Linear Algebra

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Normal Distributions

- Univariate Normal: $X \sim \mathcal{N}(\mu, \sigma^2)$ has density $f_X(x; \mu, \sigma^2) = (2\pi\sigma^2)^{-1/2} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$
  - $X \sim \mathcal{N}(\mu, \sigma^2) \implies \mathbb{E}[X] = \mu$
  - $X \sim \mathcal{N}(\mu, \sigma^2) \implies \mathrm{Var}[X] = \sigma^2$
  - Any linear combination of **independent** Normal random variables is also Normal. Example: $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2), X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2) \implies aX_1 + bX_2 \sim \mathcal{N}(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^2)$
- Multivariate Normal: $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma})$, where $\boldsymbol{\mu} \in \mathbb{R}^D$ and $\mathbf{\Sigma} \in \mathbb{R}^{D \times D}$ is positive definite has density $f_X(\mathbf{x}; \boldsymbol{\mu}, \mathbf{\Sigma}) = (2\pi)^{-D/2} \det(\mathbf{\Sigma})^{-1/2} \exp\big(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^\top \mathbf{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\big)$
  - $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma}) \implies \mathbb{E}[\mathbf{X}] = \boldsymbol{\mu}$
  - $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma}) \implies \mathrm{Var}[\mathbf{X}] = \mathbf{\Sigma}$
  - If $\mathbf{X}$ follows a multivariate Normal distribution, then any linear transformation of $\mathbf{X}$ also follows a multivariate Normal. Example: $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{\Sigma}) \implies A\mathbf{X} \sim \mathcal{N}(A\boldsymbol{\mu}, A\mathbf{\Sigma}A^\top)$

Gaussian Processes

Bayesian Optimization