习题1-1

1(2) $y'=\frac{\cos x\cdot x-1\cdot\sin x}{x^2}=\frac{x\cos x-\sin x}{x^2}$

需对 $y=\frac{\sin x}{x}$ 补充定义 $y=1\ (x=0)$

代入得：$\frac{x\cos x-\sin x}{x}+\frac{\sin x}{x}=\cos x$，表达式无误

但原微分方程定义域 $x\in(-\infty,+\infty)$，$y=\frac{\sin x}{x}$ 在其上不连续，故非解

1(4) 表达式一阶连续可导，$y'=\begin{cases}-\frac{1}{2}(x-c_1) & -\infty<x<c_1\\ 0 & c_1\le x\le c_2\\ \frac{1}{2}(x-c_2) & c_2<x<+\infty\end{cases}$，恒有 $y'\geq 0$.

代入检验：$\begin{cases}-\infty<x<c_1: & y'=-\frac{1}{2}(x-c_1)=\sqrt{|1-\frac{1}{4}(x-c_1)^2|}=\sqrt{|y|}\\ c_1\le x\le c_2: & y'=0=\sqrt{|y|}\\ c_2<x<+\infty: & y'=\frac{1}{2}(x-c_2)=\sqrt{|1-\frac{1}{4}(x-c_2)^2|}=\sqrt{|y|}\end{cases}$

~~2(1) $y'''=x \Rightarrow y''=\frac{1}{2}x^2\Rightarrow y'=\frac{1}{6}x^3$~~ 故其为通解

2(1) $y'''=x\Rightarrow y''=\frac{1}{2}x^2+c_1\Rightarrow y'=\frac{1}{6}x^3+c_1x+c_2\Rightarrow y=\frac{1}{24}x^4+\frac{1}{2}c_1x^2+c_2x+c_3$

代入初值条件得：$c_3=a_0$. $c_2=a_1$，$c_1=a_2\Rightarrow y=\frac{1}{24}x^4+\frac{1}{2}a_2x^2+a_1x+a_0$

~~2(2) $y=\int f(x)dx$ 设 $f(x)$ 原函数为 $F(x)$. 则 $y=F(x)+c$~~

~~代入 $y(0)=0\Rightarrow$~~

2(2) $\int_0^s\frac{dy}{dx}dx=\int_0^s f(x)dx+c$

$\Rightarrow y(s)-y(0)=\int_0^s f(x)dx+c \Rightarrow$ 由 $y(0)=0$，令 $s\to 0$ 得 $c=0$

$\Rightarrow y(s)=\int_0^s f(x)dx$

3(1) $y'=c+2x\Rightarrow c=y'-2x\Rightarrow y=(y'-2x)\cdot x+x^2\Rightarrow y=xy'-x^2$

3(4) $(x-a)^2+(y-b)^2=r^2\Rightarrow$ 对 x 求导 $2(x-a)+2(y-b)\cdot y'=0\Rightarrow x-a+y'(y-b)=0$

$\Rightarrow$ 对 x 求导：$1+y''(y-b)+y'\cdot y'=0$，即：$(y')^2-by''+yy''+1=0$

~~故：$x-a=(b-y)y'$，$y-b=\frac{-(y')^2-1}{y''}$~~

~~故 $(b-y)\cdot y'+y'\cdot[-\frac{(y')^2+1}{y''}]=0$，即 $y''\cdot y'\cdot(y-b)+y'[1+(y')^2]=0$~~

$\Rightarrow y-b=-\frac{(y')^2+1}{y''}\Rightarrow$ 对 x 求导：$y'=-\frac{2y'\cdot y''\cdot y''-y'''[(y')^2+1]}{(y'')^2}$

$-y'\cdot(y'')^2=2y'\cdot(y'')^2-y'''[(y')^2+1]\Rightarrow y'''[(y')^2+1]=3y'(y'')^2$

4. 局部看，每一部分的通积分均可解为通解

```
%作业题 5
clc,clear,close all
x_0=-3:0.2:3;
y_0=-3:0.2:3;
[x,y]=meshgrid(x_0,y_0);
d=sqrt(1+(4.*y.*(1-y)).^2);
u=1./d;
v=1*(4.*y.*(1-y))./d;
quiver(x,y,u,v);
xlim([-3,3])
ylim([-3,3])
hold on

z=dsolve('Dz-4*z*(1-z)=0','t')
```

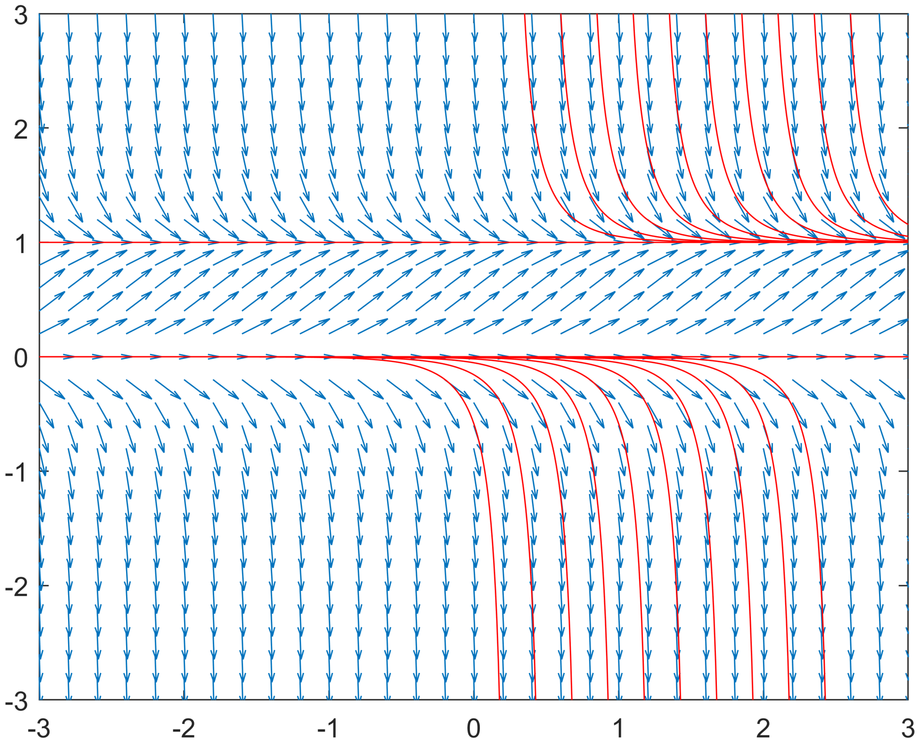
警告: Support of character vectors and strings will be removed in a future release. Use sym objects to define differential equations instead.

z =

$$\begin{pmatrix} -\frac{1}{e^{C_1-4t}-1} \\ 0 \\ 1 \end{pmatrix}$$

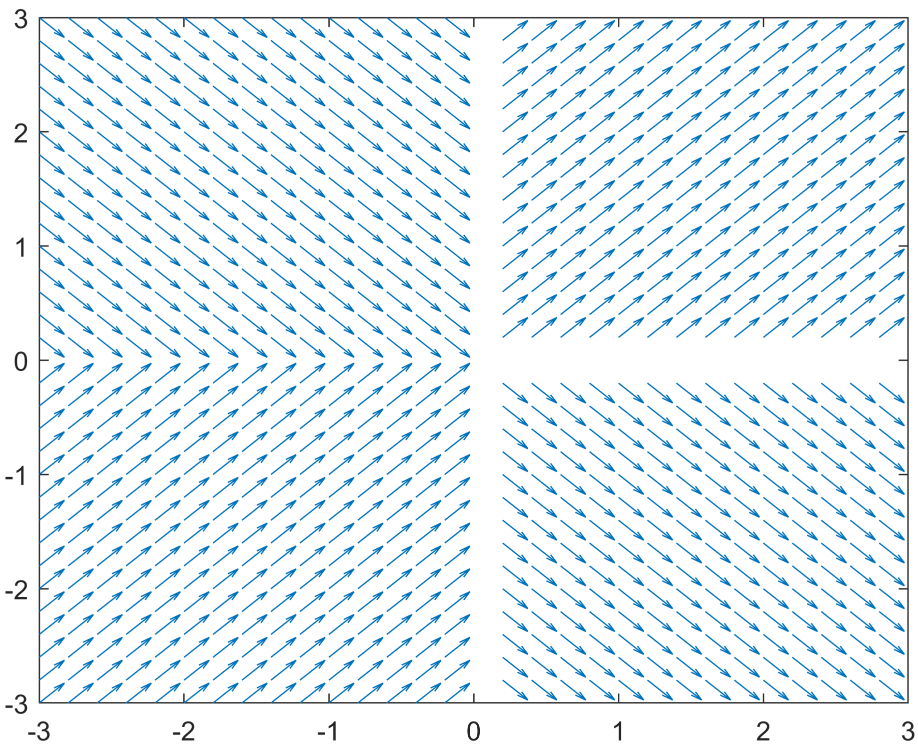
```


for i=1:10
t=-3:0.01:(i/4-0.01);
z11=(-1)./(exp(i-4.*t)-1);
plot(t,z11,'r')
hold on
end
for i=1:10
t=(i/4+0.01):0.01:3;
z12=(-1)./(exp(i-4.*t)-1);
plot(t,z12,'r')
hold on
end
size(t);
t=-3:(6/49):3;
z2=zeros(1,50);
z3=ones(1,50);
plot(t,z2,'r',t,z3,'r')
hold off
```

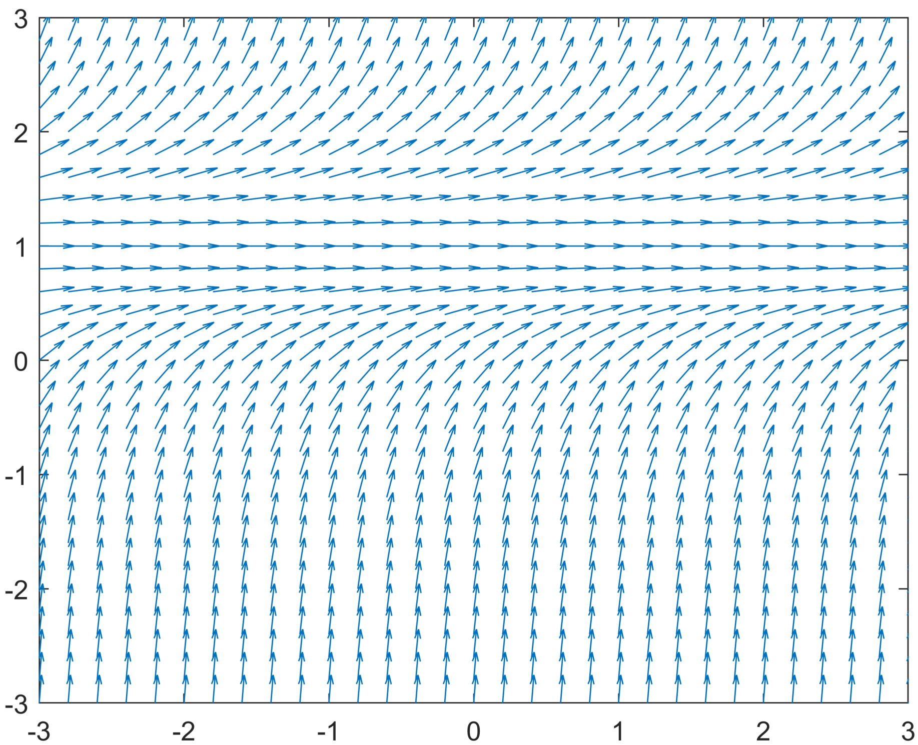


3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3

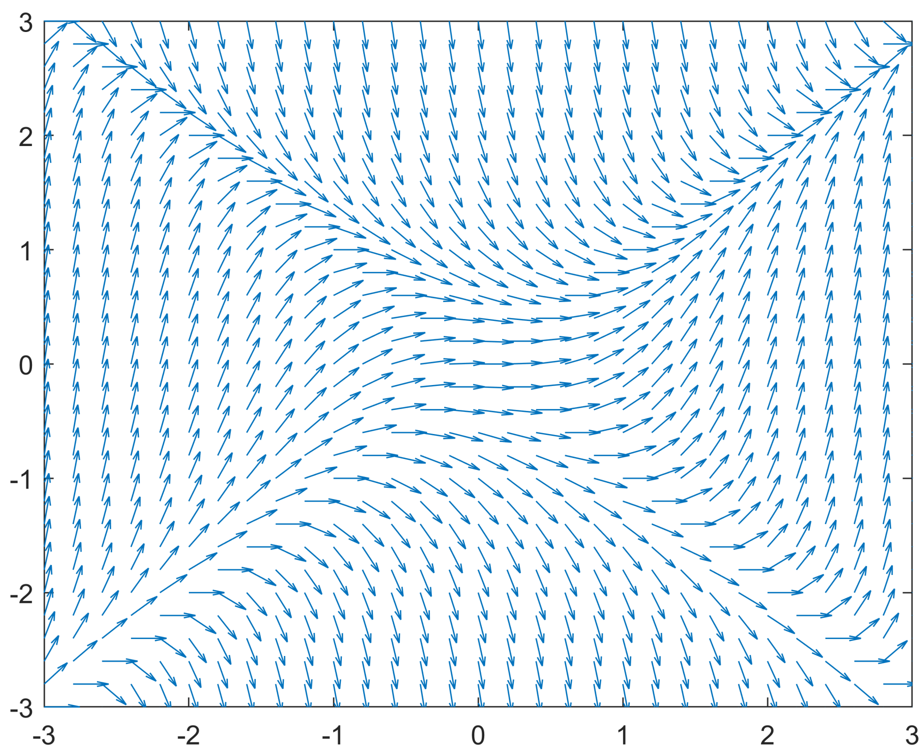
```
%作业题 6
%1-（1）
clc,clear,close all
x_0=-3:0.2:3;
y_0=-3:0.2:3;
[x,y]=meshgrid(x_0,y_0);
d=sqrt(1+((x.*y)./abs(x.*y)).^2);
u=1./d;
v=1.*((x.*y)./abs(x.*y))./d;
quiver(x,y,u,v);
xlim([-3,3])
ylim([-3,3])
```



```
%作业题 6
%1-（2）
clc,clear,close all
x_0=-3:0.2:3;
y_0=-3:0.2:3;
[x,y]=meshgrid(x_0,y_0);
d=sqrt(1+((y-1).^2).^2);
u=1./d;
v=1.*((y-1).^2)./d;
quiver(x,y,u,v);
xlim([-3,3])
ylim([-3,3])
```



```
%作业题 6
%2-(2)
clc,clear,close all
x_0=-3:0.2:3;
y_0=-3:0.2:3;
[x,y]=meshgrid(x_0,y_0);
d=sqrt(1+(x.^2-y.^2).^2);
u=1./d;
v=1*(x.^2-y.^2)./d;
quiver(x,y,u,v);
xlim([-3,3])
ylim([-3,3])
hold on
```




```
z=dsolve('Dz-t^2+z^2=0','t')
```

```
警告: Support of character vectors and strings will be removed in a future release. Use sym objects to define
differential equations instead.
z =
```

$$\left(\begin{array}{c}\dfrac{\sqrt{t}\ \sigma_2+\dfrac{\sigma_3}{2\ \sqrt{t}}}{\sqrt{t}\ \sigma_3}\\ \dfrac{\sqrt{t}\ \left(t\,\mathrm{J}_{-\frac{3}{4}}\left(\frac{t^2\,\mathrm{i}}{2}\right)\mathrm{i}-\frac{\sigma_1}{2\,t}\right)+\dfrac{\sigma_1}{2\ \sqrt{t}}+C_1\ \sqrt{t}\ \sigma_2+\dfrac{C_1\,\sigma_3}{2\ \sqrt{t}}}{\sqrt{t}\ \sigma_1+C_1\ \sqrt{t}\ \sigma_3}\end{array}\right)$$

where

$$\sigma_1=\mathrm{J}_{\frac{1}{4}}\left(\frac{t^2\,\mathrm{i}}{2}\right)$$

$$\sigma_2=t\,\mathrm{Y}_{-\frac{3}{4}}\left(\frac{t^2\,\mathrm{i}}{2}\right)\mathrm{i}-\frac{\sigma_3}{2\,t}$$

$$\sigma_3=\mathrm{Y}_{\frac{1}{4}}\left(\frac{t^2\,\mathrm{i}}{2}\right)$$

```
%方程的解较为复杂，绘制通解图像难度较高，观察方向场图像可大致判断积分曲线族
```