

Q1a  $f(x) = \frac{2-\sqrt{x}}{x-4}$

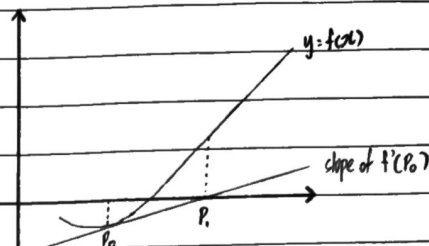
$$f(4.1) = \frac{2-\sqrt{4.1}}{4.1-4}$$

$$= \frac{2-2.025}{0.1}$$

$$= \frac{-0.025}{0.1}$$

$$= -0.25$$

Q2ai



$$Q1b \quad g(x) = \frac{2-\sqrt{x}}{x-4} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}}$$

$$= \frac{4-x}{2x+x\sqrt{x}-8-4\sqrt{x}}$$

$$= \frac{4-x}{2(x-4)+\sqrt{x}(x-4)}$$

$$= \frac{-(-4+x)}{2(x-4)+\sqrt{x}(x-4)}$$

$$= \frac{-1}{2\sqrt{x}+1}$$

$$Q1c \quad g(4.1) = \frac{-1}{2+\sqrt{4.1}}$$

$$= \frac{-1}{2+2.025}$$

$$= \frac{-1}{4.025}$$

$$= -0.248$$

$$Q1d \quad \text{actual value} = -0.2485$$

$$E_{rela} = \frac{1-0.2485 - (-0.25)}{1-0.2485}$$

$$= \frac{0.0015}{0.2485}$$

$$= 0.006$$

$$E_{relb} = \frac{1-0.2485 - (-0.248)}{1-0.2485}$$

$$= \frac{0.0005}{0.2485}$$

$$= 0.002$$

Q2aii  $f(x) = x^3 + 5x^2 - 5 = 0 \quad P_0 = 1 \quad \text{tol} = 1 \times 10^{-3} \cdot 0.001$

$f'(x) = 3x^2 + 10x$

$n=1, P_0=1$

$f(1) = 1 + 5 - 5 = 1$

$f'(1) = 3 + 10 = 13$

$P_1 = 1 - \left(\frac{1}{13}\right) = 0.923077$

$n=2, P_1 = 0.923077$

$f(0.923077) = 0.046883$

$f'(0.923077) = 11.786983$

$P_2 = 0.923077 - \left(\frac{0.046883}{11.786983}\right) = 0.919099$

$n=3, P_2 = 0.919099$

$f(0.919099) = 0.000117$

$f'(0.919099) = 11.75219$

$P_3 = 0.919099 - \left(\frac{0.000117}{11.75219}\right) = 0.919089$

$|P_3 - P_2| = |0.919089 - 0.919099| = 0.00001 < 1 \times 10^{-3}$

Stopped at  $n=3$ , the root is  $P_3 = 0.919089$ 

Q2b  $\int_{x_1}^{x_2} y(x) dx = \frac{h}{2} [y_1 + y_2]$

$y(x) = Ax + B \quad h = x_2 - x_1$

$\int_{x_1}^{x_2} y(x) dx = \int_{x_1}^{x_2} Ax + B dx$

$= \left[ \frac{A}{2} x^2 + Bx \right]_{x_1}^{x_2}$

$= \left[ \frac{A}{2} (x_2)^2 + Bx_2 \right] - \left[ \frac{A}{2} (x_1)^2 + Bx_1 \right]$

$= \frac{A}{2} (x_2^2 - x_1^2) + B(x_2 - x_1)$

$= \frac{A}{2} (x_2 - x_1)(x_2 + x_1) + B(x_2 - x_1)$

$= \frac{x_2 - x_1}{2} [A(x_2 + x_1) + B]$

$= \frac{h}{2} [Ax + B]$

$= \frac{h}{2} [y(x_1)]$

$= \frac{h}{2} [y_1 + y_2]$

$$Q2c \quad \int_{-1}^1 5 \ln(2x^2 + 1) dx \quad h = 4 \quad h = \frac{4^2}{4} = 0.6$$

$$A = \frac{0.5}{2} [f(-2) + f(4) + 2(f(-1) + f(3) + f(1) + f(5))]$$

$$= 0.25 [10.986123 + 17.482538 + 2(13.013448 + 14.722195 + 16.143592)]$$

$$= 0.25 [28.468661 + 2(43.980935)]$$

$$= 0.25 [28.468661 + 87.961870]$$

$$= 0.25 [116.430531]$$

$$= 29.107633$$

Q2d	m=0	m=1	m=2
n=0	R(0,0) = 28.468661	-	-
n=1	R(1,0) = 28.956526	R(1,1) = 29.119148	-
n=2	R(2,0) = 29.081683	R(2,1) = 29.123402	R(2,2) = 29.123686

Q3a

$$\begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & -5 & 1 & 0 \\ -3 & 8 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix} \xrightarrow{\substack{R_1+R_2 \rightarrow R_2 \\ R_2+R_3 \rightarrow R_3 \\ 3R_1+R_4 \rightarrow R_4}} \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 10 & 4 & -9 \\ 0 & -16 & -11 & 18 \end{bmatrix}$$

$$\xrightarrow{\substack{5R_2+R_3 \rightarrow R_3 \\ -8R_2+R_4 \rightarrow R_4}} \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -3 & -2 \end{bmatrix}$$

$$\xrightarrow{\substack{3R_2+R_4 \rightarrow R_4 \\ -3R_2+R_4 \rightarrow R_4}} \begin{bmatrix} 3 & -7 & -2 & 2 \\ 0 & -2 & -1 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U$$

$$Q3dii \ a. \frac{(39090)(4120) - (297220)(5547)}{(8)(39090) - (5547)^2}$$

$$= \frac{-3609080}{5804}$$

$$= -621.83$$

$$b. \frac{(8)(297220) - (5547)(4120)}{(8)(39090) - (5547)^2}$$

$$= \frac{95280}{5804}$$

$$= 16.42$$

$$y = a + bx$$

$$y = -621.83 + 16.42x$$

$$Q3b \ 4x - y - z = 3 \quad x = \frac{1}{4}(3 + y + z)$$

$$-2x + 6y + z = 9 \quad y = \frac{1}{2}(9 + 2x + z)$$

$$-x + y + 7z = -6 \quad z = \frac{1}{7}(-6 + x + y + y_{k+1})$$

n	x	y	z
0	0	0	0
1	0.75	1.75	-1
2	0.938	1.979	-1.006
3	0.993	1.999	-1.001

$$Q3diii \ y = -621.83 + 16.42(95)$$

$$= -621.83 + 1559.9$$

$$= 938.07$$

Q3e	$x_k$	$y_k$	first	second	third	fourth
	0	0				
			1			
	1	1		3		
			7		1	
	2	2		6		0
			14		1	
	3	27		9		
			31			
	4	64				

$$Q3c \ \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix} = \begin{bmatrix} 2-\lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$(2-\lambda)(-6-\lambda) - 9 = 0$$

$$-12 - 2\lambda + 6\lambda + \lambda^2 - 9 = 0$$

$$\lambda^2 + 4\lambda - 21 = 0$$

$$(\lambda + 7)(\lambda - 3) = 0$$

$$\lambda = -7, \lambda = 3$$

$$Q3eii \ P_3(x) = (x) + 3(x)(x-1) + (x)(x-1)(x-2)$$

$$= x + (3x^2 - 3x) + (x^3 - x^2)(x-2)$$

$$= 3x^2 - 2x + (x^3 - 2x^2 - x^2 + 2x)$$

$$= x^3$$

Q3d

x	y	$x^2$	$xy$
55	340	3025	18700
58	355	3364	19490
64	410	4096	26240
68	460	4624	31280
70	450	4900	31500
75	610	5625	45750
80	735	6400	58800
84	780	7056	65520
954	4120	39090	297220

$$Q3eiii \ P_3(2.5) = 2.5^3$$

$$= 15.625$$