Locally Weighted Logistic Regression

1. Weight Calculation

$$w_i = \exp\left(-\frac{\|X_{\text{train},i} - x\|^2}{2\tau^2}\right),$$

where:

- $X_{\text{train},i}$ is the i-th row of the training data matrix.
- x is the query point.
- $\|X_{\text{train},i} - x\|^2$ is the squared Euclidean distance.
- τ is the bandwidth parameter.

2. Logistic Regression Model

$$h_\theta(x) = \frac{1}{1 + \exp(-\theta^\top x)}.$$

For all training examples:

$$h = \frac{1}{1 + \exp(-X_{\text{train}}\theta)}.$$

3. Gradient of the Weighted Logistic Loss

$$g = X_{\text{train}}^\top \left(W(y_{\text{train}} - h)\right) - \lambda\theta,$$

where:

- $W = \text{diag}(w_1, w_2, \ldots, w_m)$ is a diagonal matrix of weights.
- y_{train} is the vector of training labels.
- λ is the regularization parameter ($\lambda = 1 \times 10^{-4}$).

4. Hessian Matrix

$$H = -X_{\text{train}}^\top W \, \text{diag}(h \circ (1 - h)) X_{\text{train}} - \lambda I,$$

where:

- $h \circ (1 - h)$ is the element-wise product of h and $1 - h$.
- I is the identity matrix.

5. Newton's Update Rule

$$\theta \leftarrow \theta - H^{-1} g.$$

6. Prediction

$$y = \begin{cases} 1 & \text{if } \theta^\top x > 0, \\ 0 & \text{otherwise.} \end{cases}$$