09/15 Lecture 1

上海交通大学 2022 年秋季学期

《复分析》课程教学说明

一、课程基本信息

1. 任课教师：来米加（laimijia@sjtu.edu.cn）
2. 时间地点：下院 205　周一、四 10：00-11:40
3. 参考书目：复分析：可视化方法 尼达姆著 齐民友译
4. 办公室：理科六号楼 725
5. Office hour：周四 3-5pm or by appointments, email, canvas
6. 助教：张扬　zyy2233456@sjtu.edu.cn

二、课程考核方式及说明

总评成绩=35%平时和小测+65%期末考试 （酌情调整）

三、课程大纲

Part1：复数的几何、复值函数、莫比乌斯变换（第一、二、三章）
Part2：复函数的微分、 柯西黎曼方程、环绕数与拓扑学（第四、五、七章）
Part3：复积分及其应用（第八、九章）
Part4：当复分析遇见几何、拓扑、偏微分方程....（第十、十一、十二章）


TURING · 05
VISUAL COMPLEX ANALYSIS
复分析：
可视化方法
[美] 特里斯坦 · 尼达姆 ——著
齐民友 ——译
T R I S T A N N E E D H A M
中国工信出版集团
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Why complex numbers?

Is Mathematics Invented or Discovered?

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^2} = ?$$

$$\int_0^{2\pi} \frac{dt}{1+a^2-2a\cos t} = ?$$

Riemann-Zeta function.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

if $Re(s) > 1$. $\zeta(s)$ converges

$\Downarrow$

Prime numbers.

1. 领略复变量函数别具一格的特点

2. 提升考察问题的视野

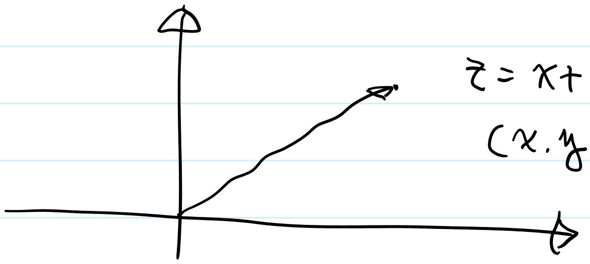
3. 联系拓扑学、微分几何、偏微分方程，开拓思维方式

Complex numbers $\mathbb{C}$

$z = x + yi$. $\quad i^2 = -1$.

$(1+2i)(3+4i) = 3 + 6i + 4i + 8\cdot(-1) = -5 + 10i$.

$\mathrm{Re}(z) = x \quad \mathrm{Im}(z) = y$.



$x^3 = 3px + 2q$ ←

$x = \sqrt[3]{q + \sqrt{q^2 - p^3}} + \sqrt[3]{q - \sqrt{q^2 - p^3}}$.

Q: what if $q^2 - p^3 < 0$?

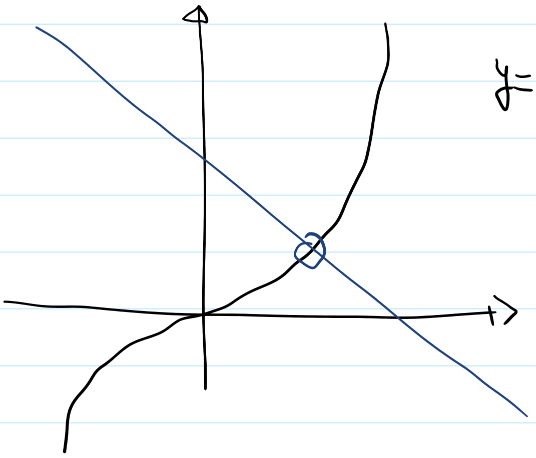
$p = 5$. $q = 2$. $\quad x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}} = 2a$.

$(a+bi)^3 = 2 + 11i$.

$(a-bi)^3 = 2 - 11i$.

Geometric meaning of solution of

$x^3 = 3px + 2q$.



$y = x^3$

$y = 3px + 2q$

欧拉公式

Euler formula:

$$\boxed{e^{i\theta} = \cos\theta + i\sin\theta} \qquad \theta \in \mathbb{R}.$$

令 $\theta = \pi$ $\qquad e^{i\pi} = -1.$

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad x \in \mathbb{R}.$$

令 $x = i\theta$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \cdots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \right)$$

$$= \cos\theta + i\sin\theta$$

$$z = x + yi = r\cos\theta + r\sin\theta\, i = r(\cos\theta + i\sin\theta)$$

$$= r e^{i\theta}.$$

$r = |z| = \sqrt{x^2 + y^2}.$

$\theta = \arg(z)$

$\operatorname{Arg}(z)$

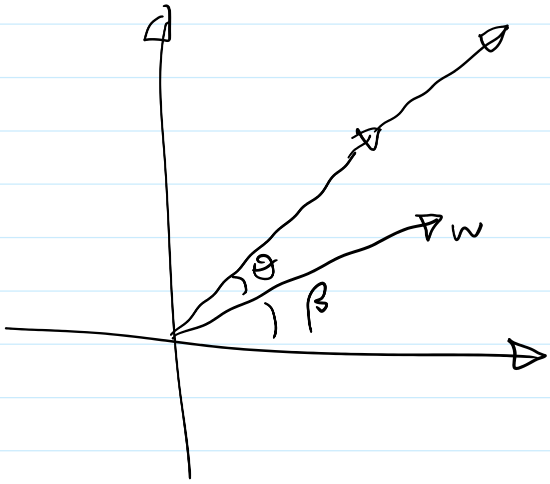
θ: Argument 辐角.

复数乘法的几何意义:

复数乘法的几何意义：

$$z \cdot w = re^{i\theta} \cdot (se^{i\beta})$$

$$= r \cdot se^{i(\theta+\beta)}$$

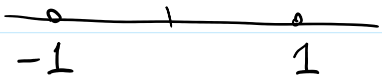


$z \cdot (\ ) \longleftrightarrow$ 旋转 + 伸缩

科茨 1716

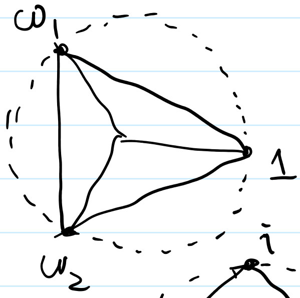
代数学基本定理 明于 1716

$$\int \frac{dx}{x^n-1}$$



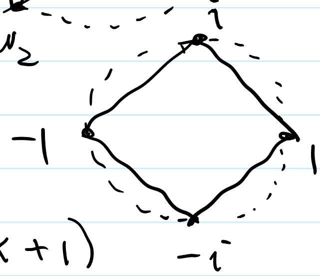
$$x^2-1=(x-1)(x+1)$$

$$x^3-1=(x-1)(x^2+x+1)=(x-1)(x-\omega_1)(x-\omega_2)$$

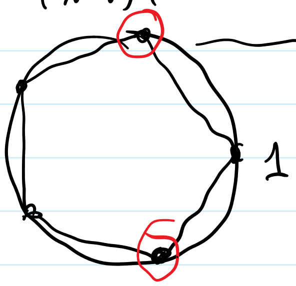


$$\omega_1=e^{i\frac{2\pi}{3}} \qquad \omega_2=e^{i\frac{4\pi}{3}}$$

$$x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x+i)(x-i)$$



$$x^5-1=(x-1)(x^4+x^3+x^2+x+1)=(x-1)\left(x^2+\frac{1+\sqrt5}{2}x+1\right)\left(x^2+\frac{1-\sqrt5}{2}x+1\right)$$



$x^n-1=0$ 有 n 个根. 令 $\omega=e^{i\frac{2\pi}{n}}$ $\qquad \omega^n=\left(e^{i\frac{2\pi}{n}}\right)^n=e^{i\cdot 2\pi}=1.$

$$x^n-1=(x-1)(x-\omega)(x-\omega^2)\cdots\cdots(x-\omega^{n-1})$$

$z=x+yi$

$\bar{z}=x-yi$

$$\big(z-(a+bi)\big)\big(z-(a-bi)\big)$$

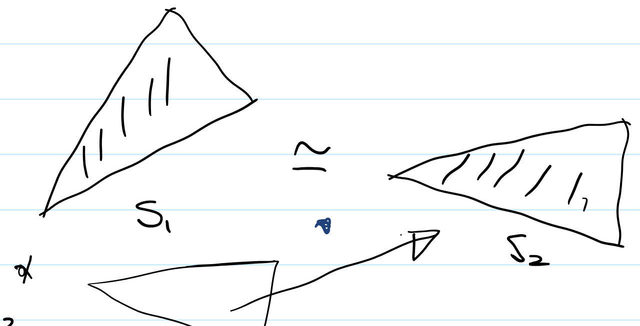
$$=z^2-2az+(a^2-b^2)$$

复数和平面变换

全等

↓

Defn

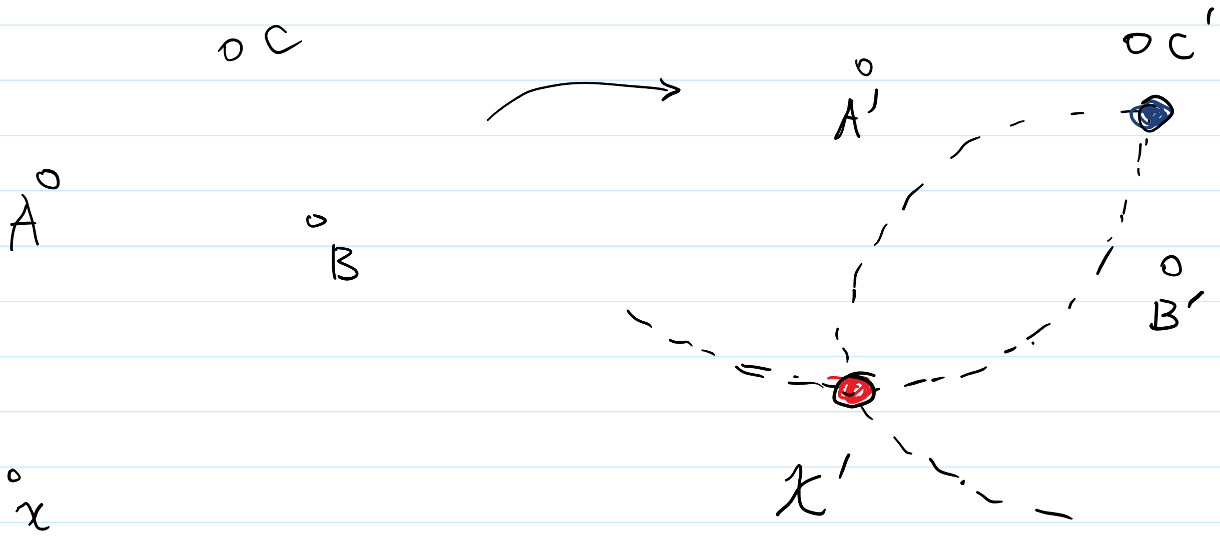


$M: \mathbb{R}^2 \to \mathbb{R}^2$ a map preserves distance. is called a Motion 运动.

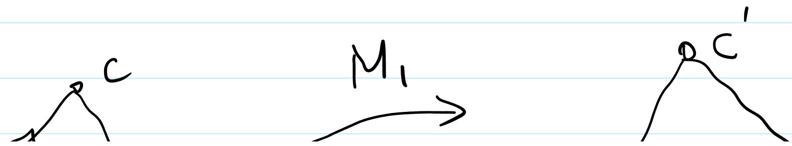
$$|p-q| = |M(p)-M(q)|$$

S_1 全等于 $S_2 \iff \exists\, M$ distance-preserving map s.t. $M(S_1)=S_2$.

Prop 1. A motion is uniquely determined by its action on three points not on a line.



Motion has two types:



保持定向运动.

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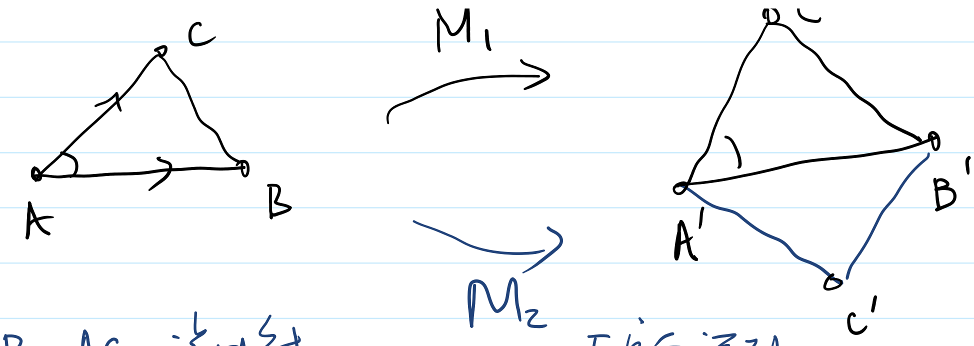


$\measuredangle\ \vec{AB}, \vec{AC}$ 逆时针

反定向运动.

$\measuredangle\ \vec{A'B'} \to \vec{A'C'}$ 顺时针.

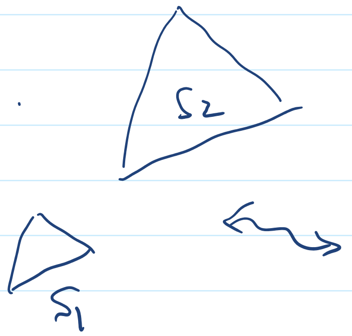
Prop 3. 保定向运动 必为 一旋转 或者 一平移

Prop 4

$$M(z) = e^{i\theta} \cdot z + w.$$

$$= e^{i\theta}(z + e^{-i\theta} w)$$

相似.



$\exists\ M: \mathbb{R}^2 \to \mathbb{R}^2$ s.t.

$$\frac{|p-q|}{|M(p)-M(q)|} = \lambda \text{ fixed.}$$

st.

$$M(S_1) = S_2$$

Such M is called a 相似变换.

Prop. $\forall$ 保定向相似变换 M is of the form

$$M(z) = r \cdot e^{i\theta} z + v.$$