Fourier Transform

Complex plane

$e^{2\pi i t}$ ↑ time has passed

Sound Σ graph → Decompose it

Wrap the signal by circle

$g(t) \cdot e^{-2\pi i f t}$

Inverse Fourier → track center of mass

(frequency)

$g(t) \rightarrow \hat{g}(f) \rightarrow$ Complex

(time) ↘ Real part $\approx \frac{1}{N} \cdot \sum_{k=1}^{N} g(t_k) \cdot e^{-2\pi i f t_k}$

$\approx \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} g(t) \cdot e^{-2\pi i f t}$ $(-\infty, +\infty)$

Heat Equation

$\frac{\partial T}{\partial t}(x,t) = \alpha \cdot \frac{\partial^2 T}{\partial x^2}(x,t)$ [1D]

(T, x, t)

$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$

$\frac{\partial \left(\frac{\partial T}{\partial x} \right)}{\partial x} = \frac{\partial^2 T}{\partial x^2}$

Laplacian $\nabla^2 T$

Second difference

$\Delta\Delta T_1$

$= \Delta T_2 - \Delta T_1$

$\frac{dT_2}{dt} = \frac{\alpha}{2} \cdot \Delta\Delta T_1$

Sound waves (time vs frequency)

Doppler radar (distance vs velocity)

Quantum particles (position vs momentum)

Time Complexity + Primitive operation

(5) $C(n) = n \in O(n)$
PO: print("*");

(6) PO: count++;

$C(n) = n \cdot \left(\frac{n}{2} + 5\right) = \frac{n^2}{2} + 5n \in O(n^2)$

(7) PO: System.out.println ...

i	0	1	...	n-1
j	n-1	n-2	...	0

$C(n)$: $n(n-1)/2 \in O(n^2)$

(8) PO: count++

i	0	1	...	n-1
j	2n	2n-1	...	n+1

$C(n)$: $2n + \dots + n+1$

$= \frac{3n^2}{2} + \frac{n}{2} + C \in O(n^2)$

(9) PO: System.out.println ... x2

i	0	1	...	$\frac{n}{2}-1$	n-1
j	$\frac{n}{2}$	$\frac{n}{2}-1$	...	1	

$C(n) = 2 \cdot \frac{1}{2} \frac{n}{2} \cdot \left(\frac{n}{2} + 1\right) = \frac{n^2}{4} + \frac{n}{2} \in O(n^2)$

(10) O(1)

(11) $O(\log_2(\frac{n}{4}) + n - 2) \quad \in O(n)$

(12)

i	0	1	...	n-1
j	1 → n-1	2 → n-1		
k	2 → n-1	3 → n-1		

$(n-1)(n-2) + (n-2)(n-3) + \dots \quad C_n^3$

$C(n) = \frac{n(n-1)(n-2)}{6} \sim \frac{n^3}{6}$

(13) $O(n + \log_3 n) \in O(n)$

(14) $O(\frac{n}{2} \cdot \log^2 n)$

(15) $O(\frac{n^2}{4} \cdot \lg n)$

(16) $S = (1 + 2 + 3 + \dots + t) \leq n$

$\frac{t(t+1)}{2} \leq n \Rightarrow t = \left\lfloor \frac{-1 + \sqrt{1+8n}}{2} \right\rfloor$

(A) $\frac{C(2n)}{C(n)} \quad C(n) \sim k n^p \rightarrow$ approach...

(c)

	1	2	i	2n − 1
j	[1..1]	[1..4]	[1..i²]	[1..(2n−1)²]
k	[1..1]			[1..(2n−1)²]

Dominated by $(2n-1)^2 \cdot (2n-1)^2 \sim O(n^4)$