Technische Universität München
Institute for Communications Engineering
Prof. Dr. Gerhard Kramer

Numerical Simulation of the Optical Fiber Channel with Higher-Order Nonlinear Effects

Ingenieurpraxis

Author: Mohamed Houcine Safta

Advisor: Francisco Javier Garcia Gomez
Supervisor: Prof. Dr. Gerhard Kramer

Submission date: June 26, 2019

Contents

1 Abstract **1**

2 Introduction **3**

3 Theory **5**

3.1 Fiber properties and pulse propagation 5

3.1.1 Pulse propagation 5

3.1.2 Fiber Impairments: Linear and nonlinear 6

3.2 Mechanism of Dispersion and Nonlinearity in optical fibers 7

3.2.1 Attenuation 7

3.2.2 Dispersion 7

3.2.3 Kerr effect: 8

3.2.4 Self-Steepening: 8

3.2.5 Stimulated Ramen Scattering: 8

3.3 Non linear Schrodinger equation 9

4 Principle of the Split-Step Fourier Method (SSFM) **13**

4.1 Numerical method 13

4.2 Linear and nonlinear Sub-Steps 15

5 Simulation results **17**

5.1 Pulse propagation with higher order non-linearities not taken into account: 17

5.2 Pulse propagation with higher order non-linearities taken into account: 22

5.2.1 Self-Steepening only 22

5.2.2 Intrapulse Raman Scattering 25

6 Conclusion: **31**

List of Tables

5.1 Experiment characteristics (DLR regime) 17
5.2 Simulation characteristics for the studied case with Self-Steepning considered 22
5.3 Experiment characteristics for the studied case with intrapulse ramen scattering considered 25

List of Figures

3.1 The constituents of a typical optical fiber communication system, which consist of an optical transmitter , the transmission medium (optical fibers), one or more repeaters and an optical receiver. 5
3.2 Illustration of a pulse propagation in SMF and MMF 6

4.1 The Split-Step Fourier Scheme 14
4.2 The symmetric Split-Step Fourier Algorithm 14

5.1 Pulse and Spectrum Evolution for the case $L_{fiber} >> L_D$ and $L_{fiber} << L_{NL}$ 18
5.2 Dispersion-induced broadening for Gaussian pulses at $z = 0$, L_D, $3L_D$, $5L_D$ and $7L_D$ 19
5.3 Pulse and Spectrum Evolution for the case $L_{fiber} << L_D$ and $L_{fiber} >> L_{NL}$ 20
5.4 Pulse and Spectrum Evolution for the case $L_{fiber} << L_D$ and $L_{fiber} << L_{NL}$ 21
5.5 Self-steepening of a Gaussian pulse in the dispersionless case. 23
5.6 Spectrum of a Gaussian pulse at a distance $z = 20L_{NL}$. 23
5.7 Pulse shapes and spectra at $z = 0.2L_D$ (upper row) and $0.4L_D$ (lower row) for a Gaussian pulse propagating in the normal-dispersion regime of the fiber. 24
5.8 Temporal and spectral evolution of an unchirped Gaussian pulse for different dispersion lengths $[z = L_D, z = 8L_D]$ in the cases of anomalous (bottom row) for which $\beta_2 = -4.10^{-5} ps^2/km$ and normal (top row) dispersion for which $\beta_2 = 4.10^{-5} ps^2/km$. 26
5.9 Description of A_1 and A_2 27
5.10 Relative error evolution for different short pulse widths and different initial powers. 29

1 Abstract

Light wave communication systems that use fibers as communication medium for data transmission started in the late 20th century. Because of the characteristics of optical fibers including low loss and wide bandwith transmission, they are ideal for carrying all different kind of signals (Voice, Images and Video).

Due to the absence of proper hardware, it is sometimes difficult to test transmission techniques on real optical fiber. This explains the necessity of simulating different pulse shape propagation with simulation tools (MATLAB ...) with which optical system design becomes easy to represent with all the realistic impairments.

In this project, an attempt is made to demonstrate the various properties of optical fiber and mechanism of signal propagation through it taking into consideration higher order nonlinear effects.

This software system can be used to illustrate and evaluate optical communication systems and therefore provide a convenient method to move past the abstract level of theory to gain insight into physical phenomena associated with optical systems.

2 Introduction

Optical fiber is a medium for carrying information from one point to another in the form of light. A basic fiber optic system consists of a transmitting device, which generates the light signal; an optical fiber cable, which carries the light; and a receiver, which attempts to recover the data from the received signal [1].

Since its invention, the use and demand of optical fiber has grown tremendously. The uses of optical fiber today are quite numerous and are used in telecommunication, medicine, automotive and much more. Its applications to telecommunications range from global networks to phone exchanges to the use of internet on desktop computers involving the transmission of voice, data, or video over distances of less than a meter to hundreds of kilometers, using one of a few standard fiber designs.

The huge bandwidth of fiber makes it the perfect choice for transmitting signals to all the wide world users and although it was assumed at the beginning of Telecommunication history that bandwidth is unlimited it was thereafter revealed that the fiber has some impairment that inhibits the transmission of huge information.

The primary intention of the project is to simulate the various linear and nonlinear properties of optical fiber and the report is presented in a manner that treats the mathematical numerical knowledge that lies behind the Simulations. Therefore a thorough introduction of the fiber properties is given in Section 3.1. When a signal is transmitted through the fiber, it encounters various impairments and all the impairments are divided into two classes: linear and nonlinear. This will be further discussed in Section 3.2. The numerical method used is then described in Chapter 4 and the different simulation results that validate the well functioning of this method will be presented in Chapter 5.

3 Theory

3.1 Fiber properties and pulse propagation

3.1.1 Pulse propagation

A basic fiber communication system primarily consists of a transmitting device, which generates the light signal; an optical fiber, which carries the light beam loaded with information; a bunch of repeaters, which boost the signal strength to overcome the fiber losses; and a receiver, which accepts the light signal transmitted. These components are shown in Figure 3.1:

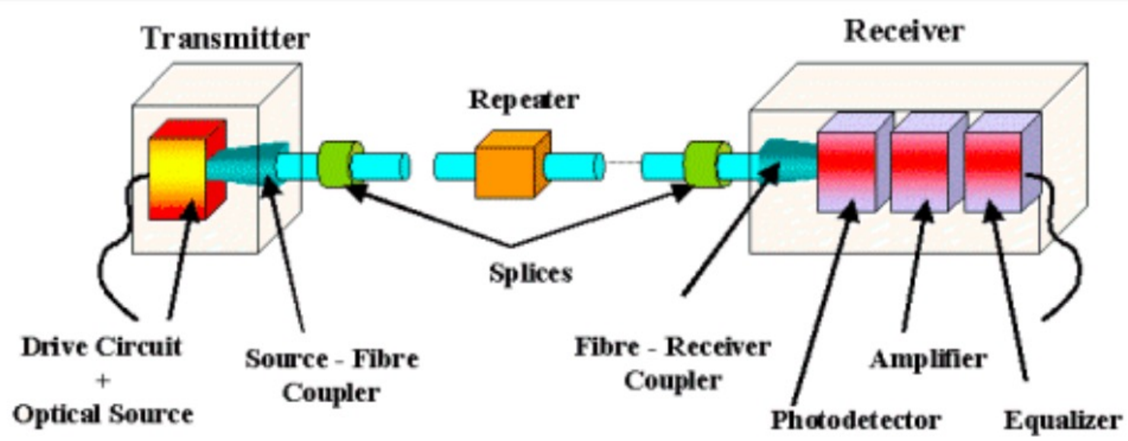


Figure 3.1: The constituents of a typical optical fiber communication system, which consist of an optical transmitter , the transmission medium (optical fibers), a repeater and an optical receiver [2] .

Optical fibers as transmission media have been developed and improved drastically over the last decades. Compared with traditional less advanced communication systems, optical fibers are advantageous in the way that they offer a wide transmission bandwidth, Long distance signal transmission and immunity to electromagnetic interference.

Based on the supported mode, optical fibers are classified into two types: single-mode fiber (SMF) and multimode fiber (MMF). SMF enables one type of light mode to be propagated at a time. It is used for example to guide light for long-distance telephony whereas MMF through which numerous modes or light rays simultaneously propagate, is used to guide light for short transmission distances. Figure 3.2 illustrates both those types of fiber.

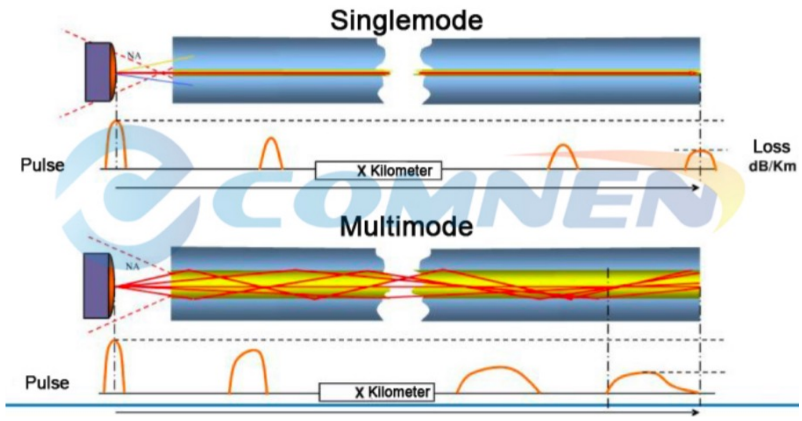


Figure 3.2: Illustration of a pulse propagation in SMF and MMF [3]

3.1.2 Fiber Impairments: Linear and nonlinear

In practice the propagation through optical fiber is beset with several limitations. As the transmission systems evolved to longer distances and higher bit rates, the linear effect of fibers, which are the attenuation and dispersion, became important limiting factor. Linear effects are wavelength dependent and nonlinear effects like Kerr effects and stimulated scattering are intensity dependent [1].

These impairments restrict the amount of information transmitted through the fiber and are divided into two classes: linear and nonlinear. For further understanding we can subdivide these this way:

1. Linear impairments:
   - Attenuation
   - Dispersion:
     - Chromatic dispersion
     - Mode dispersion
     - Polarization mode dispersion
2. Nonlinear impairments:
   - Kerr effect

  - Self Phase Modulation
  - Cross Phase Modulation
  - Four Wave Mixing
- Stimulated Scattering
  - Stimulated Brillouin scattering (SBS)
  - Stimulated Raman scattering (SRS)

3.2 Mechanism of Dispersion and Nonlinearity in optical fibers

As described in Section 3.1, different physical properties can influence the propagation of light in optical fibers. Depending on the initial characteristics of the pulse and the fiber itself, those may degrade the transmitted signal. In this section, we will briefly discuss some of those principal mechanisms which are highly relevant for the study conducted.

3.2.1 Attenuation

Attenuation represents the reduction of signal strength during transmission and is mathematically governed by the equation 3.1:

$$\alpha = (10/L)\log_{10}(P_{out}/P_{in}) \tag{3.1}$$

which have as a unit dB/km and where P_{in} the power launched at the input of the fiber with length L and P_{out} is the transmitted power. The mechanisms responsible for this attenuation are the material absorption and Rayleigh scattering but thanks to the repeaters this attenuation is overcame. In fact these repeaters are inserted along the length of fiber and are boosting the pulse intensity to reconstruct it to its original quality. In the experimental part of this report we will usually set $\alpha = 0$ because of this technological fact.

3.2.2 Dispersion

The dispersion in an optical fiber arises from the material and structural properties and is caused by the fact that the propagation velocity of light is a function of wavelength. When a beam of finite spectral width is sent to the fiber, each spectral component of the pulse travels inside the fiber at a velocity that mostly depends on its wavelength.

Dispersion represents the spreading out of a light pulse in time as it propagates down the fiber and it can be classified into two parts:

- Intramodal, or chromatic dispersion which affects all types of fibers and depends mainly on fiber materials
- Intermodal or group velocity dispersion (GVD)

One of the principal factors that limit the performance of transmission capacity of the fiber optics communication channel is the so called **chromatic dispersion** [4] which limits the amount of data that can be transported on a single fiber. A common mathematical approach that can be applied for analyzing high-order chromatic dispersion consists in finding the exact values of the derivatives of the refractive index with respect to wavelength (We limit the simulations to the 3rd order) [5]:

$$\beta(\omega) = n(\omega)\frac{\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \frac{1}{6}\beta_3(\omega - \omega_0)^3 + O((\omega - \omega_0)^4) \quad (3.2)$$

Where:

- β_1 is the group velocity parameter
- β_2 is the GVD parameter and responsible for pulse broadening
- β_3 is the third order dispersion (TOD) parameter which can be neglected for pulses of width $T_0 > 5ps$,and otherwise may be both important and relevant.

3.2.3 Kerr effect:

The Kerr effect is a nonlinear optical effect occurring when intense light propagates in crystals and glasses or gases. Its physical origin is a nonlinear polarization generated in the medium thus modifying the propagation properties of the light. The Kerr effect is the effect of an instantaneously occurring nonlinear response, which can be described as modifying the refractive index [6].

3.2.4 Self-Steepening:

The self-steepening is a phenomena that may influence the pulse edge and arises from an intensity-dependent group velocity and produces a temporal pulse distortion and an asymmetry in the pulse spectrum. It can cause an optical shock, perceived as an extremely sharp rear edge [7].

3.2.5 Stimulated Ramen Scattering:

When light propagates through a medium, the photons tend to interact with silica molecules during propagation. The photons also interact with each other and cause scattering effects, such as stimulated Raman scattering (SRS). This may result in a sporadic distribution of energy in a random direction. SRS refers to lower wavelengths pumping up the amplitude of higher wavelengths, which results in the higher wavelengths suppressing signals from the lower wavelengths and one way to mitigate the effects of SRS is to lower the input power.

3.3 Non linear Schrodinger equation

Pulse propagation within a dispersive and nonlinear medium like fiber can be modeled by the following non-linear Schrödinger equation:

$$\frac{\partial A}{\partial z} = \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3}\right) A + i\gamma \left(|A|^2 + iS\frac{1}{A}\frac{\partial}{\partial T}\left(A|A|^2\right) - T_R\frac{\partial |A|^2}{\partial T}\right) A \quad (3.3)$$

The equation describes the propagation of an optical pulse in single-mode fibers.

- $A := A(z, T)$ represents the envelope amplitude at an instant t and a certain distance z.
- α: Fiber loss
- β_2, β_3: Chromatic dispersion parameters.
- γ: nonlinearity coefficient of the fiber
- S: Governs the effects of self-steepening ($S = \lambda/(2\pi c)$) with λ the central wavelength of the pulse and c the speed of light.)
- T_R: Governs the effects of stimulated Raman scattering

When pulse widths are well in the pico-second regime and the pulse width > 5ps, $S = 0$ and $T_R = 0$ can be used and all nonlinear derivatives vanish [1].

We therefore obtain the simplified Schrodinger equation:

$$\frac{\partial A}{\partial z} = \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial T^2}\right) A + i\gamma |A|^2 A \quad (3.4)$$

Where A is the slowly varying amplitude of the pulse envelope and T is measured in a frame of reference moving at the group velocity.

For a more instructive reason let us consider a normalized version of equation 3.3. We therefore introduce a time scale normalized to the input pulse width T_0 (the input pulse width)as

$$\tau = T/T_0 \tag{3.5}$$

At the same time, we introduce a normalized amplitude U as

$$A(z,\tau) = \sqrt{P_0}\exp\left(-\alpha/2\right)U(z,\tau) \tag{3.6}$$

with P_o the peak power of the incident pulse and the exponential factor accounting for fiber losses.

Note: In the experimental simulation and due to the fact that we only consider cases for which $\alpha = 0$ the exponential term in (3.6) can be omitted.

If (3.6) is used to define the normalized amplitude U, (3.3) takes the form:

$$\frac{\partial U}{\partial z} = \left(-\frac{\alpha}{2} - i\frac{sgn(\beta_2)}{2L_D}\frac{\partial^2}{\partial\tau^2} + \frac{sgn(\beta_3)}{6L'_D}\frac{\partial^3}{\partial\tau^3}\right)U + \\ i\frac{e^{-\alpha z}}{L_{NL}}\left(|U|^2 + is\frac{1}{U}\frac{\partial}{\partial\tau}\left(U|U|^2\right) - \tau_R\frac{\partial|U|^2}{\partial\tau}\right)U \tag{3.7}$$

with L_D, L'_D and L_{NL} three length scales defined as:

- Dispersion Length $L_D = \frac{T_0^2}{|\beta_2|}$
- Higer Order Dipersion Length $L'_D = \frac{T_0^3}{|\beta_3|}$
- Nonlinear Length $L_{NL} = \frac{1}{\gamma P_0}$

The parameters s and τ_R govern the effects of self-steepening and intrapulse Raman scattering, respectively, and are defined as:

- $\tau_R = \frac{T_R}{T_o}$
- $s = \frac{S}{T_o}$

Depending on the initial width T_0 and the peak power P_0 of the incident pulse, either dispersive or nonlinear effects may dominate along the fiber.

Consequently we can induce that depending on the relative magnitude of L_{fiber} (the length of the optical fiber), L_D, L_{NL} , the propagation behaviour can be classified into the following categories:

Category 1: When the fiber length, L_{fiber} is such that $L_{fiber} << L_D$ and $L_{fiber} << L_{NL}$, then neither dispersive nor nonlinear effects play a significant role during pulse propagation. The pulse maintains its shape during propagation:$A(z,T) = A(0,T)\ \forall z$. The fiber plays a passive role in this regime and acts as a mere transporter of optical pulses. For 50km, L_D and L_{NL} should be larger than 500km for distortion free transmission [1]. This regime is called **non-dispersive linear regime (NDLR)**.

Category 2: When the fiber length is such that $L_{fiber} >> L_D$ but $L_{fiber} << L_{NL}$,then the pulse propagation is governed by GVD, and the nonlinear effect play a relatively minor role. The dispersion dominant regime is applicable whenever the fiber and pulse parameters are such that: $L_D/L_{NL} << 1$. This regime is called **dispersive linear regime (DLR)**.

Category 3: When the fiber length is such that $L_{fiber} << L_D$ but $L_{fiber} >> L_{NL}$ then the pulse evolution in the fiber is governed by the so called Self Phase Modulation (SPM) that leads to spectral broadening of the pulse. The nonlinearity-dominant regime is applicable whenever: $L_D/L_{NL} >> 1$. This regime is called **non-linear non-dispersive linear regime (NLNDR)**

<u>**Note**</u>: Self Phase Modulation is a consequence of the Kerr effect and it means that a light wave in the fiber experiences a nonlinear phase delay which depends on its own intensity. For a fiber mode, the phase change per unit optical power and unit length is described by the proportionality constant called **nonlinear coefficient** and which is often denoted with γ [8] .

Category 4: When the fiber length L_{fiber} is longer or comparable to both L_D, and L_{NL} , dispersion and nonlinearity act together as the pulse propagates along the fiber. The simultaneous interaction between GVD and SPM leads to different behaviors when compared to when the effect is manifested singly. In some configurations when both effects cancel mutually we observe the **soliton** propagation [6]. This regime is called **nonlinear dispersive regime (NLDR)**.

<u>**Note**</u>: A Soliton is a phenomena that occurs when in certain conditions where dispersion and nonlinearity are present, the frequency chirps created by those two effects are opposite. In this particular case neither the pulse nor its spectrum broadens and the undistorted pulse is called a Soliton [9]. Waves with similar features can be created inside an optical fiber.

4 Principle of the Split-Step Fourier Method (SSFM)

4.1 Numerical method

The nonlinear regime propagation equation does not offer a closed-form solution and hence it is necessary to make some approximations. The main techniques in order to solve the propagation equation are the FFT and the IFFT.

In a first approach it is possible to write the following equation:

$$\frac{\partial A}{\partial z} = \underbrace{\left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3}\right)}_{\mathcal{D}} A + \underbrace{i\gamma\left(|A|^2 + iS\frac{1}{A}\frac{\partial}{\partial T}\left(A|A|^2\right) - T_R\frac{\partial|A|^2}{\partial T}\right)}_{\mathcal{N}} A \tag{4.1}$$

Where $\mathcal{D}$(A) is the linear differential operator which considers dispersion and losses within the linear medium and $\mathcal{N}$(A) is a nonlinear operator that accounts for the fiber nonlinearities on pulse propagation. Using these definitions we can write equation (4.1) as:

$$\frac{\partial A}{\partial z} = (\mathcal{D} + \mathcal{N})A \tag{4.2}$$

In general case, the dispersion and the nonlinearity appear at the same time along the fiber axial direction. In the split step Fourier method, an approximated solution is calculated assuming that the dispersion effect and the nonlinear optical effect appears separately when light travels for a short distance, h, in an optical fiber.

The approach of this method is then to divide the optical fiber into many small steps of length h and devide each step into two sub-steps , were, in the first part, only the nonlinearities manifest and, then, in the second sub-length, only the dispersion effect is taken into account.

Under the assumption of independence between $\mathcal{D}$ and $\mathcal{N}$, and by applying the Baker-Hausdorff formula [10] to both non-commuting operators $\mathcal{D}$ and $\mathcal{N}$, (4.2) can be solved and the numerical approximated solution is:

$$A(z+h,T) = \exp(h(\mathcal{D}+\mathcal{N}))A(z,T) \approx \exp(h(\mathcal{D}))\exp(h(\mathcal{N}))A(z,T) \tag{4.3}$$

A commonly used symmetric approximation for the equation 4.3 which has been proven to be an $O(h^2)$ approximation is:

$$A(z+h,T) = \exp\left(\frac{h}{2}\mathcal{D}\right)\exp(h\mathcal{N})\exp\left(\frac{h}{2}\mathcal{D}\right)A(z,T) \tag{4.4}$$

More concretely and for a better illustration of the matter, the light propagation from z to z+h considers the following three steps, the scheme used is depicted in Figure 4.1 and the numerical algorithm employed can be illustrated in Figure 4.2:

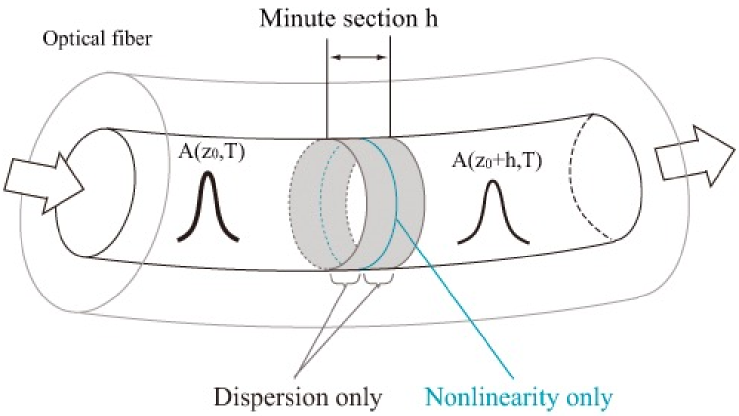


Figure 4.1: The Split-Step Fourier Scheme [11]

And the respective numerical algorithm for solving (4.1) is illustrated by:

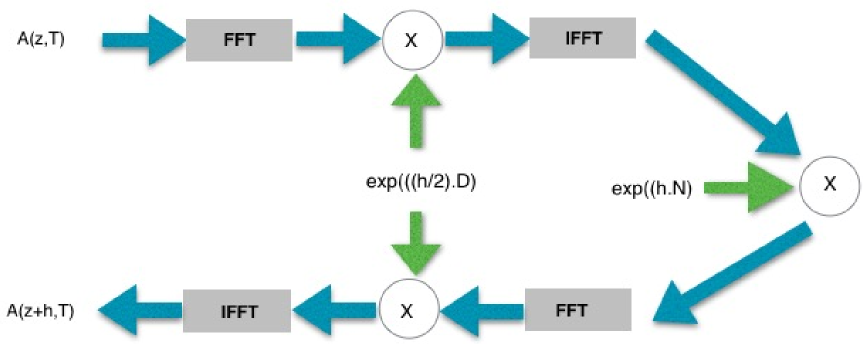


Figure 4.2: The symmetric Split-Step Fourier Algorithm

4.2 Linear and nonlinear Sub-Steps

Linear Sub-Steps:

It is computationally efficient to apply Fourier transformation in T to $\mathcal{D}$ and transform the differentials into multiplications in frequency domain.

$$\exp\left(\frac{h}{2}\mathcal{D}\right)A(z,T) = \mathcal{F}^{-1}\exp\left[\frac{h}{2}\left(-\frac{\alpha}{2}+i\frac{\beta_2}{2}\omega^2-i\frac{\beta_3}{6}\omega^3\right)\right]\mathcal{F}A(z,T) \tag{4.5}$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the direct and inverse Fourier transformations, respectively.

In the simulations conducted, one uses discrete Fourier transformation and for ω we employ the discrete frequency spectrum $\{j\omega : j \in Z, -N \leq j \leq N-1\}$ with spectral width $d\omega = 2\pi/Ndt$.

The temporal window traveling with the pulse is discretized with $2N$ points. Denoting the temporal discretization width with dt, this gives a temporal window of the extensions $[(N-1)dt, Ndt]$.

Nonlinear Sub-Steps

A straightforward approach for deriving a nonlinear operator $\mathcal{N}(\mathcal{A})$ consists in using the **Central Difference Method** employed in [12] as a benchmark. We are going next to give a brief introduction about this method for this relevance in the simulations conducted in order to test our program.

First we can realize that the operator $\mathcal{N}(\mathcal{A})$ after substituting $|A|^2 = A\bar{A}$ can be rewritten as:

$$\mathcal{N}(A) = i\gamma\left(|A|^2 + iS\bar{A}\frac{\partial A}{\partial T} + [iS - T_R]\,\frac{\partial |A|^2}{\partial T}\right) \tag{4.6}$$

We then simply approximate the temporal derivatives in (4.6) with complex-valued second-order accurate central differences of the discrete values A_j, with A_j the numerical approximation to the pulse envelope at a certain distance $A(.,jdt)$ at the discrete point with index j and we obtain the equation:

$$\mathcal{N}_j(A) \approx i\gamma\left(|A_j|^2 + iS\bar{A}_j\frac{A_{j+1}-A_{j-1}}{2\Delta T} + [iS - T_R]\,\frac{|A_{j+1}|^2 - |A_{j-1}|^2}{2\Delta T}\right) \tag{4.7}$$

The implementation of boundary conditions for the temporal window, we employ auxiliary points outside of the actual domain based on the following: $A_{-N-1} := A_{N-1}$ & $A_N := A_{-N}$

5 Simulation results

In this section the simulation analysis will be discussed. Different pulses were tested and simulated but we will mainly showcase the results obtained for the Gaussian pulse with initial shape:

$$A(0,T) = \sqrt{P_0} \exp\left(-\frac{1+iC}{2}\frac{T^2}{T_0^2}\right) \tag{5.1}$$

Where:

- P_0 is the initial power
- C is a Chirp parameter [13].
- T_0 is the $1/e$ width and which we will define as the pulse width for the remainder of this report.

5.1 Pulse propagation with higher order non-linearities not taken into account:

Dispersive linear regime (DLR)

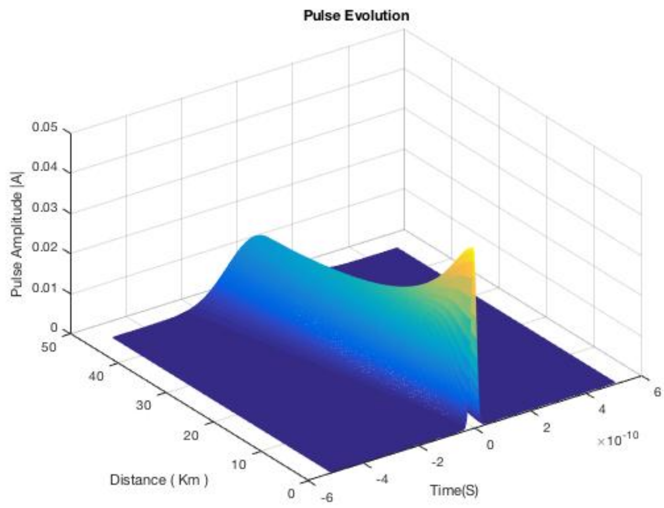
In the dispersive regime, the pulse broadens but its bandwidth remains the same. Only different frequencies get separated in time due to dispersion, but there are no new frequencies generated.

For this first simulation we use the parameters found in [14] and presented in Table 5.1 .

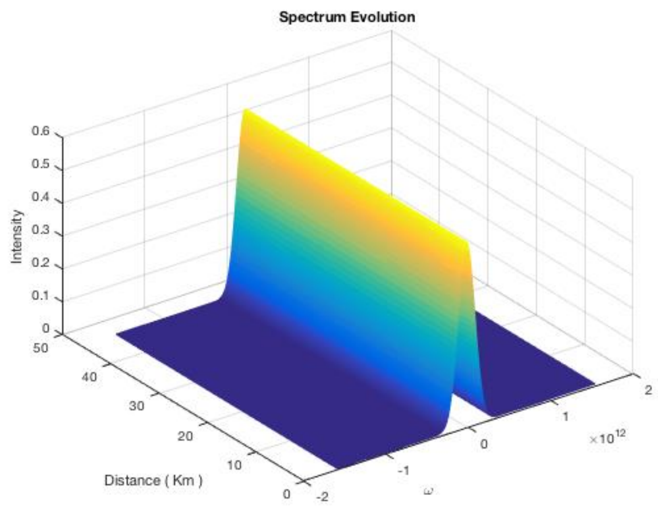
Variables	Value[Unit]
P_0 (peak power input)	2 mW
T_0 (initial pulse widh)	10 ps
α (loss parameter)	0 dB/km
β_2 (2nd order dispersion)	-21.67 ps^2/km
β_3 (3rd order dispersion)	0 ps^3/km
γ (nonlinear coefficient)	1.27 $W^{-1}km^{-1}$

Table 5.1: Experiment characteristics (DLR regime)

With these parameters we can easily calculate both: $L_D \approx 5km$ and $L_{NL} \approx 395km$. We simulate the propagation of the optical pulse for a fiber length of roughly $L_{fiber} = 10L_D \approx 50km$ and obtain the following representations in figure 5.1:



(a) Pulse Evolution in the DLR regime



(b) Spectrum Evolution in the DLR regime

Figure 5.1: Pulse and Spectrum Evolution for the case $L_{fiber} >> L_D$ and $L_{fiber} << L_{NL}$

It is mathematically proven that in this regime a Gaussian pulse stays Gaussian on propagation but its width increases and becomes $T_1 = T_0\sqrt{(1 + z/L_D)}$ [1] This has been numerically verified and we illustrate the pulse broadening for different lengths inside the optical fiber and obtain the Figure 5.2:

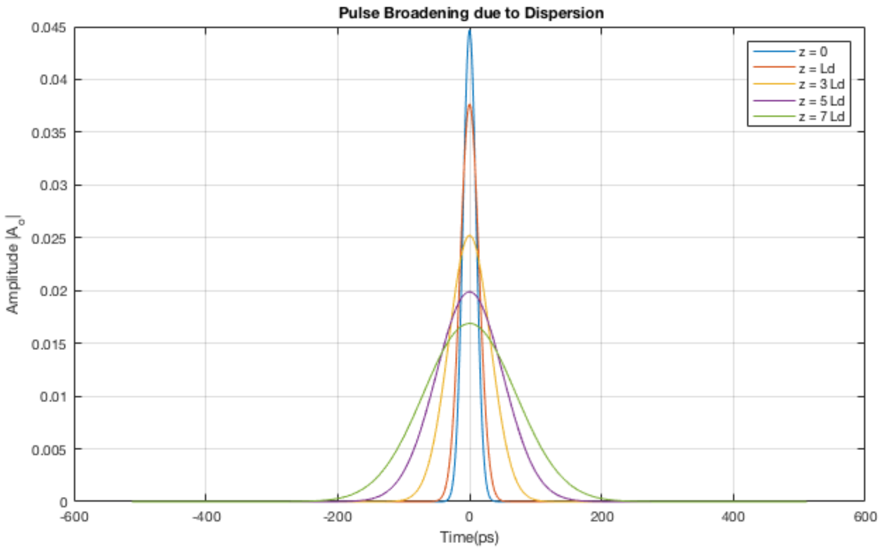


Figure 5.2: Dispersion-induced broadening for Gaussian pulses at $z = 0$, L_D, $3L_D$, $5L_D$ and $7L_D$

In the Figure 5.1(a) we see that the pulse is broadened and its amplitude decreases. This is due to the energy conservation, being the pulse width variation of a factor of $\sqrt{2}$ if we take the example of $z = L_D$. The maximum amplitude of the pulse occurs at $z = 0$ and as it moves along the fiber the pulse loses amplitude and broadens. This phenomenon is observed because of the GVD which affects the characteristics of the pulse such as the amplitude and the width.

If we simulate the evolution of the pulse′s spectrum, Figure 5.1(b) is obtained. It is verified that the magnitude of the pulse spectrum does not change along the fiber. The GVD changes the phase of each spectral component of the pulse, which depends on the frequency and also the distance, and these phase variations dont affect the pulse spectrum but only the pulse width.

Non-linear, non-dispersive linear regime(NLNDR)

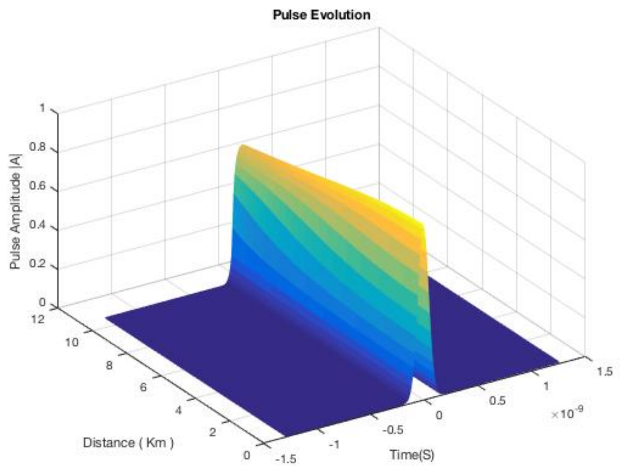
For this next simulation we use the parameters presented in table 5.1 and change:

- $P_0 = 0.8W$
- $T_0 = 50ps$

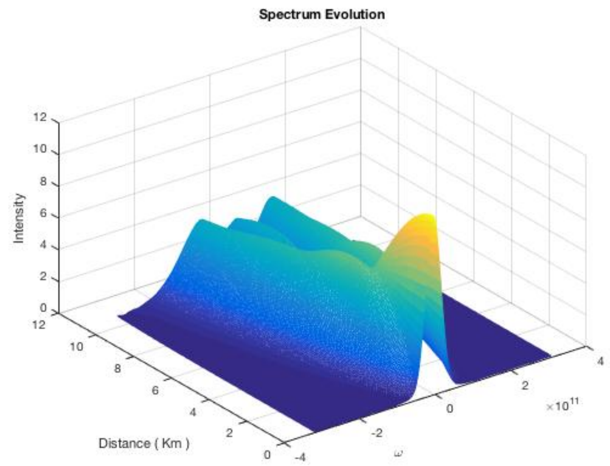
Hence we obtain:

- $L_D \approx 110Km$
- $L_{NL} \approx 1Km$

For a fiber length approximated to $L_{fiber} = 0.1L_D \approx 11Km$ we obtain the following figure 5.3



(a) Pulse Evolution in the NLNDR regime



(b) Spectrum Evolution in the NLNDR regime

Figure 5.3: Pulse and Spectrum Evolution for the case $L_{fiber} << L_D$ and $L_{fiber} >> L_{NL}$

non-dispersive linear regime (NDLR)

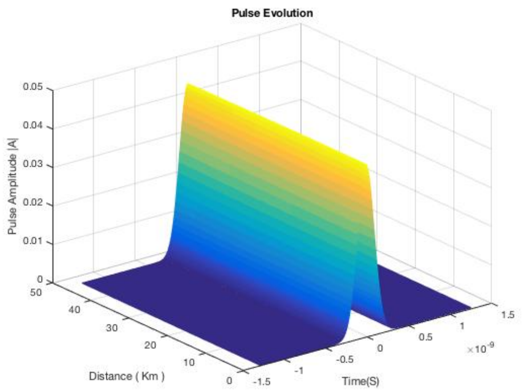
For this regime we change from Tbale 5.1 :

- $T_0 = 100ps$

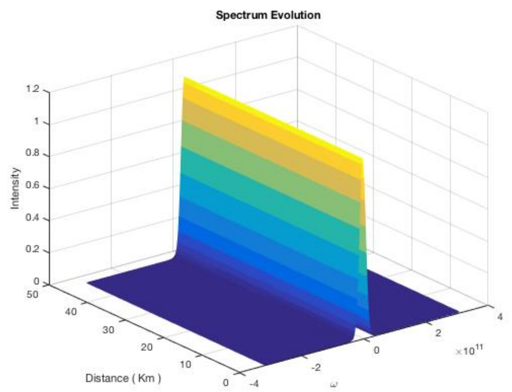
Hence we obtain:

- $L_D \approx 470Km$
- $L_{NL} \approx 400Km$

For a fiber length approximated to $L_{fiber} = 0.1L_d \approx 47Km$ we obtain the Figure 5.4



(a) Pulse Evolution in the NDLR regime



(b) Spectrum Evolution in the NDLR regime

Figure 5.4: Pulse and Spectrum Evolution for the case $L_{fiber} << L_D$ and $L_{fiber} << L_{NL}$

Conclusion: The algorithm gives results that are consistent with the theoretical findings in [1] and briefly discussed in Section (3.3) that describe the evolution of optical pulses. Different phenomena (GVD, DPM ..) were numerically tested and the output is in accordance with what was described in the literature and in different research papers.

5.2 Pulse propagation with higher order non-linearities taken into account:

In this section we mainly focus on reproducing some of the numerical results that were found and discussed by Agrawal in Chapter4 of [1] when studying the influence of higher order nonlinear effects on pulse propagation. In fact and for ultrashort pulses $T_o < 1ps$ those effects are must be considered due to their high relevance and we therefore study the influence of Self -Steepening and Raman Scattering seperately and when they are taken into account together.

5.2.1 Self-Steepening only

Self-steepening results from the intensity dependence of the group velocity. Its effects on SPM were first considered in liquid nonlinear media and later extended to optical fibers. It leads to an asymmetry in the SPM-broadened spectra of ultrashort pulses.

Let us consider the parameters in Table 5.2:

Variables	Value[Unit]
P_0 (peak power input)	0.625 mW
T_0 (initial pulse width)	80 fs
α (loss parameter)	0 dB/km
β_2 (2nd order dispersion)	$0ps^2/km$
β_3 (3rd order dispersion)	0 ps^3/km
γ (nonlinear coefficient)	1 $W^{-1}km^{-1}$
T_R	0 s
S	0.8 fs

Table 5.2: Simulation characteristics for the studied case with Self-Steepning considered

Figure 5.5 shows the calculated pulse shapes at $z = 10L_{NL}$ and $20L_{NL}$ for $S = 0.8fs$.

As the pulse propagates inside the fiber, it becomes asymmetric, with its peak shifting toward the trailing edge. As a result, the trailing edge becomes steeper and steeper with increasing z. Physically, the group velocity of the pulse is intensity dependent such that the peak moves at a lower speed than the wings.

Self-steepening of the pulse eventually creates an optical shock, analogous to the development of an acoustic shock on the leading edge of a sound wave and for femtoseconds pulses significant self-steepening of the pulse can occur in a few-centimeter-long fiber. Optical shocks with an infinitely sharp trailing edge never occur in practice because of the GVD; as the pulse edge becomes steeper, the dispersive terms in the NLSE equation become increasingly more important and cannot be ignored.

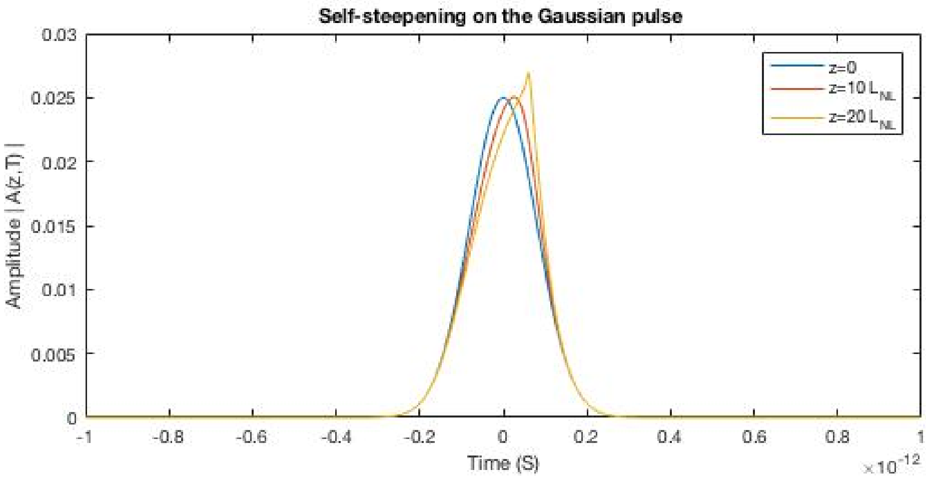


Figure 5.5: Self-steepening of a Gaussian pulse in the dispersionless case.

Self-steepening also affects SPM-induced spectral broadening. Figure 5.6 shows the calculated spectrum at $z = 20L_{NL}$. The most notable feature is spectral asymmetrythe red-shifted peaks are more intense than blue-shifted peaks. The other notable feature is that SPM-induced spectral broadening is larger on the blue side than the red side. Both of these features can be understood qualitatively from the changes in the pulse shape induced by self-steepening. The spectrum is asymmetric simply because the pulse shape is asymmetric.

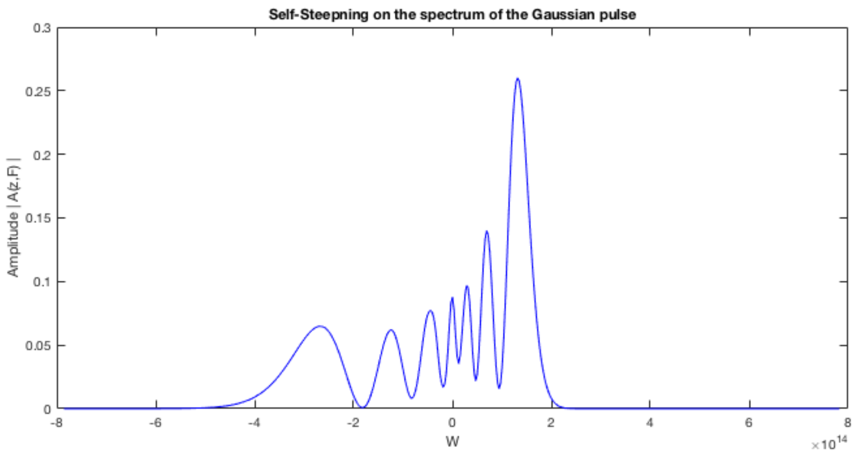


Figure 5.6: Spectrum of a Gaussian pulse at a distance $z = 20L_{NL}$.

The spectral features seen in Figure 5.6 are considerably affected by GVD, which cannot be ignored when short optical pulses propagate inside silica fibers.

The pulse evolution in this case is studied by solving the equation 3.3 numerically using the gaussian pulse shape introduced at the beginning of this chapter and using the same parameter defined in 5.2 only with a change in the value of β_2 to $\beta_2 = 4.10^{-5} ps^2/km$ which is a reasonable practical value.

Figure 5.7 shows the pulse shapes and the spectra at $z = 0.2L_D$ and $0.4L_D$ in the case of an initially unchirped Gaussian pulse. Note that with the chosen parameters we obtain the relation $L_D = 100L_{NL}$.

A direct comparison between figures 5.7, 5.5 and 5.6 shows that both the shape and spectrum are significantly affected by GVD even though the propagation distance is only a fraction of the dispersion length L_D. It is the GVD that dissipates the shock by broadening the steepened trailing edge, a feature clearly seen in the asymmetric pulse shapes of figure 5.7.

With a further increase in the propagation distance, the pulse continues to broaden while the spectrum remains nearly unchanged.

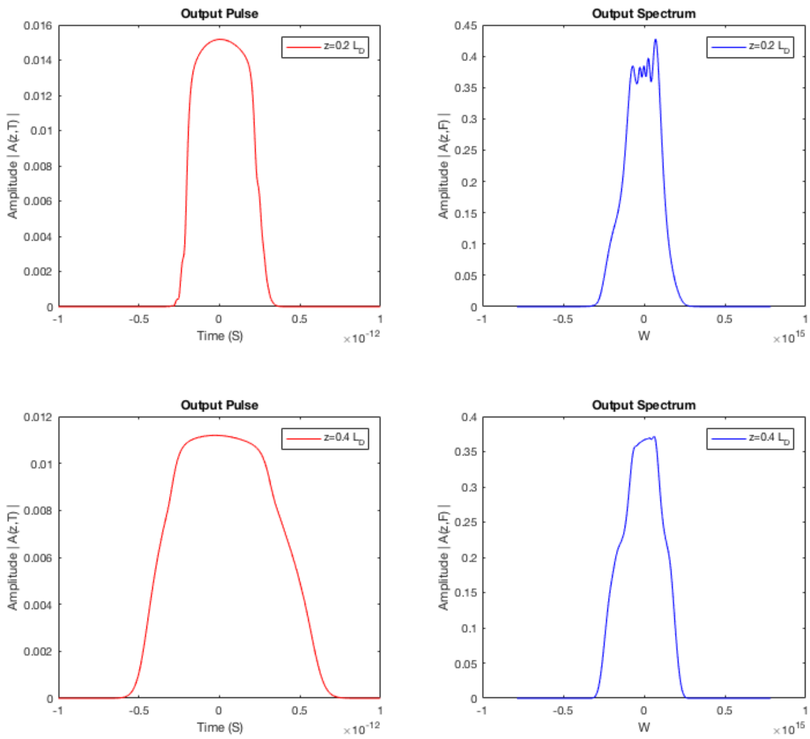


Figure 5.7: Pulse shapes and spectra at $z = 0.2L_D$ (upper row) and $0.4L_D$ (lower row) for a Gaussian pulse propagating in the normal-dispersion regime of the fiber.

5.2.2 Intrapulse Raman Scattering

The discussion so far has neglected the last term in (3.3) that is responsible for intrapulse Raman scattering. In the case of optical fibers, this term becomes quite important for ultrashort optical pulses and should be included in modeling pulse evolution of such short pulses in optical fibers.

For this case let us define the parameters of (3.3) as follows:

Variables	Value[Unit]
P_0 (peak power input)	625 mW
T_0 (initial pulse width)	15 fs
α (loss parameter)	0 dB/km
β_2 (2nd order dispersion)	$(+/-)4.10^{-5}ps^2/km$
β_3 (3rd order dispersion)	0 ps^3/km
γ (nonlinear coefficient)	1 $W^{-1}km^{-1}$
T_R	0.45 fs
S	0 fs

Table 5.3: Experiment characteristics for the studied case with intrapulse ramen scattering considered

Figure 5.8 shows the temporal and spectral evolution of an unchirped Gaussian pulse over 7 dispersion lengths in the cases of anomalous (bottom row) and normal (top row) dispersion. One can see immediately the dramatic effect of the nature of the fiber dispersion.

In the case of normal dispersion, the pulse broadens rapidly while its spectrum is broadened through SPM by a small factor.

In contrast, in the case of anomalous dispersion, the optical pulse undergoes an initial narrowing stage and then slows down, as is apparent from its bent trajectory.

The most noteworthy features of Figure 5.8 is a Raman-induced frequency shift (RIFS) in the pulse spectrum toward longer wavelengths. This is a direct consequence of intrapulse Raman scattering. As the pulse spectrum shifts through the RIFS, the pulse slows down because the group velocity of a pulse is lower at longer wavelengths in the case of $\beta_2 < 0$. This deacceleration is responsible for the bent trajectory of the pulse

Physically speaking, the Raman term shifts the carrier frequency at which pulse spectrum is centered. This spectral shift in turn shifts the pulse position in the time domain because of changes in the group velocity through fiber dispersion.

Conclusion:

- The SPM and other nonlinear effects such as stimulated Raman scattering, occurring simultaneously inside optical fibers, can broaden the spectrum of an ultrashort pulse so much that it may extend over more than 100 THz.

- In this section of the Chapter 5 we have simulated different cases that considerably demonstrate the influence of different higher order nonlinearities on the pulse shape and therefore indirectly it's energy. All the results obtained with the Matlab software program implemented for this reason show high similitude with the ones discussed by Agrawel and we have therefore verified the well functioning on the numerical program in hands.

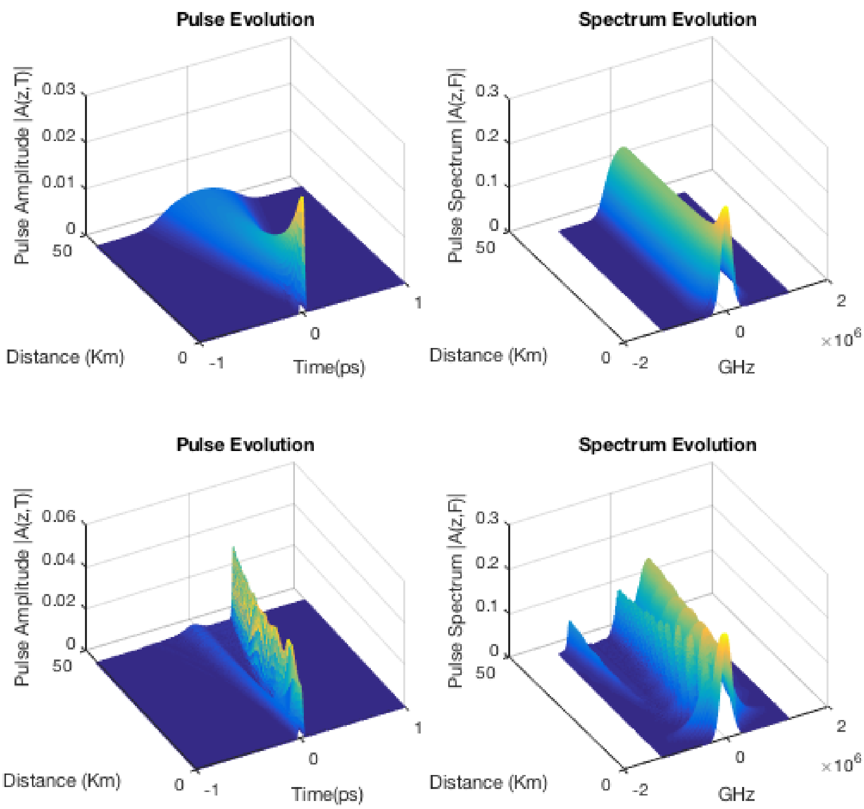


Figure 5.8: Temporal and spectral evolution of an unchirped Gaussian pulse for different dispersion lengths [$z = L_D, z = 8L_D$] in the cases of anomalous (bottom row) for which $\beta_2 = -4.10^{-5} ps^2/km$ and normal (top row) dispersion for which $\beta_2 = 4.10^{-5} ps^2/km$.

For these simulations we consider high order non-linearities (Stimulated Raman Scattering and Self-Steepening) and we therefore introduce two new walues of T_R and S described in equation 3.3[12].

We set:

- T_R= 3ps
- $S = 8.2310^{-4}ps$ (for $\lambda = 1550nm$)

It is highly interesting to identify the importance of these parameters on the pulse shape when propagating and we therefore study the relative error obtained when those parameters are introduced to when they are omitted.

To make it simple let's consider two pulses A_1 and A_2 and apply the SSFM for both pulses as described in Figure 5.9:

Figure 5.9: Description of A_1 and A_2

We also define the relative error E_{rel} as following:

$$E_{rel} = \frac{\int\limits_{-\infty}^{\infty} \mid A_2(z,t) - A_1(z,t) \mid^2 \; dt}{\int\limits_{-\infty}^{\infty} \mid A_1(z,t) \mid^2 dt} \tag{5.2}$$

We also note that for the case of $A_1(z,t)$ we use the following NLSE:

$$\frac{\partial A_1}{\partial z} = \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} \right) A_1 + i\gamma |A_1|^2 A_1 \tag{5.3}$$

to evaluate the pulse propagation. Whereas for $A_2(z,t)$ and accounting for higher order nonlinear effects we obtain (5.4) which the nonlinear operator in our simulations is approximated to the one obtained in (4.7):

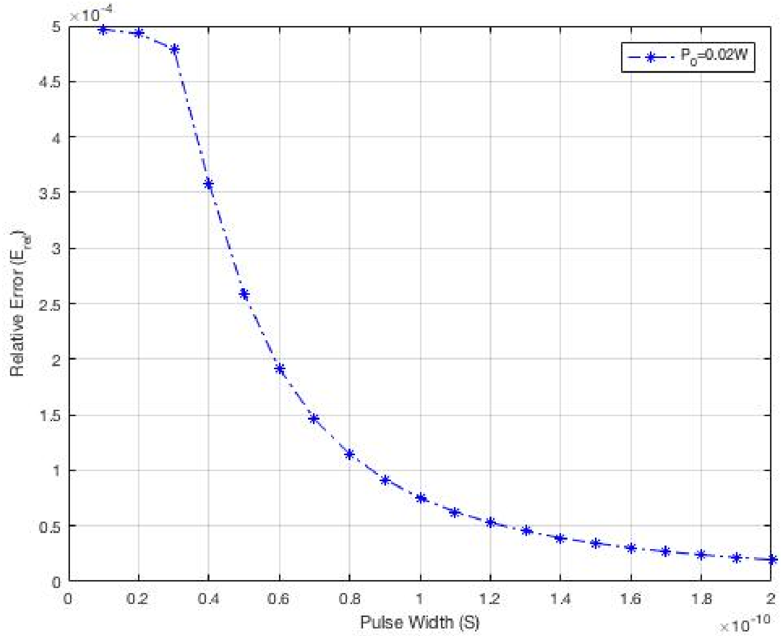
$$\frac{\partial A_2}{\partial z} = \left(-\frac{\alpha}{2} - i\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\right) A_2 + i\gamma \left(|A_2|^2 + iS\bar{A}_2\frac{\partial A_2}{\partial t} + [iS - T_R]\,\frac{\partial |A_2|^2}{\partial t}\right) \quad (5.4)$$

Running the simulation for the values of T_R and S defined previously and for different values of initial power peak we obtain different aspects for the relative error curve. It is important to signal that the fiber length L_{fiber} and P_o must be chosen carefully in order to evaluate (5.2). At $z = L_{fiber}$ the pulse should be undistorted in both of the trials (With/Without high order nonlinearities taken into account) and for all the pulses tested.

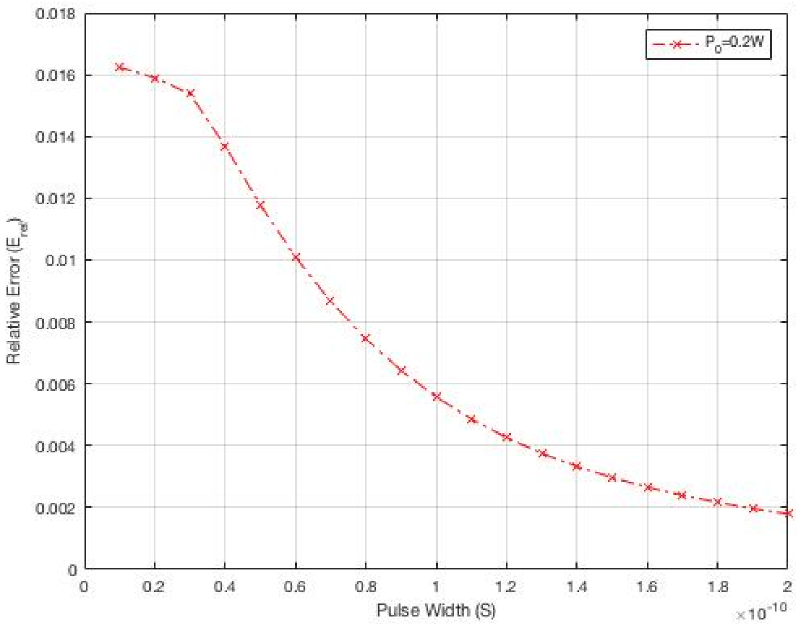
We simulate for example E_{rel} for $L_{fiber} = 10km$ for two values of P_0, for the pulse width $T_0 \in [10, 200]pS$, $\beta_2 = -21,67ps^2/km$, $\gamma = 1.27W^{-1}km^{-1}$ and obtain the following results in Figure 5.10.

We can observe that:

- A bigger T_0 make the relative error smaller, this may goes along with the fact that for large pulses S and T_R we tend to neglect those parameters and set them equal to 0. Hence there is no remarkable difference between A_1 and A_2 and $A_1(L_{fiber}, t) \approx A_2(L_{fiber}, t)$.
- With bigger P_0 the error tends to become more important. This is quite logical due to the fact that all the terms of the nonlinear operator are directly or indirectly proportional to $\mid A(z,t) \mid$ and therefore $\sqrt{P_0}$.



(a) Relative error measured between A_1 and A_2 for $P_0 = 0.02W$



(b) Relative error measured between A_1 and A_2 for $P_0 = 0.2W$

Figure 5.10: Relative error evolution for different short pulse widths and different initial powers.

6 Conclusion:

Though optical fiber represents a transmission system of huge bandwidth, still it has various impairments that we briefly discussed like attenuation, dispersion, stimulated scattering, etc. In addition, the pulse propagation inside this transmission system is described by the nonlinear Schrödinger equation (NLSE) which incorporates almost all the properties of optical fibers. All the impairments of optical fibers are classified into linear effects, which are wavelength dependent, and nonlinear effects, which are intensity (power) dependent.

In this internship, we have simulated the various linear and nonlinear properties of optical fibers with the aid of NLSE and solving it employing the Split Step Fourier Method which is found to be a good approximating algorithm for the solution. We have simulated some analytical results with predefined parameters and conditions. Overall we noted that the simulations conducted output results that are in accordance with what was found in the literature.

When introducing the parameters for high order nonlinear effects it was somehow difficult to come up with good values for the parameters that govern the pulses propagation in order to showcase the pulse behaviour for lack of knowledge about real systems. The simulations described in this report were conducted for different pulse widths and using the Central Difference Method.

While comparing different pulses we came up to the conclusion that both the pulse width and the initial peak power p_0 have influence on the pulse shape at the end of the optical fiber.

More realistic parameters need to be tested and their influence on the shape and energy of pulses at the end of the optical fibers.

Bibliography

[1] Govind Agrawal. *Nonlinear Fiber Optics.* Academic Press, 10th October 2012.

[2] Marco Costa Pereira Maria Joo Martins. Optoelectronics Course. `http://www.pef.uni-lj.si/eprolab/comlab/sttop/sttop-oe/index.htm`. [Online Course].

[3] http://www.comnen.com/whats-the-difference-between-singlemode-and-multimode-fiber-patch-cables/.

[4] Saied Hemati and M Emadi. A method for analyzing higher-order dispersion in optical fiber. In *CCECE 2003-Canadian Conference on Electrical and Computer Engineering. Toward a Caring and Humane Technology (Cat. No. 03CH37436)*, volume 1, pages 289–292. IEEE, 2003.

[5] M. Asvial and A. A. S. Paramitha M.P. Analysis of high order dispersion and non linear effects in fiber optic transmission with non linear schrodinger equation model. In *2015 International Conference on Quality in Research (QiR)*, pages 145–150, Aug 2015.

[6] Yuri S Kivshar and Govind Agrawal. *Optical solitons: from fibers to photonic crystals.* Academic press, 2003.

[7] JR De Oliveira, Marco A de Moura, J Miguel Hickmann, and ASL Gomes. Self-steepening of optical pulses in dispersive media. *JOSA B*, 9(11):2025–2027, 1992.

[8] M Brandt-Pearce, Ira Jacobs, J-H Lee, and JK Shaw. Optimal input gaussian pulse width for transmission in dispersive nonlinear fibers. *JOSA B*, 16(8):1189–1196, 1999.

[9] Fatkhulla Abdullaev, Sergei Darmanyan, Pulat Khabibullaev, and J Engelbrecht. *Optical solitons.* Springer Publishing Company, Incorporated, 2014.

[10] JA Oteo. The baker–campbell–hausdorff formula and nested commutator identities. *Journal of mathematical physics*, 32(2):419–424, 1991.

[11] https://en.symphotony.com/fiber-lasers/split-step-fourier/.

[12] R. Deiterding, R. Glowinski, H. Oliver, and S. Poole. A reliable split-step fourier method for the propagation equation of ultra-fast pulses in single-mode optical fibers. *Journal of Lightwave Technology*, 31(12):2008–2017, June 2013.

[13] Kazunori Naganuma, Kazuo Mogi, and Hajime Yamada. General method for ultrashort light pulse chirp measurement. *IEEE Journal of Quantum Electronics*, 25(6):1225–1233, 1989.

[14] R. Essiambre, G. Kramer, P. J. Winzer, G. J. Foschini, and B. Goebel. Capacity limits of optical fiber networks. *Journal of Lightwave Technology*, 28(4):662–701, Feb 2010.