AE's report on

"The Locally Gaussian Partial Correlation"

JBES-P-2019-0538

I have heard back from two referees. The views are mixed. Referee 2 is not enthusiastic about the paper and recommends rejection while Referee 1 recommends a major revision. I also read the paper carefully and understand both referees' concerns.

In his/her cover letter, Referee 2 wrote

"*Definitively the topic is very interesting. However, I do not see a clear contribution of this paper to the literature on estimating dependence and testing independence.*

I don't see the added value of this paper when is compared to the nonparametric approaches that use Kernel functions (e.g. Normal Kernel) to locally approximate joint distribution functions.

In addition, we have many available tests for conditional independence between a vector and scalar random variables (and also between two vectors of random variables) conditional on a third vector random variable. However, the test proposed in this paper is limited to testing conditional independence between two scalars only. In addition, the simulation results are definitively not better than the existing tests.

Also, I believe the asymptotic and bootstrap theories are not complete. I also made other comments, please see my report.

For the above reasons, I believe the contribution in this paper is definitively below the level of JBES."

Referee 1 enjoys reading the paper and have some comments that can be relatively easily addressed.

Given the fact that both referees think that the paper is interesting and I also feel it is interesting, I lean toward the positive referee and do not object to seeing a revision of the paper. Nevertheless, the authors should keep in mind that they must address the referees' major concerns to their satisfaction in order for the paper to pass the second round of review.

My main comments are as follows, some of which overlap those of the referees.

1. I like the idea to extend the local likelihood framework of Hjort and Jones (1996) to estimate the local correlation and to extend the measure of local Gaussian correlation of Tjøstheim and Hufthammer (2013) to the measure of local Gaussian partial correlation (LGPC).

2. As both referees mentioned (see R1's point 1 and R2's point 3), one limitation of the proposed testing procedure is that it only tests the conditional independence between two scalar variables (X_1, X_2) given a random vector $(X_3, ..., X_p)$. It is important to generalize the test to allow both X_1 and X_2 to be random vectors.

3. p.5: As Referee 1 points out, the density in (6) is not the real density of $\mathbf{Z}$. It is a local Gaussian approximation.

4. p.10: Since you claim that $\boldsymbol{b}$ is a diagonal matrix of bandwidths (not squared bandwidths). It seems that you want to define $K_{\boldsymbol{b}}(\boldsymbol{x}) = |\boldsymbol{b}|^{-1} K(\boldsymbol{b}^{-1}\boldsymbol{x})$.

5. pp.12-14:

   (a) p.13: It seems that both $\boldsymbol{J_b}$ and $\boldsymbol{M_b}$ are of exact order $O(1)$ (so that the second term in the definition of $\boldsymbol{M_b}$ is asymptotically negligible). This suggests that the estimator of the local correlations have convergence rate $\left(nb^5\right)^{-1/2}$ under certain undersmoothing bias conditions. Why is the convergence rate here much slower than the usual $\left(nb^3\right)^{-1/2}$-rate for the kernel estimator of a trivariate density? In addition, it is possible to estimate the local conditional correlation or not? Note that for nonparametric tests based on conditional distributions, typically only the dimension of the conditioning variable matters for the convergence rate.

(b) p.14, line 2: $1/\left(nb^3\right)$ should be $1/\sqrt{nb^3}$, which is the convergence rate of the kernel estimator of a trivariate density.

(c) p.14, line 4: $1/\left(nb^5\right)$ should be $1/\sqrt{nb^5}$.

(d) The results are too limited as it is only for the case with $p = 3$. A general result for the generic p is needed.

(e) Theorem 3.1 is a pointwise result. For the inference purpose, one typically needs a uniform result.

6. p.16: Is there a data-driven choice of the bandwidth? The choice of $c = 4$ seems too arbitrary.

7. p.24-26: The authors propose a new test for conditional independence based on the LGPC. The test statistic should be formally studied under the null hypothesis of conditional independence and the usual Pitman sequence of local alternatives. For this purpose, you need uniform consistent estimate of the local correlations. As Referee 2 mentions in his/her fourth comment, you also need to justify the asymptotic validity of the bootstrap.

8. p.26: For the comparison with the existing tests, you need to compare the asymptotic properties (e.g., local power properties) too.

9. The paper contains a lot of typos. Referee 1 pointed out some of them and the authors should correct others too.