of the tasks that we discussed in Section 2.2. The general pattern is to pass an `lg`-object, see the preceding section, to one of the

Argument	Explanation	Default value
x	The data, an $n \times p$ matrix or data frame	
bw_method	Method for calculating the bandwidths	"plugin"
est_method	Estimation method	"1par"
transform_to_		
marginal_normality	Transform the data?	TRUE
bw	The bandwidths to use if already calculated	NULL
plugin_constant_marginal	Prop. const. in plugin formula for marg. bw.	1.75
plugin_exponent_marginal	Exponent in plugin formula for marg. bw.	-1/5
plugin_constant_joint	Prop. const. in plugin formula for joint bw.	1.75
plugin_exponent_joint	Exponent in plugin formula for joint bw.	-1/6
tol_marginal	Abs. tolerance when optimizing $CV(b)$, marg.	10^{-3}
tol_joint	Abs. tolerance when optimizing $CV(b)$, joint	10^{-3}

Table 1: Arguments to the initialization function `lg_main()`



(a) The estimated pairwise LGC for the stock data along the diagonal.

(b) The corresponding joint density estimate of the four return series, plotted along the diagonal in \mathbb{R}^4 .

Figure 3: Important output from the `dlg()`-function.

estimation or test functions provided in the `lg`-package. We will look at some financial data in the examples: the monthly returns on the S&P500, FTSE100, DAX30 and TOPIX stock indices from January 1985 to March 2018. This data is available as an online appendix to this article.

Density estimation

We start by introducing a basic function for estimating the LGC on a grid as described by [Otneim and Tjøstheim \(2017\)](#), and thus also a probability density estimate. We create a grid, `x0`, having the same number of columns as the data in the code below. Note that we use the pipe operator `%>%` from the `magrittr`-package ([Bache and Wickham, 2014](#)) as well as functions from the `dplyr`-package ([Wickham et al., 2018](#)) for easy manipulation of data frames. We then pass the grid and the `lg`-object containing our modelling choices to the `dlg()`-function in order to do the estimation.

```
R> # Create an lg-object
R> lg_object <- lg_main(x = stock_data %>% select(-Date),
R+   est_method = "1par",
R+   bw_method = "plugin",
R+   transform_to_marginal_normality = TRUE)
R>
R> # Construct a grid diagonally through the data.
R> grid_size <- 100
R> x0 <- stock_data %>%
R+   select(-Date) %>%
R+   apply(2, function(y) seq(from = -7,
R+     to = 7,
R+     length.out = grid_size))
R>
```

Argument	Explanation	Default value
lg_object	The lg-object created by lg_main()	
grid	The evaluation points for the LGC	NULL
level	Level for confidence bands	0.95
normalization _points	The estimated density does not integrate to one by construction. dlg() will generate the given number of normal variables, having the same moments as the data, approximate $\int \hat{f}_X(x) dx$ by a Monte Carlo integral, and then normalize the density estimate accordingly	
NULL		
bootstrap	Calculate bootstrapped confidence intervals instead of asymptotic expressions	FALSE
B	Number of bootstrap replicates	500

Table 2: Arguments to the dlg()-function

```
R> # Estimate the local Gaussian correlation on the grid
R> density_object <- dlg(lg_object, grid = x0)
```

The last line of code creates a list containing a number of elements. The two most important are `$loc_cor`, which is a matrix of local correlations having one row per grid point and one column per *pair of variables* (the columns correspond to the rows in `density_object$pairs`), as well as `$f_est`, which is a vector containing the estimate $\hat{f}_X(x)$ of the joint density $f_X(x)$ in the grid points specified in `x0`. The estimated local correlations for this example is plotted in Figure 3a, and the corresponding density estimate is plotted (along the diagonal $x_1 = x_2 = x_3 = x_4 = x$) in Figure 3b.

The list `density_object` contains the estimated standard deviations of the local correlations in `$loc_cor_sd`, as well as lower and upper confidence bands `$loc_cor_lower` and `$loc_cor_upper` at the 95% level. We refer to Table 2 for a complete overview of the arguments to `dlg()`.

Note that the configuration `transform_to_marginal_normality = TRUE` and `est_method = 5par` in the bivariate case coincides with the situation considered by Tjøstheim and Hufthammer (2013). In that case, `dlg()` serves as a wrapper for the function `localgauss()` in the `localgauss`-package (Berentsen et al., 2014).

Obtaining the estimate of a conditional density using the Otneim and Tjøstheim (2018) algorithm described in Section 2.2 is very similar, but one must take particular care of the *ordering* of the variables in the data set. The estimation function, `clg()`, will *always* assume that the free variables come first, and that the conditioning variables come last. Let us illustrate this in the following code chunk by estimating the joint conditional density of S&P500 and FTSE100, given that DAX30 = TOPIX = 0.

```
R> # We must make sure that the free variables come first
R> returns1 <- stock_data %>% select(SP500, FTSE100, DAX30, TOPIX)
R>
R> # Create the lg-object
R> lg_object <- lg_main(returns1,
R+   est_method = "1par",
R+   bw_method = "plugin",
R+   transform_to_marginal_normality = TRUE)
R>
R> # Create a grid
R> x0 <- returns1 %>%
R+   select(SP500, FTSE100) %>%
R+   apply(2, function(y) seq(from = -7,
R+   to = 7,
R+   length.out = grid_size))
R>
R> # Calculate the conditional density
R> cond_density <- clg(lg_object, grid = x0, cond = c(0, 0))
```

Argument	Explanation	Default value
lg_object	The lg-object created by lg_main()	
h	The function $h(\cdot)$ in (3)	function(x) x^2
S	The integration area S in (3). Must be a logical function on potential grid points in \mathbb{R}^2	function(x) as.logical(rep(1, nrow(x)))
bootstrap	The bootstrap method, must be either "plain", "block" or "stationary"	"plain"
block	Block length for the block bootstrap,	
_length	mean block length for the stationary bootstrap. Calculated by <code>np: : b.star()</code> (Hayfield and Racine, 2008) if not provided	NULL
n_rep	Number of bootstrap replicates	1000

Table 3: Arguments to the ind_test()-function

The key argument in the call to `clg()` above is `cond = c(0, 0)`. This means that the last two variables are conditioning variables (and hence that the first $4 - 2 = 2$ variables are free), and that the value of the conditioning variables are fixed at $\text{DAX30} = 0$ and $\text{TOPIX} = 0$ respectively. This also means that the number of columns in the grid $x0$ plus the number of elements in `cond` must equal the number of variables p in the data set, and the call to `clg()` will result in an error message if this requirement is not fulfilled. Apart from `cond`, the `clg()`-function takes exactly the same arguments as `d1g()` listed in Table 2, and the conditional density estimate in our example is available in the vector `cond_density$f_est`.

Tests for independence

Three independence tests based on the LGC have appeared in the literature thus far:

1. A test for independence between the stochastic variables X_1 and X_2 based on iid data, cf. Berentsen and Tjøstheim (2014).
2. A test for serial independence between X_t and X_{t-k} within a time series $\{X_t\}$, cf. Lacal and Tjøstheim (2017).
3. A test for serial cross-dependence between X_t and Y_{t-k} for two time series $\{X_t\}$ and $\{Y_t\}$, cf. Lacal and Tjøstheim (2018).

As we noted in Section 2.2, their practical implementations are very similar, and the **lg**-package provides the function `ind_test()` to perform the tests. Let us first consider the iid case, and generate 500 observations `test_x` from the well known parabola model $X_2 = X_1^2 + \varepsilon$, where both X_1 and ε are independent and standard normal. In this case, X_1 and X_2 are uncorrelated, but obviously strongly dependent. Berentsen and Tjøstheim (2014) considers mainly the full bivariate fit to the raw data, which we easily encode into the lg-object as before. The implementation of the test using 100 bootstrap replicates is shown below.

```
R> lg_object <- lg_main(test_x,
R+   est_method = "5par",
R+   transform_to_marginal_normality = TRUE)
R> test_result <- ind_test(lg_object, n_rep = 100)
R> test_result$p_value

[1] 0
```

This may take a few minutes due to the bootstrapping. We can further specify the function h and the integration area S in the test statistic (3), see Table 3 for details.

The only difference when testing for serial independence within a time series $\{X_t\}$ is to create a two-column data set consisting of X_t and X_{t-k} . For example, if we wish to perform this test for $k = 1$ for one of the variables in the stock-exchange series, create the matrix of observations as below, and proceed exactly as in the iid case.

Argument	Explanation	Default value
lg_object_nc	The lg-object covering the non-crisis period	
lg_object_c	The lg-object covering the crisis period	
grid_range	Range of diagonal for measuring the LGC	(5%, 95%) quantiles
grid_length	The number of grid points to use	30
n_rep	Number of bootstrap replicates	1000
weight	Weight function	function(y) rep(1, nrow(y))

Table 4: Arguments to the `cont_test()`-function

```
R> returns2 <- stock_data %>% select(SP500) %>%
R+   mutate(sp500_lagged = lag(SP500))
```

Finally, the only thing that we must alter in order to perform the third test for serial cross-dependence is the bootstrap method. [Lacal and Tjøstheim \(2018\)](#) consider two options here, the block bootstrap with either fixed ([Kunsch, 1989](#)) or random ([Politis and Romano, 1994](#)) block sizes, and this choice is specified by choosing `bootstrap_type = "block"` or `bootstrap_type = "stationary"`, respectively, in the call to `ind_test()`.

Test for financial contagion

Assume that we observe two financial time series $\{X_t\}$ and $\{Y_t\}$ at times $t = 1, \dots, T$, and that some crisis occurs at time $T^* < T$. As described in Section 2.2, [Støve et al. \(2014\)](#) measure the local correlation between $\{X_t\}$ and $\{Y_t\}$ *before* and *after* T^* , and take significant differences between these measurements as evidence against the null hypothesis of no linkage, or contagion, between the time series. In order to implement this test using the `lg`-package, one must create *two* lg-objects: one for the observations covering the non crisis period and one covering the crisis period. We do not enter a discussion here how to empirically identify such time periods; this is a job that must be done by the practitioner before performing the statistical test.

Let us illustrate the implementation of this test by looking at the same financial returns data that we have used in preceding sections, but this time we will, in the spirit of [Støve et al. \(2014\)](#), concentrate on *GARCH(1,1)-filtrated daily returns on the S&P500 and FTSE100 indices from 2 January 1985 to 29 April 1987* in order to test for financial contagion between the US and UK stock markets following the global stock market crash of 19 October 1987 ("Black Monday"). Assume that these observations are loaded into the R workspace as the $n_1 \times 2$ data frame `x_nc` containing the observations covering the $n_1 = 728$ days preceding the crisis, and the $n_2 \times 2$ data frame `x_c` containing the observations covering the $n_2 = 140$ consecutive trading days starting on *Black Monday* (see the online code supplement for details concerning the GARCH-filtration and data processing). In the code below we construct one lg-object for each of these data frames with configuration matching the setup used by [Støve et al. \(2014\)](#), and perform the test by means of the `cont_test()`-function.

This function returns a list containing the estimated p -value as well as other useful statistics, including the empirical local correlations measured in the two time periods. See Table 4 for details concerning other arguments that may be passed to this function.

```
R> # Create the two lg-objects
R> lg_object_nc <- lg_main(x_nc,
R+   est_method = "5par",
R+   transform_to_marginal_normality = FALSE)
R>
R> lg_object_c <- lg_main(x_c,
R+   est_method = "5par",
R+   transform_to_marginal_normality = FALSE)
R>
R> # Run the test with a limited number of bootstrap replicates for
R> # demonstration purposes.
R> result <- cont_test(lg_object_nc, lg_object_c, n_rep = 20)
R>
R> # Print out the p-value
```

```
R> result$p_value
```

```
[1] 0
```

Partial local correlation

Consider finally the work by [Otneim and Tjøstheim \(2019\)](#) who take the off-diagonal element in the local correlation matrix corresponding to the local conditional covariance matrix (12) or (14) as a local measure of conditional dependence between two stochastic variables $X^{(1)}$ and $X^{(2)}$ given the stochastic vector $X^{(3)}$, and furthermore, in the case with data having been transformed to marginal standard normality, take the statistic

$$T_{n,b}^{CI} = \int h(\hat{\alpha}(z)) dF_n(z)$$

as a measure of *global* conditional dependence. To this end, the **lg**-package provides two functions. The first, `partial_cor()`, calculates the local partial correlations as well as their estimated standard deviations, on a specified grid in the $(x^{(1)}, x^{(2)}, x^{(3)})$ -space, and is essentially a wrapper function for the `clg()`-function presented in Section 2.4.1. See Table 5 for details. The second function, `ci_test()`, performs a test for conditional independence between the first two variables in a data set given the remaining variables.

It is well known in the econometrics literature that conditional independence tests are instrumental in the empirical detection of Granger causality ([Granger, 1980](#)). For example, if we continue to concentrate on the monthly stock returns data that we have already loaded into memory, we may test whether

$$H_0 : R_{FTSE100,t} \perp R_{SP500,t-1} \mid R_{FTSE100,t-1} \quad (17)$$

in the period starting in January 2009, the converse of which is a sufficient, but not necessary, condition for $R_{SP500,t}$ Granger causing $R_{FTSE100,t}$. We perform the test by running the code below, where `x` is a data frame having the following columns strictly ordered as $R_{FTSE100,t}$, $R_{SP500,t-1}$ and $R_{FTSE100,t-1}$ (see the online supplement for the pre-processing of data).

The critical values of this test are calculated using the bootstrap under the null hypothesis by independently resampling replicates from the conditional density estimates $\hat{f}_{X^{(1)}|X^{(3)}}(x^{(1)}|x^{(3)})$ and $\hat{f}_{X^{(2)}|X^{(3)}}(x^{(2)}|x^{(3)})$, as obtained by the `clg()`-function, using an approximated accept-reject algorithm. In order to avoid excessive optimization of the local likelihood function (1), we estimate $f_{X^{(1)}|X^{(3)}}$ and $f_{X^{(2)}|X^{(3)}}$ on the univariate regular grids $x_0^{(1)}$ and $x_0^{(2)}$ respectively (while keeping $x^{(3)}$ fixed at the observed values of $X^{(3)}$), and produce interpolating functions $\tilde{f}_{X^{(1)}|X^{(3)}}$ and $\tilde{f}_{X^{(2)}|X^{(3)}}$ using cubic splines. It is much less computationally intensive to generate replicates from \tilde{f} than directly from \hat{f} .

We refer to the documentation of the **lg**-package for details on how to finely tune the behavior of the bootstrapping algorithm by altering the arguments of the `ci_test()`-function, and rather describe the arguments most suitable for modifications by the user in Table 6.

```
R> # Create the lg-object
R> lg_object <- lg_main(returns4)
R>
R> # Perform the test
R> test_result <- ci_test(lg_object, n_rep = 100)
R>
R> # Print out result
R> test_result$p_value

[1] 0.71
```

The conditional independence test does not provide evidence against the null hypothesis (17).

Graphics

We conclude this article by describing the `corplot()` function, that may be used to draw dependence maps such as those displayed in Figure 1. [Berentsen et al. \(2014\)](#) report on such capabilities in the **localgauss**-package, but the possibility of creating dependence maps was unfortunately removed from **localgauss** in the latest version 0.4.0 due to incompatibilities with the **ggplot2** ([Wickham, 2016](#)) plotting engine. We make up for this loss by providing `corplot()`, a function that plots the estimated

local correlations as provided by `dlg()`, or the estimated local *partial* correlations as provided by `partial_cor()`.

The plotting function is highly customizable and provides a number of options covering most basic graphical options. Users well versed in the `ggplot2`-package may also modify the graphical object returned by `corplot` in the standard way by adding layers as demonstrate in the example below.

In the first example we generate a set of bivariate normally distributed data using the `mvtnorm`-package (Genz et al., 2018) and estimate the local Gaussian correlation on a regular grid using the `dlg()`-function. Passing the resulting `dlg_object` to `corplot()` without further arguments results in Figure 4.

```
R> grid <- expand.grid(seq(-3, 3, length.out = 7),
R+                     seq(-3, 3, length.out = 7))
R>
R> x <- mvtnorm::rmvnorm(500, sigma = matrix(c(1, rho, rho, 1), 2))
R> lg_object <- lg_main(x,
R+                     est_method = "5par",
R+                     transform_to_marginal_normality = FALSE,
R+                     plugin_constant_joint = 4)
R> dlg_object <- dlg(lg_object, grid = grid)
R>
R> # Make a dependence map using default setup
R> corplot(dlg_object1)
```

We may tweak the appearance of our dependence map by passing further arguments to `corplot()`. Some of the options are demonstrated in the code chunk below, in which we, for example, superimpose the observations (by setting `plot_obs = TRUE`) as well as preventing the the estimated local correlations from being plotted in areas without data. The latter option is available through the argument `plot_thres`, which works by calculating a bivariate kernel density estimate $\tilde{f}(x_1, x_2)$ for the pair of variables in question, and only allowing $\hat{\rho}(x_1, x_2)$ to be plotted if $\tilde{f}(x_1, x_2) / \max \tilde{f}(\cdot) > \text{plot_thres}$. Adding layers to a dependence map using the ordinary `ggplot2` syntax works as well, which we demonstrate in Figure 5 by changing the `ggplot2` theme.

The plotting function works in the same way when plotting the local partial correlations returned by `partial_cor()`, and the arguments of `corplot()` are summarized in Table 7.

```
R> corplot(dlg_object1,
R+         plot_obs = TRUE,
R+         plot_thres = 0.01,
R+         plot_labels = FALSE,
R+         alpha_point = 0.3,
R+         main = "",
R+         xlab = "",
R+         ylab = "") +
R+   theme_classic()
```

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Argument	Explanation	Default value
lg_object	The lg-object created by lg_main()	
grid	The evaluation points for the LGPC, must be a data frame or matrix having 2 columns	NULL
cond	Vector with fixed values for $X^{(3)}$	NULL
level	Significance level for approximated confidence bands	0.95

Table 5: Arguments to the partial_cor()-function

Argument	Explanation	Default value
lg_object	The lg-object created by lg_main()	
h	The function $h(\cdot)$ in (16)	function(x) x^2
n_rep	Number of bootstrap replicates	500

Table 6: Arguments to the ci_test()-function

Argument	Explanation	Default value
dlg_object	The object created by dlg() or partial_cor()	
pair	Which pair to plot if more than two variables	1
gaussian_scale	Logical. Plot on the marginal st. normal scale?	FALSE
plot_colormap	Logical. Plot the colormap?	TRUE
plot_obs	Logical. Superimpose observations?	FALSE
plot_labels	Logical. Plot labels on dependence map?	TRUE
plot_legend	Logical. Add legend?	FALSE
plot_thres	Threshold for plotting the estimated LGC	0
alpha_tile	Transparency of color tiles	0.8
alpha_point	Transparency of points	0.8
low_color	Color representing $\hat{\rho} = -1$	"blue"
high_color	Color representing $\hat{\rho} = +1$	"red"
break_int	Break interval for color coding	0.2
label_size	Size of labels in plot	3
font_family	Font family for labels	"sans"
point_size	Size of points, if plotted	NULL
xlim, ylim	Axis limits	NULL
xlab, ylab	Axis labels	NULL
rho_lab	Title of legend	NULL
main, subtitle	Title and subtitle of plot	NULL

Table 7: Arguments to the corplot()-function

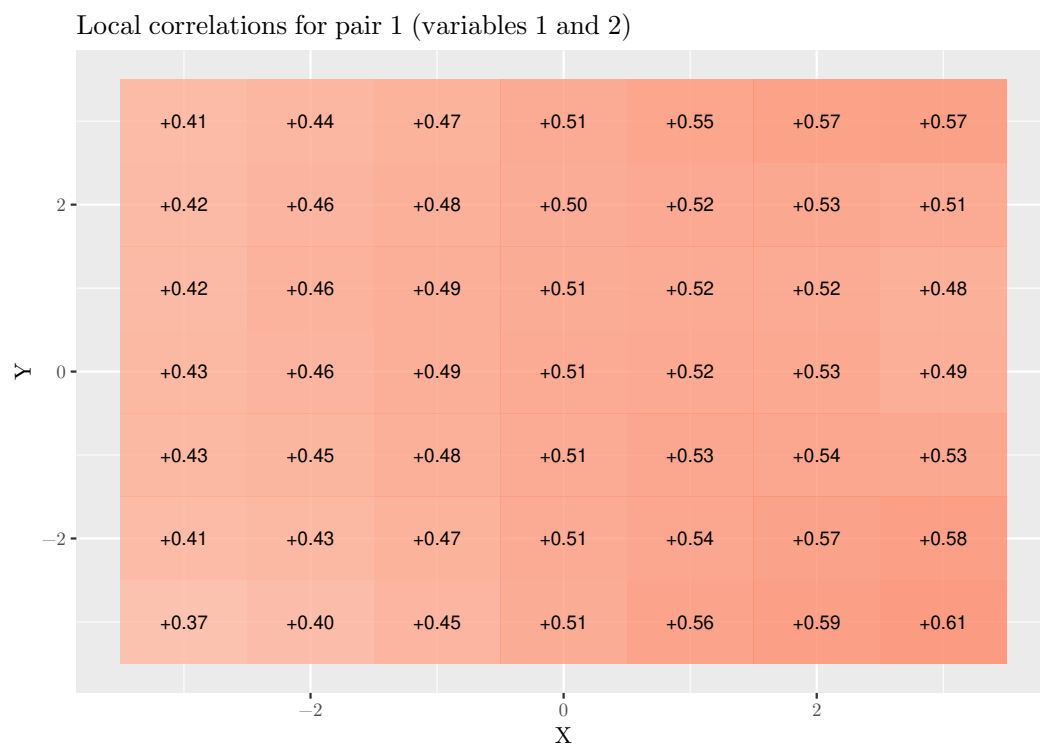


Figure 4: Dependence map produced by `corplot()` using the default configuration

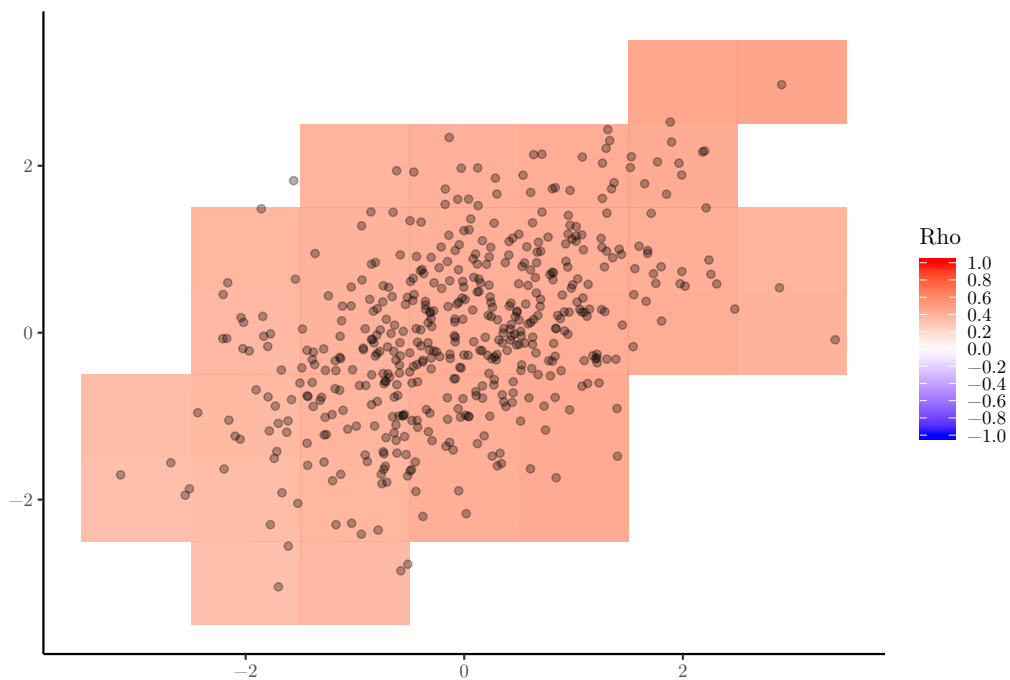


Figure 5: Dependence map produced by tweaking the arguments of `corplot()`

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