AP Physics C Mechanics notes: Newton's Laws

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1 Overview and Guide

This is a collection of notes on calculus-based physics at the **AP Physics C** level. They cover the entire 2023-2024 AP Physics C curriculum at a suitable depth for complete self-study, but since the underlying physics is the same, they can be used in accompaniment with classes or programs.

The first section of notes is on **mechanics,** split into seven units based on the official CollegeBoard course description. They assume fluency with single-variable calculus at the level of AP Calculus AB, and preferably some honors physics background. An approximate unit breakdown of the **AP Physics C Mechanics** exam is provided below for reference.

Mechanics unit	Approximate portion of exam grade (%)
Kinematics	14-20
Newton's Laws	**16-20**
Work, Energy and Power	14-17
Systems and Linear Momentum	14-17
Rotation	14-20
Oscillations	6-14
Newtonian Gravity	6-14

The second section of notes is on **electricity and magnetism,** split into five units based on the official CollegeBoard course description. They assume the same background as the first section notes, but more background with introductory physics is preferred, particularly with fields. An approximate unit breakdown of the **AP Physics C Electricity and Magnetism** exam is provided below for reference.

E & M unit	Approximate portion of exam grade (%)
Electrostatics	26-34
Electricity in matter	14-17
Circuits	17-23
Magnetism	17-23
Electromagnetism	14-20

We use the following common conventions.

- Arrowed quantities like $\vec{A}, \vec{p}, \vec{x}$ are vectors, whereas unbold quantities like v, r, t are scalars. If a vector quantity is introduced, its magnitude will often be noted by the unarrowed version of itself: $v = \|\vec{v}\|$.
- Using Newton's convention, time derivatives are denoted with dots: $\dot{v} = \frac{\mathrm{d}v}{\mathrm{d}t}$. Note that primes are *not* derivatives unless otherwise specified.
- Time-averaged quantities will be denoted with an overline; if r is precipitation, $\overline{r}$ is the time-averaged precipitation.
- Differential quantities will be treated like legitimate algebraic quantities; if $\frac{\mathrm{d}m}{\mathrm{d}t}$ is the rate of change of the mass of a snowball, then $\mathrm{d}m$ may be interpreted as the amount of mass accumulated in time $\mathrm{d}t$.

2 Newton's Laws

Newton's laws govern how objects interact with each other. Specifically, different systems may exert forces against each other, and a system's response to a force is characterized by its inertia.

2.1 Systems and forces

We call the object or the collection of objects we are interested in the **system**. The choice of system will depend on the problem at hand, and we may analyze multiple systems in a given problem.

Example 1

A student throws a closed lunchbox with an apple inside it into the air. What system would you take if you wanted to analyze the motion of the lunchbox? What about the apple?

Solution 1

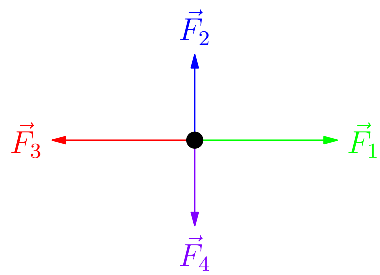
If we're interested in the lunchbox, we should take the lunchbox and apple as a system. The center of mass of the lunchbox-apple system evolves exactly by the projectile motion equations. Then the trajectory of the lunchbox is well approximated by the trajectory of the center of mass, as long as the lunchbox is small. If we are interested in the apple, we would take the apple as the system, and we would have to carefully treat the motion of the apple as it bounces around the lunchbox.

Systems allow us to absorb away any messy internal properties of objects if we're only concerned about the bulk motion of the object. We don't need to consider the motion of every clay particle in a clay ball if we only want to know how the ball rolls.

Objects in contact can exert *forces* on each other. While I'm typing, my fingers push on the keys, exerting a downwards force on them. As a ball rolls on the floor, friction exerts a backwards force, slowing it down. In general, forces *change* the motion of objects. There are many different kinds of forces that may impact the motion of objects; for example:

- The gravitational force, which causes massive objects to attract each other. The Earth exerts a gravitational force on all objects pointing towards its core.
- The normal force prevents rigid, solid objects from passing through one another, acting perpendicular to rigid surfaces. Your chair's normal force supports your weight, the ground's normal force supports your chair, and so forth.
- Friction opposes relative motion between rough surfaces, parallel to the surface and in the opposite direction of the motion.
- Air or viscous resistance oppose the motion of objects through fluids, acting in the opposite direction of the velocity of the object.
- The electrostatic Coulomb force draws opposite charges towards one another and repels like charges. We will revisit the Coulomb force in the second half of notes on electricity and magnetism.

The sum of all the forces acting on an object is called the net force. You can keep track of all the forces acting on an object by drawing a free body diagram, in which the object is a point and the force vectors are extending from that point. The free body diagram below shows an object experiencing four forces, with the net force being zero.



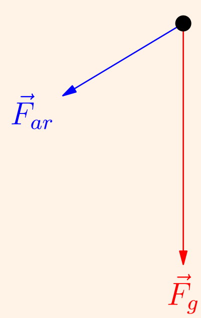
Note that the forces don't have to be right angles to each other.

Example 2

Draw the free-body diagram of a projectile during the upwards part of its trajectory. Do not neglect air resistance.

Solution 2

There are two forces: air resistance and gravity. Gravity acts downwards and air resistance acts backwards to the velocity.



2.2 Newton's laws

Newton's laws are the centerpiece of the Newtonian framework of physics. Make sure you understand them deeply, as they will be used everywhere from now on.

The first law states that an object which experiences no net (external) forces moves in a straight line at constant velocity.

- The first law also applies to systems - the Milky Way experiences (approximately) no net external forces, so it travels at constant velocity.

- Balls rolling on the floor eventually stop because they experience friction, which creates a net backwards force.
- Newton's first law allows observers to check if their frame of reference is inertial. If, in their reference frame, an object experiencing no net external force travels in a straight line at constant speed, then they are in an inertial reference frame.

The second law states that an object with mass m that experiences net external force $\vec{F}_{\text{net}}$ undergoes acceleration $\vec{a}$, and $\vec{F}_{\text{net}} = m\vec{a}$.

- In the case of systems, m becomes the total mass of the system, and $\vec{a}$ becomes the center-of-mass acceleration of the system.
- Setting $\vec{a} = 0$, we recover Newton's first law.

The third law states that for any two objects A and B, if A exerts force $\vec{F}$ on B, then B exerts force $-\vec{F}$ on A.

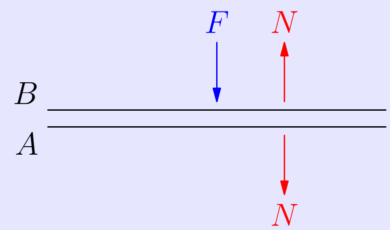
- Be careful with this one. It does not state that every force has an "equal and opposite reaction" force that act on the same object - the reaction force acts on the other object. If you push something, it pushes back equally hard.
- When you walk, you feet push backwards on the Earth via friction. By Newton's third law, the Earth then pushes you forwards via friction, allowing you to move. The same is true for biking. Note that even though the forward and backward forces are equal, the Earth barely moves because it's so massive, and $a = \frac{F}{m}$, so the acceleration of the Earth is tiny.
- Newton's third law can be tricky to apply in more complicated situations. A rocket exerts downwards force on its ejected fuel, propelling it downwards, and thus experiences an upwards force by Newton's third law. Planes counter gravity via lift, which is an upward force, which they achieve by pushing air downwards. Helium balloons experience upwards forces from the surrounding air due to buoyancy, so they must push air downwards by Newton's third law.

Newton's laws apply **in any inertial frame of reference.**

We'll first look at some applications of Newton's laws in finding contact forces.

Idea 1: Contact forces

Consider two surfaces that are being pressed together with some force F, which could be due to gravity or some external agent.



To prevent B from passing through A, surface A will exert a normal force upwards on B. By Newton's third law, B will exert an equal and opposite downwards normal force on surface A.

If you try to drag B parallel to A, friction will oppose the relative motion. There are two kinds of friction: static and kinetic friction. Static friction prevents relative motion from starting. If you have tried to push books on a rough table, you may have noticed that the books do not start moving until a certain threshold force is met. The static friction force between two surfaces obeys

$$F_s \leq \mu_s N$$

where N is the normal force between the surfaces and μ_s is the coefficient of static friction, which roughly characterizes the "roughness" of surfaces. Note that this is only an upper bound - static friction will take any value necessary to keep the surfaces at rest relative to one another, but it cannot exceed $\mu_s N$. Once static friction is broken, the surfaces will start slipping, and static friction is replaced with kinetic friction which obeys

$$F_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction, which is also determined by the "roughness" of surfaces. In general, we have $\mu_s \geq \mu_k$.

Microscopically, the normal force is due to the fact that atomic orbitals cannot overlap, by the Pauli exclusion principle. Static friction arises because the surface molecules of objects will "lock" at the interface, requiring force to pry them past each other.

Remark 1

You'll notice that the formula for kinetic friction is independent of the slipping speed. This is an idealization - for large speeds, the coefficient of kinetic friction becomes an increasing function of speed $\mu_k = \mu_k(v)$. The general dependence of friction of speed is important for engineering vehicles which slip at high speeds, like racecars.

Idea 2: Statics

If an object is stationary or moving at constant velocity, the all forces acting on it must sum to zero.

Example 3

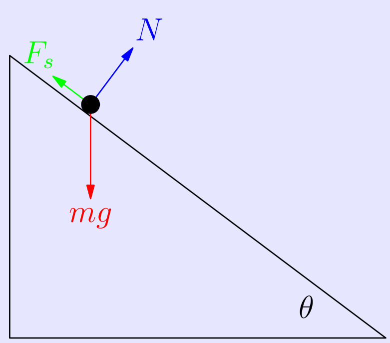
A book of mass m rests on a table. Find the magnitudes of the force of gravity and the normal force acting on the book.

Solution 3

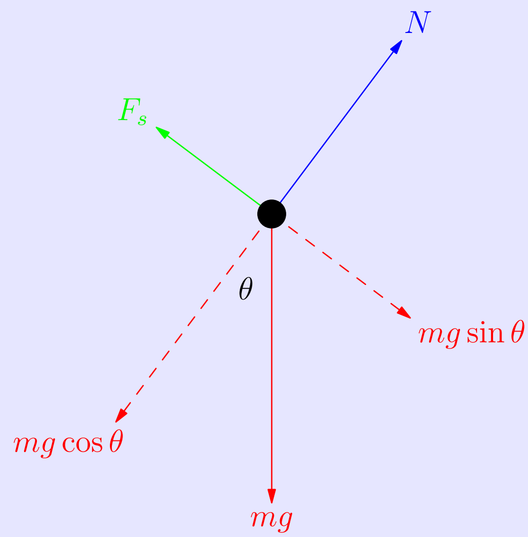
Consider any mass in freefall, experiencing only gravity. All masses accelerate downwards with magnitude g, so by Newton's second law they experience gravitational force $F_g = mg$ downwards. This is also true for objects that aren't accelerating. Since the book is stationary, the normal force must balance gravity, so $N = mg$ upwards.

Idea 3: Inclined planes

Consider a mass at rest on a rough, fixed incline plane with angle θ and coefficient of static friction μ.



As usual, we consider the forces along coordinates parallel and perpendicular to the incline.



Balancing forces then gives

$$F_s = mg\sin\theta$$
$$N = mg\cos\theta.$$

If we were looking for the minimum possible coefficient of static friction for the block to remain static, we should set $F_s = \mu_s N$, so that

$$\mu_s mg\cos\theta = mg\sin\theta \implies \mu_s = \tan\theta.$$

Therefore, for a given θ, the mass will stay at rest if $\mu_s \geq \tan\theta$. Conversely, for a given μ_s, the mass will stay at rest if $\theta \leq \arctan\mu$.

Now suppose that $\mu_s < \tan\theta$, so the mass slips and we enter the regime of kinetic friction. We now have

$$F_{\text{net, parallel}} = mg\sin\theta - \mu_k N$$
$$N = mg\cos\theta,$$

where the parallel forces won't balance by assumption. Substituting the second equation into the first and applying Newton's second law gives

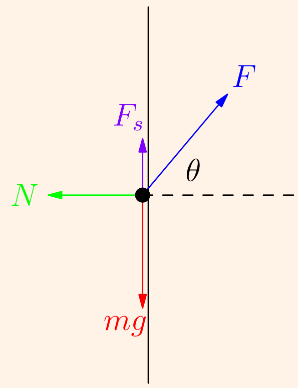
$$F_{\text{net, parallel}} = mg(\sin\theta - \mu_k\cos\theta) \implies a = g(\sin\theta - \mu_k\cos\theta).$$

If the plane is frictionless, the mass accelerates down the plane with $a = g\sin\theta$, so the acceleration is only a function of angle (and not of mass). This was shown experimentally by Galileo in the 1600's.

Example 4

A student holds a textbook up against a wall with force F, with angle θ upwards to the horizontal, with static coefficient of friction μ. What is the minimum value of F for which the textbook will remain stationary?

Solution 4

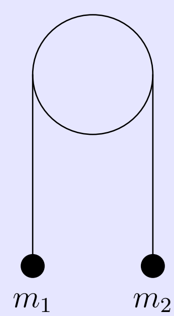


At the minimum value of F, friction will take its maximum value μN, and the vertical components of the applied force and friction must balance gravity. By force balance perpendicular to the wall, we have $N = F\cos\theta$. Then we have $F_s = \mu F\cos\theta$, so

$$\mu F\cos\theta + F\sin\theta = mg \implies F = \frac{mg}{\mu\cos\theta + \sin\theta}$$

Idea 4: Pulleys

Pulley problems can either be solved by setting up a system of equations using Newton's second law, or using generalized coordinates.



Let the tension be T. If the rope is massless and the pulley is frictionless, tension will be constant along the rope. Newton's second law gives

$$m_1 a_1 = T - m_1 g$$
$$m_2 a_2 = T - m_2 g,$$

where $a_1 = -a_2 := a$ since the rope maintains constant length. Solving the system gives

$$a = \frac{m_1 - m_2}{m_1 + m_2} g.$$

Alternatively, lay the rope flat and consider the forces acting along the rope.