ME625 Final Project Report

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1. Introduction

The objective of this project is to implement the Swept Surface algorithm in Chapter 10 in the NURBS book. Given a trajectory curve $\boldsymbol{T}(v)$ and section curve $\boldsymbol{C}(u)$, this project will create a B-Spline surface sweeping the section curve along the trajectory curve. Both $\boldsymbol{T}(v)$ and $\boldsymbol{C}(u)$ is represented by B-Spline curve initially.

2. Formulation and results

A general form of the swept surface is given by

$$\boldsymbol{S}(u,v) = \boldsymbol{T}(v) + \boldsymbol{M}(v)\boldsymbol{C}(u)$$

Where $\boldsymbol{M}(v)$ is a 3x3 matrix incorporating rotation and non-uniform scaling of $\boldsymbol{C}(u)$. In practice, most swept surfaces are one of two specific types:

1. $\boldsymbol{M}(v)$ is identity matrix for all v, that is, for each v, $\boldsymbol{C}(v)$ is just translated by $\boldsymbol{T}(v)$.

2. $\boldsymbol{M}(v)$ is not identity matrix.

This two types of swept surface are implemented in this project.

Type 1: Generalized translational sweep.

$$\boldsymbol{S}(u,v) = \boldsymbol{T}(v) + \boldsymbol{C}(u)$$

$$\boldsymbol{T}(v) = \sum_{j=0}^{m} N_{j,q}(v)T_j \qquad V = \{v_0, \dots, v_s\}$$

$$\boldsymbol{C}(u) = \sum_{j=0}^{m} N_{i,p}(u)Q_i \qquad U = \{u_0, \dots, u_r\}$$

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\,\boldsymbol{P}_{i,j} \qquad U, V \text{ same as above}$$

$$\boldsymbol{P}_{i,j} = \boldsymbol{T}_j + \boldsymbol{Q}_i \quad i = 0, \dots, n \qquad j = 0, \dots, m$$

Example 1 of Transitional Swept Surface:

Table 1 Data of Swept Surface 1

		$\boldsymbol{T}(v)$	$\boldsymbol{C}(u)$
Degree	p	2	2
Number of points	n+1	7	9
Knot vector	U	{0,0,0,0.2,0.4,0.6,0.8,1,1,1}	{0,0,0,0.15,0.3,0.4,0.6,0.7,0.85,1,1,1}

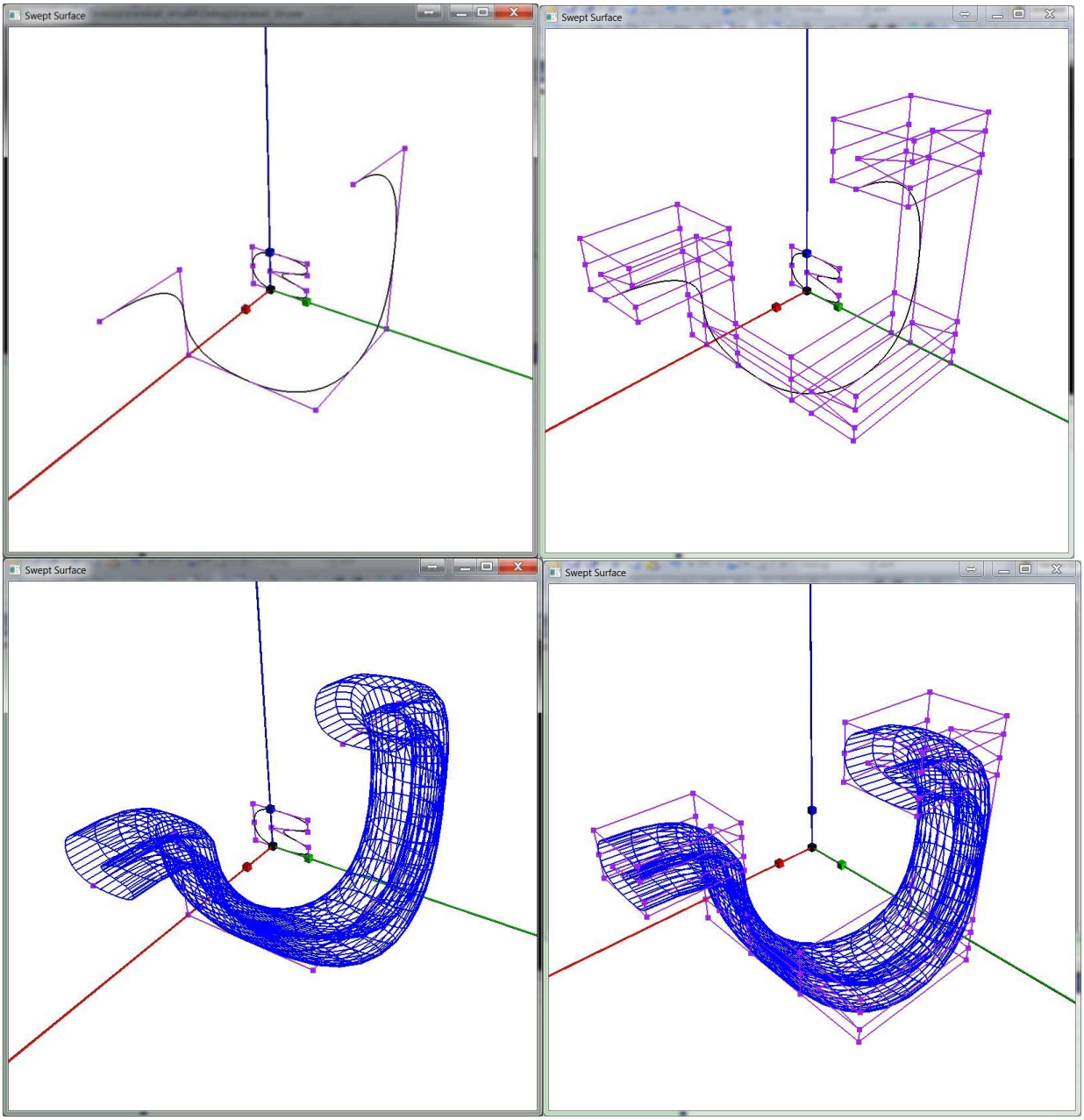


Figure 1 Transition Swept Surface 1; From left to right, from top to bottom: (1) Trajectory and section curve (2) control net (3) swept surface, (4) control net and swept surface.

Example 2 of Transitional Swept surface:

Table 2 Data of Swept Surface 2

		$\boldsymbol{T}(v)$	$\boldsymbol{C}(u)$
Degree	p	2	2
Number of points	n+1	7	6
Knot vector	U	{ 0,0,0,0.2,0.4,0.6,0.8,1,1,1}	{0,0,0,0.25,0.5,0.75,1,1,1}

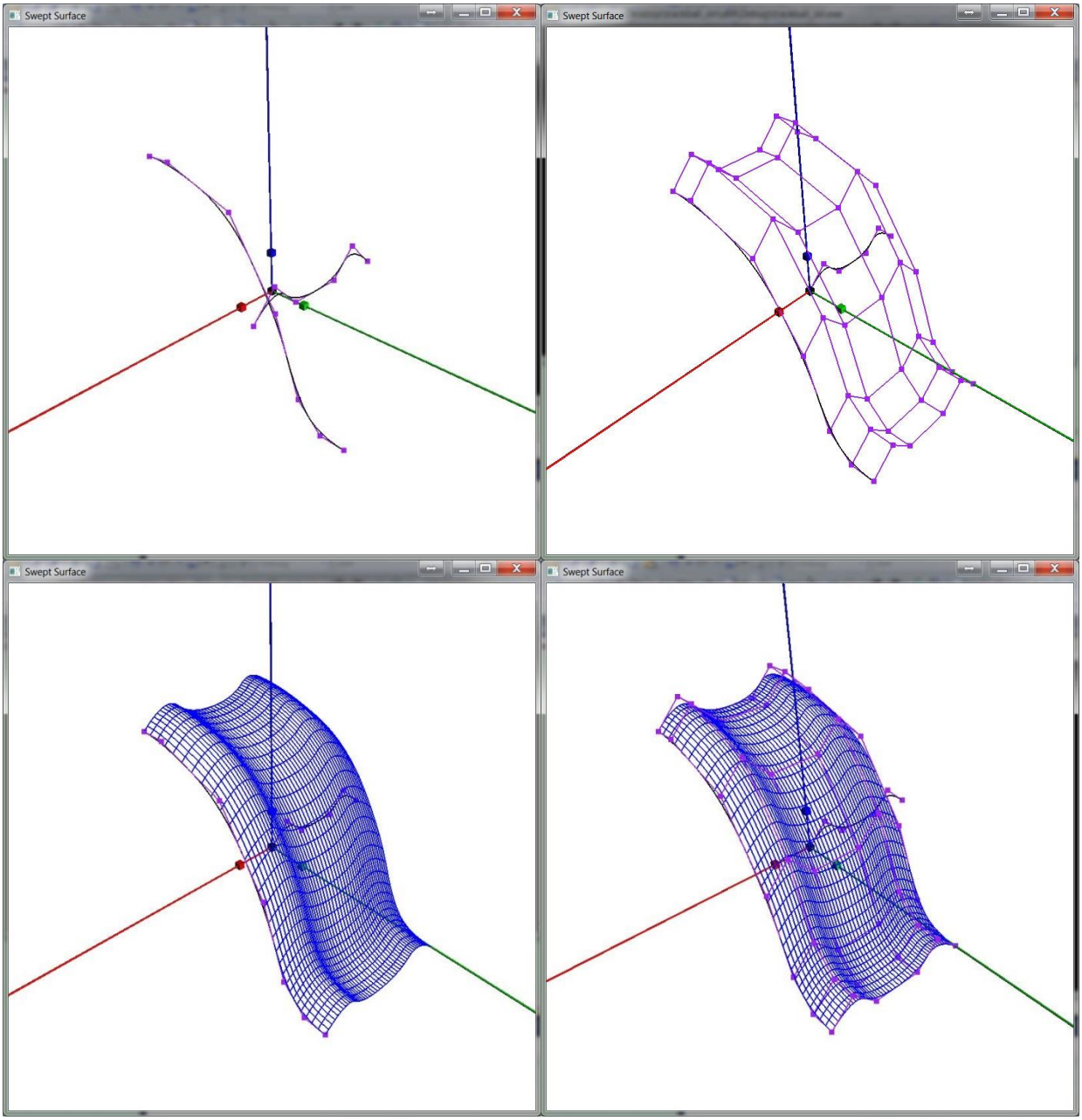


Figure 2 Transition Swept Surface 2; From left to right, from top to bottom: (1) Trajectory and section curve (2) control net (3) swept surface, (4) control net and swept surface.

Type2: Trajectory Controlled Swept Surface.

$$\boldsymbol{S}(u,v) = \boldsymbol{T}(v) + A(v)S(v)\boldsymbol{C}(u)$$

$A(v)$ is the general transformation matrix taking global coordinate system $\{\boldsymbol{O},\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z}\}$ into local orthonormal coordinate system $\{\boldsymbol{o}(v),\boldsymbol{x}(v),\boldsymbol{y}(v),\boldsymbol{z}(v)\}$, where

$$\boldsymbol{o}(v) = \boldsymbol{T}(v), \quad \boldsymbol{x}(v) = \frac{\boldsymbol{T}'(v)}{|\boldsymbol{T}'(v)|} \qquad \boldsymbol{B}(v)\cdot\boldsymbol{x}(v) = 0 \qquad \boldsymbol{z}(v) = \frac{\boldsymbol{B}(v)}{|\boldsymbol{B}(v)|}$$

$$\boldsymbol{y}(v) = \boldsymbol{z}(v)\times\boldsymbol{x}(v) \qquad\qquad s(v) = (\, s_x(v), s_y(v), s_z(v)\,)$$

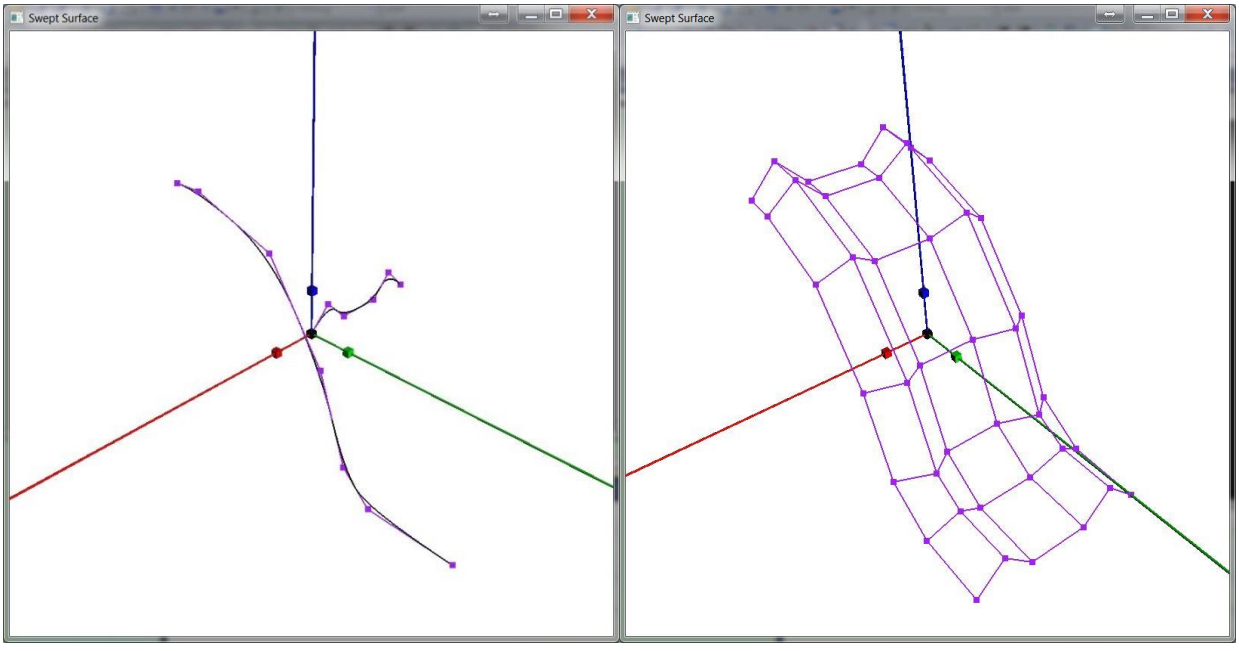
If $\boldsymbol{T}(v)$ is a planar B-Spline curve, select $\boldsymbol{B}(v)$ as the vector that is normal to the $\boldsymbol{T}(v)$ plane, else $\boldsymbol{T}(v)$ is 3D, then use the *Frenet frame:*

$$\boldsymbol{B}(v) = \frac{\boldsymbol{T}'(v)\times\boldsymbol{T}''(v)}{|\boldsymbol{T}'(v)\times\boldsymbol{T}''(v)|} \qquad\qquad \boldsymbol{N}(v) = \boldsymbol{B}(v)\times\boldsymbol{T}'(v)$$

In this project $s(v)$ is always treated as identity matrix.

Example 3 Trajectory Controlled Swept Surface (planar trajectory curve):

Data is the same of Example 2. Sections Number K+1 = 7.



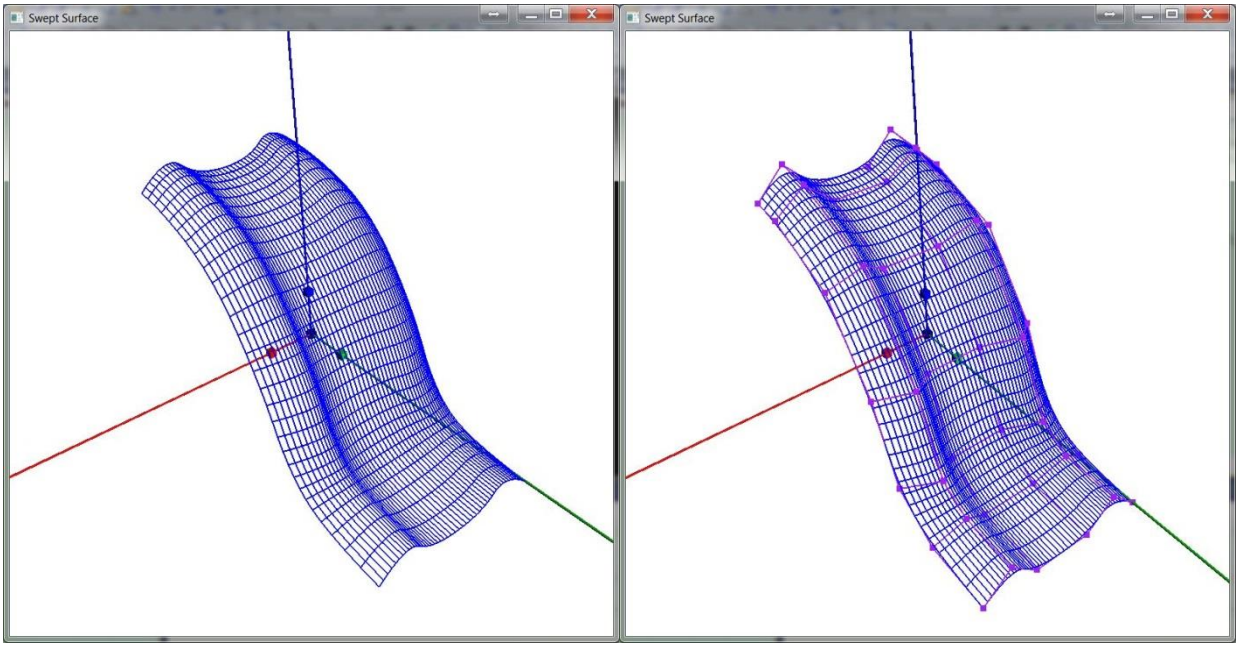


Figure 3 Trajectory Controlled Swept Surface (planar trajectory curve); From left to right, from top to bottom: (1) Trajectory and section curve (2) control net (3) swept surface, (4) control net and swept surface.

Example 4 Trajectory Controlled Swept Surface (3D trajectory curve):

Table 3 Data of Swept Surface 4

		$\boldsymbol{T}(v)$	$\boldsymbol{C}(u)$
Degree	p	2	2
Number of points	n+1	7	6
Knot vector	U	{0,0,0,0.2,0.4,0.6,0.8,1,1,1}	{0,0,0,0.25,0.5,0.75,1,1,1}
Sections number	K+1		11

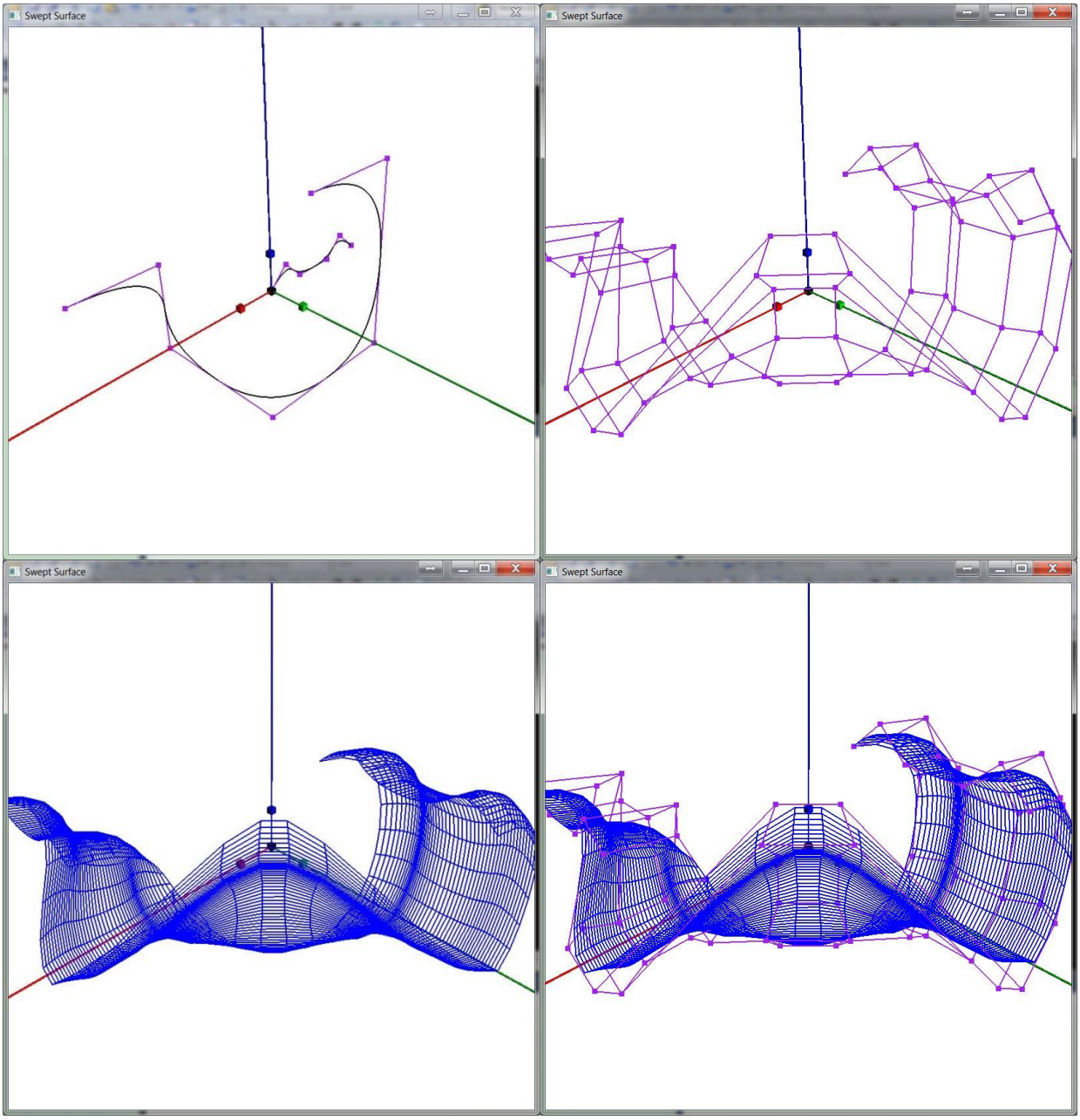


Figure 4 Transformed swept surface(Frenet frame); From left to right, from top to bottom: (1) Trajectory and section curve (2) control net (3) swept surface, (4) control net and swept surface.

3. Conclusion

The program written for this project has successfully implemented the swept surface algorithm. More specifically, It implements the transitional swept surface and the trajectory controlled swept surface for planar and 3D trajectory curves(Frenet frame). The results are in accordance with the exmaples in the NURBS book. Future work can be focused on extending this program to a rational B-Spline curves and swept surfaces, also non-uniform scaling can also be added.